# STRUCTURAL OPTIMIZATION USING ANSYS CLASSIC AND RADIAL BASIS FUNCTION BASED RESPONSE SURFACE MODELS

By

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This work is dedicated to my parents,

Mr. L.K Krishna and Mrs. Vijaya Krishna

who have been always there for me and for their love,

support and inspiration without

which my graduate study at UT Arlington could not be

possible.

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ABSTRACT

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RESPONSE SURFACE MODELS

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This thesis describes a methodology to develop an effective and simple structural optimization method using Response Surfaces. It also further demonstrates how these Response Surfaces can then be utilized for further Multi Objective Optimization. The true or exact responses were calculated using Ansys Classic while the codes for the training and simulation of Response Surface is programmed in Matlab. Initially we used a Halton sequence to generate a DOE of the design variables. Then we used Ansys Classic to generate the exact responses. Once the responses were imported the Response Surface are generated using a RBF approximation model. This RBF model is then optimized, then another design point and the optimized points are then added to the DOE. This process is carried out until the results converge. The center points and the coefficient points of each loading condition are then used for multiobjective analysis.

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# CHAPTER 1

# INTRODUCTION

*Optimization* may be defined as the process of maximizing or minimizing a desired objective function while satisfying the prevailing constraints. In every stage of design, construction and maintenance of engineering systems, engineers are bound to take certain technological and managerial decisions. The ultimate goal of all such decisions is either to minimize the effort required or maximize the desired benefit. Since either of these goals in any physical situation can be expressed as a function of certain design variables, *optimization* may also be defined as the process of finding the conditions that give the maximum or minimum value of a function. (1)

Nature provides abundance examples of optimization. For example in metals and alloys, atoms take the position of least energy to form unit cells. It is these unit cells that define the crystalline structure of materials. Another example of nature's optimization process is genetic mutation for survival. Like nature, organizations and businesses employ optimization in their work process to meet the current consumer demands and increased competitions. In engineering, optimization can be used to solve any problem. Some typical applications from different engineering disciplines are-

- Design of aircraft and aerospace structures for minimum weight.
- Vibration and noise optimization of automobile for ride quality, comfort and handling.
- Optimal design of electric networks.
- Analysis of statistical data and building empirical models from experimental results to obtain the most accurate representation of the physical phenomenon.
- Optimal production planning, controlling and scheduling etc.

A number of optimization methods are available to solve such problems. However, for engineers to apply optimization at their work place they need to understand the theory, the algorithm and the techniques behind these methods. This is because practical problems may require modifying algorithmic parameters and even scaling and adapting the existing methods to suit the specific application. Above all, the user may have to try out a number of optimization methods to find one that can be successfully applied.

## 1.1. Design Optimization

### *1.1.1 Basic Design Optimization Formulation*

Problem formulation is normally the most difficult part of the process. It is the selection of design variables, constraints, objectives, and models of the disciplines. A further consideration is the strength and breadth of the interdisciplinary coupling in the problem.

**Design variables**

A design variable is a specification that is controllable from the point of view of the designer. For instance, the thickness of a structural member can be considered a design variable. Another might be the material the member is made out of. Design variables can be continuous (such as a wing span), discrete (such as the number of ribs in a wing), or boolean (such as whether to build a monoplane or a biplane). Design problems with continuous variables are normally solved more easily.

Design variables are often bounded, that is, they often have maximum and minimum values. Depending on the solution method, these bounds can be treated as constraints or separately.

### Constraints

A constraint is a condition that must be satisfied in order for the design to be feasible. An example of a constraint in aircraft design is that the lift generated by a wing must be equal to the weight of the aircraft. In addition to physical laws, constraints can reflect resource limitations, user requirements, or bounds on the validity of the analysis models. Constraints can be used explicitly by the solution algorithm or can be incorporated into the objective using Lagrange multipliers.

### Objectives

An objective is a numerical value that is to be maximized or minimized. For example, a designer may wish to maximize profit or minimize weight. Many solution methods work only with single objectives. When using these methods, the designer normally weights the various objectives and sums them to form a single objective. Other methods allow multiobjective optimization, such as the calculation of a Pareto front.

### Models

The designer must also choose models to relate the constraints and the objectives to the design variables. These models are dependent on the discipline involved. They may be empirical models, such as a regression analysis of aircraft prices, theoretical models, such as from computational fluid dynamics, or reduced-order models of either of these. In choosing the models the designer must trade off fidelity with analysis time.

The multidisciplinary nature of most design problems complicates model choice and implementation. Often several iterations are necessary between the disciplines in order to find the values of the objectives and constraints. As an example, the aerodynamic loads on a wing affect the structural deformation of the wing. The structural deformation in turn changes the shape of the wing and the aerodynamic loads. Therefore, in analyzing a wing, the aerodynamic and structural analyses must be run a number of times in turn until the loads and deformation converge.

**Standard form**

Once the design variables, constraints, objectives, and the relationships between them have been chosen, the problem can be expressed in the following form:

Find x that minimizes J(x)

Subject to $g(x) \leq 0, h(x) = 0$ and $x_{lb} \leq x \leq x_{ub}$

Where *J* is an objective, x is a vector of design variables, g is a vector of inequality constraints, h is a vector of equality constraints, and $\mathbf{x}_{lb}$ and $\mathbf{x}_{ub}$ are vectors of lower and upper bounds on the design variables. Maximization problems can be converted to minimization problems by multiplying the objective by -1. Constraints can be reversed in a similar manner. Equality constraints can be replaced by two inequality constraints.

## 1.2. RBF Based Optimization

In recent years, response surface methodology (RSM) has become a popular tool for Multidisciplinary Design Optimization (MDO). RSM provide an overall perspective of system response within the design space and simplify the process of integrating different mathematical models required in MDO. Traditional RSM is a global approach that uses polynomials to approximate the responses. The coefficients of the polynomial models are computed using the least squares method. The computed response surface model is the 'best-fit' polynomial function from the available data. In general, the fitted function does not interpolate the available data. In this paper, we investigate the use of radial basic, function (RBF) to build response surfaces (RBF response surface) for design optimization. Essentially RBF response surface models are multidimensional interpolation functions. One basis function is chosen for each data point. Thus RBF models are very flexible. It can be built with very little data. (2)

The Multiquadratics introduced by Hardy in 1971 appears to be the first known application of RBF in approximation. They were used to approximate geographical surfaces, gravitational and magnetic anomalies. It was largely unknown to mathematicians until the publication of Franke's review paper .Frank reviewed many methods and concluded, "In terms of fitting ability and visual smoothness the most impressive in the tests is the multiquadratic method, due to Hardy". More recent review papers by McDonald et al (2001) and Jin et al (2000) have indicated that MQ can also be used as a basis to construct multivariate response surface models.

### *1.2.1 Radial Basis Function Response Surface Model*

Radial basis function response surface use the following interpolation function to approximate the response at an untried point x.

$$f(x,h) = \sum C_j \times \varphi(x,h)$$

Where $\varphi = (|r_i|^2 + h)^\beta$, $|r_i| = |r - x_i|$, $h \; and \; \beta$ are parameters for the model.

Given a set of simulation results $(x_i, y_i)$, $x_j \in R^N$, $y_i \in R$, *i*=1, the coeffecients $C_i$ can be calculated by solving the following system of linearequations

A*C = b Where the element of A, b are

$$A_{ij} = \varphi_j(x_i, h, \beta)$$

$$b_i = y_i$$

## 1.3. Objective and approach of the thesis

The Objective of this thesis was to develop a methodology for structural analysis by developing an approximated RBF Response Surface model in Matlab using a more accurate Ansys Model. The model developed through Ansys is computationally expensive. This thesis seeks to demonstrate that once an RBF Response Surface model has been developed, it can be used for approximated prediction model for single and multi objective optimization.

The approach to achieve the above mentioned objective is ,

1. The input matrix in generated for the design variables using a Halton series.
2. Then the center points are chosen and the structure is analysed for these center points using Ansys.
3. Once we have the initial input and responses, we build a response surface model and optimize it with FMINCON.
4. Now these optimal results and one more design point is added to the center points matrix and their Ansys responses are determined.
5. Step 3 is repeated until the convergence criteria for the optimal results provided by FMINCON are satisfied.
6. The Center points and the Coeff matrix is stored for this test case
7. The same procedure is carried out for the second test case and center points and coeff matrix required to simulate the conditions is also determined.
8. The matrixes for these two test cases are then passed into FMINIMAX which minimizes the maximum of these two test cases to finally give a optimal solution which satisfies both the test cases.
9. The reason the particular methodology was chosen was because Ansys can be used to verify the results at every step.

## 1.4. Outline of the thesis

The general idea of structural optimization is presented in Chapter 2. Historical Development, Application of optimization in the field of engineering design, formulation of general optimization problem, classifications of available optimization methods, the use of sequential quadratic programming for optimization, a discussion on finite element based optimization and Multi disciplinary Optimization are also discussed in this chapter. Additionally, Design and Analysis of Experiments, Guidelines for Designing Experiments is also discussed.

In Chapter 3, a detailed discussion is presented on structural optimization based on response surface methodology (RSM), use of radial basis functions (RBF) for problem approximation.

Chapter 4 explains the proposed algorithm for structural optimization using radial basis function based response surface model and Ansys Classic.

Chapter 5 presents the implementation of the proposed optimization scheme by solving different application problems.

Finally, the conclusions and recommendations for future research are presented in Chapter 6.

# CHAPTER 2

# OPTIMIZATION CONCEPTS AND METHODS

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function. It can be seen from Figure 2-1 that if a point x" corresponds to the minimum value of function f(x), the same point also corresponds to the maximum value of the negative of the function, -f(x). Thus, without loss of generality, optimization can be taken to mean minimization since the maximum of a function can be found by seeking the minimum of the negative of the same function. There is no single method available for solving all optimization problems efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problems. (3)

The optimum seeking methods are also known as mathematical programming techniques and are generally studied as a part of operations research. Operations research is a branch of mathematics concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solutions

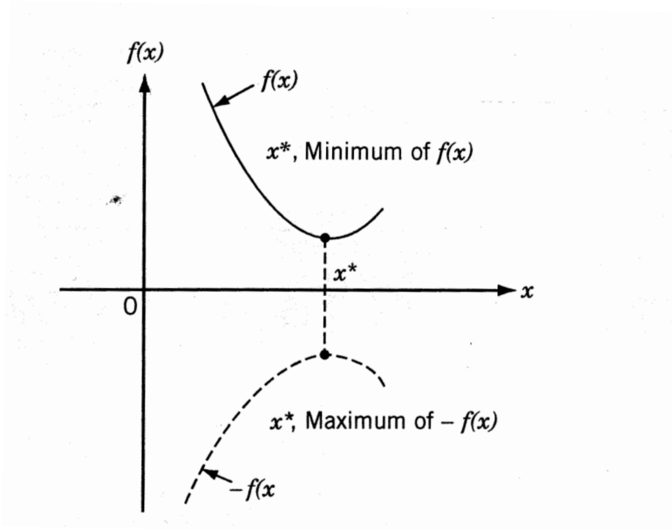



Figure 2-1 : Maxima and Minima

Mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. Stochastic process techniques can be used to analyze problems described by a set of random variables having known probability distributions. Statistical methods enable one to analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation. (4)

## 2.1. Historical Development

The existence of optimization methods can be traced to the days of Newton, Lagrange, and Cauchy. The development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus. The foundations of calculus of variations, which deals with the minimization of functionals, were laid by Bernoulli, Euler, Lagrange, and Weirstrass. The method of optimization for constrained problems, which involves the addition of unknown multipliers, became known by the name of its inventor, Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained minimization problems. Despite these early contributions, very little progress was made until the middle of the twentieth century, when high-speed digital computers made implementation of the optimization procedures possible and stimulated further research on new methods. Spectacular advances followed, producing a massive literature on optimization techniques. This advancement also resulted in the emergence of several well-defined new areas in optimization theory. (5)

It is interesting to note that the major developments in the area of numerical methods of unconstrained optimization have been made in the United Kingdom only in the l960s. The development of the simplex method by Dantzig in 1947 for linear programming problems and the annunciation of the principle of optimality in 1957 by Bellman for dynamic programming problems paved the way for development of the methods of constrained optimization Work by Kuhn and Tucker in 1951 on the necessary and sufficiency conditions for the optimal solution of programming problems laid the foundations for a great deal of later research in nonlinear programming. The contributions of Zoutendijk and Rosen to nonlinear programming during the early 1 960s have been very significant. Although no single technique has been found to be universally applicable for nonlinear programming problems, work of Carroll and Fiacco and McCormick allowed many difficult problems to be solved by using the well-known techniques of unconstrained optimization Geometric programming was developed in the l960s by Duffin, Zener, and Peterson. Gomoiy did pioneering work in integer programming, one of the most exciting and rapidly developing areas of optimization. The reason for this is that most real-world applications fall under this category of problems. Dantzig and Charnes and Cooper developed stochastic programming techniques and solved problems by assuming design parameters to be independent and normally distributed. The desire to optimize more than one objective or goal while satisfying the physical limitations led to the development of multiobjective programming methods. Goal programming is a well-known technique for solving specific types of multiobjective optimization problems. The goal programming was originally proposed for linear problems by Charnes and Cooper in 1961. The foundations of game theory were laid by von Neumann in 1928 and since then the technique has been applied to solve several mathematical economics and military problems. Only during the last few years has game theory been applied to solve engineering design problems. Simulated annealing, genetic algorithms, and neural network methods represent a new class of mathematical programming techniques that have come into prominence during the last decade.

Simulated annealing is analogous to the physical process of annealing of solids. The genetic algorithms are search techniques based on the mechanics of natural selection and natural genetics. Neural network methods are based on solving the problem using the efficient computing power of the network of interconnected "neuron" processors.

## 2.2. Engineering Applications of Optimization

Optimization, in its broadest sense, can be applied to solve any engineering problem. To indicate the wide scope of the subject, some typical applications from different engineering disciplines are given below. (6)

- Design of aircraft and aerospace structures for minimum weight
- Finding the optimal trajectories of space vehicles
- Design of civil engineering structures such as frames, foundations, bridges, towers, chimneys, and dams for minimum cost
- Minimum-weight design of structures for earthquake, wind, and other types of random loading
- Design of water resources systems for maximum benefit
- Optimal plastic design of structures
- Optimum design of linkages, cams, gears, machine tools, and other mechanical components
- Selection of machining conditions in metal-cutting processes for minimum production cost

- Design of material handling equipment such as conveyors, trucks, and cranes for minimum cost
- Design of pumps, turbines, and heat transfer equipment for maximum efficiency
- Optimum design of electrical machinery such as motors, generators, and transformers
- Analysis of statistical data and building empirical models from experimental results to obtain the most accurate representation of the physical phenomenon

## 2.3. Statement of a Optimization Problem

An optimization problem can be stated as follows.(7)

$$\text{Find } X=\begin{Bmatrix} x1 \\ x2 \\ x3 \\ . \\ . \\ . \\ xn \end{Bmatrix} \quad \text{which minimizes } f(X)$$

subject to the constraints

$$g_j\,(X) \leq 0, \quad j=1,2,\ldots\ldots,m$$

$$l_j\,(X)=0\,, \quad j=1,2,\ldots\ldots,p$$

where X is an n-dimensional vector called the design vector, f(X) is termed the objective function and $g_j$ (X) and $l_j$(X) are known as inequality and equality constraints, respectively. The number of variables n and the number of constraints m and /or p need not be related in any way. The problem stated in Eq. (1.1) is called a constrained optimization problem. Some optimization problems do not involve any constraints and can be solved as

$$\text{Find } X=\begin{Bmatrix} x1 \\ x2 \\ x3 \\ . \\ . \\ . \\ xn \end{Bmatrix} \quad \text{which minimizes } f(X)$$

Such problems are called unconstrained optimization problem.

*2.3.1 Design Variables*

Any engineering system or component is defined by a set of quantities some of which are viewed as variables during the design process. In general, certain quantities are usually fixed at the outset and these are called preassigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables $x_i$, i = 1,2,. . . ,n. The design variables are collectively represented as a design vector

$$X= =\begin{Bmatrix} x1 \\ x2 \\ x3 \\ . \\ . \\ . \\ xn \end{Bmatrix}.$$

*2.3.2 Design Constraints*

In many practical problems, the design variables cannot be chosen arbitrarily; rather, they have to satisfy certain specified functional and other requirements. The restrictions that must be satisfied to produce an acceptable design are collectively called design constraints. Constraints that represent limitations on the behavior or performance of the system are termed behavior or functional constraints. Constraints that represent physical limitations on design variables such as availability, fabricability, and transportability are known as geometric or side constraints.

*2.3.3 Constraint Surface*

For illustration, consider an optimization problem with only inequality constraints $g_j(X) \leq 0$. The set of values of X that satisfy the equation $g_j(X)= 0$ forms a hypersurface in the design space and is called a constraint surface. Note that this is an (n-1)dimensional subspace, where n is the number of design variables. The constraint surface divides the design space into two regions: one in which $g_j (X) < 0$ and the other in which $g_j (X) > 0$. Thus the points lying on the hypersurface will satisfy the constraint $g_j (X)$ critically, whereas the points lying in the region where $g_j (X) > 0$ are infeasible or unacceptable, and the points lying in the region where $g_j(X) < 0$ are feasible or acceptable. The collection of all the constraint surfaces $g_j(X) = 0$, j =1,2,. . . ,m, which separates the acceptable region is called the composite constraint surface.

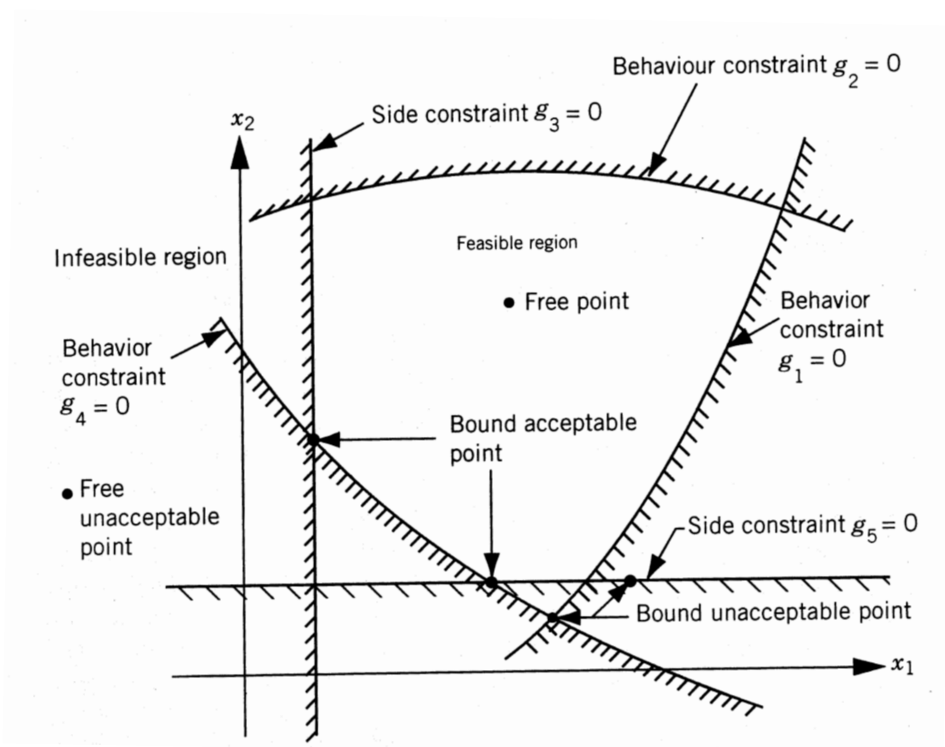



Figure 2-2 : Constraint Surfaces

Figure 2-2 shows a hypothetical 2D design space where the infeasible region is indicated by hatched lines. A design point that lies on one or more constraint surfaces is known as a node point and its associated constraint as an *active constraint*. Those points that do not lie on the constraint surface are known as *free points*. Depending on the location of a design point on the design space, it can be classified into four as:

1. A free and acceptable point
2. A free and unacceptable point
3. A bound and acceptable point and
4. A bound and unacceptable point.

*2.3.4 Objective Function*

The conventional design procedures aim at finding an acceptable or adequate design which merely satisfies the functional and other requirements of the problem. In general, there will be more than one acceptable design, and the purpose of optimization is to choose the best one of the many acceptable designs available. Thus a criterion has to be chosen for comparing the different alternative acceptable designs and for selecting the best one. The criterion with respect to which the design is optimized, when expressed as a function of the design variables, is known as the criterion or merit or objective function The choice of objective function is governed by the nature of problem. The objective function for minimization is generally taken as weight in aircraft and aerospace structural design problems. In civil engineering structural designs, the objective is usually taken as the minimization of cost. The maximization of mechanical efficiency is the obvious choice of an objective in mechanical engineering systems design. Thus the choice of the objective function appears to be straightforward in most design problems. However, there may be cases where the optimization with respect to a particular criterion may lead to results that may not be satisfactory with respect to another criterion. For example, in mechanical design, a gearbox transmitting the maximum power may not have the minimum weight. Similarly, in structural design, the minimum-weight design may not correspond to minimum stress design, and the minimum stress design, again, may not correspond to maximum frequency design. Thus the selection of the objective function can be one of the most important decisions in the whole optimum design process.

In some situations, there may be more than one criterion to be satisfied simultaneously. For example, a gear pair may have to be designed for minimum weight and maximum efficiency while transmitting a specified horse-power. An optimization problem involving multiple objective functions is known as a multiobjective programming problem. With multiple objectives there arises a possibility of conflict, and one simple way to handle the problem is to construct an overall objective function as a linear combination of the conflicting multiple objective functions. Thus if f1(X) and f2(X) denote two objective functions, construct a new (overall) objective function for optimization as $f(X) = \alpha_1 f_1(X) + \alpha_2 f_2(X)$ where $\alpha_1$ and $\alpha_2$ are constants whose values indicate the relative importance of one objective function relative to the other.

### *2.3.5 Objective Function surfaces*

The locus of all the points satisfying f(x) = c = constant forms a hypersurface in the design space, and for each value of c there corresponds a different member of a family and surfaces. These surfaces, called objective function surfaces are shown in a hypothetical two dimensional design space in Figure 2-3.

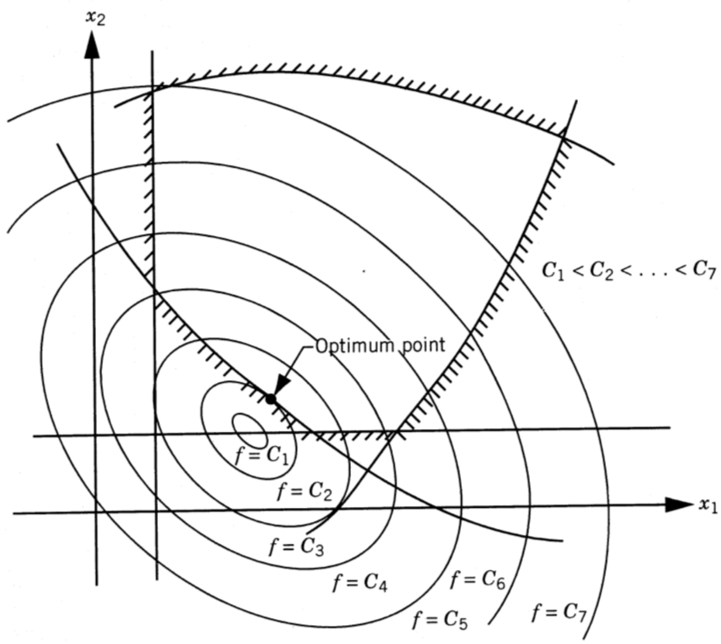



Figure 2-3 : Objective Function Surfaces

Once the objective function surfaces are drawn along the constraint surfaces, the optimum point can be determined without much difficulty.

