ANALYSIS OF A COMPOSITE BEAM WITH UNSYMMETRICAL C CROSSECTION

by

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Abstract

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UNSYMMETRICAL C CROSSECTION

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The research work focuses on analysis of composite beam, where a closed form analytical solution was developed to determine the sectional properties of composite beam with unsymmetrical C cross section. The sectional properties such as centroid, equivalent axial stiffness and equivalent bending stiffness are computed. A parametric study of shear center and centroid with different layup sequences was conducted using the developed solution. The ply stresses of uneven flanges of the C beam subjected to axial load and bending moment is also calculated analytically and is verified by finite element analysis. The result from the proposed theory gives excellent agreement with the ANSYS™.

Table of Contents

List of Illustrations

List of Tables

Chapter 1

Introduction

1.1 Composite Material Overview

Composite materials are materials made from two or more constituent materials with significantly different physical property, that when combined, produce a material with characteristics different from the individual components. The individual components remain separate and distinct within the finished structure. The new materials are superior to those of the constituent materials acting independently. The properties such as high specific stiffness, high specific strength, low density, corrosion resistance, easy fabrication, low thermal expansion and design flexibilities etc. are the properties which make composite materials preferable over isotropic materials.

Composite materials are widely used in aviation industry for making different aircraft components. Initially composite materials were used in manufacturing of secondary structure of aircraft but nowadays, due to the technological advancement in composite industry, composite materials are now even used for manufacturing of primary structure as well. Composite materials are also used in other areas such as automobile, sports and civil industries.

Most composite structures are designed as assemblies of beams, column, plates and shell. Beams are structural members that carry bending loads and have one dimension much larger than the other two dimensions whereas the plates and shells are two dimensional elements. In aviation industry, thin walled beams of isotropic and composite structure with closed and open cross-section are widely used as stiffeners, stringers and as primary load carrying members. The most commonly used stiffener cross sections are I, C and hat sections.

Due to the complexity of the structure and limitations of the closed form analytical solutions composite beam structures are normally validated by testing, which is very expensive and tedious process. The other alternate method for validation is using Finite Element Method

(FEM) which uses software such as ANSYS, MSC PATRAN etc. that can analyze complex composite structures with high accuracy. However, the accuracy of FEM is dependent upon the quality of modeling and boundary conditions.

1.2 Literature Review

There have been many researches going on composite beams in the past, which focuses mainly in different areas of analytical studies and finite element analysis. Some of the research work is limited to finding the structural properties such as stiffness's, centroid, shear center and equivalent stiffness's whereas there is also research work on composite beams for finding ply stresses under different loading conditions. Most of the work is based on some assumptions such as it is limited only to symmetrical laminates, specific fiber orientations, symmetrical geometries etc. and only few research works were found on torsional analysis of composite beams and is also limited symmetric geometries.

Craddock and Yen [1] in their study obtained the relationship for equivalent bending stiffness for a symmetric I- beams. However the bending stiffness relationship is calculated using the axial stiffness A_{11} only which ignores the stiffness due to coupling and Poisson's ratio effect. Drummond and Chan [2] also their research analytically and experimentally to determine the bending stiffness for I-beam which also includes the spandrels at the intersection of flange and web.

Lee [3] combined the classical lamination theory with the Vlasov and Gjelsivk theory of thin walled elastic beam to find the closed form solution for center of gravity (C.G.) and shear center. The method is applicable to mono symmetric cross-section as well as any arbitrary layup. He showed that the location of C.G. and shear center is dependent on the material properties

Parambil et al. [4] developed the closed form solution for finding the ply stresses developed in the Composite I beam under axial and bending loads applied at the centroid of the beam. He also determine the equivalent axial and bending stiffness and the centroid location for the composite I beam and validated all the results using the finite element analysis

Rios and Chan [5] started his research on simple laminate composite plate and extended it to develop sectional properties of laminated composite with a stiffener boned together, both stiffener aligned and unaligned with centerline of laminate width. The analysis focuses on centroid location, axial and bending stiffnesses, and the ply stresses of the structure. In addition to that he also extends his analysis to z-stiffener, circular cross-section beam and airfoil composite beam. The results were compared with the finite element method.

Sanghavi and Chan [6] in his research determine the shear center, equivalent torsional stiffness, equivalent warping stiffness and equivalent bending stiffness with respect to z-z axis for a mono symmetric composite I-beam. He found the shear center for a composite structure is depended not only on the cross section of the geometry like in isotropic material but also depended on the material properties, stacking sequence, fiber orientation. The study also included the coupling behavior and also shows more accuracy than the smeared property approach. He also found that if the web laminate is symmetric, change in fiber orientation of the web laminate will not affect the shear center location but the change in fiber orientation of the flange laminate will affect the shear center location. He also highlighted the inaccuracy of finding the shear center using the complete ABD matrix approach to find the shear center of the composite cross-section.

Kumpton [7] developed an analytical closed form solution to find the centroid, axial stiffness, bending stiffness and ply stresses in composite C-beam with uneven flange cross-section. The analytical solution is an extension of classical lamination theory and is valid for symmetrical and unsymmetrical layup sequences. The analytical solution is validated by finite element analysis by using ANSYS. It is found that the closed for solutions shows excellent agreement with ANSYS solution.

A procedure for calculating the shear center for the geometrically unsymmetrical isotropic C channel is given in Ugural [8]. Sanghavi and chan [6] also has developed analytical solution to find the shear center for the symmetric I beam but no work has been found on unsymmetrical composite cross section

1.3 Objective and Approach

The objective of the research is to develop an analytical method to analyze composite C-beam with uneven flanges. The closed form expressions of sectional properties such as centroid, axial and bending stiffness as well as shear center are also developed. An ANSYS finite element model is also developed to obtain the ply stresses of the C-beam under loading. The results of finite element analysis are used to compare the developed analytical method.

1.4 Outline of the Thesis

Chapter 2 deals with constitutive equations of laminated plates and beam and explain the stress/strain relationship in lamina, laminate level as well as narrow and wide beams.

Chapter 3 outlines the geometry of the composite C-beam with unsymmetrical C cross-section and describes the development of the analytical method to calculate the sectional properties and ply stresses.

Chapter 4 details the analytical method to find the shear center for the isotropic as well as composite C beam

Chapter 5 describes the various aspects in finite element methods with ANSYS such as the preprocessing and post processing.

The results of the analysis is included in Chapter 6

Chapter 7 contains the conclusion and future work of the research.

Chapter 2

Constitutive Equations of Laminated Plates and Beams

This chapter gives brief introduction to classical lamination theory and explains the ply stress/strain relationship and the general constitutive equation of composite laminated plates and beams.

2.1 Coordinate System of Composite Lamina

The composite laminate composed of many plies with difference fiber orientation that perfectly bonded together. Two types of coordinate system are used in composite analysis, local coordinate system and global coordinate system.1-2-3 coordinate represents material coordinate system and x-y-z coordinate represents global coordinate system. The 1, 2 &3 in the local coordinate system refers to fiber direction, transverse fiber direction and direction perpendicular to in-plane ply. Since the composite laminate is considered as thin plate, the plane stress condition is enforced ($\sigma_3 = \tau_{13} = \tau_{23} = 0$). And the laminated composite coordinates are reduced from 3-D to 2-D (1-2 coordinate and x-y coordinate) as shown in figure 2-1

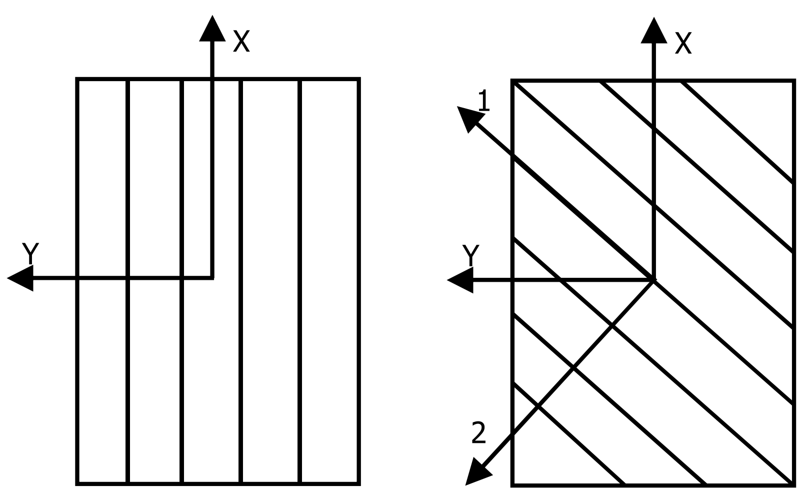

Figure 2-1 Global coordinates and local coordinate

2.2 Stress-Strain Relationship of Lamina

For a composite laminate, each layer can have different material orientations or material coordinates and hence can be treated as orthotropic material. The stress-strain relationship for 2-D composite lamina can be expressed in matrix form as follows:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \text{ or } [\varepsilon]_{1-2} = [S]_{1-2}[\sigma]_{1-2} \qquad (2.1)$$

Inverse of the reduced compliance matrix of a lamina is reduced stiffness matrix.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \text{ or } [\sigma]_{1-2} = [Q]_{1-2}[\varepsilon]_{1-2} \qquad (2.2)$$

The components in $[S]_{1-2}$ and $[Q]_{1-2}$ matrix can also be expressed in terms of elastic properties as follows:

$$S_{11} = {}^{1}/_{E_1}$$

$$S_{22} = {}^{1}/_{E_2}$$

$$S_{12} = {}^{-v_{12}}/_{E_1} = {}^{-v_{21}}/_{E_2}$$

$$S_{66} = {}^{1}/_{G_{12}} \qquad (2.3)$$

$$Q_{11} = {}^{E_1}/_{(1 - v_{12}v_{21})}$$

$$Q_{22} = {}^{E_2}/_{(1 - v_{12}v_{21})}$$

$$Q_{12} = {}^{v_{21}E_1}/_{(1 - v_{12}v_{21})} = {}^{v_{12}E_2}/_{(1 - v_{12}v_{21})}$$

$$Q_{66} = {}^{E_2}/_{(1 - v_{12}v_{21})}$$

and [Q] $_{1\text{-}2}$ = [S$_{1\text{-}2}$]$^{-1}$ (2.4)

