

STRUCTURAL OPTIMIZATION USING FEMLAB AND
SMOOTH SUPPORT VECTOR REGRESSION

by

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ABSTRACT

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In recent years support vector machine (SVM) has been emerging as a popular tool for function approximation. Application of SVM for approximation of mathematical functions and complex engineering analysis has been represented by Palancz et al and Clarke et al, respectively. However the training of the original SVM involves the solution of a quadratic programming (QP) problem. This makes the application of SVM to large problem computationally expensive. To circumvent this difficulty, Lee et al developed a more efficient SVM formulation namely ϵ -SSVR which drastically improved the training efficiency of SVM.

In this research the SSVR is used to build a metamodel for structural optimization. In the proposed method, Quasi Monte Carlo (QMC) technique is used for the selection of training data in the design space. SSVR using a radial basis function kernel is used to build the metamodel for structural optimization. The structural responses are evaluated by a commercial finite element package, FEMLAB (recently renamed as COMSOL). Several structural optimization examples are presented to illustrate the effectiveness of the proposed approach.

TABLE OF CONTENTS

ACKNOWLEDGMENTS	iv
ABSTRACT	v
LIST OF FIGURES	x
Chapter	
1. INTRODUCTION	1
1.1 Introduction to Optimization.....	1
1.2 Support Vector Regression for Optimization.....	2
1.3 Objective and Scope.....	3
1.4 Outline of Thesis.....	3
2. STRUCTURAL OPTIMIZATION	5
2.1 Introduction	5
2.2 Historic Development	5
2.3 Applications of Optimization	6
2.4 Statement and Solution of an Optimization Problem	7
2.5 Structural Optimization Methods	9

2.6 Femlab for Optimization	12
3. SUPPORT VECTOR MACHINES FOR REGRESSION	14
3.1 Introduction	14
3.2 History of Support Vector Machines	15
3.3 Some Important Features of SVM	15
3.4 Regression Estimation Problem	16
3.5 Support Vector Regression	17
3.5.1 Linear Support Vector Regression	18
3.5.2 Nonlinear Support Vector Regression	22
3.6 Smooth Support Vector Regression	23
3.7 Kernel Functions	25
3.7.1 Polynomial Kernel	27
3.7.2 Radial Basis Function Kernel	28
3.8 Quasi-Monte Carlo Sampling	29
4. GENERAL ALGORITHM FOR STRUCTURAL OPTIMIZATION USING SMOOTH SUPPORT REGRESSION.....	30
4.1 Introduction	30
4.2 Algorithm of Metamodel	30

5. NUMERICAL RESULTS AND COMPARISONS	37
5.1 Implementation of Metamodel for Optimization	37
5.2 Two Dimensional Model of Trapezoidal Plate	37
5.3 Three Dimensional Trapezoidal Plate	42
5.4 Rectangular Plate with Circular Hole Fixed at Centre	46
5.5 Rectangular Plate for Optimum Location and Radius of Circular Hole	52
6. CONCLUSIONS AND FUTURE WORK	59
6.1 Conclusions.....	59
6.2 Future Work	60
REFERENCES.....	61
BIOGRAPHICAL INFORMATION	64

LIST OF FIGURES

Figure	Page
2.1 Various methods of Optimization	11
3.1 Loss functions used in SVMs. (a). Quadratic loss function, (b). Laplace loss function, (c). Huber loss function and (d). ϵ -insensitive loss function.	18
3.2 Accounting for the Slack Variables	20
3.3 Kernel Mapping	26
3.4 An Example of Polynomial kernel.....	27
3.5 An Example of RBF kernel.....	28
4.1 Simple black box model of SSVR	32
4.2 Flow chart of the algorithm for structural optimization with SSVR function	36
5.1 Trapezoidal plate - initial design.....	38
5.2 Plot of input variables and output responses during convergence using SSVR for two dimensional model of trapezoidal plate	39
5.3 Convergence plot of volume of plate using SSVR for two dimensional model of trapezoidal plate	40
5.4 Plot of input variables and output responses during convergence using fmincon for two dimensional model of trapezoidal plate	40
5.5 Convergence plot of volume of plate using fmincon for two dimensional model of trapezoidal plate	41
5.6 Optimum design of two dimensional model of trapezoidal plate	41

5.7	Plot of input variables and output responses during convergence using SSVR for three dimensional trapezoidal plate	44
5.8	Convergence plot of volume of plate using SSVR for three dimensional trapezoidal plate	44
5.9	Optimum design of three dimensional trapezoidal plate	45
5.10	Plot of input variables and output responses during convergence using fmincon for three dimensional trapezoidal plate	45
5.11	Convergence plot of volume of plate using fmincon for three dimensional trapezoidal plate	46
5.12	Rectangular plate with circular hole fixed at the centre	47
5.13	Plot of input variable (radius of hole in inch) during convergence using SSVR for rectangular plate with circular hole at centre	49
5.14	Convergence of volume of the plate using SSVR for rectangular plate with circular hole at centre	49
5.15	Optimum design of rect. plate with circular hole at centre by SSVR	50
5.16	Plot of input variable (radius of hole in inch) during convergence using fmincon for rectangular plate with circular hole at centre	50
5.17	Plot of output response during convergence using fmincon for rectangular plate with circular hole at centre	51
5.18	Convergence of volume using fmincon for rectangular plate with circular hole at centre	51
5.19	Rectangular plate with hole	53
5.20	QMC samples used in SSVR model for rectangular plate with hole	54
5.21	Optimum input design values created by SSVR during the optimization process for rectangular plate with hole	55

5.22 Plot of output response created by SSVR during optimization for rectangular plate with hole	55
5.23 Convergence of volume using SSVR for rectangular plate with hole	56
5.24 Optimum design of rectangular plate with hole by SSVR for rectangular plate with hole	56
5.25 Plot of input data used during optimization using fmincon for rectangular plate with hole	57
5.26 Plot of output response during optimization using fmincon for rectangular plate with hole	57
5.27 Convergence of volume using fmincon for rectangular plate with hole	58

CHAPTER 1

INTRODUCTION

1.1 Introduction to Optimization

Engineering Design has become a major challenge in today's increasingly complex world. Engineers are called upon to design complex structures like aerodynamic shapes, mechanical systems, controllers and likewise, that need to fulfill a number of user defined design and performance objectives. This rigorous exercise of arriving at a best or optimum design under a predefined design window is termed as the process of optimization. Quite a large number of mathematical techniques have been developed in an attempt to solve the optimization problems.

The advent of high speed digital computer has provided engineers with incredible opportunities to find solutions to complex engineering problems. This has eventually resulted in development of Finite Element (FE), Finite Difference, and other discretization methods in the field of design analysis and various numerical search techniques in the field of optimization. A noticeable feature of these search techniques is that the problem is formulated using algebraic equations and not the integral or differential equations and is easily solved with the aid of computers. Although various optimization techniques are available for solving design problems, it is completely

dependent on the engineer to choose appropriate solution technique by proper recognition of characteristics of problem and in a well defined design window.

1.2 Support Vector Regression for Optimization

Structural optimization problems usually involve finite element analysis to get the deflection, stress, vibration, buckling and many other behavioral aspects of engineering structures. And each variation in the design takes significant amount of time to evaluate using finite element analysis. Hence the process of optimization is excessively time consuming. One possible approach to reduce this iteration time is by running a small set of analysis for design variations and then fit approximations to the relationship between properties of objective functions and corresponding constraint functions. This approximation is used in finding the optimum values. Thus the computationally expensive and time consuming process is replaced by a simplified mathematical approximation method. Such a mathematical approximation of the original complex analysis is termed as metamodel or in short model of a model. There are various metamodeling techniques like response surface methodology, kriging, radial basis functions, and multivariate adaptive regression splines available for engineering design [7]. Support vector machines with their simplicity and high efficiency combined with the flexibility of statistical methods have gained prominence in the field of optimization. In the current research smooth support vector regression with radial basis function kernel is used to build the metamodel for structural optimization.

1.3 Objective and Scope

The essential objective of current research work is to develop a metamodel for structural optimization using smooth support vector regression (SSVR) technique. The SSVR along with the radial basis function (RBF) kernel is used to build this metamodel for structural optimization. A design optimization code is written in the Matlab programming language while using commercial Finite Element package, FEMLAB 3.1 for obtaining the structural responses.

The results obtained from this metamodel are then compared with those obtained by using ‘fmincon’, the inbuilt optimization tool of Matlab [20]. Various optimization examples are considered for analysis.

1.4 Outline of Thesis

The present work is documented into six chapters.

Chapter Two presents the basic idea of structural optimization, general application along with formulation of optimization problem, various methods of optimization available in practice and use of Femlab for optimization.

Chapter Three focuses on support vector machines (SVM’s) for regression analysis. It gives a brief view of SVM’s and their salient features along with the formulation of regression problem. The concepts of linear and nonlinear support vector regression, smooth support vector regression, kernel function and QMC sampling technique are discussed.

Chapter Four emphasizes on algorithm of structural optimization using SSVR.

Chapter Five discusses implementation of SSVR to some application problems and the results compared with those obtained using Matlab optimization tool, fmincon.

Chapter Six presents the conclusions and future work.

CHAPTER 2

STRUCTURAL OPTIMIZATION

2.1 Introduction

Optimization is the process of obtaining maximum or minimum value of an objective function while subjected to various constraints. Thus the motive of optimal design is to obtain best possible design following a set of pre-selected measures of effectiveness. The awareness of the scarcity of natural resources drives us towards light weight and low cost structures, thus emphasizing the need for weight and cost optimization of structures [6].

2.2 Historical Development

The idea of optimal design can be traced back to the days of Galileo. Galileo Galilei's formulation of the strongest beam in bending and constant shear was an optimization approach for minimum weight under constant stress conditions. However, the development of mathematical optimization was possible by contribution of Newton, Leibnitz in the field of calculus. Bernoulli, Euler and Lagrange laid the foundation for variational calculus. Cauchy developed the steepest descent method to solve unconstrained minimization problem. Regardless of these contributions in the 17th and 18th centuries not much progress was observed until the middle of the 20th century in the field of optimization [11].

Development of simplex method (1947) by Dantzig and the principle of optimality (1957) by Bellman contributed towards the method of constrained optimization. Kuhn and Tucker in 1951 worked on the necessary and sufficient conditions for optimal solution, paved the way for non-linear programming. Geometric programming concept was introduced by Duffin, Zener and Peterson in the 1960s. Dantzig, Charnes & Cooper developed the stochastic programming methodologies. Goal programming was proposed by Charnes & Cooper in 1961. Simulated annealing, genetic algorithms and neural network methods are a very recent mathematical programming techniques developed during last decade [9].

2.3 Applications of Optimization

Optimization can be used to solve many engineering problems. In our daily life we come across innumerable examples of optimization present in nature. A liquid droplet of perfect spherical shape is an ideal example of least surface area for a given volume. The honeycomb structure stands as one of the most optimal arrangements for packaging.

It is very important that engineers understand the basic theory, algorithms and the techniques of optimization, so that they can suitably modify the techniques and apply to practical problems. Some of the typical applications related to various diversified fields are listed as follows [9].

1. Design of civil engineering structures like frames, bridges, towers, dams for minimum cost.
2. Minimum weight design of structures for earthquake, wind and other random loadings.

3. Design of space vehicles for minimum weight and thrust applications.
4. Optimum design of linkages, cams, machine tools and other mechanical components.
5. Design of pumps, turbines and water cooling equipment for maximum efficiency.
6. Optimization in production, planning and control.
7. Allocation of resources and services among several activities to maximize the benefits.
8. Optimum design of control systems, chemical processing plants, etc.

2.4 Statement and Solution of an Optimization Problem

Looking at the general outline of optimization approach, problem formulation and problem solution are the two major steps. The problem formulation involves statement of the problem characterized by variables, objective and constraint functions. The problem solution concerns with the selection of suitable optimization technique, and setting up of the solver for repeated analysis to get optimal solution.

An optimization or otherwise mathematical programming problem can be stated as follows.

Find $X = \{x_1, x_2, x_3, \dots, x_n\}$ which maximizes or minimizes the function $f(X)$ subjected to inequality constraints

$$g_i(X) \leq 0, \quad i=1,2,\dots,m$$

and equality constraints

$$h_i(X) = 0, \quad i=1,2,\dots,n$$

where X is an n -dimensional vector called design vector and $f(X)$ is termed as the objective function.

The constraints on the variables themselves are defined as

$$XL \leq X \leq XU$$

with general terminology of XL as lower bound and XU as the upper bound for the design vector 'X'. The number of variables 'n' and the number of inequality constraints 'i' are not at all related to each other.

X^* constitutes the optimum solution for the problem in n-dimensional space. Any optimization problem stated as above is named as a constrained optimization problem. If there are no equality or inequality constraints it is an unconstrained problem.

The formulation of the objective function is based on the nature of the problem. The objective function of minimization is generally taken as weight for structural design problems. An optimization problem involving multiple objectives is known as a multi-objective programming problem.

The locus of all points satisfying $f(X) = C = \text{constant}$, forms a hyper surface in the design space and for each value of C there is a different hyper surface. These surfaces are called objective function surfaces.

In many practical optimization problems the design variables cannot be selected arbitrarily. The conditions that must be satisfied to produce an acceptable design are called design constraints. Limitations imposed on the behavior or performance of the system constitutes behavior or functional constraint respectively. Apart from these there are other geometry or side constraints for some problems.

Consider an optimization problem with inequality constraint $g_i(X) \leq 0$. The set of values of X that satisfy the equation $g_i(X) = 0$ forms a hyper surface in design space and is called constraint surface. This constraint surface is a dividing line for the regions of $g_i(X) < 0$ and $g_i(X) > 0$ in the design space. Thus all points on the hyper surface satisfy the constraint $g_i(X) \leq 0$ critically. The points in the region $g_i(X) > 0$ are unacceptable, while the points in the region $g_i(X) < 0$ are acceptable. A design point lying on one or more constraint surfaces is called a bound point and the constraint associated with it called active constraint. Design point not lying on any constraint surface is a free point. Based on whether the design point lies in acceptable or unacceptable region, it is being classified into free-acceptable point, free-unacceptable point, bound-acceptable point, bound-unacceptable point [9].

2.5 Structural Optimization Methods

Optimization techniques, also known as mathematical programming techniques are part of operations research, which is a branch of mathematics concerned with the application of scientific methods and techniques to decision making problem and obtain best or optimal solutions. Various optimization techniques, developed from the area of operations research are classified as follows [9].

1. **Mathematical Programming Techniques:** Mathematical programming techniques are generally used in finding the minimum value of an objective function of several variables being subjected to a set of constraint functions. Calculus methods, Linear and

non-linear programming, Dynamic programming, Multi-Objective programming, CPM – PERT methods, Genetic Algorithms are few such techniques.

2. Stochastic Process Techniques: This optimization technique is used to analyze problems prescribed by a set of random variables with known probability distribution. Statistical decision theory, Markov process are some among these.

3. Statistical Methods: As the name indicates these methods analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation. Clutter analysis, Regression analysis, Discriminate analysis are few of the statistical methods in use.

Fig. 2.1 shows various methods of optimization in practice [9].