## 2.4. Discussion on Commonly used Optimization Techniques

Optimization techniques are studied as a part of operations research. The optimum seeking methods of operations research are categorized as (8)

### *2.4.1* Mathematical Programming Techniques

Mathematical Programming Techniques are used to find the minimum of a function of several variables under a prescribed set of constraints. Examples of such methods are: Calculus methods; nonlinear programming; geometric programming; quadratic programming; sequential quadratic programming (SQP); linear programming; genetic algorithm etc.

### *2.4.2 Stochastic Process Techniques*

They are used to analyze problems described by a set of random variables with known probability distribution. Examples are: Markov Process; queuing theory; statistical decision theory etc.

### *2.4.3 Statistical Techniques*

Statistical methods are used to build empirical models from experimental data through analysis in order to obtain the most accurate representation of the physical situation. Examples are: Regression analysis; Design of Experiments (DOE) etc.

The choice of methods depends on the type of problems being solved. There is no single method to solve all optimization problems efficiently. Hence one has to try various methods in order to choose the best one that proves to be computationally efficient and accurate. The subsequent subsections of this topic provides a brief discussion on the optimization methods used in this research work along with a discussion on one of the optimality criterion known as *SubProblem* employed in ANSYS Classic to find the best optimal solution for the design problem.

## 2.5. Sequential Quadratic Programming

Sequential Quadratic Programming (SQP) has received a lot of attention in the recent years owing to the superior rate of convergence. It represents a state of the art in non linear programming methods. Finite element based problems, that involve relatively large number of degrees of freedom and design variables are quite effectively solved using SQP. The formulation of an SQP is based on Newton’s method and Karush-Kuhn-Tucker (KKT) optimality conditions for constrained problems. This method was first published by Pshenichny in 1970 and was called a “linearization method” . While this method has received some attention in the past for engineering applications, the algorithms have deviated from the theory originally presented by

Pshenichny (9); consequently the algorithms work well on certain problems but fail on others. All gradient methods involve two major tasks:

- Direction finding or where to go in the design space.
- Step size selection or how far to go.

Once these two parameters are determined, a new and improved design point on the design space can be obtained as

$$x_{k+1} = x_k + \alpha_k d_k$$

where $d_k$ signifies the direction vector at the given point $x_k$ and $\alpha_k$ represent the step size. A number of iterations are performed to reach at the best optimum point. Iteration involves solving a quadratic programming sub problem to find the direction vector at the point $x_k$. The step length parameter $\alpha_k$ is determined by an appropriate line search algorithm that minimizes the merit function.

### *2.5.1 Attractions of SQP*

- The starting point can be infeasible.
- Gradients of only active constraints are needed.
- Equality constraints can be handled in addition to the inequalities.
- Non linear constrained problems can be solved in less iteration than an unconstrained problem.

One of the reasons for this is that, because of the limits on the feasible area, the optimizer can make informed decisions regarding directions of search and step length. On account of these factors, in this research work a MATLAB optimization tool called “fmincon” based on SQP was employed.

## 2.6. Genetic Algorithm

Genetic Algorithms are search algorithms based on the principle of evolution and survival of the fittest. John Holland, from the University of Michigan began his work on genetic algorithms at the beginning of the 60s. The computational techniques developed by Holland simulated the evolution process and applied it to mathematical programming. These algorithms guide the evolution of a set of randomly selected design variables from the design space towards a near and in some cases to an optimal solution. (10)

Genetic Algorithms differ from more traditional optimization techniques; in that they involve a search from a "population" of solutions, not from a single point. Each iteration or generation of a Genetic Algorithm involves a competitive selection that weeds out poor solutions. The solutions with high "fitness" are "recombined" with other solutions by swapping parts of a solution with another. Solutions are also "mutated" by making a small change to a single element of the solution. Recombination and mutation are used to generate new solutions that are biased towards regions of the space for which good solutions have already been seen. But such algorithms have proved to be computationally expensive when solving complex design problems. However the development in computer technology and the nature of such algorithms have rendered them suitable for implementation on parallel processing machines. The general steps followed by a Genetic Algorithm process can be summarized as:

1. Initialize the population
2. Evaluate initial population
3. Perform competitive selection
4. Apply genetic operators to generate new solutions
5. Evaluate solutions in the population
6. Repeat steps 3 through 5 until some convergence criteria are satisfied.

Zero order approximations in ANSYS Classic optimization module is used to find optimal solutions to the application problems in this research .Results of the proposed MQR optimization method is then compared with the results obtained from ANSYS , to check for the accuracy of the MQR method.

## 2.7. Optimization based on Finite Elements

Today's engineering structures are often analyzed using Finite Elements which is a well-known tool for structural analysis. Finite elements are applied to capture the dynamic response, heat transfer, fluid flow and other phenomena of a system and also to determine the deformation and stresses in a structure subjected to loads and boundary conditions. Mathematically it may be considered as a numerical tool to analyze problems governed by partial differential equations that describe the behavior of the system being studied.

Of all the engineering disciplines, structural designs have seen tremendous development and application of numerical optimization methods. It was Lucien Schmit in 1960 that recognized the potential for combining optimization techniques in structural design. He was the first to introduce nonlinear programming techniques to the design of elastic structures .Today, various commercial finite element codes are available that have optimization capabilities inbuilt to it. Research is also being conducted to explore possibility of solving complex problems by integrating modern optimization tools like MATLAB with finite element packages such as ANSYS.

### *2.7.1 Formulation*

Optimization problems based on finite element can generally be expressed as :

$$\text{Minimize } f(x,U)$$

$$\text{subject to } g_i(x,U) \leq 0 \quad i = 1, \dots., m$$

$$\text{and } h_j(x,U) = 0 \quad j = 1, \dots\dots, l$$

where U is an (*ndof* × 1) nodal displacement vector from which the displacement field u(*x, y, z*) is readily determined. '*ndof*' refers to the number of degrees of freedom in the structure and x corresponds to the design variable set. It is to be noted here that U is an implicit function of x .i.e. any change made to the element parameter *xi* will affect the displacement. The relation between U and x is governed by partial differential equations of equilibrium.

Based on Finite element theory these differential equations can be expressed as:

$$K(x)\, U = F(x)$$

where K is a (*ndof* × *ndof*) square stiffness matrix and F is a (*ndof* × 1) load vector. The functions *f* ,*gi* ,*hj* are all implicit functions of design variables x. They depend explicitly on x, and also implicitly through U

### *2.7.2 Classification of Finite Element based Optimization problems*

Depending on the type of design variables x finite element based optimization may be classified as *parameter or size, shape and topology* optimization. In *parameter or size* optimization the objective function *f* is typically the weight of the structure, and *gi* are the constraints reflecting limits on stress and displacement. The design variable set x can take various forms. In the case of a pin-jointed truss section, *xi* can be the cross sectional area or the length of the truss member. In the plane stress case or in a shell structure, *xi* can be the thickness of each finite element used to mesh the region or in case of a beam cross-section *xi* can represent moment of inertia. It is important to note that, when formulating any finite element based optimization problem, the constraints are to be expressed in normalized form. .i.e. If the stress developed in a structure has to be less that 10,000 psi, then the stress constraint can be expressed as:

$$g \equiv \frac{\sigma}{10000} - \leq 0$$

This way it ensures that the constraints when satisfied will have values lying in the interval [0, 1].

*Shape optimization* problems deals with determining the outline of a body, shape and/or size of a hole, etc. In a '*sizing*' problem mesh geometry is unchanged as the parameters that are changed are those that affect K and F whereas in '*shape*' problems, the *X, Y, Z* coordinates of the nodes or grid points in the finite element model are changed iteratively. The main concept involved in shape optimization is *mesh parameterization* i.e. how can the coordinates of the grid points be related to a finite number of parameters. A common experience observed by analyst in shape optimization of CAD dependent parametric model is inconsistency of mesh pattern. If the mesh pattern cannot be kept, there may be unexpected variation of stress in addition to that caused by parametric changes. This observed phenomenon that usually causes problems in the optimization process is termed "stress oscillation". This difficulty is overcome by using CAD-Independent parametric modeling technique such as "Contour Natural Shape Function" . The main idea behind such an approach is to make parametric changes in the structure without changing the existing mesh connectivity and pattern. (11)

*Topology optimization* on the other hand has to do with distribution of material, creation of holes, ribs or stiffeners, creation/deletion of elements, etc., in the structure. By contrast, in shape optimization of continua, the genus of the body is unchanged .By genus it means the number of cuts necessary to separate the body. While shape and size optimization is quite well known, topology optimization is beginning to gain its importance in commercial optimization codes. Ideally, shape, size and topology optimization should be integrated. However such a capability has been an area of current research.

## 2.8. Multidisciplinary Design optimization

Multidisciplinary design optimization (MDO) is a field of engineering that uses optimization methods to solve design problems incorporating a number of disciplines. It is also known as multidisciplinary optimization and multidisciplinary system design optimization (MSDO).

MDO allows designers to incorporate all relevant disciplines simultaneously. The optimum of the simultaneous problem is superior to the design found by optimizing each discipline sequentially, since it can exploit the interactions between the disciplines. However, including all disciplines simultaneously significantly increases the complexity of the problem.

These techniques have been used in a number of fields, including automobile design, naval architecture, electronics, computers, and electricity distribution.

### *2.8.1 Gradient-based methods*

There were two schools of structural optimization practitioners using gradient-based methods during the 1960s and 1970s: optimality criteria and mathematical programming. The optimality criteria school derived recursive formulas based on the Karush-Kuhn-Tucker (KKT) necessary conditions for an optimal design. The KKT conditions were applied to classes of structural problems such as minimum weight design with constraints on stresses, displacements, buckling, or frequencies [Rozvany, Berke, Venkayya, Khot, et al.] to derive resizing expressions particular to each class. The mathematical programming school employed classical gradient-based methods to structural optimization problems. The method of usable feasible directions, Rosen's gradient projection (generalized reduce gradient) method, sequential unconstrained minimization techniques, sequential linear programming and eventually sequential quadratic programming methods were common choices. Schittkowski et al. reviewed the methods current by the early 1990s.

The gradient methods unique to the MDO community derive from the combination of optimality criteria with math programming, first recognized in the seminal work of Fleury and Schmit who constructed a framework of approximation concepts for structural optimization. They recognized that optimality criteria were so successful for stress and displacement constraints, because that approach amounted to solving the dual problem for Lagrange multipliers using linear Taylor series approximations in the reciprocal design space. In combination with other techniques to improve efficiency, such as constraint deletion, regionalization, and design variable linking, they succeeded in uniting the work of both schools. This approximation concepts based approach forms the basis of the optimization modules in modern structural design software ASTROS, MSC.Nastran, Genesis, I-DEAS, iSight.

Approximations for structural optimization were initiated by the reciprocal approximation Schmit and Miura for stress and displacement response functions. Other intermediate variables were employed for plates. Combining linear and reciprocal variables, Starnes and Haftka developed a conservative approximation to improve buckling approximations. Fadel chose an appropriate intermediate design variable for each function based on a gradient matching condition for the previous point. Vanderplaats initiated a second generation of high quality approximations when he developed the force approximation as an intermediate response approximation to improve the approximation of stress constraints. Canfield developed a Rayleigh Quotient approximation to improve the accuracy of eigenvalue approximations. Barthelemy and Haftka published a comprehensive review of approximations in 1993

### *2.8.2 Non-gradient-based methods*

MDO practitioners have investigated optimization methods in several broad areas in the last dozen years. These include decomposition methods, approximation methods, evolutionary algorithms, memetic algorithms, response surface methodology, reliability-based optimization, and multiobjective optimization approaches.

The exploration of decomposition methods has continued in the last dozen years with the development and comparison of a number of approaches, classified variously as hierarchic and non-hierarchic, or collaborative and non-collaborative. Approximation methods spanned a diverse set of approaches, including the development of approximations for surrogate models, variable fidelity models, and trust region management strategies. The development of multipoint approximations blurred the distinction with response surface methods. Kriging methods became popular.

Response surface methodology, developed extensively by the operations research community, received much attention in the MDO community in the last dozen years. A driving force for their use has been the development of massively parallel systems for high performance computing, which are naturally suited to distributing the function evaluations from multiple disciplines that are required for the construction of response surfaces. Distributed processing is particularly suited to the design process of complex systems in which analysis of different disciplines may be accomplished naturally on different computing platforms and even by different teams.

Evolutionary methods led the way in the exploration of non-gradient methods for MDO applications. They also have benefited from the availability of massively parallel high performance computers, since they inherently require many more function evaluations than gradient-based methods. Their primary benefit lies in their ability to handle discrete design variables and the potential to find globally optimal solutions.

Reliability-based optimization (RBO) is a growing area of interest in MDO. Like response surface methods and evolutionary algorithms, RBO benefits from parallel computation, because the numeric integration to calculate the probability of failure requires many function evaluations. One of the first approaches employed approximation concepts to integrate the probability of failure. The classical first-order reliability method (FORM) and second-order reliability method (SORM) are still popular. Grandhi used appropriate normalized variables about the most probable point of failure, found by a two-point adaptive nonlinear approximation to improve the accuracy and efficiency. Southwest Research Institute has figured prominently in the development of RBO, implementing state-of-the-art reliability methods in commercial software. RBO has reached sufficient maturity to appear in commercial structural analysis programs like MSC.Nastran.

## 2.9. Design and Analysis of experiments

Design of experiments, or experimental design, is the design of all information-gathering exercises where variation is present, whether under the full control of the experimenter or not. (The latter situation is usually called an observational study.) Often the experimenter is interested in the effect of some process or intervention (the "analysis") on some objects (the "experimental models"), which may be structures. Design of experiments is thus a discipline that has very broad application across all the natural, social and engineering sciences. (12)

### *2.9.1 Strategy of Experimentation*

Investigators perform experiments in virtually all fields of inquiry usually to discover something about a particular process or system. Literally, an experiment is a test. More formally, we can define an experiment as a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response.

Our focus is on experiments in engineering and science. Experimentation plays an important role in product realization activities, which consist of new product design and formulation, manufacturing process development, and process improvement. The objective in many cases may be to develop a robust process, that is, a process affected minimally by external sources of variability.

As an example of an experiment, suppose that a metallurgical engineer is interested in studying the effect of two different hardening processes, oil quenching and saltwater quenching, on an aluminum alloy. Here the objective of the experimenter is to determine which quenching solution produces the maximum hardness for this particular alloy. The engineer decides to subject a number of alloy specimens or test coupons to each quenching medium and measure the hardness of the specimens after quenching. The average hardness of the specimens treated in each quenching solution will be used to determine which solution is best.

As we consider this simple experiment, a number of important questions come to mind:

1. Are these two solutions the only quenching media of potential interest?
2. Are there any other factors that might affect hardness that should be investigated or controlled in this experiment?
3. How many coupons of alloy should be tested in each quenching solution?
4. How should the test coupons be assigned to the quenching solutions, and in what order should the data be collected?
5. What method of data analysis should be used?
6. What difference in average observed hardness between the two quenching media will be considered important?

Experimentation is a vital part of the scientific (or engineering) method. Now there are certainly situations where the scientific phenomena are so well understood that useful results including mathematical models can be developed directly by applying these well-understood principles. The models of such phenomena that follow directly from the physical mechanism are usually called mechanistic models. A simple example is the familiar equation for current flow in an electrical circuit, Ohm’s law. However, most problems in science and engineering require observation of the system at work and experimentation to elucidate information about why and how it works. Well-designed experiments can often lead to a model of system performance;

such experimentally determined models are called empirical models. These empirical models can be manipulated by a scientist or engineer just as a mechanistic model can.

A well-designed experiment is important because the results and conclusions that can be drawn from the experiment depend to a large extent on the manner in which the data were collected. To illustrate this point, suppose that the metallurgical engineer in the above experiment used specimens from one heat in the oil quench and specimens from a second heat in the saltwater quench. Now, when the mean hardness is compared, the engineer is unable to say how much of the observed difference is the result of the quenching media and how much is the result of inherent differences between the heats. Thus, the method of data collection has adversely affected the conclusions that can be drawn from the experiment.

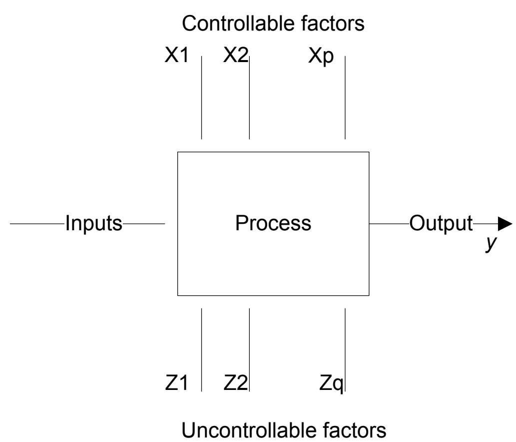



Figure 2-4 : General Model of a Process

In general, experiments are used to study the performance of processes and systems. The process or system can be represented by the model shown in Figure 2-4. We can usually visualize the process as a combination of operations machines, methods, people, and other resources that transforms some input (often a material) into an output that has one or more observable response variables. Some of the process variables and material properties x1, x2,…..$x_p$ are controllable whereas other variables z1, z2, . . ., $Z_q$ are uncontrollable (although they may be controllable for purposes of a test). The objectives of the experiment may include the following:

1. Determining which variables are most influential on the response y.
2. Determining where to set the influential x's so that y is almost always near the desired nominal value.
3. Determining where to set the influential x's so that variability in y is small.
4. Determining where to set the influential x's so that the effects of the uncontrollable variables z1, z2, . . . ,$z_q$ are minimized.

As you can see from the foregoing discussion, experiments often involve several factors. Usually, an objective of the person conducting the experiment, called the experimenter, is to determine the influence that these factors have on the output response of the system. The general approach to planning and conducting the experiment is called the strategy of experimentation. An experimenter can use several strategies.

### *2.9.2 Experimental Design in Engineering*

Experimental design methods have found broad application in many disciplines. As noted previously, we may view experimentation as part of the scientific process and as one of the ways we learn about how systems or processes work. Generally, we learn through a series of activities in which we make conjectures about a process, perform experiments to generate data from the process, and then use the information from the experiment to establish new conjectures, which lead to new experiments, and so on.

Experimental design is a critically important tool in the scientific and engineering world for improving the product realization process. Critical components of these activities are in new manufacturing process design and development, and process management. The application of experimental design techniques early in process development can result in

1. Improved process yields
2. Reduced variability and closer conformance to nominal or target requirements
3. Reduced development time
4. Reduced overall costs

Experimental design methods are also of fundamental importance in engineering design activities, where new products are developed and existing ones improved. Some applications of experimental design in engineering design include

1. Evaluation and comparison of basic design configurations
2. Evaluation of material alternatives
3. Selection of design parameters so that the product will work well under a wide variety of field conditions, that is, so that the product is robust
4. Determination of key product design parameters that impact product performance
5. Formulation of new products

The use of experimental design in product realization can result in products that are easier to manufacture, products that have enhanced field performance and reliability, lower product cost, and shorter product design and development time.

### *2.9.3 Basic Principles*

Statistical design of experiments refers to the process of planning the experiment so that appropriate data that can be analyzed by statistical methods will be collected, resulting in valid and objective conclusions. The statistical approach to experimental design is necessary if we wish to draw meaningful conclusions from the data. When the problem involves data that are subject to experimental errors, statistical methods are the only objective approach to analysis.

Thus, there are two aspects to any experimental problem: the design of the experiment and the statistical analysis of the data. These two subjects are closely related because the method of analysis depends directly on the design employed.

The three basic principles of experimental design are randomization, replication, and blocking. Randomization is the cornerstone underlying the use of statistical methods in experimental design. By randomization we mean that both the allocation of the experimental material and the order in which the individual runs or trials of the experiment are to be performed are randomly determined. Statistical methods require that the observations(or errors) be independently distributed random variables. Randomization usually makes this assumption valid. By properly randomizing the experiment, we also assist in "averaging out" the effects of extraneous factors that may be present.

Computer software programs are widely used to assist experimenters in selecting and constructing experimental designs. These programs often present the runs in the experimental design in random order. This random order is created by using a random number generator. Even with such a computer program, it is still often necessary to assign units of experimental material, operators, gauges or measurement devices, and so forth, for use in the experiment.

Sometimes experimenters encounter situations where randomization of some aspect of the experiment is difficult. For example, in a chemical process, temperature may be a very hard-to-change variable as we may want to change it less often than we change the levels of other factors. In an experiment of this type complete randomization would be difficult because it would add time and cost. There are statistical design methods for dealing with restrictions on randomization.

By replication we mean an independent repeat of each factor combination. Replication has two important properties. First, it allows the experimenter to obtain an estimate of the experimental error. This estimate of error becomes a basic unit of measurement for determining whether observed differences in the data are really statistically different. Second, if the sample mean is used to estimate the true response for one of the factor levels in the experiment, replication permits the experimenter to obtain a more precise estimate of this parameter.

## 2.10. Guidelines for Designing Experiments

To use the statistical approach in designing and analyzing an experiment it is necessary for everyone involved in the experiment to have a clear idea in advance of exactly what is to be studied, how the data are to be collected, and at least a qualitative understanding of how these data are to be analyzed.

### *2.10.1 Recognition of and statement of the problem.*

This may seem to be a rather obvious point, but in practice it is often not simple to realize that a problem requiring experimentation exists, nor is it simple to develop a clear and generally accepted statement of this problem. It is necessary to develop all ideas about the objectives of the experiment. Usually, it is important to solicit input from all concerned parties: engineering, quality assurance, manufacturing, marketing, management, the customer, and operating personnel (who usually have much insight and who are too often ignored). For this reason a team approach to designing experiments is recommended.

It is usually helpful to prepare a list of specific problems or questions that are to be addressed by the experiment. A clear statement of the problem often contributes substantially to better understanding of the phenomenon being studied and the final solution of the problem. It is also important to keep the overall objective in mind. An important aspect of problem formulation is recognition that one large comprehensive experiment is unlikely to answer the key questions satisfactorily. A single comprehensive experiment requires the experimenters to know the answers to a lot of questions, and if they are wrong, the results will be disappointing. This leads to wasting time, materials, and other resources, and may result in never answering the original research questions satisfactorily. A sequential approach employing a series of smaller experiments, each with a specific objective, such as factor screening, is a better strategy.

