EXPRESSIONS FOR BOND STRESS OF A TENSION SPLICE IN STEEL REINFORCED CONCRETE

by

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ABSTRACT

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The aim of the research is to develop expressions for the bond stress at a tension splice in reinforced concrete. Many experiments have been conducted inorder to ascertain the relationship between appropriate variables and develop an expression to evaluate the bond stress. The scope of this research is to use existing experimental data to establish a relation and develop an expression.

Before the preprocessing begins, the data are organized into different categories such as bottom, top and side bars, and bars with transverse reinforcement.

The existing experimental data is evaluated for statistical validation. It is then subjected to individual and relational variance tests to ascertain the variation of individual variables in comparison with each and every variable. It is also tested for significance of presence.

This preprocessed data is subjected to correlation tests. Proper variables are then selected whose 'contributions' are significant to the endogenous variable which in this case is the bond stress at splice length normalized with respect to the square root of the characteristic compressive strength of the concrete.

Once, the desired variables are established, a linear model is built and the coefficients of the desired variables are evaluated. The linear model is then tested for errors. Since significant error is to be expected due to the omission of number of variables, the equation is discretized over a discrete interval of a selected parameter.Thus, the error is minimized and a fairly accurate expressions are developed.

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CHAPTER 1

INTRODUCTION

In a reinforced concrete member, the strength of the member essentially depends on the bond between the steel reinforcing bar and the concrete. Concrete is not a homogenous or ductile material. There are important qualities for any material used in structures. Ductility can be designed into behavior of concrete member by the appropriate introduction of reinforcement bars.

In most of the texts related to reinforced concrete, steel reinforcement is considered to impart ductility to the concrete member. Generally no discussion is given about the mechanism involved behind this process. Numerous experiments have been conducted in order to describe the parameters such as volume of concrete, number of stirrups, splice length, longitudinal and transverse bar diameter etc. affecting the bond strength.

The aim of this project is not to discuss the mechanism of bond in detail but to develop expressions for bond from existing data. The data from various sources are collected. They are analyzed using statistical procedure and their relationships are analyzed. Then, a linear relationship is developed between the desired variable and other desired independent variables.

Chapter 2 describes the data arrangement that aids in better analysis. This chapter discusses various types of variables and their importance in the analysis. It also describes various possible relationships between the variables and how each variable is analyzed and organized according to their importance in the relationship.

Chapter 3 describes the preliminary processing of the data. Analysis of variance of the variable and how it affects the processing of data for regression are discussed.

The significance of presence of each variable and its proportion of contribution to the endogenous variable is evaluated. The types of errors that may possibly occur due to omission of certain tests and wrong assumptions are also discussed. The data are tested and probable variables for representing the endogenous variables are identified.

Chapter 4 gives a detailed account of regression analysis. General linear form is described and derived and then extended to multiple regressions. The variables to be involved in the regression analysis are treated for normality. Various processes and methods to remedy any abnormality are discussed in detail. The upper bound and lower bound values for the coefficients of the regression equation are evaluated.

Chapter 5 analyzes the procedure for optimizing the coefficients of the expression to generate least error.

Chapter 6 describes the accuracy of the equations. The error quotient is calculated for random set of data and their accuracies are checked.

Chapter 7 gives the conclusion. Possible scope of this project in future is also discussed.

1.1 Linear Model - Why?

The important fact about linear functions is that they are easy to work with. They are easy to solve, easy to plot, and easy to understand. So when looking for a function to approximate the behavior of something in the real world, a linear function is generally tried first; and only if that proves to be an unacceptable model, other kinds of functions are then examined.

There are already many equations and theories present involving non-linear approaches for bond expressions. Here, only linear approach is considered to show that it is easy to use and also produces results of higher accuracy compared to non-linear models.

1.2 Importance of Bond

In traditional design capacity, bond between steel and concrete is achieved by providing adequate length of bar or lap splice and to anchor the bar. This ensures that the bar is capable of reaching yield under design loading. Usual approach to establish a bond relationship is to deduce from experimental data for bond strength using empirical relations.

Compatibility of deformations between concrete and the anchored bar becomes an important issue when design criteria are based on the performance of the structure. The increasing significance of performance based design has motivated new considerations in bond design. The need to establish the performance of the existing buildings are getting higher when new buildings are to be constructed nearby and when they are to be connected structurally. While there are many approaches for designing bond strength, there need to be an easy way to establish the bond stress. Since there is always a slip in the bar although minimum development length is provided, need to know the bond stress and its strength becomes important.[19]

1.3 Influence of Bond on Structural Performance

In general, building performance in dynamic loading conditions such as seismic forces or sudden impacts should be satisfactory. In other words, under these loads, the buildings should survive without any loss of strength and with little or no damage to the structure. Damage to the structure due to heavy load occurs when plastic hinge forms. This is due to insufficient moment resisting capacity. Adequate reinforcement and ensuring bond between concrete and reinforcement can meet the requirements of rotations of the member and avoid damage. [19]

1.4 Observations on Bond Modeling

The requirement of bond modeling is to ensure the engagement of the bar deformations in concrete are sufficient to anchor the bar when it develops its full force capacity given the detailing and section geometry. Since there is movement of bar observed when the member is loaded, bond slip occurs between the steel reinforcement and concrete. This increases the tensile strength in the concrete leading to cracks which migrate from the surface of the steel to the free surface of the concrete. Long bar embedment or increased cover or more transverse steel can minimize the cracks to a good extent.

1.5 Oversupply of Bond Resistance

Mode of failure is as important as the strength of the member. In concrete structures, beams are designed so that it is tension controlled and has ductile failure. It is to be noted that bond improved beyond ability of the member to sustain it will result in the change of mode of failure from pullout to splitting. To prevent this, additional transverse reinforcement will be required. The increased bond also decreases the plastic hinge rotation which leads to premature failure of structures.

CHAPTER 2

DATA ARRANGEMENT AND ORGANIZATION

2.1 Data Source

The project is based upon a set of data generated from various experiments performed on various elements. The project is mainly concerned with the data itself and not on the experiments from which the values are obtained.

The performance of reinforced concrete structures depends on adequate bond strength between concrete and reinforcing steel. This project statistically analyzes bond and development length of straight reinforcing bars under tensile load. Bond behavior and the factors affecting bond are discussed, including concrete cover and bar spacing, bar size, transverse reinforcement, bar geometry, concrete properties, steel stress and yield strength, bar surface condition, bar casting position, development and splice length, distance between spliced bars, and concrete consolidation. Equations for development and splice strength are presented and compared using a large database of test results.[1]

Experiments including pull-out tests[2, 25, 26, 14], flexure test of beam with single[17, 22, 26, 4, 16], two point[1, 20, 23, 11, 13] and three point loading [12, 10, 15, 5, 21, 6] are performed to gather the data for analysis.

2.2 Application to Development Lengths

Similar behavior in cracking and splitting has been observed in tests for development lengths of a single bar and lap splices. The mode of failure should be the same if the bar is isolated or is adjacent to another bar as in the case of a splice. It seems, therefore, that the empirical equation for splice strength should be applicable to development lengths as well as splices.[1]

In order to check this, extensive tests were conducted by Ferguson, Thompson, and Chamberlain.[7, 23, 10] These tests lead to a conclusion that for the same given parameters, the same length is required for a lap splice as for development length of a single bar. As a result, the same basic equation can be used for determining development lengths of a single straignt bar as well as lap splices.[23, 8]

2.3 Data Basics

2.3.1 Variables

Variables are numeric representatives of parameters that measure, modify or control research. They are significant in that they can take the role of any quantity to be measured and aid in assessing the desired quantities by qualitative or quantitative measures.

2.3.2 Variables - Types

Based on the relationship between the variables involved in the research, variables are of two kinds. They are as follows:

- Independent variables
- Dependent variables

Independent variables are measures whose values are bound to change but are not affected in any way by any other measures bound to change. Independent variables are sometimes referred to as manipulative variables in that they control or modify the values of other variables namely dependent variables. The aforementioned kinds of variables get their measure from direct observation of an experiment or from actual measurement. For example, a diameter of a steel rod measured constitutes a measure for an independent variable.

Dependent variables are measures whose values are calculated based on the independent variables based on their relationships.

2.3.3 Variables - Kinds

Not all variables are measured accurately. Hence, most of the variables have inherent error associated with them. Hence, in the analysis and research based upon these variables, there has to be room for accomodating the error to find the value of the depending variable more accurately.

- Nominal variable
- Ordinal variable
- Interval variable
- Ratio variable

Nominal variables allow for only qualitative classification. That is, they can be measured only in terms of whether the individual items belong to some distinctively different categories, but we cannot quantify or even rank order those categories.

Ordinal variables allow us to rank order the items we measure in terms of which has less and which has more of the quality represented by the variable, but still they do not allow us to say quantify.

Interval variables allow us not only to rank order the items that are measured, but also to quantify and compare the sizes of differences between them.

Ratio variables are very similar to interval variables; in addition to all the properties of interval variables, they feature an identifiable absolute zero point, thus they allow for statements such as x is two times more than y.

The type of variable that occurs in this research is nominal variable. This variable gives the category that it represents and the quality measure of it and not any relationship between any of other measures. In all practical relevance, variables are either referred as dependent or independent variables.

2.3.4 Variables - Relationships

Among the variables involved in an experiment, there always exist a relationship between them. The relationship would be evident from the distribution of the values of the variables in a consistent manner. In any scientific experiment involved, there is no such measure that does not have any link to other measures though how small it may be. In order to explain the behaviour of any, say, variable, relationship of the interested variable with other possible related variables must be established and studied. This would give a good knowledge about the working of the system and gives the proportion of contribution of each variable to the ultimate factor.

2.3.5 Properties of relation between variables

There are two distinguished properties exhibited by the variables considered in the relationship.

- Magnitude or Size: This property indicates the physical relationship between the variables. If two variables x and y are considered, this property would show if x is greater than y or such.
- Reliability: This property indicates the extent to which the value can be used in further research. In effect, it determines if the measure of this variable is closer to accuracy. One way to measure the reliability is to use some significance tests.

2.4 Typical Data

List of variables observed related to the material property are as follows:

1. Concrete strength ksi. $f_c^{'}$
2. Bar yield strength ksi. f_y
3. Stirrup yield strength ksi. f_{yt}

List of variables observed related to the geometric property of bar and its placement are as follows:

1. Splice length in. l_s
2. Diameter of the longitudinal bar in. d_b
3. Relative rib area. R_r
4. Number of spliced or developed bars. N_b
5. Nominal stirrup diameter in. d_{tr}
6. Number of stirrups along splice or development length. N_s
7. Number of legs per stirrup. N_l
8. Bar area sq.in. A_b
9. Area of one leg of a stirrup. A_t

List of variables observed related to the placement of the bar are as follows:

1. Side cover for reinforcement in. c_{so}
2. One-half clear spacing between bars in. c_{si}
3. Bottom cover in. c_b

List of variables observed related to the geometric property concrete are as follows:

1. Breadth of concrete element in. b
2. Height of concrete element in. h
3. Depth of concrete element in. d

The aim of the research is to develop an expression for the bond stress at tension splice. Many experiments have been conducted in order to ascertain the relationship between appropriate variables and develop an expression to evaluate the bar stress. The scope of this research is to use the experimental data already recorded to establish a relation and develop an expression.

An expression should preferably be simple and easy to calculate. At the same time, it should also have accurate answers without compromising on the quality and quantity of the value evaluated. It is best to observe and record all possible parameters in an experimental research. However, it is not possible to include all those parameters in the expression. Hence there should be a way to assess what variables or parameters would find a place in the expression.[1]

2.5 Scatter Plots

Scatter plots are two dimensional graph. It involves two variables.