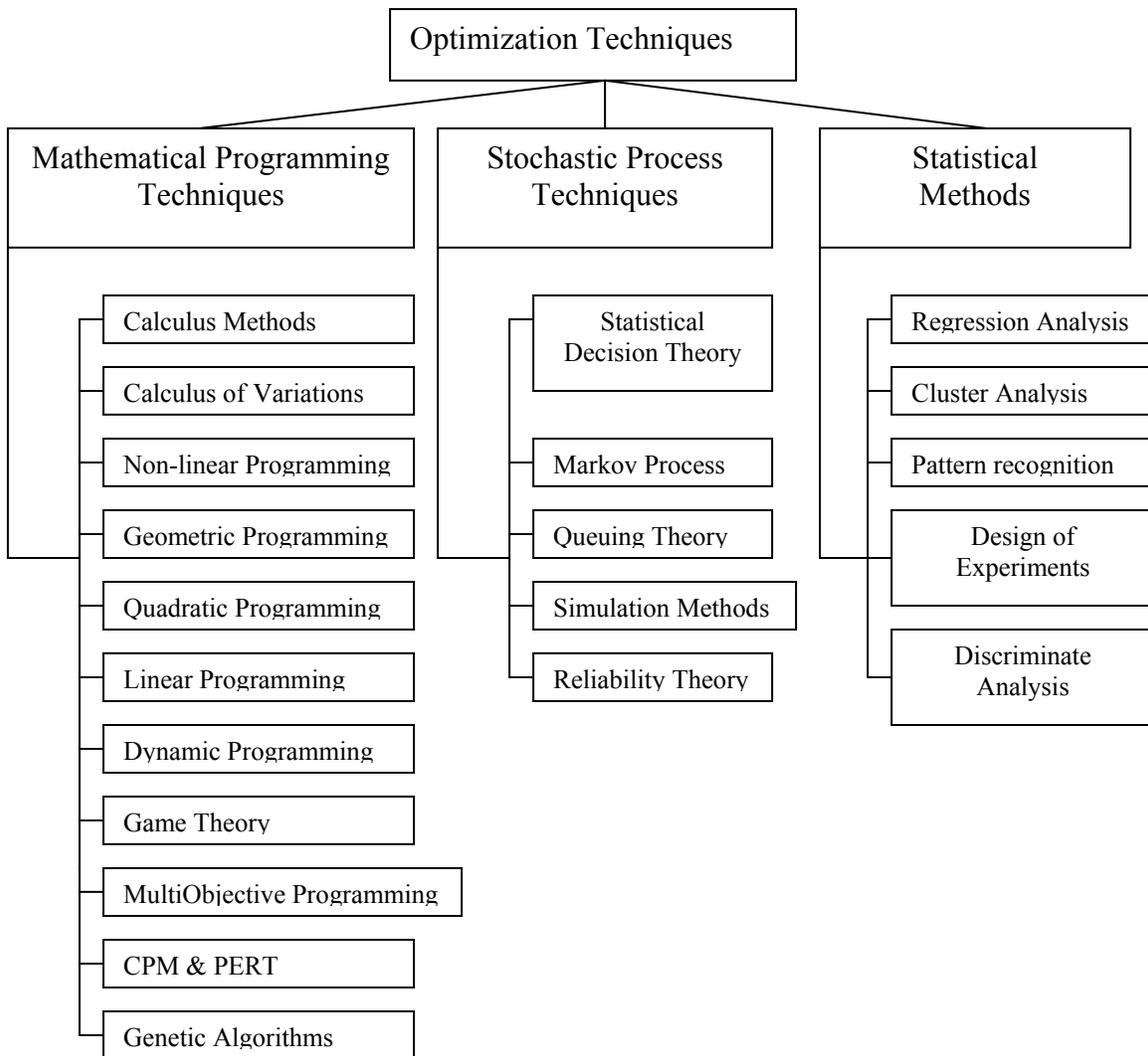


Fig. 2.1 Various methods of Optimization

2.6 Femlab for Optimization

Femlab is powerful and user-friendly finite element software used for modeling and solving scientific and engineering problems. The ease with which the conventional models of one field of science can be extended to state of art multiphysics problems involving various other fields of science and engineering demonstrates its versatile nature as multiphysics software. The results to modeled problems are obtained based on solution to partial differential equations pertinent to that specific model. The design problem can be stated in two different ways. One is through the user defined partial differential equations (PDE's) which requires in-depth knowledge of mathematics and numerical analysis on part of the user. The other way is Femlab itself defines a set of PDE's that represent user developed model along with various boundary & loading specifications and the various solutions required of the analysis [19].

The PDE's representing the model can be written in three forms:

1. Coefficient form which is very convenient for linear models,
2. General form for nonlinear models and
3. Weak form which is a high level modeling language.

Femlab can be run in the stand alone mode or it can be implemented from the Matlab command line or both. This multiphysics tool is useful for solving variety of problems in structural mechanics, heat transfer, acoustics, electromagnetics, fluid flow, chemical engineering, general mutliphysics, PDE's, etc. Owing to all the above

characteristics of the finite element solver, Femlab is chosen in this research for finding structural responses to design problems.

Literature survey reveals that there are only a few instances when Femlab has been implemented for solving problems in optimization. Ziu et al used Femlab for solving the problem of compliance minimization structural topology optimization using level set method [15].

CHAPTER 3

SUPPORT VECTOR MACHINES FOR REGRESSION

3.1 Introduction

In the last few years, the amount of information being processed has increased enormously and there is a great necessity for foreseeing and predicting responses of the system to inputs that have yet to be observed. Thus the problem of empirical data modeling has originated in many engineering applications. In the empirical data modeling, a model of the system is built which is based on the past responses of the system. It is very important to note that the quality and quantity of these input data are crucial and decisive for proper and efficient empirical model generation. The sample data used for building the model is limited and sparsely distributed in the input space. A variety of optimization algorithms have been developed for the parameter selection towards a reliable empirical model.

The support vector machines, SVMs in short, developed by Vapnik have gained popularity due to many attractive features and promising empirical performance in the area [1]. SVMs serve as a means of developing functions from a set of training data. Initially these were developed to solve major classification problems in data processing and problems of pattern recognition, but now, the SVMs are being used extensively for solving the regression problems as well [8].

3.2 History of Support Vector Machines

The support vector algorithm is a nonlinear generalization of the generalized portrait algorithm developed in Russia in the 1960s (Vapnik and Lerner 1963, Vapnik and Chervonenkis 1964). This algorithm is based on the statistical learning theory or the VC theory in short, developed by Vapnik and Chervonenkis in 1974. SVMs got modified to their present form with further contribution by Vapnik and his colleagues at the AT&T Bell Laboratories in 1990s [8]. Smola and Scholkopt contribute in extending the implementation of vector machines to regression analysis. Of recent, smoothing strategies are used to develop smooth support vector machines for classification and the ε -smooth support vector machine for regression problems [12, 13].

3.3 Some Important Features of SVM

The following are some of the important features of SVM which makes it very efficient for learning problems.

1. SV algorithms are very simple to implement and are very efficient from the computation point of view, like the linear algorithms and also possess the flexibility of nonlinear systems, like networking algorithms. This makes them very efficient and handy for the users.
2. It is very convenient to deal with large data sets.
3. Being laid on the principles of statistical learning theory, they are highly resistant to over fitting, when other classical algorithms are incompetent due to the curse of dimensionality.

4. As SVM is a non-parametric technique there is no necessity to specify the number of basis functions before hand.
5. The input data of SV algorithms need not always be in the form of vectors. Strings, trees and images are also acceptable forms of input.

3.4 Regression Estimation Problem

Consider two sets of elements X and Y. These two sets are said to be functionally dependent on each other if there exists any vector $x \in X$ corresponding to a unique scalar element $y \in Y$.

Mathematically, for every vector $x \in X$, there exists a scalar $y \in Y$. Thus a stochastic dependency exists between X and Y such that for every 'x' there is a unique 'y'. A distribution $F(y/x)$ is defined on Y for each 'x', depending on which value of 'y' is selected. The function of conditional probability expresses this stochastic relationship between 'y' and 'x'.

Now let the vector 'x' be chosen in a random and independent fashion in accordance with another distribution of 'x' say $F(x)$. Also the corresponding values of 'y' are obtainable from $F(y/x)$. Thus there exists a joint distribution function $F(x, y)$ such that the pairs

$$(y_1, x_1), (y_2, x_2) \dots\dots\dots (y_f, x_f)$$

are all random and independent of one another. Estimation of stochastic dependence based on the empirical data thus obtained means the estimation of conditional distribution function $F(y/x)$, which is a tough problem. But it is not necessary to possess the complete

knowledge of the function $F(y/x)$. The problem can be simplified to determining one of the characteristics, like say, the function of conditional mathematical expectation

$$r(x) = \int y dF(y/x).$$

The function $r(x)$ stated above is called the regression and its estimation in the set of functions $f(x, \alpha)$, where $\alpha \in A$ is the problem of regression estimation [4].

3.5 Support Vector Regression

Support vector machines earlier used for classification problems, can be applied to problems of regression by introducing a loss function. In simple words the problem of regression is to find a function which approximates mapping from the input domain to corresponding real values as closely as possible, with the aid of training data set [2]. The loss function is one that determines the accuracy to which the regression function can attain output values compared to the true output responses observable from a system. There are four possible loss functions.

Quadratic loss function is based on the least square error criterion is shown in fig. 3.1(a). The Laplacian loss function shown in fig. 3.1(b) is less responsive to outliers when compared with quadratic function. The third loss function in fig. 3.1(c), Huber loss function is a robust function with optimal properties when the underlying data distribution is not known. But the three loss functions stated here do not account for the sparseness in a support vector. Fig. 3.1(d) represents the ϵ -insensitive loss function which is an approximation to Huber's loss function and this enables a sparse set of support vectors to be obtained [1].

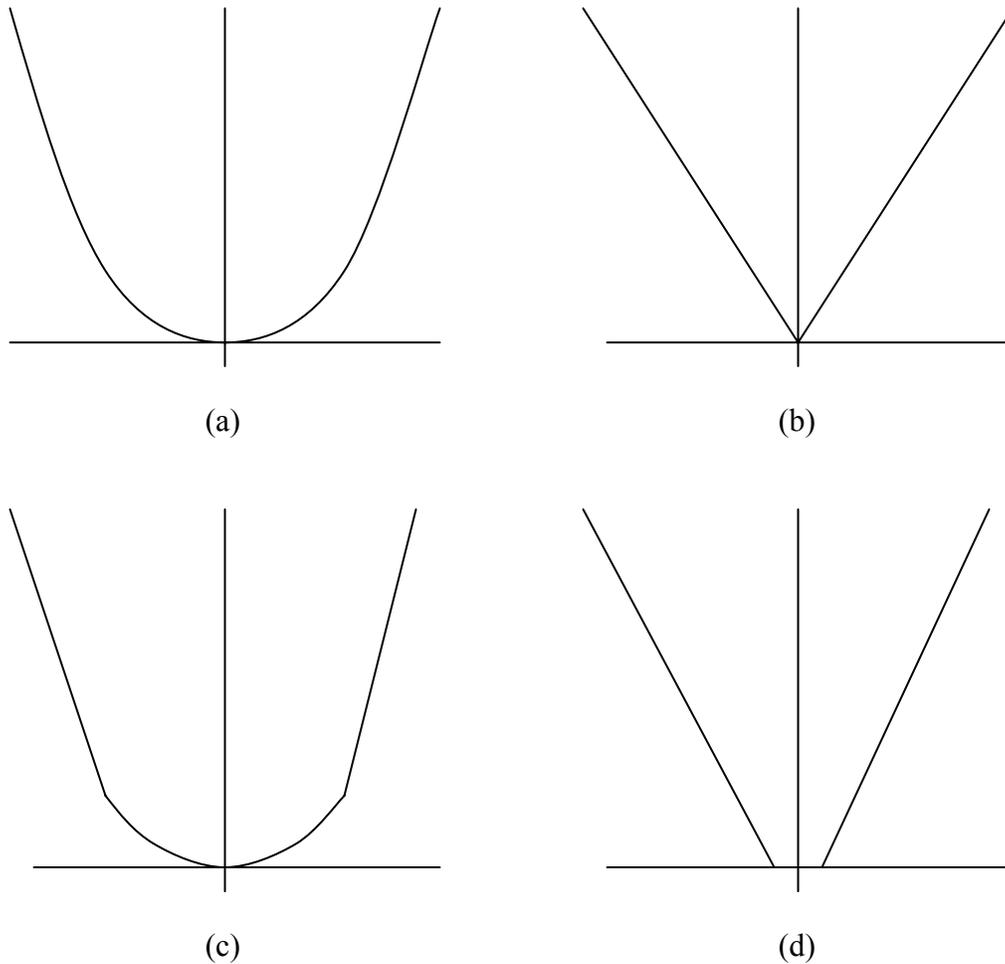


Fig. 3.1 Loss functions used in SVMs. (a). Quadratic loss function, (b). Laplace loss function, (c). Huber loss function and (d). ϵ -insensitive loss function.

3.5.1 Linear Support Vector Regression

Support vector regression (SVR) is aimed at finding a function $f(x)$ which tolerates a maximum of ' ϵ ' deviation in the response of the training inputs. The value ϵ is

due to both approximation and randomness errors. Thus the function $f(x)$ is represented as

$$f(x) = \langle w, x \rangle + b$$

where $\langle w, x \rangle$ is the dot product representation of w and x . Also the function $f(x)$ needs to be as flat as possible. When the flattest function is chosen in the feature space, a smooth function results in the input space. Small value of 'w' is an indication of the flatness of the function $f(x)$.

Thus the optimization problem is formulated as follows for SVR approximation

$$\text{Minimize} \quad \frac{1}{2} \|w\|^2$$

subjected to following constraints

$$y_i - \langle w, x_i \rangle - b \leq \varepsilon$$

$$\langle w, x_i \rangle + b - y_i \leq \varepsilon$$

A very important assumption made in this algorithm is that function $f(x)$ approximates all the input pairs (x_i, y_i) with a precision of 'ε', but the problem may sometimes demand for greater tolerance values. For this purpose the slack variables ξ_i and ξ_i^* are used (fig 3.2) and the optimization problem is restated as:

$$\text{Minimize} \quad \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{\ell} (\xi_i + \xi_i^*)$$

$$\text{subjected to} \quad y_i - \langle w, x_i \rangle - b \leq \varepsilon + \xi_i$$

$$\langle w, x_i \rangle + b - y_i \leq \varepsilon + \xi_i^*$$

$$\xi_i, \xi_i^* \geq 0$$

where the constant term $C > 0$, also known as weighted parameter is a tradeoff value, that is allowable between the flatness and the degree to which deviations in the slack variable are tolerable. This is referred to as the ε -insensitive loss function, which enables a sparse set of support vectors to be obtained for regression analysis.