2.3 Stress-Strain Transformation Matrices

The global coordinate is always used as the reference coordinate and is located at the mid-plane of the laminate, so the local ordinate must coincide with global coordinate to represent the stresses in terms of global coordinate. For example, in order to represent the stresses generated in the angle ply, in terms of global coordinate one has to transform the stresses in the local coordinate to global coordinate and the transformation matrix is used to do so.

$$[\sigma]_{1-2} = [T_\sigma(\theta)][\sigma]_{x-y} \tag{2.5}$$

$$[\varepsilon]_{1-2} = [T_\varepsilon(\theta)][\varepsilon]_{x-y} \tag{2.6}$$

Where $[T_\sigma(\theta)]$ and $[T_\varepsilon(\theta)]$ are transformation matrices for stress and strain, respectively

$$[T_\sigma(\theta)] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \tag{2.7}$$

$$[T_\varepsilon(\theta)] = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \tag{2.8}$$

Where $m = \mathrm{Cos}\theta$ and $n = \mathrm{Sin}\theta$

The reduced stiffness matrices [Q] are generally calculated in term of material coordinate. The 0° ply is considered as global coordinate system, while the other angles transform to coincide with global coordinate system. Then, the $[Q]_{1-2}$ matrices also transform to $[\overline{Q}]_{x-y}$ matrices by using transformation matrices.

$$[\bar{Q}]_{x-y} = [T_\sigma(-\theta)][Q]_{1-2}[T_\varepsilon(\theta)] \tag{2.9}$$

2.4 Classical Lamination Theory

The laminated composite consists of multiple laminas (layers) with various fiber orientations bonded together to form a laminate as shown in figure 2-2. To analyze the behavior of laminated composite, a coordinate system common to all of laminas is introduced. It is usually set at the mid-plane of laminate. Then, the strain of any point can be calculated in term of the mid-plane strain and curvatures in global coordinated system. We define each ply by k^{th} layer from bottom to top of laminated. The strain at each ply can be calculated by using the following relationship:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_{k^{th}} = \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + z_{k^{th}} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{2.10}$$

Where, ε_x^o, ε_y^o and γ_{xy}^o are the mid-plane strain, κ_x, κ_y and κ_{xy} are the mid plane curvatures, z is the distance from mid-plane to any point of layer.

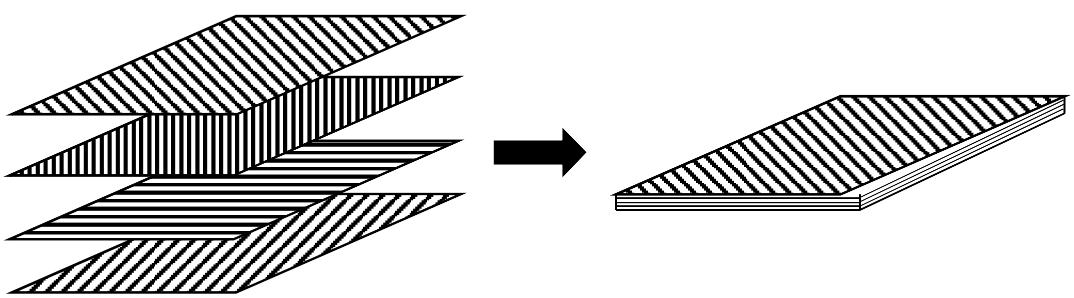

Figure 2-2 Plies with different fiber orientation bonded together perfectly

Substituting Equation (2.10) into Equation (2.2), the stress of k^{th} ply can be expressed in term of mid-plane and curvature as follows:

$$\begin{bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{bmatrix}_{k^{th}} = [\bar{Q}]_{k^{th}}\begin{bmatrix}\varepsilon_x\\\varepsilon_y\\\gamma_{xy}\end{bmatrix}_{k^{th}} = [\bar{Q}]_{k^{th}}\left\{\begin{bmatrix}\varepsilon_x^o\\\varepsilon_y^o\\\gamma_{xy}^o\end{bmatrix} + z_{k^{th}}\begin{bmatrix}\kappa_x\\\kappa_y\\\kappa_{xy}\end{bmatrix}\right\} \tag{2.11}$$

2.5 Constitutive Equation of Laminated Plate

The in-plane forces [N] and moments [M] per unit width of laminate can be calculated by integrating forces in each ply through the thickness of laminate

$$\begin{bmatrix}N_x\\N_y\\N_{xy}\end{bmatrix} = \sum_{k=1}^{n}\int_{-h_{k-1}}^{h_k}\begin{bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{bmatrix}dz \tag{2.12}$$

$$\begin{bmatrix}M_x\\M_y\\M_{xy}\end{bmatrix} = \sum_{k=1}^{n}\int_{-h}^{h}\begin{bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{bmatrix}zdz \tag{2.13}$$

Where h is a distance from mid-plane to any k^{th} ply

The positive in-plane forces and moments are shown in figure 2.3 and M_x is the moment pointing to the positive Y- direction and M_y is the moment pointing to negative X- direction.

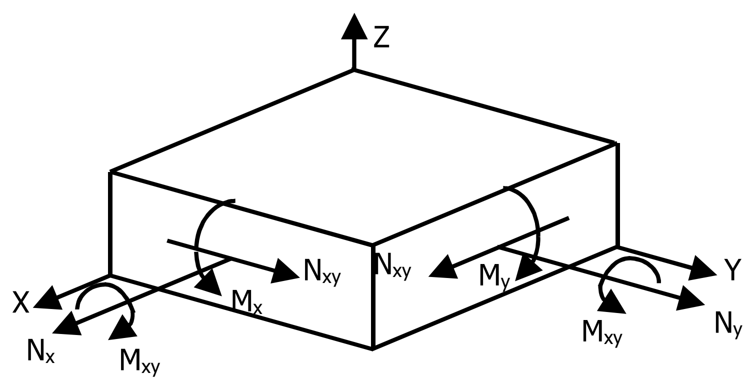

Figure 2-3 Loading components for in plane laminate

Substituting Equation (2.9) in Equation (2.12 & 2.13), the constitutive equation of laminate can be expresses as follow:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \text{ or } \begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^\circ \\ K \end{bmatrix} \quad (2.14)$$

Where [A] is extensional stiffness matrix, [B] is extensional-bending coupling stiffness matrix and [D] is the bending stiffness matrix. The stiffness matrices [A], [B], and [D] can also be expressed as follows:

$$[A] = \sum_{k=1}^{n} [\overline{Q}]_k \cdot (h_k - h_{k-1}) \quad (2.15)$$

$$[B] = \frac{1}{2} \sum_{k=1}^{n} [\overline{Q}]_k \cdot (h_k^2 - h_{k-1}^2) \quad (2.16)$$

$$[D] = \frac{1}{3} \sum_{k=1}^{n} [\overline{Q}]_k \cdot (h_k^3 - h_{k-1}^3) \quad (2.17)$$

Where the subscript k indicates the layer number, (Figure 2-4) and h_k and h_{k-1} are the upper and lower surface locations of the k_{th} layer.

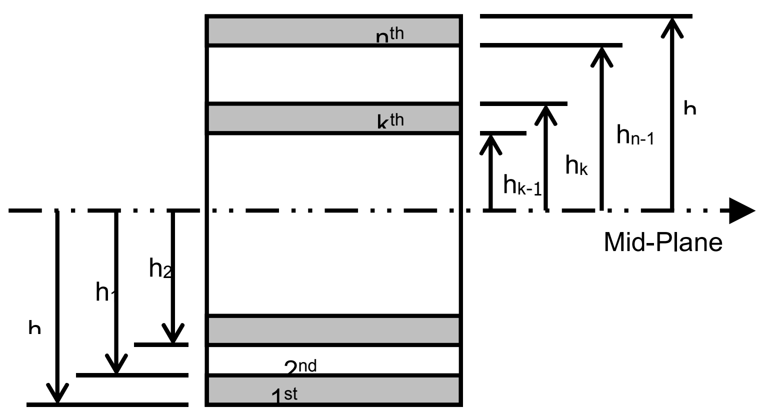

Figure 2-4 Geometry of an n-layer laminate

2.6 Narrow Beam VS Wide Beam

Figure 2-5 illustrates the deformed shape of the narrow beam and wide beam under loading. The narrow beam have induced lateral curvature ($K_y \neq 0$) due to Poison's effect, the lateral moment is ignored. While the wide beam have a large width-to-thickness ratio and therefore the curvature is produced only in the edge of beam. It has no curvature exist except the near edges, so the induced lateral curvature is insignificant and the lateral moment need to be consider ($K_y = 0$, $M_y \neq 0$).

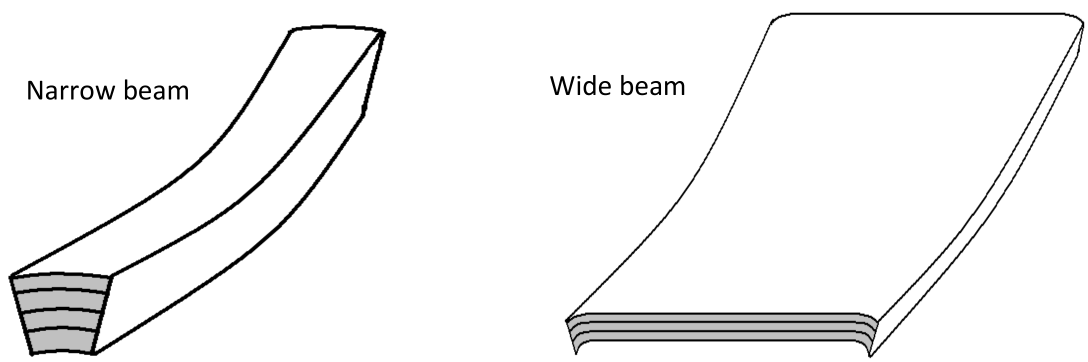

Figure 2-5 Narrow Beam VS Wide Beam

For C-channel case, the beam is considered as narrow beam. The equation (2.14) can be modified as

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \begin{bmatrix} A* & B* \\ B* & D* \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \kappa_x \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_x^0 \\ \kappa_x \end{bmatrix} = \begin{bmatrix} a^* & b^* \\ b^* & d^* \end{bmatrix} \begin{bmatrix} N_x \\ M_x \end{bmatrix} \quad (2.18)$$

Where,

$[a^{*}] = a_{11} - b_{16}^{2}/d_{66}$ $\quad$ $[b^{*}] = b_{11} - b_{16}d_{16}/d_{66}$ $\quad$ $[d^{*}] = d_{11} - d_{16}^{2}/d_{66}$

The constitutive equation for narrow beam can be rewrite as follow:

$$N_x = A^{*}\varepsilon_x^{0} + B^{*}\kappa_x \tag{2.19}$$

$$M_x = B^{*}\varepsilon_x^{0} + D^{*}\kappa_x \tag{2.20}$$

These equations were derived in Ref. [4].