- Explanatory variable or Independent variable
- Response variable or Dependent variable

Generally, explanatory variable is plotted on the X-axis and the response variable on Y-axis. Scatter plots are used if very large amount of data are to be plotted for analysis.For scatter plots, it is general procedure to standardize the X and Y axis with respect to the mean value of the particular variable. This allows for better analysis and comparison than using a standard X and Y axis.

Scatter plots provides information about the following:

- strength
- shape
- direction
- presence of outliers

This graph will have clusters of data points along the line explaining the correlation between the variables.

2.5.1 Cause and Effect

It should be noted that scatter plot need not necessarily signify a direct relationship between the two variables involved. Indirect relationship through a third uninvolved variable would also produce a scatter plot falsely signifying a relationship between the involved variables. The term involved variables would include the explanatory and response variables only.

2.5.2 Purpose

The purpose of the scatter plot is to determine if any relationship between the variables is possible. The scatter plotting is usually performed even before data analysis is considered or before testing the regression fitting model.

CHAPTER 3

PRELIMINARY ANALYSIS

3.1 ANOVA

Analysis of variance (ANOVA) is used to test hypotheses about differences between two or more means. The t-test based on the standard error of the difference between two means can only be used to test differences between two means. When there are more than two means, it is possible to compare each mean with each other mean using t-tests. However, conducting multiple t-tests can lead to severe inflation of the Type I error rate.

3.1.1 Type I and Type II Errors

There are two kinds of errors that can be made in significance testing: (1) a true null hypothesis can be incorrectly rejected and (2) a false null hypothesis can fail to be rejected. The former error is called a Type I error and the latter error is called a Type II error. The probability of a Type I error is designated by the Greek letter alpha and is called the Type I error rate; the probability of a Type II error (the Type II error rate) is designated by the Greek letter beta.

A Type II error is only an error in the sense that an opportunity to reject the null hypothesis correctly was lost. It is not an error in the sense that an incorrect conclusion was drawn since no conclusion is drawn when the null hypothesis is not rejected. [18]

3.1.2 Sample Size

If the sample size is the same for all of the treatment groups, then the letter 'n' (without a subscript) is used to indicate the number of subjects in each group. The total number of subjects across all groups is indicated by 'N.' If the sample sizes are equal then N = (a)(n); otherwise, N = $n_1 + n_2 + ... + n_a$.

3.1.3 Assumptions

Analysis of variance assumes normal distributions and homogeneity of variance. Therefore, in a one-factor ANOVA, it is assumed that each of the populations is normally distributed with the same variance σ^2.

In between-subjects analyses, it is assumed that each score is sampled randomly and independently.

3.2 Statistical Significance

In the process of developing relationship, there always exists a chance that the relationship between the two said variables is by chance. Hence for all practical considerations, no such relationship exists between the two variables.

In order to determine such false relationship, stastical significance of the result is to be considered. This gives something about the truth of the relationship.

The p-value represents the reliability of a result in a decreasing order. The lower the p-value, the more we can believe that the observed relation between the variable in the sample is a reliable indicator of the relation between the respective variables in the population.

In other words, the probability of error involved in accepting our observed result as valid is represented by the p-value.

For example, a p-value of .05 (i.e.,1/20) indicates that there is a 5 percent probability that the relation between the variables found in our sample is a 'fluke.' In other words, assuming that in the population there was no relation between those variables whatsoever, and we were repeating experiments like ours one after another, we could expect that approximately in every 20 replications of the experiment there would be one in which the relation between the variables in question would be equal or stronger than in ours.

There is no way to avoid arbitrariness in the final decision as to what level of significance will be treated as really 'significant.' That is, the selection of some level of significance, up to which the results will be rejected as invalid, is arbitrary. In practice, the final decision usually depends on whether the outcome was predicted a priori or only found post hoc in the course of many analyses and comparisons performed on the data set, on the total amount of consistent supportive evidence in the entire data set, and on 'traditions' existing in the particular area of research. Typically, in many sciences, results that yield

$$p \leq .05 \tag{3.1}$$

are considered borderline statistically significant but remember that this level of significance still involves a pretty high probability of error (5 %). Results that are significant at the

$$p \leq .01 \tag{3.2}$$

level are commonly considered statistically significant, and

$$p \leq .005 \tag{3.3}$$

or

$$p \leq .001 \tag{3.4}$$

levels are often called 'highly' significant. But remember that those classifications represent nothing else but arbitrary conventions that are only informally based on general research experience.

The number of analyses being performed would make the relations meet the expected significance level. For example, calculation of correlations of ten variables would yield about two correlation coefficient at

$$p \leq 0.05 \tag{3.5}$$

though the values of the variables donot follow any significant pattern or otherwise completely random and not in harmony with the distribution of other variables.

There is no said method to find the significance level without any error and establish the importance of the variable in the analyses to be conducted. There is always a chance of an error occuring in these unexpected findings.

3.3 Analysis of data with transverse reinforcement

3.3.1 Sigma test values Bottom Bar

The variable U denotes the bond stress of the tension splice.[1] The description of the variables are given in section 2.4

Inorder to find variables eligible for the expression, the sigma 1-tailed test values are to be analyzed first. The value 0.05 represents an error percentage of 5. Hence, lower the value, better it will serve by its purpose in the equation.

In this way, the qualifying variables are

Secondly, another short list would be made with variables exhibiting higher correlation values.

The variables are

$$l_s$$

$$d_{tr}$$

$$N_s$$

Table 3.1. Sig-value and correlation-bottom bar

parameter	pearson r	sigma 1-t
U	1.000	-
l_s	0.477	0.000
d_b	-0.191	0.067
b	0.101	0.216
h	0.139	0.139
d	0.311	0.007
c_{so}	-0.181	0.078
c_{si}	-0.036	0.390
c_b	0.007	0.477
N_b	0.278	0.014
d_{tr}	0.426	0.000
N_s	0.518	0.000
N_l	-	0.000
A_b	-0.191	0.067
A_t	0.419	0.000
f_y	0.314	0.006
f_{yt}	-0.117	0.181
R_r	0.382	0.001

$$A_t$$

If the correlation value

$$r \geq 0.4 \tag{3.6}$$

then, they are said to have 'significant' correlation.

$$r \leq 0.4 \tag{3.7}$$

signifies weak correlation and

$$r \leq 0.2 \tag{3.8}$$

signifies very weak or nil correlation.

Comparing the two short lists, the variable

$$N_i$$

is dropped as it exhibited very random distribution in scatter plot.

Since

$$A_t \subset d_{tr}, d_{tr}$$

being a more independent and easily measurable variable, the variable

$$d_{tr}$$

would be preferred in the expression.

Hence, the variables

$$l_s$$

$$d_{tr}$$

$$N_s$$

are the three variables to be considered.

Also, since, all these variables are nominal variables and are measurable in the field directly, this would make the equation not desk bound.

The value of

$$f_c^{'}$$

is not considered since the value of the stress is normalized with respect to

$$\sqrt{f_c^{'}}.$$

From structural point of view,

$$l_s$$

$$l_d$$

Table 3.2. Sig-value and correlation-top bar

parameter	pearson r	sigma 1-t
U	1.000	-
l_s	0.775	0.000
d_{tr}	0.791	0.000
N_s	-0.245	0.050

Table 3.3. Sig-value and correlation-side bar

parameter	pearson r	sigma 1-t
U	1.000	-
l_s	0.775	0.000
d_{tr}	0.791	0.000
N_s	-0.245	0.050

is a function of

$$f_y$$

$$d_b$$

which will reiterate the fact that

$$f_y$$

$$d_b$$

are indirectly affecting the said expression.

The correlation and significance test values for top and side bars are tabulated below:

3.4 Data Screening

3.4.1 Multivariate normality

The estimation method assumes that all the univariate distributions are normal, the joint distribution of any pair of the variables is bivariate normal and all bivariate scatter

Table 3.4. Sig-value and correlation-top bar without transverse reinforcement

parameter	pearson r	sigma 1-t
U	1.000	-
d_b	-0.388	0.001
N_b	-0.648	0.000
f_y	0.788	0.050

Table 3.5. Sig-value and correlation-side bar without transverse reinforcement

parameter	pearson r	sigma 1-t
U	1.000	-
d_b	-0.053	0.386
N_b	-0.108	0.277
f_y	0.117	0.262

plots are linear and homoscedastic. Since it is often impractical to examine all joint frequency distributions, it can be difficult to asses all aspects of multivariate normality. Most of the data used in the analysis are found to be univariate.

3.4.2 Transformations

One way to deal with univariate nonnormality is with transformations. It is a process of converting with a mathematical operation to new ones that may be more normally distributed. Since transformations alter the shape of the distribution, they can also be useful for dealing with outliers (values scattered off the expected curve of interest). Since transformations would involve the use of polynomial or trigonometric usage, and make the equation less than simple, it is not used. The effects that transformations produce are effected upon at the end by obtaining the range and optimizing the predictor coefficients to achieve coefficients with 95 % confidence interval.

3.4.3 Outliers

The individual scatter plots of mean and standard deviations are used to identify the outliers and eliminate them before analysis.

3.4.4 Multicollinearity

Another cause of singular covariance matrices is multicollinearity which occurs when intercorrelations among some variables are so high that certain mathematical operations are either impossible or unstable because some denominators are close to 0. For the aforementioned reason, the variables chosen as independent variables or predictors have average correlation between themselves and also the dependent or criterion variable.[1]

CHAPTER 4

REGRESSION ANALYSIS AND RESULTS

4.1 Multiple Regression

The general purpose of multiple regression is to analyze the relationship between several independent or predictor variables and a dependent or criterion variable.

The computational problem that needs to be solved in multiple regression analysis is to fit a straight line (or plane in an n-dimensional space, where n is the number of independent variables) to a number of points. In the simplest case – one dependent and one independent variable – one can visualize this in a scatterplot (scatterplots are two-dimensional plots of the scores on a pair of variables). It is used as either a hypothesis testing or exploratory method.

4.1.1 General form

A one dimensional surface in a two dimensional or two-variable space is a line defined by the equation

$$Y = b_0 + b_1 * X \tag{4.1}$$

According to this equation, the Y variable can be expressed in terms of or as a function of a constant and a slope times the X variable. The constant is also referred to as the intercept, and the slope as the regression coefficient.

In general then, multiple regression procedures will estimate a linear equation of the form:

$\mathrm{Y} = \mathrm{b}_0 + b_1X_1 + b_2X_2 + ... + b_kX_k$

where k is the number of predictors. Note that in this equation, the regression coefficients or

$$b_1$$

.....

$$b_k$$

coefficients represent the independent contributions of each in dependent variable to the prediction of the dependent variable. Another way to express this fact is to say that, for example, variable X1 is correlated with the Y variable, after controlling for all other independent variables. This type of correlation is also referred to as a partial correlation

The regression surface(a line in simple regression, a plane or higher-dimensional surface in multiple regression) expresses the best prediction of the dependent variable (Y), given the independent variables (X's). The deviation of a particular point from the nearest corresponding point on the predicted regression surface (its predicted value) is called the residual value. Since the goal of linear regression procedures is to fit a surface, which is a linear function of the X variables, as closely as possible to the observed Y variable, the residual values for the observed points can be used to devise a criterion for the 'best fit.' Specifically, in regression problems the surface is computed for which the sum of the squared deviations of the observed points from that surface are minimized. Thus, this general procedure is sometimes also referred to as least squares estimation.