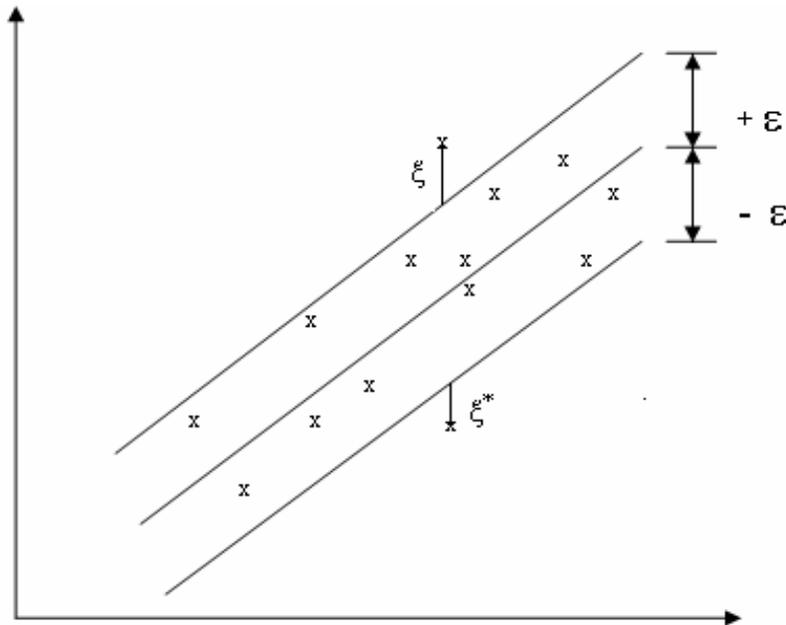


Fig. 3.2 Accounting for the Slack Variables [7]

The Lagrange form of the optimization function along with the linear constraints is as follows

$$L = \frac{1}{2} |w|^2 + C \sum_{i=1}^{\ell} (\xi_i + \xi_i^*) - \alpha_i (\varepsilon + \xi_i - y_i + \langle w \cdot x_i \rangle + b) - \sum_{i=1}^{\ell} \alpha_i^* (\varepsilon + \xi_i^* + y_i - \langle w \cdot x_i \rangle - b) - \sum_{i=1}^{\ell} (\eta_i \xi_i + \eta_i^* \xi_i^*)$$

The necessary conditions for α to be a solution to the original optimization problem are as follows

$$\partial_b L = \sum_{i=1}^{\ell} (\alpha_i^* - \alpha_i) = 0$$

$$\partial_w L = w - \sum_{i=1}^{\ell} (\alpha_i^* - \alpha_i) x_i = 0$$

$$\partial_{\xi_i} L = C - \alpha_i - \eta_i = 0$$

$$\partial_{\xi_i^*} L = C - \alpha_i^* - \eta_i^* = 0$$

Thus, it is now required to

$$\text{Maximize} \quad - \frac{1}{2} \sum_{i,j=1}^{\ell} (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) \langle x_i \cdot x_j \rangle - \varepsilon \sum_{i=1}^{\ell} (\alpha_i + \alpha_i^*) + \sum_{i=1}^{\ell} y_i (\alpha_i - \alpha_i^*)$$

such that

$$\sum_{i=1}^{\ell} (\alpha_i - \alpha_i^*) = 0$$

and $(\alpha_i - \alpha_i^*) \in [0, C]$

Now we have

$$w = \sum_{i=1}^{\ell} (\alpha_i^* - \alpha_i) x_i$$

and the linear regression function $f(x)$ is expressed as

$$f(x) = \sum_{i=1}^{\ell} (\alpha_i - \alpha_i^*) \langle x_i, x \rangle + b$$

Hence $f(x)$ and the whole optimization problem is expressed in terms of dot products. This defines the SVR optimization problem as a quadratic programming problem with linear constraints and a positive definite Hessian matrix with one single global optimum value as the output response. There are other highly effective and thoroughly tested quadratic solvers available for such problems. Also the dual formulation ensures the input vectors to appear in the dot product only. This also ensures the dot product which is a scalar value for each pair of inputs to be stored in a separate quadratic matrix, thus nullifying the curse of dimensionality prevailing in many other algorithms [7].

3.5.2 Nonlinear Support Vector Regression

With the linear support vector regression in the dual form, the nonlinear support vector regression approach can be easily formulated by replacing the dot product of input vectors with a nonlinear transformation on the input vectors. This nonlinear

transformation function is called the kernel function and is denoted by $k(x_i, x_j)$, where x_i and x_j are the input vectors.

Optimization problem gets transformed to

$$\text{Maximizing} \quad -\frac{1}{2} \sum_{i,j=1}^{\ell} (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) k(x_i, x_j) - \varepsilon \sum_{i=1}^{\ell} (\alpha_i + \alpha_i^*) + \sum_{i=1}^{\ell} y_i (\alpha_i - \alpha_i^*)$$

subject to

$$\sum_{i=1}^{\ell} (\alpha_i - \alpha_i^*) = 0 \quad \text{and}$$

$$\alpha_i - \alpha_i^* \in [0, C]$$

The non linear SVR approximation function $f(x)$ becomes

$$f(x) = \sum_{i=1}^{\ell} (\alpha_i - \alpha_i^*) k(x_i, x_j) + b$$

It is very important to ensure that the kernel matrix is positive definite to obtain a unique optimum global solution to the quadratic optimization problem [7, 8].

3.6 Smooth Support Vector Regression

The concept of applying smoothing strategies for solving support vector regression is introduced by Lee et al [13]. Smoothing techniques are in extensive use nowadays for solving important mathematical problems. As already seen in the previous disruption, support vector regression is formulated as constrained minimization problem, i.e. the convex quadratic programming problem. In smooth support vector regression, the

ϵ -insensitive loss function is replaced by an accurate smooth approximation and with few modifications the problem is solved as an unconstrained minimization problem. In the SVR approximation stated earlier, single norm of ϵ -insensitive loss function with the weight parameter C was used in the objective function. Now using smoothing approach, the single norm is replaced by $\| \langle w, x_i \rangle + b - y_i \|^2$ and the weight parameter is reduced to $C/2$. Also the term $\frac{1}{2} b^2$ is added to the objective function to ensure that the problem converges to a unique global optimal solution [13].

The optimization problem is redefined as unconstrained minimization problem:

$$\text{Minimize} \quad \frac{1}{2} (\| w \|^2 + b^2) + \frac{1}{2} C \sum_{i=1}^{\ell} \| \langle w, x_i \rangle + b - y_i \|^2$$

Now the square of the ϵ -insensitive loss function in the above formulation is rewritten in terms of the approximation smooth function, p_ϵ^2 which is stated as follows by Lee et al [13].

$$p_\epsilon^2(x, \alpha) = (p(x - \epsilon, \alpha))^2 + (p(-x - \epsilon, \alpha))^2$$

where $p(x, \alpha) = x + (1/\alpha) \log(1 + e^{-\alpha x})$ and α is the smoothing parameter. Thus the p^2 function replaces the loss function to arrive at the following linear smooth support vector regression problem formulation as follows:

$$\text{Minimize} \quad \frac{1}{2} (\| w \|^2 + b^2) + \frac{1}{2} C \sum_{i=1}^{\ell} p_\epsilon^2(\langle w, x_i \rangle + b - y_i, \alpha)^2$$

and the linear regression function $f(x)$ is expressed as

$$f(x) = \langle w, x \rangle + b$$

Thus in the SSVR approach (with linear regression), a linear system of equations is solved whereas for the conventional linear SVR discussed in previous sections quadratic problem was solved.

Similarly, for the case of nonlinear SSVR the minimization problem formulation is as follows:

$$\text{Minimize } \frac{1}{2} (\|w\|^2 + b^2) + \frac{1}{2} C \sum_{i=1}^{\ell} p_{\epsilon}^2 (k(x_i, x_j)w + b - y_i, \alpha)^2$$

and the linear regression function $f(x)$ is expressed as

$$f(x) = k(x_i, x_j)w + b.$$

where $k(x_i, x_j)$ is the kernel function.

3.7 Kernel Functions

$K(x_i, x_j) \equiv \Phi(x_i)^T \Phi(x_j)$ is called the kernel function. Kernel function is basically a weighted function applicable for nonparametric function estimations. Different kernels developed so far are either dot product functions or distance functions. Linear, Polynomial and Sigmoid kernels are dot product kernels while the Radial basis function (RBF) kernel, Multiquadratic kernel, etc form the distance kernels. A kernel mapping of input vectors, to the corresponding feature space is shown in Fig.3.3.

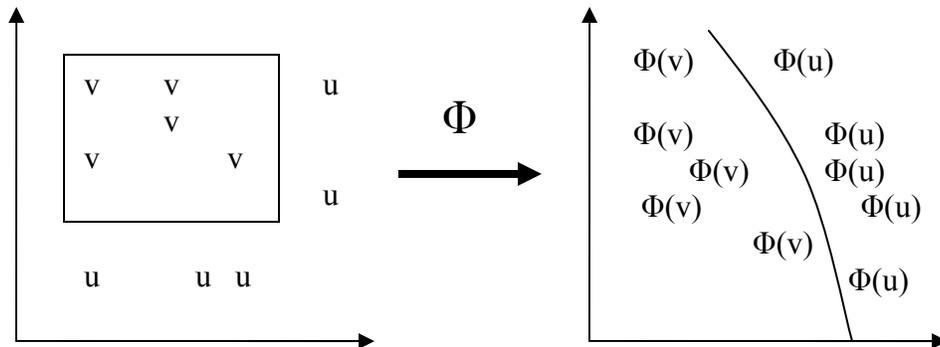


Fig. 3.3 Kernel Mapping [16]

The four basic kernels used in SVM are [3, 14, 15] :

1. Linear kernel- It is represented as $K(x_i, x_j) = x_i^T \cdot x_j$
2. Polynomial kernel- It is mathematically represented as

$$K(x_i, x_j) = (\gamma x_i^T \cdot x_j + r)^d \quad \text{and } r > 0.$$

3. Radial Basis Function (RBF) kernel- This is symbolically represented as

$$K(x_i, x_j) = e^{(-\gamma \|x_i - x_j\|^2)} \quad \text{and } r > 0.$$

4. Sigmoid Kernel – This kernel is represented as

$$K(x_i, x_j) = \tanh(\gamma x_i^T \cdot x_j + r).$$

where γ , r and d are various kernel parameters.

All the SVM kernels satisfy the Mercer's condition, i.e. they generate a positive integral operator. Of the above, Polynomial and RBF kernel are most extensively used.

3.7.1 Polynomial Kernel

A polynomial kernel function is directional in nature. The output product is dependent on the direction of the two vectors in the input space and all the vectors with same direction will have higher output for the kernel. It is also clear that the magnitude of input vectors influence the magnitude of output. Thus polynomial kernels are suited for problems where all the training data is normalized. A simple representation of the polynomial kernel is given in fig. 3.4.

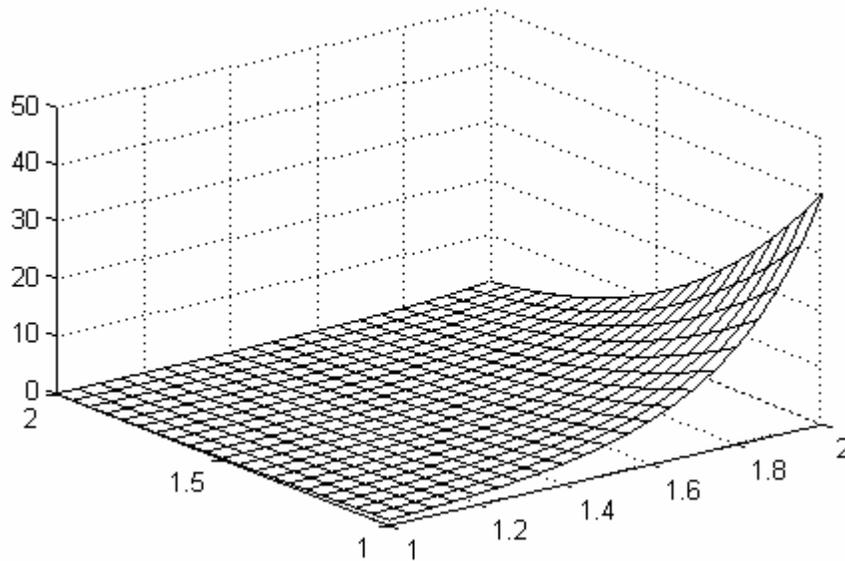


Fig. 3.4 An Example of Polynomial kernel

3.7.2 Radial Basis Function Kernel

In most cases a gaussian function is used as RBF kernel. This kernel uses the euclidean distance between two vectors in input space to find suitable correlation in the feature space. Points close to each other have good correlation while points far away each other have uncorrelated image in the feature space. It has got a very good feature of strong learning ability. An example of RBF kernel is given in fig.3.5.

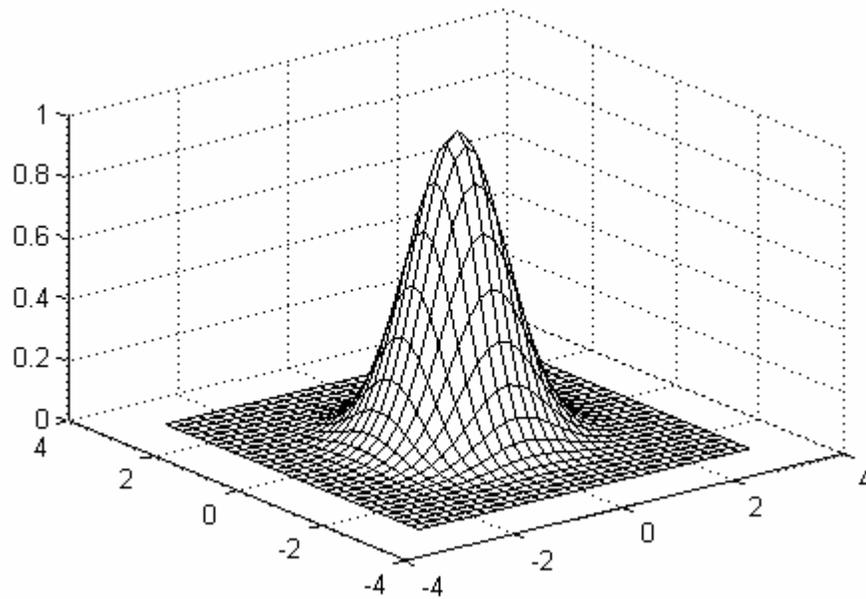


Fig. 3.5 An Example of RBF kernel

The RBF kernel performs a nonlinear mapping of the input vectors into a high dimensional feature space, which is not possible with the linear kernel. Linear and Sigmoid kernels are special cases of RBF kernel with certain critical values of kernel parameters. Also RBF kernel has less numerical difficulty when compared to polynomial

kernel. Thus RBF kernel gives good performance using general smoothing assumptions and is very widely used for support vector regression [9, 15 and 18].

3.8 Quasi-Monte Carlo Sampling

In general, a Monte-Carlo method is any method that uses random numbers as input for its application. But one great disadvantage of random numbers is their non-uniform distribution over the design space, resulting in cluster of points. Quasi-Monte Carlo (QMC) method has provide us with a solution to this randomness, by spreading out the sample data points all over the design space.

In this research work, the training data sampling is based on the QMC method with halton sequencing. In the QMC method the point selection is deterministic in nature, hence resulting in best possible spread of data. This deterministic sequence of data sampling is termed as low-discrepancy sequence [21]. Halton sequence developed by Halton in 1960s, Sobol sequence by Sobol in 1982, etc are some of them. In this work, halton sequence is used as it is practically very appealing. It has been noticed that QMC method yields more accurate results than the Monte Carlo methods, with less number of sample points and using less computational time.