Chapter 3

Basic Equation for Composite Laminated Beam

3.1 Geometry of Composite Laminated C -Beam

The beam is divided into three sections that contain three sub-laminates top flange, bottom flange, and web as shown in figure 3-1.

Where b_{f1}, b_{f2}, and h_w are width of top flange, bottom flange and height of web, respectively and Z_1, Z_2, and Z_3 are the distance from Y-axis to centroid of top flange, bottom flange and web, respectively.

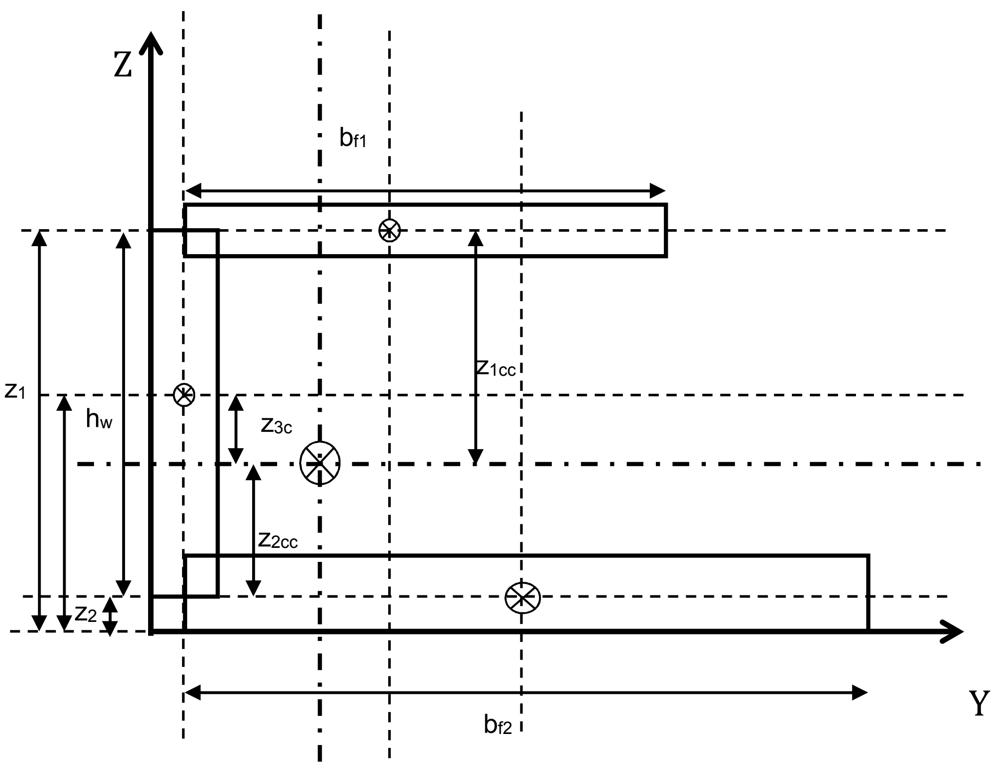

Figure 3-1 Geometry of composite beam with C-Channel cross-section

3.2 Centroid of Composite Laminated C- Beam

The centroid is an important sectional property that is used to determine the structural response. The centroid of a structural cross-section is defined as the average location of forces acting on each part of the cross section. At the centroid, an axial load N_x^c does not cause a change in curvatures (κ_x^c & κ_z^c), and the bending moments ($\bar{M}_x$ & $\bar{M}_z$) acting at this location do not produce any axial strain (ε_x^c). To calculate centroid, set Y-axis at the most bottom of the bottom flange and Z-axis at the most left of web as shown in figure 3-2. Apply axial force on the centroid of each laminate. The total force will be acting on the centroid.

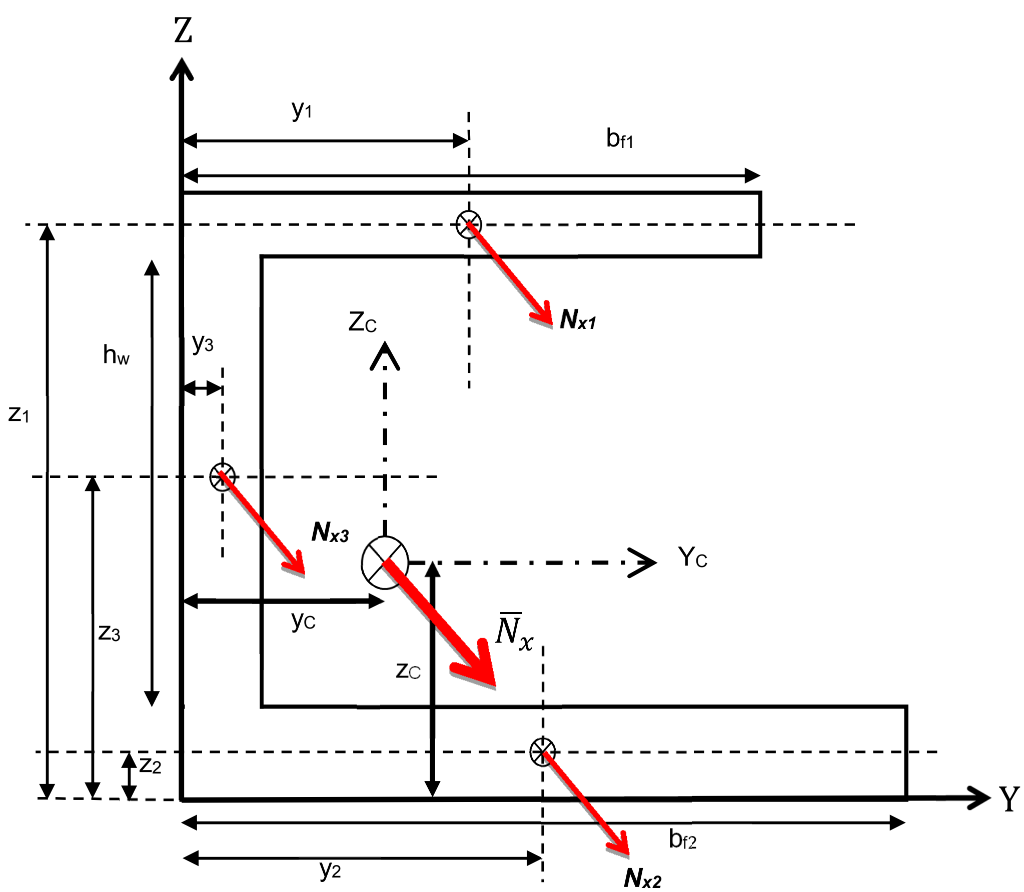

Figure 3-2 Axial forces on centroid of each sub-laminate

Total moment and total axial force for Y-axis are:

$$\overline{N}_x z_c = N_{x1} b_{f1} z_1 + N_{x2} b_{f2} z_2 + N_{x3} h_w z_3 \tag{3.1}$$

$$\overline{N}_x = N_{x1} b_{f1} + N_{x2} b_{f2} + N_{x3} h_w \tag{3.2}$$

Then, Z_c can be obtained as

$$z_c = \frac{N_{x1} b_{f1} z_1 + N_{x2} b_{f2} z_2 + N_{x3} b_w z_3}{N_{x1} b_{f1} + N_{x2} b_{f2} + N_{x3} b_w} \tag{3.3}$$

Applying constitutive equation for narrow laminate beam, so equation (3.3) can be modified as:

$$z_c = \frac{A_{f1}^* b_{f1} z_1 + A_{f2}^* b_{f2} z_2 + A_w^* b_w z_3}{A_{f1}^* b_{f1} + A_{f2}^* b_{f2} + A_w^* b_w} \tag{3.4a}$$

Similarly, Y_c can be obtained by the same procedure as follow:

$$y_c = \frac{A_{f1}^* b_{f1} y_1 + A_{f2}^* b_{f2} y_2 + A_w^* b_w y_3}{A_{f1}^* b_{f1} + A_{f2}^* b_{f2} + A_w^* b_w} \tag{3.4b}$$

If the entire beam has identical layup, then equation (3.4a) and (3.4b) can be reduced to

$$z_c = \frac{b_{f1} z_1 + b_{f2} z_2 + b_w z_3}{b_{f1} + b_{f2} + b_w} \tag{3.5a}$$

$$y_c = \frac{b_{f1} y_1 + b_{f2} y_2 + b_w y_3}{b_{f1} + b_{f2} + b_w} \tag{3.5b}$$

It should be noted that above expression become the geometric dependence.

3.3 Equivalent Axial Stiffness and Bending Stiffness's

The stiffness is the property of the material which resists the deformation in response to an applied force. There are three types of stiffness's: axial, bending and torsion stiffness. The axial, bending and torsional stiffness's are used for predicting the response of structure under different loading conditions. To evaluate the equivalent stiffness's, axial force & bi-axial bending moment are applied at the centroid of cross-section, the load components were shown on the figure 3-3 and figure 3-4.