*2.10.2 Selection of the response variable.*

In selecting the response variable, the experimenter should be certain that this variable really provides useful information about the process under study. Most often, the average or standard deviation (or both) of the measured characteristic will be the response variable. Multiple responses are not unusual. Gauge capability (or measurement error) is also an important factor. If gauge capability is inadequate, only relatively large factor effects will be detected by the experiment or perhaps additional replication will be required. In some situations where gauge capability is poor, the experimenter may decide to measure each experimental unit several times and use the average of the repeated measurements as the observed response. It is usually critically important to identify issues related to defining the responses of interest and how they are to be measured before conducting the experiment. Sometimes designed experiments are employed to study and improve the performance of measurement systems.

*2.10.3 Choice of factors, levels, and range.*

When considering the factors that may influence the performance of a process or system, the experimenter usually discovers that these factors can be classified as either potential design factors or nuisance factors. The potential design factors are those factors that the experimenter may wish to vary in the experiment. Often we find that there are a lot of potential design factors, and some further classification of them is helpful. Some useful classifications are design factors, held-constant factors, and allowed-to-vary factors. The design factors are the factors actually selected for study in the experiment. Held-constant factors are variables that may exert some effect on the response, but for purposes of the present experiment these factors are not of interest, so they will be held at a specific level.

Nuisance factors are often classified as controllable, uncontrollable, or noise factors. A controllable nuisance factor is one whose levels may be set by the experimenter. For example, the experimenter can select different batches of raw material or different days of the week when conducting the experiment. The blocking principle, discussed in the previous section, is often useful in dealing with controllable nuisance factors. If a nuisance factor is uncontrollable in the experiment, but it can be measured, an analysis procedure called the analysis of covariance can often be used to compensate for its effect. For example, the relative humidity in the process environment may affect process performance, and if the humidity cannot be controlled, it probably can be measured and treated as a covariate. When a factor that varies naturally and uncontrollably in the process can be controlled for purposes of an experiment, we often call it a noise factor. In such situations, our objective is usually to find the settings of the controllable design factors that minimize the variability transmitted from the noise factors. This is sometimes called a process robustness study or a robust design problem. Blocking, analysis of covariance, and process robustness studies are discussed later in the text.

Once the experimenter has selected the design factors, he or she must choose the ranges over which these factors will be varied, and the specific levels at which runs will be made. Thought must also be given to how these factors are to be controlled at the desired values and how they are to be measured. Process knowledge is required to do this. This process knowledge is usually a combination of practical experience and theoretical understanding It is important to investigate all factors that may be of importance and to not be overly influenced by past experience, particularly when we are in the early stages of experimentation or when the process is not very mature.

When the objective of the experiment is factor screening or process characterization, it is usually best to keep the number of factor levels low. Generally, two levels work very well in factor screening studies. Choosing the region of interest is also important. In factor screening, the region of interest should be relatively large—that is, the range over which the factors are varied should be broad.

*2.10.4 Choice of experimental design.*

If the pre-experimental planning activities above are done correctly, this step is relatively easy. Choice of design involves consideration of sample size (number of replicates), selection of a suitable run order for the experimental trials, and determination of whether or not blocking or other randomization restrictions are involved.

There are also several interactive statistical software packages that support this phase of experimental design. The experimenter can enter information about the number of factors, levels, and ranges, and these programs will either present a selection of designs for consideration or recommend a particular design. These programs will usually also provide a worksheet (with the order of the runs randomized) for use in conducting the experiment.

In selecting the design, it is important to keep the experimental objectives in mind. In many engineering experiments, we already know at the outset that some of the factor levels will result in different values for the response. Consequently, we are interested in identifying which factors cause this difference and in estimating the magnitude of the response change. In other situations, we may be more interested in verifying uniformity.

*2.10.5 Performing the experiment.*

When running the experiment, it is vital to monitor the process carefully to ensure that everything is being done according to plan. Errors in experimental procedure at this stage will usually destroy experimental validity. Up-front planning is crucial to success. It is easy to underestimate the logistical and planning aspects of running a designed experiment in a complex manufacturing or research and development environment.

Coleman and Montgomery (1993) suggest that prior to conducting the experiment a few trial runs or pilot runs are often helpful. These runs provide information about consistency of experimental material, a check on the measurement system, a rough idea of experimental error, and a chance to practice the overall experimental technique. This also provides an opportunity to revisit the decisions made in steps 1—4, if necessary.

*2.10.6 Statistical analysis of the data.*

Statistical methods should be used to analyze the data so that results and conclusions are objective rather than judgmental in nature. If the experiment has been designed correctly and if it has been performed according to the design, the statistical methods required are not elaborate. There are many excellent software packages designed to assist in data analysis, and many of the programs used in step 4 to select the design provide a seamless, direct interface to the statistical analysis. Often we find that simple graphical methods play an important role in data analysis and interpretation. Because many of the questions that the experimenter wants to answer can be cast into an hypothesis testing framework, hypothesis testing and confidence interval estimation procedures are very Useful in analyzing data from a designed experiment. It is also usually very helpful to present the results of many experiments in terms of an empirical model, that is, an equation derived from the data that expresses the relationship between the response and the important design factors. Residual analysis and model adequacy checking are also important analysis techniques.

Statistical methods cannot prove that a factor (or factors) has a particular effect. They only provide guidelines as to the reliability and validity of results. Properly applied, statistical methods do not allow anything to be proved experimentally, but they do allow us to measure the likely error in a conclusion or to attach a level of confidence to a statement. The primary advantage of statistical methods is that they add objectivity to the decision-making process. Statistical techniques coupled with good engineering or process knowledge and common sense will usually lead to sound conclusions.

*2.10.7 Conclusions and recommendations.*

Once the data have been analyzed, the experimenter must draw practical conclusions about the results and recommend a course of action. Graphical methods are often useful in this stage, particularly in presenting the results to others. Follow-up runs and confirmation testing should also be performed to validate the conclusions from the experiment.

Throughout this entire process, it is important to keep in mind that experimentation is an important part of the learning process, where we tentatively formulate hypotheses about a system, perform experiments to investigate these hypotheses, and on the basis of the results formulate new hypotheses, and so on. This suggests that experimentation is iterative. It is usually a major mistake to design a single, large, comprehensive experiment at the start of a study. A successful experiment requires knowledge of the important factors, the ranges over which these factors should be varied, the appropriate number of levels to use, and the proper units of measurement for these variables. As an experimental program progresses, we often drop some input variables, add others, change the region of exploration for some factors, or add new response variables. Consequently, we usually experiment sequentially, and as a general rule, no more than about 25 percent of the available resources should be invested in the first experiment. This will ensure that sufficient resources are available to perform confirmation runs and ultimately accomplish the final objective of the experiment.

# CHAPTER 3

# RESPONSE SURFACE METHODOLOGY IN STRUCTURAL OPTIMIZATION

## 3.1. Introduction

Response surface methodology is a method for constructing global approximations to system behavior based on results calculated at various points in the design space. The strength of the method is in applications where the calculation of the design sensitivity information is difficult or impossible to compute, as well as in cases with noisy functions, where the sensitivity information is not reliable, or when the function values are inaccurate. In the past, global approximations have been utilized by others in multidisciplinary optimization (13) and in reliability calculations (14).

An important objective in response surface construction is to achieve an acceptable level of accuracy while attempting to minimize the computational effort, i.e. the number of function evaluations. In this study the following strategies commonly used to improve accuracy and efficiency are investigated:

1. The use of engineering knowledge to select the form of the response surface.
2. Eliminating terms which have poorly estimated coefficients.
3. Increasing the number of points.
4. Reducing the design region using reasonable design criteria or through the use of a moving window.

The selection of the functions used to represent the behavior of a system may have a strong influence on the accuracy of the approximation. Hence, the chosen functions must be able to approximate the non-linearity of the actual response as accurately as possible, e.g. a traditional approach in structural sizing has been the use of reciprocal variables as an attempt to capture the non-linearity associated with stress constraints. A more sophisticated approach could be to use variable complexity modeling (15) in which an approximate model is used in conjunction with a more accurate model to obtain accurate global approximations. In the current study, intermediate response quantities were used to develop response surfaces over a large range of the variables.

The selection of points in the design space where designs will be evaluated, i.e. the design of the experiment has an important influence on the accuracy and the cost of computing the response surface. The D-optimality design criterion as implemented in this study has been an attractive option to researchers in structural optimization. Increasing the number of experimental points could improve accuracy. However, using a large number of points is expensive and the potential accuracy may be inhibited by other factors such as the order of the approximating functions, the selection of intermediate functions and the sub region size under investigation.

After the set of experiments has been run, the functions are fitted to the results of the experiments using least-squares techniques. The terms comprising the response surface may then be manipulated using techniques for the selection of the `best' regression equation (16).

The response surfaces are only valid over a part of the design space called the region of interest. An important consideration is the strategy used to find an effective size and location of this region. Initially, the region of interest might not contain the optimal design point. It may therefore be necessary to use a succession of linear approximations to find the region containing the optimal design. A variation of this method of moving the region of interest has been used with success in structural optimization (17). A quadratic response surface valid over a larger region of interest, being consequently less accurate, may also be used to give an indication of the location and size of the sub region needed. The optimal design point can then be found to the desired accuracy through the successive reduction of the sub region size and/or the use of higher-order functions. Reducing the size of the region may sometimes be the only way of improving the accuracy. Kaufman (18) proposed a method to constrain the region of interest to a part of the design space containing only designs judged reasonable in terms of cost and constraint function values. This may be preferable to reducing the design space arbitrarily by imposing simple move limits. In this study the method is evaluated to assess its capabilities for yielding better approximations.

After stating the optimization problem in the next section, the present study continues to detail the strategies for constructing effective response surfaces. The following section illustrates these strategies by means of three truss optimization examples. The study is concluded with the final recommendations.

## 3.2. Design optimization

The optimization problem is stated as

$$\text{Miminize } F(x), x \in R^n$$

$$\text{Subjected to } L_j \leq g_j(x) \leq U_j, \quad j = 1,2,3, \dots, m$$

$$x_i^L \leq x_i \leq x_i^U, \quad i = 1,2,3, \dots, n$$

$F(x)$ is the objective function and x are the design variables. $L_j$ and $U_j$ are respectively the lower and upper bound on the $j^{th}$ non-linear constraint, $g_j(x)$. $x_i^L$ and $x_i^U$ are, respectively, the lower and upper bound on the design variables.

If the objective function or the constraint functions can be approximated over the whole (or a part of the) design space, a (series of) sub problem(s) may be solved to and the solution to the optimization problem. The sub problem is

$$\text{Miminize } \mathrm{F}(x), x \in R^n$$

$$\text{Subjected to } L_j \leq G_j(x) \leq U_j, \quad j = 1,2,3, \dots, m$$

$$x_i^{L`} \leq x_i \leq x_i^{U`}, \quad i = 1,2,3, \dots, n$$

Where F(x) and G(x) are the approximations to the objective and the constraint functions respectively. $x_i^{L`}$ and $x_i^{U`}$ are the lower and upper bounds on the variables for the sub problem.

## 3.3. Response Surface Methodology

### *3.3.1 Approximation and error*

Techniques from experimental design and response surface methodology (19) are used to build the approximate models.

Consider a response y dependent on a set of variables $\xi$. The exact functional relationship between them is

$$y = \eta(\xi)$$

One wants to use an approximation for the functional relationship

$$\eta(\xi) \cong f(\xi)$$

Over some region of interest $R(\xi)$

The response is evaluated at $\xi_1, \xi_2, \ldots., \xi_p$ in $R(\xi)$ for a total of p experiments. This set of experimental points are commonly referred to as the experimental design. At the experimental points we have $Y = (y_1, y_2, \ldots, y_p)^T$ , and $f(\xi) = \{f(\xi_1), f(\xi_2), \ldots, f(\xi_p)\}^T$, and the error, $\varepsilon^r = (\varepsilon_1^r, \varepsilon_2^r, \ldots, \varepsilon_p^r)^T$. The model can now be written as

$$Y = \eta(\xi) + \varepsilon^r$$

Using the approximating function with

$$Y = f(\xi) + \delta(\xi) + \varepsilon^r, \quad where \quad \delta(\xi) = \eta(\xi) - f(\xi)$$

There are therefore two types of errors,

1. The modeling (bias) errors,$\delta(\xi) = \eta(\xi) - f(\xi)$, the difference between the approximating function, $f(\xi)$ and the exact functional response $\eta(\xi)$.
2. The random errors $\varepsilon^r$.

The modeling error, unlike the random error, is dependent only on the choice of approximating function and sub region size.

There has been a tendency to ignore the modeling errors (bias) and to concentrate on the random errors (19). This is because elegant mathematical results are possible if this is done. With computer experiments, however, the modeling error is of primary importance.

*3.3.2 The approximating function*

The approximating function, f can be decomposed as follows for linear regression analysis,

$$f(\xi) = \sum_{i=0}^{L} a_i \phi_i(\xi)$$

Where L is the number of functions, and $\phi$ are the basis functions that constitute the model.

The constants, a, are selected to minimize the function

$$\sum_{P=1}^{P} \left\{ w_p \left[ y(\xi_p) - f(\xi_p) \right]^2 \right\} = \sum_{P=1}^{P} \left\{ w_p \left[ y(\xi_p) - \sum_{i=0}^{L} a_i \phi_i(\xi) \right]^2 \right\}$$

where the P experimental points, with y the exact response at the experimental point $\xi_p$ and $w_p$ is the weight coeffcient for the pth experimental point.

If no weighting is used, then the approximation can be written in matrix form as

$$y = X\beta + \varepsilon^l$$

Where $\varepsilon^l$ is the residual error associated with the least-squares calculations and X are the values of the basis function at the experimental points, x, i.e.

$$X = \{X_{ui}\} = \{f_i(\xi_u)\}$$

The least squares estimation b to $\beta$ is found with

$$b = (X^T X)^{-1} X_y$$

The choice of the basis functions $\phi$can influence the accuracy of the approximation. In particular, $\phi$ are often chosen as the monomials for the quadratic approximation commonly used. A good choice of $\phi$ will make the approximation more accurate, and applicable over a wider region of the design space. If possible, $\phi$ should therefore be chosen using engineering knowledge of the true functional form of the response. Another approach is variable-complexity modeling (15) in which two models are used in the structural optimization. The optimization is done on a simple and inexpensive model. The simple model is updated to be more accurate through a response surface with the results from a more accurate and expensive model. The response surface should then be smooth and may be chosen as a quadratic polynomial.

The summative form of the approximation function (12) need not be the only form that can be chosen. Other choices may result in a non-linear least-squares problem. Intrinsical linear forms which can be transformed into a linear problem do exist. The multiplicative, exponential, and power functions can be transformed into a linear problem by taking the logarithm or raising the function to a power.

### *3.3.3 Experimental design*

The selection of points in the design space where the response must be evaluated is commonly called design of experiments. The choice of the experimental design can have a large influence on the accuracy of the approximation and the cost of constructing the response surface.

A commonly used experimental design is the factorial design, which in its simplest form, is a hypercube in the design space consisting of the points $[\pm 1, \pm 1, \ldots, \pm 1]$. A total of $2^n$ experimental points must therefore be examined, where n is the number of design variables. The $2^n$ design only estimates the first-order and interaction terms of the approximating function. For a quadratic approximation three levels of the design variables must be considered, giving the $3^n$ factorial experimental design.

The D-optimality criterion for experimental design, although more complicated than competing methods, is the method of choice for some researchers. The D-optimality criterion states that the best set of experimental points maximizes $|X^T X|$. This selection minimizes the variance of the parameter estimates. Some added advantages above other experimental design methods are:

1. Design regions with irregular shapes can be considered.
2. Any number of experimental points can be considered.
3. Experimental points analysed during previous design iterations may be incorporated into a new experimental design.

A disadvantage of the D-optimality criterion is that it is a `variance method' and disregards the possible effects of bias. However, if the design is limited to a region of interest, the difference between the spread of the design points for an all-bias design and an all-variance design is often not large (20). Researchers in structural optimization (21) reported that the use of the D-optimality criterion leads to a lower maximum error and a higher average error relative to a minimum-bias criterion. The D-optimality criterion appears not to be unrealistic when the model is correct or when the design is to be restricted to the region of interest, or both (20). Some methods considering bias do exist but they require an a priori knowledge of the functional form of the response. This knowledge might not be available or if available it might be utilized by using intermediate functions or a variable complexity approach. The response surface will, in general, be used only to approximate the part of the response for which the true functional relationship is not available, too difficult to calculate or too noisy to be useful.

### *3.3.4 Selecting the `best' regression model*

Model building is used in various disciplines. A detailed treatment is given by Draper and Smith (16). The model building process is used to select from the set of all possible basis functions, $\phi_1, \ldots, \phi_L$, the subset of basis equations best suited to the purpose of the model. Two applications of a fitted model are:

a. Using the model to predict the response of the system.
b. Identifying the functional form of the response

Two conflicting criteria come into play for the selection of the `best' regression model.

a. One would like as many terms as possible in the regression equation for accurate prediction of the response.

b. The use of too many terms may lead to overfitting which may reduce the predictive capabilities of the model.

This process of making a compromise between the above criteria is referred to as the selection of the `best' regression model. In some applications it may be desirable to have only the important terms in the model in order to have fewer experiments.

*3.3.5 Reasonable design space*

The use of experimental points reasonable in terms of function and constraint values will result in a smaller design space with less curvature.

Kaufman (18) proposed that an unreasonable design, x, should be moved so that it resides on the edge of the reasonable design space:

$$x' = \alpha(x - x_c) + x_c$$

where $x$ and $x_c$ are the locations of the design point and the centre of the current design space respectively, and $0 \leq \alpha \leq 1$ is the parameter adjusted to make the design feasible.

Some important considerations for the use of a reasonable design space are:

i. The resulting experimental design will likely have poorer properties than the original design, thus making the use of the D-optimality design criterion for the selection of a good subset of points attractive.

ii. As many as possible responses should be used in order to constrain the subregion in all possible directions, e.g. the use of a weight constraint only may lead to an experimental design stretched out in the direction of constant weight.

iii. It may be necessary to use the least expensive functions first to move the experimental points, thus reducing computing time.

iv. The result is a smaller subregion with less curvature. This may conflict with the use of an intermediate function which will attempt to remove the curvature from an already smooth function, thus yielding a non-linear function in the transformed space.

*3.3.6 Intermediate variables and response quantities*

In the choice of an approximating function one should consider the functional form of the response under consideration, since there might be an analytical relationship that can be utilized. The general formulation for intermediate variables and response quantities is given by Barthelemy and Haftka. If the functional form $\eta(\xi)$ can be written in terms of an intermediate response $\eta_I$ and of intermediate variables $\xi_I$, the functional form can be written as

$$\eta(\xi) = \eta\{\eta_I[\xi_I(\xi)]\}$$

If the relationship $\eta = \eta(\eta_I)$ an the $\xi_I = \xi_I(\xi)$ are known analytically and if an approximation $f_I(\xi_I)$ exists, the approximation $f(\xi)$ is given by

$$\eta(\xi) \cong f(\xi) = \eta\{f_I[\xi_I(\xi)]\}$$

In general, all three nested relationship may be approximated,

$$\eta(\xi) \cong f(\xi) = f\{f_I[\hat{\xi}_I(\xi)]\}$$

Where $\hat{\xi}_I$ is an approximation to the intermediate variables.

The stress and buckling constraints in a truss member are frequently approximated using intermediate functions in this study. The stress in a truss member is

$$\sigma_i = \sigma_i(x_i, f_i(x))$$

Where $\sigma_i$ is the stress in member *i* with cross-section $x_i$ under the action of the force, $f_i$. One may write

$$\sigma_i = \frac{1}{x_i} f_i(x)$$

If a high-quality approximation can be found for the force, the stress can be predicted very accurately with this intermediate reciprocal approximation. This behavior has also been exploited with the use of reciprocal variables in local approximation techniques.

# CHAPTER 4

# METHODOLOGY OF THE STUCTURAL OPTIMIZATION PROCESS

The main purpose of this thesis was to build a process to analyse and build a mathematical model for a multi objective optimization in MATLAB. The DOE for the structure is built in Matlab using the Halton Process. The main finite element modeling and analysis is carried out in Ansys Classic. These responses are then imported into Ansys and a mathematical model is built using Response Surface Methodology with Radial Basis Functions as the approximation model. This mathematical Model can then be used for multi objective optimization. The analysis was carried out Intel Core 2 Duo with Vista 64 bit. The main softwares used were Matlab R2008b and Ansys v11.0 Classic.

The main steps in the process is listed below with explanations,

## 4.1. Definition of the Problem

The finite element is first defined in Ansys. A APDL file is written which takes in a set of input variables and stores them. It then generates the structure for the given boundary conditions, loads, material properties and for each variation of the input variables the structure is analsyed for the requested responses like Displacements, Stresses, Frequencies etc. These responses are then saved for each run and then written in a results file

## 4.2. Generate the DOE in Matlab

First the design variables that can be modified in a structure is identified and its upper [ XU ] and lower limit [ XL ] is fixed. Depending on the number of the design variables a normalized Halton matrix [ XQMC ] is generated and then the individual design variables are then mapped onto it. This Design matrix [ Xdata ] now consists of a large number of variations [ nPoints ] of the design variables.

The center points are also determined from the Design Matrix. We initially take the first 10 points [ Ninit] since this would roughly cover about 90% of the entire spectrum of the Design Variables. Additionally the important parameters like the number and type of responses [ Nresp ], the Limits for the responses, constraints matrix [ Cy,Cb ], the convergence criteria are initialized.

### 4.3. Generation of the responses

Once the input ccd matrix is generated, the entire matrix is imported into Ansys and a APDL Macro is used to analyse the structure. Responses for each set of input are calculated and stored. Once the analysis is run for the entire input data set, the entire result matrix is written to a file [ results.txt ]. This file is then imported into Matlab and divided into the appropriate response set like Displacements, Stress, Volume, Frequency etc . Additionally all the responses are also stored in a matrix [ Yc ].

Once these responses have been sorted out, we then start to train a RBF response surface model. The coefficient matrix is first initialized and then we pass the center Points [ Xc ] , the responses [ Yc ] and the points for which we have to find the responses, which in this case is the center points itself. This generates a RBF model [ C ] from which we can pass the unknown points and get approximated response.

### 4.4. Run the optimization Scheme

Once the center points and the coefficient matrix [ c ] have been determined we can then code a program which will take the unknown design variable points [ x ] and then generate the responses [ Yc ] by simulating the RBF function. We divide the responses into the objective and the constraints function as required and then using the FMINCON solver in Matlab, optimize the Design variables to generate the optimized solution [ Xrsm ].

We start by using an appropriate starting point and then solve the following design optimization problem

Minimize : $f = A_x \times X_c + A_y \times Y_c$

Subject to : $g = C_x \times X_c + C_y \times Y_c - C_b \leq 0$

$$0.1 \leq A_i \leq 2.0 \quad i = 1,2,3, \ldots.$$

Where $C_x, C_y, C_b$ are the constraint matrixes,

$A_x, A_y$ are the Objective matrixes,

$X_c, Y_c$ are the center points and the responses respectively

$A_i$ are *i* number of design variables

An optimized solution [ Xrsm ] is generated that satisfies the constraints. These variables are then stored in a design variable history matrix so that we can plot out the design convergence and history.

## 4.5. Check for convergence

Once the optimized solution [ Xrsm ] is generated for the first time, which point and another point from the Design matrix [ Xdata ] is added to the centre points [ Xc ]. The responses for these two new data points are once again generated through Ansys. A new rbf model is trained again from these center points and responses to get the new Coefficient Matrix [ c ].

