

EFFECTS OF EDGE DISTANCE, HOLE SIZE RATIO AND HOLE SPACING
ON PEAK STRESSES OF COMPOSITE LAMINATE
WITH MULTIPLE HOLES

by

MANISHKUMAR KHERADIYA

Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

DECEMBER 2008

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ACKNOWLEDGEMENTS

I would like to express my sincere thanks and appreciation to Dr. Wen Chan for being my research advisor as well as providing me guidance, support and encouragement throughout the research work. He has always guided me towards the right approach, understanding and troubleshooting problems. I would like to thank Dr. Kent Lawrence and Dr. B.P. Wang for their advice and assistance during my master's coursework and thesis. My special thanks to Dr. Haiying Huang for being my committee member and to my colleagues Gianfranco Rios, Chia-Wei Su and Vijay Krishna for their help and support.

November 12, 2008

ABSTRACT

EFFECTS OF EDGE DISTANCE, HOLE SIZE RATIO AND HOLE SPACING ON PEAK STRESSES OF COMPOSITE LAMINATE WITH MULTIPLE HOLES

Manishkumar Kheradiya, M.S.

The University of Texas at Arlington, 2008

Supervising Professor: Wen S. Chan

Unlike isotropic material, the stress distribution of laminate with a hole varies with its size, its material properties and fiber orientations of each layer. Moreover, presence of a hole in the neighborhood of another hole may also affect their stress distribution, too. The major objective of this study is to determine stress distribution of finite width laminated composite having multiple holes. The effect of stress concentration due to presence of holes in each angle ply of laminated composite is a focus of this study.

Three dimensional finite element models are developed to determine the stress distribution in laminate using commercial software package ANSYS 10.0 classic. The

material used for laminated composite is T300/977-2 graphite/epoxy with $[0_3/\pm 45/90]_S$ laminate layup.

The stress distribution of composite laminate with single, two and three holes is investigated. The effect of proximity of single hole to laminate edge is also discussed. The stress concentration for “*two holes oriented in-line with load*” and “*two holes oriented side by side transverse to load*” are discussed. The effects of “*hole size ratio*” and “*edge distance between two holes*” on stress distribution are also investigated. The patterns of three holes placed at three vertices of a triangle with different edge distance between holes are also been included in this study.

The maximum stress concentration found at the point on hole periphery where fibers are tangent to the hole. The patterns of the hole orientation and holes size ratio as well as the edge distance between holes play important role on the magnitude of stress concentration of composite laminates.

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CHAPTER 1

INTRODUCTION

1.1 Background

The behavior of material with stress concentration is of great importance to design engineer because of the resulting reduced strength of components and higher amount of damage around this region. Stress concentration is an important parameter to be taken into consideration for structures design, because the point near maximum stress concentration is often the location of initialization of damage in the structure.

Composite material has been widely used in many applications because they offer high strength-to-weight and stiffness-to-weight ratio; moreover they can be tailored in design to meet strength/stiffness requirement. The most common applications of composites are to manufacture aircrafts, automotive, biomedical, marine and sporting goods. Majority of these applications require holes for joints, repair, accessibility, etc. Unlike isotropic materials, maximum stresses in composite structure with presence of a hole not only depend on the material properties, fiber orientation and stacking sequence of the laminate but also depend on the size of the hole.

Stress concentration around periphery of hole can be determined by analytical elasticity analysis, finite element methods and experimental methods. Lekhnitskii, Tan and Brian ESP, have used linear elastic theories to give closed form solution for stress

concentration in composite laminates having holes. But major limitations of such solutions are that they consider average effective laminate properties and some of these can only be used for infinite size of composite laminate. Because of this the stress concentrations in each angle ply of laminate and hence exact prediction of failure of laminate is difficult.

Stress distribution and stress concentration for a finite width composite laminate is often considered in structural design, for example repairing damaged aircraft structure by bolting a patch, and designing opening for windows and door in aircraft structure. In the repair cases always question arise that how far a hole should be located from edge of laminated composite? How the magnitude of stress concentration will be changed with change in diameter for finite width composite laminate? Which pattern of multiple holes orientation will minimize the stress concentration for finite width laminated composite? How far holes should be placed to reduce effect of holes interaction (when stress zones of two holes affect each other) on stress concentration?

Stress concentration factor for isotropic material infinite plate having single circular hole is well defined. But for the case of anisotropic composite laminate having finite width and having multiple holes makes the problem complicated and difficult to define exact close form solution to problem and it does not exist.

Finite element method can be useful and considerably accurate tool to solve such problem by generating 3-D model and which accounts for interlaminar and intralaminar stresses generated in finite width laminated composite having either single or multiple

holes, as well as gives the stress distribution of each angle ply of laminate. This resembles more close to the physical model.

Lekhniskii [1] in 1968 used anisotropic elasticity with complex variable method to give close form solution to compute stress distributions around a circular hole in an infinite anisotropic plate. Average laminate elastic properties to compute average stress distribution in the infinite size laminate with hole was used.

Fan and Wu [2] in 1988, employed Faber series expansion to obtain the stress concentration factor for an infinite size laminate with multiple holes. The results were based on average laminate properties.

Hansaw, Sorem and Glaessgen [4] in 1997, conducted finite element analysis of ply-by-ply to obtain equivalent stress concentrations for infinite size composite laminate with multiple holes under tensile and shear loading. The comparisons of stress distribution around holes for infinite size laminate with three holes using finite element method were given.

Tan [3] in 1994, extended Lekhniskii's method to obtain average stress concentration factor of finite width composite laminate having elliptical opening. A finite width correction factor for stress concentration was also given. Ochoa and Reddy [5] in 1992, used finite element analysis to determine stress concentration factor of composite laminate having three holes placed in-line with tensile load.

Neelkanthan, Shah and Chan [6] in 1997, investigated stress distribution around multiple circular loaded holes in a stiffener reinforced laminate. Later, Vendhagiri and Chan [7] in 2001 obtained stress distributions around the hole in composite bonded and

bolted joint. Esp [8] in 2007, using least square boundary collocation method for anisotropic composite laminate and investigated failure of composite laminate with two unloaded holes placed in close proximity.

1.2 Objectives and Approach to the Thesis

The primary objective of this study is to investigate the stress concentration due to presence of holes in laminate. Study was focused to find the effects of the maximum stress at the edge of the hole due to the distance between the laminate edge and the hole and between the holes. Effects of the maximum stress due to the pattern of the single, double and triple holes arrangements in laminate are also investigated.

3-D finite element models of ANSYS 10.0 were developed to determine the stress distribution in laminate. The material used for composite laminate is T300/977-2 carbon-fiber-reinforced epoxy laminate.

This study intended to provide an understanding of the minimum distance between holes and laminate edge to the hole that ensures sufficient laminate strength in design.

1.3 General Outline

Chapter 2 lists the hole pattern geometry in laminates used in this study. The procedure to develop the 3-D finite element model and boundary conditions are described. The validation of the model is also included in this chapter.

Chapter 3 presents the stress distribution of laminate with single hole. The locations of maximum stress concentration points around periphery of hole for different

angle plies are identified and discussed. Effect of hole stress interaction with the edge of laminated composite, on the magnitude of stress concentration is also investigated.

The results for multiple holes, the holes size ratio and the holes edge interaction effect on the stress concentration of composite laminate are presented in chapter 4. A comparison of maximum stress concentration results due to hole arrangement when two oriented side by side transverse to the loading direction and when two holes placed in-line to the loading direction are investigated. Discussion of laminate with three holes placed in a pattern of three vertices of a triangle is also included.

Chapter 5 includes comparisons and conclusions.

CHAPTER 2

FINITE ELEMENT MODEL AND VALIDATION

Finite element method has been used to investigate effect of stress concentration around periphery of hole in finite width laminated composites having single or multiple holes patterns at various locations. ANSYS 10.0 classic has been used to develop the required 3-D finite element model. This chapter will describe in detail the geometry and material of laminated composite used, the steps to develop 3-D finite element model and boundary condition as well as loading conditions for the model. The major challenge in developing 3-D finite element model, for such kind of problems is the mapped meshing.

2.1 Geometry and Material Used

2.1.1 Material of Composite Laminate

The material used for composite laminate is T300/977-2 graphite/epoxy laminate. The layup of the laminate chosen to be symmetrical and balanced to eliminate the coupling effects of bending and shear. The stacking sequence of laminate is $[0_3/\pm 45/90]_S$. The laminate consists of 12 layers, with ply thickness 0.005" each. The unidirectional layer orthotropic material properties for the material are given as follows

$$E_{11}=21.75 \times 10^6 \text{ psi}, \quad E_{22}=1.595 \times 10^6 \text{ psi}, \quad E_{33}=1.595 \times 10^6 \text{ psi},$$

$$\nu_{12}=0.25, \quad \nu_{23}=0.45, \quad \nu_{13}=0.25,$$

$$G_{12}=0.8702 \times 10^6 \text{ psi}, G_{23}=0.5366 \times 10^6 \text{ psi}, G_{13}=0.8702 \times 10^6 \text{ psi}$$

E_{11} , E_{22} and E_{33} are the young's moduli of composite ply among the material coordinates. The subscripts 1, 2 and 3 are along fiber direction, transverse and perpendicular to the plane respectively. G_{12} , G_{23} and G_{13} are shear moduli with respect to 1-2, 2-3 and 1-3 planes respectively. ν_{12} , ν_{23} and ν_{13} are the poisson's ratio.

2.1.2 Geometry of Laminate

The study focus on finite width composite laminate of rectangular shape; having single or multiple circular holes having different sizes, locations and holes orientation pattern within the laminate. The size of the laminate used is 5" length, 1.5" width and total thickness 0.06". Diameter of holes varies from 0.125" to 0.5". The main four types of holes orientation patterns have been used for this study are

- 1) Single hole
- 2) Two holes – in-line with load
- 3) Two holes – side by side transverse to load
- 4) Three holes.

These configurations are shown in the Figure 2.1 below. Table 2.1 lists sizes of holes and the locations of the centre of the hole

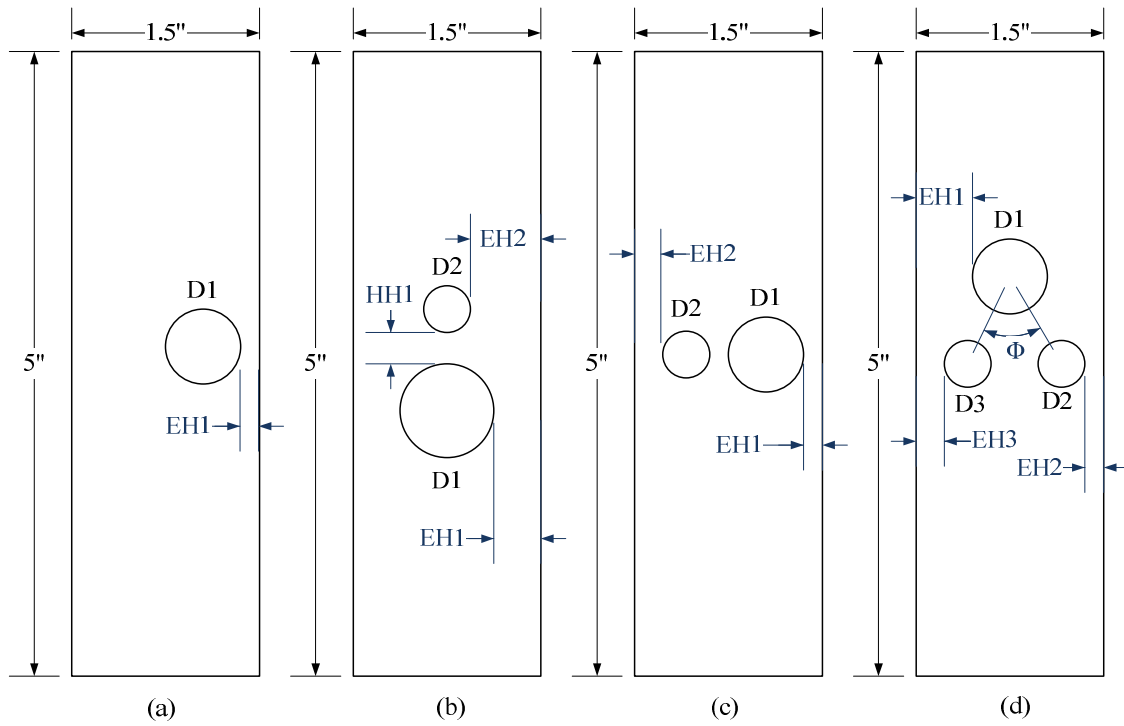


Figure 2. 1 Various Hole Orientation Patterns Geometries used for (a) Single Hole (b) Two Holes In-line With Load (c) Two Holes Side by Side transverse to load (d) Three Holes

Table 2. 1 Geometric Parameters for Various Hole Orientation Pattern

Configuration	Diameter of hole (inch)			Laminate to hole dge distance (inch)			Hole space (inch)	
	D1	D2	D3	EH1	EH2	EH3	HH1	Φ
Single Hole	0.5	-	-	0.5	-	-	-	-
	0.5	-	-	0.25	-	-	-	-
	0.5	-	-	0.125	-	-	-	-
Two Holes- Top and Bottom	0.5	0.5	-	0.5	0.5	-	0.5	-
	0.5	0.5	-	0.5	0.5	-	0.25	-
	0.5	0.5	-	0.5	0.5	-	0.125	-
	0.5	0.25	-	0.5	0.625	-	0.5	-
	0.5	0.25	-	0.5	0.625	-	0.25	-
	0.5	0.25	-	0.5	0.625	-	0.125	-
	0.5	0.125	-	0.5	0.6875	-	0.5	-
	0.5	0.125	-	0.5	0.6875	-	0.25	-
0.5	0.125	-	0.5	0.6875	-	0.125	-	
Two Holes- Side by Side	0.5	0.5	-	0.125	0.125	-	0.25	-
	0.5	0.5	-	0.125	0.25	-	0.125	-
	0.5	0.5	-	0.125	0.3125	-	0.0625	-
	0.5	0.25	-	0.125	0.375	-	0.25	-
	0.5	0.25	-	0.125	0.5	-	0.125	-
	0.5	0.25	-	0.125	0.5625	-	0.0625	-
Three Holes	0.25	0.25	0.25	0.625	0.25	0.25	-	45°
	0.25	0.25	0.25	0.625	0.25	0.25	-	60°
	0.5	0.25	0.25	0.5	0.25	0.25	-	45°
	0.5	0.25	0.25	0.5	0.25	0.25	-	60°

2.2 Development of Finite Element Model

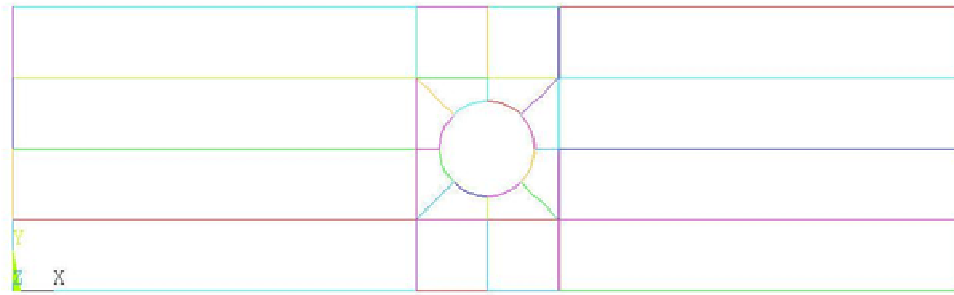
2.2.1 Finite Elements Used

ANSYS 10.0 classic have been used to develop the finite element model. Higher order 3-D 20-node solid elements of SOLID186 are used. The element in which quadratic shape function is used to achieve higher accuracy in result. All the models developed for study are 3-D. Moreover, each of the layers in the laminate is meshed with separate hexahedron elements along thickness direction. Mapped meshing is used for higher accuracy. Element sizes have been selected to maintain proper aspect ratio of less than 20 for all elements generated. Applied in-plane nominal tensile stress to laminate is 100 psi.

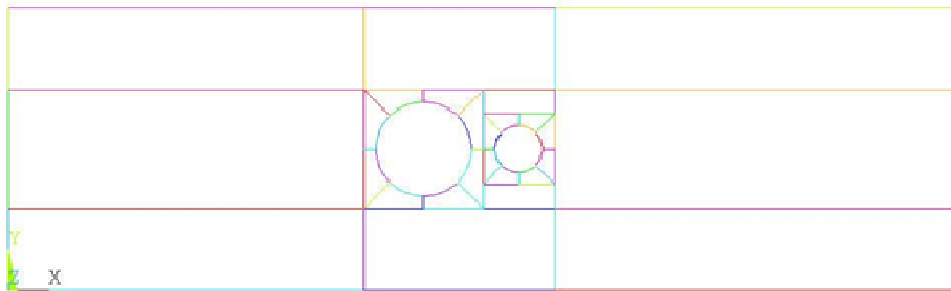
2.2.2 Modeling and Mesh Generation

The following is the procedure used for generating 3-D finite element models for this study:

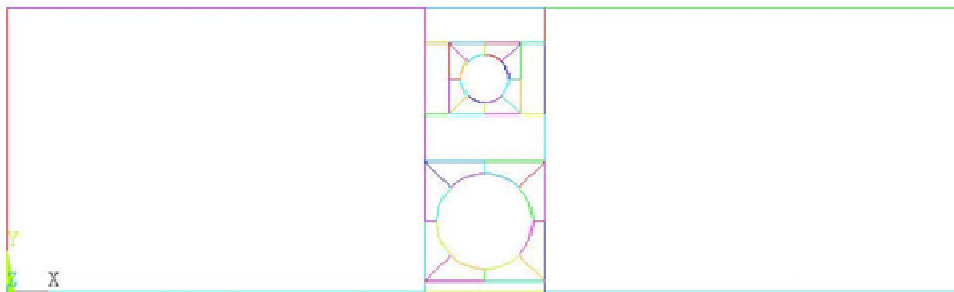
- 1) Define 8-node PLANE183 element as element type 1 and 20-node SOLID186 element as element type 2, define the unidirectional orthotropic material properties for lamina.
- 2) To generate 3-D model having SOLID186 elements for individual layers, we will first generate 2-D plot in XY plane at Z=0. Then will extrude it along Z-axis to create hexahedron elements. So first generate various key points as per dimension of the laminate and plot the lines and areas, as shown in Figure 2.2 below.



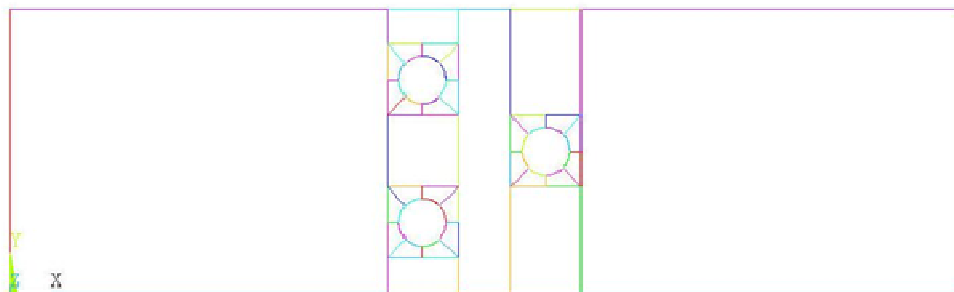
(a)



(b)



(c)



(d)

Figure 2. 2 Area Generation for Mapped Meshing for (a) Single Hole (b) Two Holes In-line With Load (c) Two Holes Side by Side transverse to load (d) Three Holes

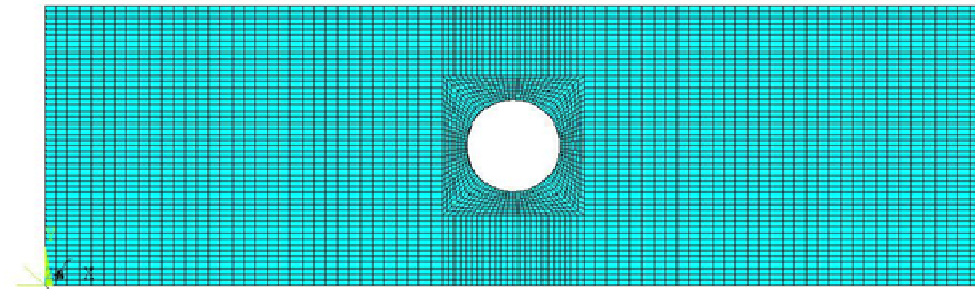
Figure 2.2 shows the base areas to be generated for one, two or three holes. Glue all the areas generated. A square area has been used around each hole, to achieve mapped meshing in such geometries. Size of square taken such that, we can generate approximately same numbers and size of elements, for various sizes and locations of holes.

3) For mapped meshing of each area generated, select the lines and define the number of divisions required on each line for mapped meshing. Each layer has been meshed with single individual elements along the thickness direction of laminate i.e. Z-axis. Size of each element has been kept as per thickness 0.005", such that aspect ratio for each element generated should not exceed 20 which is the limiting value in ANSYS.

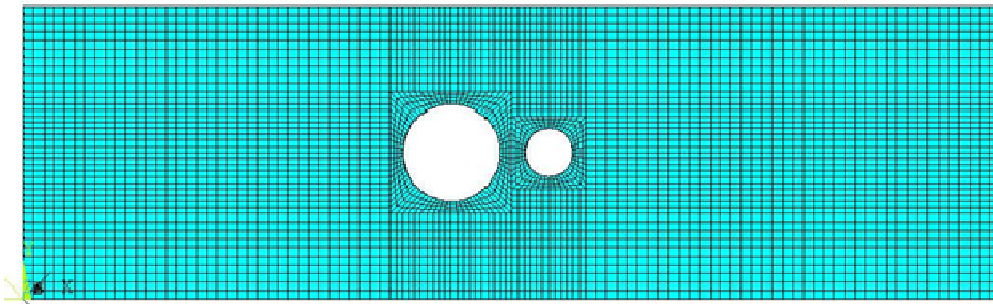
4) Mapped mesh all the areas. Remember to select element type as already defined element PLANE 183. After mapped meshing all areas, with 2-D elements mesh generated in XY plane at Z=0 will be seen, as shown in Figure 2.3 below.

5) As composite laminate is symmetric about thickness direction, only half of the laminate(6 layers) has been used for the model. Now select all the areas (along with elements) at Z=0 and copy 6 times, for 6 different layers and maintain distance between each of them equal to a ply thickness. This will take care that x-y co-ordinates of each nodes, for various layers remain exactly the same. And later on we can merge adjacent nodes of two layers one below the other.

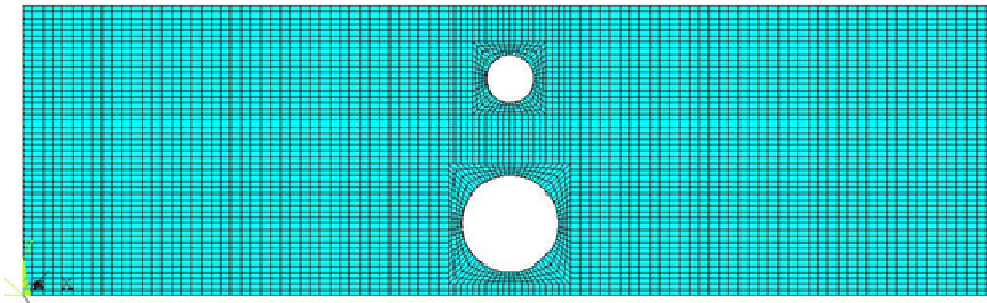
6) Define 6 element co-ordinates system, corresponding to angle of fiber orientation of each layer of laminated composite. Then again, set coordinate system to global Cartesian co-ordinate system.



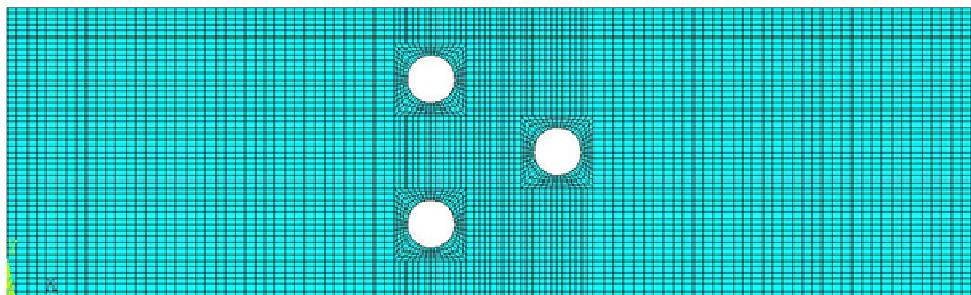
(a)



(b)



(c)



(d)

Figure 2. 3 Sample of Mapped Meshing for (a) Single Hole (b) Two Holes In-line With Load (c) Two Holes Side by Side transverse to load (d) Three Holes

7) Now first select element coordinate systems corresponding to layer at $Z=0$, set element type as Solid186, and extrude the 2-D elements at $Z=0$, these will generate 3-D element mesh with tetrahedron element, which corresponds to the 1st layer. Repeat similar procedure for all the six layers; do not forget to select corresponding element coordinate system for each layer before extruding it. After extruding all layers, the meshing will be as shown in Figure 2.4.

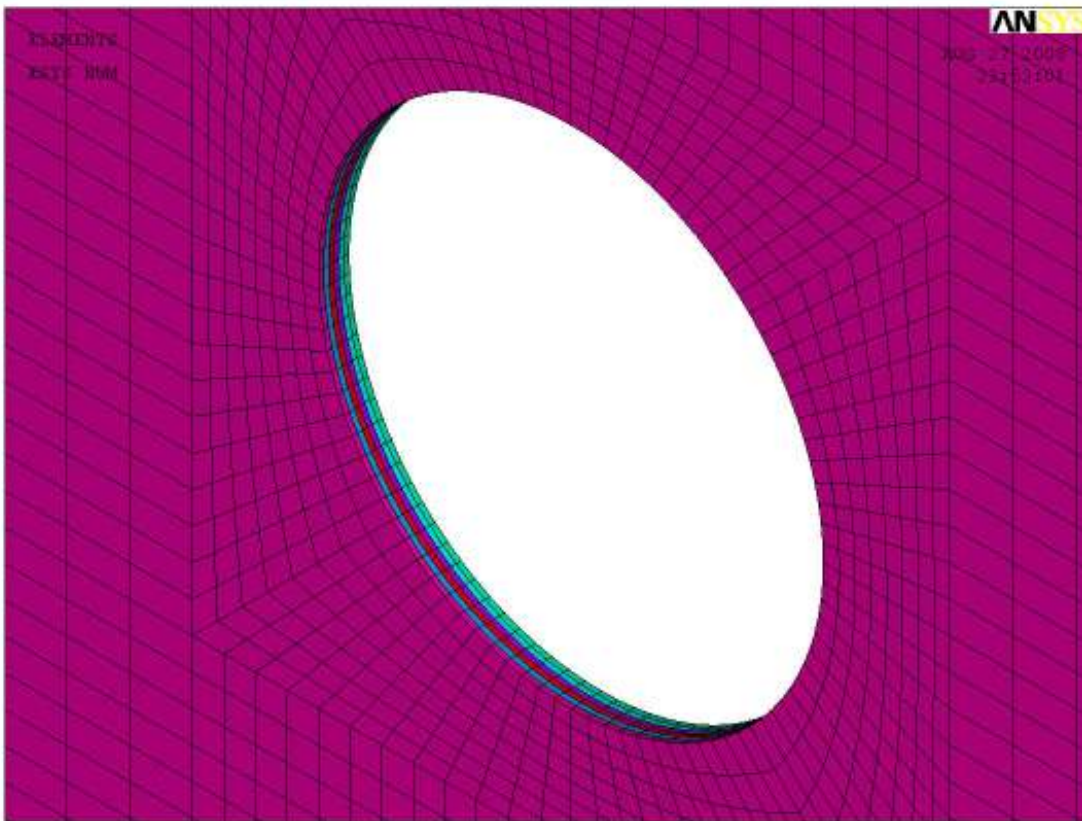


Figure 2. 4 Different Element Co-ordinate Systems Represented by different Colors for 3-D Mesh

8) Now merge all the corresponding nodes and key-points for two adjacent layers faces, which are in contact with each other. This will generate bonding between each layer. And the desired 3-D finite element model is ready.