CHAPTER 4

GENERAL ALGORITHM FOR STRUCTURAL OPTIMIZATION USING SMOOTH SUPPORT VECTOR REGRESSION

4.1 Introduction

This chapter discusses the general algorithm of the metamodel for structural optimization using smooth support vector regression (SSVR). Femlab finite element model is solved to get the structural responses for various design problems. SSVR implements the radial basis function (RBF) kernel to generate the approximation functions for the objectives and constraints of the original optimization problem. Quasi-Monte Carlo (QMC) method is used for the selection of sampling data.

A design optimization code is written in Matlab to implement the process of optimization using the proposed metamodel. This code includes the QMC sampling program, SSVR code, RBF kernel approximation and the parametric model of the design problem in Femlab 3.1. This parametric model is written in the Matlab programming language. All the experiments are run on personal computer with Pentium 4 processor and 1 GHz memory in Windows 2000 environment.

4.2 Algorithm of Metamodel

A brief description of the steps involved in the implementation of proposed metamodel for structural optimization is given here. A very important factor to be considered before the implementation of this algorithm is the selection of appropriate

design window for the model. First of all there should be clear understanding of the model regarding the objectives, constraints and type of solution for that problem. The feasibility of finding a solution to the problem for specified bounds should be carefully analyzed.

The algorithm for optimization is broadly divided into 8 stages which are explained as follows:

1. Specifying the Design problem

The first stage of the algorithm begins with the statement of the optimization problem. The lower and upper bounds of the input data XL and XU, total number of input samples NC considered for optimization, number of samples taken initially for generation of responses NO , initial value of the input design variables XO, are all defined. The objective and constraints for the optimization problem are also stated along with the stopping criterion for the objective function, constraint function and weight parameter.

The optimization problem requires that the objectives and constraints be specified in matrix format as follows:

$$f = A_{xObj} * X + A_{yObj} * Y$$

$$g = A_{xCon} * X + A_{yCon} * Y - B$$

where f is the set of objective functions, g is the set of constraint functions, X is set of input design variables and Y is set of output responses. B is the matrix of numerical values in the constraint functions.

A_{xObj} represents the objectives corresponding to input variables.

AyObj represents the objectives corresponding to output responses.

AxCon represents the constraints corresponding to input variables.

AyCon represents the constraints corresponding to output responses.

All the objectives and constraints mentioned above are expressed in matrix form for convenience. A simple black box representation of SSVR model is shown in fig. 4.1.

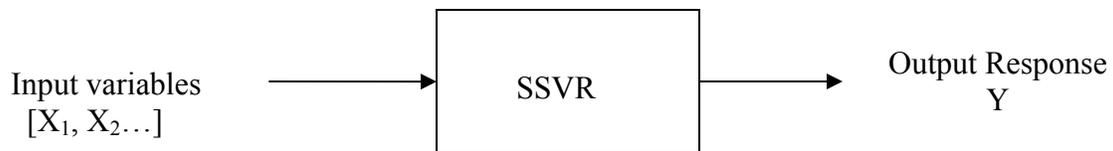


Fig. 4.1 Simple black box model of SSVR

2. Femlab analysis of typical design

The design problem is first modeled for a set of design values and solved for the output response in Femlab software. The geometry is built, boundary conditions are specified and the loading is defined in the preprocessor phase. The problem is then solved for static, dynamic, frequency responses, etc. in the solution phase. In the post processing stage the output responses are obtained. All this finite element analysis is saved as an m-file for further use.

3. Building the Femlab parametric model (FPM)

The m-file corresponding to the typical design problem solved in the previous stage will be modified into a parametric model. This will be used in SSVR code for generating output responses to any set of input variables.

4. Generation of Input design variables

The Quasi-Monte Carlo (QMC) sampling technique is used to generate NC number of input variables. These NC input variables are stored in a matrix named XCAIldata. QMC method uses the halton sequencing approach for generation of these variables. Of these NC inputs, initially a small set NO number of input variables is taken. Now the output responses are obtained for these NO input variables using the FPM. The entire sample of this NO input variables and the corresponding output responses are stores in two separate matrices, which are termed as the input data matrix and output response matrix respectively.

5. Starting iteration for optimal results

Now that the design variables have been specified clearly, the criterion of convergence of optimization is known, the Femlab parametric model is constructed, input design variables have been generated, the next step is to start the process of optimization. A *while* loop is initiated with the value of i set to zero initially and maximum value of i is set to NC-NO.

6. Structural optimization using SSVR

This step of the algorithm has two essential parts.

a). Generation of approximation functions

The value of i is incremented to 1 and the $i+NO^{\text{th}}$ input data variable is evaluated to get the $i+NO^{\text{th}}$ output response. These values are added to the already existing input data matrix and the output response matrix. These modified input and output response

matrices, weight parameter and shift parameter for optimization are fed into an m-file to find the approximation functions for the objectives and constraints. This m-file has the RBF kernel incorporated in it whose purpose is to map the input data into a high dimensional feature space.

b). Optimization process

This is a crucial stage where the optimization process is implemented using the SSVR. All of the design data specified in the previous stages i.e. the bounds of design variables, NC, NO, objectives and constraints of the input as well as responses, FPM model code, approximation functions generated in previous step, various stopping criterion and the shift parameter are all input into the Matlab file that does the optimization using SSVR. Using the principles laid down for SSVR by Lee et al [13] which has been described in brief in previous chapter, we arrive at the quadratic problem formulation. In next stage, the Matlab optimization tool ‘fmincon’ is used to find the optimum solution to the quadratic problem. Fmincon uses the sequential quadratic programming approach to solve for the optimum values. The bounds of design variables, initial design variable, newly obtained approximation functions for objectives and constraints and matrix forms of initial objective and constraints of X and Y are all input to the optimization tool. After getting the optimum values using fmincon, the true output response for this optimum value of design is computed and the optimum values are added to the input variable matrix as well as the output response matrix.

7. Check for convergence of solution

After obtaining the optimum value from the SSVR model, we need to check whether the solution has actually converged or not. For this we take the difference of objective functions of two successive iterations and compare it with the stopping criterion, which is the tolerance allowable for the objective function. If the difference is within the tolerance limits we go further to check for constraint function. If the tolerance criterion is met for the constraint function as well, we conclude that convergence is attained and terminate the optimization loop.

8. If the problem does not converge then value of i is incremented by 1, the optimum input values X_{opt} and corresponding output response Y_{opt} , are added to input data matrix and output response matrix respectively and the previous two steps are repeated till convergence is attained. If the maximum number of iterations is reached and yet the problem does not converge then the design parameters need to be checked.

A flow chart of the algorithm is given in the next page in Fig. 4.2.

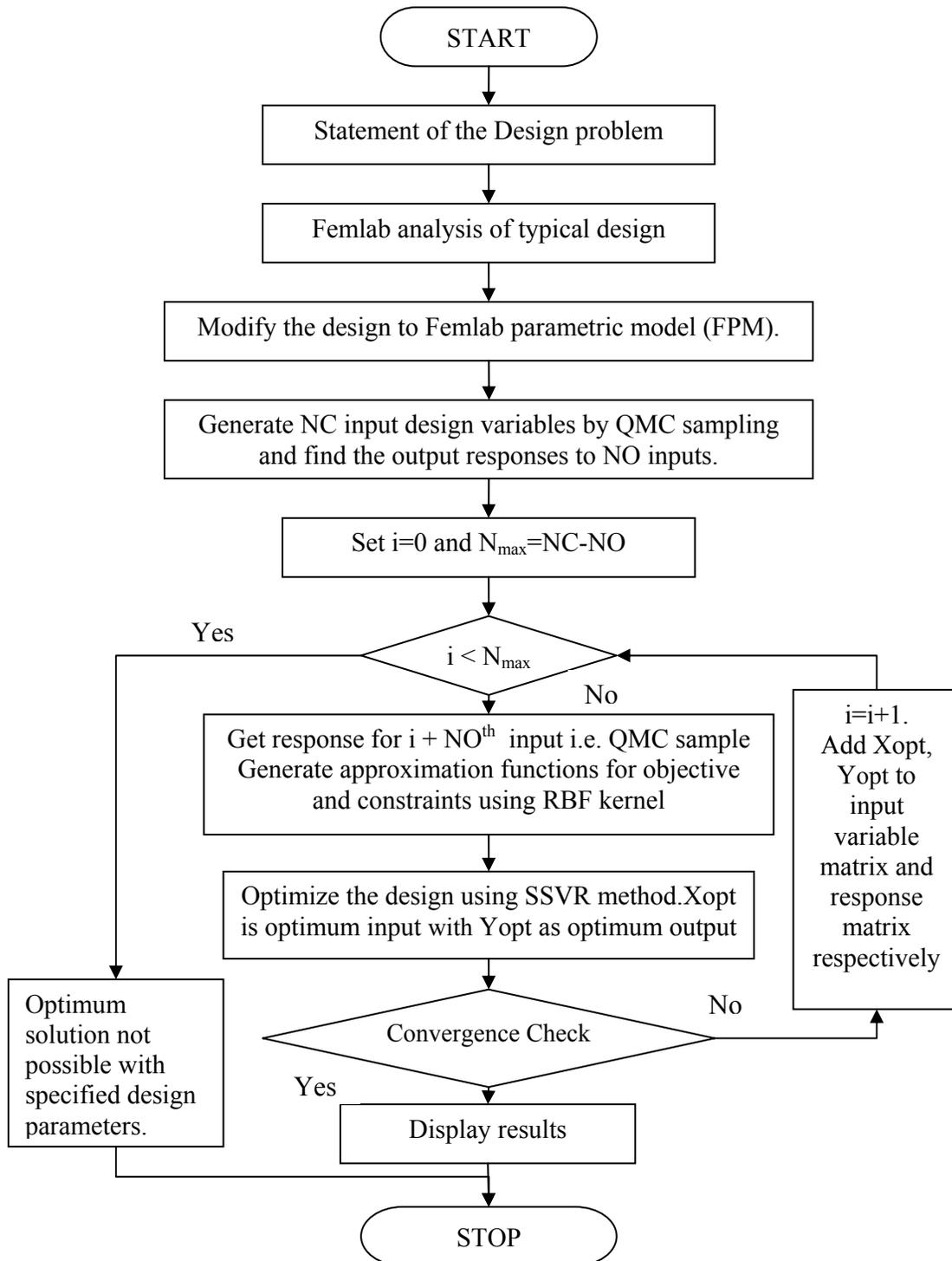


Fig. 4.2 Flow chart of the algorithm for structural optimization with SSVR function

CHAPTER 5

NUMERICAL RESULTS AND COMPARISONS

5.1 Implementation of Metamodel for Optimization

The metamodel developed in this research has been successfully implemented for different applications. The accuracy of optimum design obtained using the metamodel are compared to that obtained directly using the Matlab optimization tool, fmincon. It can be seen that the metamodel results are very accurate with small error when compared to the results of fmincon. In one of the cases metamodel obtained better results than the Matlab tool. It is significant to note that the number iterations required for convergence using metamodel is significantly low when compared to iterations run using fmincon tool.

The application problems solved in this research are described in detail in following sections of this chapter.

5.2 Two Dimensional Model of Trapezoidal Plate

The first application solved in this work is the problem of two dimensional model of trapezoidal plate. The objective of this optimization problem is to design the trapezoidal plate for minimum volume subjected to constraint that the first (in-plane) natural frequency should be greater than or equal to 100 Hz. The plate is made up of aluminum with the following material properties.

Young's modulus = $1e7$ psi

Yield strength =11000 psi

Poisson ratio=0.3

Weight Density =0.1 lbf/in³

The plate is not subjected to any external loading. It is completely fixed at one end and freely hanging on the other end. Fig. 5.1 shows the model of plate. 2D triangular elements are considered in the finite element analysis using Femlab3.1. The length of the plate is fixed value of 100 inch with unit thickness. It is required to find optimum length of the two offset sides of the trapezoidal plate, so that the total volume of the plate is minimum and the first eigen value i.e. the natural frequency of the plate should be greater than or equal to 100 Hz. The bounds for the offset sides i.e. fixed and free edges are 10 to 100 inch and 1 to 10 inch respectively.

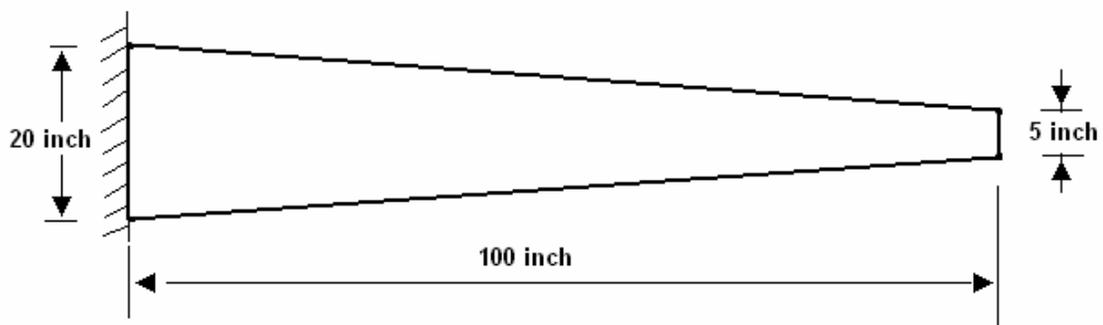


Fig. 5.1 Trapezoidal plate - initial design

The lengths of the two offset sides of the trapezoid are the input design variables. The initial value of inputs is taken as [20, 5].

The parameters used in smooth support vector regression are shift parameter $h=0.5$, stopping criteria for objective function $fEPS=0.3$, stopping criteria for constraint function $gEPS=0$, move-limits for the QMC sampling $MLPara=0.9$. Also $NC=300$ and $NO=10$.

The optimization process converged in 96 function evaluations and minimum volume is 1188.3571 cubic inch and the optimum lengths of the fixed and free edge are 22.7671 inch and 1 inch respectively. Note that for the SSVR process, number of function evaluations is twice the number of iterations plus the value of NO and for $fmincon$, the number of function evaluations is same as number of iterations. The optimum response i.e. first natural frequency is 99.3512 Hz. Following figures show the plots of inputs and outputs during convergence using SSVR and $fmincon$.