The actual computations involved in solving regression problems can be expressed compactly and conveniently using matrix notation. Suppose that there are n observed values of Y and n associated observed values for each of k different X variables. Then

$$Y_i$$

$$X_{ik}$$

e_i

can represent the ith observation of the Y variable, the ith observation of each of the X variables, and the ith unknown residual value, respectively. The multiple regression model in matrix notation then can be expressed as

Y = X*b + e

where b is a column vector of 1 (for the intercept) + k unknown regression coefficients. Recall that the goal of multiple regression is to minimize the sum of the squared residuals. Regression coefficients that satisfy this criterion are found by solving the set of normal equations

X'Xb = X'Y

When the X variables are linearly independent (i.e., they are nonredundant, yielding an X'X matrix which is of full rank) there is a unique solution to the normal equations. Premultiplying both sides of the matrix formula for the normal equations by the inverse of X'X gives

(X'X)$^{-1}X'Xb = (X'X)^{-1}X'Y$

or

b = (X'X)$^{-1} * X'Y$

This last result is very satisfying in view of its simplicity and its generality. With regard to its simplicity, it expresses the solution for the regression equation in terms just 2 matrices (X and Y) and 3 basic matrix operations, (1) matrix transposition, which involves interchanging the elements in the rows and columns of a matrix, (2) matrix multiplication, which involves finding the sum of the products of the elements for each row and column combination of two conformable (i.e., multipliable) matrices, and (3) matrix inversion, which involves finding the matrix equivalent of a numeric reciprocal, that is, the matrix that satisfies

A$^{-1}AA = A$

for a matrix A.

With regard to the generality of the multiple regression model, its only notable limitations are that (1) it can be used to analyze only a single dependent variable, (2) it cannot provide a solution for the regression coefficients when the X variables are not linearly independent and the inverse of X'X therefore does not exist. These restrictions, however, can be overcome, and in doing so the multiple regression model is transformed into the general linear model.

4.1.2 Extension of Linear Model

One way in which the general linear model differs from the multiple regression model is in terms of the number of dependent variables that can be analyzed. The Y vector of n observations of a single Y variable can be replaced by a Y matrix of n observations of m different Y variables. Similarly, the b vector of regression coefficients for a single Y variable can be replaced by a 'b' matrix of regression coefficients, with one vector of b coefficients for each of the m dependent variables. These substitutions yield the multivariate regression model, but it should be emphasized that the matrix formulations of the multiple and multivariate regression models are identical, except for the number of columns in the Y and b matrices. The method for solving for the b coefficients is also identical, that is, m different sets of regression coefficients are separately found for the m different dependent variables in the multivariate regression model.

The general linear model goes a step beyond the multivariate regression model by allowing for linear transformations or linear combinations of multiple dependent variables. This extension gives the general linear model important advantages over the multiple and the so-called multivariate regression models, both of which are inherently univariate (single dependent variable) methods. One advantage is that multivariate tests of significance can be employed when responses on multiple dependent variables are correlated. Separate univariate tests of significance for correlated dependent variables are not independent and may not be appropriate. Multivariate tests of significance of independent linear combinations of multiple dependent variables also can give insight into which dimensions of the response variables are, and are not, related to the predictor variables. Another advantage is the ability to analyze effects of repeated measure factors. Repeated measure designs, or within-subject designs, have traditionally been analyzed using ANOVA techniques. Linear combinations of responses reflecting a repeated measure effect (for example, the difference of responses on a measure under differing conditions) can be constructed and tested for significance using either the univariate or multivariate approach to analyzing repeated measures in the general linear model.

A second important way in which the general linear model differs from the multiple regression model is in its ability to provide a solution for the normal equations when the X variables are not linearly independent and the inverse of X'X does not exist. Redundancy of the X variables may be incidental (e.g., two predictor variables might happen to be perfectly correlated in a small data set), accidental (e.g., two copies of the same variable might unintentionally be used in an analysis) or designed. Finding the regular inverse of a non-full-rank matrix is reminiscent of the problem of finding the reciprocal of 0 in ordinary arithmetic. No such inverse or reciprocal exists because division by 0 is not permitted. This problem is solved in the general linear model by using a generalized inverse of the X'X matrix in solving the normal equations. A generalized inverse is any matrix that satisfies

$\mathrm{AA}^{-1}A = A$

for a matrix A.

A generalized inverse is unique and is the same as the regular inverse only if the matrix A is full rank. A generalized inverse for a non-full-rank matrix can be computed by the simple expedient of zeroing the elements in redundant rows and columns of the matrix. Suppose that an X'X matrix with r non-redundant columns is partitioned as

$$\mathrm{X}'X = \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix}$$

where Aij (i=j=1) is an r by r matrix of rank r. Then the regular inverse of A11 exists and a generalized inverse of X'X is

$$(\mathrm{X}'X)^{-1} = \begin{matrix} (A_{11})^{-1} & 0_{12} \\ 0_{21} & 0_{22} \end{matrix}$$

where each 0 (null) matrix is a matrix of 0's (zeroes) and has the same dimensions as the corresponding A matrix.

In practice, however, a particular generalized inverse of X'X for finding a solution to the normal equations is usually computed using the sweep operator. This generalized inverse, called a g2 inverse, has two important properties. One is that zeroing of the elements in redundant rows is unnecessary. Another is that partitioning or reordering of the columns of X'X is unnecessary, so that the matrix can be inverted 'in place.'

There are infinitely many generalized inverses of a non-full-rank X'X matrix, and thus, infinitely many solutions to the normal equations. This can make it difficult to understand the nature of the relationships of the predictor variables to responses on the dependent variables, because the regression coefficients can change depending on the particular generalized inverse chosen for solving the normal equations. [18]

4.1.3 Unstandardized Variable

In order to compare the measure of the variables and to find relational values between them, the measures of the variables in its same scale should not be used. The measures of the variables as obtained from different experiments are referred to as unstandardized variables. For any analyses, they have to have a similar distribution and

Table 4.1. Unstandardized Coefficients bottom bar

parameter	B Value	UnStd Coeff.	Std Coeff β
l_s	0.604	0.172	0.368
d_{tr}	35.483	12.314	0.306
N_s	1.773	0.753	.267
constant	13.925	6.019	-

should be standardized. In other words, instead of the variables having a unit appropriate from the experiments or observations, it should have statistical units.

4.1.4 Standardized Variable

A standardized variable is a variable that has been transformed so that its mean is 0 and its standard deviation is 1.0. The standard way to standardize a variable is to convert its raw scores to z scores or otherwise called normal deviates. A raw variable, as it is called before being standardization, say X, is converted to a normal deviate with the formula

$$z = \frac{X - M}{SD} \tag{4.2}$$

Where X = experimental statistical values M = sample mean SD = standard deviation

Statistical results computed with standardized variables are called standardized estimates. They are interpreted in the same way for all variables. Unstandardized estimates are derived with variables with its raw data obtained without treating it or processing it with the statistical parameters. The standardized coefficients are evaluated with lower bound and upper bound values for 95 percent confidence interval.

The table values gives the coefficients for the equation with 95% confidence interval.

Table 4.2. Unstandardized Coefficients top bar

parameter	B Value	UnStd Coeff.	Std Coeff β
l_s	0.533	0.274	0.274
d_{tr}	27.591	16.032	0.182
N_s	1.072	0.428	0.428
constant	13.928	-	-

Table 4.3. Unstandardized Coefficients side bar

parameter	B Value	UnStd Coeff.	Std Coeff β
l_s	0.388	0.156	0.646
d_{tr}	34.846	29.208	0.269
N_s	-0.017	0.455	-0.010
constant	35.743	12.025	-

Table 4.4. Bound Values Coeff bottombar

parameter	lowerbound	upperbound
l_s	0.260	0.947
d_{tr}	10.843	60.123
N_s	0.266	3.280
constant	1.881	25.968

Table 4.5. Bound Values Coeff top bar

parameter	lowerbound	upperbound
l_s	-0.022	1.087
d_{tr}	-4.864	60.046
N_s	0.206	1.938
constant	-2.424	30.280

Table 4.6. Bound Values Coeff side bar

parameter	lowerbound	upperbound
l_s	-0.022	1.087
d_{tr}	-4.864	60.046
N_s	0.206	1.938
constant	-2.424	30.280

CHAPTER 5

OPTIMIZATION OF STATISTICAL VALUES

5.1 Post Processing

Once the upper bound and lower bound values of the B coefficients of the multiple regression are tabulated, the optimization of the variables should be carried out. The optimization is a process wherein the coefficients are obtained that the equation satisfies the statistical data to a maxima i.e., the equation provides value close to the data sets.

5.1.1 Least Squares Optimization

In this variant process, the boundary values (upper bound and lower bound) are divided into equally spaced regions. The equation is then substituted for these values of each variable in each regions till the data set values are checked for error. The error is then squared and summed up and identified for each region. Once all the regions are run through the data set, the squared error list is picked for a least value. The region associated with the least squared value would be the optimized values for the coefficients.

5.1.2 Discretization

Once the equations with optimized coefficients are obtained, the process of discretization begin. This is because of the nature of the data set. If the optimized equation is expected to represent the whole dataset without any further modification, then, all the variables have to be considered in order to ascertain the proportional contribution of each and every one of them. And also, the nature of the equation would also be considerably changed and will probably an exponential or a polynomial of third degree or higher. To avoid such lengthy equations and also to provide a simplified set of equations, linear nature is assumed over a discrete set of data.

Here, the discreteness of the data over a parameter is to be established. Since the parameter is unknown, it is required to establish the parameter first before proceeding to discretize the equation.[1]

The difference in the equation produced values and the experimental values are listed and percentage of variance calculated. The variance in the list is then categorized according to the significant intervals. The dataset corresponding to the significant interval are identified. For the variables involved in the analysis, a correlation test is performed for each and every variable with the error %. This test identifies the parameter over which the equation is to be discretized.[5, 23]

Once the parameter is established, re-optimization of the coefficients takes place in order to re-minimize the squared error. Thus, different sets of equations over discrete intervals are obtained.

5.2 Equation Summary

The equations obtained for discrete intervals are given below. The value of U in ksi. is normalized to the square root of the characteristic compressive strength of concrete.

5.2.1 Bottom Bar with Transverse Reinforcement

$$\frac{U}{\sqrt{f_c'}} = 0.2 * l_s + 2 * d_{tr} + 0.5 * N_s + 20 \tag{5.1}$$

for $f_c' \leq 8ksi$.[20, 22, 16, 21, 1]

$$\frac{U}{\sqrt{f_c'}} = 0.071 * l_s + 5 * d_{tr} + 0.5 * N_s + 22 \tag{5.2}$$

for $f_c^{'} > 8ksi$.[20, 1, 22, 16, 21]

5.2.2 Top Bar with Transverse Reinforcement

$$\frac{U}{\sqrt{f_c^{'}}} = 0.2 * l_s + 2 * d_{tr} + 0.5 * N_s + 10 \tag{5.3}$$

[25, 17, 1, 14]

5.2.3 Side Bar with Transverse Reinforcement

$$\frac{U}{\sqrt{f_c^{'}}} = 0.2 * l_s + 16 * d_{tr} + 0.1 * N_s + 15 \tag{5.4}$$

[1, 12, 11, 13]

5.2.4 Bottom Bar without Transverse Reinforcement

$$\frac{U}{\sqrt{f_c^{'}}} = 0.5 * d_b + 1.0 * N_b + 0.5 * f_y + 3 \tag{5.5}$$

for $f_c^{'} \leq 8ksi$.[9, 1]

$$\frac{U}{\sqrt{f_c^{'}}} = -10 * d_b - 3 * N_b + 0.5 * f_y + 3 \tag{5.6}$$

for $f_c^{'} > 8ksi$.[1, 7, 6, 3]

5.2.5 Top Bar without Transverse Reinforcement

$$\frac{U}{\sqrt{f_c^{'}}} = 0.5 * d_b + 1.0 * N_b + 0.5 * f_y + 3 \tag{5.7}$$

[24, 1, 7, 20, 2]

5.2.6 Side Bar without Transverse Reinforcement

$$\frac{U}{\sqrt{f_c'}} = -2.2 * d_b + 1.2 * N_b - 0.4 * f_y + 55 \tag{5.8}$$

[12, 11, 1]

The bottom bar has two equations whereas the top and side bars have only one equation. This is because single equation for bottom bar showed much deviation from expected values. When two equations are used, accuracy of the values generated improved. The values of diameter of bar should be used in the equation such that if you are using no. 8 bar, then input 8 into the equation for diameter of the bar-both longitudinal as well as transverse bar.