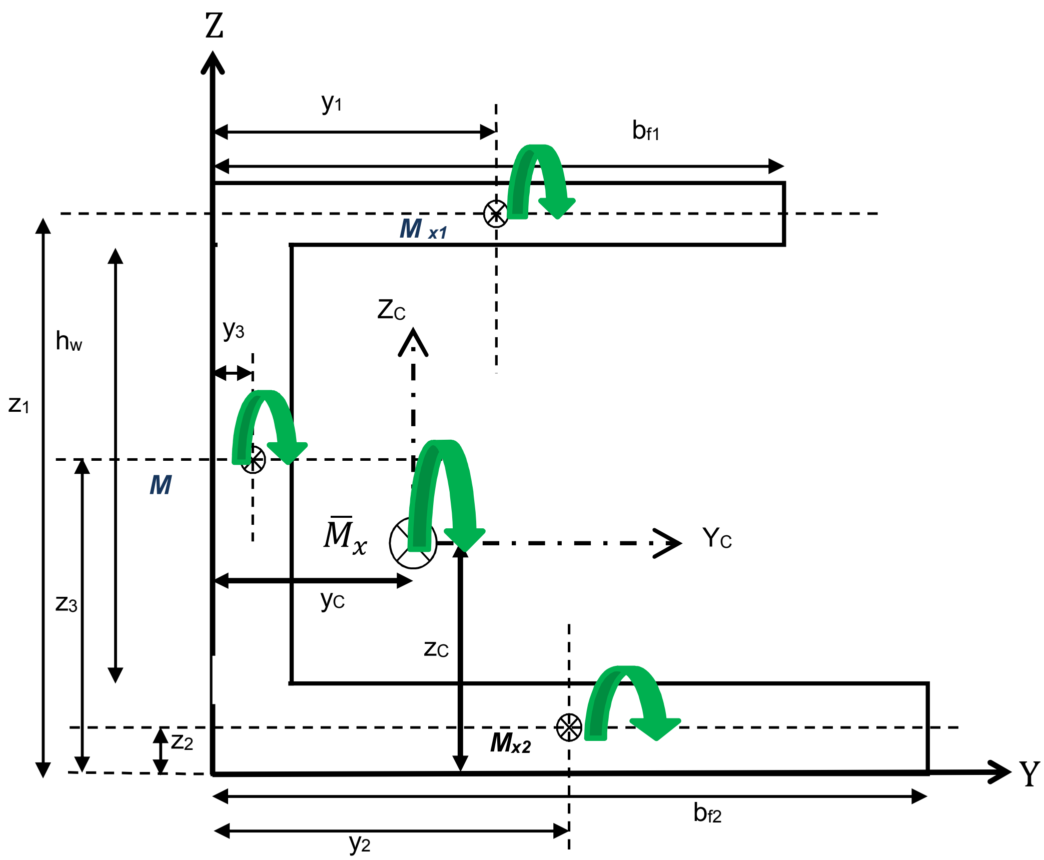

Figure 3-3 Bending loads on centroid of each sub-laminate

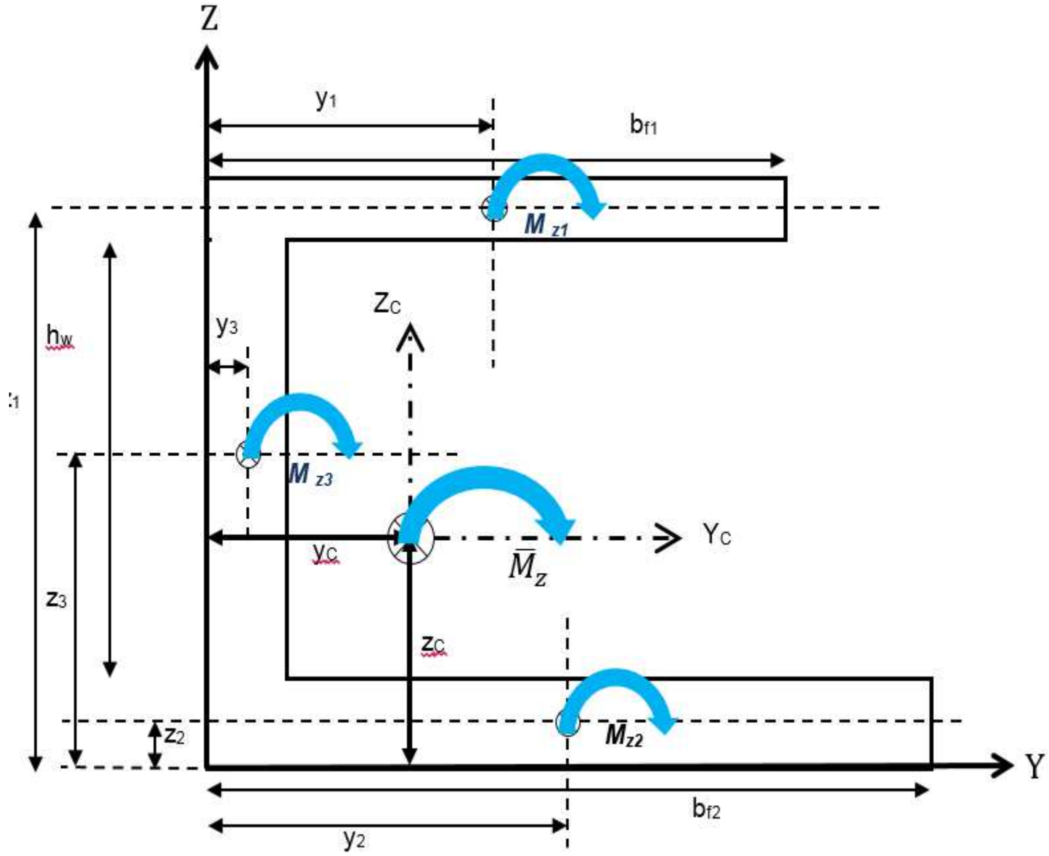

Figure 3-4 Bending moment with respect to z-axis on centroid of each sub-laminate

The governing equation is given by

$$\begin{bmatrix} \overline{N}_x \\ \overline{M}_x \\ \overline{M}_z \end{bmatrix} = \begin{bmatrix} \overline{EA} & 0 & 0 \\ 0 & \overline{D}_x & \overline{D}_{xy} \\ 0 & \overline{D}_{xy} & \overline{D}_y \end{bmatrix} \begin{bmatrix} \varepsilon_x^c \\ \kappa_x^c \\ \kappa_z^c \end{bmatrix} \tag{3.6}$$

Where positive $\overline{M}_x$ is pointing to the positive Y-direction and the positive $\overline{M}_z$ is pointing to the negative Z- direction. They are shown in figure 3-5.

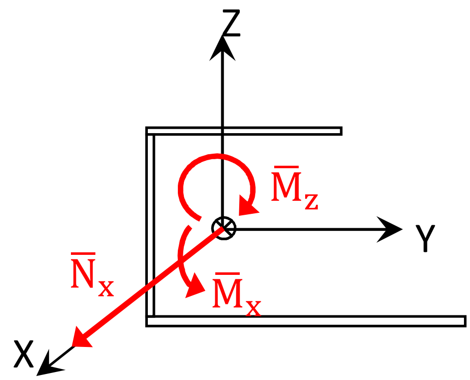

Figure 3-5 Loads components

The mid-plane strain of a beam subjected to bi-axial bending can be written in term of centroid strain and bi-axial curvature as following:

$$\varepsilon_x^o = \varepsilon_x^c + z\kappa_x^c + y\kappa_z^c \tag{3.7}$$

3.3.1 Axial Stiffness, $\overline{EA}$

The axial stiffness can be defined as the resistance of the structure to deform axially against applied load.

Let assume the total axial force acting on the structure, that is all the three sub-laminates' axial force were applied at the centroid of the C-channel cross-section.

From the 1st equation of equation (3.6), the net force acting on the structure can be written as following

$$\overline{N}_x = \overline{EA}\varepsilon_x^c \tag{3.8}$$

The total axial force is equal to summation of axial forces in the sub-laminates'. We have

$$\overline{N}_x = b_{f1}\overline{N}_{x,f1} + b_{f2}\overline{N}_{x,f2} + h_w\overline{N}_{x,w} \tag{3.9}$$

From the constitutive equation of narrow beam;

$$\overline{N}_x = b_{f1}(A^*_{f1}\varepsilon^o_{x,f1} + B^*_{f1}\kappa_{x,f1}) + b_{f2}(A^*_{f2}\varepsilon^o_{x,f2} + B^*_{f2}\kappa_{x,f2}) + h_w(A^*_w\varepsilon^o_{x,w}) \tag{3.10}$$

The strains produced in all laminates are equal the total strain along the x-axis and since the axial load is applied at the centroid of the entire cross-section, the induced curvature is zero. The constitutive equation can be rewritten as:

$$\overline{N}_x = \{A^*_{f1}b_{f1} + A^*_{f2}b_{f2} + A^*_w h_w\}\varepsilon^c_x \tag{3.11}$$

Considering equation (3.11) and (3.8), the axial stiffness can be written as:

$$\overline{EA} = A^*_{f1}b_{f1} + A^*_{f2}b_{f2} + A^*_w h_w \tag{3.12}$$

3.3.2 Bending Stiffness, $\overline{D}_x$, $\overline{D}_y$, *and* $\overline{D}_{xy}$

To evaluate $\overline{D}_x$, only moment in y-direction ($\overline{M}_x$) that produce κ^c_x is included. Then

$$\overline{M}_x = b_{f1}(N_{x1}z_{1cc} + M_{x1}) + b_{f2}(N_{x2}z_{2cc} + M_{x2}) + \left\{\int_{-\left(\frac{h_w}{2}-z_{3cc}\right)}^{\left(\frac{h_w}{2}+z_{3cc}\right)} zN_{x3}dz\right\} \tag{3.13}$$

Applying constitutive equation for narrow beam it can be found that for sub laminate 1 (top flange);

$$b_{f1}(N_{x1}z_{1cc} + M_{x1}) = b_{f1}\{(A^*_{f1}\varepsilon^o_{x,f1} + B^*_{f1}\kappa_{x,f1})z_{1cc} + (B^*_{f1}\varepsilon^o_{x,f1} + D^*_{f1}\kappa_{x,f1})\} \tag{3.14}$$

The mid-plane strain, $\varepsilon^o_{x,f1}$ related to κ^c_x is

$$\varepsilon^o_{x,f1} = z_{1cc}\kappa^c_x \tag{3.16}$$

At any point, the curvatures about x-direction is the same as the curvature at the centroid, $\kappa_{x,f1} = \kappa^c_x$. The equation (3.14) can be modified as follows;

$$b_{f1}(N_{x1}z_{1cc} + M_{x1}) = b_{f1}(A^*_{f1}z^2_{1cc} + 2B^*_{f1}z_{1cc} + D^*_{f1})\kappa^c_x \quad (3.16)$$

Where z_{1mc} is the distance from mid-plane of top flange to centroid of the cross-section.