The new model is then optimized again to get new values of the optimized solution [ Xrsm ]. These optimized values are then compared with its previous optimized values for convergence.

If it satisfies the convergence criteria then the program exits out of the loop and plots the design variable history convergence plots. In case the convergence criteria is not fulfilled then the program repeats the steps in the first and second paragraph until the convergence criteria are met.

## 4.6. Multiobjective Design Optimization

Once the above outlined processes are carried out for all the single objective cases, we save the center points [ Xc ] and the coefficient matrixes [ c ] for those cases. These matrices represent the approximated response of the single objective optimization for any unknown design variables.

We can now use these matrixes to simulate the RBF function and thereby simulating the responses .The matrixes are then coded as individual functions that return the responses. A multi objective optimization is then carried out by using these function to give the overall optimized value for the structure.

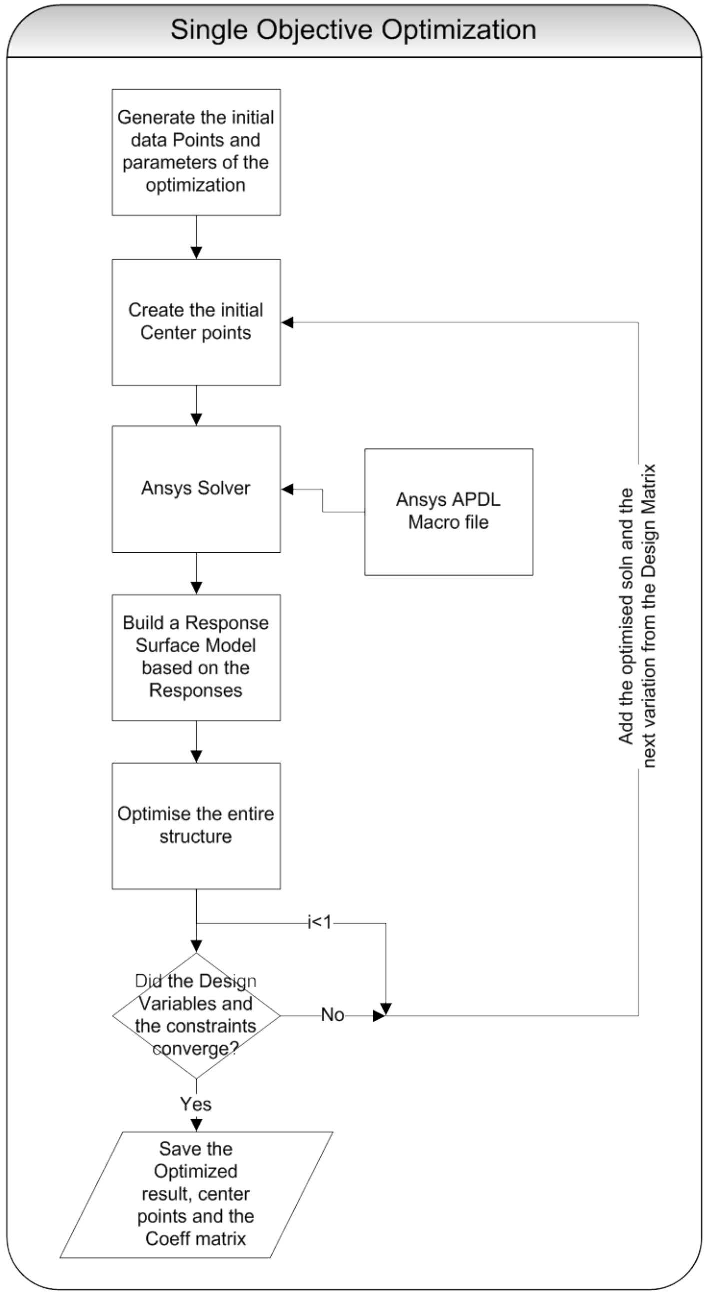



Figure 4-1 : Single Objective Optimization

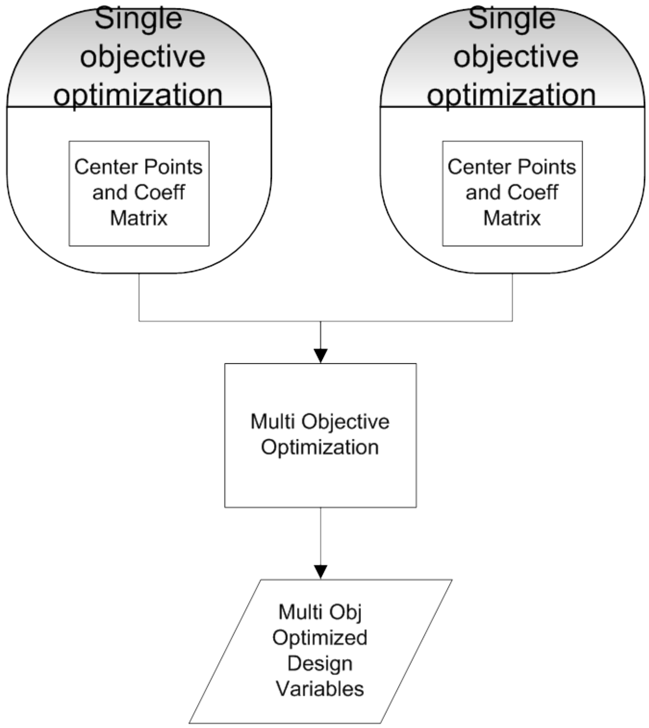



Figure 4-2 : Multi Objective Optimization

# CHAPTER 5

# IMPLEMENTATION OF RBF RESPONSE SURFACE METHODOLOGY

The following test cases demonstrates and verifies a methodology for multi objective optimization in Matlab using Response Surface Methodology. The Initial responses were calculated using Ansys and imported into Matlab for optimization. We have considered models from 2 design variable to 7 design variables. The structures designed consisted of Frame Elements, Surface Element and Solid Elements. The type of analysis carried out was Structural, Thermal and Modal. The test cases are described below. All of the test cases considered were kept as standardized as possible so that the results could be verified.

1. Static analysis of a 7 Truss element subjected to structural forces and analyzed for stresses and displacements.
2. Static analysis of a 9 Truss element subjected to structural forces and analyzed for stresses and displacements.
3. Multiobjective Static analysis of a 25 truss element subjected to structural forces and analyzed for stresses and displacements.
4. Multiobjective Static analysis of a plate with a hole subjected to structural pressure, Temperatures and analyzed for stresses and Heat Flux.

5.1. Static Analysis of a 7 Element truss model

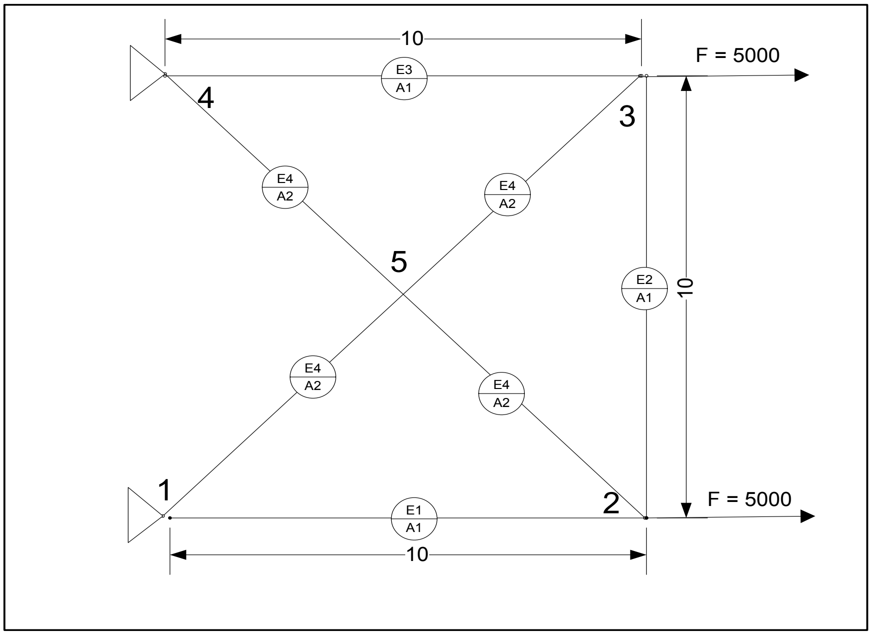



Figure 5-1 : 7 Element Truss Structure

A truss model is as shown below. It is a 7 Truss element model with 5 nodes. It is fixed at Node 1 and 4. A load is applied at Node 2 and 3 in the x direction to a magnitude of 5000 lbf. The lengths of the outer elements are 10 in. The outer 3 trusses have a common cross-sectional Area A1 while the inner trusses have a common cross-sectional Area A2.

The material properties of the steel structure is Young's Modulus of elasticity, $E = 2.973 \times 10^7\, psi$ , Poisson's ratio, $\upsilon = 0.334$ and Yield strength , $\sigma_Y = 7500\, psi$.

The objective of this analysis is to find the optimal cross-sectional Areas so that the total volume of the structure is minimized, while keeping the max stresses within the one third of the yield stress and the max displacement within the allowable displacement limits of 0.001 in.

### *5.1.1 Problem Formulation*

The Optimization analysis is formulated as below

Objective Function : Minimize the Total Volume

Subjected to: Von Mises stress, $\sigma_{VM} \leq 2500\ psi$

Max Displacements, $U_{x,y} \leq 0.001\ in$

Design Constraints: A1 and A2

Design Bounds : Upper limit of 2 $in^2$ and Lower limit of 0.1 $in^2$, i.e

$$0.1\ in^2 \leq A_1, A_2 \leq 2.0\ in^2$$

### *5.1.2 Problem Results and Tabulations*

The analysis is run with an Objective Convergence of 0.01 and an Constraint Convergence of 0.03. An Haltom model is developed first for about 1000 design points and then the initial 10 points are taken as the center points and analyzed for responses through Ansys Classic. The input is exported and the responses are imported through Ansys Classic Batch Mode. The displacements and von mises stresses of all the elements are taken into account as the responses. The center points for both the area groups after the analysis is given in Figure 5-2.

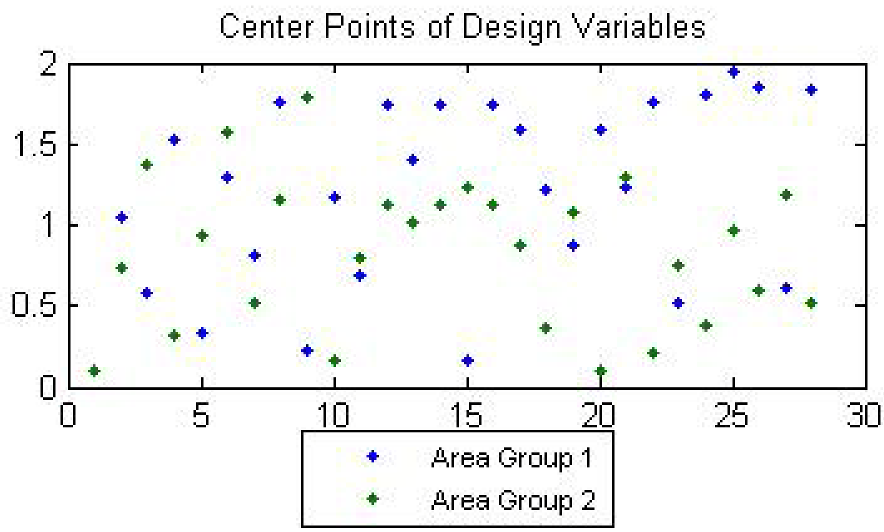



Figure 5-2 : Center Points of a 7 Element Structure

The results converged after 9 iterations with an optimal volume of $69.3621\ in^3$. The total numbers of center points considered were 28 as shown in Figure 5-2. The optimal values are tabulated in Table 5-1. The same optimization was run in Ansys Classic with the same convergence values to verify the results and the optimal values are tabulated in Table 5-1.

Table 5-1: Results for a 7 Truss element

| | Ansys Classic | Matlab |
|---|---|---|
| Iterations | 24 | 9 |
| Area Group 1 | $1.9656\ in^2$ | $1.8334\ in^2$ |
| Area Group 2 | $0.10393\ in^2$ | $0.5077\ in^2$ |
| Total Structure Volume | $61.908\ in^3$ | $69.3621\ in^3$ |
| Max Stress | $2497.5\ psi$ | $2500\ psi$ |
| Max Disp | $0.00084\ in$ | $0.0008409\ in$ |

It can be seen that the RBF approximation model gives better results than the Ansys Classic optimization model. The convergence plots for the Design Variables in Matlab are shown in Figure 5-3 and the Volume Convergence plots in Matlab are shown in Figure 5-4. The convergence plots for the Design Variables in Ansys are shown in Figure 5-6 and the Volume Convergence plots in Ansys are shown in Figure 5-5.

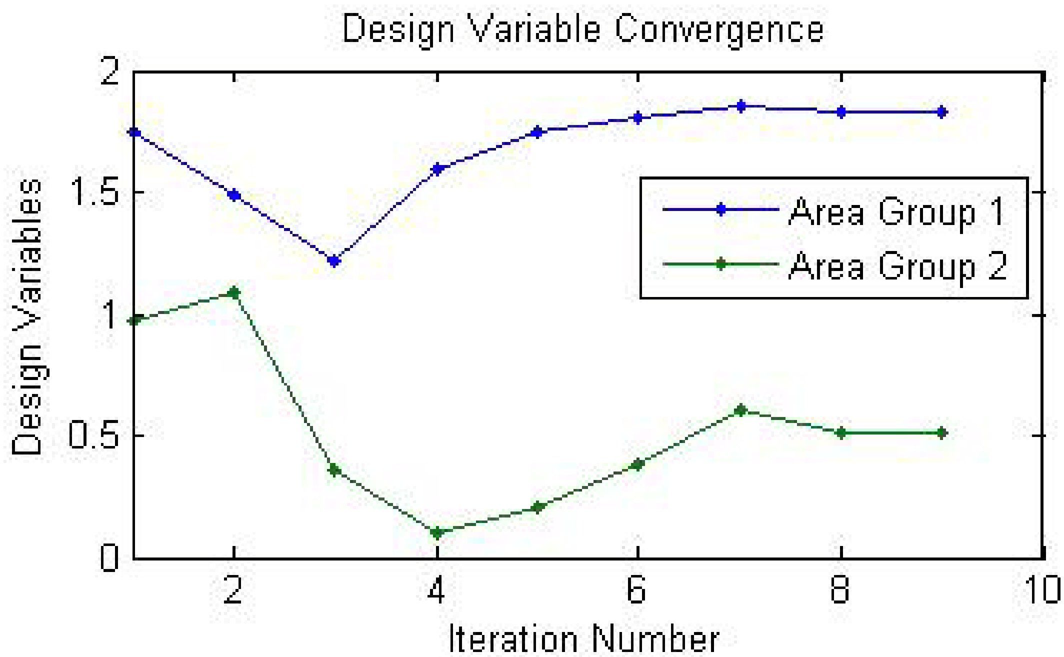



Figure 5-3 : Design Variable Convergence in Matlab

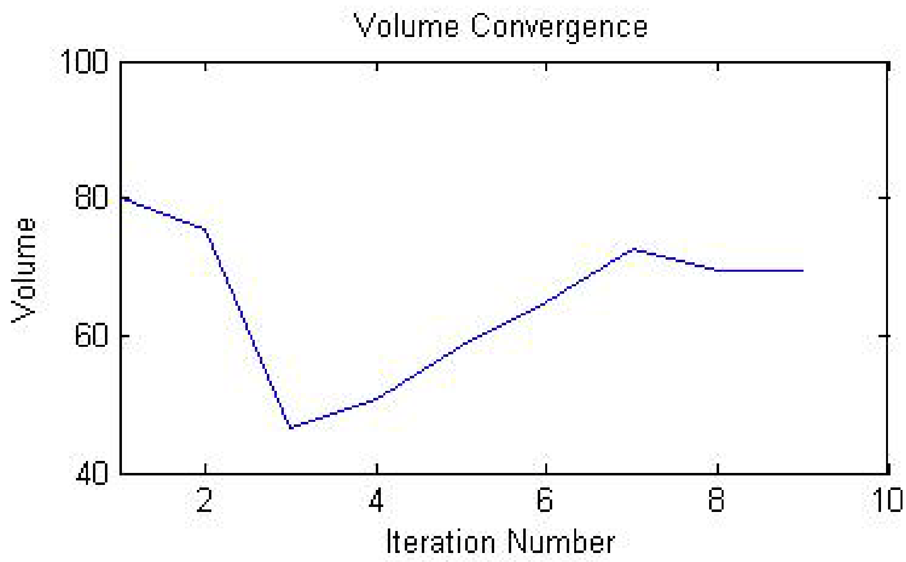



Figure 5-4 : Volume Convergence in Matlab

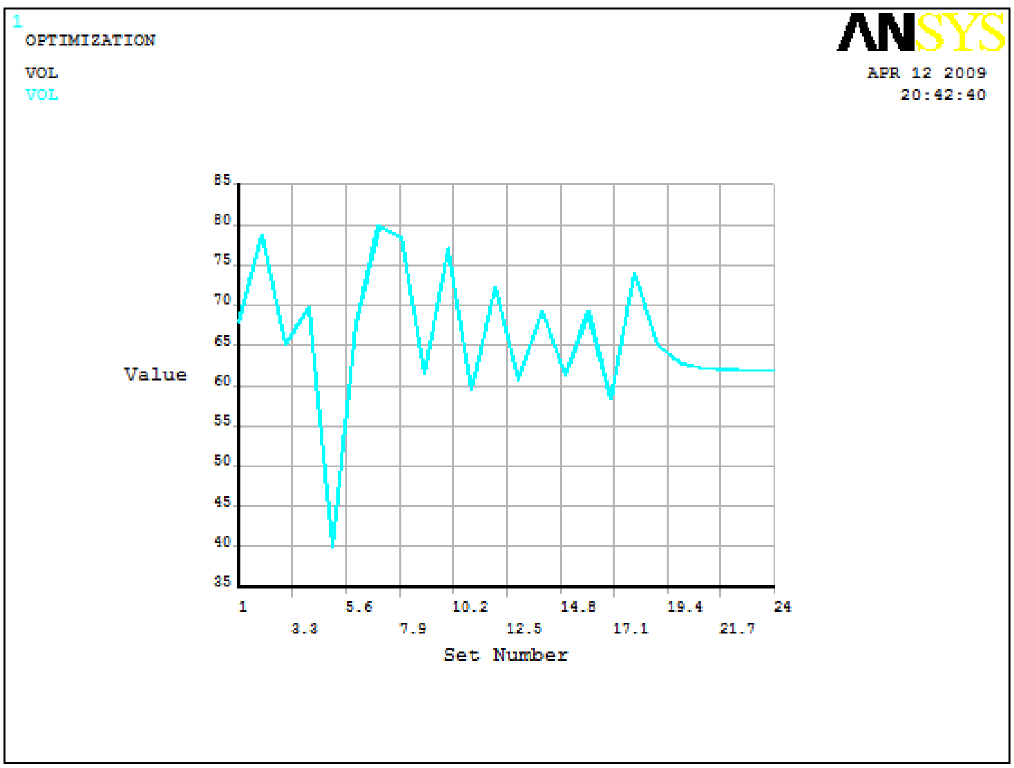



Figure 5-5 : Volume Convergence in Ansys

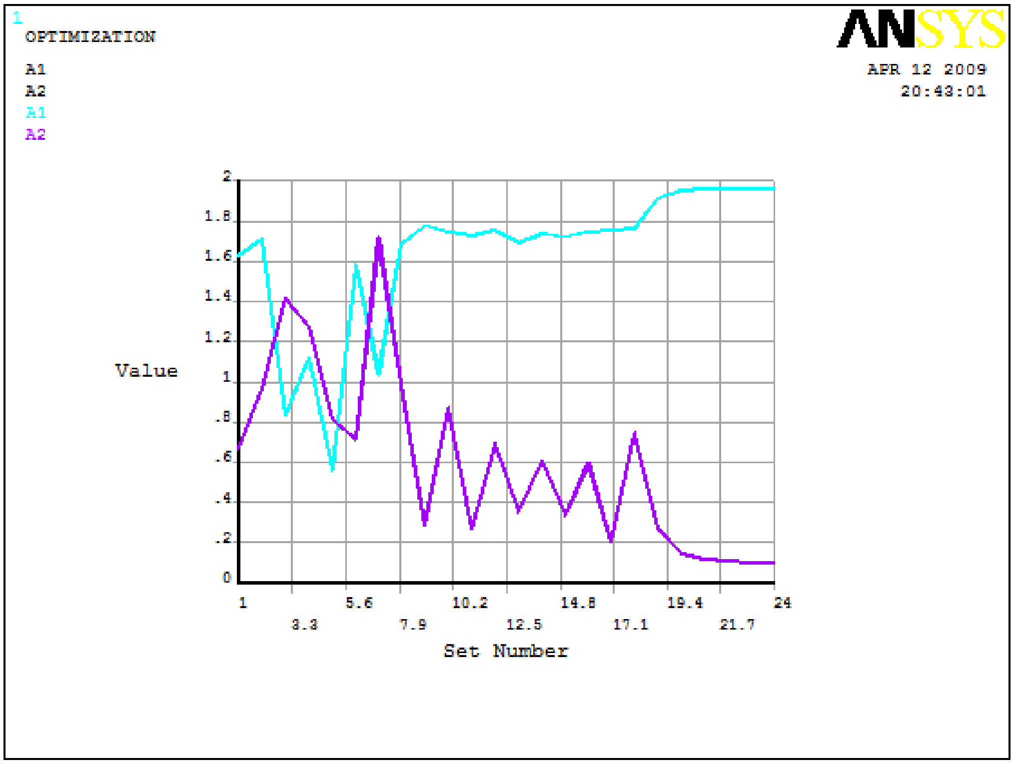



Figure 5-6 : Design Variables convergence in Ansys

5.2. Static Analysis of a 9 Truss element structure

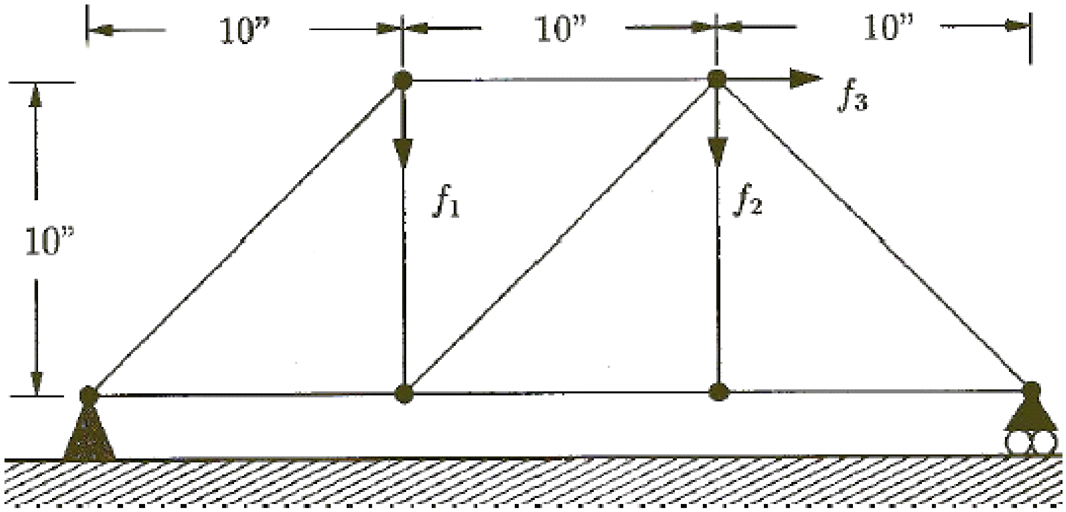



Figure 5-7 : 9 Truss Structure

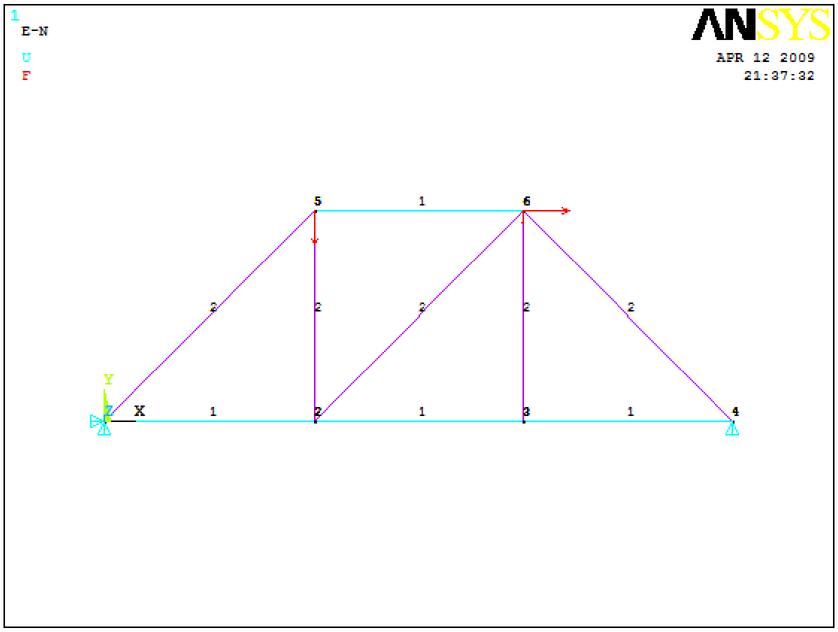



Figure 5-8 : 9 Truss Element with Boundary Conditions

A truss model is as shown below. It is a 9 Truss element model with 6 nodes. It is fixed at Node 1 in all directions and at Node 4 in UY direction. The three applied loads are $f_1 = -5000\ lbs$, $f_2 = -2000\ lbs$ and $f_3 = 7000\ lbs$. The lengths of the top and bottom elements are 10 in while the interior elements have a vertical height of 10 in. There are two area groups in the structure; the upper and lower trusses have a common cross-sectional Area A1 while the inner trusses have a common cross-sectional Area A2.