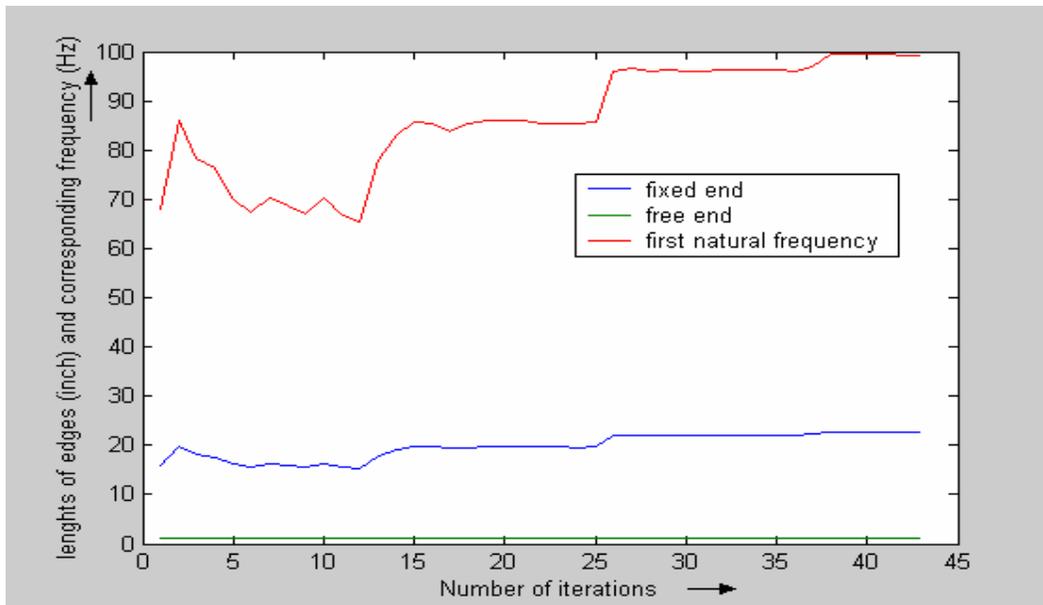


Fig. 5.2 Plot of input variables and output responses during convergence using SSVR for two dimensional model of trapezoidal plate

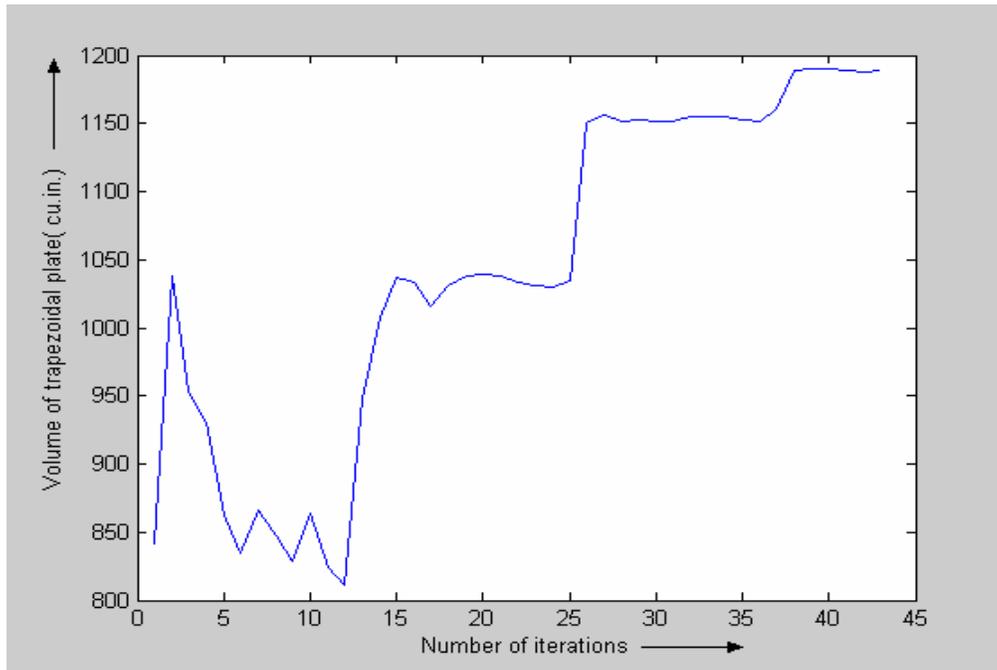


Fig. 5.3 Convergence plot of volume of plate using SSVR for two dimensional model of trapezoidal plate

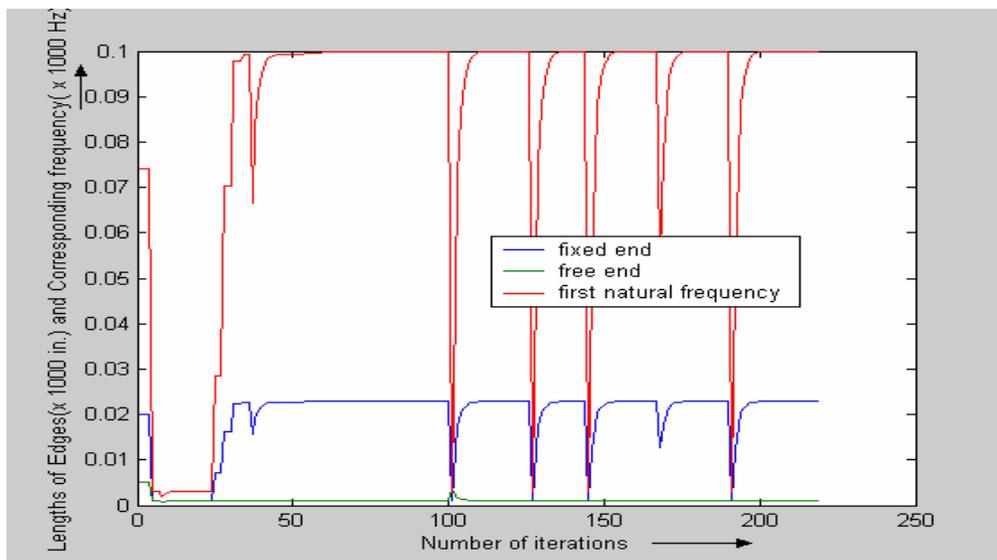


Fig. 5.4 Plot of input variables and output responses during convergence using fmincon for two dimensional model of trapezoidal plate

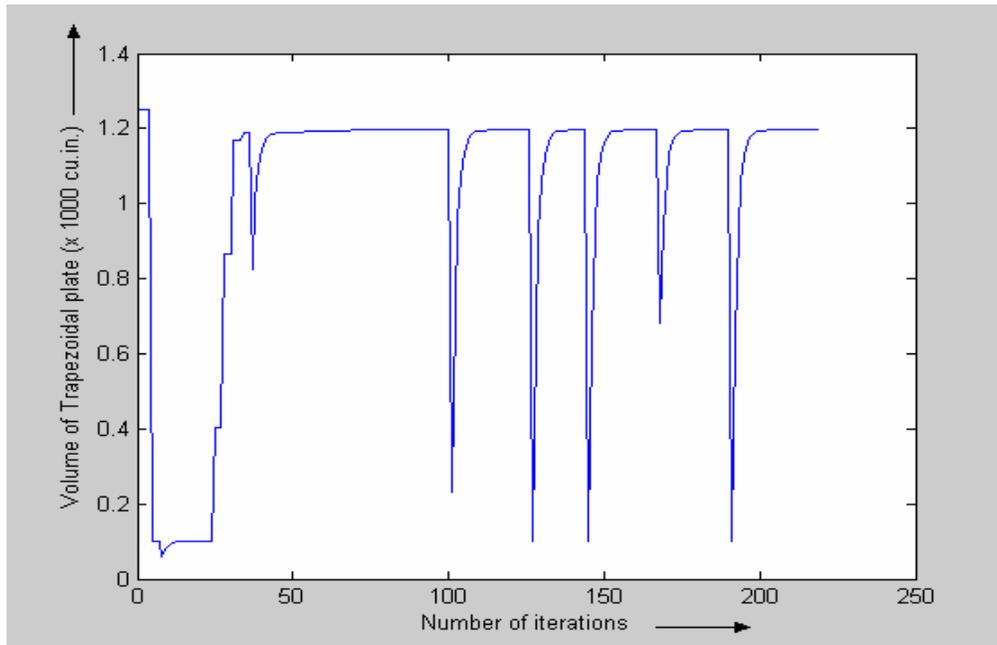


Fig. 5.5 Convergence plot of volume of plate using fmincon for two dimensional model of trapezoidal plate

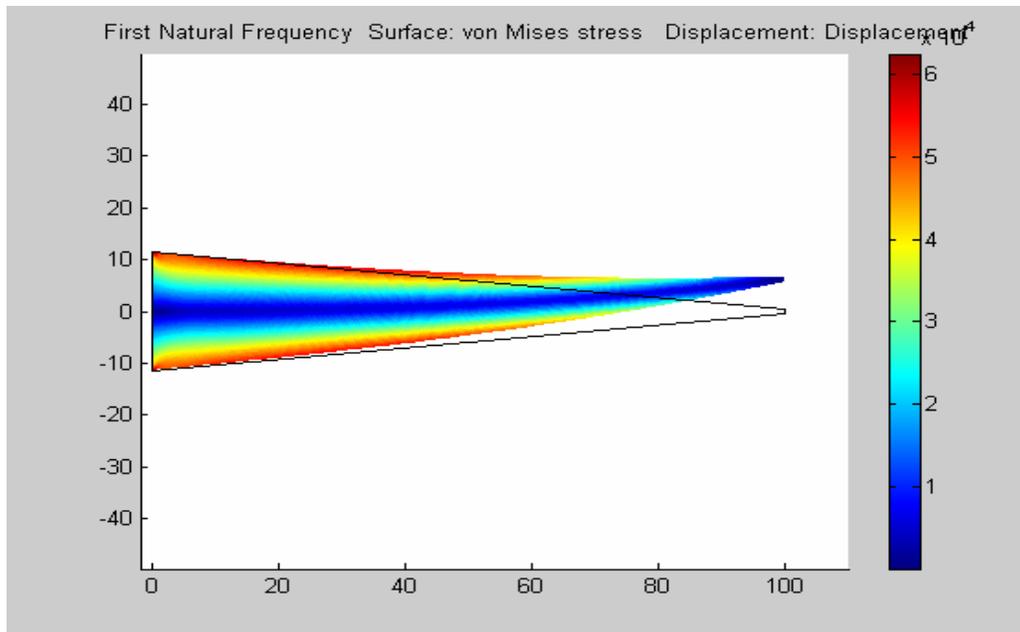


Fig. 5.6 Optimum design of two dimensional model of trapezoidal plate

The Matlab optimization tool fmincon gave the convergence in 247 function evaluations and minimum volume is 1195.67 cubic inch and the optimum lengths of the fixed and free edge are 22.9135 inch and 1 inch respectively with output of 100.001Hz.

5.3 Three Dimensional Trapezoidal Plate

In this problem, 3D trapezoidal plate is to be optimized for minimum volume. It is required to find the length of the offset sides of trapezoid such that the total volume of the plate is minimum. The constraints for this design are the first natural frequency should be greater than or equal to 100 Hz and the second natural frequency should be a multiple i.e. 1.25 times greater than the value of second natural frequency of a certain optimum design with first natural frequency of 100 Hz.

These constraints can be mathematically stated as follows:

Let eig_1 represent the first natural frequency and eig_2 be second natural frequency.

For $eig_1 = 100$ Hz, let eig_2 value be f_2 Hz.

Now in our problem the constraints will be

$eig_1 \geq 100$ Hz and

$eig_2 \geq 1.25$ times f_2 .

The plate material is aluminum with Young's modulus value $1e7$ psi, yield strength of 11000 psi, poison ratio 0.3 and weight density of 0.1 lbf/in³. Again there is no external load on the plate. The model is similar to the one shown in Fig. 5.1 with greater edge completely fixed and smaller edge is freely hanging. The plate has length of 100 inch and constant unit thickness. The bounds for the offset sides i.e. fixed and free edges

are 10 to 100 inch and 1 to 10 inch respectively. This model is similar to the 2D plate, but this is built as a 3D model, with tetrahedral elements used for meshing and different constraints of frequency. The input variables are the lengths of two offset sides with bounds of 10 to 100 inch, 1 to 10 inch for the fixed and free edges respectively. Initial design value is [20, 5].

For this problem, constants used in SSVR are $h=0.5$, $fEPS=0.3$, $gEPS=0.03$, $MLPara=0.5$. Also $NC=150$ and $NO=10$.

Number of function evaluations for convergence using SSVR model is 64. The minimum volume is 1496.2758 cubic inch and the optimum lengths of the fixed and free edge are 28.9255 inch and 1.0000 inch respectively. The optimum response i.e. first and second natural frequencies are 126.1979 Hz and 348.2976 Hz respectively. The results obtained for this model are illustrated in the following figures.

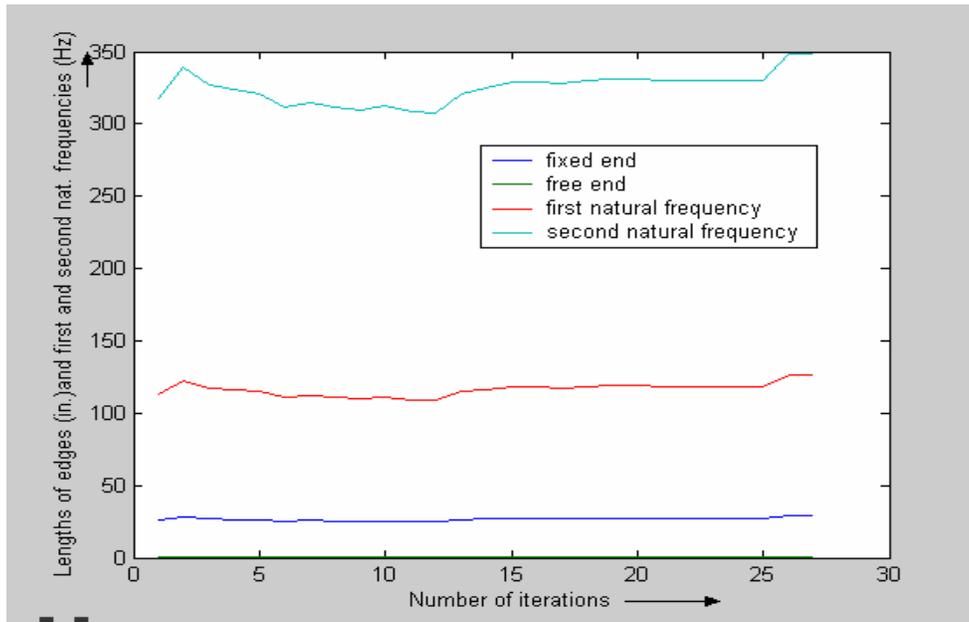


Fig. 5.7 Plot of input variables and output responses during convergence using SSVR for three dimensional trapezoidal plate

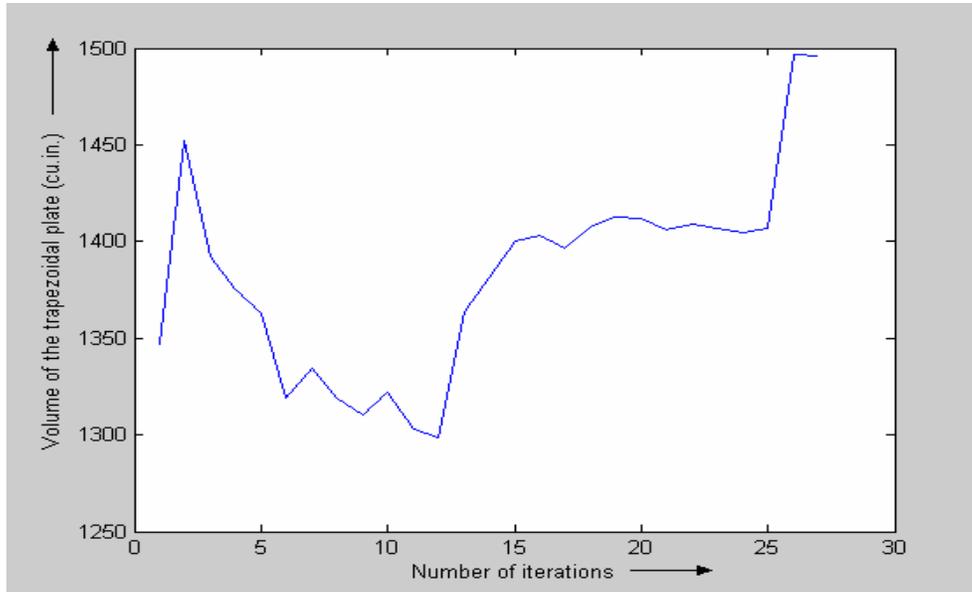


Fig. 5.8 Convergence plot of volume of plate using SSVR for three dimensional trapezoidal plate