In the same way, for sub-laminate 2 (bottom flange),

$$b_{f2}(N_{x2}z_{2cc} + M_{x2}) = b_{f2}(A^*_{f2}z^2_{1cc} + 2B^*_{f1}z_{1cc} + D^*_{f1})\kappa^c_x \quad (3.17)$$

And for sub-laminate 3 (web), by integrating along the width of the web,

$$\int_{-\left(\frac{h_w}{2}-z_{3cc}\right)}^{\left(\frac{h_w}{2}+z_{3cc}\right)} zN_{x3}dz \;= A^*_w\left\{\frac{h^3_w}{12} + h_w z^2_{3cc}\right\}\kappa^c_x \quad (3.18)$$

Substitute equations (3.16) to (3.18) into equation (3.14), we get

$$\overline{M}_x = \left[b_{f1}(A^*_{f1}z^2_{1cc} + 2B^*_{f1}z_{1cc} + D^*_{f1}) + b_{f2}(A^*_{f2}z^2_{2cc} + 2B^*_{f2}z_{2cc} + D^*_{f2}) + A^*_w\left(\frac{h^3_w}{12} + h_w z^2_{3cc}\right)\right]\kappa^c_x \quad (3.19)$$

Comparing equation (3.19) with equation (3.16), the bending stiffness can be written as,

$$\overline{D}_x = b_{f1}(A^*_{f1}z^2_{1cc} + 2B^*_{f1}z_{1cc} + D^*_{f1}) + b_{f2}(A^*_{f2}z^2_{2cc} + 2B^*_{f2}z_{2cc} + D^*_{f2}) + A^*_w\left(\frac{h^3_w}{12} + h_w z^2_{3cc}\right) \quad (3.20)$$

In the evaluation of bending stiffness, $\overline{D}_y$ we can find $\overline{M}_z$ that produces curvature κ^c_z

$$\overline{M}_z = \overline{D}_y\kappa^c_z \quad (3.21)$$

$$\overline{M}_z = \left\{\int_{-(\frac{b_{f1}}{2}-y_{1cc})}^{(\frac{b_{f1}}{2}+y_{1cc})} yN_{x1}dy\right\} + \left\{\int_{-(\frac{b_{f2}}{2}-y_{2cc})}^{(\frac{b_{f2}}{2}+y_{2cc})} yN_{x2}dy\right\} + \{N_{x3}h_w y_{3cc} + M_{x1}h_w\} \quad (3.22)$$

The same procedure as $\overline{D}_x$ is used to evaluate bending stiffness, $\overline{D}_y$ .Then the result can be obtained as follow;

$$\overline{D}_y = A^*_{f1}\left\{\frac{b^3_{f1}}{12} + b_{f1}y^2_{1cc}\right\} + A^*_{f2}\left\{\frac{b^3_{f2}}{12} + b_{f2}y^2_{2cc}\right\} + \{A^*_w y^2_{3cc} + 2B^*_w y_{3cc} + D^*_w\} \quad (3.23)$$

To calculate $\overline{D}_{xy}$, we extract $\overline{M}_x$ that produce κ^c_z

$$\overline{M}_x = \overline{D}_{xy}\kappa_z^c \tag{3.24}$$

From equation (3.14), The mid-plane strain for top flange, bottom flange and web can be obtained by lamination theory as, $\varepsilon_{x,f1}^o = \varepsilon_x^c + y_{1mc}\kappa_z^c$, but $\varepsilon_z^c = 0$ because there is no strain at centroid. Curvatures at any point are the same as curvature at the centroid, $\kappa_{x,f1} = \kappa_z^c$.

$$b_{f1}(N_{x1}z_{1cc} + M_{x1}) = b_{f1}(A_{f1}^* z_{1cc} + B_{f1}^*)y_{1cc}\kappa_z^c \tag{3.25}$$

$$b_{f2}(N_{x2}z_{2cc} + M_{x2}) = b_{f2}(A_{f2}^* z_{2cc} + B_{f2}^*)y_{2cc}\kappa_z^c \tag{3.26}$$

$$\int_{-\left(\frac{h_w}{2}-z_{3cc}\right)}^{\left(\frac{h_w}{2}+z_{3cc}\right)} zN_{x3}dz \;= (A_w^* y_{3cc} + B_w^*)h_w z_{3cc}\kappa_z^c \tag{3.27}$$

Substitute equations (3.25) to (3.27) into equation (3.14) and comparing it with equation (3.21), the bending stiffness can be written as:

$$\overline{D}_{xy} = (A_{f1}^* z_{1cc} + B_{f1}^*)b_{f1}y_{1cc} + (A_{f2}^* z_{2cc} + B_{f2}^*)b_{f2}y_{2cc} + (A_w^* y_{3cc} + B_w^*)h_w z_{3cc} \tag{3.28}$$

3.4 Ply Stress Analysis

The strains and curvatures at the centroid of each laminates are calculated. The load acting at the centroid decouples the structural response between axial extension and bending. ε_x^c, κ_x^c and κ_z^c can be obtaining by modifying three equations in equation (3.6) follows:

$$\varepsilon_x^c = \frac{\overline{N}_x}{\overline{EA}} \tag{3.29}$$

$$\kappa_x^c = \frac{\overline{M}_y\overline{D}_y - \overline{M}_z\overline{D}_{xy}}{\overline{D}_x\overline{D}_y - \overline{D}_{xy}^2} \tag{3.30}$$

$$\kappa_z^c = \frac{\overline{M}_z\overline{D}_x - \overline{M}_y\overline{D}_{xy}}{\overline{D}_x\overline{D}_y - \overline{D}_{xy}^2} \tag{3.31}$$

3.4.1 Top Flange (sub-laminate 1)

From constitutive equation of narrow beam, the axial force and bending moment acting on the top flange can be written as:

$$\mathrm{N_{x,f1}} = A^*_{1,f1}\varepsilon^0_{\mathrm{x,f1}} + B^*_{1,f1}\kappa_{\mathrm{x,f1}} \tag{3.32}$$

$$\mathrm{M_{x,f1}} = B^*_{1,f1}\varepsilon^0_{\mathrm{x,f1}} + D^*_{1,f1}\kappa_{\mathrm{x,f1}} \tag{3.33}$$

Where $\varepsilon^0_{\mathrm{x,f1}} = \varepsilon^c_x + z_{1mc}\kappa^c_x + y_1\kappa^c_z$ and $\kappa_{\mathrm{x,f1}} = \kappa^c_x$

z_{1mc} : distance from centroid to mid-plane of top flange

y_1: distance from centroid to any point of top flange

Equation (3.29) and equation (3.31) become:

$$\mathrm{N_{x,f1}} = A^*_{1,f1}(\varepsilon^c_x + z_{1mc}\kappa^c_x + y_1\kappa^c_z) + B^*_{1,f1}\kappa^c_x \tag{3.34}$$

$$\mathrm{M_{x,f1}} = B^*_{1,f1}(\varepsilon^c_x + z_{1mc}\kappa^c_x + y_1\kappa^c_z) + D^*_{1,f1}\kappa^c_x \tag{3.35}$$

The mid-plane strains and curvatures of top flange can be written as:

$$\begin{bmatrix} \varepsilon^0_x \\ \varepsilon^0_y \\ \gamma^0_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}_{f1} = \begin{bmatrix} a_{11} & b_{11} & b_{16} \\ a_{12} & b_{12} & b_{26} \\ a_{16} & b_{16} & b_{66} \\ b_{11} & d_{11} & d_{16} \\ b_{12} & d_{12} & d_{26} \\ b_{16} & d_{16} & d_{66} \end{bmatrix}_{f1} \begin{bmatrix} \mathrm{N_{x,f1}} \\ \mathrm{M_{x,f1}} \\ \mathrm{M_{xy,f1}} \end{bmatrix} \tag{3.36}$$

From the 6th equation: $\kappa_{xy} = b_{16}\mathrm{N_{x,f1}} + d_{61}\mathrm{M_{x,f1}} + d_{66}\mathrm{M_{xy,f1}} = 0$

$$\mathrm{M_{xy,f1}} = -\frac{(b_{16,f1}\mathrm{N_{x,f1}} + d_{61,f1}\mathrm{M_{x,f1}})}{d_{66,f1}} \tag{3.37}$$

From equation (3.36) and equation (3.37) we can calculate ε^0_x, ε^0_y, γ^0_{xy}, κ_x and κ_y

$$\varepsilon_x^0 = a_{11,f1}N_{x,f1} + b_{11,f1}M_{x,f1} - \frac{b_{16,f1}(b_{16,f1}N_{x,f1}+d_{61,f1}M_{x,f1})}{d_{66,f1}} \tag{3.38}$$

$$\varepsilon_y^0 = a_{12,f1}N_{x,f1} + b_{12,f1}M_{x,f1} - \frac{b_{26,f1}(b_{16,f1}N_{x,f1}+d_{61,f1}M_{x,f1})}{d_{66,f1}} \tag{3.39}$$

$$\gamma_{xy}^0 = a_{16,f1}N_{x,f1} + b_{16,f1}M_{x,f1} - \frac{b_{66,f1}(b_{16,f1}N_{x,f1}+d_{61,f1}M_{x,f1})}{d_{66,f1}} \tag{3.40}$$

$$\kappa_x = b_{11,f1}N_{x,f1} + d_{11,f1}M_{x,f1} - \frac{d_{16,f1}(b_{16,f1}N_{x,f1}+d_{61,f1}M_{x,f1})}{d_{66,f1}} \tag{3.41}$$

$$\kappa_y = b_{12,f1}N_{x,f1} + d_{12,f1}M_{x,f1} - \frac{d_{26,f1}(b_{16,f1}N_{x,f1}+d_{61,f1}M_{x,f1})}{d_{66,f1}} \tag{3.42}$$

The strain of the ply in flange 1 laminate can be obtained as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_{k^{th},f1} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{f1} + z_{k^{th},f1} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}_{f1} \tag{3.43}$$

We can determine stresses on each ply by using stains in equation (3.43)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{k^{th},f1} = [\bar{Q}]_{k^{th},f1} \left\{ \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{f1} + z_{k^{th},f1} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}_{f1} \right\} \tag{3.44}$$