The material properties of the steel structure is Young's Modulus of elasticity, $E = 2.973 \times 10^7\ psi$ , Poisson's ratio, $\upsilon = 0.334$ and Yield strength , $\sigma_Y = 7500\ psi$.

The objective of this analysis is to find the optimal cross-sectional Areas so that the total volume of the structure is minimized, while keeping the max stresses within the allowable stress limits of 10000psi and the max displacement within the allowable displacement limits of 0.05 in.

*5.2.1 Problem Formulation*

The Optimization analysis is formulated as below

Objective Function : Minimize the Total Volume

Subjected to: Von Mises stress, $\sigma_{VM} \leq 10000\ psi$

Max Displacements, $U_{x,y} \leq 0.05\ in$

Design Constraints: A1 and A2

Design Bounds : Upper limit of 2 $in^2$ and Lower limit of 0.1 $in^2$, i.e

$$0.1\ in^2 \leq A_1, A_2 \leq 2.0\ in^2$$

*5.2.2 Problem Results and Tabulations*

The analysis is run with an Objective Convergence of 0.01 and an Constraint Convergence of 0.03. An Haltom model is developed first for about 1000 design points and then the initial 10 points are taken as the center points and analyzed for responses through Ansys Classic. The input is exported and the responses are imported through Ansys Classic Batch Mode. The displacements and von mises stresses of all the elements are taken into account as the responses. The center points for both the area groups after the analysis is given below.

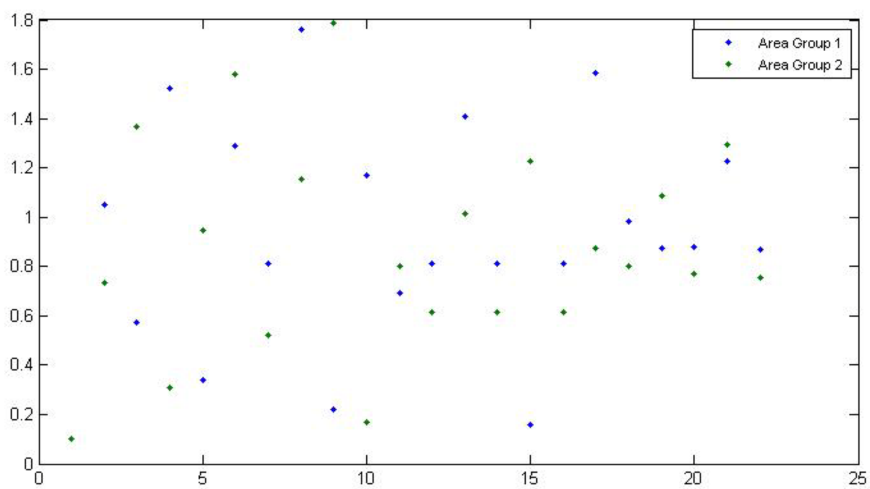



Figure 5-9 : Center Points of a 9 Element Structure

The results converged after 6 iterations with an optimal volume of $81.7657\ in^3$. The total numbers of center points considered were 22 as shown in Figure 5-9. The optimal values are tabulated in Table 5-2. The same optimization was run in Ansys Classic with the same convergence values to verify the results and the optimal values are tabulated in Table 5-2.

Table 5-2: Results for a 9 Truss element

| | Ansys Classic | Matlab |
|---|---|---|
| Iterations | 14 | 6 |
| Area Group 1 | $0.87313\ in^2$ | $0.8665\ in^2$ |
| Area Group 2 | $0.755\ in^2$ | $0.7546\ in^2$ |
| Total Structure Volume | $82.057\ in^3$ | $81.7657\ in^3$ |
| Max Stress | $9926\ psi$ | $10000\ psi$ |
| Max Disp | $0.012125\ in$ | $0.0075\ in$ |

It can be seen that the RBF approximation model gives better results than the Ansys Classic optimization model. The convergence plots for the Design Variables in Matlab are shown in Figure 5-10 and the Volume Convergence plots in Matlab are shown in Figure 5-11. The convergence plots for the Design Variables in Ansys are shown in Figure 5-13 and the Volume Convergence plots in Ansys are shown in Figure 5-12.

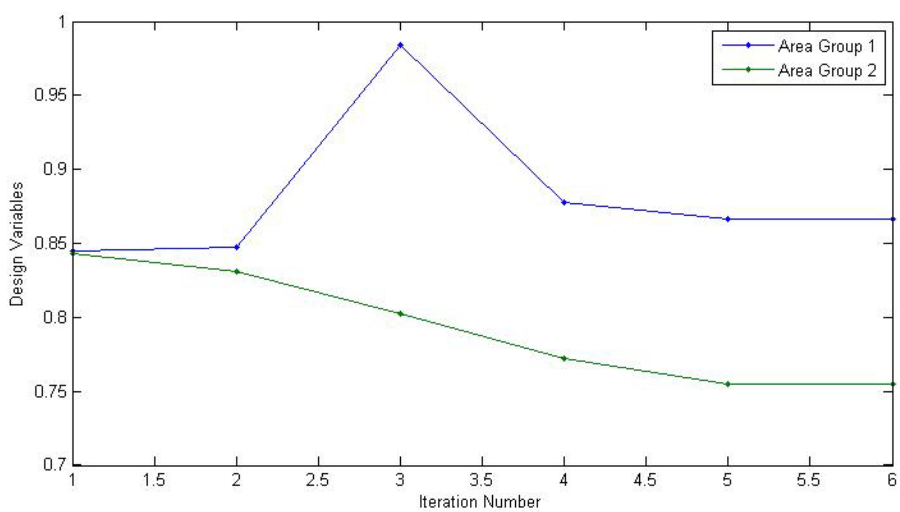



Figure 5-10 : Design Variable Convergence in Matlab

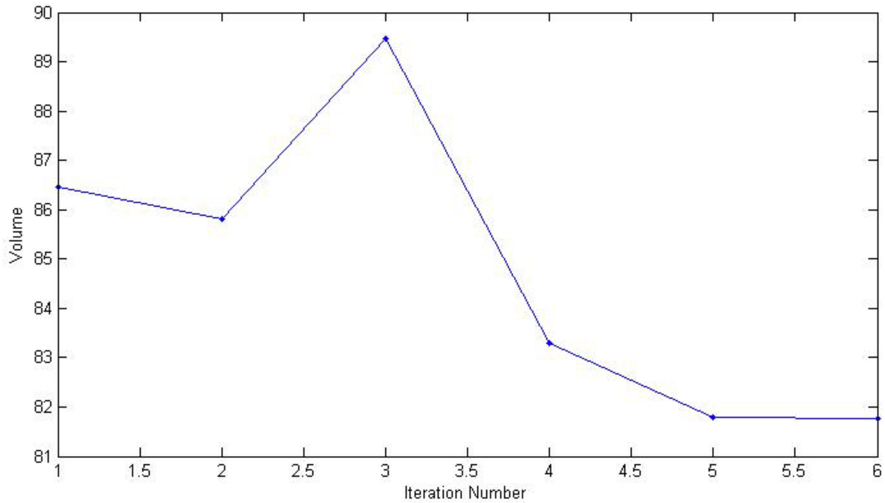



Figure 5-11 : Total Volume Convergence in Matlab

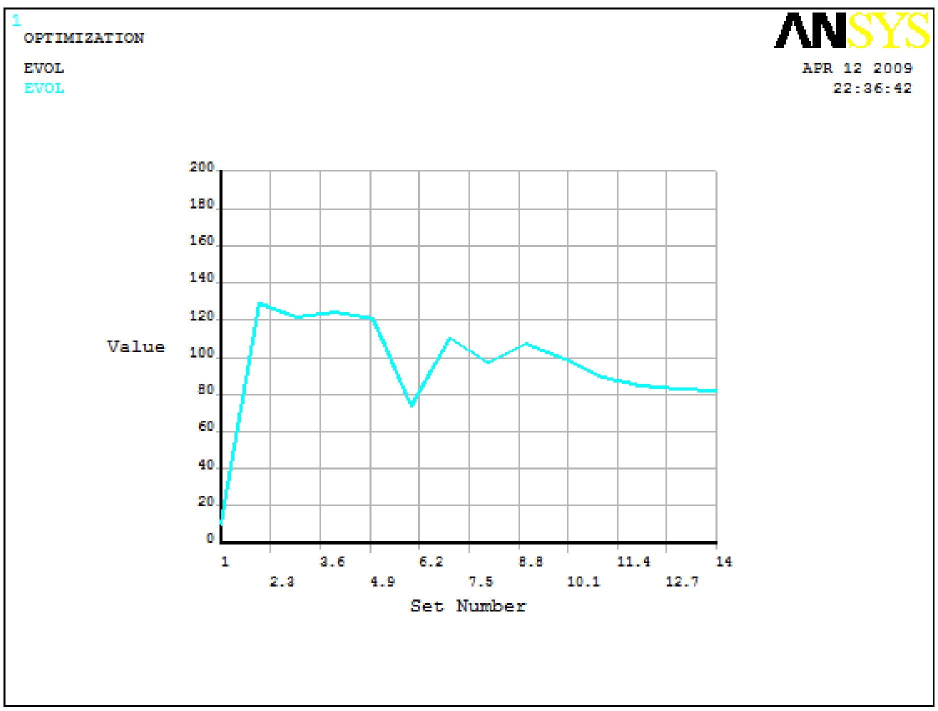



Figure 5-12 : Total Volume Convergence in Ansys

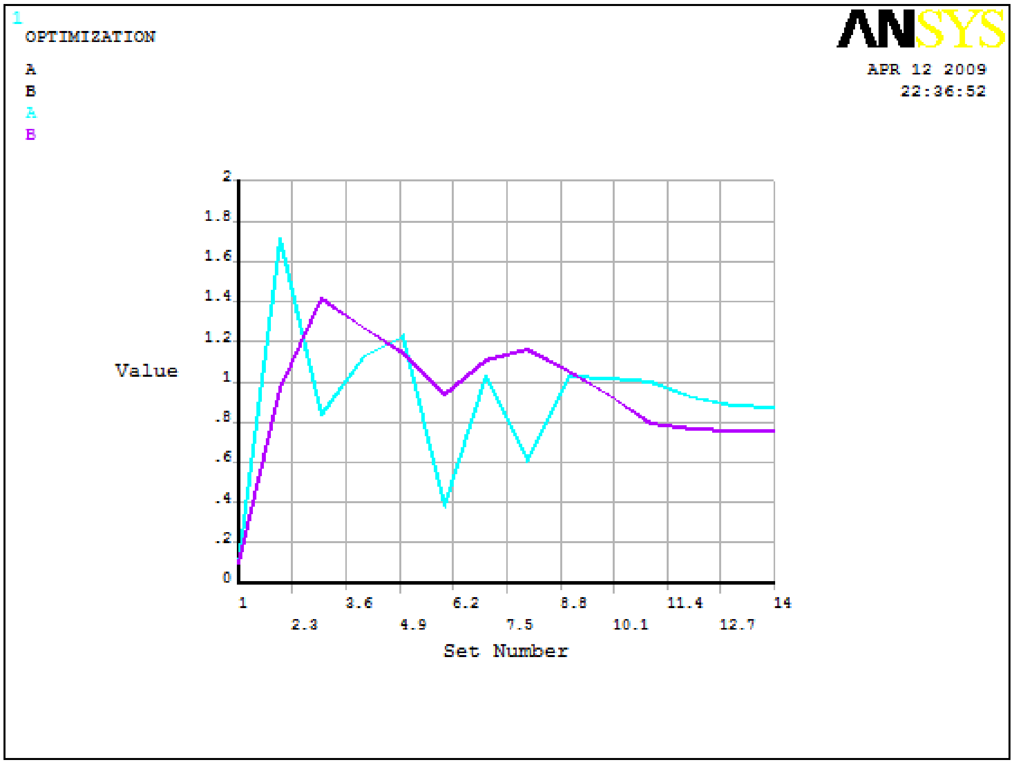



Figure 5-13 : Design Variable Convergence in Ansys

5.3. Multi Objective optimization of a 25 truss element Structure

The model chosen is a 25 truss element structure as shown below.

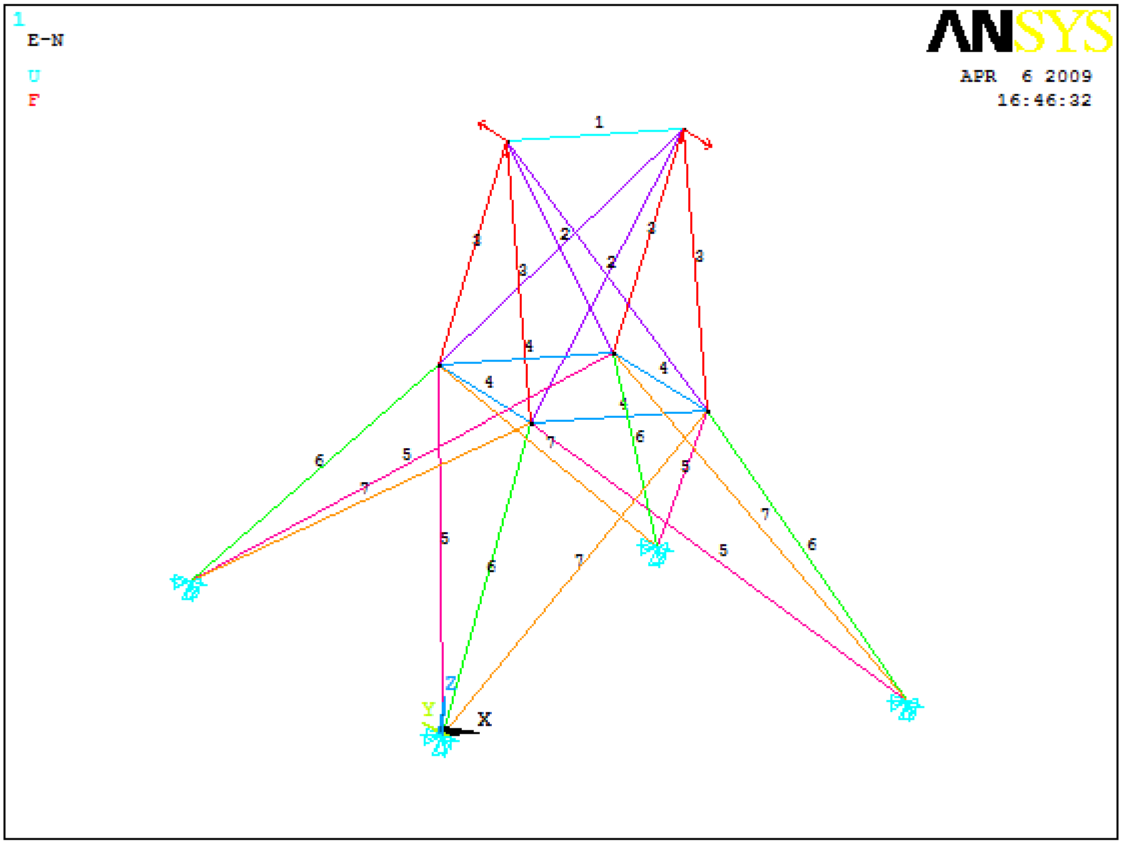



Figure 5-14: Area Groups of a 25 Bar Truss Structure

The 25 truss structure above has a 7 Area groups which are to be optimized. The different area groups are shown in different colors as shown in Figure 5-14. The entire structure is fixed at the base and four forces at the opposite end of the topmost truss. The structure is loaded to a force of 5000 at both the ends and 20000 in the horizontal plane but in opposite directions.

The material properties of the Aluminum structure is Young's Modulus of elasticity, $E = 1.048 \times 10^7 \, psi$ , Poisson's ratio, $\upsilon = 0.334$.

Two test cases defined are firstly, the structure is analyzed with all the loads for static conditions and secondly, the structure is again analyzed for thermal stresses with the same boundary conditions. The test cases are discussed in detail below.

*5.3.1 Stresses with Static Loading*

The objective of this analysis is to find the optimal cross-sectional Areas so that the total volume of the structure is minimized, while keeping the max stresses within the allowable stress limits of 40000 psi .

Problem Formulation

The Optimization analysis is formulated as below

Objective Function : Minimize the Total Volume

Subjected to: Von Mises stress, $\sigma_{VM} \leq 40000 \, psi$

Design Constraints: A1 to A2

Design Bound : Upper limit of 2 $in^2$ and Lower limit of 0.1 $in^2$, i.e

$$0.1 \, in^2 \leq A_1, A_2, A_3, A_4, A_5, A_6, A_7 \leq 2.0 \, in^2$$

Problem Results and Tabulations

The analysis is run with an Objective Convergence of 0.01 and an Constraint Convergence of 0.01. An Haltom model is developed first for about 1000 design points and then the initial 10 points are taken as the center points and analyzed for responses through Ansys Classic. The input is exported and the responses are imported through Ansys Classic Batch Mode. The displacements and von mises stresses of all the elements are taken into account as the responses. The center points for both the area groups after the analysis is given below.

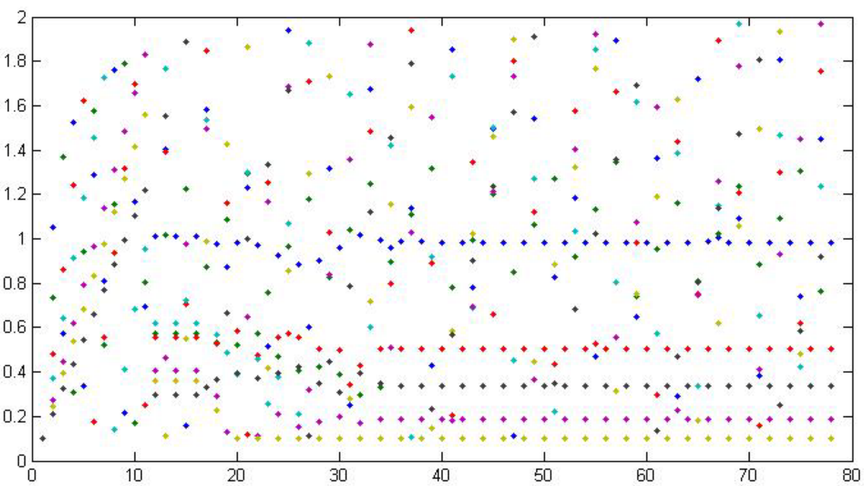



Figure 5-15 : Center Points of Design Variables

The results converged after 34 iterations with an optimal volume of $927.615\ in^3$. The total numbers of center points considered were 78 as shown in Figure 5-15. The optimal values are tabulated in Table 5-3. The same optimization was run in Ansys Classic with the same convergence values to verify the results and the optimal values are tabulated in Table 5-3.

Table 5-3: Results of Stresses of 25 Truss Structure

| | Matlab | Ansys |
|---|---|---|
| Iterations | 34 | 45 |
| X1 | 0.9845 $in^2$ | 0.10389 $in^2$ |
| X2 | 0.3364 $in^2$ | 0.34060 $in^2$ |
| X3 | 0.5036 $in^2$ | 0.53032 $in^2$ |
| X4 | 0.1 $in^2$ | 0.10414 $in^2$ |

Table 5.3 – *Continued*

| | | |
|---|---|---|
| X5 | 0.1883 $in^2$ | 0.21595 $in^2$ |
| X6 | 0.1 $in^2$ | 0.10386 $in^2$ |
| X7 | 0.3356 $in^2$ | 0.40915 $in^2$ |
| Volume | 927.6150 $in^3$ | 951.76 $in^3$ |
| Max Stress | 40000 psi | 40002 psi |

The center points and the Coeff matrix was a 78 * 7 and 78 * 44 respectively. They are stored separately in a Matlab MAT file for further multiobjective analysis.

It can be seen that the RBF approximation model gives better results than the Ansys Classic optimization model. The convergence plots for the Design Variables in Matlab are shown in Figure 5-16 and the Volume Convergence plots in Matlab are shown in Figure 5-17. The convergence plots for the Design Variables in Ansys are shown in Figure 5-19 and the Volume Convergence plots in Ansys are shown in Figure 5-18.