2.2.3 Boundary Conditions

The following three boundary conditions are enforced

- 1) Because of symmetric laminate, only half of the laminate is modeled. Hence translation degree of freedom in Z-axis is constrained ($U_z=0$) for all nodes at $Z=0$.
- 2) All nodes at one of the end face is constrained along the longitudinal direction($U_x=0$).
- 3) All nodes at vertical centre line on the x-constrained surface are constrained along y-direction ($U_y=0$)

2.2.4 Loading Condition

An in-plane nominal tensile stress of 100 psi is applied to all the cases considered. The stresses are applied to the end face along longitudinal direction on which boundary conditions are not applied. Only tensile load is considered for this study.

2.3 Validation of the Model

Since stress concentration around hole for finite width isotropic plate having a hole at the centre is well defined, the model developed for study is validated using isotropic material properties. The model size, meshing, elements, boundary conditions, and loading conditions will remain same as, that used orthotropic laminate. Hence, isotropic material properties presented below are used for validation of model.

$$E_{11} = E_{22} = E_{33} = 21.75 \times 10^6 \text{ psi,}$$

$$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$$

$$G_{12} = G_{13} = G_{23} = \frac{E_{11}}{2(1 + \nu_{12})}$$

The results of stress distribution in the loading direction, X-axis is shown in the Figure2.5,

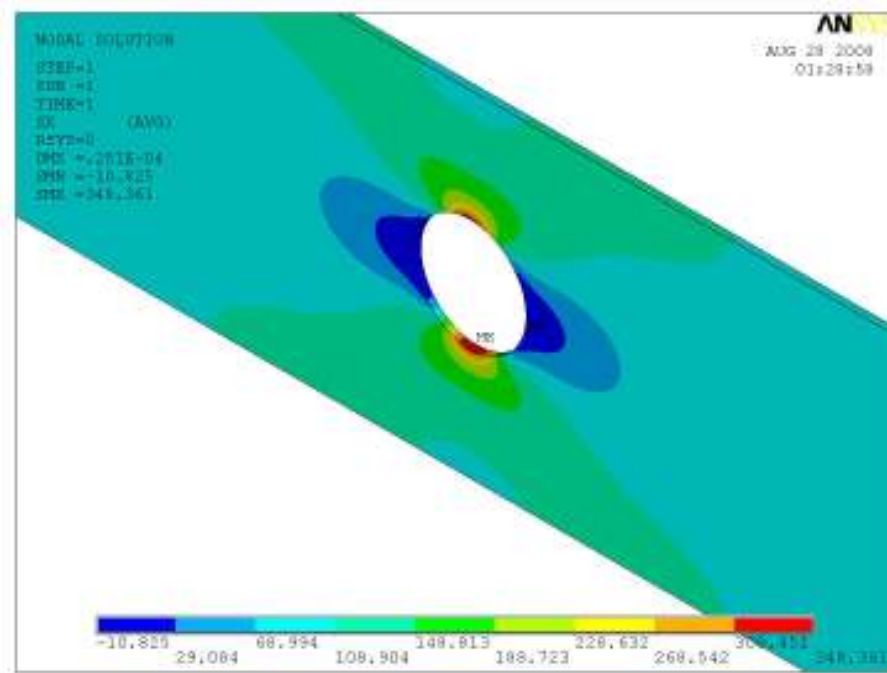


Figure 2. 5 Stress Contours of σ_x For Finite Width Isotropic Plate

From the above analysis, we can see that maximum stress at the periphery of hole is 348 psi. And applied in-plane nominal tensile stress is 100 psi.

Stress Concentration Factor

$$= \frac{\text{Maximum Magnitude of Stress Around Hole Periphery}}{\text{Applied in - plane nominal stress}}$$

$$= \frac{348}{100} = 3.48 \dots\dots\dots (1)$$

Now for finite width isotropic plate, subjected to uniaxial tensile loading, the stress concentration factor is given as [10],

$$Kt = \frac{2 + \left(1 - \frac{D}{W}\right)^3}{\left(1 - \frac{D}{W}\right)}$$

Where

D= hole diameter

W=width of plate

Substituting D=0.5” and W=1.5” in above equation we get

$$Kt = 3.44 \dots\dots\dots(2)$$

Comparing analytical result from equation (2), the FEM result from equation (1) gives only 1.16 % difference.

2.4 Convergence

A convergence study was conducted for composite laminate with single hole located at centre of width of laminate. Figure 2.7 shows the graph of maximum magnitude of σ_x versus No. of elements along hole periphery. Since the region around hole periphery has high stress gradient, the number of elements around hole periphery used for convergence study. Figure 2.6 shows the meshes used for this study. Only a quarter of hole meshes is shown. The number of elements around the hole ranges from 16 to 196. For example Case A shows 16 elements used around the hole periphery but only 4 elements is shown. The maximum magnitude of stress at hole periphery of composite laminate is used for comparison. From the graph in the Figure 2.7, it can be concluded that with increasing the number of elements at hole periphery, initially the maximum magnitude of stress σ_x increases. The slope of the graph gradually decreases with further

increasing number of elements around the hole periphery, which tend to converge the solution of σ_x . Hence, the mesh with 96 elements at the hole periphery is selected for the entire study.

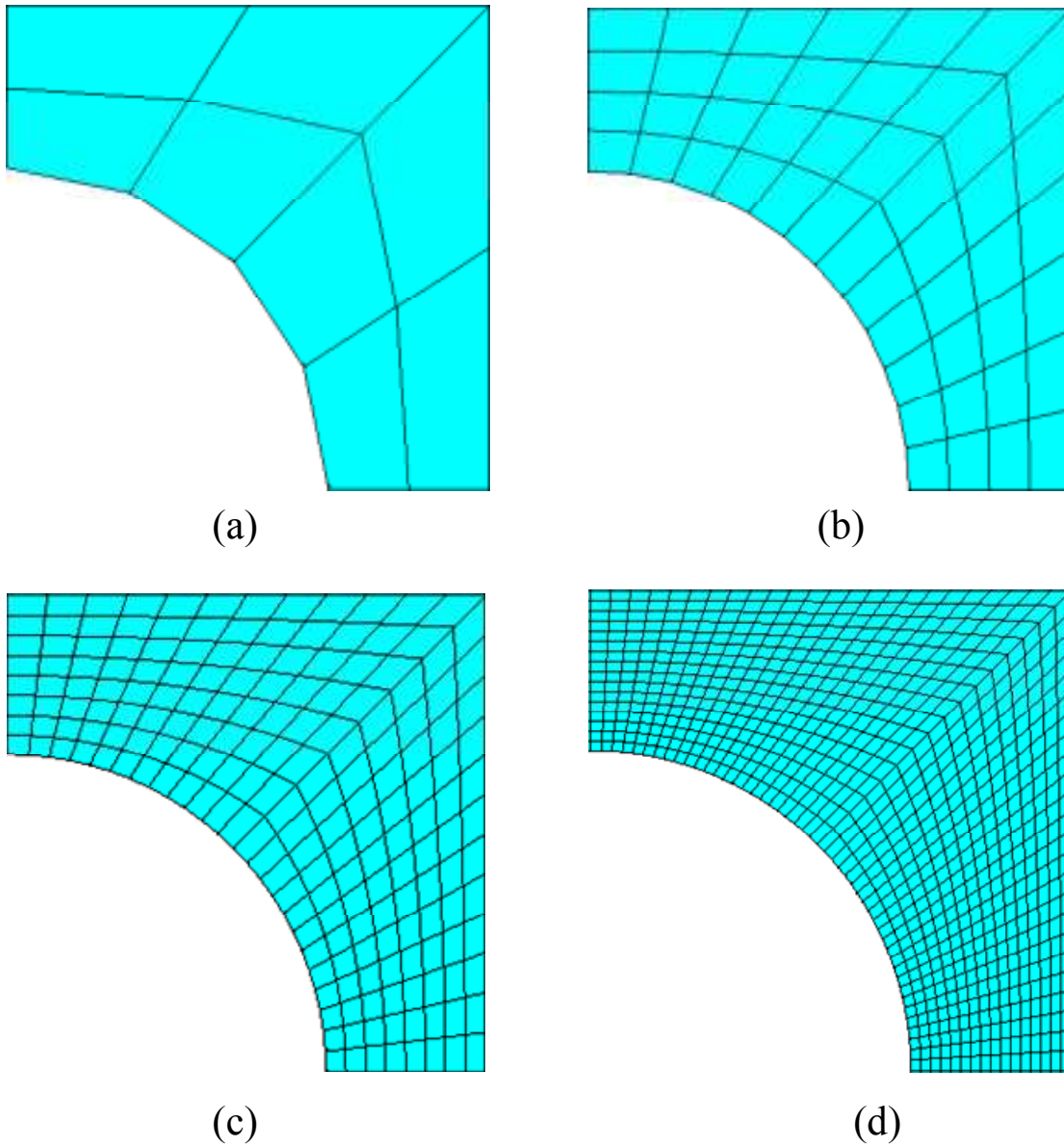


Figure 2. 6 Meshing Used for Parametric Study (a) 16 Elements (4 in a quarter) (b) 48 Elements (12 in a quarter) (C) 96 Elements (16 in a quarter) (d) 192 Elements (32 in a quarter)

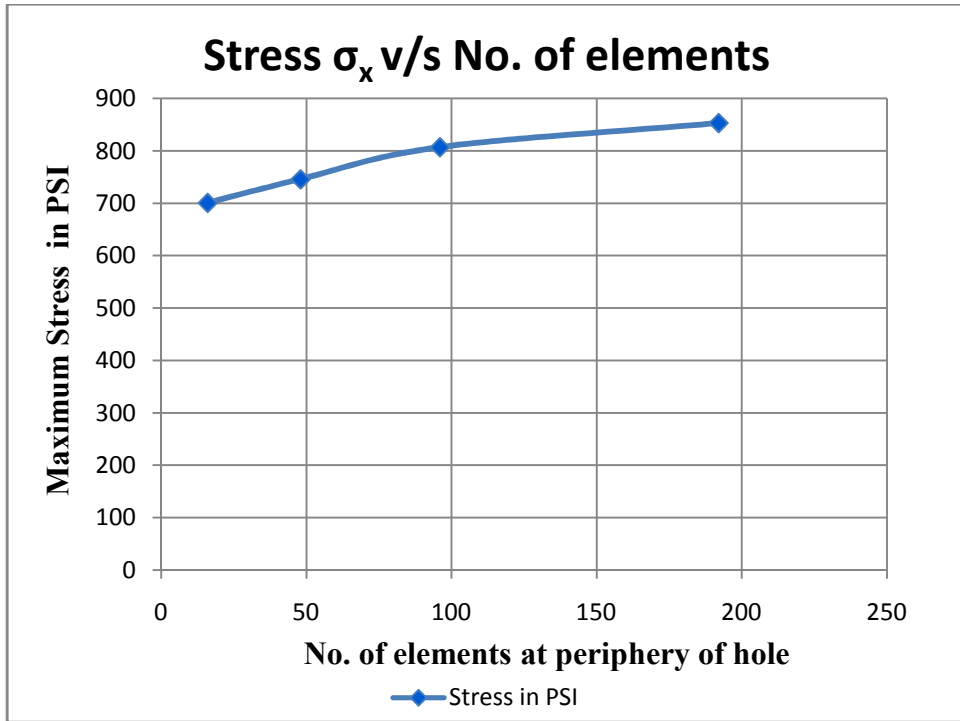


Figure 2. 7 Graph for Maximum Magnitude of σ_x Versus No. of Elements Along Hole Periphery

CHAPTER 3

SINGLE HOLE

The effect of the hole edge to laminate edge distance on the stress concentration around a single circular hole in the composite laminate is investigated in this chapter. The change in the magnitude of stress concentrations around hole periphery as the hole is located closer to the edge of the laminate (hole to laminate edge distances decreases), has been discussed in detail. A fixed value for the size of hole, size of laminate and stacking sequence are used for all cases considered for study.

3.1 Cases Considered for the Study

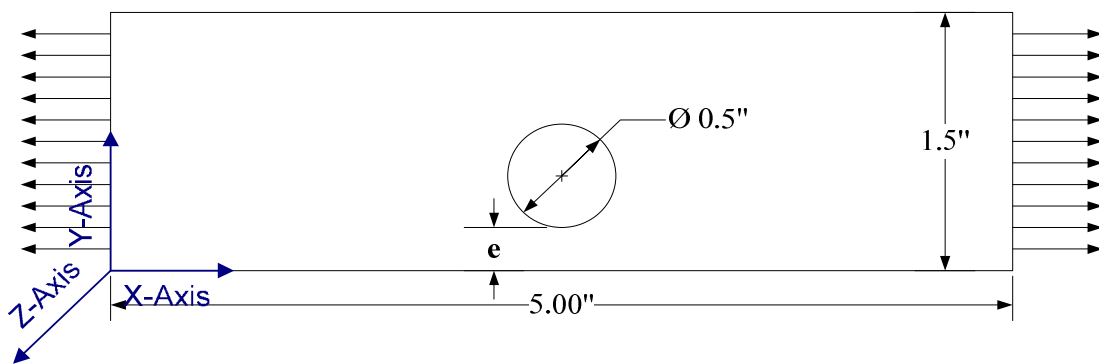


Figure 3. 1 Geometric Parameters for Laminate with Single Hole

T300/977-2 graphite/epoxy laminate having stacking sequence $[0_3/\pm 45/90]_S$ was used for all cases. The dimensions of the laminate are 5 inch length x 1.5 inch width x 0.06 inch thickness. The diameter of hole is 0.5" for each case. Applied in-plane nominal tensile stress $\sigma_0 = 100$ psi. Same boundary conditions have been used for all cases considered here. Various location of hole in the laminate was chosen. Different values of minimum distance between the hole edge to laminate edge (e) as shown in Figure 3.1, was chosen in terms of various fractions of diameter of hole as follows:

1) $e = 0.5" = D$

2) $e = 0.25" = \frac{D}{2}$

3) $e = 0.125" = \frac{D}{4}$

4) $e = 0.0625" = \frac{D}{8}$

3.2 FEM Model Mapped Meshing

Figure 3.2 shows top view of the meshing of the 3-D finite element models used. The method need to create the mapped meshing is described in Section 2.2 of Chapter 2.

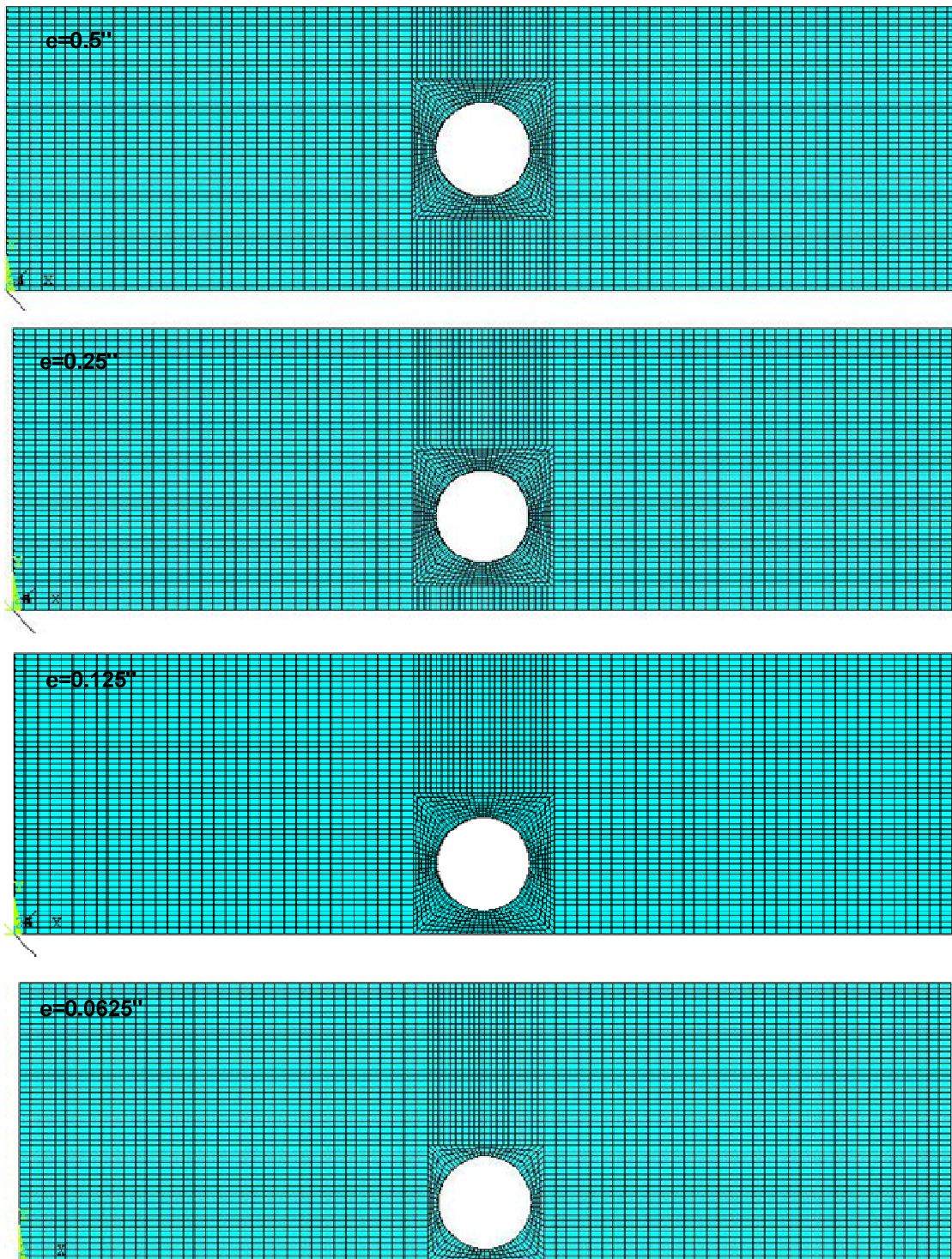


Figure 3. 2 Top View of Meshing for Laminate with Single Hole and Having Various Hole Edge to Laminate Edge Distances

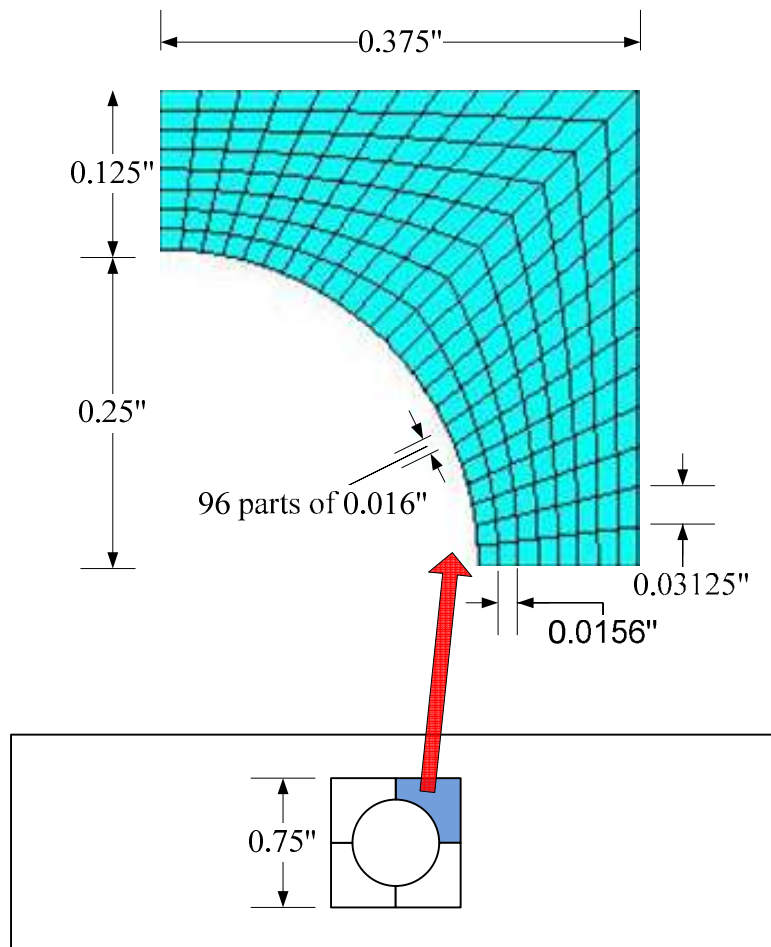


Figure 3. 3 Closer Look at the Element Sizing for Mapped Meshing

Region closer the periphery of the hole is subjected to higher stress concentration. As shown in Figure 3.3, to maintain symmetry of model and consistency of results an attempt has been made to use a square of size 0.75" x 0.75" around the hole, and periphery of the hole has been divided into 96 equal parts of 0.016" each, giving the elements around periphery of hole 0.016" x 0.0156" x 0.005" size, where 0.005" is thickness of each ply (In the thickness direction each ply has been meshed with a single

element). This selection of size gives the aspect ratio 3.2 for elements around periphery of the hole.

Following Figure 3.4 shows elements of the 3-D finite element model developed for this study. A closer look of the model in the neighborhood of the hole is shown in Figure 3.5.

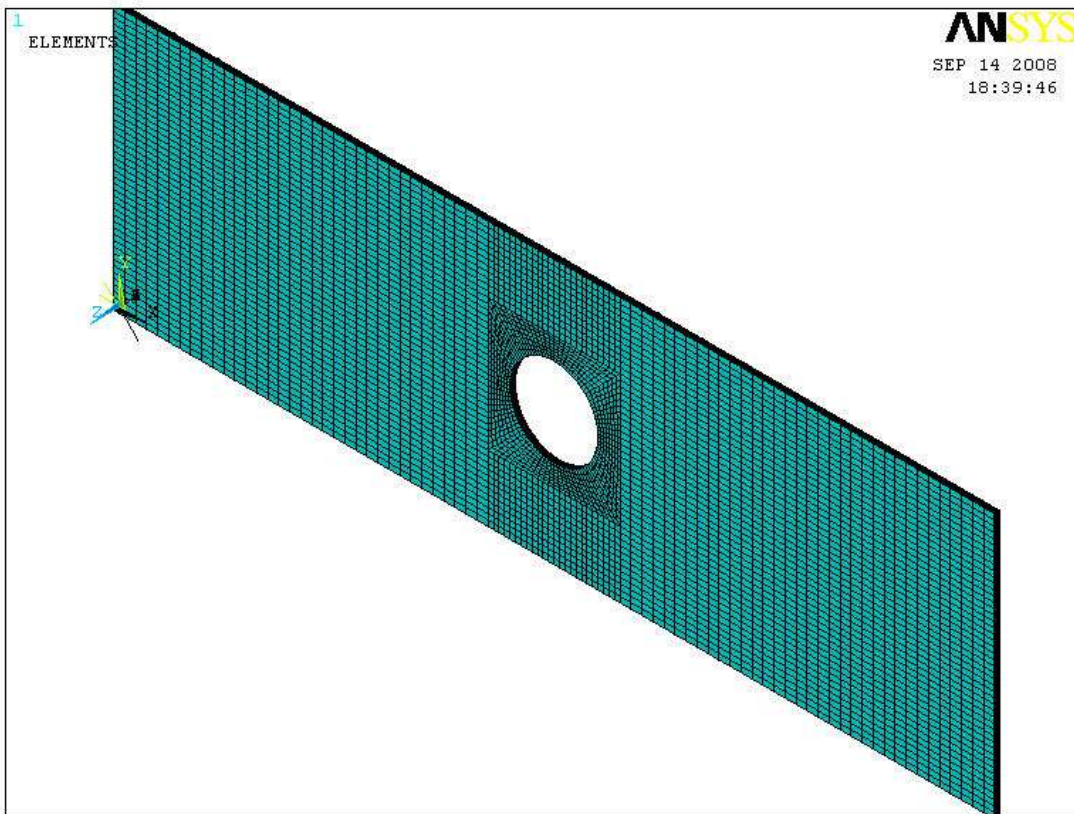


Figure 3. 4 3-D Finite Element Model with Mapped Meshing

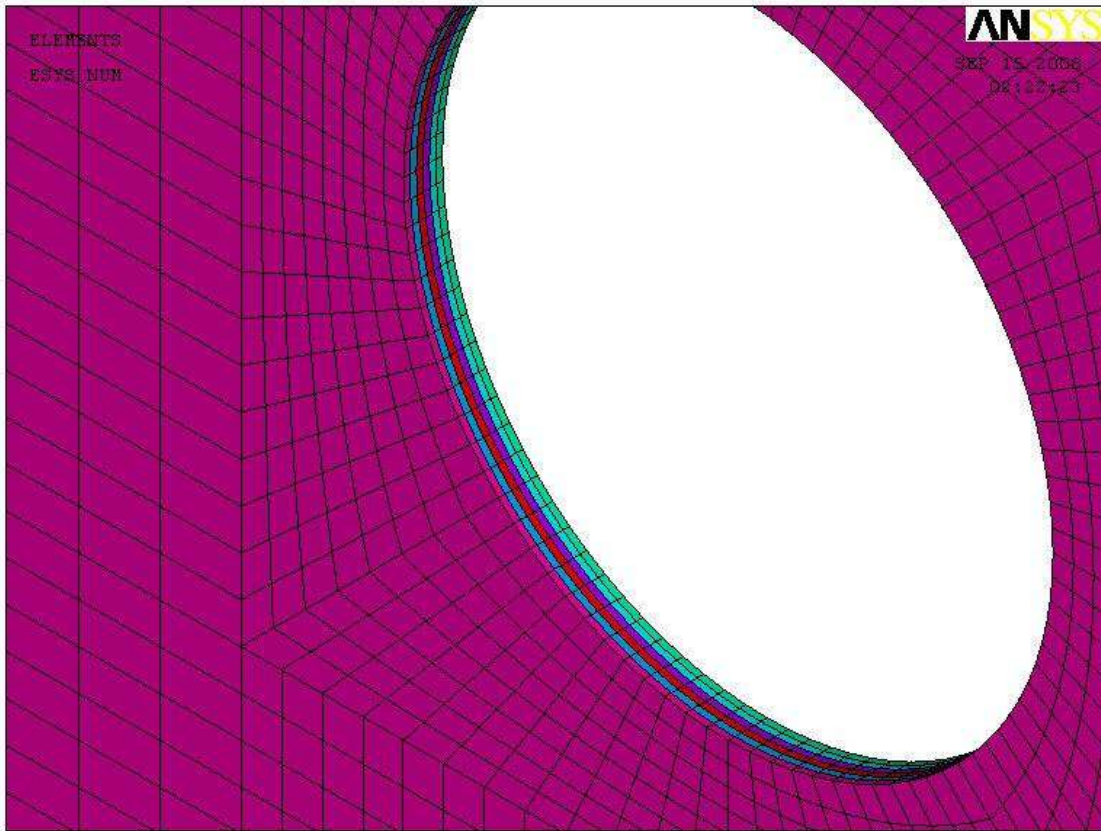


Figure 3. 5 Close View of Elements with Different Colors Corresponding to Different Element Co-ordinate Systems

The total no. of elements and nodes used for various cases considered in this study is listed in Table 3.1.

Table 3. 1 Total No. of Nodes and Elements for Single Laminate with Single Hole

Case	e	No. of Elements	No. of Nodes
1	0.5	27648	128176
2	0.25	27648	128176
3	0.125	27648	128176
4	0.0625	25440	118040

3.3 General Discussion on Stress Profiles

For a laminate subjected to tension, we can imagine constant force flux lines running from one end of the laminate through the other end for a 0° ply as shown in Figure 3.6. In each angle ply of the laminate a constant value of stress σ_x exists which depends upon fiber orientation.