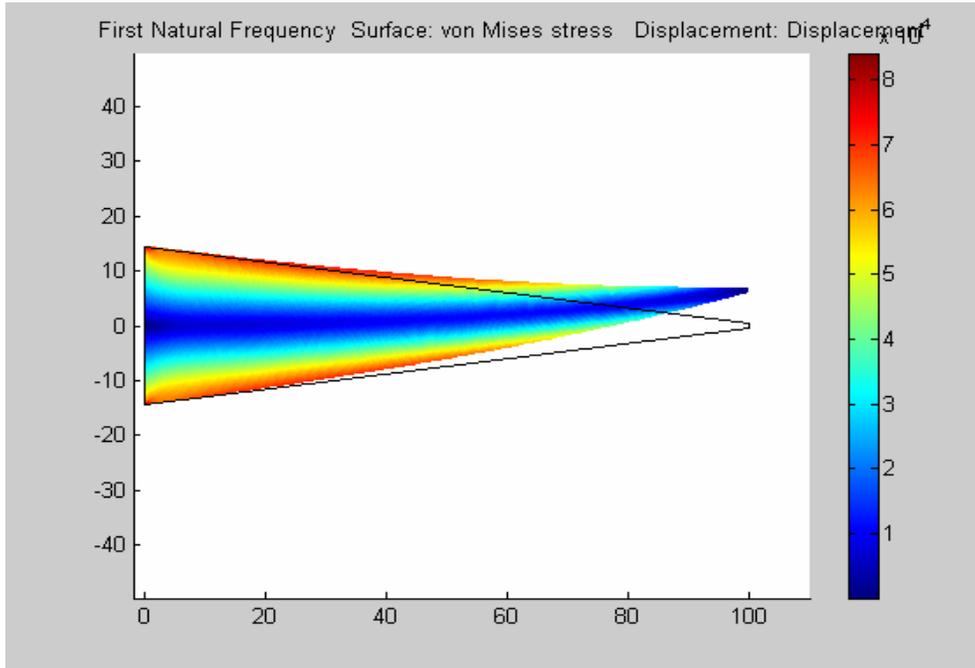


Fig. 5.9 Optimum design of three dimensional trapezoidal plate

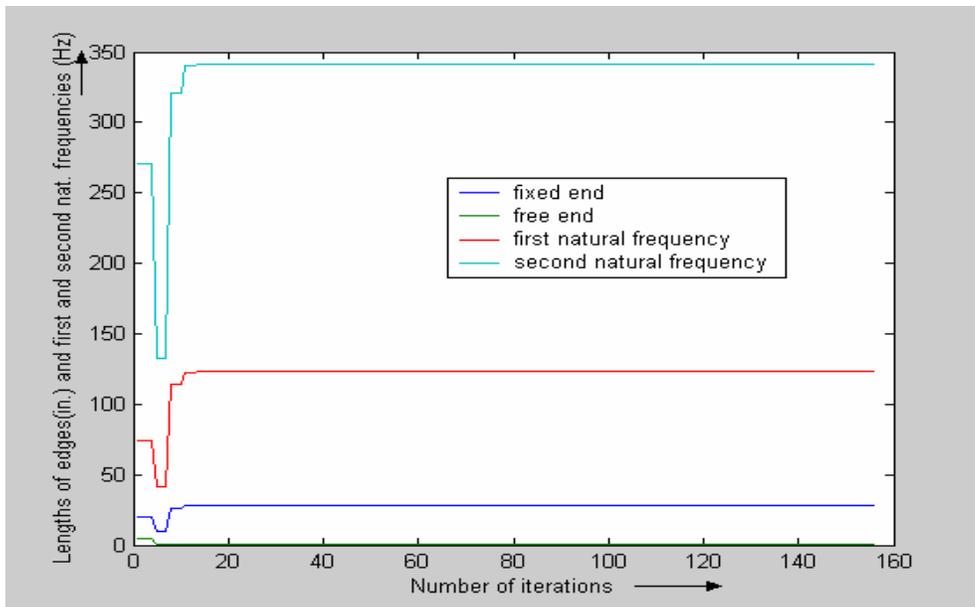


Fig. 5.10 Plot of input variables and output responses during convergence using fmincon for three dimensional trapezoidal plate

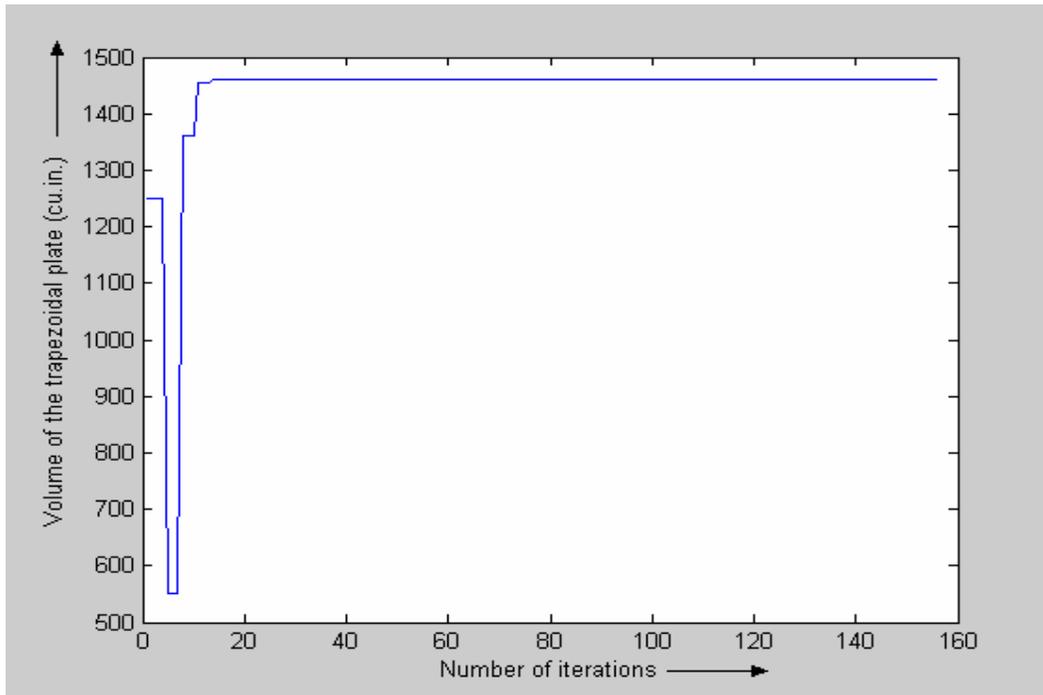


Fig. 5.11 Convergence plot of volume of plate using fmincon for three dimensional trapezoidal plate

Fmincon showed the optimum results after 162 function evaluations. The minimum volume is 1460.08 cubic inch and the optimum lengths of the fixed and free edge are 28.2013 inch and 1.0003 inch respectively. The optimum values for first natural and second natural frequencies are 123.0917 Hz and 341.0003 Hz respectively.

5.4 Rectangular Plate with Circular Hole Fixed at Centre

The next problem considered for analysis is rectangular plate with circular hole fixed at the centre. The objective is to find maximum radius of the hole or in other words minimize the volume of the plate. The constraint for the problem is the maximum shear stress of the plate.

The plate has dimensions of 50 X 30 X 0.5 cubic inch. The plate is made of aluminum with Young's modulus value $1e7$ psi, yield strength of 11000psi, poisson ratio 0.3 and density of 0.1 lbf/in^3 . The plate is fixed at one end and other end is subjected to a tensile loading of 1000 psi applied on the edge. The maximum allowable stress coming on the plate should be less than the yield strength value. Hence our constraint for this problem is that the maximum stress on the plate should be less than 11000 psi. The following figure shows the loading conditions on the plate. The plate is modeled using triangular elements.

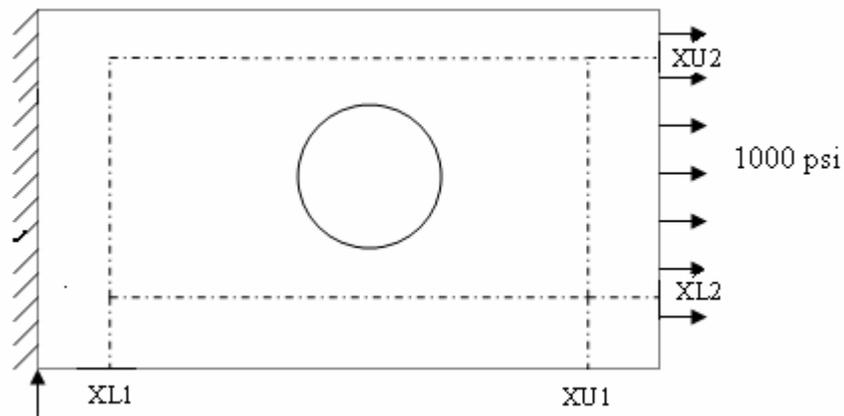


Fig. 5.12 Rectangular plate with circular hole fixed at the centre

The centre of the hole is fixed at $[25, 15]$, the centre of the plate. Now it is required to find the optimum radius of the hole to with stand the load and give minimum volume design of the plate. It is to be noted that the hole has to lie inside the plate so that the finite element analysis can be performed well. Our input design variable is the radius

of circular hole. So in this problem a tolerance of 5 units is given in the x-dimensions and 2 units for the y-dimensions so that the area within which the circle can move is defined by the parameters XL1 (x=5), XU1(x=45), XL2(y=2) and XU2(y=28). This gives us a rectangular region defined by the two diagonally opposite vertices (5, 2) and (45, 28). Hence our constraint is that the circular hole should lie in this new rectangular region. The radius of circle is allowed to vary from 1 to 14 inches.

The initial input design variable is 12 inch of radius. For this problem again, the constants used in SSVR are $h=0.5$, $fEPS=0.3$, $gEPS=0.03$, $MLPara=0.5$. Also $NC=150$ and $NO=10$.

Number of function evaluations for convergence using SSVR model is 58. The minimum volume is 523.0978 cubic inch and the optimum radius obtained is 12.0218 inch. The maximum value of shear stress for this optimum design is $1.0679e+004$ psi. The following figures show the results obtained.

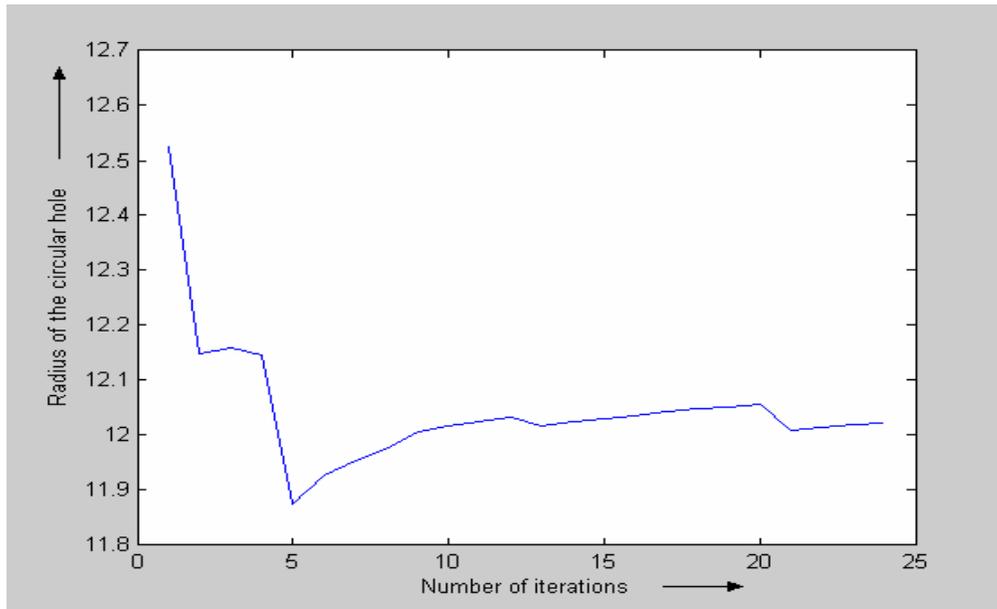


Fig. 5.13 Plot of input variable (radius of hole in inch) during convergence using SSVR for rectangular plate with circular hole at centre

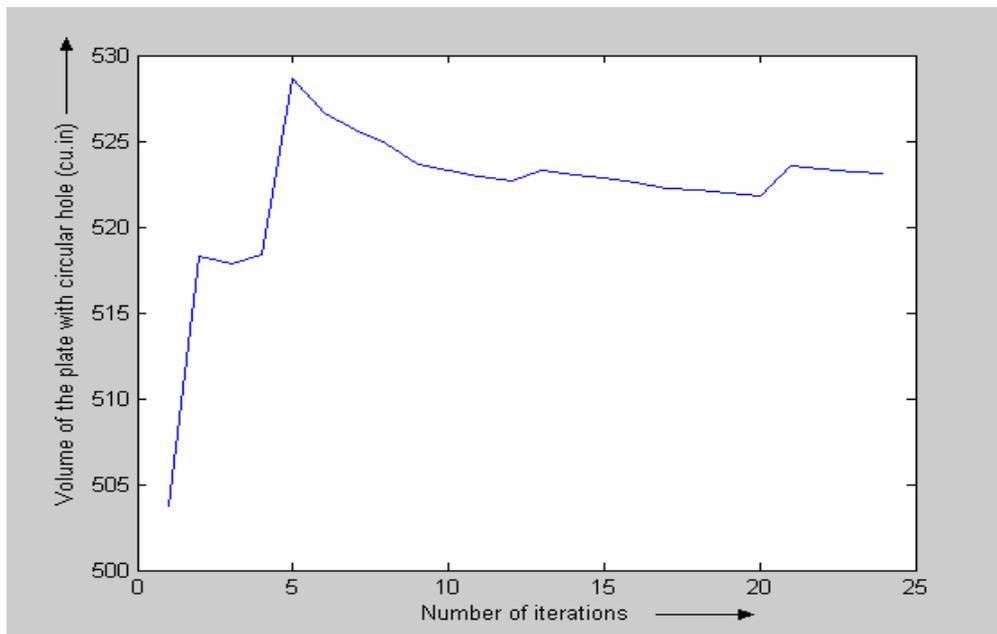


Fig. 5.14 Convergence of volume of the plate using SSVR for rectangular plate with circular hole at centre