CHAPTER 6

ACCURACY OF EQUATIONS

6.1 Accuracy of Model

In this chapter, the error associated with the obtained expression and the equations in use are studied. Since a large database was involved in this analytical process, it would be convenient to represent the error associated in the form of table with sub-sample set selected at random from the complete sample set.

In these tables, u1 represents the bond stress calculated and tabulated along with the data using ACI 318 and procedures currently in use and u2 represents the bond stress calculated using the newly developed expression.

The last column, error quotient

$$eq = u_1/u_2 \tag{6.1}$$

was calculated in order to give an idea as to how the newly developed expression gives values relevant to the calculated values. The closer the ratio is to 1.0, the closer the equation predicts the data value.

For bottom bar, there are four tables viz with transverse reinforcement and for concrete compressive strength less than or equal to 8 ksi. [20, 22, 16, 21, 1], with transverse reinforcement and for concrete compressive strength greater than 8 ksi.[20, 1, 22, 16, 21], without transverse reinforcement and for concrete compressive strength less than or equal to 8 ksi. [9, 1] and without transverse reinforcement and for concrete compressive strength greater than 8 ksi.[1, 7, 6, 3].

Table 6.1. Bottom Bar Without Transverse ReinforcementL8

l_s	d_b	N_b	f'_c	f_y	u1	u2	eq
42.00	1.00	2	2660	63.50	40.42	37.25	1.08
50.75	1.41	2	2730	89.00	47.42	50.20	0.95
66.00	1.41	2	3140	73.00	42.07	42.21	0.99
82.50	1.41	2	3460	93.00	47.20	52.21	0.91
57.75	1.41	2	3530	65.00	33.86	38.20	0.89
39.00	1.00	2	3650	63.50	38.16	37.25	1.02
80.00	1.00	2	3740	99.00	49.85	55.00	0.91
42.00	1.00	2	3830	63.50	37.58	37.25	1.01
82.50	1.41	2	4090	65.00	38.58	38.21	1.01

Table 6.2. Bottom Bar With Transverse ReinforcementL8

l_s	d_b	d_{tr}	N_s	f'_c	f_y	f_{yt}	u1	u2	eq
14.96	0.75	0.37	02	3072	64.10	59.29	24.81	24.70	1.00
16.00	1.00	0.38	02	3820	81.00	64.55	24.99	24.91	1.00
18.00	1.00	0.50	05	4160	60.00	84.70	27.37	27.05	1.01
24.00	1.00	0.38	07	4190	75.00	69.92	28.76	29.01	0.99
36.00	1.00	0.38	03	4200	60.00	64.55	29.26	29.41	0.99
24.00	1.00	0.38	02	4230	79.00	64.55	26.86	26.51	1.01
21.77	0.99	0.31	07	4349	65.54	62.08	28.57	28.45	1.00
40.00	1.41	0.38	06	4700	81.00	64.55	31.98	31.71	1.01
18.70	0.99	0.31	13	5219	65.54	62.08	31.10	30.83	1.01
40.00	1.41	0.38	10	5250	81.00	64.55	33.58	33.71	1.00

Table 6.3. Bottom Bar Without Transverse ReinforcementG8

l_s	d_b	N_b	f'_c	f_y	u1	u2	eq
12.01	0.98	2	09514	61.79	17.38	18.05	0.96
45.00	1.41	3	10900	70.80	14.78	15.30	0.97
28.00	1.41	2	14400	66.69	16.18	16.25	0.99
36.00	1.41	2	14450	71.45	18.60	18.62	0.99
28.00	1.41	2	15034	71.45	18.03	18.60	0.96
45.00	1.41	2	15513	70.80	17.65	18.30	0.96

Table 6.4. Bottom Bar With Transverse ReinforcementG8

l_s	d_b	d_{tr}	N_s	f_c'	f_y	f_{yt}	u1	u2	eq
17.50	1.00	0.50	5	8370	80.57	64.92	27.20	28.16	0.97
14.96	0.75	0.37	4	8832	76.87	59.29	24.56	26.86	0.91
22.44	0.75	0.37	3	8832	95.14	59.29	30.82	26.86	1.15
14.96	0.75	0.37	2	8931	102.68	59.29	27.97	25.79	1.08
21.00	1.00	0.38	4	9080	80.57	71.25	26.39	27.31	0.97
22.44	0.75	0.25	7	9216	122.23	199.12	28.12	28.48	0.99
20.00	1.00	0.38	5	10620	77.96	71.25	25.58	27.78	0.92

Table 6.5. Top Bar Without Transverse Reinforcement

l_s	d_b	N_b	f_c'	f_y	u_1	u_2	eq
21.00	0.88	1	2380	87.50	42.98	46.18	0.93
35.00	0.88	1	2810	87.50	48.62	46.19	1.05
28.00	0.88	1	3340	87.50	50.01	46.19	1.08
35.00	0.88	1	3360	87.50	42.33	46.18	0.91
21.00	0.88	1	3720	87.50	44.80	46.18	0.97
18.00	0.75	3	3740	68.20	33.90	34.47	0.98
28.00	0.88	1	3780	87.50	44.44	46.19	0.96
16.50	0.63	3	4490	62.98	30.19	31.80	0.94

Table 6.6. Side Bar Without Transverse Reinforcement

l_s	d_b	N_b	f_c'	f_y	u_1	u_2	eq
47.00	1.00	2	2775	70.00	30.62	28.40	1.07
98.00	2.26	2	2860	61.30	31.43	29.37	1.07
44.00	1.41	2	3060	65.00	31.44	29.58	1.06
45.00	1.41	3	3120	71.70	30.17	28.10	1.07
57.50	1.41	2	3250	65.00	31.06	29.58	1.05
36.00	1.41	1	3280	67.50	29.88	27.38	1.09
50.00	1.41	2	3550	65.00	31.58	29.58	1.06
57.50	1.41	2	3720	65.00	30.85	29.58	1.04
85.00	1.41	2	3900	65.00	30.64	29.58	1.03
32.00	1.00	2	3920	70.00	29.80	28.40	1.04

Table 6.7. Top Bar With Transverse Reinforcement

l_s	d_b	d_{tr}	N_s	f'_c	f_y	f_{yt}	u_1	u_2	eq
33.80	1.41	0.25	17	2960	89.00	-	26.74	25.74	1.04
50.75	1.41	0.25	30	3430	89.00	-	34.99	35.63	0.98
22.00	1.41	0.38	4	3700	60.10	60.30	17.94	17.11	1.05
16.00	1.00	0.63	2	5100	69.00	-	16.81	15.39	1.09
18.00	1.13	0.38	3	8610	70.35	78.58	16.48	15.81	1.04

Table 6.8. Side Bar With Transverse Reinforcement

l_s	d_b	d_{tr}	N_s	f'_c	f_y	f_{yt}	u_1	u_2	eq
60.00	2.26	0.38	13	2620	52.60	56.60	33.58	34.30	0.97
30.00	1.69	0.25	13	3200	50.00	62.00	28.73	26.33	1.09
42.30	1.41	0.38	8	3200	70.00	56.50	33.65	30.26	1.11
60.00	2.26	0.38	10	3220	62.20	60.00	35.57	34.00	1.04
42.30	1.41	0.38	8	3340	65.00	56.50	34.86	30.26	1.15
54.00	1.69	0.25	19	3345	59.50	51.00	33.83	31.73	1.06
48.00	2.26	0.38	16	3400	59.40	60.00	33.05	32.20	1.02
57.50	1.41	0.25	8	3610	65.00	49.00	34.21	31.33	1.09
60.00	2.26	0.50	20	3940	66.00	50.50	33.97	37.00	0.91

For top bars, two tables are given - with transverse reinforcement [25, 17, 1, 14] and with out transverse reinforcement [24, 1, 7, 20, 2].

Similarly for side bars, two tables - with transverse reinforcement [1, 12, 11, 13] and without transverse reinforcement [12, 11, 1] are given.

6.2 Comparison with ACI equation

The model is compared with the ACI 318-R02 equation for development length. The ACI equation is given below:

$$\frac{f_y}{\sqrt{f'_c}} = \frac{40}{3} * \frac{\frac{c+K_{tr}}{d_b}}{\alpha\beta\gamma\lambda} * \frac{l_d}{d_b} \tag{6.2}$$

The following two equations are used for comparison with transverse bar and without transverse bar respectively.

$$\frac{U}{\sqrt{f_c'}} = 0.2 * l_s + 2 * d_{tr} + 0.5 * N_s + 20 \tag{6.3}$$

for $f_c' \leq 8ksi$.

$$\frac{U}{\sqrt{f_c'}} = 0.5 * d_b + 1.0 * N_b + 0.5 * f_y + 3 \tag{6.4}$$

for $f_c' \leq 8ksi$.

The graphs for comparison with ACI equations show that the developed equations in this project gives values well below the ACI predicted values combining a safety factor in it. In other words, the new equations give a safe lower bound values. The bottom bar data are considered for both with and without transverse reinforcement categories.

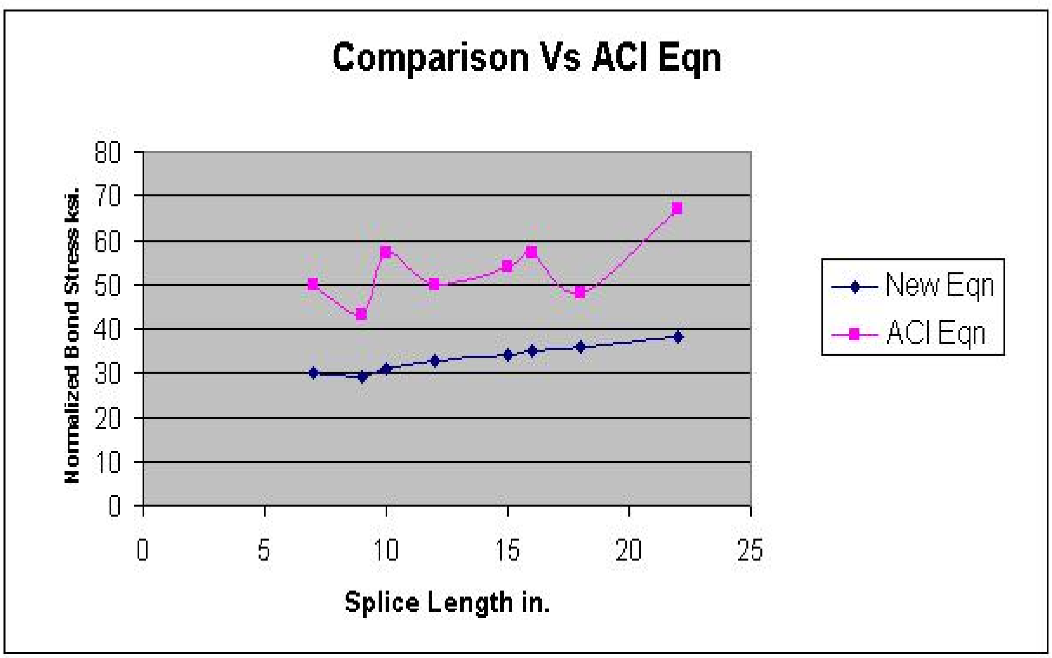

Figure 6.1. Comparison with ACI Equation - with Transverse Reinforcement.