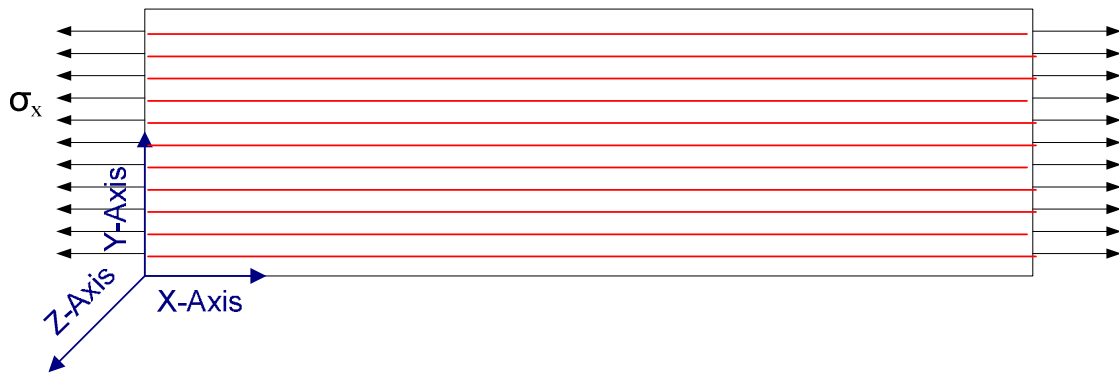


Figure 3.6 Constant Force Flux Lines in 0° Ply for Composite Laminate Without Hole

The presence of hole in laminate causes the imaginary force flux lines redistributed as shown in Figure 3.7.

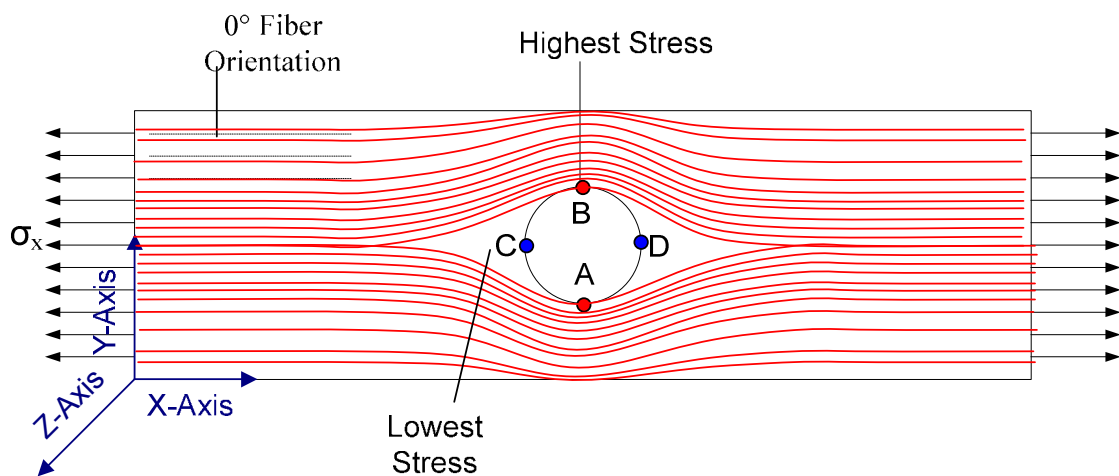


Figure 3.7 Constant Force Flux Lines in 0° Ply for Composite Laminate with Single Hole

At hole neighborhood, the imaginary force lines turn into net section area and become denser near points A and B on the hole periphery as shown in Figure 3.7. As a result, the regions of stress contours near points A and B on the periphery of hole becomes denser. Hence the maximum stress concentrations occur at points A and B at the periphery of the hole. While the region near points C and D, no force lines pass through the hole and result in a zero stress location.

3.4 Analysis Results

The stress distribution contours of in-plane stress σ_x for the composite laminate having single hole with $e=0.5''$ is shown in Figure 3.8.

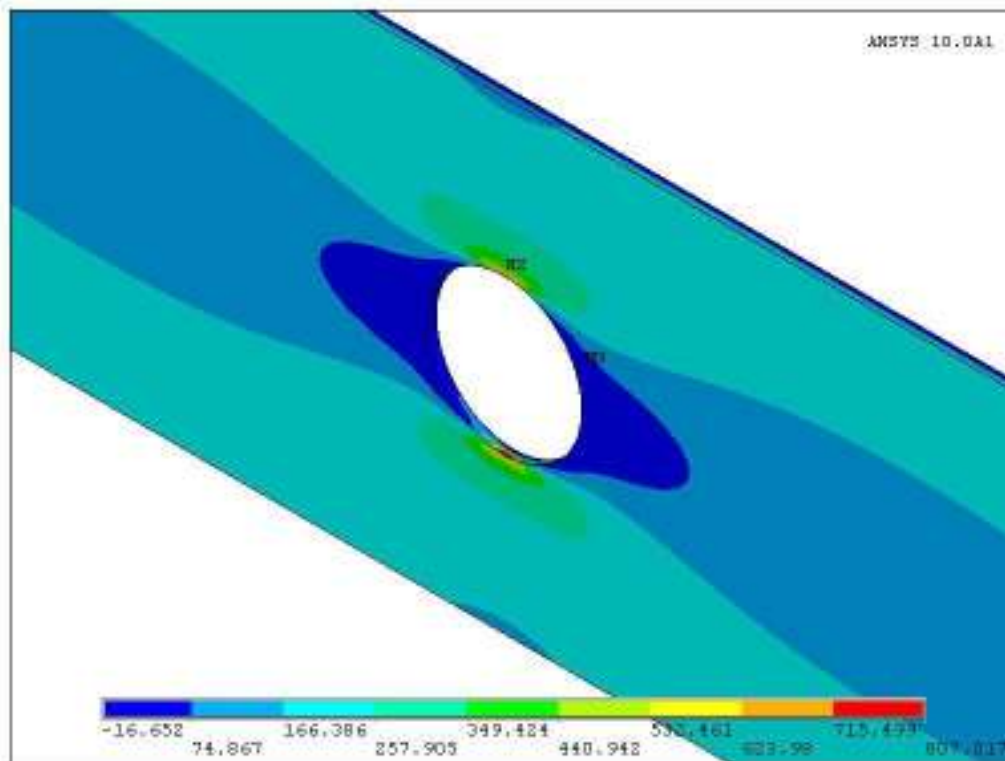


Figure 3. 8 Contour Plot of σ_x for $e=0.5''$ for $[0_3/\pm 45/90]_S$ Laminate

As shown in Figure 3.7, two points A and B of 0° layer on the periphery of the hole, are the location where the maximum magnitude of stress σ_x occurs. While points C and D on the periphery of hole indicate minimum magnitude of stresses σ_x acting on them. As indicated, the equal magnitude of stress σ_x occurs at points A and B. A closer view of stress distribution is shown in Figure 3.9. As shown, the maximum stress does appear in the 0° ply, not the other angle plies.

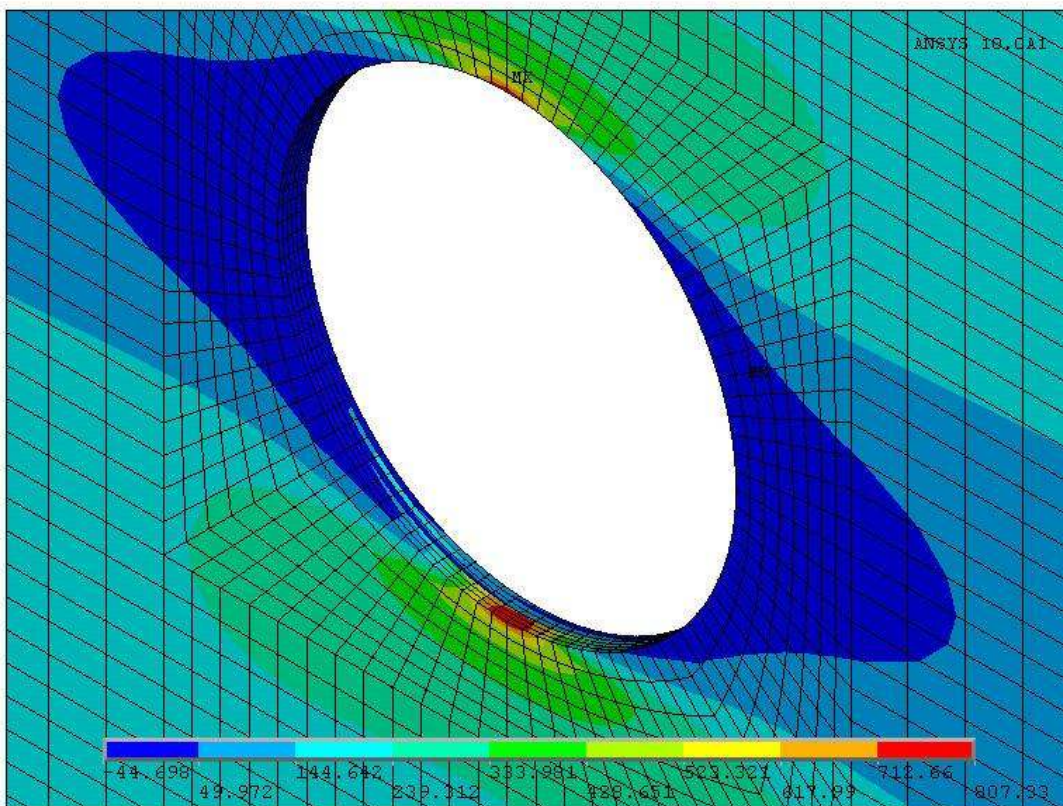


Figure 3.9 Closer Look at Stress Concentration Around Periphery of Hole with $e=0.5''$

3.5 Results for 0° plies

The following stress results are based upon the applied stress of 100 psi along the x-direction. A comparison of stress contours for 0° ply of laminated composite for different values of hole to laminate edge distances (e) has been shown in Figure 3.10.

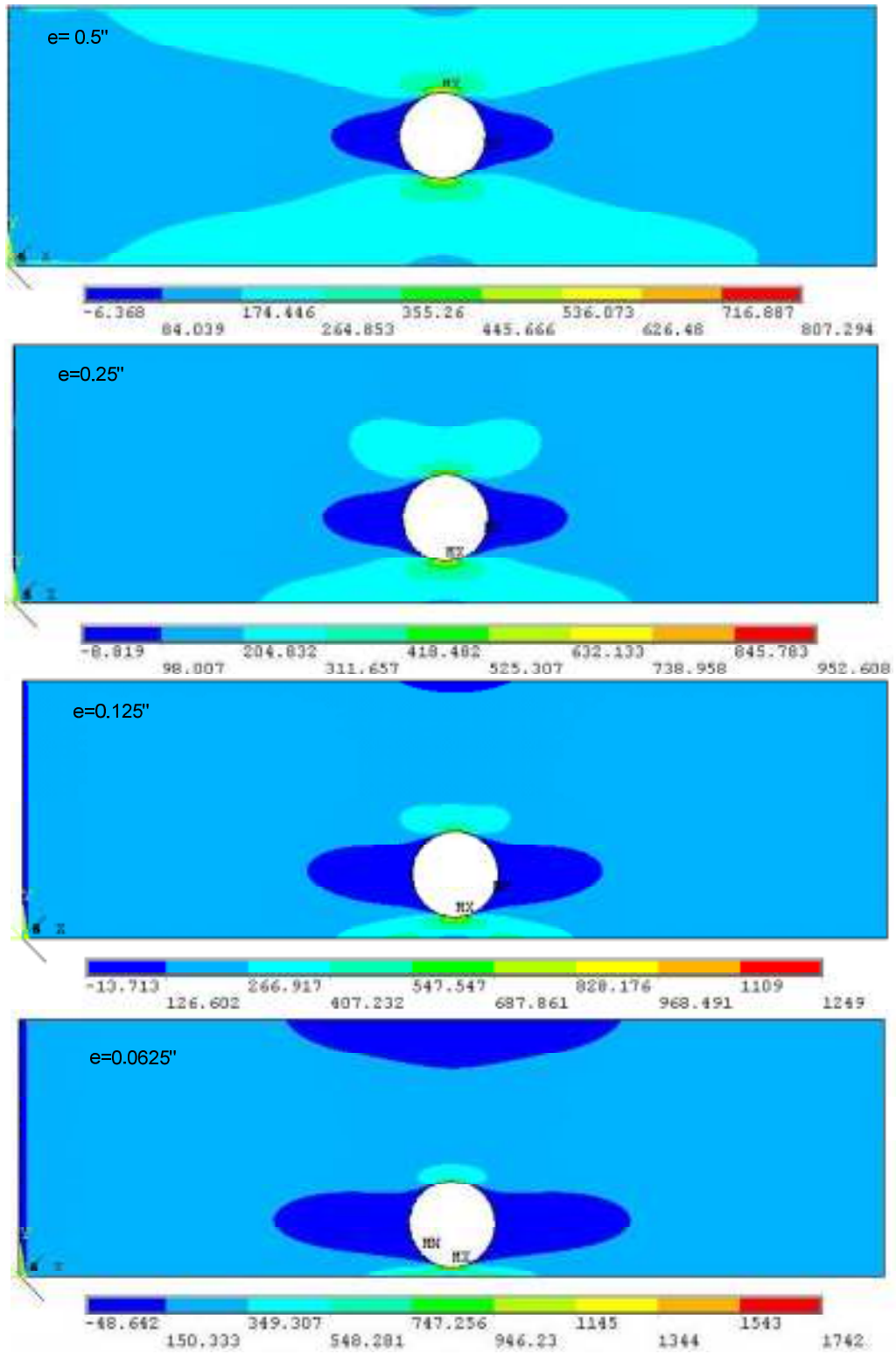


Figure 3. 10 Comparison of Stress Contours of σ_x for Different Values of “e” for 0° Ply of $[0_3/\pm 45/90]_S$

From comparisons of results for 0° plies we observe:

- 1) Among all the plies of composite laminate $[0_3/\pm 45/90]_s$, the highest magnitude stress occurs in 0° plies.
- 2) The highest magnitude of stress σ_x at a given ply occurs at the points on the hole periphery where fiber direction is tangential to the hole. For example, the higher stresses of 0° ply observed at $\theta=90^\circ$ and -90° at points A and B on the hole periphery as shown in Figure 3.11. For 45° ply highest magnitude stresses occurs at the points $\theta=135^\circ$ and -45° at points E and F on hole periphery as shown in Figure 3.11.

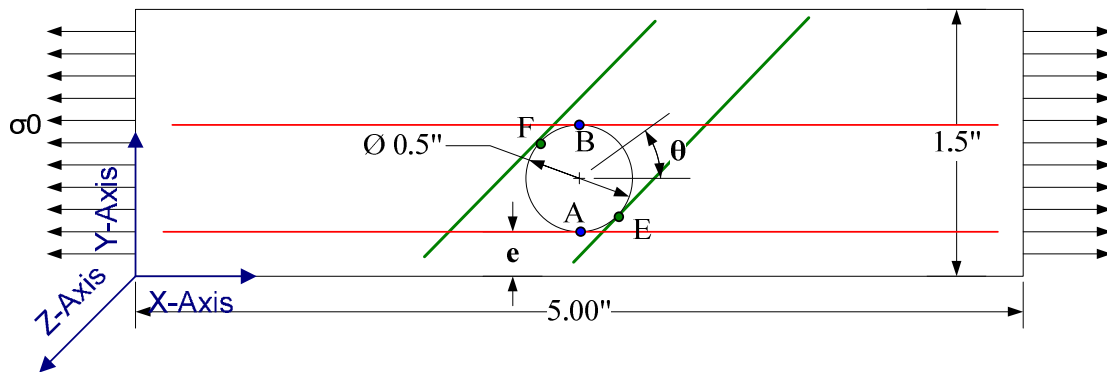


Figure 3. 11 Mechanics Showing First Fiber Tangent to Hole Periphery for 0° and 45° Plies

- 3) The magnitude of stress σ_x at points A and B for 0° layer increases, as the distance between hole to laminate edge (e) decreases. As the distance between hole to laminate edge (e) decreases, the net cross-section from hole edge to laminate reduces that causes increase in stress concentration. As shown in figure 3.10 above, the stress contour sky blue in color is the nominal stress region, stress contour in cyan color is higher stress region and stress contours shown by navy blue color is the zero to compression stress

region. We can observed from comparison that as “e” reduces, stress contours shown for zero to compression stress(navy blue) increases and higher stress concentration is occurs in the region shown by cyan color to maintain force equilibrium.

3.5.1 Effect of Edge Distance on 0° Ply Stress

1) The maximum stress of 0° ply (at points A and B as shown in Figure 3.11) versus the edge distance (e) is plotted in Figure 3.12. As indicated, the magnitude of stress at both points increases as the edge distance (e) reduces. It is also observed that the stress at point A increases more drastically than that of point B. By comparing magnitude of stress σ_x at point A when hole is located at centre of width of laminate with $e=0.5$ ”, if value of “e” is reduced by 50% stress σ_x at point A increases by 19%, if “e” is reduced by 75% stress σ_x at point A increases by 54 % and when “e” is reduced by 87.5% value stress σ_x at point A increases by 117%.

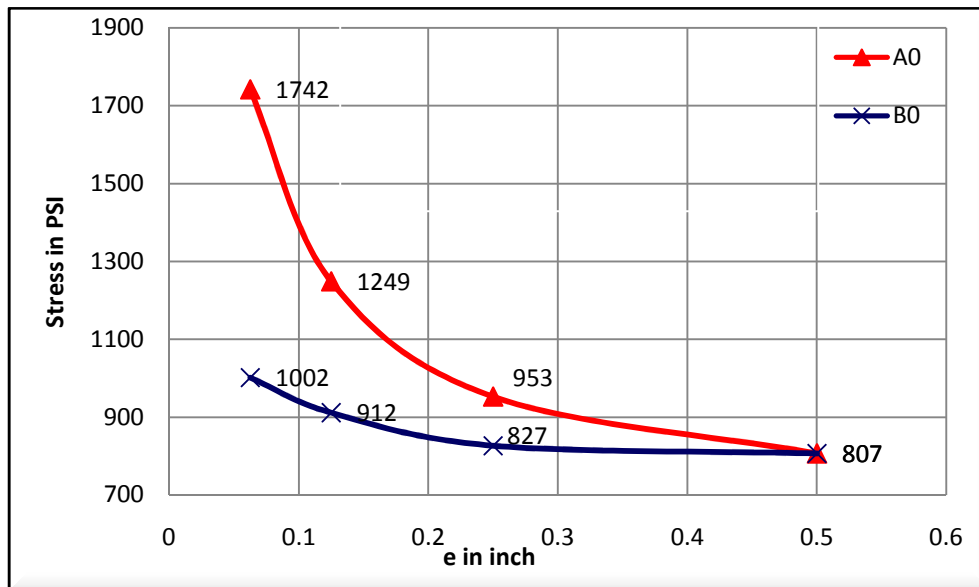


Figure 3. 12 Maximum Magnitude of Stress σ_x in 0° Ply Versus Hole to Laminate Edge Distance (e)

2) Figure 3.13 shows comparison of stress σ_x distribution along path CD transverse to the direction of loading, for various values of hole to laminate edge distances (e). As shown, σ_x distribution is steeper across the width of laminate when the edge distance reduces.

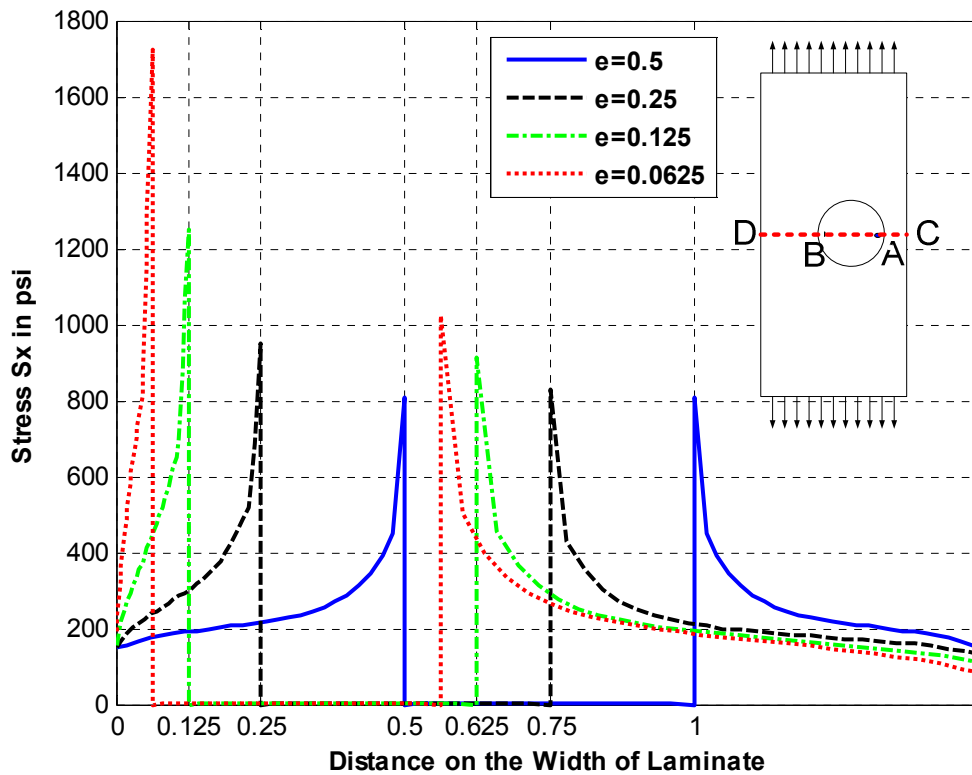


Figure 3. 13 Comparison of Stress σ_x Distribution of 0° Ply Across the Laminate Width for Various Hole to Laminate Edge Distances

3.5.2 Stress Distribution of 0° Ply Around the Hole

The stress σ_x distribution around periphery of the hole is shown in Figure 3.14. It is shown that the high stress gradient occurs at the neighborhood of $\theta = 90^\circ$ and 270° . The zero or near zero stress zone appears at the neighborhood of $\theta = 0^\circ$ and 180° . We can observe the positive stresses from approximately 45° to 135° and 225° to 315° .

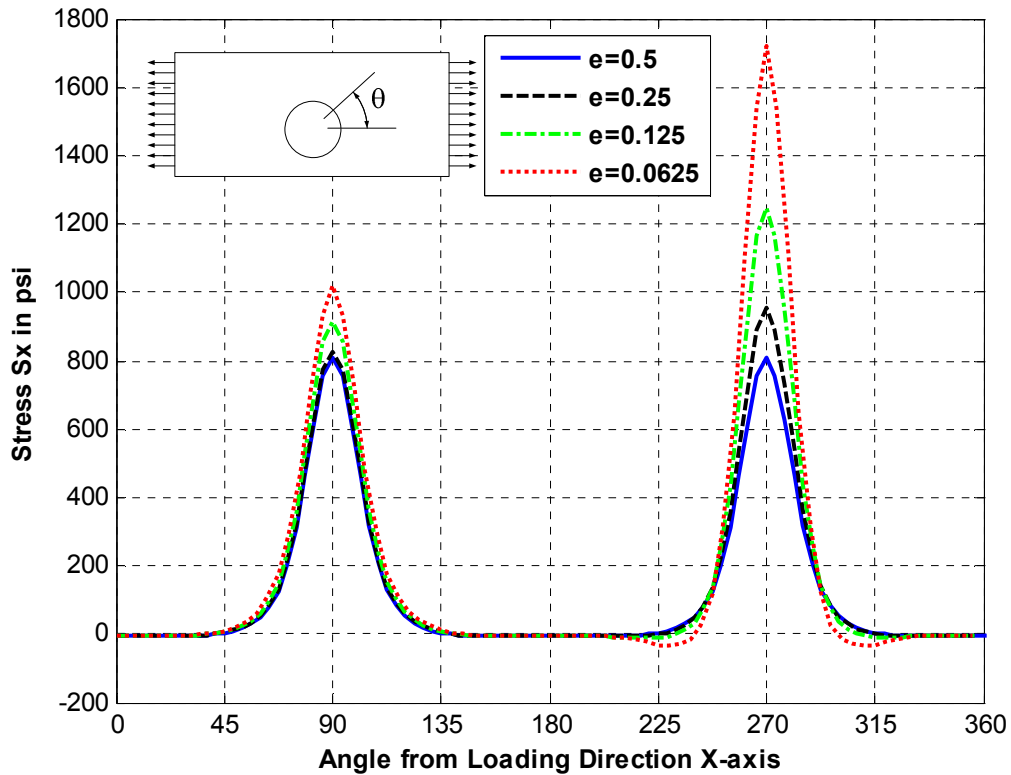


Figure 3. 14 Comparison of Magnitude of Stress σ_x Around Hole Periphery for Various Hole to Laminate Edge Distances

3.6 Results for $\pm 45^\circ$ Plies

A comparison of stress contours of stress σ_x , in $+45^\circ$ and -45° plies of $[0_3/\pm 45/90]_S$ laminate for various hole to laminate edge distances is shown in Figure 3.15 and 3.16 respectively. An unsymmetrical stress contours with respect to the width of the laminate is observed in both the figures. This is strongly suggested that the whole modeling of the entire laminate is essential. The maximum magnitude of stress at the periphery of hole is induced at points A and B located at angles $\theta_{A=}$ -67.5° and $\theta_B = 112.5^\circ$ respectively measured from loading direction x –axis. The shift of the angle from $\theta_{A=}$ -45° and $\theta_B = 135^\circ$ in the unreformed locations to the $\theta_{A=}$ -67.5° and $\theta_B = 112.5^\circ$ in

the deformed location is due to shear rotation of the 45° plies. This is also observed in the -45° plies.