3.4.2 Bottom Flange (sub-laminate 2)

The stresses developed in the bottom flange can be calculated using the same procedure used for calculating the stresses in the top flange

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{k^{th},f2} = [\bar{Q}]_{k^{th},f2} \left\{ \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{f2} + z_{k^{th},f2} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}_{f2} \right\} \tag{3.45}$$

3.4.3 Web (sub-laminate 3)

The constitutive equation for narrow beam of the sub-laminate loads can be express as in term of curvature about z-axis, ($\kappa_{z,w}$).

$$N_{x,w} = A^{*}\varepsilon_{x,w}^{0} + B^{*}\kappa_{z,w} \tag{3.46}$$

$$M_{x,w} = B^{*}\varepsilon_{x,w}^{0} + D^{*}\kappa_{z,w} \tag{3.47}$$

Where, $\varepsilon_{x,f1}^{0} = \varepsilon_{x}^{c} + z_3\kappa_{x}^{c} + y_{3mc}\kappa_{z}^{c}$ and $\kappa_{x,w} = \kappa_{z}^{c}$

$$N_{x,f1} = A^{*}(\varepsilon_{x}^{c} + z_3\kappa_{x}^{c} + y_{3mc}\kappa_{z}^{c}) + B^{*}\kappa_{z}^{c} \tag{3.48}$$

$$M_{x,f1} = B^{*}(\varepsilon_{x}^{c} + z_3\kappa_{x}^{c} + y_{3mc}\kappa_{z}^{c}) + D^{*}\kappa_{z}^{c} \tag{3.49}$$

Where,

z_3: distance from centroid to any point of web

y_{3mc}: distance from centroid to mid-plane of web

The procedure for calculating the mid-plane strains and curvature in the k^{th} ply of the web laminate is the same as top flange. But y-axis distance is considered instead of z-axis. The stress of k^{th} ply can be expressed as;

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{k^{th},w} = [\bar{Q}]_{k^{th},w} \left\{ \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_w + y_{k^{th},w} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}_w \right\} \tag{3.50}$$

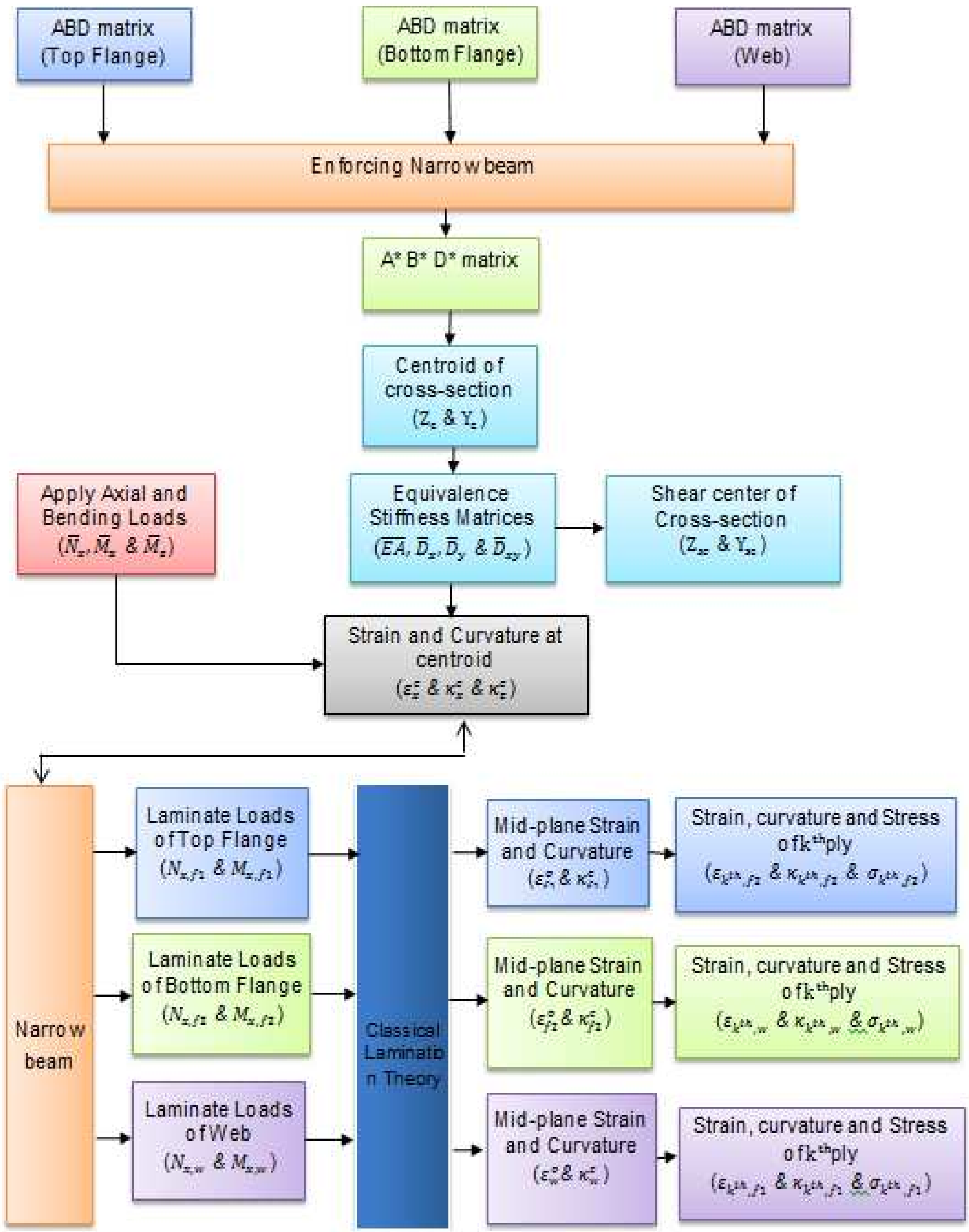

Figure 3-6 Analytical procedure can be shortly expresses in diagram below.

Chapter 4

Shear Center for Geometrically Unsymmetrical C Beam

4.1 Review on Shear Center of Isotropic Unsymmetrical C-Beam

Shear Center is defined as the point in the cross section where the bending and torsion are decoupled. That is if the lateral or transverse load pass through this point it produces only bending without twisting.

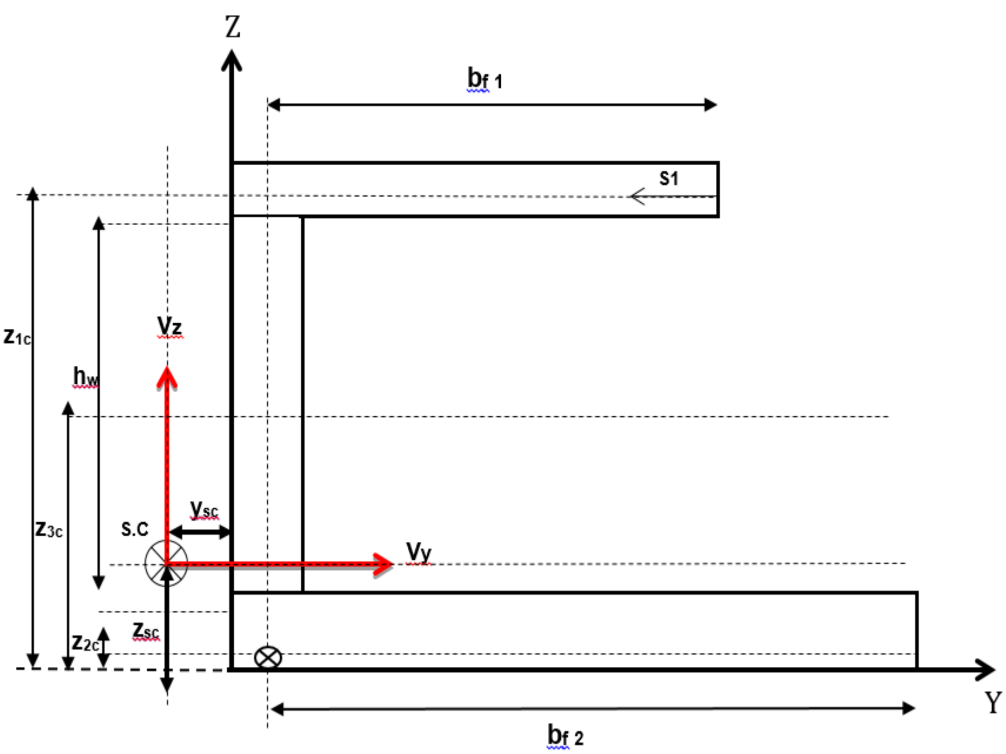

Figure 4-1 Shear center for isotropic beam with unsymmetrical C cross section

It may also be shown by the use of reciprocal theorem that, this point is also the center of twist of section subjected to torsion. In most of the cases it is difficult to guarantee that a shear load will act through the shear center. But the shear load may be represented by the combination of shear load through the shear center and torque. The stresses can then be super positioned. Therefore, it is essential to calculate and locate the shear center in the cross section. When a cross-section has an axis of symmetry the shear center must lie on that axis.