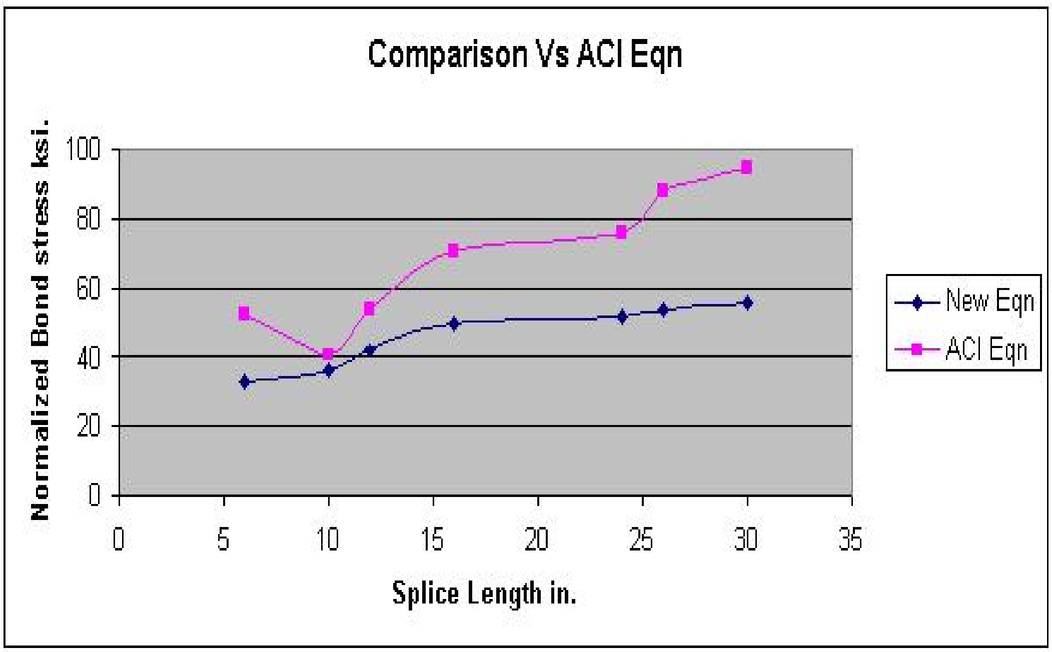

Figure 6.2. Comparison with ACI Equation - without Transverse Reinforcement.

CHAPTER 7

CONCLUSION AND FUTURE WORKS

7.1 Conclusion

In this research, we have studied the linear approach to develop expressions for the bond stress in a tension lap splice in reinforced concrete. The importance of bond the reason for choosing linear model were studied. It was shown that bond stress plays an important role due to its link with the development length which is being used now.

The different data types were studied. The experimental data were identified for their data types. The relationship between types of variables and their significance were assessed. Scatter plot and its importance in selection of eligible variables were explained.

The data were reviewed and categorized. Preliminary analysis was performed to assess the statistical significance of the data. Type I and Type II errors were identified and appropriate methods applied to eliminate them before the statistical test. Regression values and correlation values between the variables were evaluated and their relationships are determined.

The method of linear modelling was seen in detail. Multiple regression and its terminilogy were studied and the same applied to the statistically treated dataset. The variables were again categorized into standardized and unstandardized variables before forming the linear regression model.

The processed dataset and model were then refined for accuracy using procedures such as least squares method, discretization etc. The summary of the equations developed were given. The accuracy of the equations were shown by comparing the values obained to the values evaluated using existing code. The scatter plots and the probability plots were also given to graphically explain the nature and behavior of the variables involved in the research.

7.2 Future Work

The data that were used, though sufficient to generate a linear model, were less in number. The research involved dividing the data into two groups viz with concrete strength less than 8000psi and with concrete strength greater than 8000 psi. More experimental data would have permitted more detailed analysis by higher divisions. The accuracy of the equations could be made as high as 99.9%.

The data when categorized implicitly exhibits a pattern that can be programmed. For each of the variable involved, a existing behavior pattern can be established and their regression analyzed in terms of its behavior rather than its quantity. Once the behavior is established, its correlation with other variables would establish the boundary conditions of the expected variable's domain. In that way, no outliers exist and even the odd behavior of the variable is present in the analysis thus contributing completely to the correct behavior model. The output model can be expected to be linear without any constraints in the input data it supports.

APPENDIX A

PROBABILITY AND SCATTER PLOTS

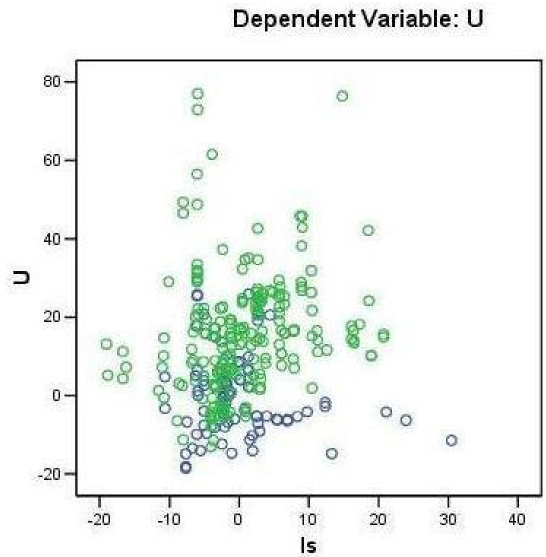

Figure A.1. Partial Reg. Plot U Vs ls bottom bar with transverse bars.

Data obtained from references [20, 22, 16, 21, 1, 20, 1, 22, 16, 21] are used for plots for all variables regarding bottom bar with transverse reinforcement.Data obtained from references [9, 1, 7, 6, 3] are used for plots for all variables regarding bottom bar with transverse reinforcement.

Data obtained from references [25, 17, 1, 14] are used for plots for all variables regarding top bar with transverse reinforcement. Data obtained from references [24, 1, 7, 20, 2] are used for plots for all variables regarding top bar with transverse reinforcement.

Data obtained from references [1, 12, 11, 13] are used for plots for all variables regarding side bar with transverse reinforcement. Data obtained from references [12, 11, 1] are used for plots for all variables regarding side bar with transverse reinforcement.

In all the plots, the dependent variable U is plotted in Y axis against the independent or primary parameters on the X axis. In scatter plots, the X axis and Y axis values are different than the normal X and Y axis values. The values on the X axis are generally

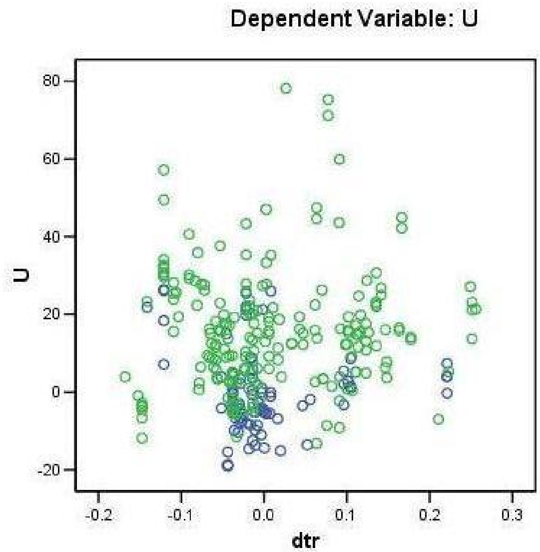

Figure A.2. Partial Reg. Plot U Vs dtr bottom bar with transverse bars.

normalized with respect to the mean value of the range of the variable. This offers for better understanding from statistical view point instead of normal X and Y axis starting with 0.

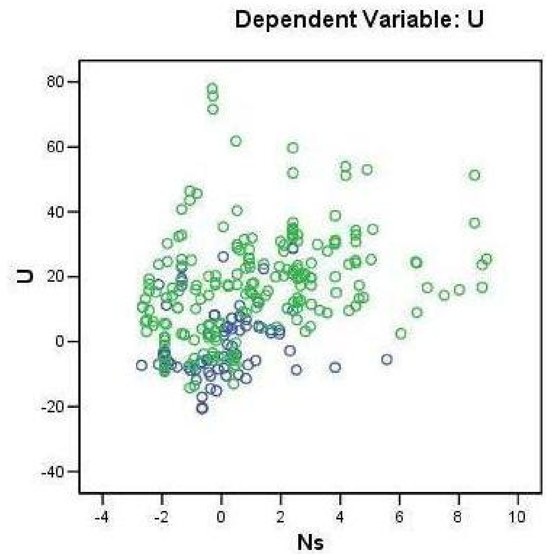

Figure A.3. Partial Reg. Plot U Vs Ns bottom bar with transverse bars.

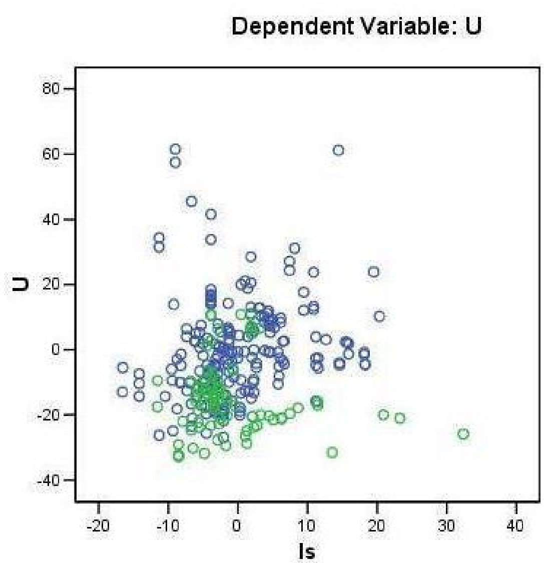

Figure A.4. Partial Reg. Plot U Vs ls bottom bar with transverse bars.

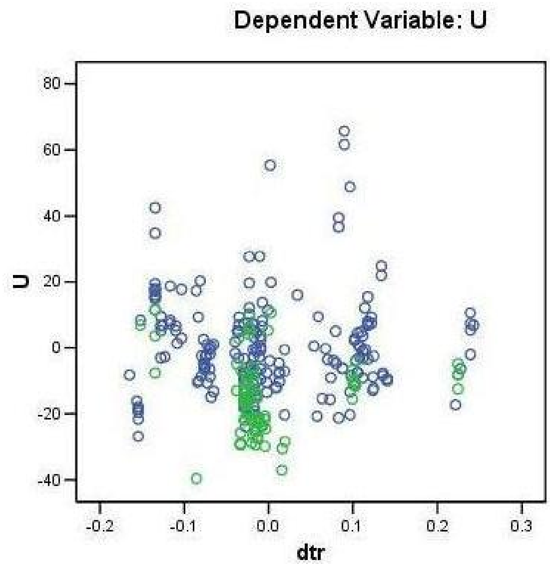

Figure A.5. Partial Reg. Plot U Vs dtr bottom bar with transverse bars.