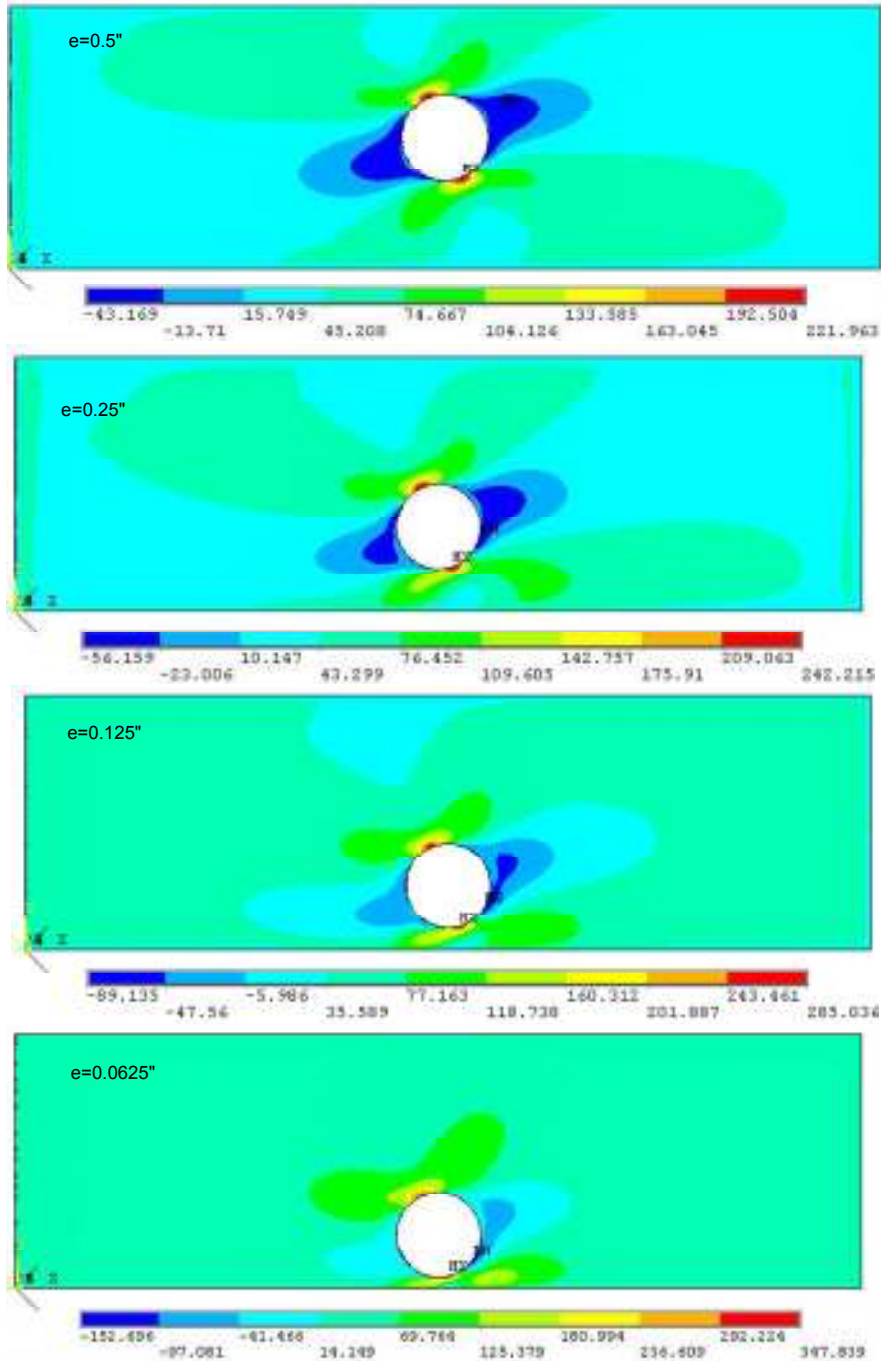


Figure 3. 15 Stress Contours of σ_x in 45° Ply of $[0_3/\pm 45/90]_S$ for Various Hole to Laminate Edge Distance

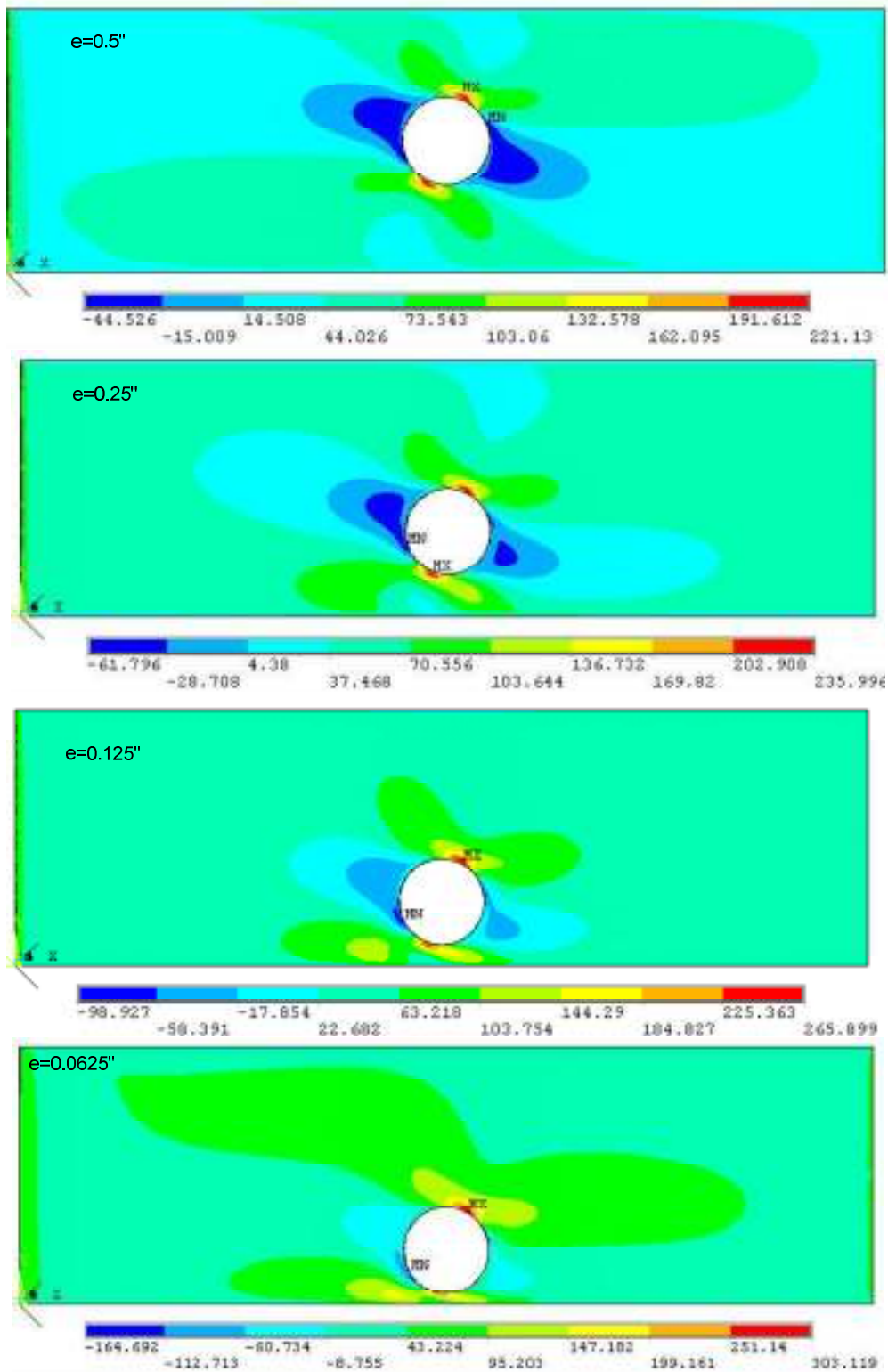


Figure 3. 16 Stress Contours of σ_x in -45° Ply of $[0_3/\pm 45/90]_s$ for Various Hole to Laminate Edge Distance

3.6.1 Effect of Edge Distance on σ_x in $\pm 45^\circ$ Plies

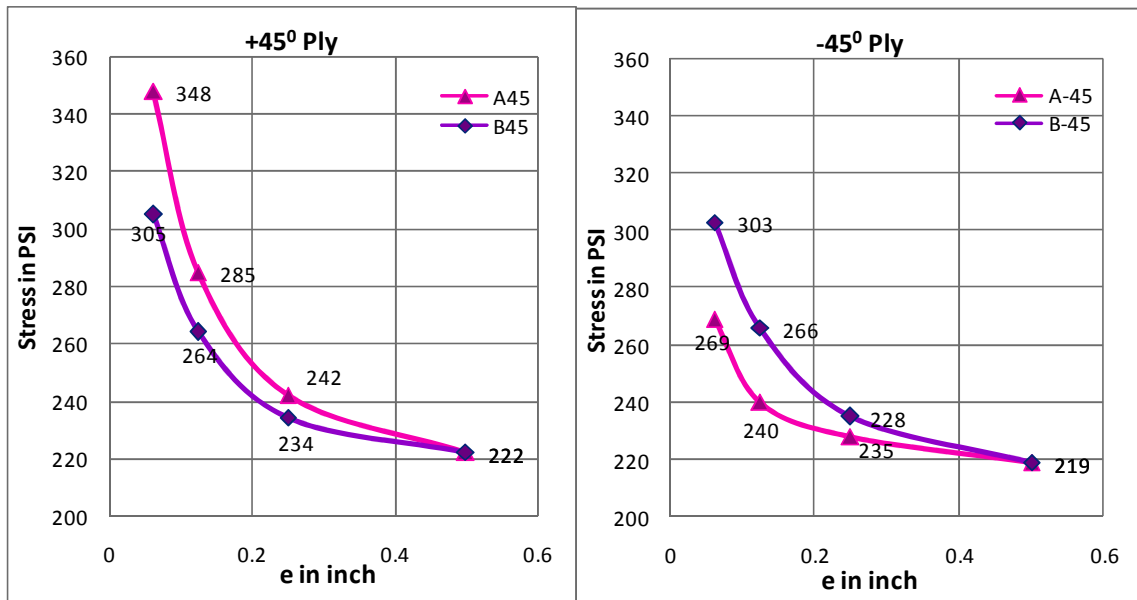


Figure 3.17 Graph Showing Comparison of Maximum Magnitude of Stress σ_x Versus Hole to Laminate Edge Distance for $\pm 45^\circ$ Plies

Figure 3.17 shows the graph of maximum magnitude of stress σ_x versus the edge distance e , for $+45^\circ$ and -45° ply respectively. It is seen that stress σ_x at point A in 45° ply is higher than point B in the same ply when the edge distance decreases. However, the trend is reversed for -45° ply. At $e=0.5''$, the maximum stress is almost identical for both $+45^\circ$ and -45° plies.

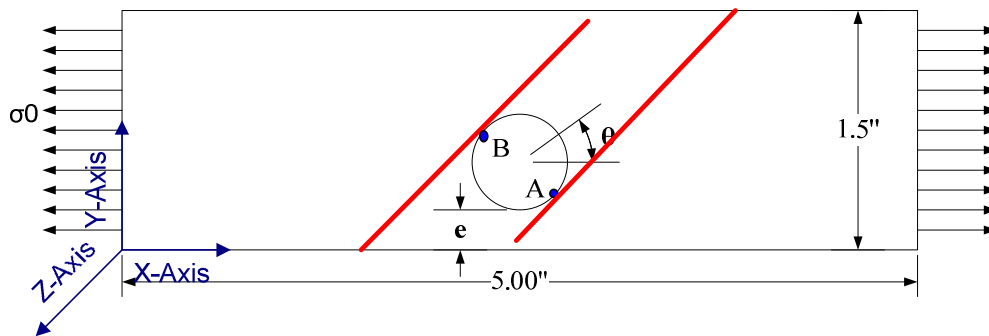


Figure 3.18 Mechanics Showing First Fiber Tangent to Hole Periphery for 45° Plies

As fibers in $+45^\circ$ ply (see Figure 3.18) are oriented at an angle, the first full length fibers tangent to hole periphery at $\theta=135^\circ$ at point B and at $\theta= - 45^\circ$ point A in undeformed shape. When $e=0.5''$, the maximum magnitude of stress on the periphery of hole occurs at a point located at angle $\theta_A= -67.5^\circ$. As hole edge comes closer to the laminate edge, at $e=0.0625''$ the angle increases to $\theta_A= -78^\circ$.

If hole is shifted from $e=0.5''$ to $e=0.0625''$, against 118% increase in magnitude of σ_x for 0° ply, for $+ 45^\circ$ ply the increase is only 56%. Thus change in value of stress concentration effect due to reduced hole to laminate edge distance for $+45^\circ$ ply is lower compared to 0° plies. As magnitude of maximum stresses induced in 0° plies is approximately 3 to 5 times greater than that of $+45^\circ$ plies, of $[0_3/\pm 45/90]_S$ laminate.

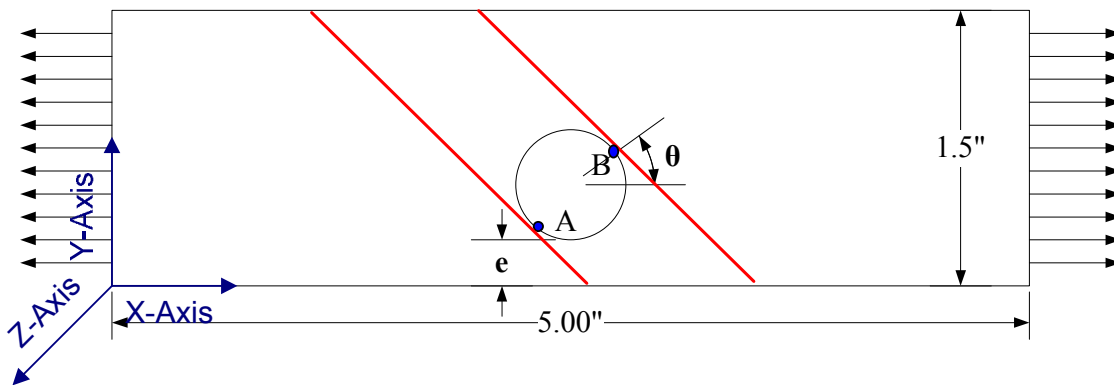


Figure 3. 19 Mechanics Showing First Fiber Tangent to Hole Periphery for -45° Plies

For $- 45^\circ$ ply(see Figure 3.19), the first full length fibers tangent to the hole periphery in undeformed state is at $\theta_A= - 135^\circ$ and at $\theta_B= 45^\circ$. For -45° ply the magnitude of maximum stress at the periphery of hole is shifted to the location where $\theta_A= -120^\circ$ and $\theta_B = 60^\circ$ respectively.

For the hole not located at centre of width of composite, maximum stress σ_x induced at point B is greater than at point A. For the $e=0.5''$, the point A on the hole periphery is located at an angle $\theta_A = -120^\circ$. As the edge of hole comes closer to laminate edge, at $e=0.0625''$, the angle $\theta_A = 104^\circ$.

3.7 Stress Concentration

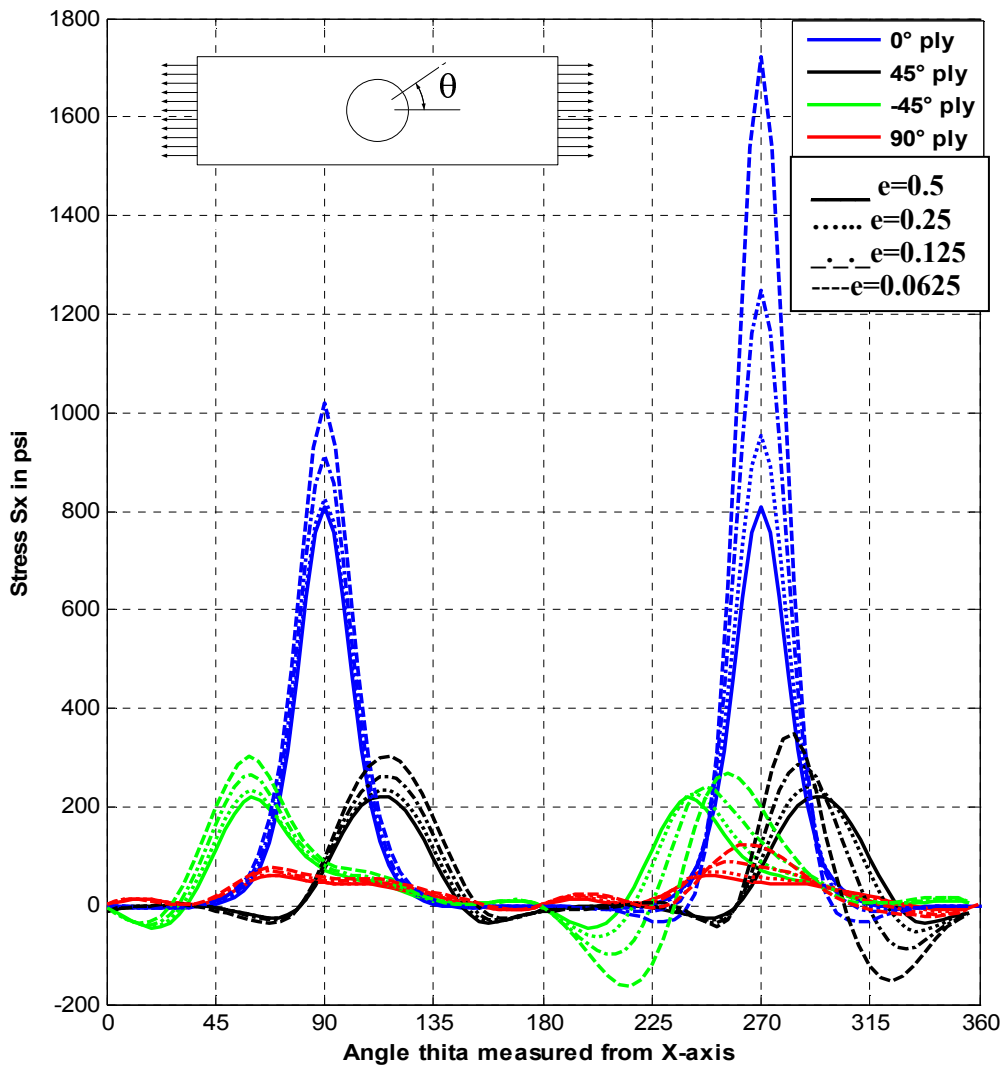


Figure 3. 20 σ_x Distribution of Each Ply at Periphery of Hole With Various Edge Distances

Figure 3.20 compiles all the ply stress distribution of stress σ_x along the periphery of hole with respect to different hole to laminate edge distances. In general, the peak stress of each ply increases as the edge distance reduces. The higher peak stress for a given value of e , the larger compressive stress near the neighborhood is observed at periphery of hole.

Table 3.2, lists the peak magnitude of stress σ_x at the hole periphery of each ply of composite laminate $[0_3/\pm 45/90]_S$, subjected to nominal stress $\sigma_0 = 100$ psi. The highest magnitude of induced stress σ_x in 0° plies is 3 to 5 times greater than that of $\pm 45^\circ$ plies. And 13 to 23 times greater than that of 90° plies.

Table 3. 2 Peak Magnitude Stress σ_x in Each Plies at the Hole Periphery

Angle of Fiber Orientation	Location	Edge to Edge Dist (e) in inch			
		0.0625	0.125	0.25	0.5
0	A0	1742	1249	953	807
	B0	1002	912	827	807
45	A45	348	285	242	222
	B45	305	264	234	222
-45	A-45	269	240	228	219
	B-45	303	266	235	219
90	A90	133	90	69	61
	B90	77	70	63	61

The value of stress σ_x in each angle ply of laminate $[0_3/\pm 45/90]_S$ without a hole and subjected to nominal stress of 100 psi is tabulated in Table 3.3. This stresses are considered as value of nominal stress for each individual ply of laminate.

Table 3. 3 Ply Stresses of Laminate $[0_3/\pm 45/90]_S$ without a Hole

Angle of Fiber Orientation	σ_x in psi
0	168.30
45	41.79
-45	41.79
90	11.49

The ply stress concentration for each ply is obtained by the peak stress divided by the ply nominal stress. These values are listed in Table 3.4.

Table 3. 4 Comparison of Stress Concentration Factor for Each Plies of Laminate

Angle of Fiber Orientation	Location	Stress Concentration factor for different (e) in inch			
		0.0625	0.125	0.25	0.5
0	A0	10.35	7.42	5.66	4.80
45	A45	8.33	6.82	5.79	5.31
-45	B-45	7.25	6.37	5.62	5.24
90	A90	11.58	7.83	6.01	5.31

Table 3.4 illustrates that higher stress concentration factor occurs at 90° and 0° plies. The table indicates that the effect of edge distance is most pronounced in 90° ply and next in 0° ply. The $\pm 45^\circ$ plies are less significant. Since the 90° ply is less stressed, it is often ignored when the total laminate failure is considered.

3.8 Interlaminar Stresses σ_z , τ_{xz} and τ_{yz} for Single Hole

Figure 3.21 represents the stress distribution of σ_z through the laminate thickness at the points A and B located on the periphery of hole. Stress distribution of σ_z for various hole to laminate edge distances are also included. As shown, the maximum σ_z occurs at mid plane of laminate.

The results are also shown that σ_z becomes higher when the hole is closer to the laminate edge (smaller). Comparing the magnitude of stress σ_z between points A and B, we found that the stress at point A is generally greater than that at point B. This indicates that the free edge of laminate plays significant effect on the stress σ_z at point A.

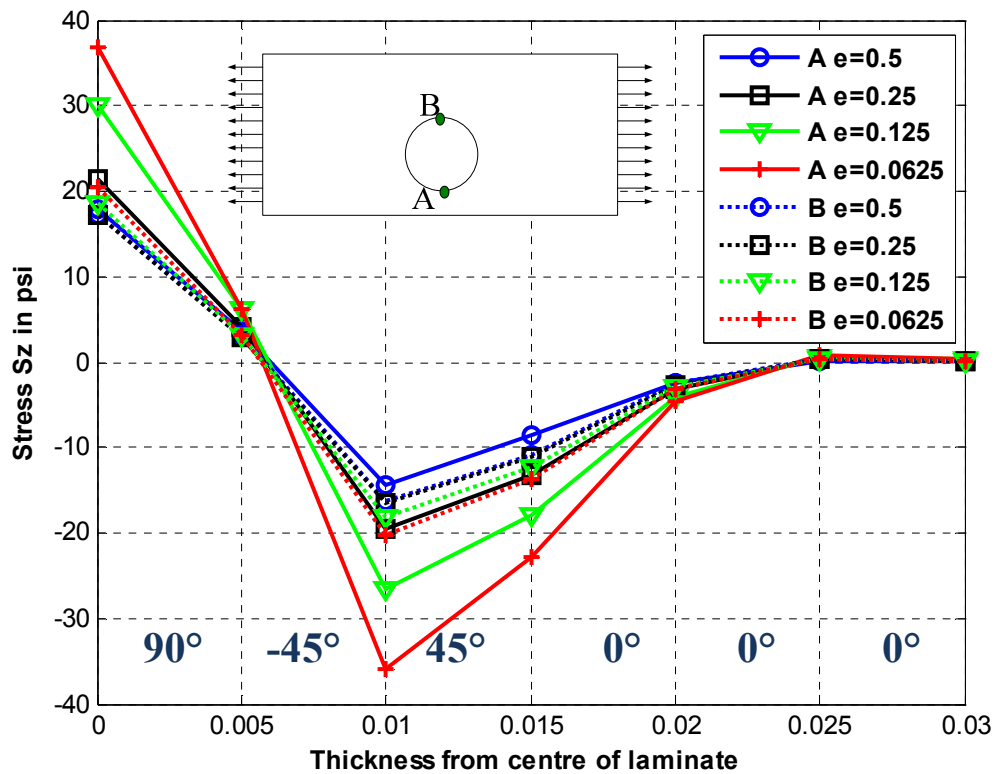


Figure 3. 21 σ_z Distribution Through Thickness of Laminate

Figure 3.22 and 3.23 show the interlaminar shear stress distribution of τ_{xz} and τ_{yz} , respectively. Because the shear stress allowable is irrelevant to tension and compression, the sign of shear stress is ignored. It is observed that the maximum magnitude of τ_{xz} occurs at interface of $-45^\circ/+45^\circ$ plies for the smallest edge distance ($e=0.0625$). However, the maximum magnitude of τ_{yz} occurs at the interface of $90^\circ/-45^\circ$ plies for the smallest edge distance ($e=0.0625$). It is also interesting to note that the magnitude of shear stresses at point A is greater than that at point B. This also suggests the free edge of laminate does affect the interlaminar shear stress distribution.

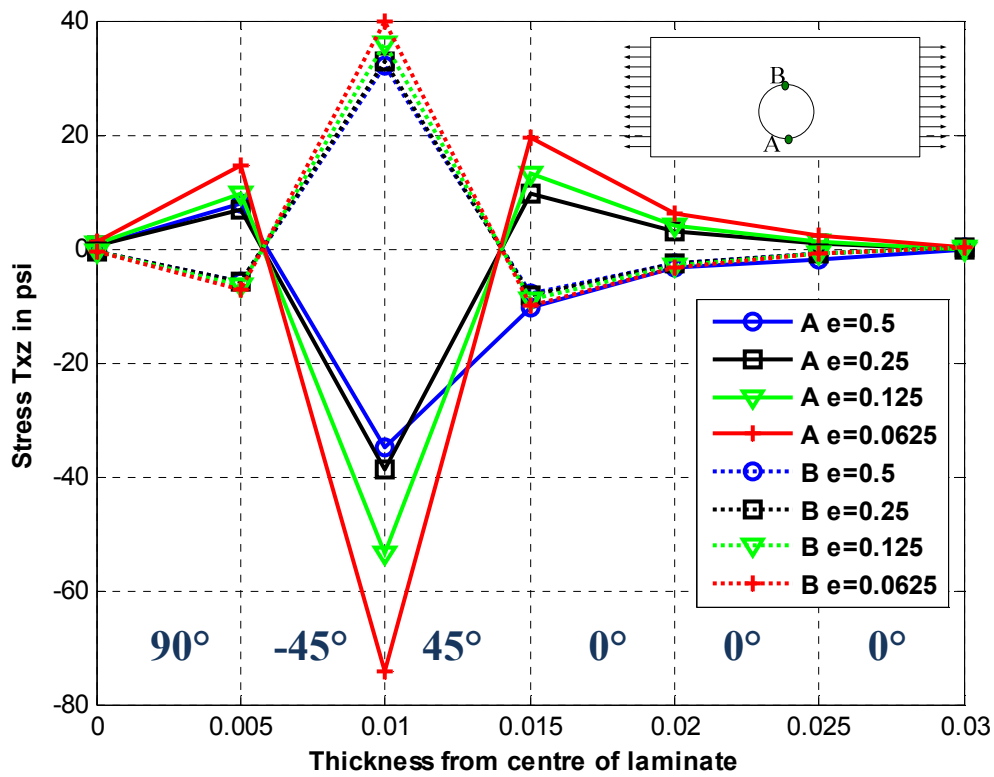


Figure 3. 22 τ_{xz} Distribution Through Thickness of Laminate

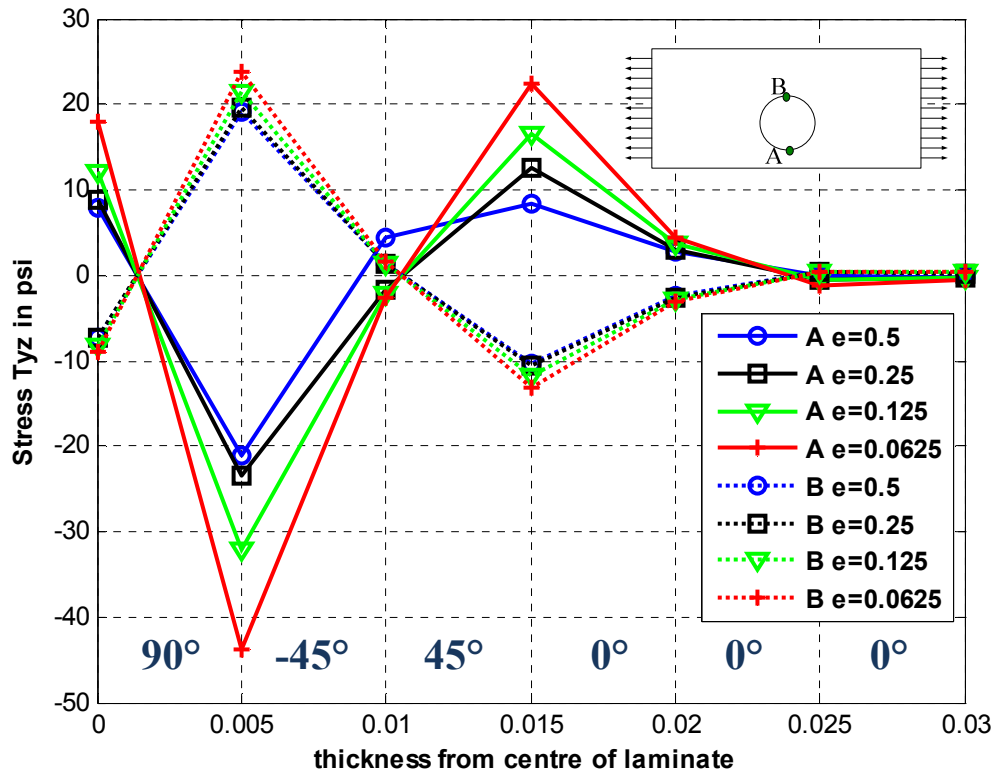


Figure 3. 23 τ_{yz} Distribution Through Thickness of Laminate

CHAPTER 4

MULTIPLE HOLES

Stress distribution and stress concentrator factor of finite width composite laminate with multiple holes are discussed in this chapter. The following patterns of multiple holes are discussed:

- 1) Two holes oriented in line with the loading direction
- 2) Two holes oriented transverse side by side to loading direction
- 3) Three holes placed at three vertices of triangle.