Thus if we assume that the cross section supports the shear loads V_x and V_y such that there is no twisting of the cross section and also as there are no hoop stresses in the beam the shear flow and direct stresses acting on an element of the beam wall are related by the below mentioned equilibrium equation:

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_x}{\partial x} = 0 \tag{4.1}$$

Where,

$$\sigma_x = \left(\frac{M_z I_y - M_x I_{yz}}{I_y I_z - I_{yz}^2}\right) y + \left(\frac{M_x I_z - M_z I_{yz}}{I_y I_z - I_{yz}^2}\right) z \tag{4.2}$$

q = shear flow = shear force per unit length = $\tau * t$

τ = shear force

t = thickness

σ_x = axial stress

M_x = Moment about y axis

M_z = Moment about z axis

I_z = Moment of Inertia about z –z Axis

I_y = Moment of Inertia about y –y Axis

I_{yz} = Product Moment of Inertia

V_y = Shear Force in y – direction

V_z = Shear Force in z - direction

Therefore we get,

$$\frac{\partial \sigma_x}{\partial x} = \left(\frac{\frac{\partial M_Z}{\partial x} I_y - \frac{\partial M_x}{\partial x} I_{yz}}{I_y I_z - I_{yz}^2} \right) y + \left(\frac{\frac{\partial M_x}{\partial x} I_z - \frac{\partial M_z}{\partial x} I_{yz}}{I_y I_z - I_{yz}^2} \right) z \tag{4.3}$$

We also have

$$V_z = \frac{\partial M_x}{\partial x} \tag{4.4}$$

$$V_y = \frac{\partial M_z}{\partial x} \tag{4.5}$$

From equation (4.3), (4.4) and (4.5) we get,

$$\frac{\partial \sigma_x}{\partial x} = \left(V_y * K_y - V_z * K_{yz}\right) y + \left(V_z * K_z - V_y * K_{yz}\right) z \tag{4.6}$$

Where,

$$K_y = \frac{I_y}{I_y I_{yz} - I_{yz}^2}, \qquad K_z = \frac{I_z}{I_y I_{yz} - I_{yz}^2}, \qquad K_{yz} = \frac{I_{yz}}{I_y I_{yz} - I_{yz}^2}$$

Substituting equation (4.6) in (4.1) we get,

$$\frac{\partial q}{\partial s} = -\left(V_y * K_y - V_z * K_{yz}\right) ty - \left(V_z * K_z - V_y * K_{yz}\right) tz \tag{4.7}$$

Integrating from s = 0 to s =s which would be the integration of complete cross – section we have,

$$\int_0^s \frac{\partial q}{\partial s} ds = -\left(V_y * K_y - V_z * K_{yz}\right) \int_0^s ty\, ds - \left(V_z * K_z - V_y * K_{yz}\right) \int_0^s tz\, ds \tag{4.8}$$

If the origin for s is taken at the open edge of the cross – section, then q = 0 when s = 0 and equation (4.8) becomes,

$$q_s = -\left(V_y * K_y - V_z * K_{yz}\right) \int_0^s ty\, ds - \left(V_z * K_z - V_y * K_{yz}\right) \int_0^s tz\, ds \tag{4.9}$$

Now, since the C- beam under consideration is geometrically unsymmetrical, we will have to apply both V_z and V_y separately. To find the shear center we first apply V_y alone at the shear center and find the shear flow in the top flange using equation (4.9) and will follow the same procedure while applying V_z.

Shear flow in the top flange, when V_y is applied at the shear center is given by

$$q_{1y} = -\left(V_y * K_y\right)\int_0^s ty\,ds - \left(-V_y * K_{yz}\right)\int_0^s tz\,ds \qquad (4.10)$$

Shear flow in the top flange, when V_z is applied at the shear center is given by

$$q_{1z} = -\left(-V_z * K_{yz}\right)\int_0^s ty\,ds - \left(V_z * K_z\right)\int_0^s tz\,ds \qquad (4.11)$$

Where,

$y = (b_{f1} - y_c) - s_1$

$z = z_{1c}$

Shear stress in the top flange, when V_y and V_z is applied at the shear center can be determined by using the Eq (4.10) & (4.11)

$$\tau_y = \frac{-\left(V_y * K_y\right)\int_0^s ty\,ds - \left(-V_y * K_{yz}\right)\int_0^s tz\,ds}{t} \qquad (4.12)$$

$$\tau_z = \frac{-\left(-V_z * K_{yz}\right)\int_0^s ty\,ds - \left(V_z * K_z\right)\int_0^s tz\,ds}{t} \qquad (4.13)$$

The shear force F_y & F_z acting on the top flange can be determined by multiplying Eq (4.12) & (4.13) with the cross sessional area (t ds). Considering force balance at the origin

$$F_y = \int_0^s \tau_y * t * ds \qquad (4.14)$$

$$F_z = \int_0^s \tau_z * t * ds \tag{4.15}$$

$$V_y * Z_{sc} = F_y * Z_{1c} \tag{4.16}$$

$$V_z * Y_{sc} = F_z * Z_{1c} \tag{4.17}$$

$$Z_{sc} = \frac{F_y * Z_{1c}}{V_y} \tag{4.18}$$

$$Y_{sc} = \frac{F_z * Z_{1c}}{V_z} \tag{4.19}$$

4.2 Shear Center for Composite Beam with Unsymmetrical C Cross Section

The shear center of isotropic beams depends only on the geometry of the cross-section but for composite beams the shear center also depends on the material properties and the stacking sequence and it should include the relevant coupling behaviors. The shear center is an important sectional property that has to be considered in the analysis, because the bending and torsion are uncoupled at the shear center. That is, when a bending moment is applied at the shear center it will only induce curvature and it will not produce any twisting in the structure and when a torsional loads is applied at the shear center of the structure, it will not induce any curvature, the structure will only twist.

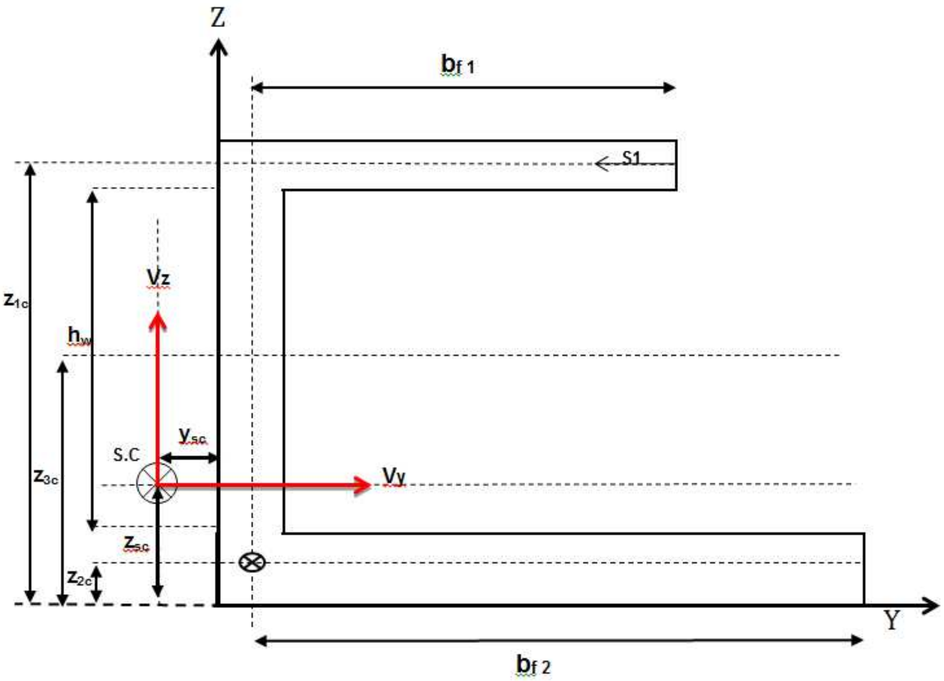

Figure 4-2 Shear center for the Composite Laminated C-Beam

To calculate the shear center we first need to understand and develop an expression for shear flow in the unsymmetrical composite C-Beam (see figure 4-2).

4.2.1 Expression for Shear Flow in C-Beam

If there is no load applied in the axial direction, the equilibrium equation is,

$$\frac{\partial q}{\partial s} + \frac{\partial N_x}{\partial x} = 0 \tag{4.19}$$

Where,

q = Shear flow

s = The flow direction

$N_x =$ Total force in x-direction, units (lb/in)

We have to find the shear force, V_y in y-direction and V_z in the z direction since the C-Beam is unsymmetrical, the procedure followed here has an approach similar to the one mentioned for the isotropic C-Beam with the introduction of material properties, stacking sequence and coupling effects.

4.2.2 Shear Flow in the Top Flange

From we have to find the equation for $N_{x,f1}$

$$\varepsilon^0_{x,f1} = \varepsilon^0_x + y * \kappa^c_z + z * \kappa^c_x$$

$$\kappa_{x,f1} = 0 \tag{4.20}$$

$$\varepsilon^0_x = \frac{\overline{N}_x}{EA}, \kappa^c_x = \frac{\overline{M}_x\overline{D}_y - \overline{M}_z\overline{D}_{xy}}{\overline{D}_x\overline{D}_y - \overline{D}^2_{xy}}, \kappa^c_z = \frac{\overline{M}_z\overline{D}_x - \overline{M}_x\overline{D}_{xy}}{\overline{D}_x\overline{D}_y - \overline{D}^2_{xy}}$$

We apply only $\overline{M}_z$ as we need to create only shear force in y-direction.

$EA =$ Equivalent axial stiffness of the composite I-Beam

Thus, from equations (4.20) and (3.34) we get,

$$N_{x,f1} = A^*_{1,f1}\left(\frac{\overline{N}_x}{EA} + y * \kappa^c_z + z\kappa^c_x\right) + B^*_{1,f1}\,\kappa_{x,f1} \tag{4.21}$$

Differentiating equation (4.21) with respect to x and $K_{x,f1}$=0 we get,

$$\frac{\partial N_{x,f1}}{\partial x} = \frac{\partial}{\partial x}\left\{A^*_{1,f1}\left(\frac{\overline{N}_x}{EA} + y * \kappa^c_z + z\kappa^c_x\right)\right\}$$

$$\frac{\partial \overline{N}_x}{\partial x} = 0; \frac{\partial \overline{M}_z}{\partial x} = V_y \ \& \ \frac{\partial \overline{M}_x}{\partial x} = V_z \ (shear\ force) \tag{4.21}$$

Thus we get,

$$\frac{\partial N_{x,f1}}{\partial x} = A^*_{1,f1}\left(y * (V_y * K_y - V_z * K_{yz}) - z * (V_z * K_z - V_y * K_{yz})\right) \tag{4.22}$$

Where,

$$K_y = \left(\frac{D_x}{D_y * D_x - D^2_{xy}}\right), K_z = \left(\frac{D_y}{D_y * D_x - D^2_{xy}}\right), K_{yz} = \left(\frac{D_{xy}}{D_y * D_x - D^2_{xy}}\right)$$

From equation (4.22) and (4.19) we get,

$$\frac{\partial q_{f1}}{\partial s_1} = -\frac{\partial N_{x,f1}}{\partial x}$$

$$\frac{\partial q_{f1}}{\partial s_1} = -\left(A^*_{1,f1}\left(y(V_y * K_y - V_z * K_{yz}) - z(V_z * K_z - V_y * K_{yz})\right)\right) \tag{4.23}$$

Integrating both the sides,

$$q_{f1} = \int_0^{s_1} -\left(A^*_{1,f1}\left(y * (V_y * K_y - V_z * K_{yz}) - z * (V_z * K_z - V_y * K_{yz})\right)\right) ds_1 \tag{4.24}$$

Now, since the C- beam under consideration is geometrically unsymmetrical, we will have to apply both V_y and V_z separately. To find the shear center we first apply V_y alone at the shear center and find the shear flow in the top flange using equation (3.14) and will follow the same procedure while applying V_z.