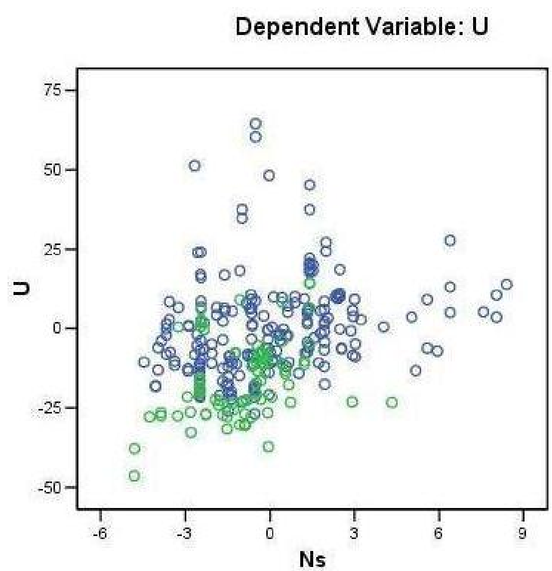

Figure A.6. Partial Reg. Plot U Vs Ns bottom bar with transverse bars.

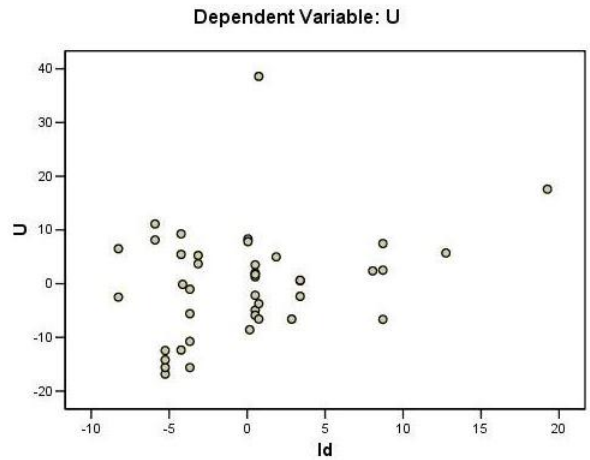

Figure A.7. Partial Reg. Plot U vs Ls top bar with transverse bars.

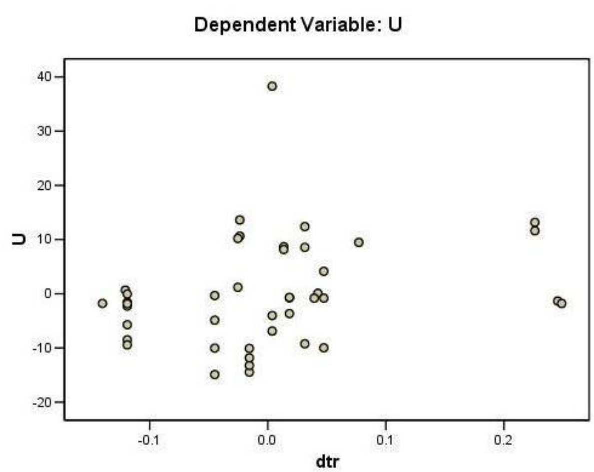

Figure A.8. Partial Reg. Plot U vs dtr top bar with transverse bars.

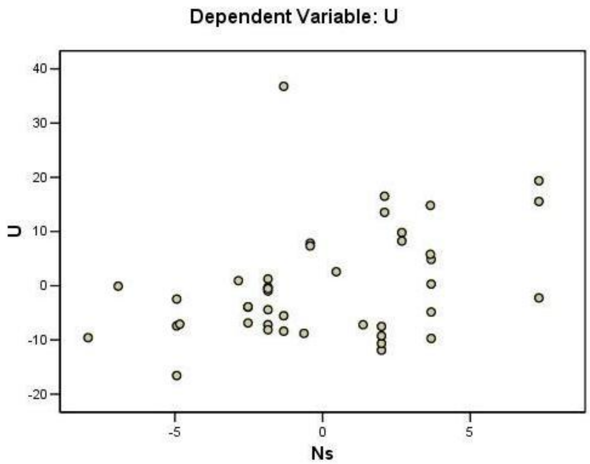

Figure A.9. Partial Reg. Plot U vs ns top bar with transverse bars.

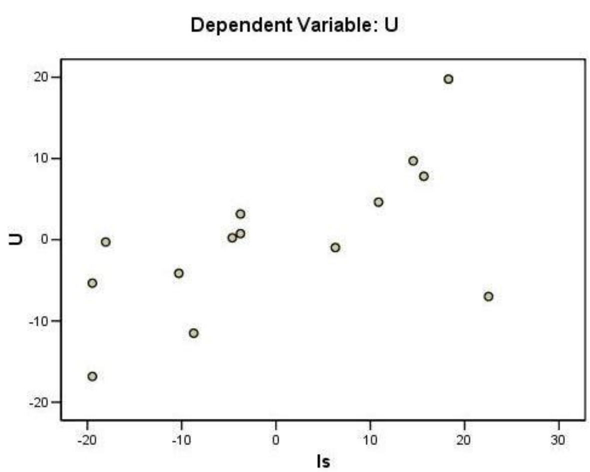

Figure A.10. Partial Reg. Plot U vs Ls side bar with transverse bars.

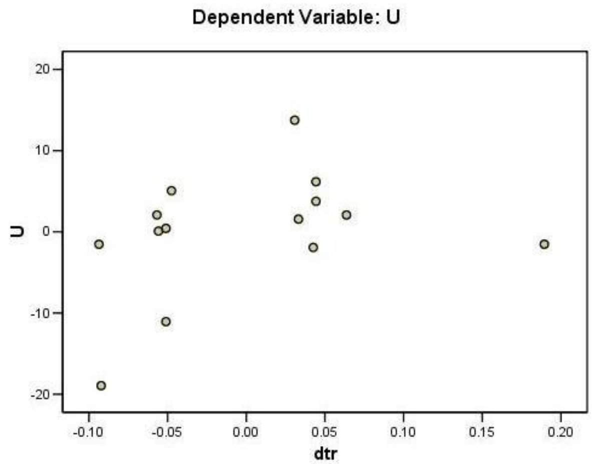

Figure A.11. Partial Reg. Plot U vs dtr side bar with transverse bars.

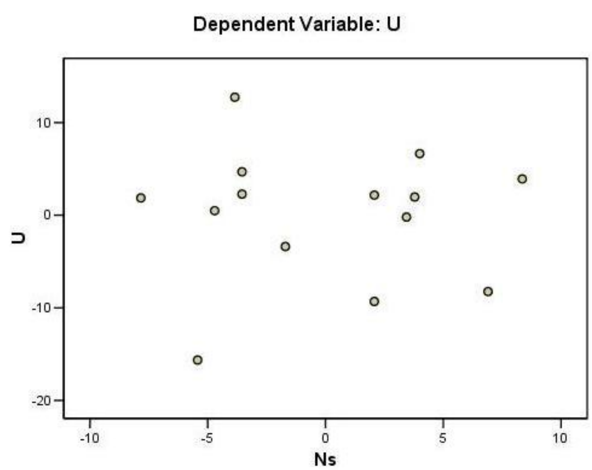

Figure A.12. Partial Reg. Plot U vs ns side bar with transverse bars.

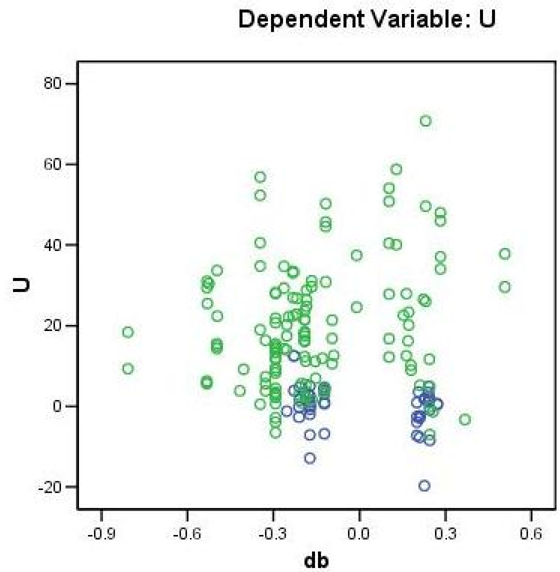

Figure A.13. Partial Reg. Plot U Vs db Bottom Bar, no T/Bars.

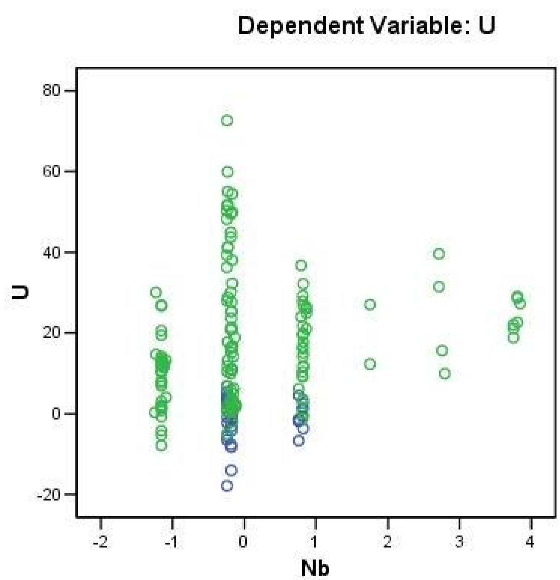

Figure A.14. Partial Reg. Plot U Vs Nb Bottom Bar, no T/Bars.

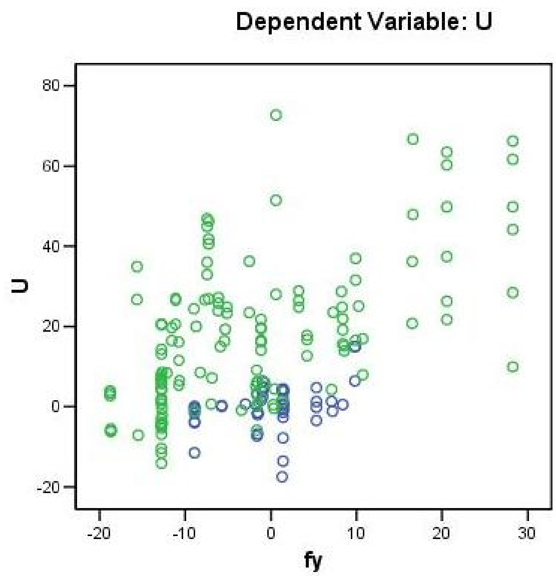

Figure A.15. Partial Reg. Plot U Vs fy Bottom Bar, no T/Bars.

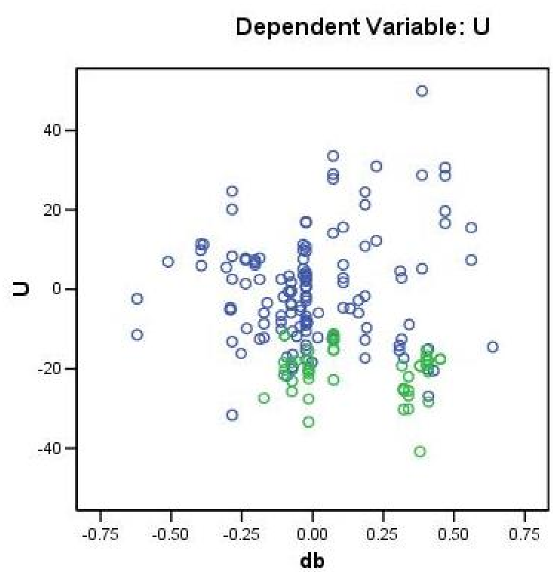

Figure A.16. Partial Reg. Plot U Vs db Bottom Bar, no T/Bars.