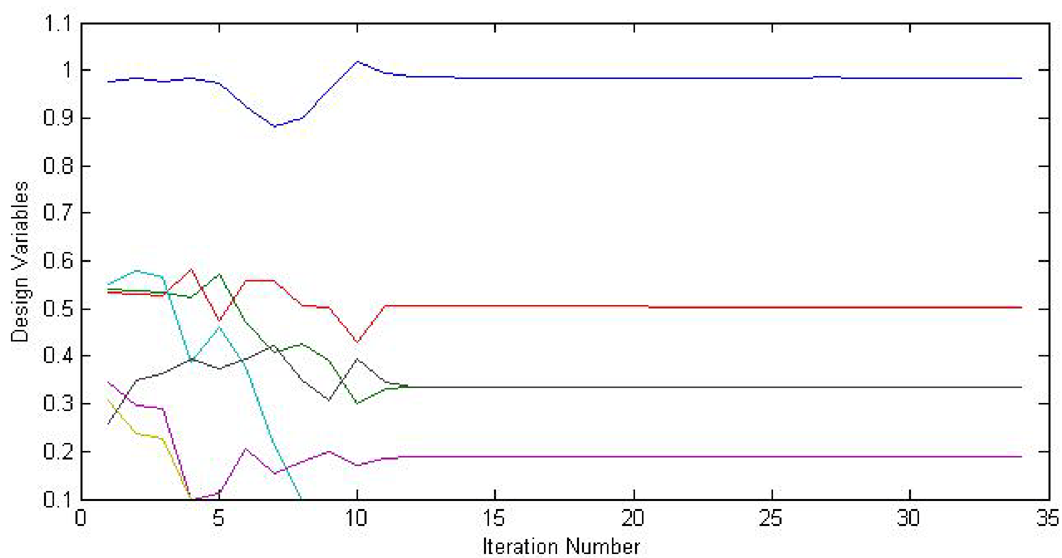



Figure 5-16: DV Conv in Matlab for Stress

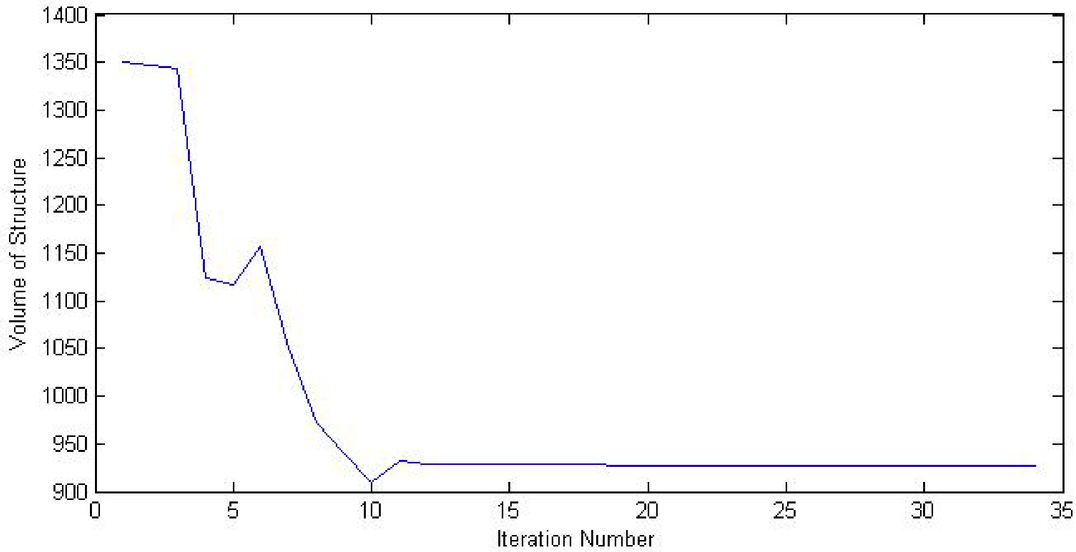



Figure 5-17 : Vol Conv in Matlab for Stress

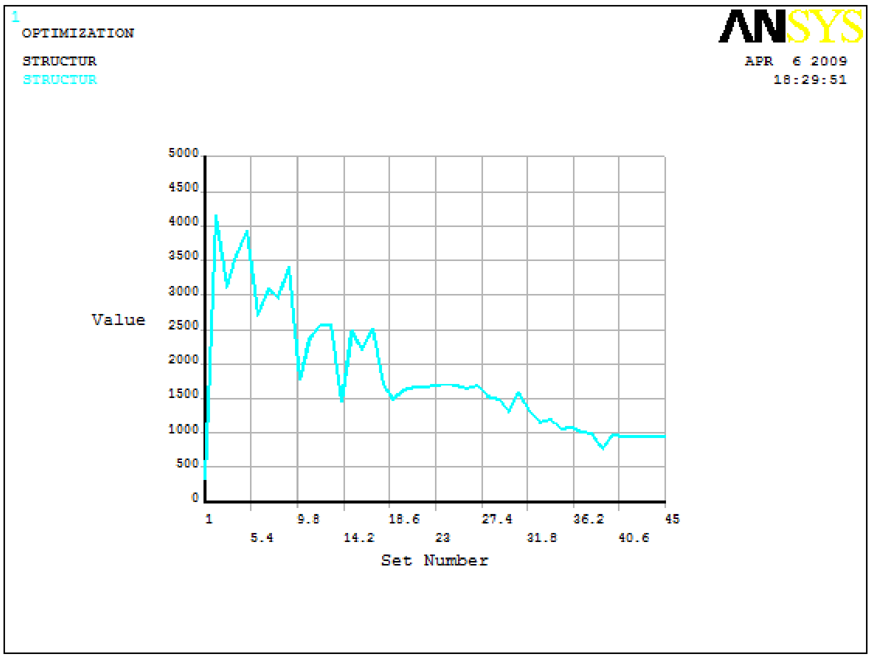



Figure 5-18 : Vol Conv in Ansys For Stress

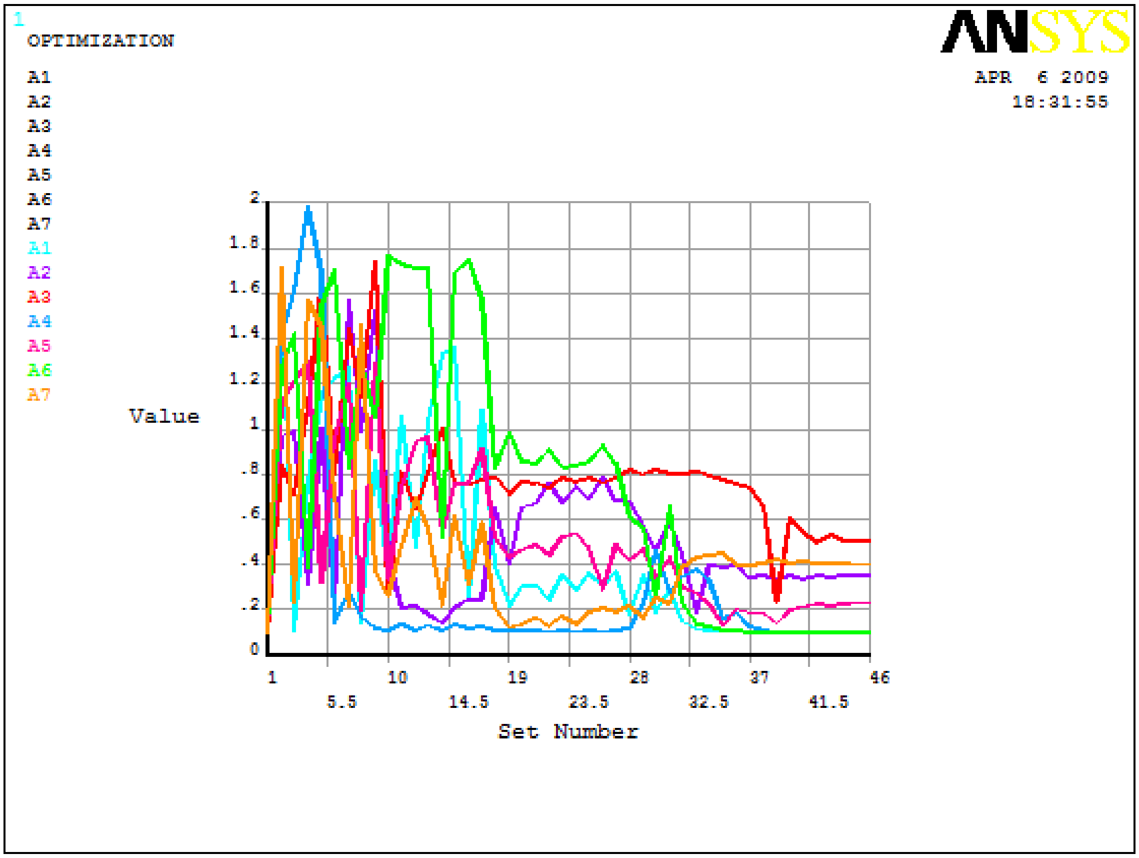



Figure 5-19 : DV Conv in Ansys for Stress

### *5.3.2 Thermal Stresses with Static Loading*

The objective of this analysis is to find the optimal cross-sectional Areas so that the total volume of the structure is minimized, while keeping the max thermal stresses within the allowable stress limits of 40000 psi .

Problem Formulation

The Optimization analysis is formulated as below

Objective Function : Minimize the Total Volume

Subjected to: Thermal stress, $\sigma_{TH} \leq 40000$ psi

Design Variables: A1 to A2

Design Bounds : Upper limit of 2 $in^2$ and Lower limit of 0.1 $in^2$,

i.e $0.1\ in^2 \leq A_1, A_2, A_3, A_4, A_5, A_6, A_7 \leq 2.0\ in^2$

Problem Results and Tabulations

The analysis is run with an Objective Convergence of 0.01 and an Constraint Convergence of 0.01. An Haltom model is developed first for about 1000 design points and then the initial 10 points are taken as the center points and analyzed for responses through Ansys Classic. The input is exported and the responses are imported through Ansys Classic Batch Mode. The displacements and von mises stresses of all the elements are taken into account as the responses. The center points for both the area groups after the analysis is given below.

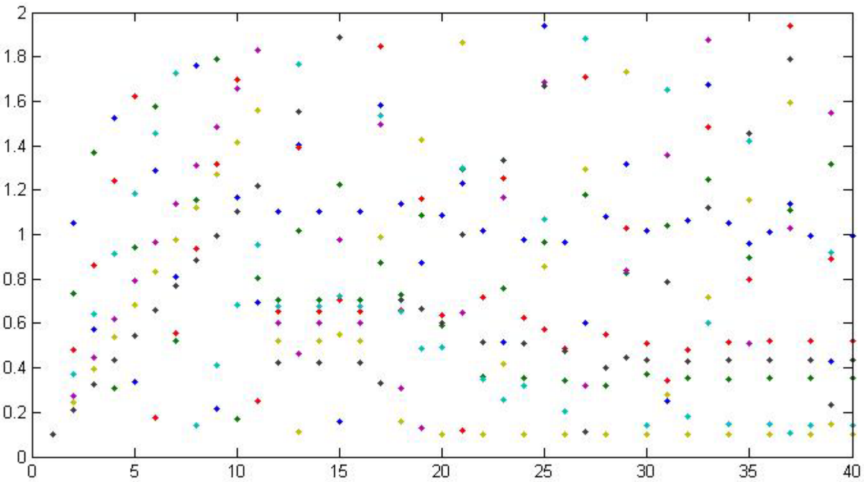



Figure 5-20 : Center Points for Thermal Stresses

The results converged after 15 iterations with an optimal volume of $965.6484\ in^3$. The total numbers of center points considered were 40 as shown in Figure 5-20. The optimal values are tabulated in Table 5-4. The same optimization was run in Ansys Classic with the same convergence values to verify the results and the optimal values are tabulated in Table 5-4.

Table 5-4 : Results of Thermal Stresses of 25 Truss Structure

| | Matlab | Ansys |
|---|---|---|
| Iterations | 15 | 50 |
| X1 | 0.9885 $in^2$ | 0.10386 $in^2$ |
| X2 | 0.3550 $in^2$ | 0.34978 $in^2$ |
| X3 | 0.5210 $in^2$ | 0.56377 $in^2$ |
| X4 | 0.1417 $in^2$ | 0.2595 $in^2$ |
| X5 | 0.1 $in^2$ | 0.17475 $in^2$ |
| X6 | 0.1 $in^2$ | 0.10475 $in^2$ |
| X7 | 0.4352 $in^2$ | 0.36099 $in^2$ |

Table 5.4 – *Continued*

| | | |
|---|---|---|
| Volume | 965.6484 $in^3$ | 953.17 $in^3$ |
| Max Stresses | 40000 psi | 39989 psi |

The center points and the Coeff matrix was a 40 * 7 and 40 * 44 respectively. They are stored separately in a Matlab MAT file for further multiobjective analysis.

It can be seen that the RBF approximation model gives better results than the Ansys Classic optimization model. The convergence plots for the Design Variables in Matlab are shown in Figure 5-21 and the Volume Convergence plots in Matlab are shown in Figure 5-22. The convergence plots for the Design Variables in Ansys are shown in Figure 5-24 and the Volume Convergence plots in Ansys are shown in Figure 5-23.

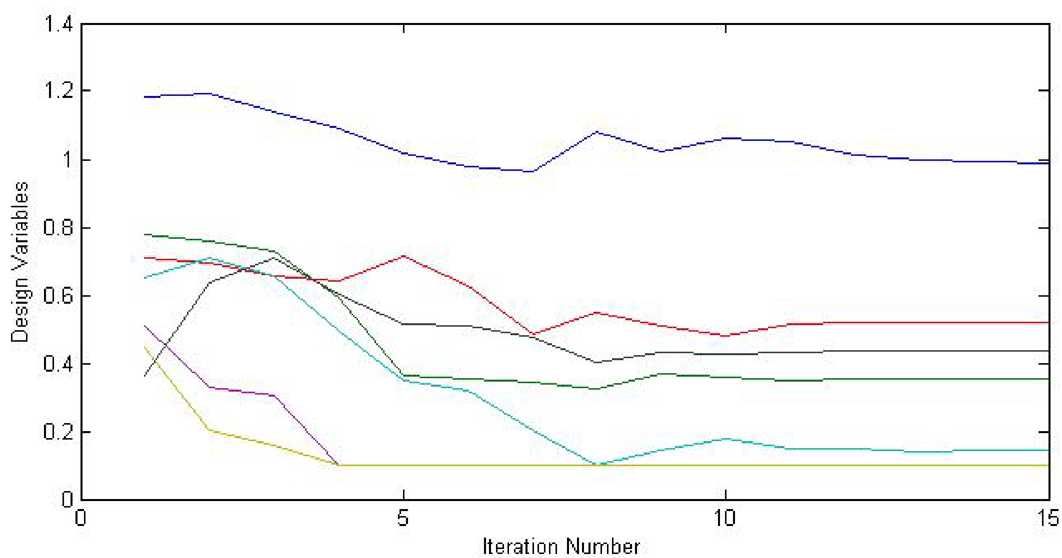



Figure 5-21: DV Conv in Matlab for Thermal Stresses

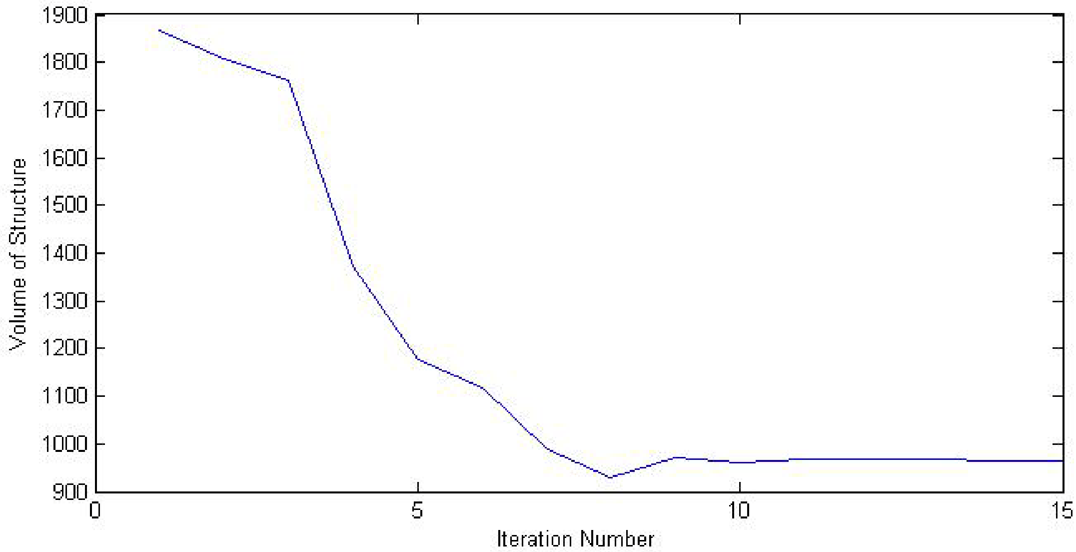



Figure 5-22 : Vol Conv in Matlab for Thermal Stresses

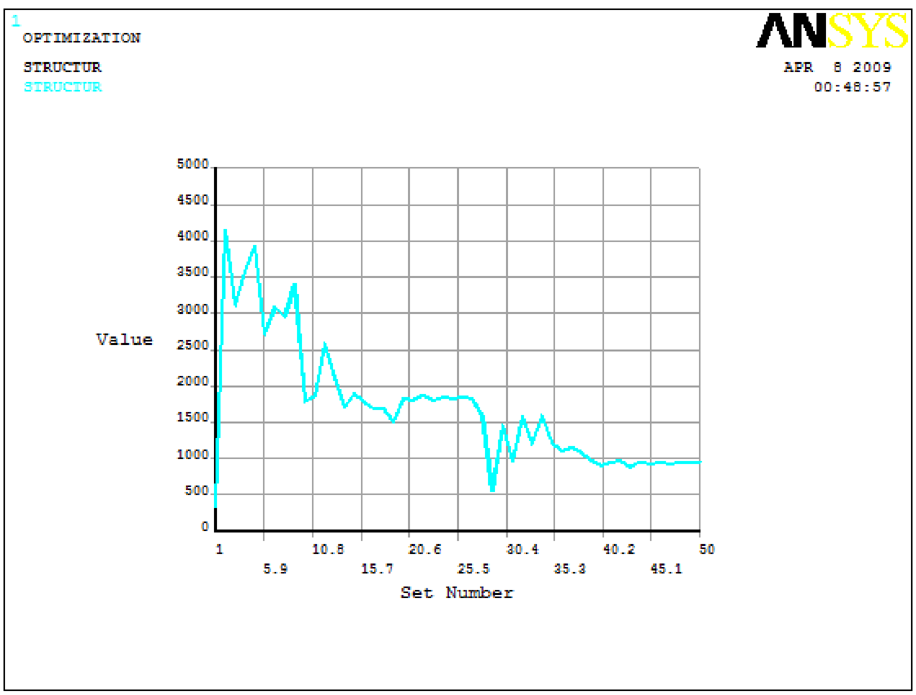



Figure 5-23 : Vol Conv in Ansys for Thermal Stresses

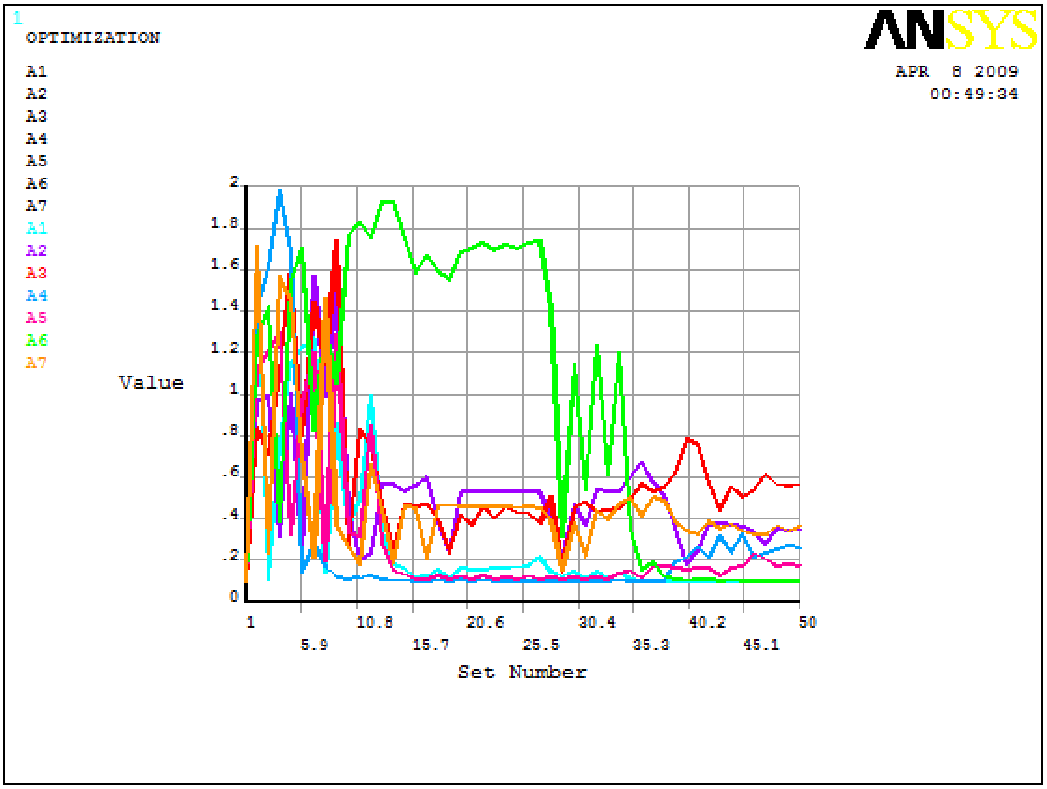



Figure 5-24 : DV Conv in Ansys for Thermal Stresses

*5.3.3 Multi Objective Optimization*

After we finish running both the test cases, the center points and the coeffecients for each is saved and multi objective min max algorithm is run that gives the minimum of the worst case among both the loading conditions.

Problem Formulation

The Optimization analysis is formulated as below

Objective Function : Minimize the Total Volume

Subjected to: : Von Mises stress, $\sigma_{VM} \leq 40000\, psi$

Thermal stress, $\sigma_{TH} \leq 40000\, psi$

Design Variables: A1 to A2

Design Limits : Upper limit of 2 $in^2$ and Lower limit of 0.1 $in^2$,

i.e $0.1\, in^2 \leq A_1, A_2, A_3, A_4, A_5, A_6, A_7 \leq 2.0\, in^2$

Problem Results and Tabulations

The Multiobjective Program converged with the following optimal Values

| DV | Area ($in^2$) |
|---|---|
| A1 | 0.96 |
| A2 | 0.36 |
| A3 | 0.5 |
| A4 | 0.13 |
| A5 | 0.16 |
| A6 | 0.1 |
| A7 | 0.44 |

With a final structure volume of 1003.2 $in^3$, as shown in Figure 5-25.