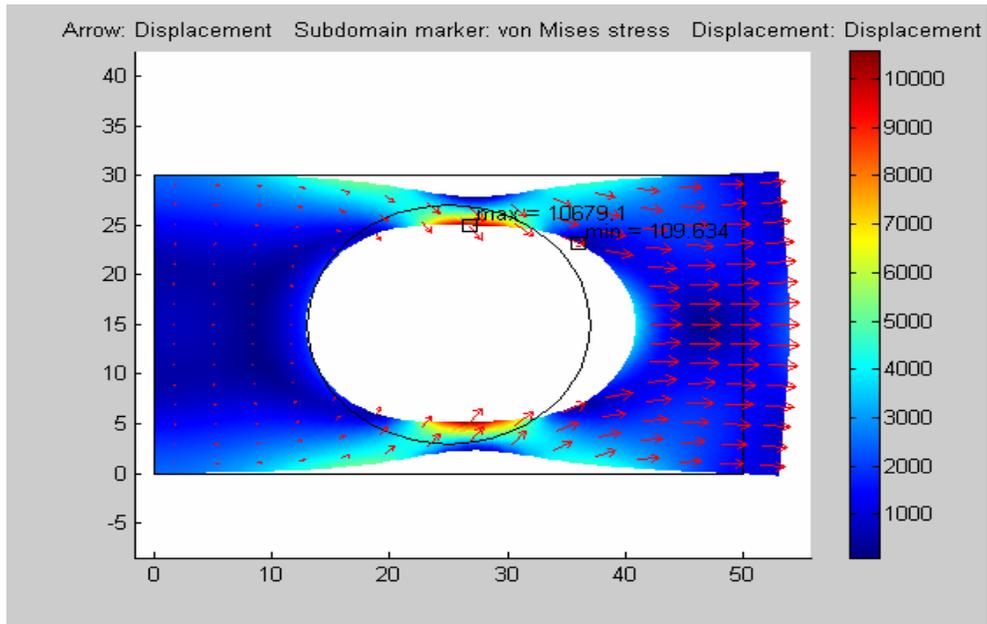


Fig. 5.15 Optimum design of rect. plate with circular hole at centre by SSVR

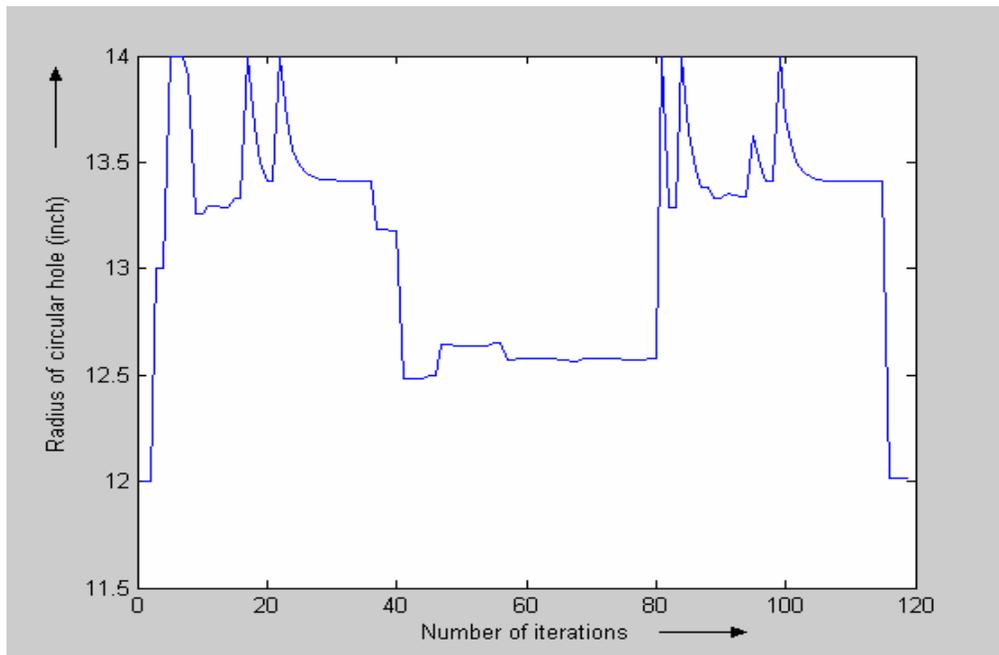


Fig. 5.16 Plot of input variable (radius of hole in inch) during convergence using fmincon for rectangular plate with circular hole at centre

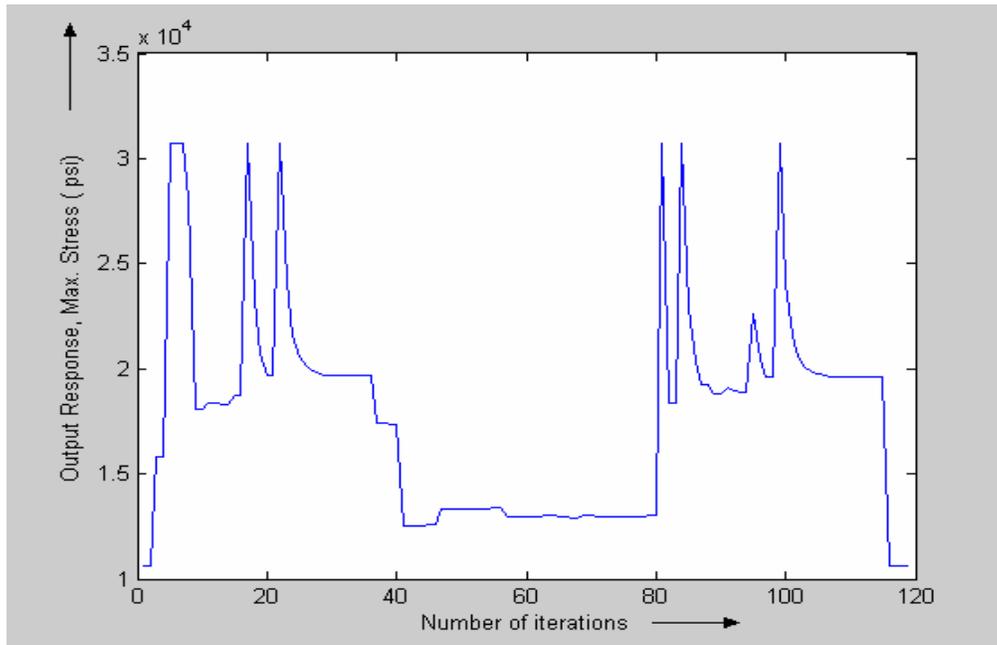


Fig. 5.17 Plot of output response during convergence using fmincon for rectangular plate with circular hole at centre

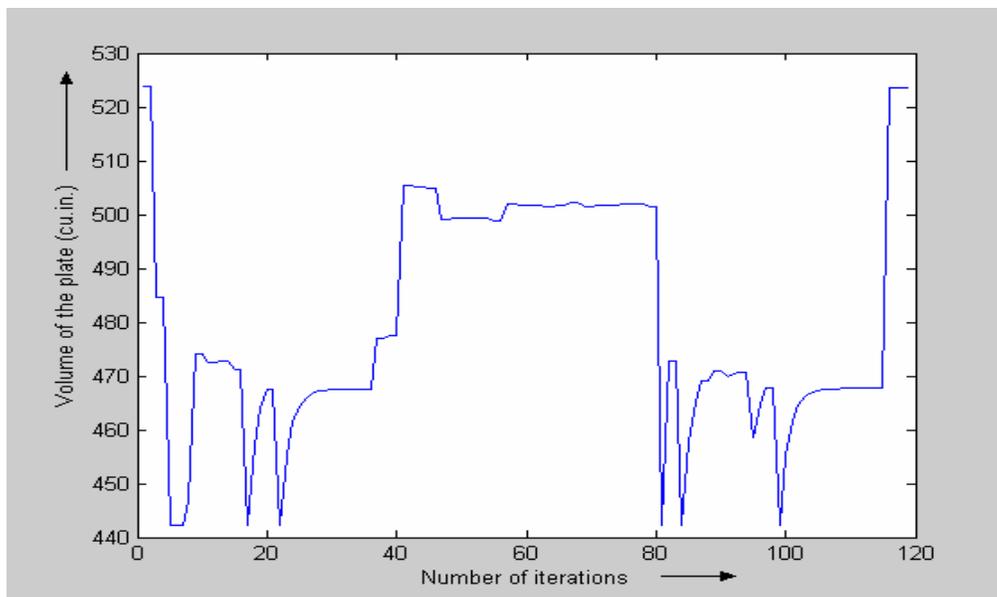


Fig. 5.18 Convergence of volume using fmincon for rectangular plate with circular hole at centre

Number of iteration for convergence using fmincon is 159 function evaluations. The minimum volume is 523.0525 cubic inch and the optimum radius obtained is 12.023 inch. The maximum value of shear stress for this optimum design is 1.078e+004 psi.

5.5 Rectangular Plate for Optimum Location and Radius of Circular Hole

In this problem, rectangular plate with circular hole is optimized for the location as well as radius. The objective is to find maximum radius of the hole or in other words minimize the volume of the plate. The constraint for the problem is the maximum shear stress of the plate. Also there is constraint on the position of the hole which needs to be within the plate.

The plate has dimensions of 50 X 30 X 0.5 cubic inch. The plate is made of aluminum with Young's modulus value $1e7$ psi, yield strength of 11000 psi, poisson ratio 0.3 and density of 0.1 lbf/in^3 . The plate is fixed at one end and other end is subjected to a tensile loading of 1000 psi applied on its edge. The maximum allowable stress coming on the plate should be less than the yield strength value. The plate is modeled using triangular elements. The following figure shows the loading conditions on the plate.

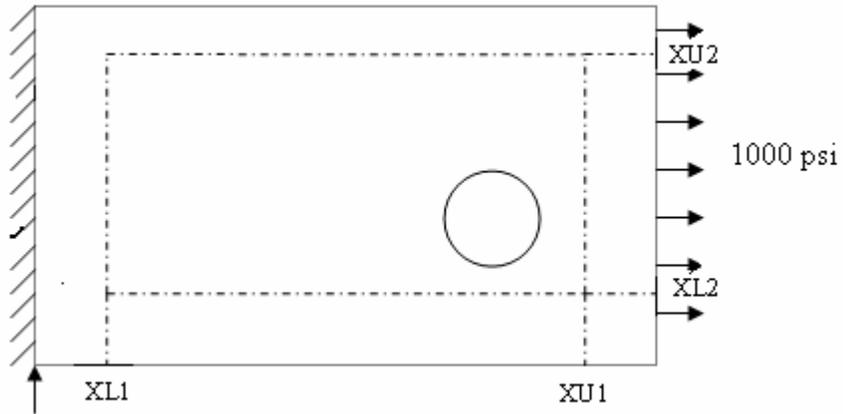


Fig. 5.19 Rectangular plate with hole

The centre of the hole is free to move as long as the hole lies entirely within the plate. Now it is required to find the optimum radius of the hole to with stand the load and give minimum volume design of the plate. Our input design variable is the radius of circular hole along with the location of centre. We have our input as the x-centre, y-centre and the radius. Giving a tolerance of 5 units in the x-dimensions and 2 units for the y dimensions so that the area within which the circle can move is defined by the parameters XL1 (x=5), XU1(x=45), XL2(y=2) and XU2(y =28). This gives us rectangular region defined by the two diagonally opposite vertices (5, 2) and (45, 28). The radius of circle is allowed to vary from 1 to 13 inches.

Now the lower and upper bounds of the input design variable are XL= [5 2 1] and XU= [45 28 13].The initial input design variable is X0= [25 15 12]. For this problem again, the constants used in SSVR are h=0.5, fEPS=0.3, gEPS=0.03, MLPPara=0.5. Also

NC=30 and NO =10. In selecting the QMC points first the x and y coordinates of the centre are obtained from the QMC method and then for each centre location the maximum possible radius is taken. The set of QMC samples thus taken for this analysis are shown in Fig. 5.20.

Number of iteration for convergence using SSVR model is 25 function evaluations. The minimum volume is 523.8333 cubic inch and the optimum radius obtained is 12.0023 inch. The best location of the centre of the hole (values are in inches) is at (24.4570, 14.9230). The maximum value of shear stress for this optimum design is 1.0873×10^4 psi. The results obtained for this model are plotted and shown the following figures.

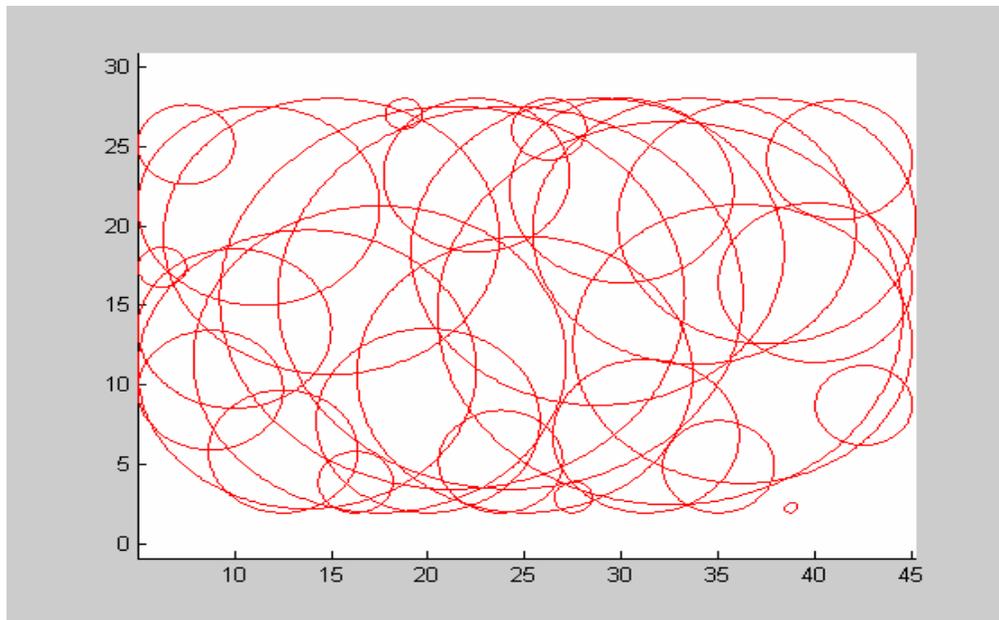


Fig. 5.20 QMC samples used in SSVR model for rectangular plate with hole

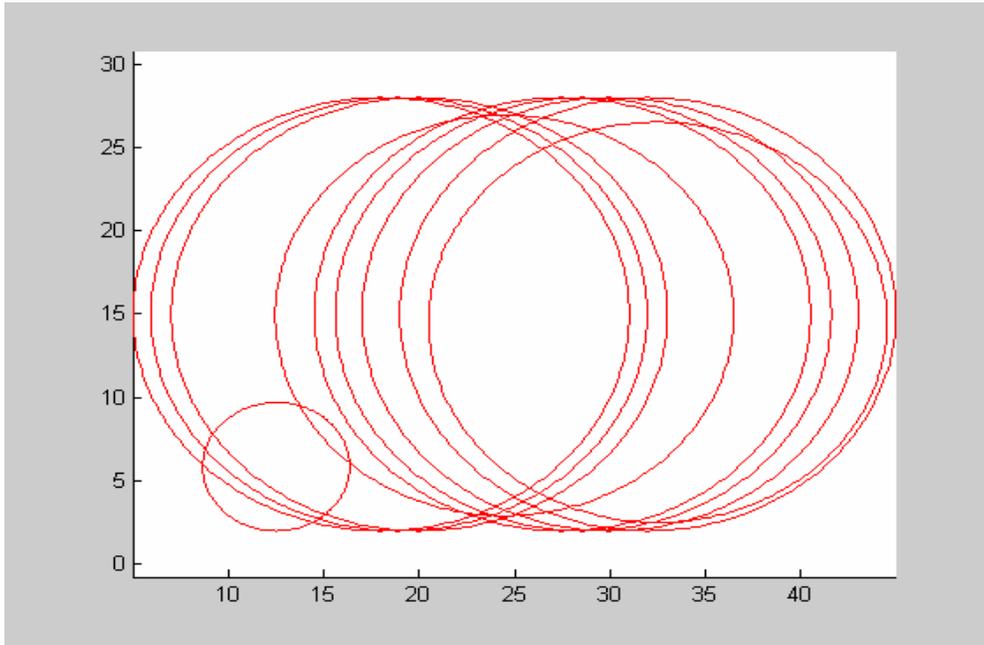


Fig. 5.21 Optimum input design values created by SSVR during the optimization process for rectangular plate with hole