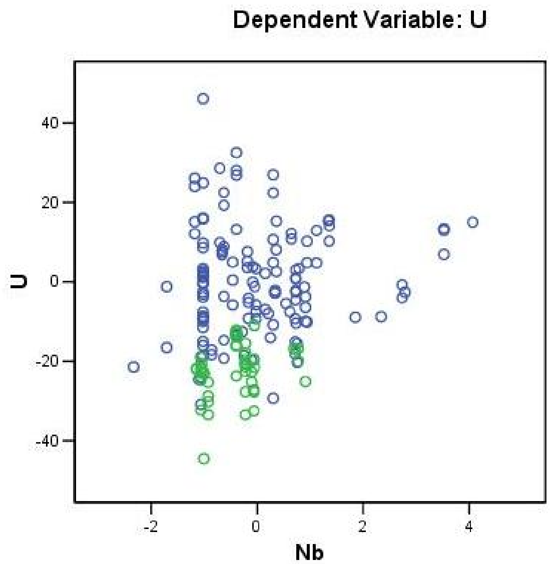

Figure A.17. Partial Reg. Plot U Vs Nb Bottom Bar, no T/Bars.

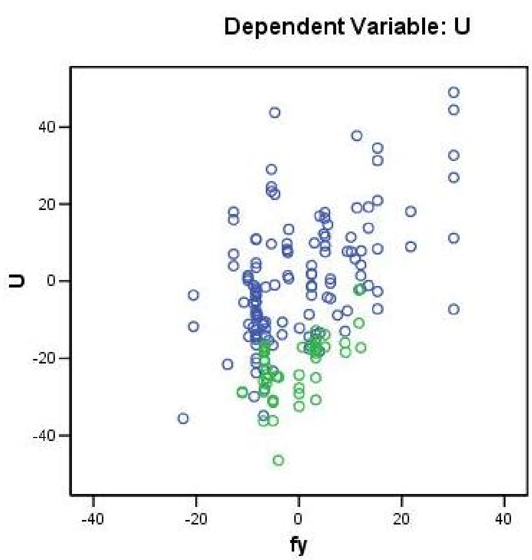

Figure A.18. Partial Reg. Plot U Vs fy Bottom Bar, no T/Bars.

APPENDIX B

NORMAL PROBABILITY

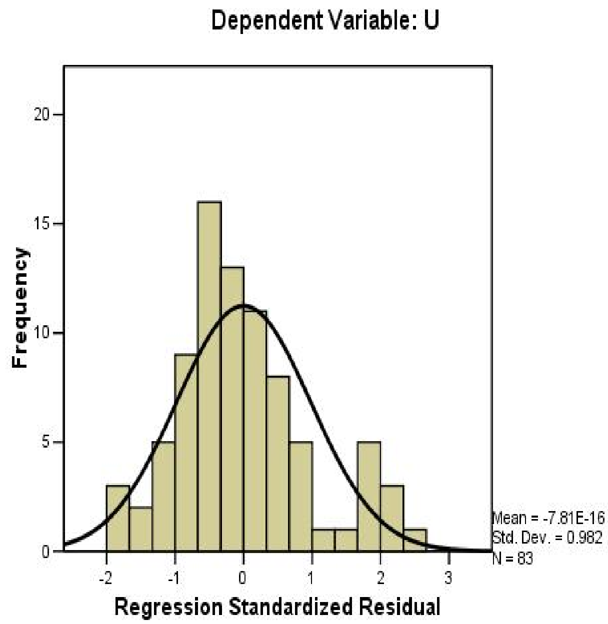

Figure B.1. Normal Residual Bottom Bar with Transverse Reinforcements.

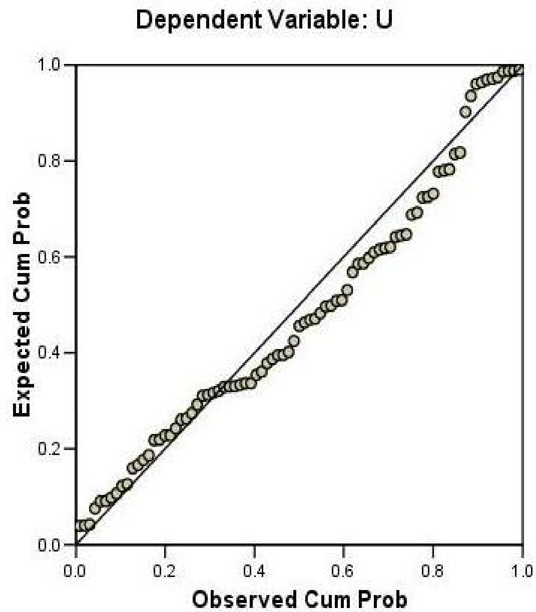

Figure B.2. Normal Residual probability Bottom Bar with Transverse Reinforcements.

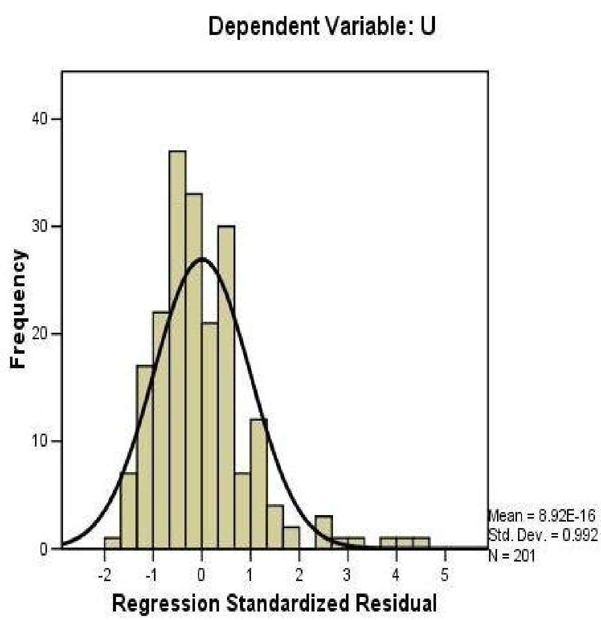

Figure B.3. Normal Residual Bottom Bar with Transverse Reinforcements.

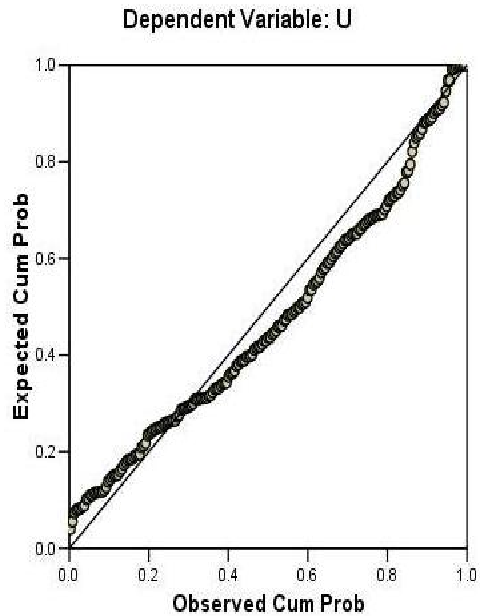

Figure B.4. Normal probability Bottom Bar with Transverse Reinforcements.

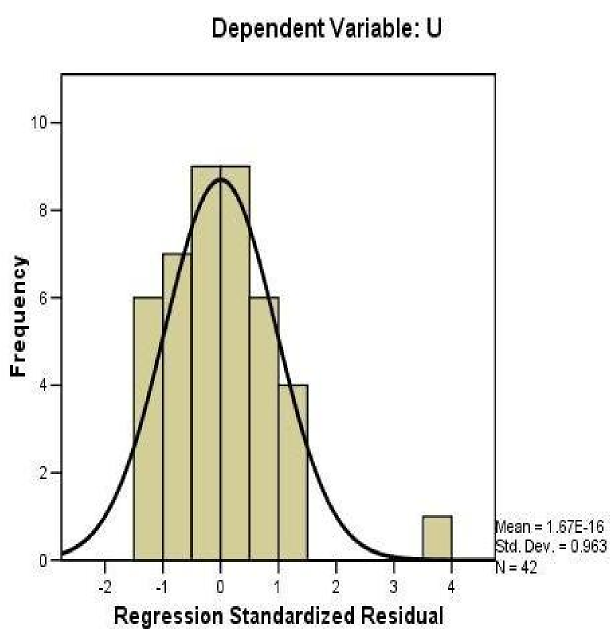

Figure B.5. Normal Residual Top bar with Transverse Reinforcements.

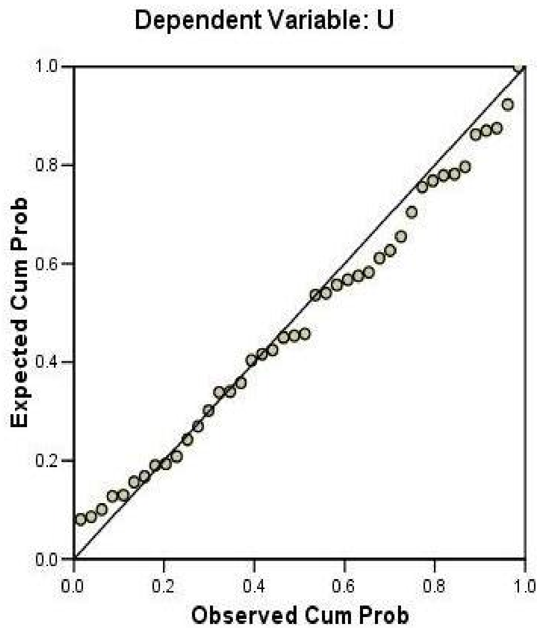

Figure B.6. Normal probability with Transverse Reinforcements.

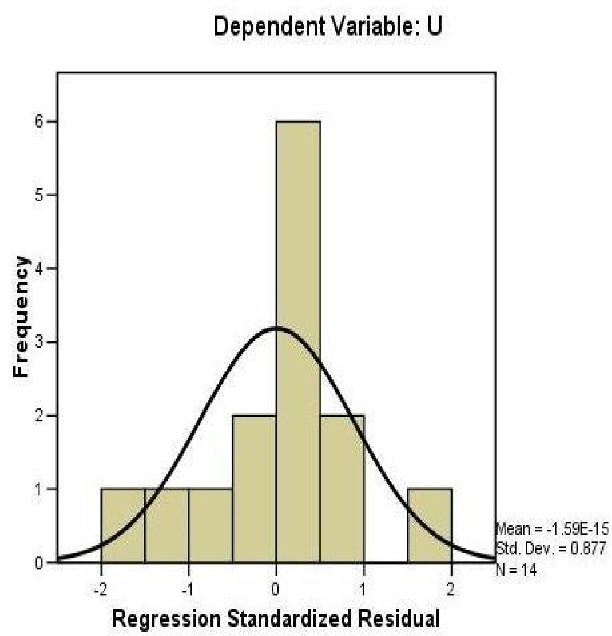

Figure B.7. Normal Residual side bar with Transverse Reinforcements.