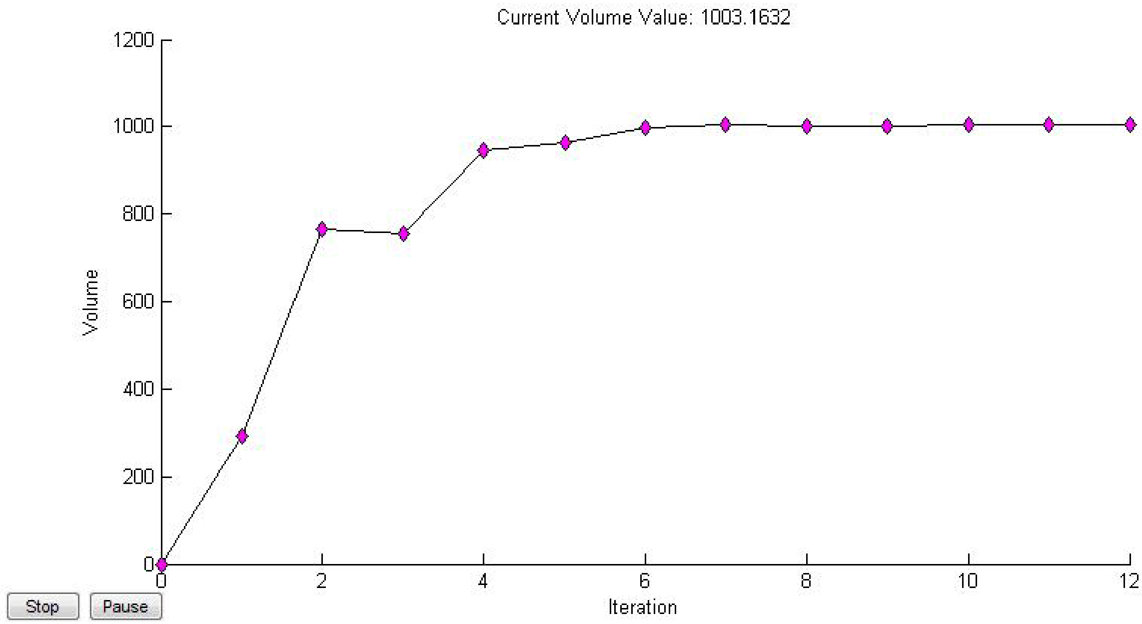



Figure 5-25 :  Multi Obj Vol Convergence

The final Design variables were once again analysed with Ansys and the results were verified with a maximum tensile stress of 39673 Mpa, a minimum compressive stress of 39394 Mpa and a maximum thermal tensile stress of 39094 Mpa, a minimum thermal compressive stress of 40000 Mpa.

5.4. Multi objective optimization of a plate with a hole

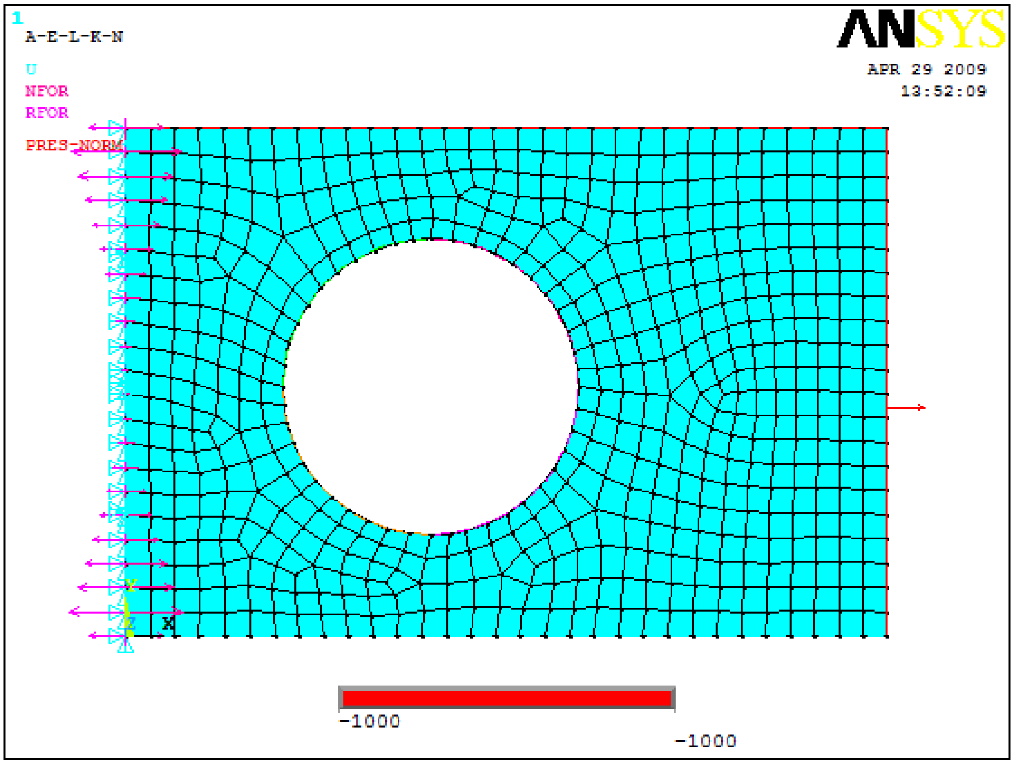



Figure 5-26 : Plate with Hole

*5.4.1 Structural Stresses with Static Loading*

A plate element model with a hole is as shown above in 5-26. It has length of 30 in, height of 20 in and thickness of 1 in. The design variables that are to be varied in this model are the x - coordinate (x), y - coordinate (y) and the Radius (R) of the circle in the center. Hence there are three design variable group that has to be varied. It is completely fixed at the left hand vertical side and a pressure and a force is applied on the right hand vertical side.

The material properties of the steel plate is Young's Modulus of elasticity, $E = 29 \times 10^6\ psi$ , Poisson's ratio, $\upsilon = 0.31$, Material Density , $\delta = 0.285\ lbs/in^3$, thermal conductivity, $k = 30 \frac{btu}{ft*f}$ and yield strength, $\sigma_y = 36000\ psi$.

The objective of this analysis is to find the optimal Design Variable values so that the total volume of the structure is minimized, and the maximum temperature on the outer edge is minimized while keeping the max stress within one third of the yield stress i.e, 12000 psi. The main difference between this model and the truss model's is that here one maximum stress value is considered while in the truss model's all the stresses at the elements and the displacement at the nodes are considered. Since the responses are significantly lesser than the other models, it is expected that the results converge much more sooner than the other models.

## Problem Formulation

The Optimization analysis for the structural case is formulated as below

Objective Function : Minimize the Total Volume

Stress Constraint: Von Mises stress, $\sigma_{VM} \leq 12000\, psi$

Geometric Constraint: $x - r \geq 2, y - r \geq 2, x + r \leq 28, y + r \leq 18$

Design Variables: x coordinate, y coordinate, Radius

Bounds : Lower limit of [2 2 1] in and Upper limit of [28 18 8] in

## Problem Results and Tabulations

The analysis is run with an Objective Convergence of 0.01 and an Constraint Convergence of 0.01. An Haltom model is developed first for about 1000 design points and then the initial 20 points are taken as the center points and analyzed for responses through Ansys Classic. The input is exported and the responses are imported through Ansys Classic Batch Mode. Only the max displacements and max von mises stress of all the elements are taken into account as the responses. The normalized center points for all the design variable groups after the analysis is given in Figure 5-27.

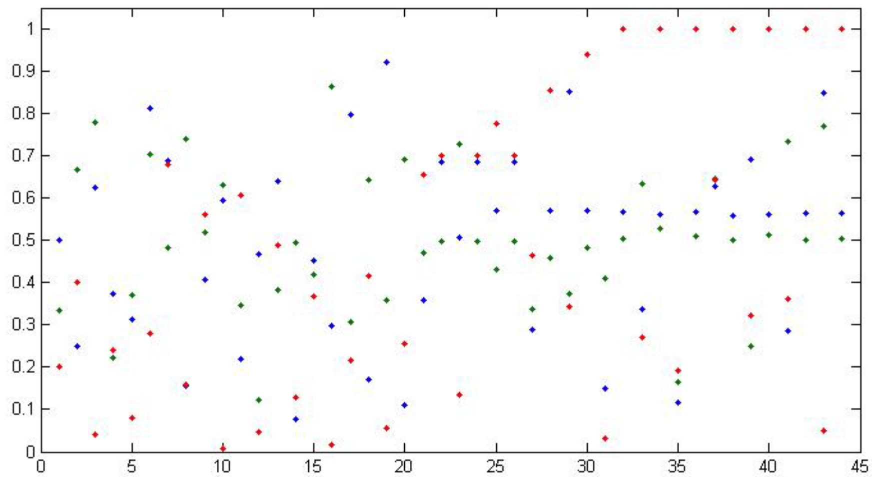



Figure 5-27 : Center Points

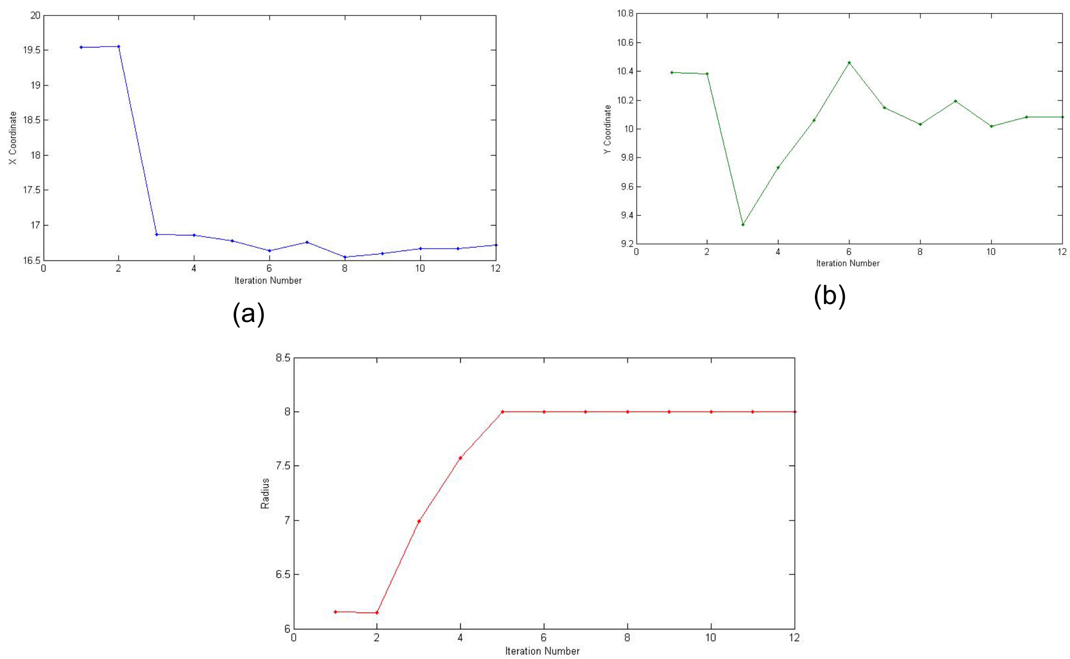
The results converged after 12 iterations with an optimal volume of $399\ in^3$. The total numbers of center points considered were 36 as shown in Figure 5-27. The optimal values are tabulated in Table 5-5. The same optimization was compared with the known values to verify the results and the optimal values are tabulated in Table 5-5.

Table 5-5: Results of Stresses of plate

| | Matlab | Known Values |
|---|---|---|
| Iterations | 12 | 22 |
| X coordinate | 16.7 in | 16.35 in |
| Y coordinate | 10 in | 10 in |
| Radius | 8 in | 8 in |
| Volume | 399 $in^3$ | 398.9 $in^3$ |
| Max Stress | 11576 psi | 10610 psi |

The center points and the Coeff matrix was a 44 * 3 and 44 * 2 respectively. They are stored separately in a Matlab MAT file for further multiobjective analysis.

It can be seen that the RBF approximation model compares very well with the known values. The convergence plots for the Design Variables in Matlab are shown in Figure 5-28 and the Volume Convergence plots in Matlab are shown in Figure 5-29.




(a) (b) (c)

Figure 5-28 : Convergence of Design Variables in Matlab by (a) x coordinate, (b) y coordinate and (c) Radius of the circle

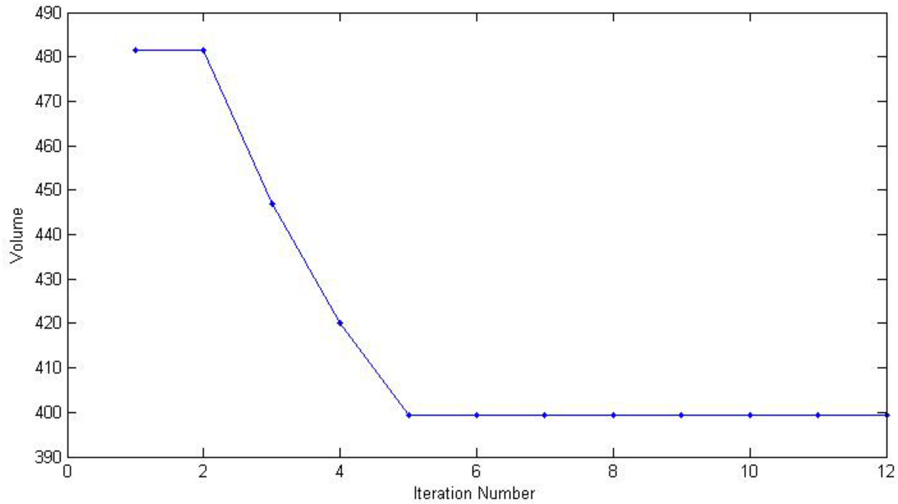



Figure 5-29 : Vol Convergence in Matlab

*5.4.2 Thermal Optimization with Static Loading*

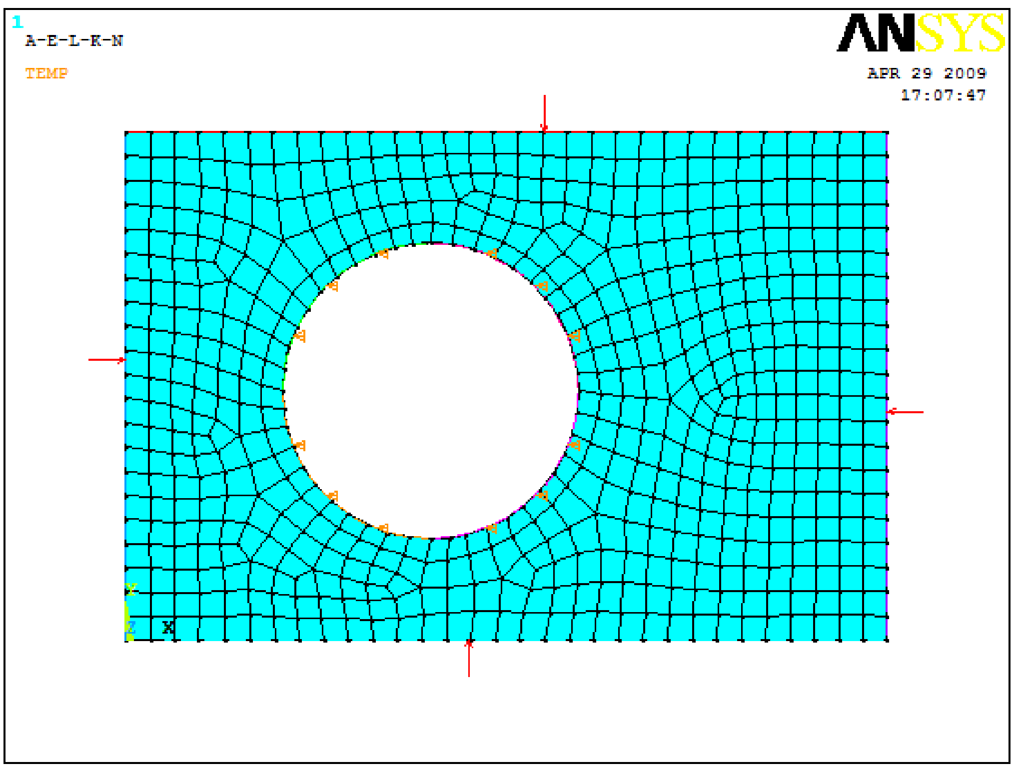



Figure 5-30 : Thermal Boundary Conditions

A plate element model with a hole is as shown above in Figure 5-30. It has length of 30 in, height of 20 in and thickness of 1 in. The design variables that are to be varied in this model are the x - coordinate (x), y - coordinate (y) and the Radius (R) of the circle in the center. Hence there are three design variable group that has to be varied. It is completely fixed at the left hand vertical side and a pressure and a force is applied on the right hand vertical side.

The material properties of the steel plate is Young's Modulus of elasticity, $E = 29 \times 10^6\ psi$ , Poisson's ratio, $\upsilon = 0.31$, Material Density , $\delta = 0.285\ lbs/in^3$, thermal conductivity, $k = 30\frac{btu}{ft*f}$ and yield strength, $\sigma_y = 36000\ psi$.

The objective of this analysis is to find the optimal Design Variable values so that the Max Outer Temperature is minimized while keeping the max heat flux within 500 Btu/Hr .

Problem Formulation

The Optimization analysis for the structural case is formulated as below

Objective Function : Minimize the Max Outer Temperature

Heat Flux Constraint: Max Heat Flux $\leq 500\ Btu/Hr$

Geometric Constraint: $x - r \geq 2, y - r \geq 2, x + r \leq 28, y + r \leq 18$

Design Variables: x coordinate, y coordinate, Radius

Bounds : Lower limit of [2 2 1] in and Upper limit of [28 18 8] in

Problem Results and Tabulations

The analysis is run with an Objective Convergence of 0.01 and an Constraint Convergence of 0.01. An Haltom model is developed first for about 1000 design points and then the initial 20 points are taken as the center points and analyzed for responses through Ansys Classic. The input is exported and the responses are imported through Ansys Classic Batch Mode. Only the max temperature of all the outer elements are taken into account as the responses. The normalized center points for all the design variable groups after the analysis is given in Figure 5-31.

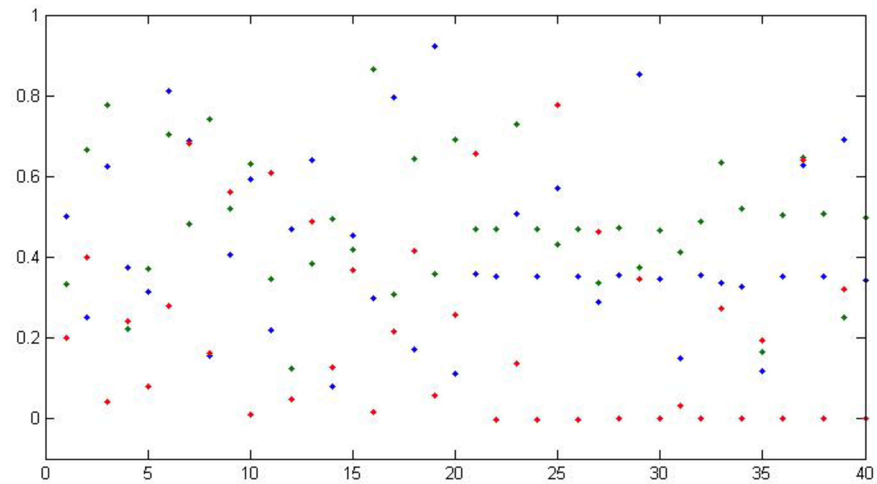


Figure 5-31 : Center Points

The results converged after 20 iterations with an optimal temperature of $71.4\ K$. The total numbers of center points considered were 46 as shown in Figure 5-31. The optimal values are tabulated in. The same optimization was run in Ansys Classic with the same convergence values to verify the results and the optimal values are tabulated in Table 5-6.

Table 5-6: Thermal Results of plate

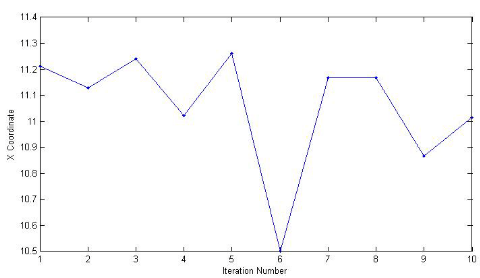
| | Matlab | Known Values |
|---|---|---|
| Iterations | 10 | |
| X coordinate | 11 in | 11.24 in |
| Y coordinate | 9.97 in | 9.55 in |

Table 5.6 – *Continued*

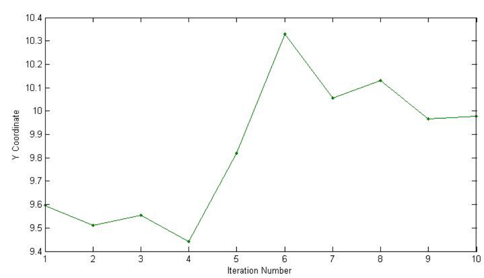
| | | |
|---|---|---|
| Radius | 6 in | 6.1 in |
| Temperature | 75 K | 75.4 K |
| Heat Flux | 467.64 Btu/Hr | 456 Btu/Hr |

The center points and the Coeff matrix was a 40 * 3 and 40 * 2 respectively. They are stored separately in a Matlab MAT file for further multiobjective analysis.

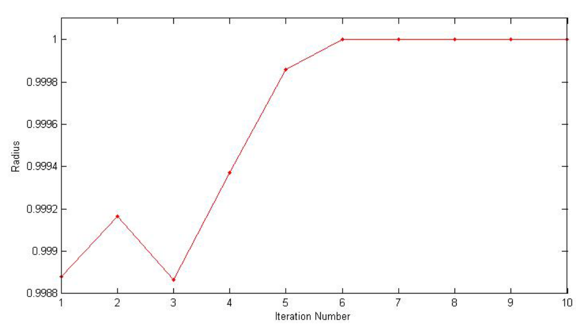
It can be seen that the RBF approximation model gives faster results than the Ansys Classic optimization model. The convergence plots for the Design Variables in Matlab are shown in Figure 5-32 and the Frequency Convergence plots in Matlab are shown in Figure 5-33.




(a)




(b)




(c)

Figure 5-32 : Convergence of Design Variables in Matlab by (a) x coordinate, (b) y coordinate and (c) Radius of the circle

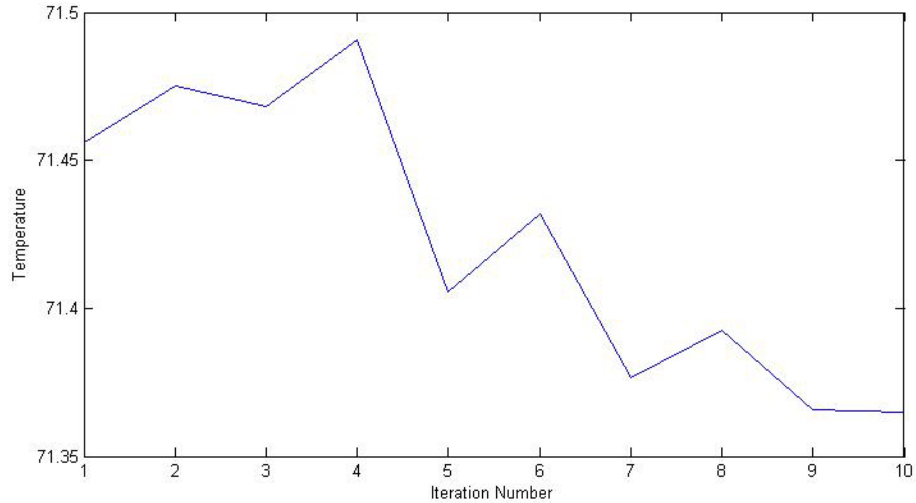



Figure 5-33 : Temperature Convergence in Matlab

*5.4.3 Multi Objective Optimization*

After we finish running both the test cases, the center points and the coeffecients for each is saved and multi objective min max algorithm is run that gives the minimum of the worst case among both the loading conditions.