4.1 Two Holes Oriented In-Line to Load

4.1.1 Geometrical Parameters and Finite Element Meshes

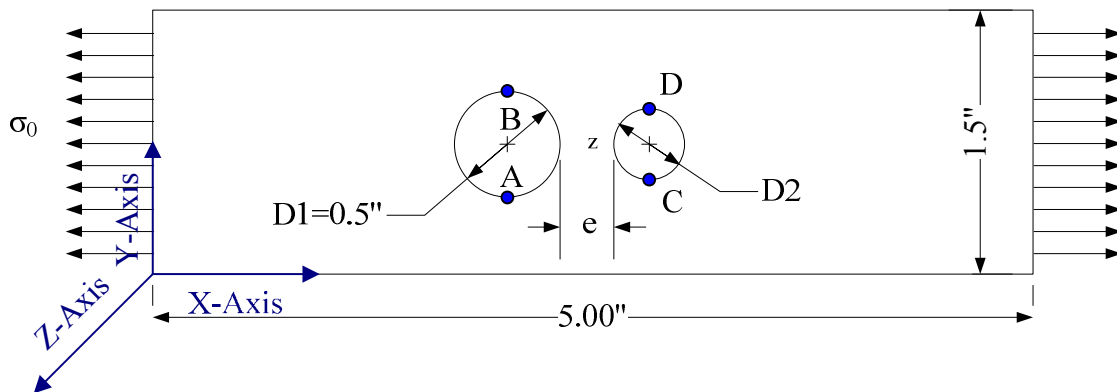


Figure 4. 1 Geometrical Parameters for Two Holes Placed In-Line with Loading Direction

Figure 4.1 shows various geometrical parameters used for the case study of two holes placed in-line with loading direction along x-axis. The finite element model used for study, material properties, boundary conditions and loading conditions were discussed in Chapter 2. The in-plane nominal stress $\sigma_0=100$ psi is applied to the laminate along x-axis. $\frac{D1}{D2}$ (Hole size ratio) is the ratio of diameter of larger hole to smaller hole. The hole size ratio used in this study ranges from 1,2 and 4. Diameter of larger hole $D1=0.5''$ is fixed in the entire study. Edge distance (e) which is the distance between two holes, is in term of size of large hole diameter.

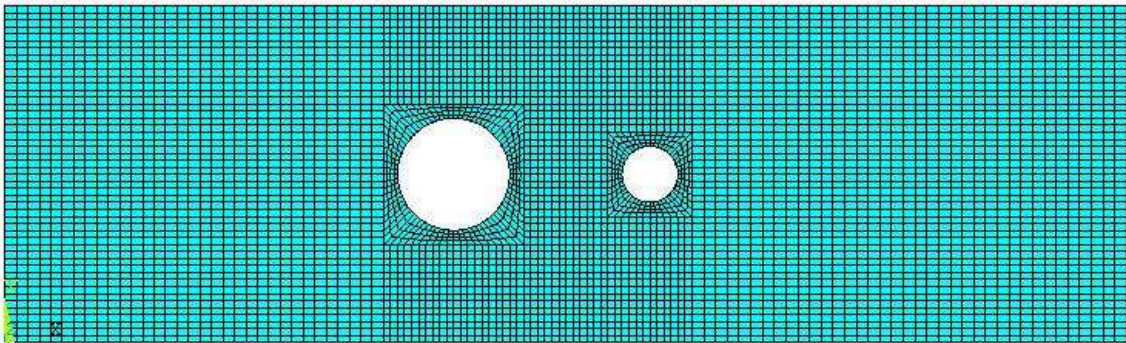


Figure 4. 2 Finite Element Mesh with $D1/D2= 0.5, e=0.5''$

Figure 4.2 shows a typical meshing used for the finite element model for $\frac{D1}{D2} = 0.5$ and $e = 0.5''$. The mesh size has been selected as discussed in Chapter 3(refer 3.2).

4.1.2 Peak Stresses for Each Ply

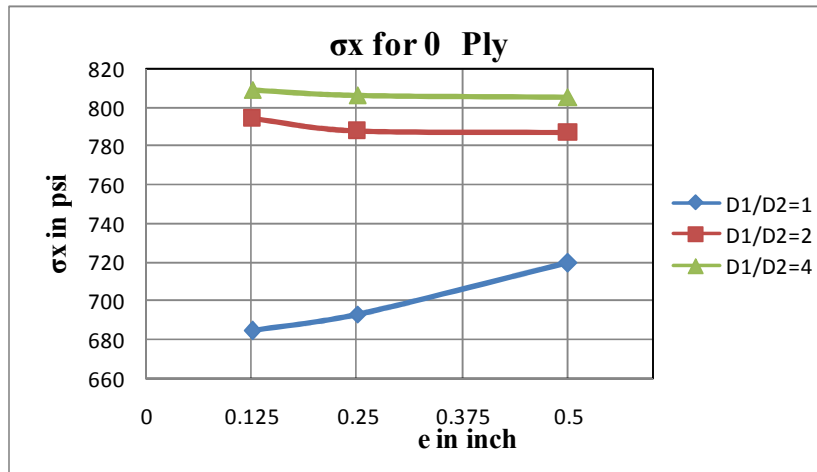
Table 4.1 summarizes the peak stress magnitude of each ply. The data in the table are graphed in Figure 4.3. The results illustrate that for two equal holes (hole size ratio equal to 1) the peak stresses reduces as the hole spacing reduces. While for unequal holes

the peak stresses increases as the hole spacing reduces Moreover, for a given hole spacing, the peak stresses increases as the hole size ratio increases.

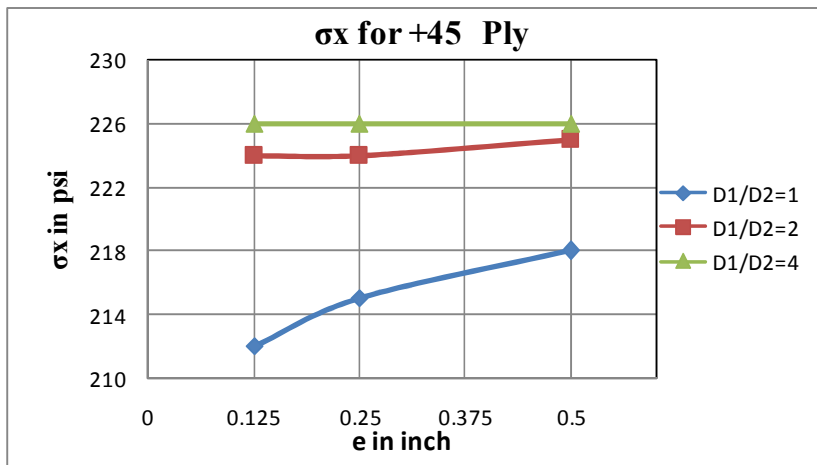
Table 4. 1 Peak Stress σ_x of Each Ply with Different Hole Size Ratio and Hole Spacing

Fiber Orientation	σ_x in psi for different "hole spacing" for two holes placed in-line to load		
	0.125	0.25	0.5
	D1/D2=1		
0	684	693	719
45	212	215	218
-45	208	211	215
90	54	55	56
	D1/D2=2		
0	794	788	787
45	224	224	225
-45	216	216	217
90	57	58	59
	D1/D2=4		
0	809	806	805
45	226	226	226
-45	217	217	217
90	59	59	59

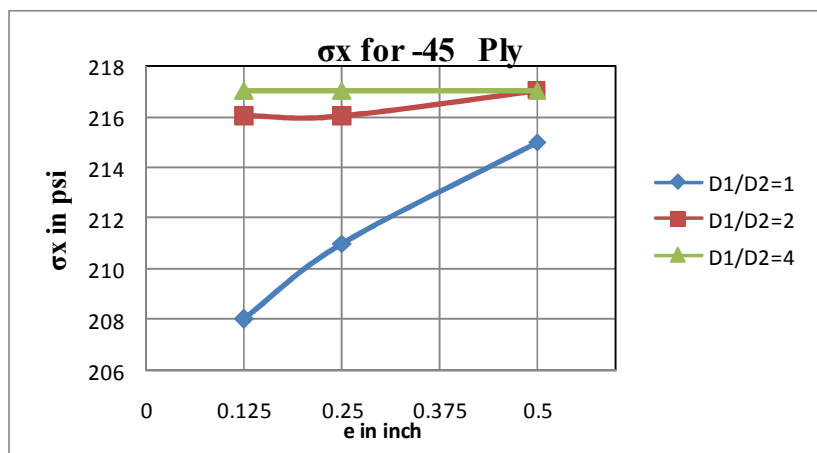
As shown in Figure 4.3, for 0° ply the peak magnitude of stress σ_x exhibits significant increase as two identical sized holes move apart. And remains insignificant decrease if smaller hole move away from the larger hole. For both $+45^\circ$ and -45° plies, the peak magnitude of stress σ_x changes insignificantly regardless the hole spacing.



(a)



(b)



(c)

Figure 4. 3 Peak Magnitude of σ_x for Various Hole Size Ratios Versus Hole Spacing (a) 0° Ply (b) +45° Ply (c) -45° Ply

4.1.3 Effect of Hole Size Ratio, $\frac{D1}{D2}$

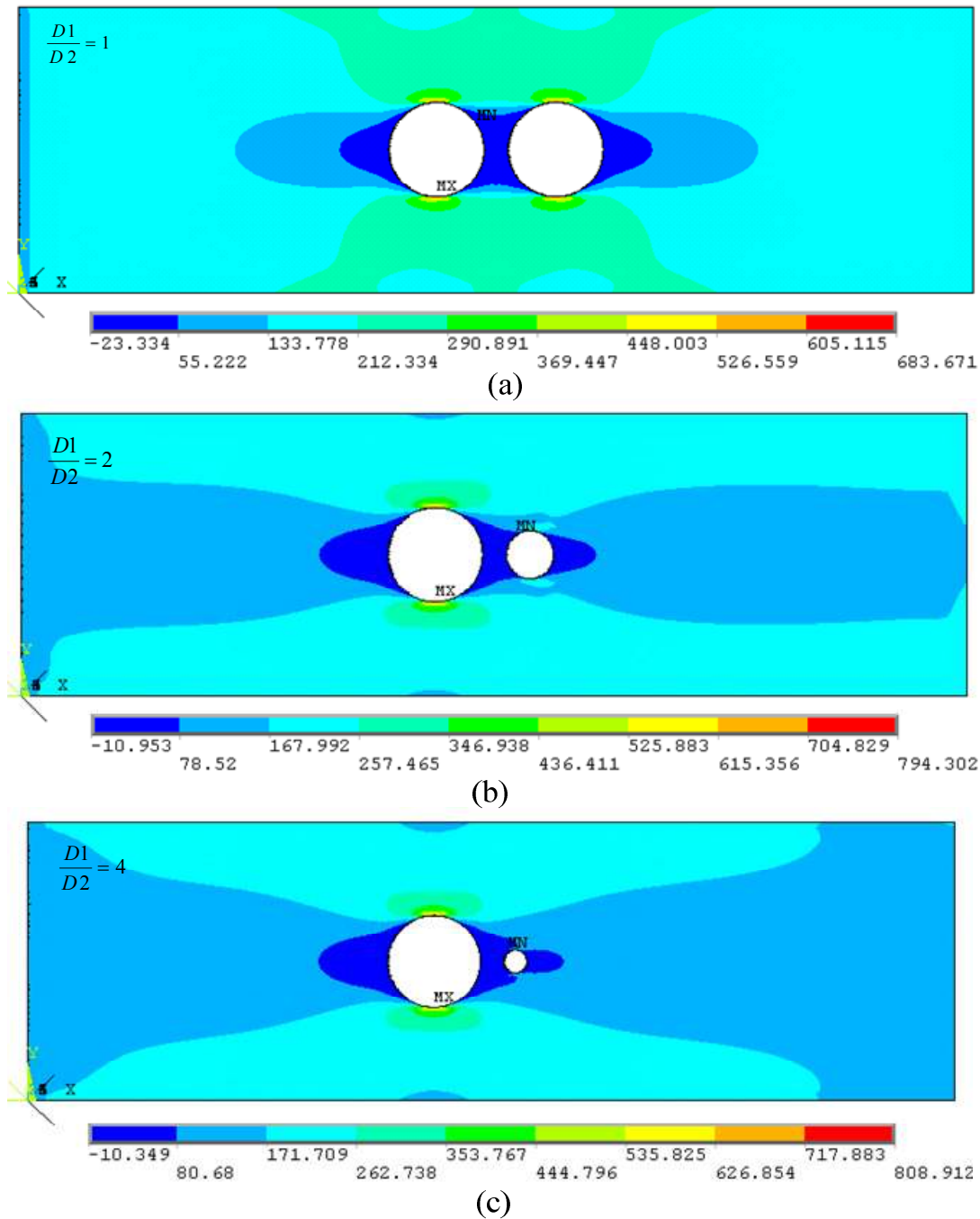


Figure 4. 4 Contour Plot of Stress σ_x with $e=0.125''$ and $D1=0.5''$ for 0° Ply for (a) $\frac{D1}{D2} = 1$ (b) $\frac{D1}{D2} = 2$ (c) $\frac{D1}{D2} = 4$

Figure 4.4 Shows the stress σ_x contours of 2nd 0° ply of laminate $[0_3/\pm 45/90]_s$ with different hole size ratios. The hole size ratios, $\frac{D_1}{D_2}$ ranging from 1 to 4 while keeping the hole spacing of $\frac{D_1}{4}$ ($= 0.125$). It is observed that the peak stress always occurs at the larger hole. The peak stress increases as the hole size ratio increases. As shown in Figure 3.7 of Chapter 3, the imaginary force flux lines encounter around hole periphery, pass through the net section area. When another hole is present, the force flux lines redistribute into the net section (see Figure 4.5). If the two holes are identical, both the net sections are of the same size (see Figure 4.8). Hence force flux lines becomes relatively uniform compared with the case of different hole sizes. As results peak stress is reduced compared with a single hole.

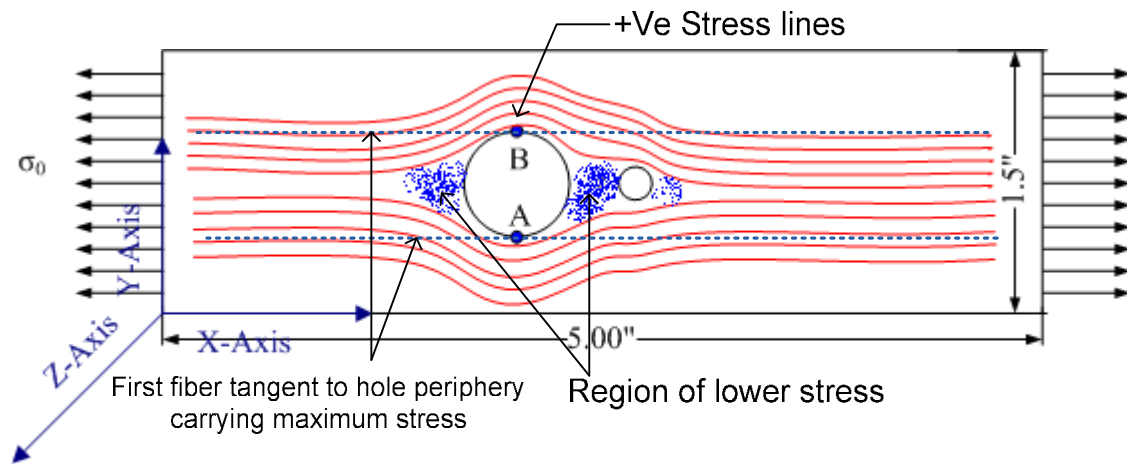


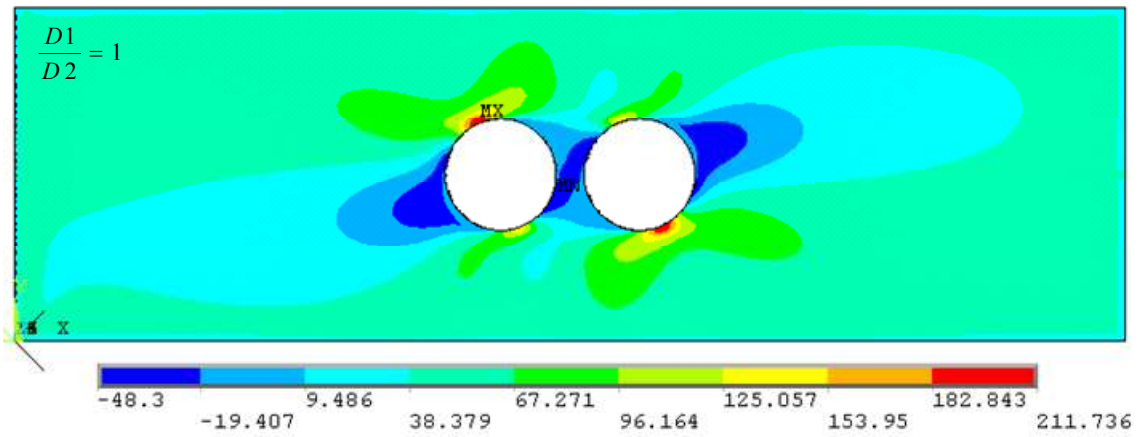
Figure 4. 5 Imaginary Constant Force Flux Lines in 0° Ply Around Two Holes

It is also noted that the peak stress of $\sigma_x = 809$ psi for 0° plies with $\frac{D_1}{D_2} = 4$, is nearly same as that for single hole $\sigma_x = 807$ psi. This means that, no considerable effect on

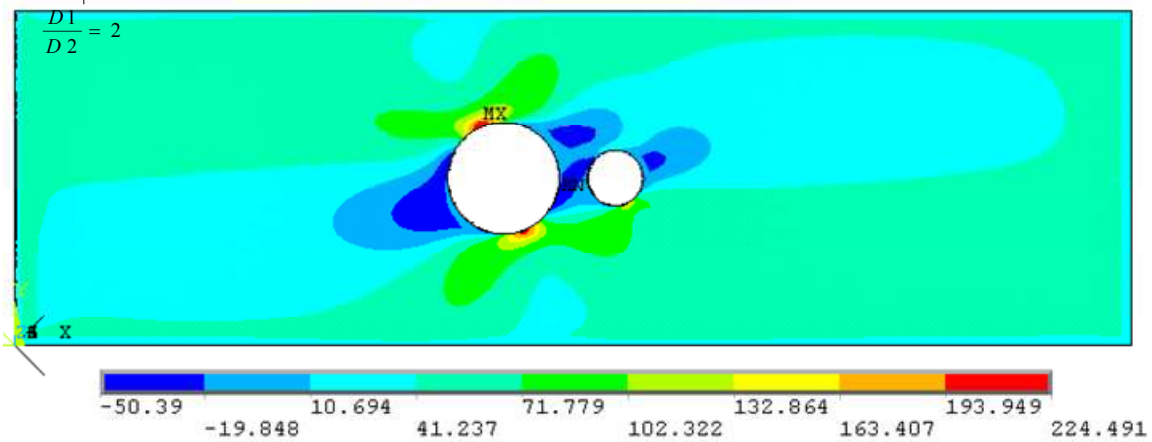
the stress concentration of laminated composite due to presence of smaller hole, if diameter ratio $\frac{D1}{D2} \geq 4$.

For $\pm 45^\circ$ plies as fibers are not oriented in direction of loading, these plies carry lower magnitude of stresses in $[0_3/\pm 45/90]_S$ laminate. The maximum value of stress σ_x in 0° ply is approximately 3 times greater than $\pm 45^\circ$ plies. Figure 4.6 shows comparisons of stress contours of σ_x , for various hole size ratios for $+45^\circ$ ply. From Table 4.1, it can be concluded that maximum value of stress σ_x do not change significantly with change in hole size ratio for $\pm 45^\circ$ plies, because the 3 layers of 0° plies carry majority of stresses applied to laminate.

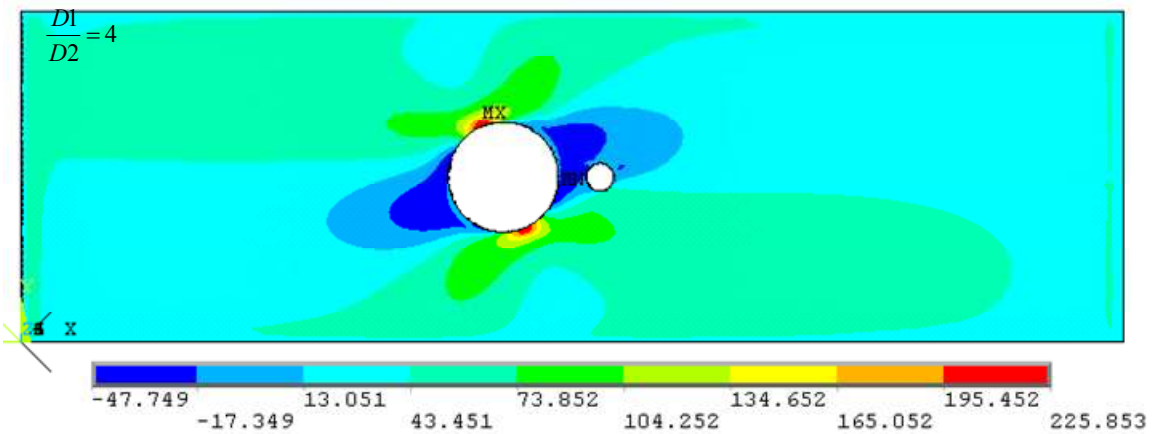
Figure 4.6 shows the stress contours of σ_x for $+45^\circ$ plies. As indicated, the peak stress σ_x of these plies occurs at point where their fibers do not interface by other hole. It is also shown that the peak stress is located at the point slightly away from the point where fiber is tangent to hole periphery in the undeformed state. The point shift is due to shear of $+45^\circ$ ply.



(a)



(b)



(c)

Figure 4. 6 Contour Plot of Stress σ_x for +45° Ply with $e=0.125''$, $D1=0.5''$ and (a) $\frac{D1}{D2} = 1$ (b) $\frac{D1}{D2} = 2$ (c) $\frac{D1}{D2} = 4$

4.1.4 Effect of Hole Spacing (e)

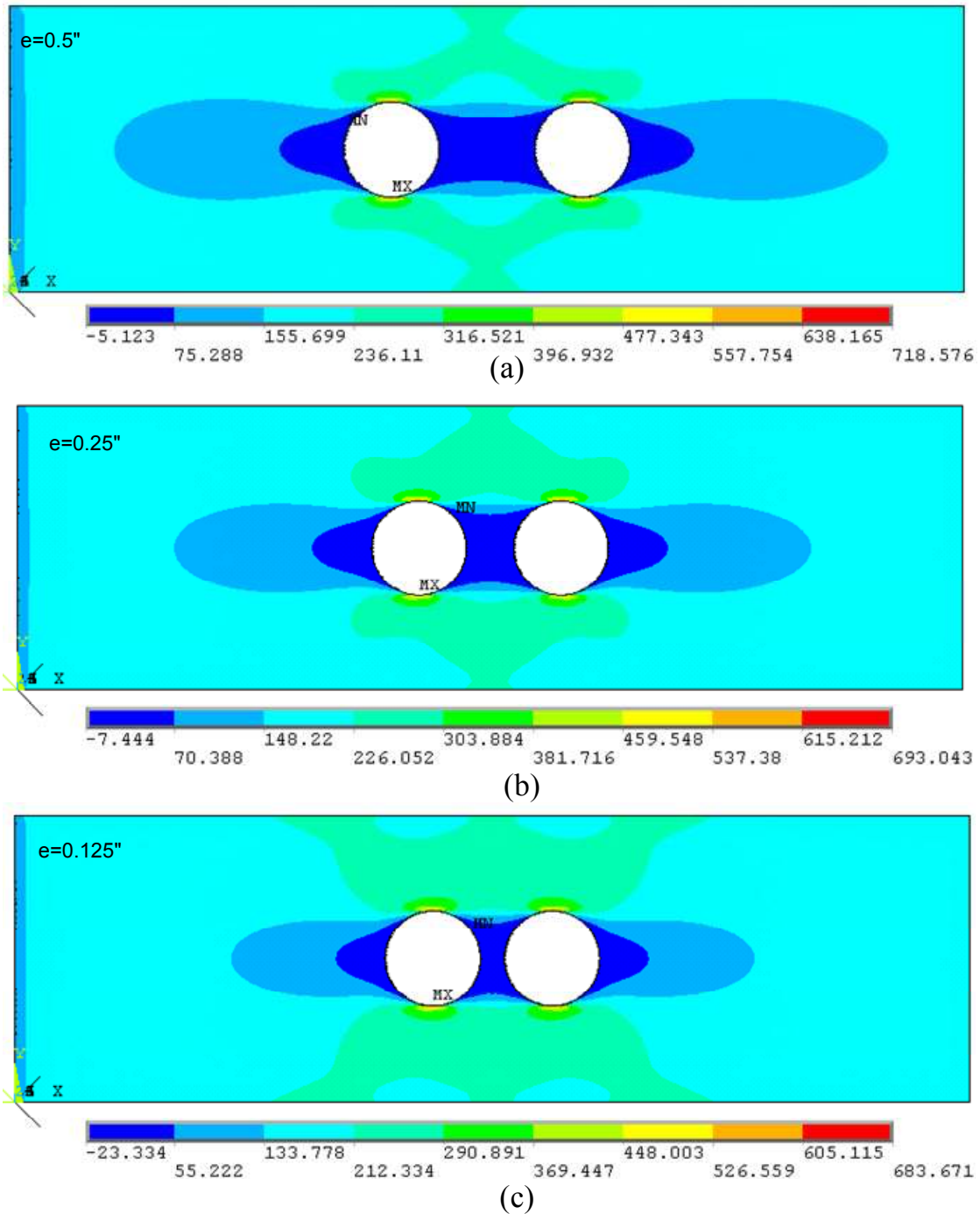


Figure 4.7 Contour Plot of Stress σ_x for 0° Ply with $\frac{D1}{D2} = 1$ and $D1=0.5''$ when (a) $e=0.5''$ (b) $e=0.25''$ (c) $e=0.125''$

For laminate with two equal diameter holes ($\frac{D_1}{D_2}=1$), as the hole spacing (e) decreases (holes comes closer), the maximum value of stress σ_x also decreases for all angle plies of laminate. This can be observed from stress contours of σ_x for 0° ply as shown in Figure 4.7. As hole spacing (e) increases, the ineffective region of fibers which do not carry the load increases (see Figure 4.8). Subsequently the force flux redistributes within this zone. If two identical size holes are located closer to each other, the force flux will distribute more uniformly. As a result, the peak stress reduces.

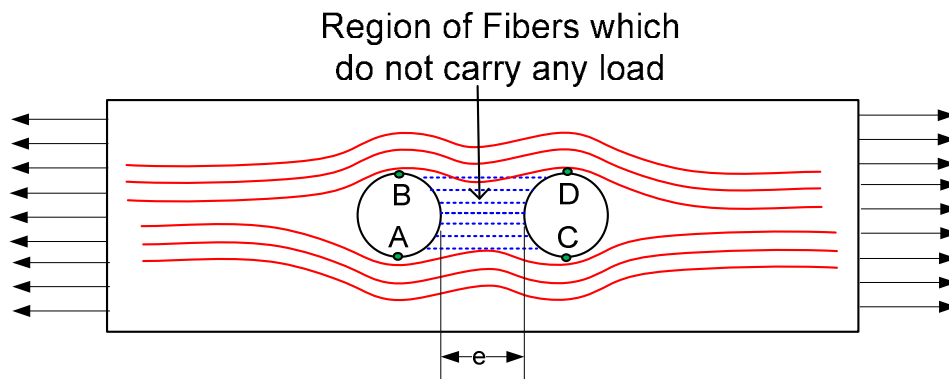


Figure 4. 8 Imaginary Force Flux Lines for Two Equal Size Holes

For hole size ratio $\frac{D_1}{D_2} > 1$, as hole spacing (e) decreases the peak stress σ_x changes insignificantly. Because as holes comes closer, greater portion of the smaller hole is covered by (lowest value of) stress contours generated by larger hole (seen by dark blue region in Figure 4.9). For hole size ratio $\frac{D_1}{D_2} > 4$, the peak stress σ_x exhibits as the value for a single hole.