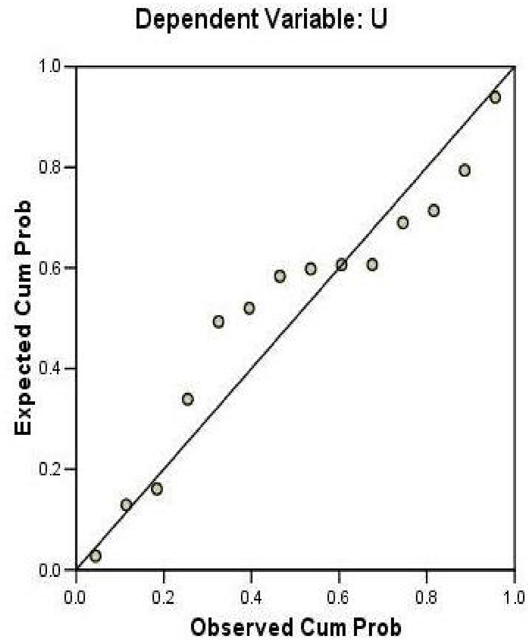

Figure B.8. Normal probability side bar with Transverse Reinforcements.

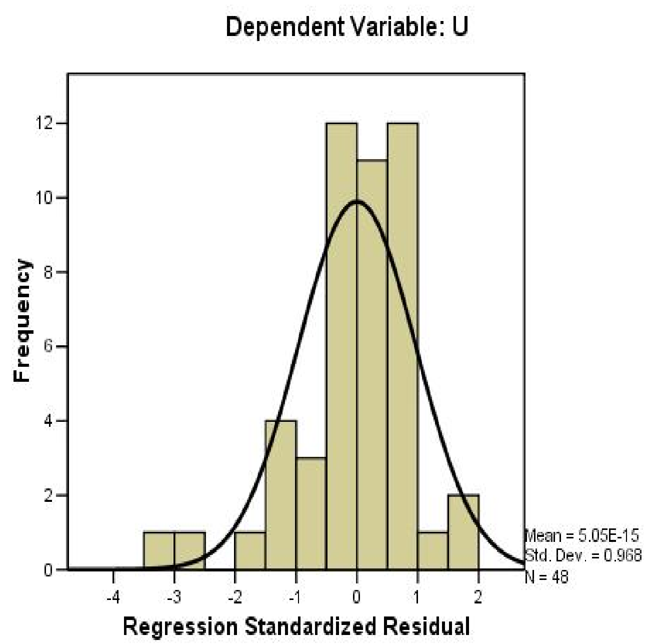

Figure B.9. Normal Residual Bottom Bar without Transverse Reinforcements.

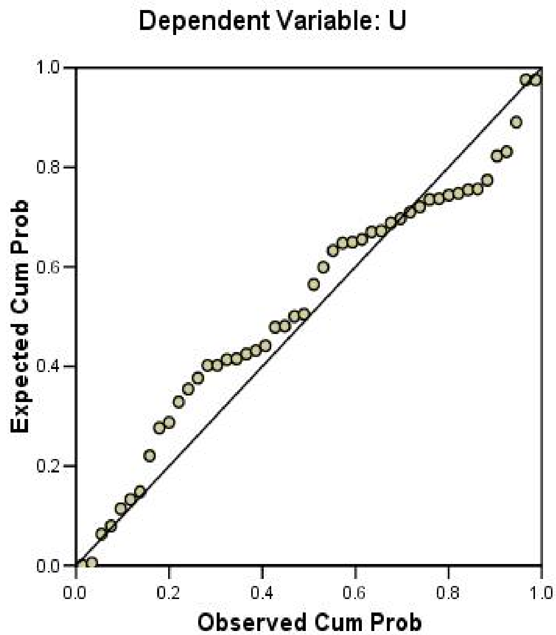

Figure B.10. Normal Residual probability Bottom Bar without Transverse Reinforcements.

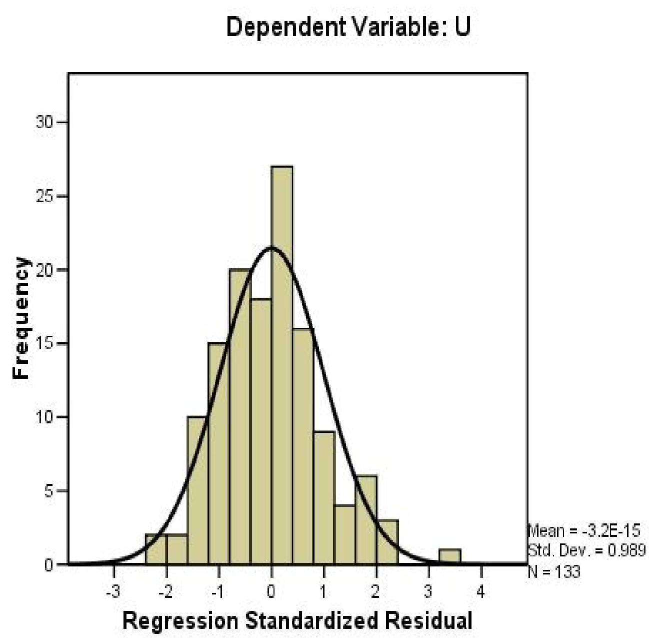

Figure B.11. Normal Residual Bottom Bar without Transverse Reinforcements.

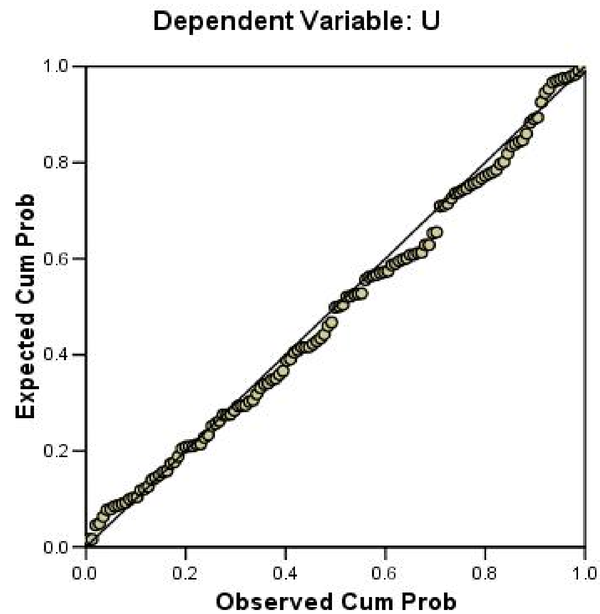

Figure B.12. Normal probability Bottom Bar without Transverse Reinforcements.

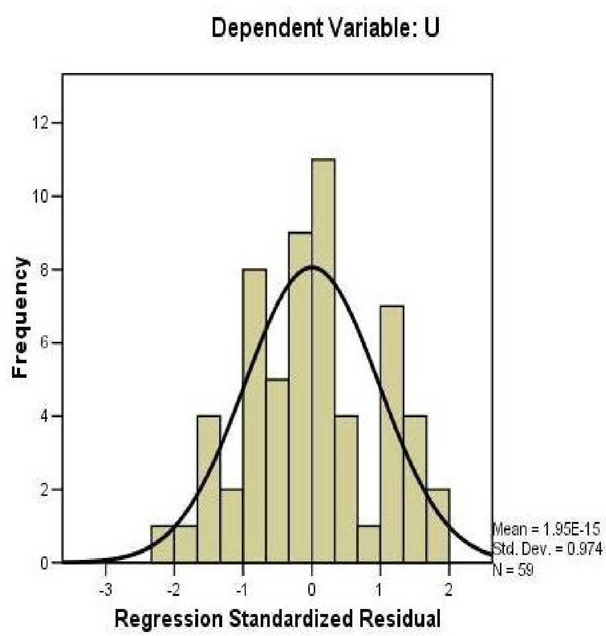

Figure B.13. Normal Residual Top bar without Transverse Reinforcements.

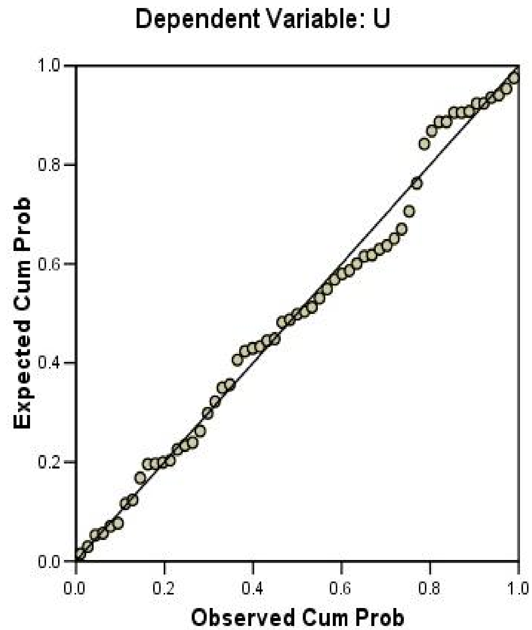

Figure B.14. Normal probability Top bar without Transverse Reinforcements.

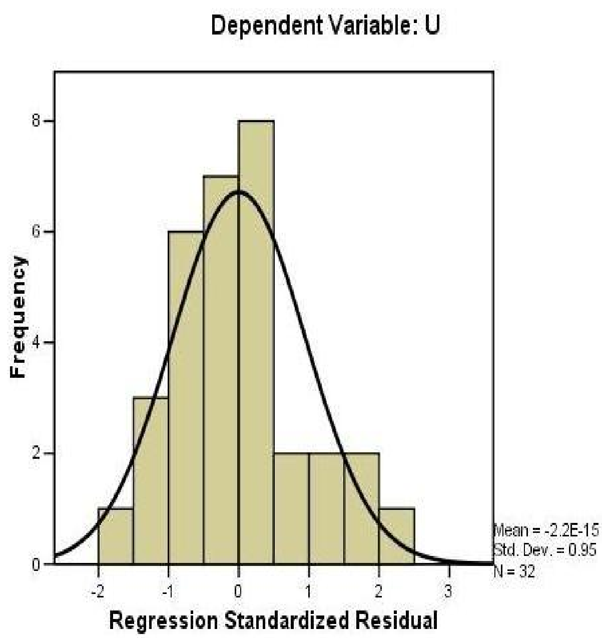

Figure B.15. Normal Residual Side bar without Transverse Reinforcements.

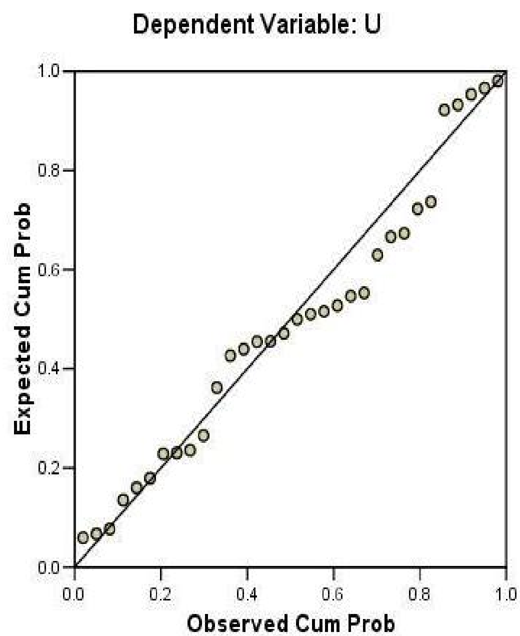

Figure B.16. Normal probability Side bar without Transverse Reinforcements.

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BIOGRAPHICAL STATEMENT

Jkumar Gopalarathnam was born in Trichy, India, in 1981. He did his schooling in his native place. He participated in various social service activities. He received his B.S. degree from Bharathidasan University, Trichy, in 2003. He then joined the University of Texas at Arlington to pursue his Masters degree in civil engineering. His research interests are in the area of prestressed concrete design and reinforced concrete design optimization. He also actively involves himself in Indian cultural events.