Problem Formulation

The Optimization analysis is formulated as below

| |
|---|
| Objective Function : Minimize the Total Volume |
| Objective Function : Minimize the Max Outer Temperature |
| Stress Constraint: Von Mises stress, $\sigma_{VM} \leq 12000\, psi$ |
| Heat Flux Constraint: Max Heat Flux $\leq 500$ Btu/hr |
| Geometric Constraint: $x - r \geq 2, y - r \geq 2, x + r \leq 28, y + r \leq 18$ |
| Design Variables: x coordinate, y coordinate, Radius |
| Bounds : Lower limit of [2 2 1] in and Upper limit of [28 18 8] in |

Problem Results and Tabulations

The Multiobjective Program converged with  the following optimal Values

| DV | Value |
|---|---|
| Width | 12 in |
| Height | 9.8 in |
| Radius | 5.83 in |

With a final structure volume of $493.6\ in^3$ , max stress of 4753.2 psi, a max temperature of 74.98 K, and a max Heat Flux value of 481.8 Btu/Hr.

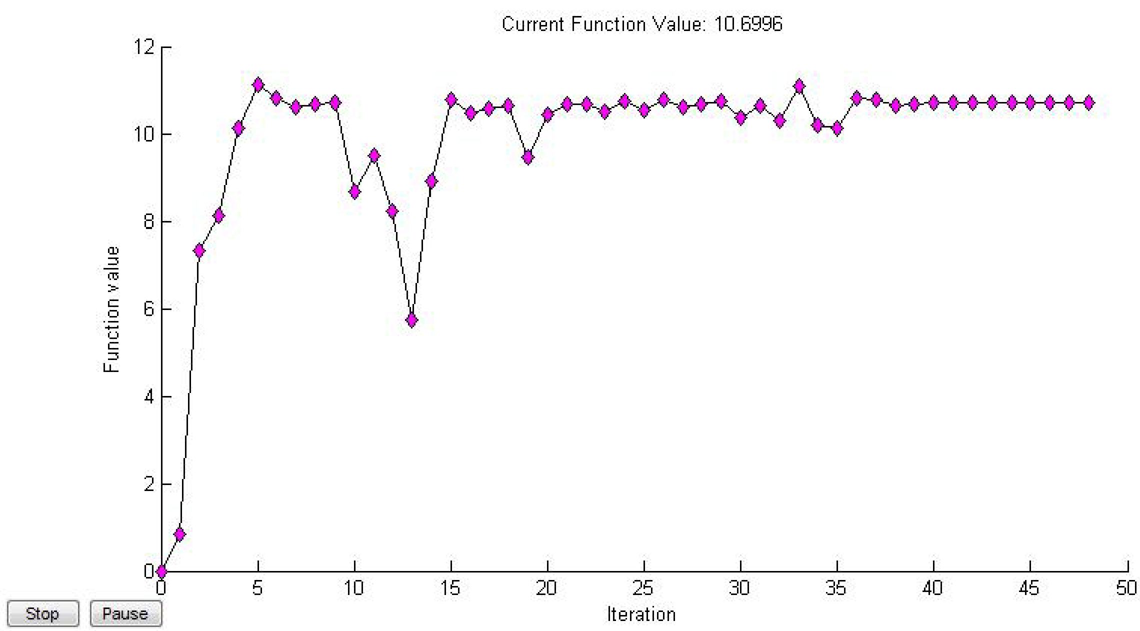



Figure 5-34 :  Multi Obj Vol Convergence

The Plots for the stress, temperature and heat flux is shown below in Figure 5-35, Figure 5-36 and Figure 5-37 respectively.

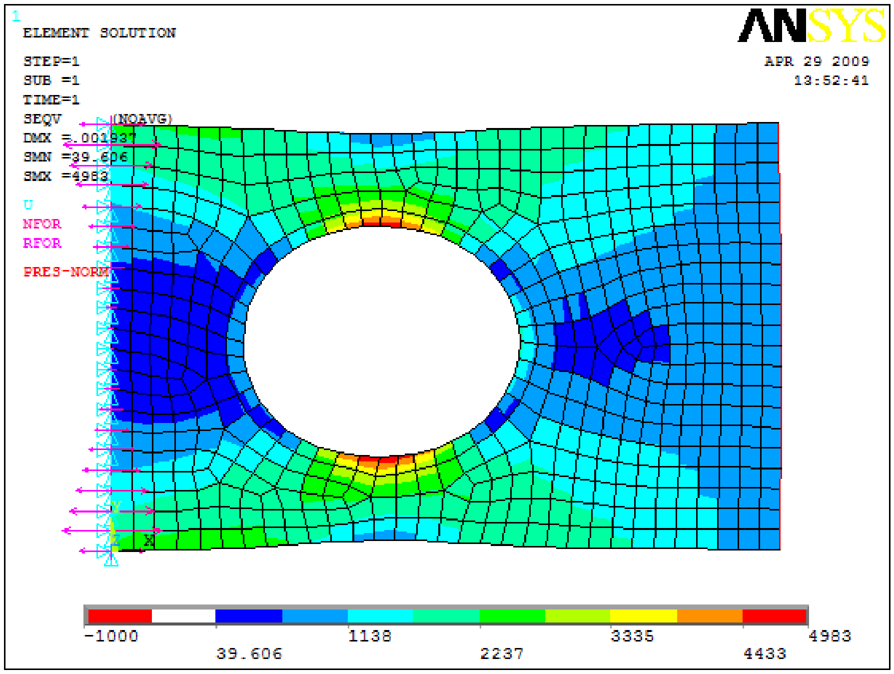



Figure 5-35 : Stress Plots of Optimized values

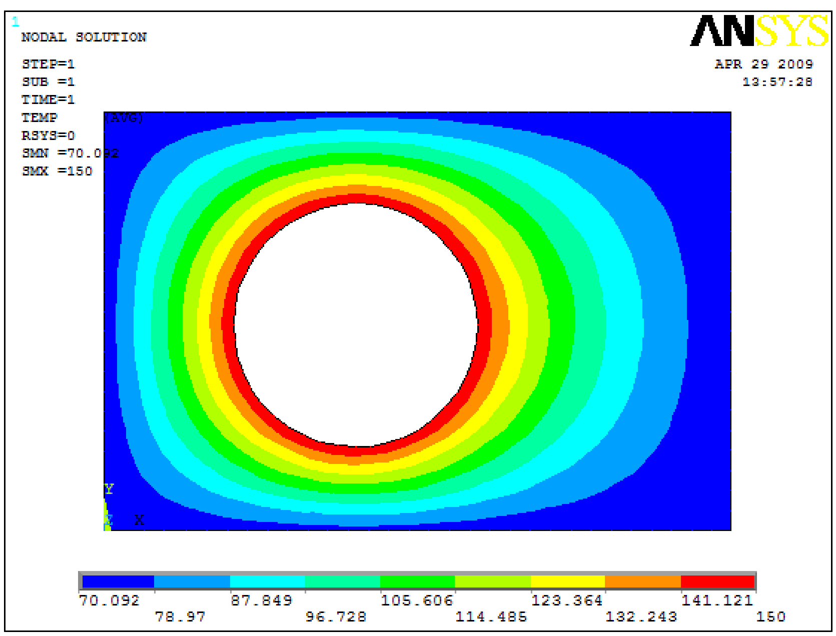



Figure 5-36 : Temperature Distribution of Optimized values

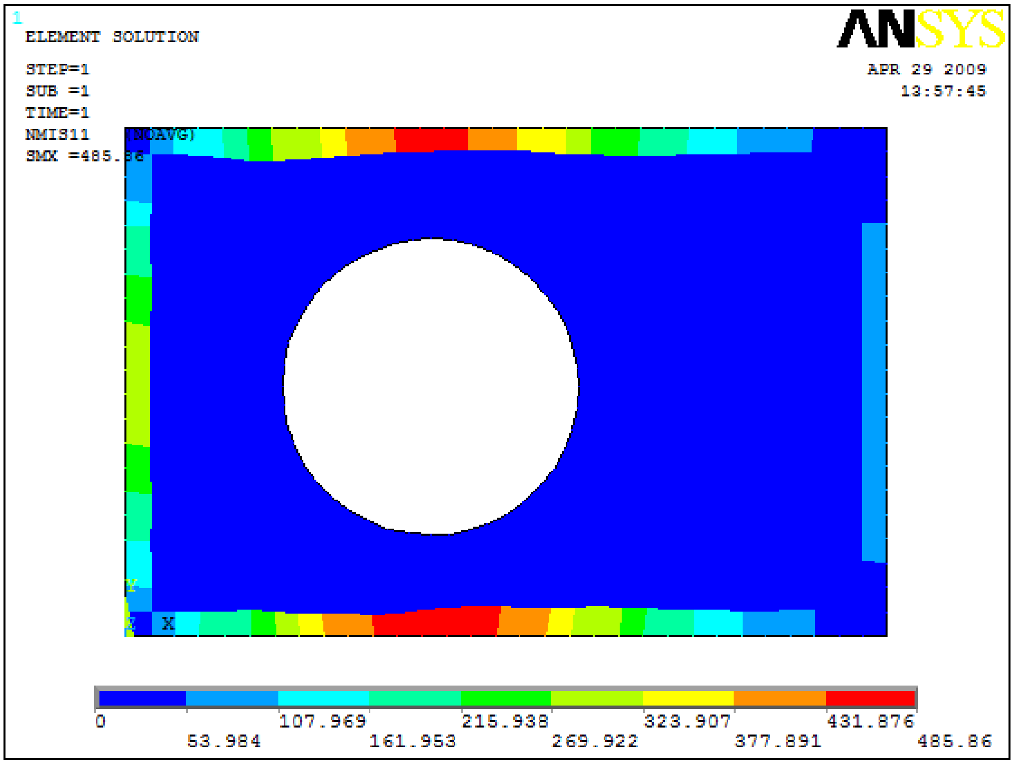



Figure 5-37 : Heat Flux flow of Optimized values

# CHAPTER 6

# RESULTS AND CONCLUSIONS

The main objective of this work was to define a design automation process in MATLAB to solve computationally expensive design optimization problems using a non conventional optimization process that integrates a *DOE, a  response surface* modeling tool (MQR) and ANSYS, a powerful finite element solver. Further the we demonstrate that the response surface model generated can be used for multi objective analysis. The design automation process in MATLAB was successfully implemented by integrating the MQR based optimization process and ANSYS the finite element solver. Several well documented optimization problems were considered and the results from the proposed method were compared with the conventional optimization process in ANSYS based on first order optimization.

## 6.1. Conclusions

We considered 4 different problems in this thesis. All these were well documented cases from which we could have a fair idea of the model accuracy. In addition to it the models were also optimized with Ansys to compare the values with ones we got from the response surface analysis in Matlab.

In all of the problems the Response Surface Analysis gave us the results in less number of iteration and was much faster. In the first two problems we minimized the volume to stress and displacement constraints using FMINCON. The differences in the responses were comparable to the Ansys Classic results.

In the third test case we had to generate two response surface models that could be used for multiobjective analysis. We chose a structural stress constraint for the first case and thermal stress constraint for the second case. The objective was to minimize the volume. Then the response surface models were optimized using FMINIMAX for both type of stresses. The optimized results were then verified using Ansys. After multi objective optimization we can see that the minimum thermal compressive stresses were equal to the stress constraints. A solution of this type was expected of the FMINIMAX solver which minimizes the worst of the objective functions.

The fourth test case was a plate with a hole and was optimized for volume and again for maximum Outer temp subjected to stress constraints. The minimum volume was compared with known values. The values agreed very well with the known values. In the second situation we wanted to minimize the maximum outer temperature and the expected result was that the radius of the hole must be as less as possible. The results from both Ansys and Matlab were almost similar. A multiobjective analysis was conducted on both response surfaces. The optimum values were then verified in Ansys with the Heat Flux values coming very close to the Heat Flux constraint value.

## 6.2. Future Research

The suggested topics for further study is

1. The concepts presented in this thesis can be expanded to include transient or dynamic analysis.
2. The limitation of large limits in RSM can be eliminated by developing a bracketing methodology.
3. Coupling of Matlab with a statistical software like MiniTab for the generation of DOE can improve accuracy.

4. A methodology for iteratively increasing or decreasing the number of design points added per iteration.
5. Global search algorithms like the Genetic Algorithm can be implemented on distributed computing grids.

# APPENDIX A

## SAMPLE MATLAB AND ANSYS CODE FOR OPTIMIZATION OF A 25 ELEMENT STRUCTURE.

Matlab Main File

```
%%  <<<<<  Define Data for Response surface model  >>>>>
% Select space-filling low discrepancy model
   METHOD='halton';  % LDS method
%  Define range of data
%       Lower bounds
    XL=[.1 .1 .1 .1 .1 .1 .1];
%       Upper bounds
    XU=[2 2 2 2 2 2 2];

%  Define number of points ( to be generated)
   nPoints=1000;
%
% Define Analysis function
     OUT.RespFun='AnsysResp'; % This function outputs  the following responses
%                              [Volume displacements stresses]
% Select RBF-RSM Model : 1=MQI;2=MQR; 3=GaussI;4=GaussR;
     MODEL_No=1;
     % Define initial model size
   Ninit=10;
%%  <<<<  Define optimization problem definition data  >>>>>
% Define coefficient matrices for objective function
      nu=18;            % number of displacements
      ns=25;            % number of stresses
      NResp=1+ns+nu;    % total number of responses
%
%  Define matrices for objective function
      Ax=[0 0];
          z15=zeros(1,NResp);Ay=z15;
     Ay(1)=[1];
% Generate the DOE Matrix
      [XData,XQMC,OUT]=DOE(METHOD,XL,XU,nPoints);
      %% Define Allowable displacement
% Allowable dtress in tension
   Stall=40000;
% Allowable dtress in compression
   Scall=-40000;

%Define matrices for constraint functions
   Cx=zeros(2*(nu+ns),2);
      E14=eye(nu+ns);
         E14(1:nu,:)=E14(1:nu,:)/Uall;
         E14((1+nu):(nu+ns),:)=E14((1+nu):(nu+ns),:)/Stall;
      z14=zeros(nu+ns,1);
   CC1=[z14 E14];
      E14=eye(nu+ns);
         E14(1:nu,:)=E14(1:nu,:)/Uall;
         E14((1+nu):(nu+ns),:)=E14((1+nu):(nu+ns),:)/Scall;
      z14=zeros(nu+ns,1);
   CC2=[z14 E14];
   Cy=[CC1;CC2];
```

```
       uSU=[ones(nu,1);ones(ns,1)];
       uSL=[ones(nu,1);ones(ns,1)];
%
% Define right hand side of constraint functions
   Cb=[uSU;uSL];
OUT.Cy = Cy;OUT.Cb = Cb;

%% <<<<<  Define convergence criterion >>>>>
% Change in objective function
   fEps=0.01;
% Feasible constraint bound
   gEps=0.01;
% Define number of data to be added from LDS data

Nadd=1;
%%      Stasrt RSM-SQP procedures
% Calculate initial data base
XC = XData(1:Ninit,:);
[YC,OBJExt]=AnsysResp(XC); %Generate the exact Responses
XCs=scaledtonorm(XC,OUT.A,OUT.B) ;
    OUT.XC=XC;
    OUT.XCs=XCs;
    OUT.YC=YC;
    OUT.OBJ=OBJExt';
    OUT.Ninit=Ninit;
    OUT.Nresp=NResp;

    %% Select model and define model para,eters

Model='MQR';h=1;r=1/1000;
[c]=train_rbf(Model,r,h,OUT);
OUT.h = h;OUT.r = r;OUT.Model = Model;OUT.C = c;OUT.RespFun='Ansys Resp';


%% SQP use current data base
XRSM = vijay_SQP(OUT,Ax,Ay,Cx,Cy,Cb)
Nadd = 1; OUT.Nadd=Nadd;

%% Iteration loop
ITER=0;
       NITER=40;
while ITER<NITER
   clear OBJadd
ITER=ITER+1;
    for i=1:Nadd
       ii=Ninit+ITER+i-1 ;
       x=XData(ii,:);
       [Yadd(i,:),OBJadd(i,1)]=AnsysResp(x);
       XCadd(i,:)=x;
    end

     i=1+Nadd;x = XRSM;
```

```
    [Yadd(i,:),OBJadd(i,1)]=AnsysResp(x);
    XCadd(i,:)=x;
% update data base
    XC=[XC;XCadd];
    YC=[YC;Yadd];
    OBJExt=OUT.OBJ;
[n_R,n_C]=size(OBJExt); if n_R>n_C;OBJExt=OBJExt';end
[n_R,n_C]=size(OBJadd); if n_R>n_C;OBJadd=OBJadd';end

    OBJExt=[OBJExt OBJadd];

    XCs=scaledtonorm(XC,OUT.A,OUT.B) ;
    OUT.XC=XC;
    OUT.XCs=XCs;
    OUT.YC=YC;
    OUT.OBJ=OBJExt;

    Ninit = Ninit+Nadd+1; OUT.Ninit = Ninit;
XCs=OUT.XCs;
 YC=OUT.YC ;
 [C] = train_rbf(MODEL_No,r,h,OUT);
   OUT.h=h;
   OUT.r=r;
   OUT.C=C;
  OUT.Model=MODEL_No;
  OUT.RBFRSM=ModelName{MODEL_No};
    Xsqp=vijay_SQP(OUT,Ax,Ay,Cx,Cy,Cb);
XHIS(ITER,:)=Xsqp;
    [RESP,Volume,U,Stress]=AnsysResp(Xsqp);

   %% Find current best design

[fBEST,gBEST,xBEST]=vijay_currentbest(OUT,gEps,Ax,Ay,Cx,Cy,Cb);

%% Convergence check

OUT.Xsqp=Xsqp;
    if ITER>2
%   RBFSQP_Convergence
    [ITER,OUT,XRSM]=vijay_Convergence(OUT,gEps,fEps,ITER,XHIS);

    end
end

Yrsm=sim_rbf(OUT.XC,XRSM,OUT.C,OUT.h);
Yansys=AnsysResp(XRSM);
Yfinal=[Yrsm;Yansys];
```

Ansys Codes

```
/PREP7

*dim,asize,,1,1
*vread,asize,Asize,txt,,IJK,1,1
(f22.14)

*dim,xareas1,,asize(1,1),1
*vread,xareas1(1,1),Area1,txt,,IJK,asize(1,1),1
(E22.14)

*dim,xareas2,,asize(1,1),1
*vread,xareas2(1,1),Area2,txt,,IJK,asize(1,1),1
(E22.14)

*dim,xareas3,,asize(1,1),1
*vread,xareas3(1,1),Area3,txt,,IJK,asize(1,1),1
(E22.14)

*dim,xareas4,,asize(1,1),1
*vread,xareas4(1,1),Area4,txt,,IJK,asize(1,1),1
(E22.14)

*dim,xareas5,,asize(1,1),1
*vread,xareas5(1,1),Area5,txt,,IJK,asize(1,1),1
(E22.14)

*dim,xareas6,,asize(1,1),1
*vread,xareas6(1,1),Area6,txt,,IJK,asize(1,1),1
(E22.14)

*dim,xareas7,,asize(1,1),1
*vread,xareas7(1,1),Area7,txt,,IJK,asize(1,1),1
(E22.14)

ET,1,Link8
*do,i,1,asize,1
r,i,1
*enddo

MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,1.048E7
MPDATA,PRXY,1,,0.334

n,1,0,0,0
n,2,200,0,0
n,3,200,200,0
n,4,0,200,0
n,5,62.5,62.5,100
```

```
n,6,137.5,62.5,100
n,7,137.5,137.5,100
n,8,62.5,137.5,100
n,9,62.5,100,200
n,10,137.5,100,200

real,1

e,9,10

real,2

e,9,6
e,9,7
e,10,5
e,10,8

real,3

e,9,5
e,9,8
e,10,6
e,10,7

real,4

e,5,6
e,6,7
e,7,8
e,8,5

real,5

e,1,8
e,2,5
e,3,6
e,4,7

real,6

e,1,5
e,2,6
e,3,7
e,4,8

real,7

e,1,6
e,2,7
e,3,8
e,4,5
```

```
D,1,all,0
D,2,all,0
D,3,all,0
D,4,all,0

f,9,fy,20000
f,9,fz,-5000
f,10,fy,-20000
f,10,fz,-5000

FINISH

*get,ncount,node,,count
*dim,ndispx,array,ncount,1
*dim,ndispy,array,ncount,1
*dim,ndispz,array,ncount,1

*get,ecount,elem,,count
*dim,stress,array,ecount,1

*CFOPEN,results,txt,,APPEND

*do,i,1,asize(1,1),1
/prep7

r,1,xareas1(i,1)
r,2,xareas2(i,1)
r,3,xareas3(i,1)
r,4,xareas4(i,1)
r,5,xareas5(i,1)
r,6,xareas6(i,1)
r,7,xareas7(i,1)

/solu
antype,static
solve
finish

/post1

set,last
Etable,stress,ls,1
Etable,volume,volu
ssum

*vget,ndispx(1,1),node,1,u,x
*vwrite,ndispx(1,1)
(E22.14)

*vget,ndispy(1,1),node,1,u,y
*vwrite,ndispy(1,1)
(E22.14)
```

```
*vget,ndispz(1,1),node,1,u,z
*vwrite,ndispz(1,1)
(E22.14)

*VGET,stress,ELEM,1,ETAB,STRESS, ,2
*vwrite,stress(1)
(E22.14)

*GET,tvol,SSUM, ,ITEM,VOLUME
*vwrite,tvol
(E22.14)

*enddo
*cfclos
Finish

/DELETE,file,EMAT
/DELETE,file,ESAV
/DELETE,file,FULL
/DELETE,file,RST
/DELETE,file,MNTR
/DELETE,file,PVTS
/DELETE,file,BCS
/DELETE,file,log
/DELETE,file,stat

/EXIT, NOSAVE
```

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# BIOGRAPHICAL INFORMATION

Vijay Krishna, the eldest son of L.K Krishna and Vijaya Lakshmi, was born in June 19, 1982, in Bangalore, India. He earned his Bachelor of Science in Mechanical Engineering Degree in January, 2005 from Visveswaraiah Technological University, Karnataka, India. After his undergraduate studies, he joined in Engineers and Engineers, the leading Mechanical Training and Consultancy firm in Bangalore in July, 2004 and worked there till July, 2005 as an Training Coordinator. Then he worked as a Technical Lead in Osys Technologies from July 2005 to Dec 2006.

He started his graduate studies in the University of Texas at Arlington in January, 2007 and earned Master of Science in Mechanical Engineering Degree in May, 2009. During Masters studies, Vijay conducted his research on the development and implementation of a Multi Disciplinary optimization scheme using radial basis function based response surface model and Ansys under the supervision of Dr. Bo Ping Wang.