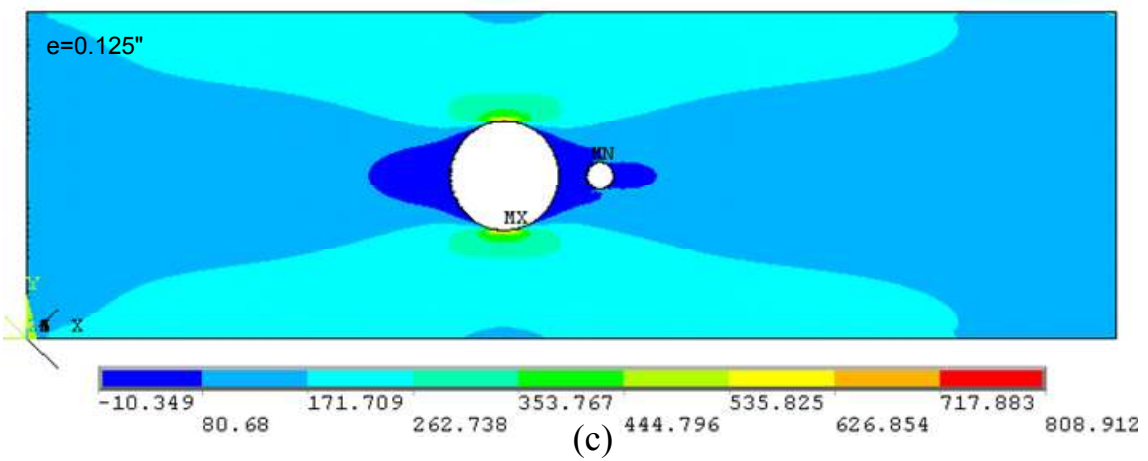
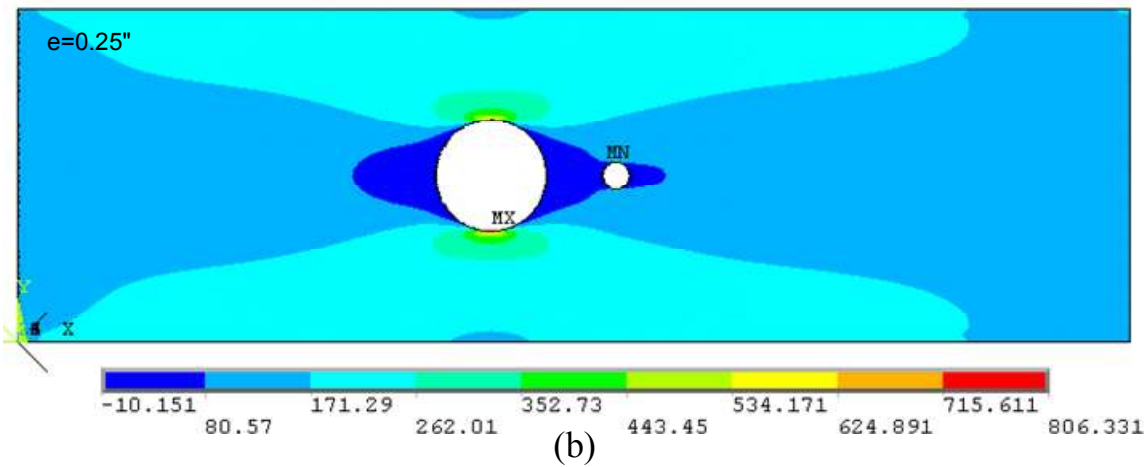
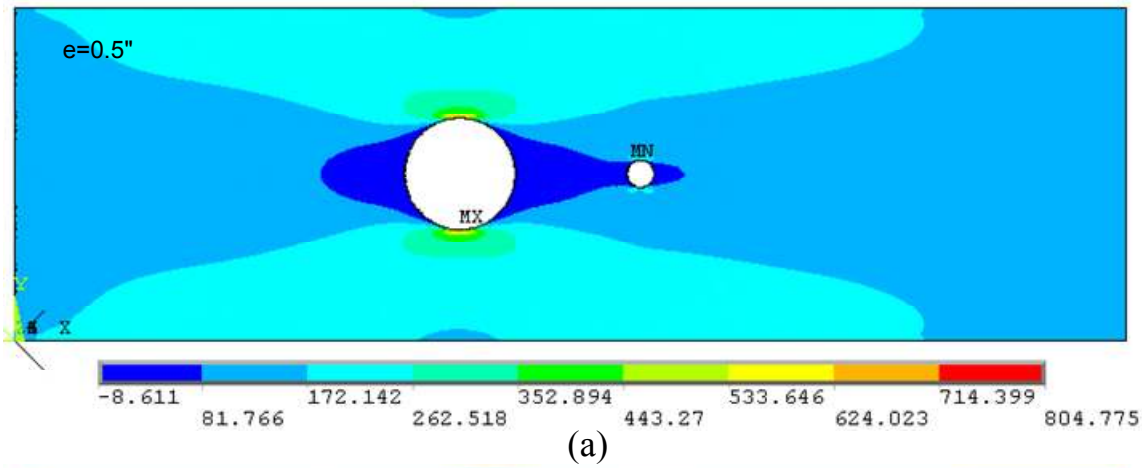


Figure 4. 9 Contour Plot of Stress σ_x for 0° Ply with $\frac{D1}{D2} = 4$ and $D1=0.5''$ and (a) $e=0.5''$ (b) $e=0.25''$ (c) $e=0.125''$

4.2 Two Holes Oriented Side by Side Transverse to Load

4.2.1 Geometric Parameters and Finite Element Meshes

Figure 4.10 shows various geometrical parameters of two holes placed side by side transverse to the loading direction, x-axis. The finite element model used for study, material properties, boundary conditions and loading conditions has been already discussed in Chapter 2. The in-plane nominal stress $\sigma_0=100$ psi, applied to the laminate along x-direction. As shown in figure, the size of larger hole ($D1$) and the distance of its edge to laminate edge is fixed as $D1=0.5''$ and $0.125''$, respectively. Hole size ratio ($\frac{D1}{D2}$) is ratio of diameter of larger hole to smaller hole. The hole spacing (e) is the distance between edges of holes placed side by side. The hole size ratios of 1 and 2 were considered and hole spacing (e) ranges from 0.0625, 0.125 and 0.25 were used in this study.

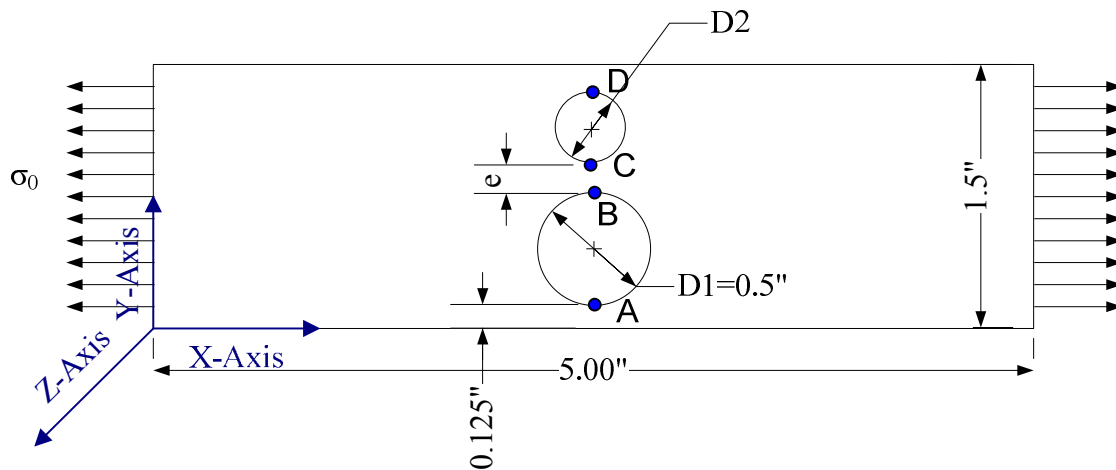
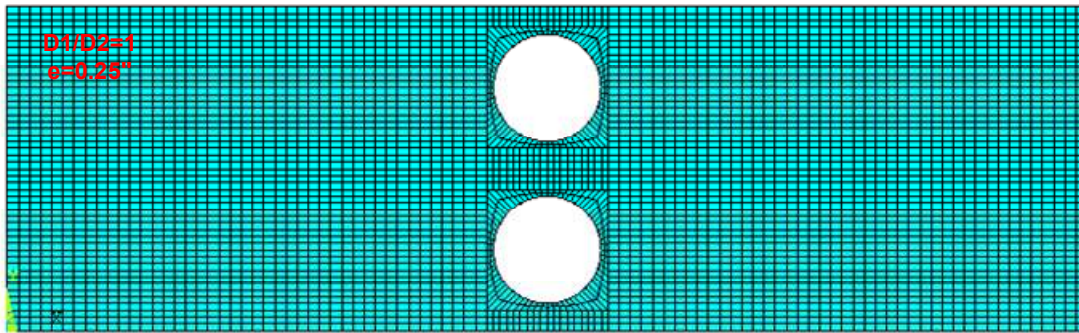
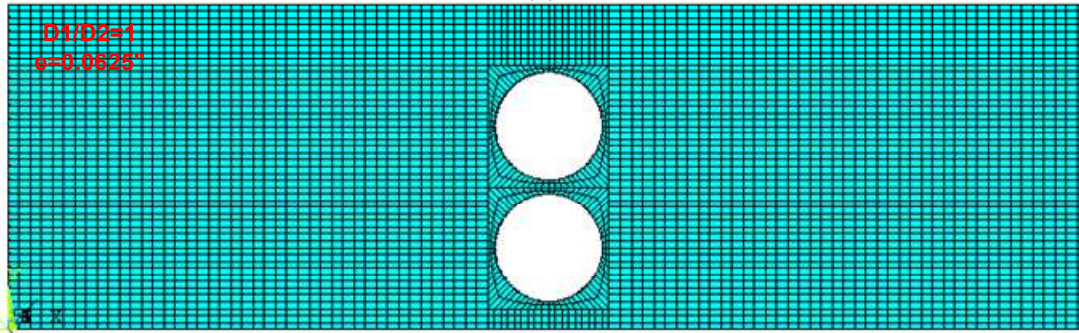


Figure 4. 10 Geometric Parameters Used for Two Holes Transverse to Load

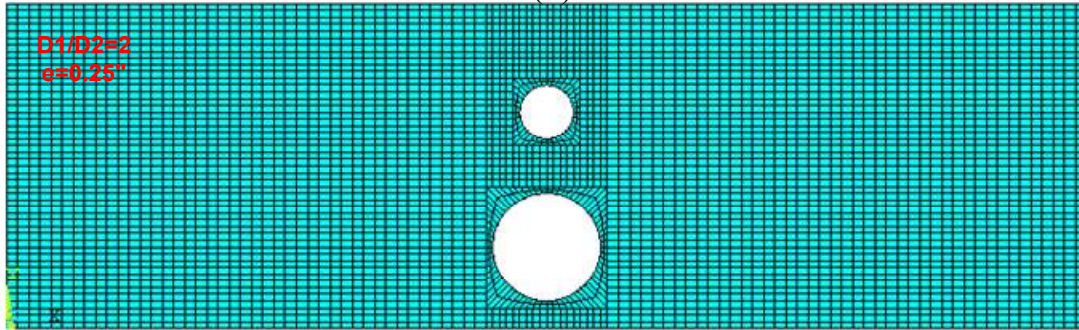
Figure 4.11 shows the meshes for various cases used in this study. The aspect ratio of elements is the same as used in previous section. Table 4.2 shows the number of elements and nodes used for each kind of model.



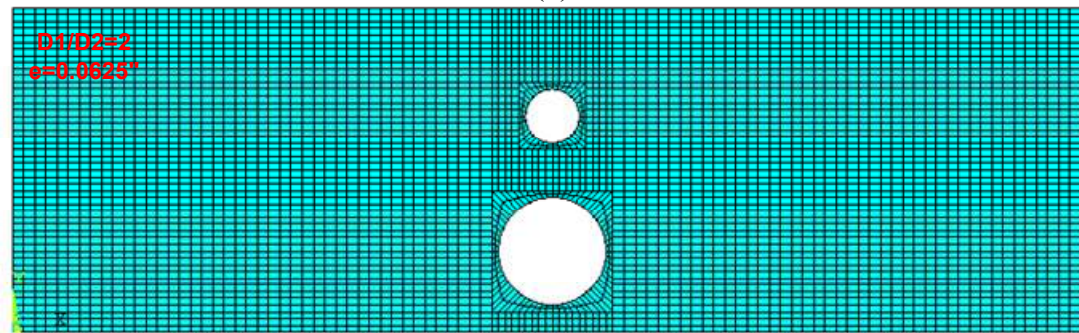
(a)



(b)



(c)



(d)

Figure 4. 11 Finite Element Meshes Used for (a) $\frac{D1}{D2} = 1$, $e=0.25$ ” (b) $\frac{D1}{D2} = 1$, $e=0.0625$ ” (c) $\frac{D1}{D2} = 2$, $e=0.25$ ”
 (d) $\frac{D1}{D2} = 2$, $e=0.0625$ ”

Table 4. 2 No. of Elements and Nodes used with Two Holes Transverse to Load

	e=0.25		e=0.125		e=0.0625	
	D1/D2=1	D1/D2=2	D1/D2=1	D1/D2=2	D1/D2=1	D1/D2=2
Elements	27792	28752	27792	28752	27792	28752
Nodes	129531	133531	129531	133531	129531	133531

4.2.2 Peak Value of σ_x

Table 4.3 shows peak values of stress σ_x , for each ply of $[0_3/\pm 45/90]_S$ laminate with different hole size ratio and hole spacing. The peak stress σ_x for second layer of 0° ply has listed in table. The variation of σ_x among 0° plies is very small. For $\pm 45^\circ$ ply, angular location of point A,B, C and D varies, than that shown in figure 4.10 (as discussed in Section 3.6).

Table 4. 3 Maximum Stress σ_x at the Hole Periphery with various Hole Size ratios and Hole Spacing

Fiber orientation for ply	Maximum value of stress σ_x											
	e=0.25				e=0.125				e=0.0625			
	A	B	C	D	A	B	C	D	A	B	C	D
	D1/D2=1											
0	1227	1048	1048	1227	1412	1317	1252	1052	1568	1772	1708	1079
45	288	334	334	288	330	384	370	281	367	434	427	297
-45	242	328	328	242	278	372	360	256	309	398	392	276
90	78	87	87	78	100	93	89	73	112	130	125	76
	D1/D2=2											
0	1284	960	863	792	1343	1059	1119	887	1406	1228	1585	1001
45	300	301	288	241	314	337	349	278	329	389	411	317
-45	251	290	261	219	263	322	319	253	276	358	355	289
90	92	71	64	58	96	80	83	66	100	98	118	73

4.2.3 Effect of Hole Size Ratio ($\frac{D1}{D2}$) and Hole Spacing (e) for 0° Plies

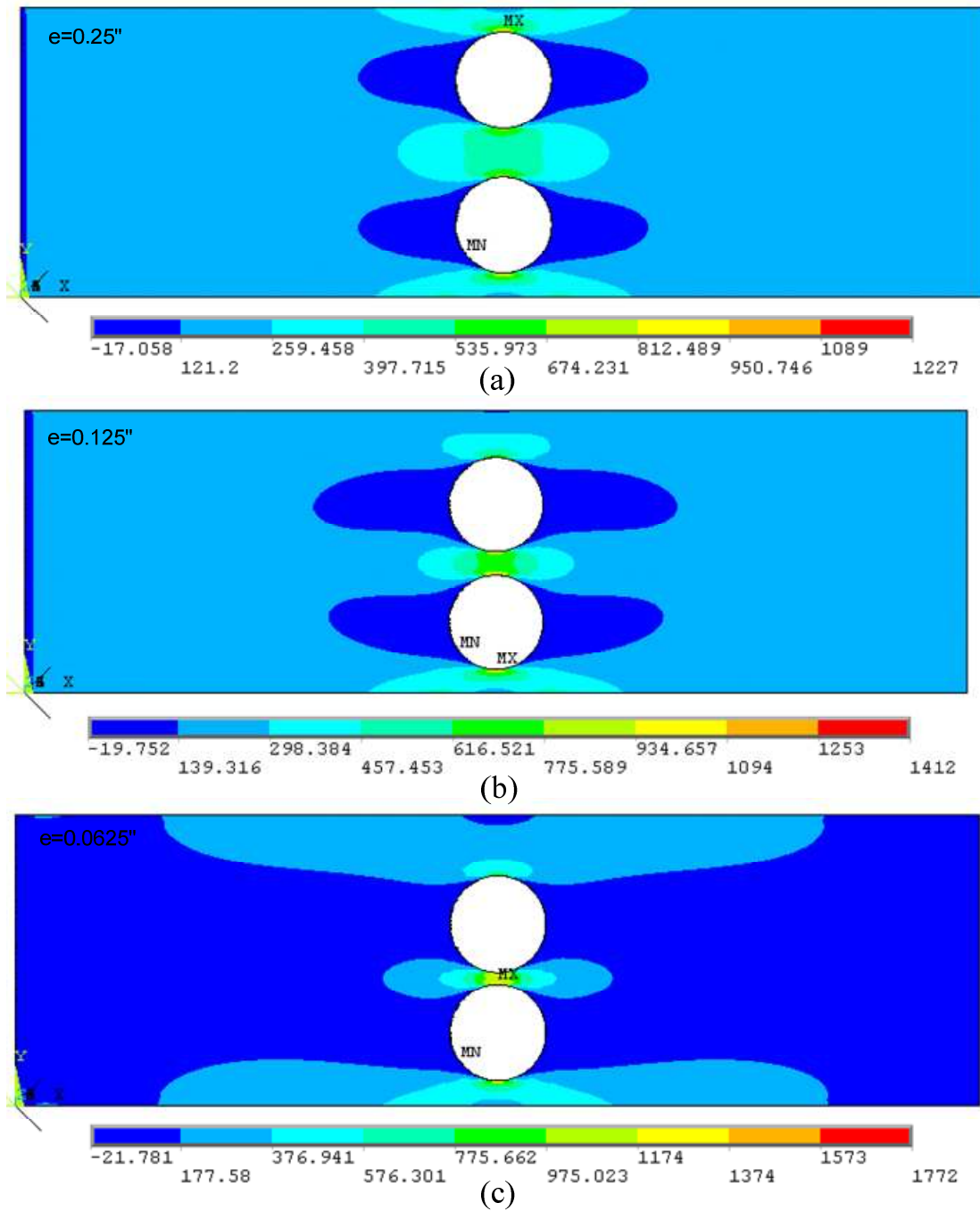


Figure 4. 12 Comparison of Stress Contours of σ_x for 0° Ply with $\frac{D1}{D2} = 1$ and (a) $e= 0.25$ (b) $e=0.125$ (c) $e= 0.0625$

Figure 4.12 shows stress contours of σ_x for 0° ply, $\frac{D1}{D2} = 1$ with various hole spacing while keeping fixed distance of one hole edge to laminate edge. As shown, the maximum stress of σ_x occurs at the point of hole edge closest to the laminate edge. However if the spacing of two holes is 0.0625, the maximum stress occurs at the point where the two holes face each other. Comparing the maximum magnitude of σ_x , we found that reducing the hole spacing between two identical sized holes results in increase of the maximum stress magnitude.

When two equal size holes placed side by side with edge distance between holes $e=0.25''$, the magnitude of stress σ_x at point A remain nearly same as that of single hole. While stress σ_x at point B is greater than that of a single hole. Because the stress concentration effect at periphery of both holes (points B and C shown in figure 4.26) interact with each other. More flux lines passes through region between two holes.

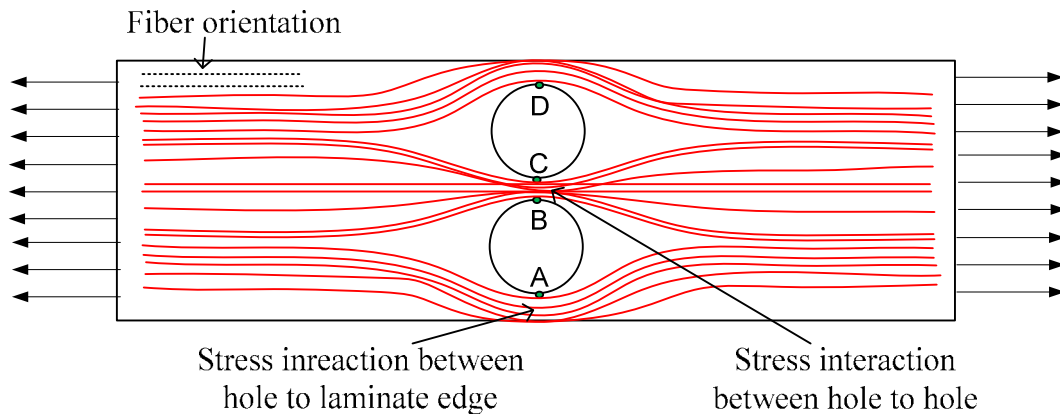


Figure 4. 13 Imaginary Constant Force Lines for Two Holes Transverse to Load

With $e=0.25''$ the stress magnitude at point A is greater than at B, because distance between point A to laminate edge is $0.125''$ compared to distance between the points B and C of two holes which is $0.25''$. As net cross- section between hole to

laminated edge is low, more stress flux lines concentrate in region between point A to laminated edge as shown in figure 4.13.

Stress interaction between holes becomes more severe as the hole comes closer further. So when $e=0.125''$ the value of stresses at point B and C increases further but still stress concentration at point A is higher than at B. But when $e=0.0625''$ hole interaction effect is predominant and value of stresses at point B is greater than at A. Magnitude of stress σ_x at point B is 26% higher for $e=0.125''$ than that of σ_x when $e=0.25''$, and 69% higher when $e=0.0625''$ than that of σ_x when $e=0.25''$. As shown in figure 4.24, the dark blue region increases as hole spacing (e) reduces. Dark blue region shows lowest value of stress contour, as distance between holes reduces region of low stress increases and stress concentration occurs at periphery of holes increases to maintain force equilibrium in the laminate.

When the size of second hole is reduced, the magnitude of stress σ_x is also reduced. This is the different case when two holes are lined along the loading direction (see values in Table 4.3). The stress contours of this case are shown in Figure 4.14. For two unequal sized holes with diameter ratio $\frac{D_1}{D_2} = 2$, the maximum magnitude of stress σ_x is less compared to that for two equal size holes and same value of hole spacing (e). As larger net cross-section to resist stress applied in this case for fixed width (1.5'') of laminate composite used in both case. For two unequal sized holes ($\frac{D_1}{D_2} = 2$) with $e=0.125''$ the maximum value of σ_x occurred at point C is 30% greater than that for $e=0.25''$ and for $e=0.0625''$ it is 84% greater than that for $e=0.25''$.

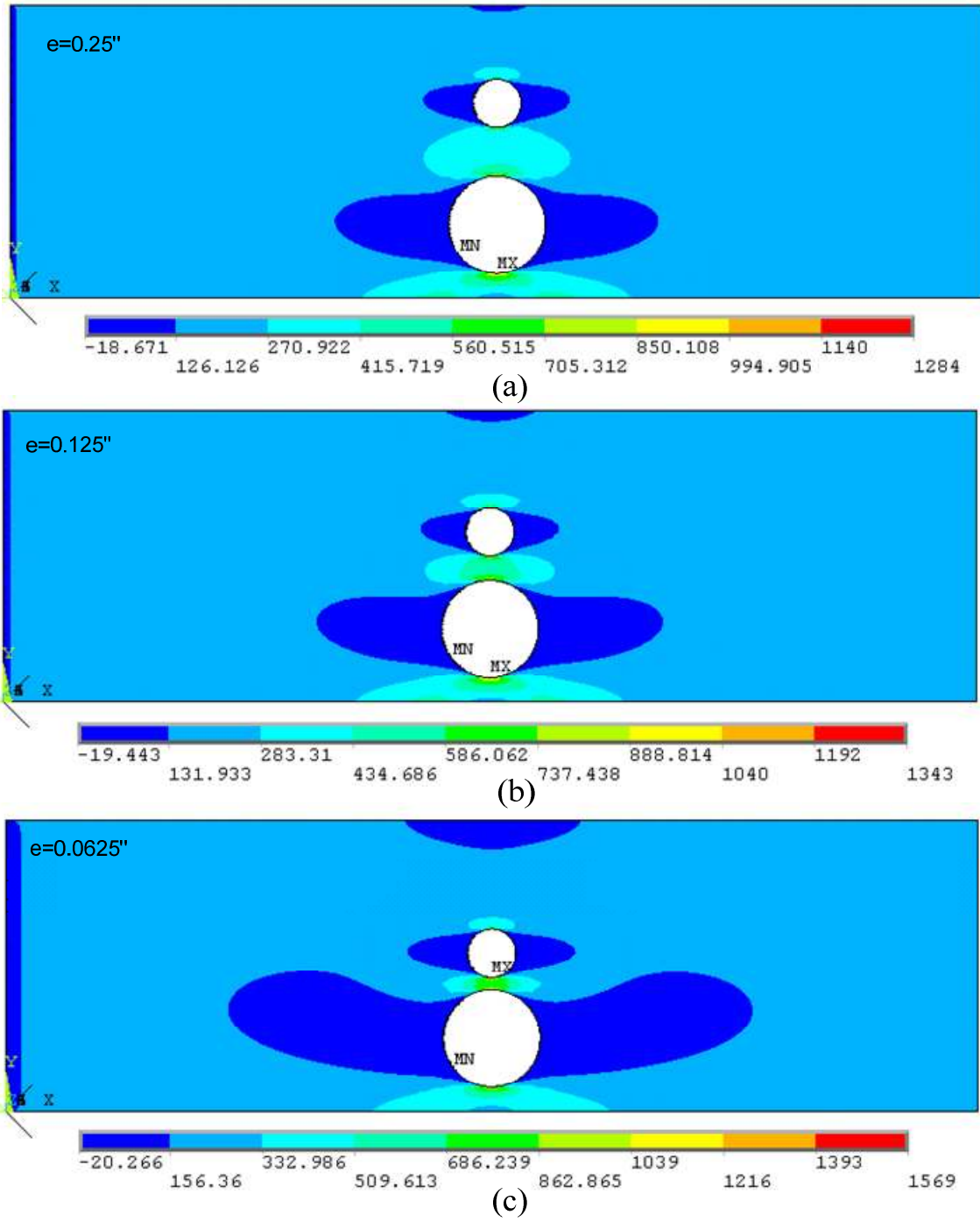


Figure 4. 14 Comparison of Stress Contours of σ_x for 0° Ply with $\frac{D1}{D2} = 2$ and (a) $e= 0.25$ (b) $e=0.125$ (c) $e= 0.0625$

4.2.5 Effect of Hole Size Ratio and Hole Spacing for $\pm 45^\circ$ and 90° Plies

Figures 4.15 and 4.16 shows comparison of stress contours of σ_x for $\pm 45^\circ$ plies, for different values of hole size ratios and hole spacing.

For all hole size ratios the peak stress σ_x , increases as holes comes closer to each other. Although the change in magnitude of stress σ_x for $\pm 45^\circ$ plies is less than compared to that for 0° plies. It is observed that the effect of hole size ratio and hole spacing on the maximum magnitude of σ_x for $\pm 45^\circ$ and 90° plies is insignificant comparing with the effect of 0° ply due to these parameters.