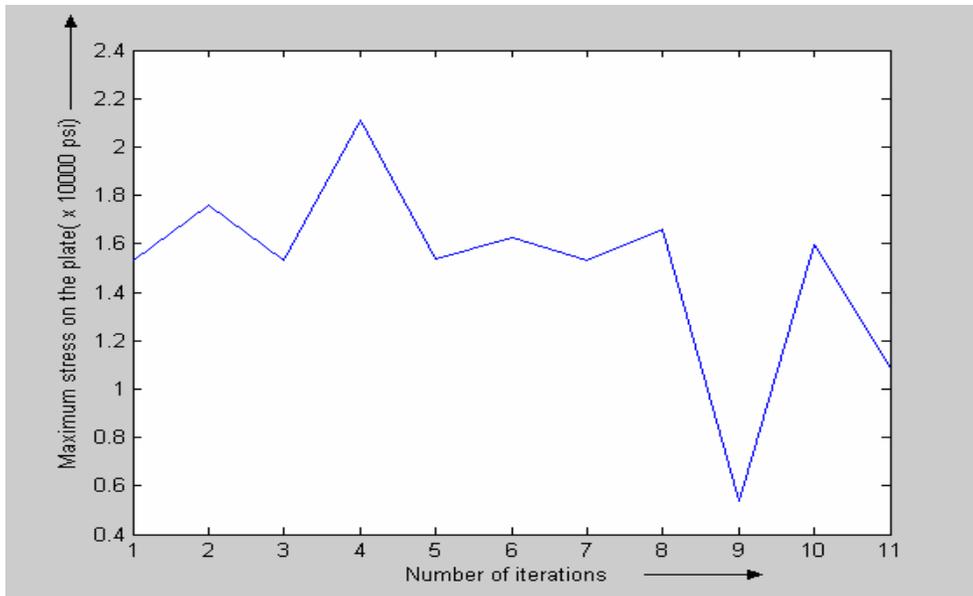


Fig. 5.22 Plot of output response created by SSVR during optimization for rectangular plate with hole

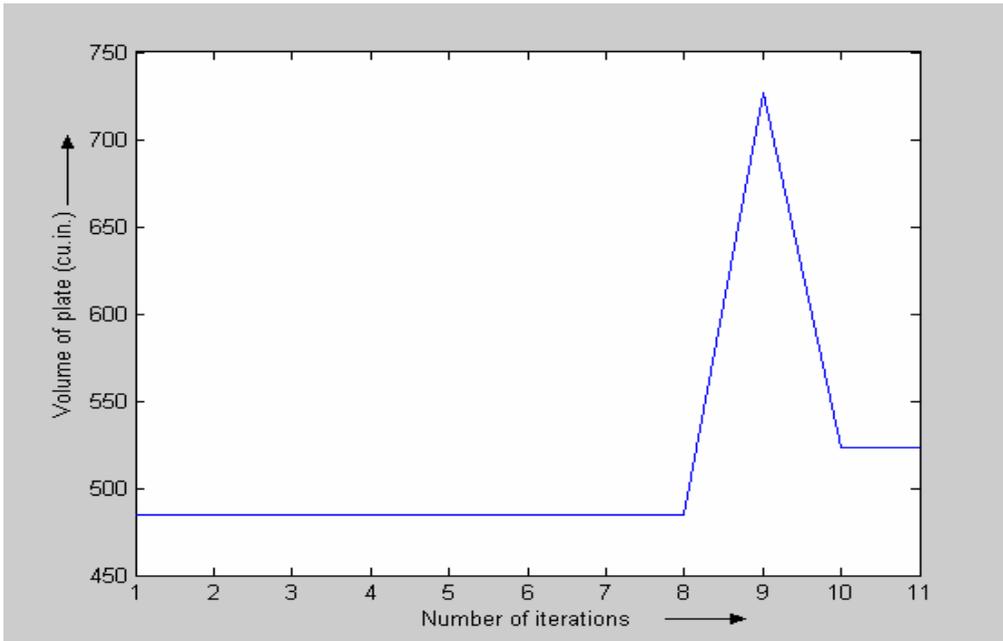


Fig. 5.23 Convergence of volume using SSVR for rectangular plate with hole

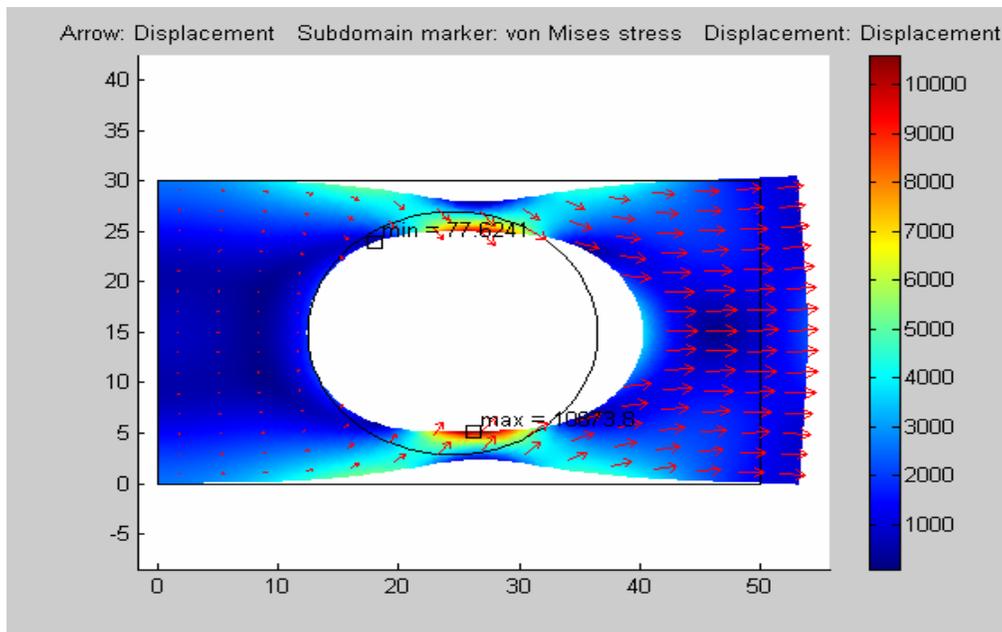


Fig. 5.24 Optimum design of rectangular plate with hole by SSVR

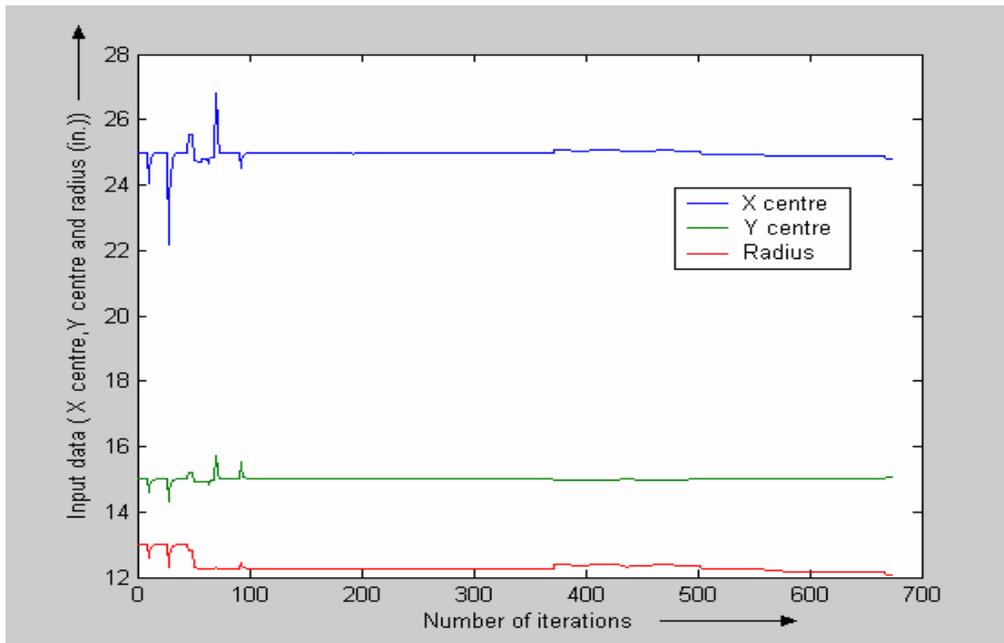


Fig. 5.25 Plot of input data used during optimization using fmincon for rectangular plate with hole

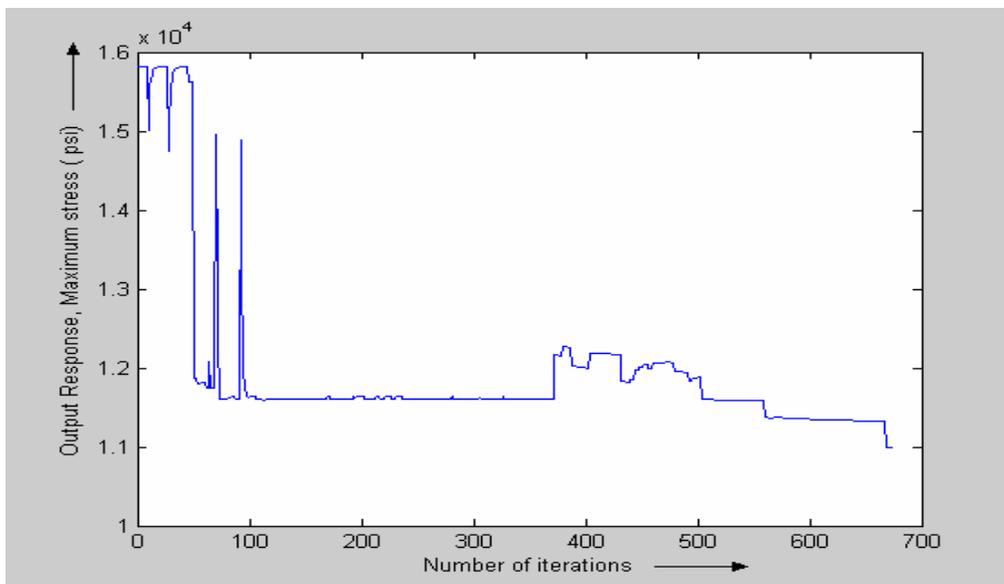


Fig. 5.26 Plot of output response during optimization using fmincon for rectangular plate with hole

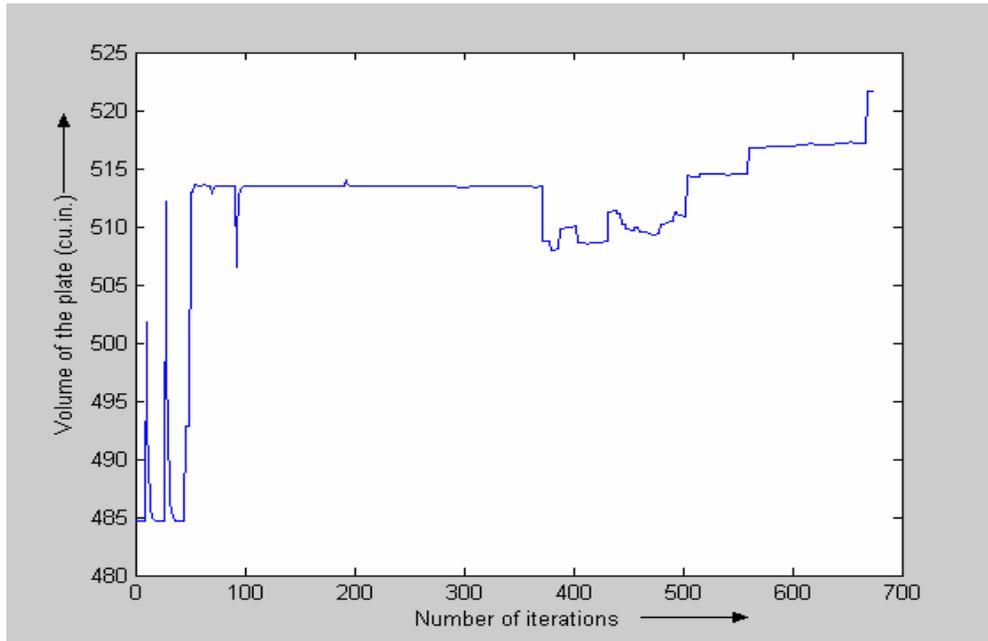


Fig. 5.27 Convergence of volume using fmincon for rectangular plate with hole

Number of iteration for convergence using fmincon is 808 function evaluations. The minimum volume is 521.6990 cubic inch and the optimum radius obtained is 12.0588 inch. The best location of the centre of the hole (values are in inches) is at position (24.7886, 15.0474). The maximum value of shear stress for this optimum design is 1.0993e+004 psi.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

In this research, smooth support vector regression (SSVR) has been used to build the metamodel for solving problem of structural optimization. Quasi-Monte Carlo sampling technique with halton sequencing has been applied to find the input data for the SSVR. The Radial basis function kernel was used to build the approximation functions for regression model. This metamodel has been successfully implemented to various application problems and the results are very accurate to that of the values obtained from the Matlab optimization tool, `fmincon`.

By solving for the optimal design using the metamodel developed in this research, there is a great amount of reduction in the number of function evaluations needed for arriving at the optimum solution. This is very important from the point of view that the users will obtain the results in much lesser time when compared to the time taken by the `fmincon` tool. Although there is a small percentage of error in the results obtained from the metamodel approach, this is tolerable because the methodology solves an approximation function, and the results very accurate. The accuracy of the results was greatly dependent on the parameters used in SSVR. As the parameters, various stopping criterion and other smoothing factors became less and less, the number of evaluations for

convergence increased. Hence a trade off between accuracy and function evaluations played a crucial role for selection of SSVR parameters and successful implementation of the metamodel using SSVR.

6.2 Future work

Based on the study and implementation of the metamodel for structural optimization, the following suggestions are given to further exploit the content of present work.

1. The Matlab tool, `fmincon` was used for solving the quadratic problem formulation of SSVR. Other quadratic problem solving methods can be investigated for faster results.
2. This approach can be extended to solving problems of higher dimensions of design variables.
3. The possibility of modifying the SSVR to formulate the non-linear regression function using kernel function to other kind of mathematical programming technique can be well thought-out.
4. The algorithm can be modified to solve problem of shape optimization and topology optimization.

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BIOGRAPHICAL INFORMATION

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