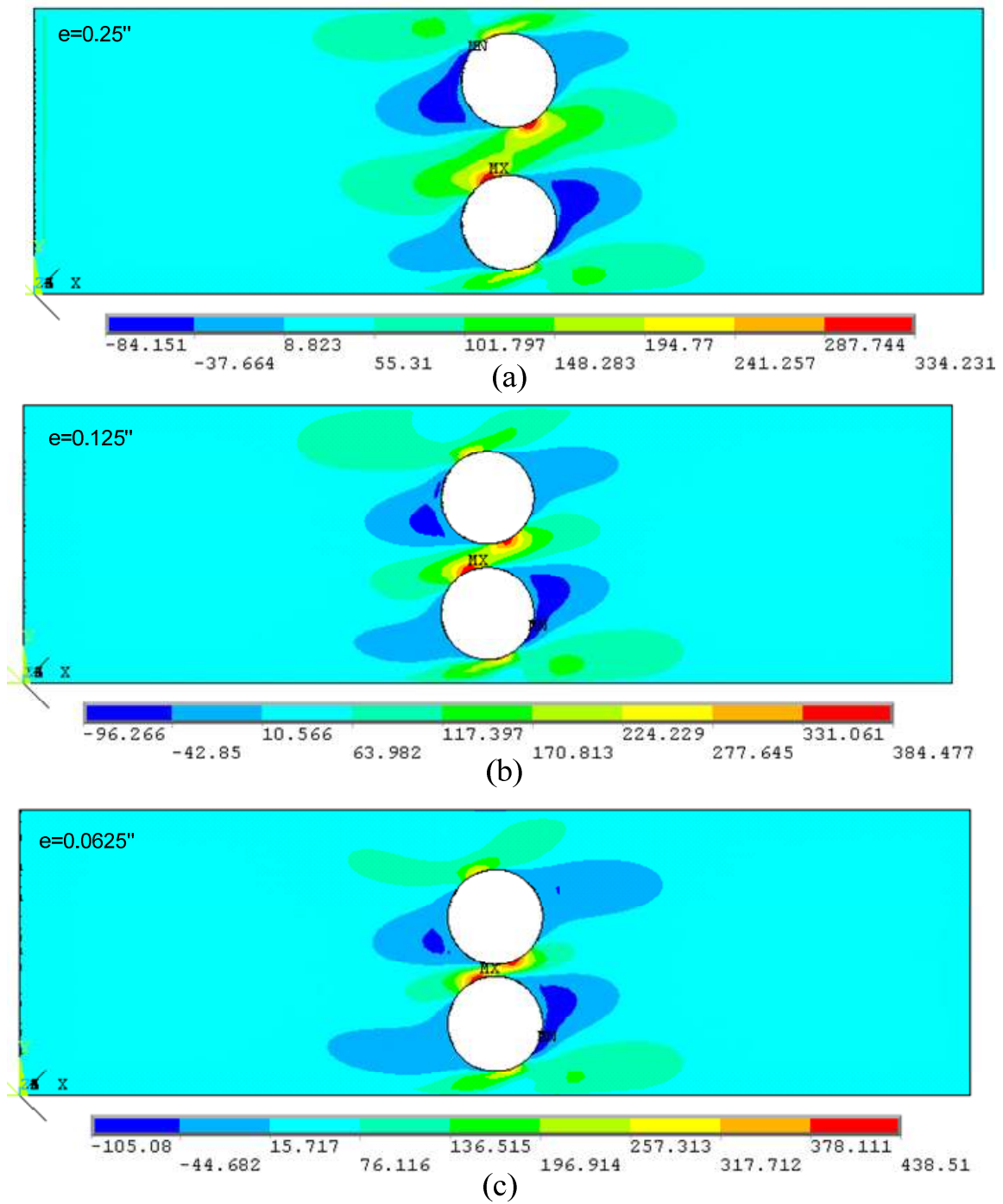


Figure 4. 15 Comparison of Stress Contours of σ_x for 45° Ply with $\frac{D_1}{D_2} = 1$ and (a) $e= 0.25$ (b) $e=0.125$ (c) $e= 0.0625$

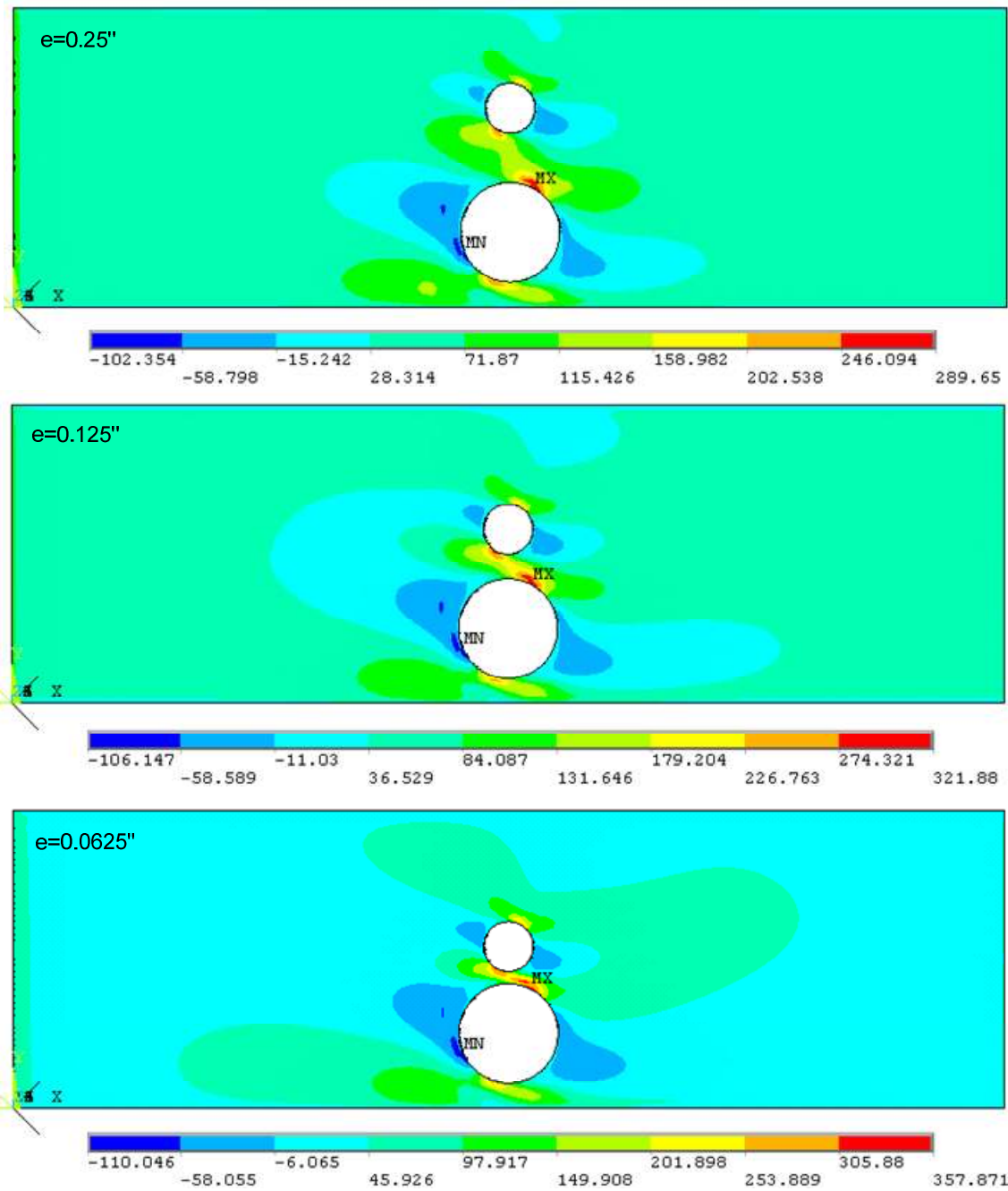


Figure 4. 16 Comparison of Stress Contours of σ_x for -45° Ply with $\frac{D1}{D2} = 2$ and (a) $e= 0.25$ (b) $e=0.125$ (c) $e= 0.0625$

4.3 Three Holes

4.3.1 Geometrical Parameters and Finite Element Meshes

The geometric parameters of 3-hole laminate used in this study are shown in Figure 4.17. As shown, D2 and D3 were kept in same size of 0.25". They are placed side by side with hole edge to laminate edge distance of 0.25". This arrangement gives the two holes located at equal space within the laminate. The hole with diameter D1 has been placed at centre of width of composite laminate; and centre lines connecting centers of holes D1 to D2 and D1 to D3 makes an angle Φ , as shown in the figure. $\Phi=45^\circ$ and 60° are considered in this study. The composite laminate model used for study, boundary condition, material properties, and modeling has been already discussed in Chapter 2. The in-plane nominal stress $\sigma_0=100$ psi is applied along x-direction.

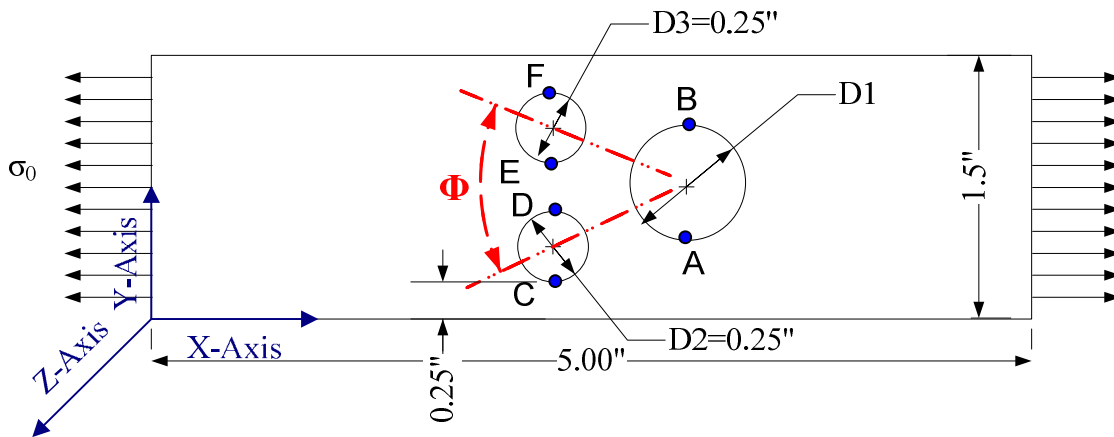
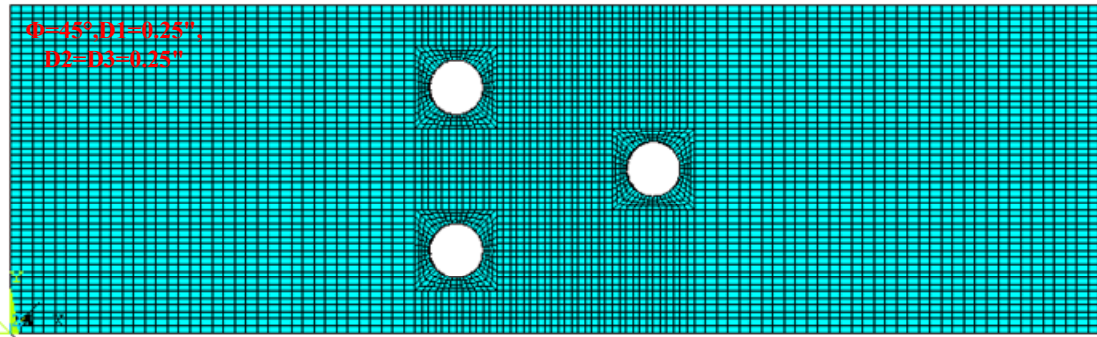
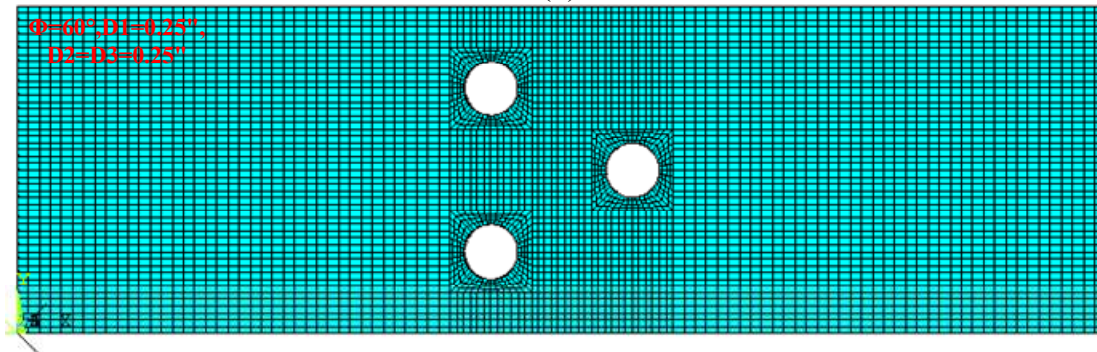


Figure 4. 17 Geometric Parameters Used for Three-Hole Laminates

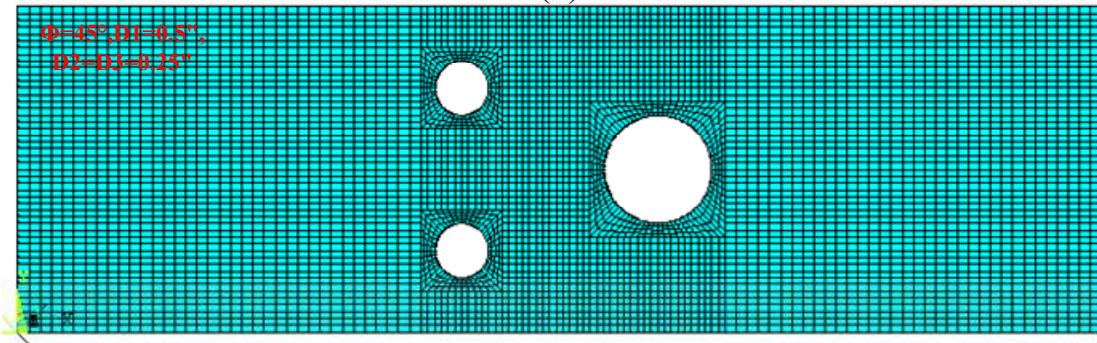
Figure 4.18 shows meshes used for study. Table 4.4 shows numbers of nodes and elements used for each case.



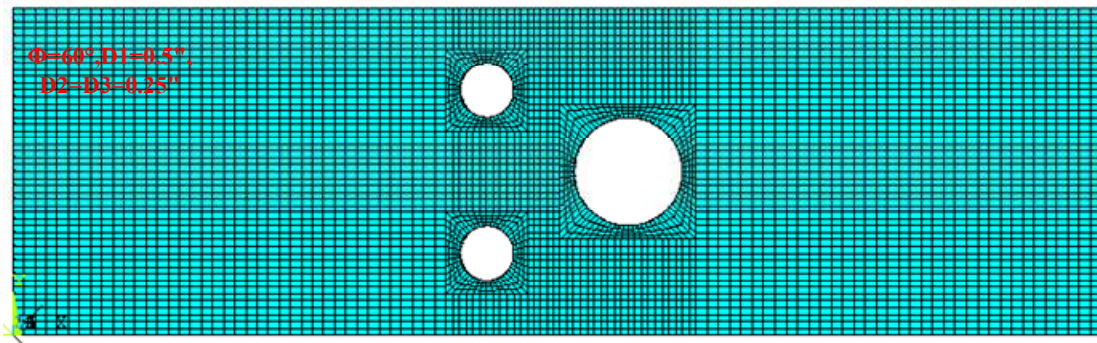
(a)



(b)



(c)



(d)

Figure 4. 18 Meshes Used for Three-Hole Laminate (a) $\Phi=45^\circ, D1=D2=D3=0.25''$ (b) $\Phi=60^\circ, D1=D2=D3=0.25''$ (c) $\Phi=45^\circ, D1=0.5''$, $D2=D3=0.25''$ (d) $\Phi=60^\circ, D1=0.5''$ $D2=D3=0.25''$

Table 4. 4 No. of Elements and Nodes Used for Three-Hole Patterns

	D1=0.25		D1=0.5	
	$\Phi=45^\circ$	$\Phi=60^\circ$	$\Phi=45^\circ$	$\Phi=60^\circ$
Elements	33408	32832	33216	32532
Nodes	154970	152338	154466	150518

4.3.2 Peak Value of σ_x in 3-Hole Laminate

Table 4. 5 Peak Magnitude of Stress σ_x in Each Plies at the Hole Periphery for Three-Hole Patterns

Fiber orientation for ply	Maximum value of stress σ_x					
	A	B	C	D	E	F
$\Phi=45^\circ, D1=D2=D3=0.25''$						
0	719	719	780	731	731	780
45	213	224	222	216	235	233
-45	207	195	213	227	201	197
90	55	52	58	57	53	55
$\Phi=60^\circ, D1=D2=D3=0.25''$						
0	756	756	799	750	750	799
45	216	245	231	216	250	239
-45	230	193	216	241	197	204
90	60	53	59	61	53	57
$\Phi=45^\circ, D1=0.5, D2=D3=0.25''$						
0	798	798	828	701	701	828
45	224	238	216	188	274	267
-45	234	215	250	264	166	180
90	61	59	66	62	56	46
$\Phi=60^\circ, D1=0.5, D2=D3=0.25''$						
0	810	810	912	758	758	912
45	218	279	250	177	329	283
-45	280	208	261	317	145	211
90	68	58	73	70	64	47

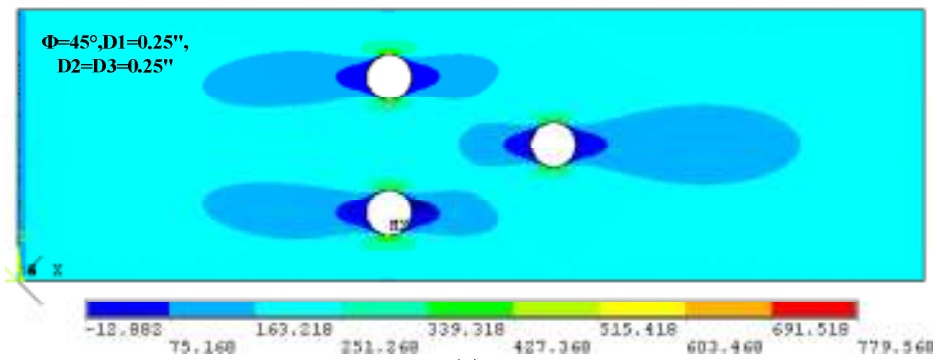
Table 4.5, shows maximum stress σ_x for all plies of Three-Hole laminate with different hole patterns. The effect of hole size ratio and its pattern will be discussed in the next section.

4.3.4 Effect of Hole Size Ratios and Hole Stress Interaction for 0° Plies

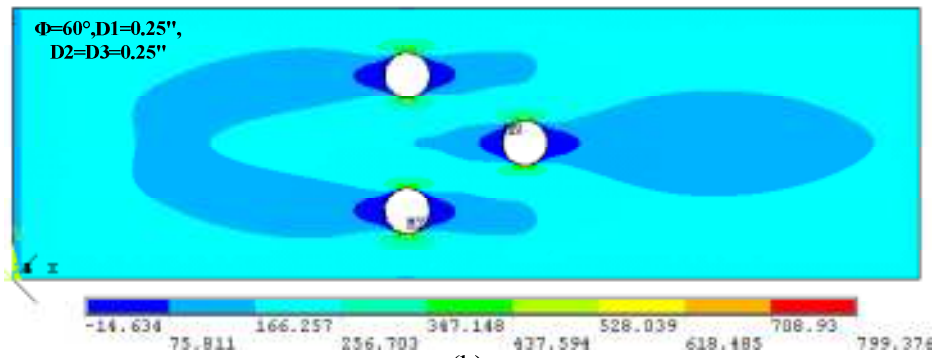
Figure 4.19 shows the stress contours of 3-Hole laminates with $\Phi=45^\circ$ and 60° respectively. The stress contours with two different hole size ratios were also included. For laminate with only two holes(hole 2 and 3) at dimensions and locations given above, the induced stress $\sigma_x=762$ psi at points C & F and induced stress $\sigma_x=738$ psi at points D & E for 0° plies. Also we have discussed the results for single hole placed at centre of width of laminate when $D=0.5''$ the maximum induced stress $\sigma_x=807$ psi at points A and B and with $D=0.25''$ the stress $\sigma_x=700$ psi at points A and B.

For three holes of size $0.25''$ each, with $\Phi=45^\circ$ has lowest stress concentration among all four cases considered, as the holes spacing are highest among all cases. With $\Phi=60^\circ$ the more effect of hole interaction is observed as hole 1 comes further closer to holes 2 and 3.

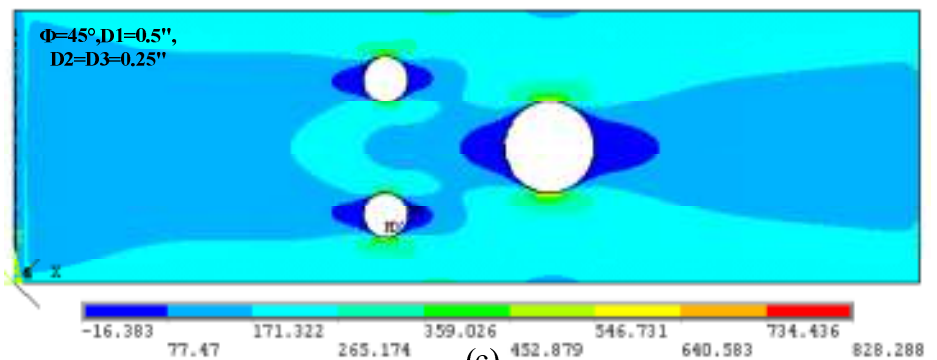
For $D1=0.5''$ and $D2=D3=0.25''$ case, when $\Phi=60^\circ$ highest values of stresses observed at points A to F. Because larger diameter of hole 1 and $\Phi=60^\circ$ hole spacing between holes are lowest among all cases considered, and hence highest amount of hole stress interaction effect occurs in this case.



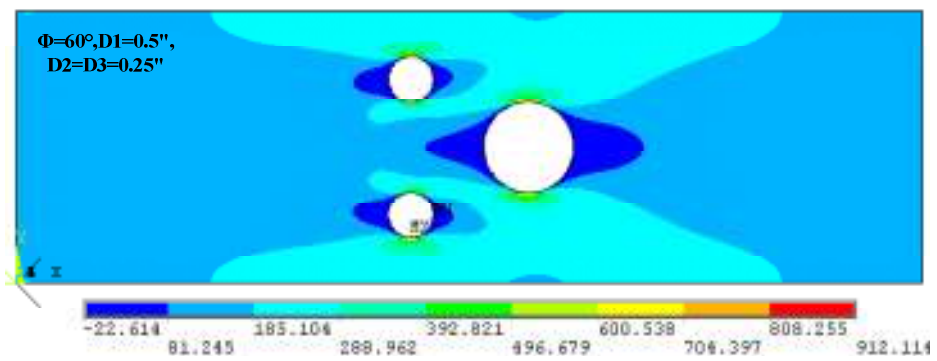
(a)



(b)



(c)



(d)

Figure 4. 19 Comparison of Stress Contour of σ_x for 0° Ply for (a) $\Phi=45^\circ, D1=D2=D3=0.25"$ (b) $\Phi=60^\circ, D1=D2=D3=0.25"$ (c) $\Phi=45^\circ, D1=0.5", D2=D3=0.25"$ (d) $\Phi=60^\circ, D1=0.5", D2=D3=0.25"$

4.3.5 Effect of Hole Size Ratios and Hole Stress Interaction for $\pm 45^\circ$ Plies

The comparison of hole interaction and hole size ratio for $\pm 45^\circ$ plies has shown in the figure 4.20 and 4.21, respectively.

For $+45^\circ$ ply, the maximum value of stress σ_x is observed at location of point E and minimum value at point D. This is due to more irregularities in path of flux, as two holes comes in path of fibers oriented at $+45^\circ$.

For -45° ply, maximum value of stress σ_x is observed at location of point E and minimum value at point D. This is due to more irregularities in path of flux, as two holes comes in path of fibers oriented at -45° .

For the cases with $D1=0.5''$ or $\Phi=60^\circ$ more effect of hole interaction observed in terms of higher values of maximum stress σ_x in ply.

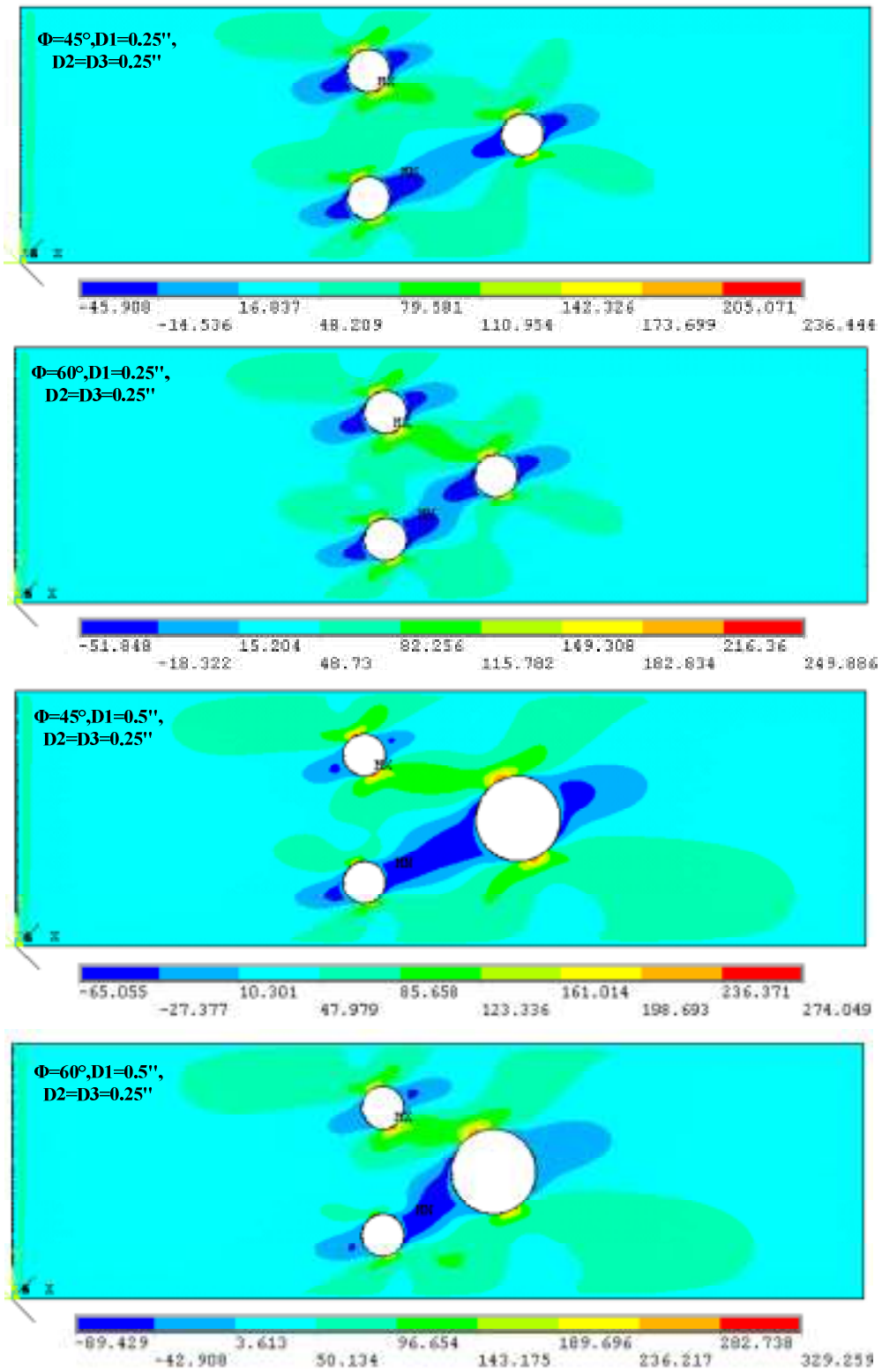


Figure 4. 18 Comparison of Stress Contour σ_x for 45° Ply for Three-Hole Patterns

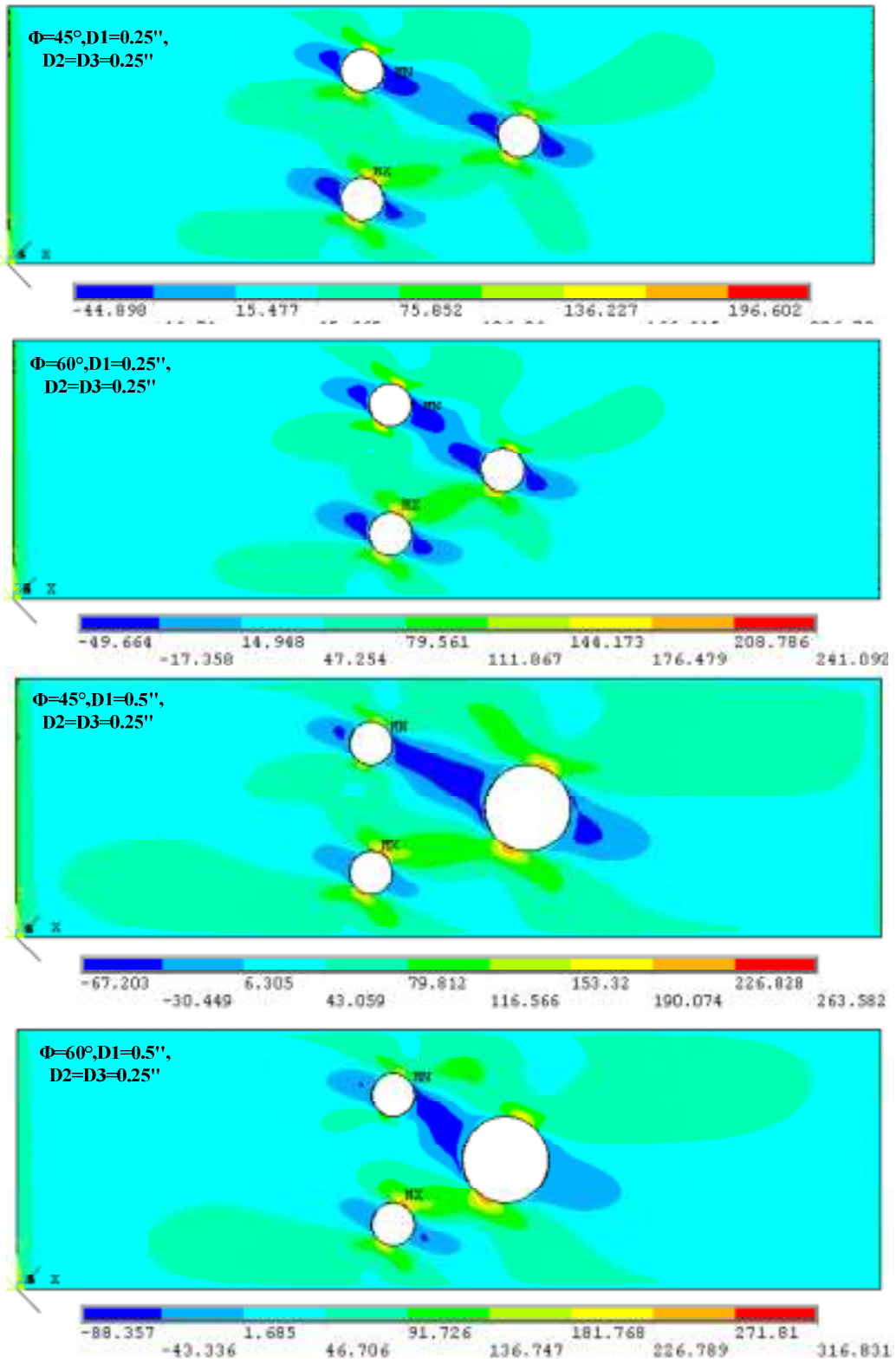


Figure 4. 19 Comparison of Stress Contour σ_x for -45° Ply for Three-Hole Patterns

CHAPTER 5

CONCLUSIONS

Three dimensional finite element models have been developed to investigate in-plane stresses of the $[0_3/\pm 45/90]_S$ laminate with multiple holes subjected to tension. The stress distribution of laminated composite with single, two and three holes were obtained. The effect of the maximum stress due to the distance between holes and laminate edge to the hole was studied. The effect of the maximum stress due to *hole size ratio* and *hole patterns* was also observed.

The maximum stress concentration was found at point on hole periphery where fibers are tangent to the hole. Holes orientation, holes size ratio and edge distance between holes plays important role on value of stress concentration factor of composite laminate.

For the $[0_3/\pm 45/90]_S$ laminate, 0° plies carries highest magnitude of stresses among all plies in the laminate. The maximum magnitude of stress σ_x in 0° ply is approximately 3 to 5 times greater than that of $\pm 45^\circ$ plies, and 11 to 13 times greater than that of 90° plies. The maximum σ_x stress magnitude of 0° ply occurs at the periphery of hole at points where the fibers of that ply are tangent to hole periphery.

The following conclusions were drawn from this study.

For Single hole:

- As the hole to laminate edge distance decreases, the magnitude of the maximum stress in 0^0 ply increases.
- As the hole moves off the center of the laminate width, the effect of the bending is induced due to eccentricity of the load path. As a result, the stress at both points of the hole edges are increased.

For two holes placed in-line:

- The maximum stress magnitude of 0^0 ply increases as the hole size ratio increases.
- For the equal size of holes in laminate, the maximum stress magnitude decreases as the the distance between two holes decreases.
- The effect of the stress magnitude due to the distance between these two hole is insignificant when the hole size ratio increases.

For two holes placed side by side:

- The maximum stress magnitude of 0^0 ply increases as the distance between two holes decreases.
- For a given distance between two holes, the maximum stress magnitude of 0^0 ply decreases as the hole size ratio increases.

For three holes in 45^0 and 60^0 arrays:

- The maximum stress magnitude of 0^0 ply occurs at the point of the outer hole edge.
- For equal size of the holes, the maximum stress magnitude of 0^0 ply is higher for the hole array in 60^0 compared to the array in 45^0 ,
- For a given hole array, the maximum stress magnitude of 0^0 ply increases as the hole size ratio increases.

APPENDIX A

BATCH MODE ANSYS INPUT DATA FILE FOR FINITE ELEMENT MODEL

1) Single Hole

! Hole Edge to Laminate Edge Distance $e = 0.25$ "
! Analysis with centre distance from edge $c = 0.5$ ".

/filename, e_0.25
/prep7

! Element Type
ET, 1, PLANE183
ET, 2, SOLID186

! Geometrical Parameters
L=5 !Laminate length
W=1.5 !Laminate Width
C1=0.5 !Centre of hole in y-direction
R1=0.25 !radius of hole
e=C1-R1 !Hole edge to laminate edge distance
C=0.375 !Half size of Side of square around circle

!Define Keypoints
k,1,0,0,0
k,2,L,0,0
k,3,L,W,0
k,4,0,W,0
k,5,L/2-C,0,0
k,6,L/2-C,0.125,0
k,7,L/2-C,0.125+C,0
k,8,L/2-C,0.125+2*C,0
k,9,L/2-C,W,0
k,10,L/2,0,0
k,11,L/2,0.125,0
k,12,L/2,0.125+C,0
k,13,L/2,0.125+2*C,0
k,14,L/2,W,0
k,15,L/2+C,0,0
k,16,L/2+C,0.125,0
k,17,L/2+C,0.125+C,0
k,18,L/2+C,0.125+2*C,0
k,19,L/2+C,W,0

k,20,0,0.125,0
k,21,0,0.125+C,0
k,22,0,0.125+2*C,0
k,23,L,0.125,0
k,24,L,0.125+C,0
k,25,L,0.125+2*C,0

! GENERATION OF AREAS

a,1,5,6,20
a,20,6,7,21
a,21,7,8,22
a,22,8,9,4
a,5,10,11,6
a,6,11,12
a,6,12,7
a,7,12,8
a,8,12,13
a,8,13,14,9
a,13,18,19,14
a,12,18,13
a,12,17,18
a,12,16,17
a,11,16,12
a,10,15,16,11
a,15,2,23,16
a,16,23,24,17
a,17,24,25,18
a,18,25,3,19

NUMCMP,ALL

!CREATING CIRCLE AND SUBTRACTION

CYL4, L/2,C1,R1
FLST, 2, 8, 5, ORDE, 4
FITEM, 2, 6
FITEM, 2,-9
FITEM, 2, 12
FITEM, 2,-15
ASBA, P51X, 21

NUMCMP, ALL

aglue, all ! Glue All Areas

! Mapped Meshing
!8 parts of "e" each 0.0156", 12 parts of 0.03125 in C dimension, 34 parts in large lengths
each 0.0625

```
lsel,s,length, ,0.125  
lesize,all,,8  
allsel  
lsel,s,length,C  
lesize,all,0.03125  
allsel  
lsel,s,length,L/2-C  
lesize,all,0.0625  
allsel  
lsel,a,length,0.625  
lesize,all,0.03125  
allsel
```

```
mshkey,1  
amesh,all
```

```
!COPYING ALL 20 AREAS 6 TIMES  
AGEN,6,all, , , ,0.005, ,0  
numcmp,all
```

```
!LOCAL CORDINATES SYSTEM FOR FIBER ORIENTATION  
local,11,,0,0,0,90  
local,12,,0,0,0,-45  
local,13,,0,0,0,45  
local,14,,0,0,0,0  
local,15,,0,0,0,0  
local,16,,0,0,0,0  
csys,0
```

```
!!!material properties  
MPTEMP,,,,,,,,  
MPTEMP,1,0  
MPDATA,EX,1,,21.75e6  
MPDATA,EY,1,,15.95e5  
MPDATA,EZ,1,,15.95e5  
MPDATA,PRXY,1,,0.25  
MPDATA,PRYZ,1,,0.45  
MPDATA,PRXZ,1,,0.25  
MPDATA,GXY,1,,87.02e4
```

```
MPDATA,GYZ,1,,53.66e4
MPDATA,GXZ,1,,87.02e4
```

```
numcmp,all
```

```
!GENERATING VOLUMES
```

```
*do,i,1,6,1
TYPE, 2
EXTOPT,ESIZE,1,0,
EXTOPT,ACLEAR,1
EXTOPT,ATTR,0,0,0
MAT,1
REAL,_Z4
ESYS,10+i
asel,s,area,,(i-1)*20+1,i*20
vext,all,,,,,0.005
allsel
*enddo
```

```
nummrg,node      ! Merging Co-incident nodes
nummrg,kp        ! Merging Co-incident key points
```

```
! Boundary Conditions
```

```
nset,s,loc,z,0    ! Selecting all nodes at Z=0
d,all,uz,0        !Symmetry b.c. for Z-dir
allsel
nset,s,loc,x,5    !Selecting all nodes at X=5
d,all,ux,0        ! DOF for 0 movement in X-dir
nset,r,loc,y,0.75 !Reselecting nodes at X=0 and Y=0.75
d,all,uy,0        ! DOF for 0 movement in Y-dir
allsel
```

```
! Loading Conditions
```

```
asel,s,loc,x,0    ! Selecting surfaces at x=0
SFA,all,1,PRES,-100 ! Surface pressure
```

```
FINISH
/SOL
SOLVE
```

2) Two Holes oriented in-line with load

!Edge to Edge Distance between holes $e = 0.5$ "
!d=0.5/0.125" for diff size holes located in-line with load

/filename,2h_0.5d0.125_ee0.5
/prep7

ET,1,PLANE183 !Element type
ET,2,SOLID186

L=5 !Length of Laminate
W=1.5 !Width of Laminate
C1=0.75 !Centre of hole in width direction
R1=0.25 !radius of hole1
R2=0.0625 !radius of hole2
S1=0.3125 !Half size of Side of square around circle 1
S2=0.125 !Half size of Side of square around circle 2
m=0.1875 !half of distance between squares $=(e-0.125)/2$

!Define Keypoints

k,1,0,0,0

k,2,L,0,0

k,3,L,W,0

k,4,0,W,0

k,5,L/2-2*S1-m,0,0

k,6,L/2-2*S1-m,C1-S1,0

k,7,L/2-2*S1-m,C1,0

k,8,L/2-2*S1-m,C1+S1,0

k,9,L/2-2*S1-m,W,0

k,10,L/2-S1-m,C1-S1,0

k,11,L/2-S1-m,C1,0

k,12,L/2-S1-m,C1+S1,0

k,13,L/2-m,0,0

k,14,L/2-m,C1-S1,0

k,15,L/2-m,C1,0

k,16,L/2-m,C1+S1,0

k,17,L/2-m,W,0

k,18,L/2+m,0,0

k,19,L/2+m,C1-S2,0

k,20,L/2+m,C1,0
k,21,L/2+m,C1+S2,0
k,22,L/2+m,W,0

k,23,L/2+S2+m,C1-S2,0
k,24,L/2+S2+m,C1,0
k,25,L/2+S2+m,C1+S2,0

k,26,L/2+2*S2+m,0,0
k,27,L/2+2*S2+m,C1-S2,0
k,28,L/2+2*S2+m,C1,0
k,29,L/2+2*S2+m,C1+S2,0
k,30,L/2+2*S2+m,W,0

k,31,0,C1-S1,0
k,32,0,C1+S1,0

k,33,L,C1-S2,0
k,34,L,C1+S2,0

!GENERATION OF AREAS

a,1,5,6,31
a,31,6,8,32
a,32,8,9,4
a,5,13,14,6
a,8,16,17,9
a,13,18,22,17
a,18,26,27,19
a,21,29,30,22
a,26,2,33,27
a,27,33,34,29
a,29,34,3,30

a,11,6,10
a,11,7,6
a,11,7,8
a,11,8,12
a,11,12,16
a,11,16,15
a,11,15,14
a,11,14,10

a,24,23,19
a,24,20,19

a,24,21,20
a,24,25,21
a,24,29,25
a,24,28,29
a,24,28,27
a,24,27,23

NUMCMP,ALL

!CREATING CIRCLES AND SUBTRACTION

CYL4,L/2-S1-m,C1,R1

CYL4,L/2+S2+m,C1,R2

FLST,2,8,5,ORDE,2

FITEM,2,12

FITEM,2,-19

ASBA,P51X, 28

FLST,2,8,5,ORDE,2

FITEM,2,20

FITEM,2,-27

ASBA,P51X, 29

NUMCMP,ALL

agluae,all

numcmp,all

!take a break and decide size

!taken 4 parts of 0.0156 in E1, and 10 parts of 0.03125 in S1 dimension,

!4 parts of 0.03125 in S2 dimension,

lsel,s,length, ,0.0625

lesize,all,,4

allsel

lsel,s,length,,S1

lsel,s,length,,S2

lsel,s,length,,2*S1

lsel,s,length,,2*S2

lesize,all,0.03125

allsel

lsel,s,length,,L/2-m-2*S1


```
lsel,s,length,,L/2+m+2*S2
lesize,all,0.0625
allsel
```

```
lsel,s,length,,W/2-S1
lsel,a,length,,S1-S2
lesize,all,0.03125
allsel
```

```
mshkey,1
amesh,all
```

```
numcmp,all
```

```
!COPYING ALL 27 AREAS 6 TIMES
AGEN,6,all,, , , ,0.005, ,0
numcmp,all
```

```
!CORDINATES SYSTEM FOR FIBER ORIENTATION AND MATERIAL PROP
local,11,,0,0,0,90
local,12,,0,0,0,-45
local,13,,0,0,0,45
local,14,,0,0,0,0
local,15,,0,0,0,0
local,16,,0,0,0,0
csys,0
```

```
!!!material properties
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,21.75e6
MPDATA,EY,1,,15.95e5
MPDATA,EZ,1,,15.95e5
MPDATA,PRXY,1,,0.25
MPDATA,PRYZ,1,,0.45
MPDATA,PRXZ,1,,0.25
MPDATA,GXY,1,,87.02e4
MPDATA,GYZ,1,,53.66e4
MPDATA,GXZ,1,,87.02e4
```

```
numcmp,all
```

```
!GENERATING VOLUMES
*do,i,1,6,1
```

```

TYPE, 2
EXTOPT,ESIZE,1,0,
EXTOPT,ACLEAR,1
EXTOPT,ATTR,0,0,0
MAT,1
REAL,_Z4
ESYS,10+i
asel,s,area,,(i-1)*27+1,i*27
vext,all,,,,,0.005
allsel
*enddo

```

```

nummrg,node      ! Merging Co-incident nodes
nummrg,kp        ! Merging Co-incident key points

```

! Boundary Conditions

```

nsel,s,loc,z,0    ! Selecting all nodes at Z=0
d,all,uz,0        ! Symmetry b.c. for Z-dir
allsel

```

```

nsel,s,loc,x,5    ! Selecting all nodes at X=5
d,all,ux,0        ! DOF for 0 movement in X-dir

```

```

nsel,r,loc,y,0.75 ! Reselecting nodes at X=0 and Y=0.75
d,all,uy,0        ! DOF for 0 movement in Y-dir
allsel

```

! Loading Conditions

```

asel,s,loc,x,0    ! Selecting surfaces at x=0
SFA,all,1,PRES,-100 ! Surface pressure

```

```

FINISH
/SOL
SOLVE

```

3) Two holes oriented side by side transverse to the load

!d=0.5 for same size holes located side by side transverse to load with e=0.0625"
/filename,2hsbs_0.5d_ee0.0625

```

/prep7

ET,1,PLANE183 !Element type
ET,2,SOLID186

L=5      !Length of Laminate
W=1.5    !Width of Laminate
C1=0.375 !Centre of hole1 in width direction
C2=0.9375 !Centre of hole2 in width direction
R1=0.25  !radius of hole1
R2=0.25  !radius of hole2
S1=0.28125 !Half size of Side of square around circle 1
S2=0.28125 !Half size of Side of square around circle 2

!Define Keypoints
k,1,0,0,0
k,2,L,0,0
k,3,L,W,0
k,4,0,W,0

k,5,L/2-S1,0,0
k,6,L/2+S1,0,0
k,7,L/2+S1,W,0
k,8,L/2-S1,W,0

k,9,L/2-S1,C1-S1,0
k,10,L/2-S1,C1,0
k,11,L/2-S1,C1+S1,0

k,12,L/2,C1-S1,0
k,13,L/2,C1,0
k,14,L/2,C1+S1,0

k,15,L/2+S1,C1-S1,0
k,16,L/2+S1,C1,0
k,17,L/2+S1,C1+S1,0

k,18,L/2-S2,C2,0
k,19,L/2-S2,C2+S2,0

k,20,L/2,C2,0
k,21,L/2,C2+S2,0

```

k,22,L/2+S2,C2,0
k,23,L/2+S2,C2+S2,0

!GENERATION OF AREAS

a,1,5,8,4
a,6,2,3,7

a,13,12,9
a,13,10,9
a,13,11,10
a,13,14,11
a,13,17,14
a,13,16,17
a,13,15,16
a,13,12,15

a,20,11,14
a,20,18,11
a,20,18,19
a,20,21,19
a,20,21,23
a,20,22,23
a,20,22,17
a,20,17,14

a,5,6,15,9
a,19,23,7,8

NUMCMP,ALL

!CREATING CIRCLES AND SUBTRACTION

CYL4,L/2,C1,R1
CYL4,L/2,C2,R2

FLST,2,8,5,ORDE,2
FITEM,2,3
FITEM,2,-10
ASBA,P51X, 21

FLST,2,8,5,ORDE,2
FITEM,2,11
FITEM,2,-18

ASBA,P51X, 22

numcmp,all

aglua,all
NUMCMP,ALL

!take a break and decide size
!taken 2 parts of 0.0156 in E1, and arts of 0.03125 in S1 and S2 dimension,
!large lenghts each 0.052

lsel,s,length,,0.03125
lesize,all,,2
allsel
lsel,s,length,,s1
lsel,a,length,,s2
lsel,a,length,,w
lesize,all,0.03125
allsel

lsel,s,length,,L/2-S1
lesize,all,0.052
allsel

lsel,s,length,,W/2-2*s1
lsel,a,length,,W/2-2*s2
lsel,a,length,,2*s1
lsel,a,length,,2*s2
lesize,all,0.03125
allsel

mshkey,1
amesh,all

numcmp,all

!COPYING ALL 20 AREAS 6 TIMES
AGEN,6,all, , , , ,0.005, ,0
numcmp,all

!CORDINATES SYSTEM FOR FIBER ORIENTATION AND MATERIAL PROP
local,11,,0,0,0,90
local,12,,0,0,0,-45
local,13,,0,0,0,45

```
local,14,,0,0,0,0
local,15,,0,0,0,0
local,16,,0,0,0,0
csys,0
```

```
!!!material properties
```

```
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,21.75e6
MPDATA,EY,1,,15.95e5
MPDATA,EZ,1,,15.95e5
MPDATA,PRXY,1,,0.25
MPDATA,PRYZ,1,,0.45
MPDATA,PRXZ,1,,0.25
MPDATA,GXY,1,,87.02e4
MPDATA,GYZ,1,,53.66e4
MPDATA,GXZ,1,,87.02e4
```

```
numcmp,all
```

```
!GENERATING VOLUMES
```

```
*do,i,1,6,1
TYPE, 2
EXTOPT,ESIZE,1,0,
EXTOPT,ACLEAR,1
EXTOPT,ATTR,0,0,0
MAT,1
REAL,_Z4
ESYS,10+i
asel,s,area,,(i-1)*20+1,i*20
vext,all,,,,,0.005
allsel
*enddo
```

```
nummrg,node      ! Merging Co-incident nodes
nummrg,kp        ! Merging Co-incident key points
```

```
! Boundary Conditions
```

```
nset,s,loc,z,0    ! Selecting all nodes at Z=0
d,all,uz,0        !Symmetry b.c. for Z-dir
allsel
nset,s,loc,x,5    !Selecting all nodes at X=5
d,all,ux,0        ! DOF for 0 movement in X-dir
nset,r,loc,y,0.75 !Reselecting nodes at X=0 and Y=0.75
```

```
d,all,uy,0      ! DOF for 0 movement in Y-dir
allsel
```

```
! Loading Conditions
asel,s,loc,x,0  ! Selecting surfaces at x=0
SFA,all,1,PRES,-100  ! Surface pressure
```

```
FINISH
/SOL
SOLVE
```

4) Three Holes at three vertices of a triangle

```
!d=0.5/0.25 for diff size holes located at 45 degrees
/filename,3hole_0.5d0.25_45
/prep7
```

```
ET,1,PLANE183  ! Element Type
ET,2,SOLID186
```

```
!Geometric parameters
L=5      !Length of Laminate
W=1.5    !Width of Laminate
```

```
!trianle h=0.375*tan67.5=0.90533
x1=L/2+0.45266  !Centre of hole1
y1=W/2
x2=L/2-0.45266  !Centre of hole2
y2=3*W/4
x3=L/2-0.45266  !Centre of hole3
y3=W/4
```

```
R1=0.25  !radius of hole1
R2=0.125 !radius of hole2
R3=0.125 !radius of hole3
```

```
S1=0.3125 !Half size of Side of square around circle 1
S2=0.1875 !Half size of Side of square around circle 2
S3=0.1875 !Half size of Side of square around circle 3
```

```
!Define Keypoints
```

k,1,0,0,0
k,2,L,0,0
k,3,L,W,0
k,4,0,W,0

k,5,x3-S3,0,0
k,6,x3+S3,0,0
k,7,x3+S3,W,0
k,8,x3-S3,W,0

k,9,x3-S3,y3-S3,0
k,10,x3-S3,y3,0
k,11,x3-S3,y3+S3,0

k,12,x3,y3-s3,0
k,13,x3,y3,0
k,14,x3,y3+s3,0

k,15,x3+S3,y3-S3,0
k,16,x3+S3,y3,0
k,17,x3+S3,y3+S3,0

k,18,x2-S2,y2-S2,0
k,19,x2-S2,y2,0
k,20,x2-S2,y2+S2,0

k,21,x2,y2-s3,0
k,22,x2,y2,0
k,23,x2,y2+s2,0

k,24,x2+S2,y2-S2,0
k,25,x3+S2,y2,0
k,26,x2+S2,y2+S2,0

k,27,x1-S1,0,0
k,28,x1+S1,0,0
k,29,x1+S1,W,0
k,30,x1-S1,W,0

k,31,x1-S1,y1-S1,0
k,32,x1-S1,y1,0
k,33,x1-S1,y1+S1,0

k,34,x1,y1-s1,0
k,35,x1,y1,0
k,36,x1,y1+s1,0

k,37,x1+S1,y1-S1,0
k,38,x1+S1,y1,0
k,39,x1+S1,y1+S1,0

!GENERATION OF AREAS

a,1,5,8,4

a,13,12,9
a,13,10,9
a,13,11,10
a,13,14,11
a,13,17,14
a,13,16,17
a,13,15,16
a,13,12,15

a,22,21,18
a,22,19,18
a,22,20,19
a,22,23,20
a,22,26,23
a,22,25,26
a,22,24,25
a,22,21,24

a,5,6,15,9
a,11,17,24,18
a,20,26,7,8
a,6,27,30,7
a,27,28,37,31
a,33,39,29,30
a,28,2,3,29

a,35,32,31
a,35,32,33
a,35,36,33
a,35,36,39
a,35,38,39
a,35,38,37

a,35,34,37
a,35,34,31

NUMCMP,ALL

!CREATING CIRCLES AND SUBTRACTION

CYL4,x1,y1,R1

CYL4,x2,y2,R2

CYL4,x3,y3,R3

FLST,2,8,5,ORDE,2

FITEM,2,25

FITEM,2,-32

ASBA,P51X, 33

FLST,2,8,5,ORDE,2

FITEM,2,10

FITEM,2,-17

ASBA,P51X, 34

FLST,2,8,5,ORDE,2

FITEM,2,2

FITEM,2,-9

ASBA,P51X, 35

numcmp,all

aglu,all

NUMCMP,ALL

!take a break and decide size

!taken 4 parts of 0.0156 in E1, and parts of 0.03125 in S1,S2 and S3 dimension,

!large lengths each 0.052

lsel,s,length, ,0.0625

lesize,all,,,4

allsel

lsel,s,length,,s1

lsel,a,length,,s2

lsel,a,length,,s3

lsel,a,length,,w

lesize,all,0.03125

allsel

lsel,s,length,,x1+S1

lsel,s,length,,x3-S1

lesize,all,0.052

allsel

lsel,s,length,,W/2-s1

lsel,a,length,,W/2-s2

lsel,a,length,,2*s1

lsel,a,length,,2*s2

lsel,a,length,,2*s3

lesize,all,0.03125

allsel

mshkey,1

amesh,all

numcmp,all

!COPYING ALL 32 AREAS 6 TIMES

AGEN,6,all, , , ,0.005, ,0

numcmp,all

!CORDINATES SYSTEM FOR FIBER ORIENTATION AND MATERIAL PROP

local,11,,0,0,0,90

local,12,,0,0,0,-45

local,13,,0,0,0,45

local,14,,0,0,0,0

local,15,,0,0,0,0

local,16,,0,0,0,0

csys,0

!!!material properties

MPTEMP,,,,,,,,

MPTEMP,1,0

MPDATA,EX,1,,21.75e6

MPDATA,EY,1,,15.95e5

MPDATA,EZ,1,,15.95e5

MPDATA,PRXY,1,,0.25

MPDATA,PRYZ,1,,0.45

MPDATA,PRXZ,1,,0.25

MPDATA,GXY,1,,87.02e4

MPDATA,GYZ,1,,53.66e4

```
MPDATA,GXZ,1,,87.02e4
```

```
numcmp,all
```

```
!GENERATING VOLUMES
```

```
*do,i,1,6,1
```

```
TYPE, 2
```

```
EXTOPT,ESIZE,1,0,
```

```
EXTOPT,ACLEAR,1
```

```
!*
```

```
EXTOPT,ATTR,0,0,0
```

```
MAT,1
```

```
REAL,_Z4
```

```
ESYS,10+i
```

```
asel,s,area,,(i-1)*32+1,i*32
```

```
vext,all,,,,,0.005
```

```
allsel
```

```
*enddo
```

```
nummrg,node ! Merging Co-incident nodes
```

```
nummrg,kp ! Merging Co-incident key points
```

```
! Boundary Conditions
```

```
nset,s,loc,z,0 ! Selecting all nodes at Z=0
```

```
d,all,uz,0 !Symmetry b.c. for Z-dir
```

```
allsel
```

```
nset,s,loc,x,5 !Selecting all nodes at X=5
```

```
d,all,ux,0 ! DOF for 0 movement in X-dir
```

```
nset,r,loc,y,0.75 !Reselecting nodes at X=0 and Y=0.75
```

```
d,all,uy,0 ! DOF for 0 movement in Y-dir
```

```
allsel
```

```
! Loading Conditions
```

```
asel,s,loc,x,0 ! Selecting surfaces at x=0
```

```
SFA,all,1,PRES,-100 ! Surface pressure
```

```
FINISH
```

```
/SOL
```

```
SOLVE
```

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BIOGRAPHICAL INFORMATION

Manishkumar Kheradiya received his Bachelor's degree in Mechanical Engineering from Maharaja Sayajirao University of Baroda, India in 2003. He worked as an engineer for three years in operation and maintenance department for a company transporting natural gas through pipeline.

He started his Master's in Mechanical Engineering at University of Texas at Arlington in January 2007. His research interests include structure analysis, design optimization, finite element methods and composite structure. Manishkumar Kheradiya received Master of Science degree in Mechanical Engineering from University of Texas at Arlington in December 2008.