Shear flow in the top flange, when V_y is applied at the shear center is given by

$$q_{1y} = -\left(V_y * K_y\right)\int_0^s A^*_{1,f1} * y\, ds - \left(-V_y * K_{yz}\right)\int_0^s A^*_{1,f1} * z\, ds \tag{4.25}$$

Shear flow in the top flange, when V_z is applied at the shear center is given by

$$q_{1z} = -\left(-V_z * K_{yz}\right)\int_0^s A^*_{1,f1} * y\, ds - \left(V_z * K_z\right)\int_0^s A^*_{1,f1} * z\, ds \tag{4.26}$$

Where,

$y = (bf1 - y_c) - s_1$

$z = z_{1c}$

Shear stress in the in the top flange when V_y and V_z is applied at the shear center can be determined by using the Eq (4.25) & (4.26)

$$\tau_y = \frac{-\left(V_y * K_y\right)\int_0^s A_{1,f1}^* * y\, ds - \left(-V_y * K_{yz}\right)\int_0^s A_{1,f1}^* * z\, ds}{t_{f1}} \tag{4.27}$$

$$\tau_z = \frac{-\left(-V_z * K_{yz}\right)\int_0^s A_{1,f1}^* * y\, ds - \left(V_z * K_z\right)\int_0^s A_{1,f1}^* * z\, ds}{t_{f1}} \tag{4.28}$$

τ_y and τ_z are the shear stresses developed in the top flange when shear force V_y and V_z are applied at the shear center of the composite C beam

The shear force F_y & F_z acting on the top flange can be determined by multiplying Eq (4.27) & (4.28) with the cross sessional area (t_{f1} *ds). Considering force balance at the origin

$$F_y = \int_0^s \tau_y * t_{f1} * ds \tag{4.29}$$

$$F_z = \int_0^s \tau_z * t_{f1} * ds \tag{4.30}$$

$$V_y * Z_{sc} = F_y * Z_{1c} \tag{4.31}$$

$$V_z * Y_{sc} = F_z * Z_{1c} \tag{4.32}$$

$$Z_{sc} = \frac{F_y * Z_{1c}}{V_y} \tag{4.33}$$

$$Y_{sc} = \frac{F_z * Z_{1c}}{V_z} \tag{4.34}$$

Chapter 5

Finite Element Analysis

This chapter gives brief description finite element analysis of composite C beam. Finite element analysis is used for validating the analytical work. The finite element analysis is comprised of pre-processing, solution and post-processing phases. The goals of pre-processing are to develop an appropriate finite element mesh, assign suitable material properties, and apply boundary conditions in the form of restraints and loads. This chapter also gives the detail of geometry, material properties, and laminate stacking sequence.

The finite element analysis software ANSYS 15 is used determine the ply stresses of composite laminated C-Channel in this research.

5.1 Preprocessing

5.1.1 Geometry of Composite Laminate

This section describes modelling of C-Channel of laminated composite beam. A two-dimension shell element, SHELL181, which has 4 nodes with 6 degrees of freedom at each node, is used in the modelling. The reason for using this element is, it supports the composite modelling in ANSYS and is suitable for thin to moderately thick shell structure. Each laminated is treated as area element, formed by 4 nodes.

To model a three-dimension model, 8 key points were defined in the shape of a C-Channel and 1 key point was defined at centroid of cross-section. The length of model is 10 inches. Figure 5-1 shows that three areas were created base on 8 key points; A1 is top flange, A2 is bottom flange and A3 is web. The width of web, top and bottom flange is 1, 1 and 2 inches respectively.

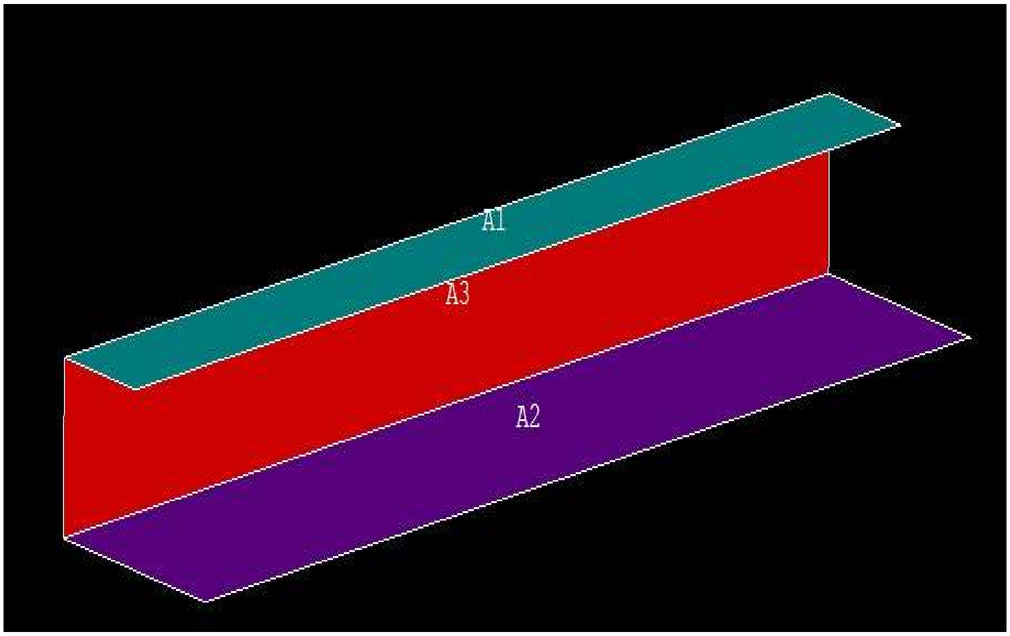

Figure 5-1 Geometry of C-channel beam

5.1.2 Material Properties

The material used in the analysis is AS/3501 graphite/epoxy and the material properties are as shown below:

E_1 = 20.00 x 10^6 psi E_2 = 1.3 x 10^6 psi

V_{12} = 0.30

G_{12} = 1.03 x 10^6 psi

Where E_1 and E_2, are elastic moduli along fiber and transverse direction respectively,

The isotropic material property used in the analysis is

$$E = E_{11} = E_{22} = 1.02 \text{ x } 10^7 \text{ psi}$$

$$\nu_{12} = 0.25$$

$$G_{12} = \frac{E}{2(1+\nu)} = 4.08 \text{ x } 10^6 \text{ psi}$$

5.1.3 Laminated Configuration

The properties of element of SHELL181 can be set specifying number of plies, its fiber orientations and layer thickness. The lay-up sequence used for the stress analysis is [45/-45/0/90]s for the top flange, bottom flange and web sub laminate.

The shell configuration set for top flange, the #1 ply or the most bottom ply is inside the model and the last ply or the most top ply is outside the model. Bottom flange, the #1 ply or the most bottom ply is outside the model and the last ply or the most top ply is inside the model. Web, the #1 ply or the most left ply is outside the model and the last ply or the most right ply is inside the model.

5.1.4 Meshing

The areas were meshed using mapped meshing option. Each laminated was divided into its width x 10 pieces along width direction and 100 pieces along the length (X-direction).The meshed model is shown in figure 5-2. Convergence study is conducted before finalizing the mesh density to make sure the stresses converges. Convergence study is very important do before starting an analysis, because if the stresses do not converges it may result in wrong results, even if all the procedures are correct.

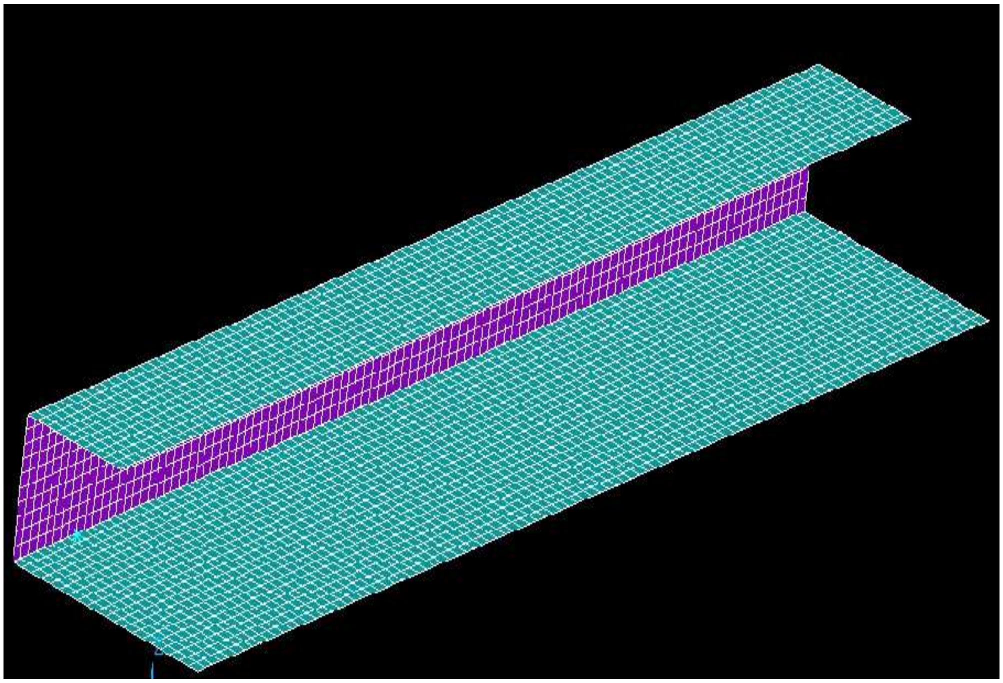

Figure 5-2 Mapped mesh model

5.1.5 Loads and Boundary Conditions

The axial load of 1 lb is applied at the centroid, since there is no node available at the centroid, a MASS21 element is created at the centroid and is connected to the all nodes at the distance x equals to 10 inch using multi point constraints as shown in figure 5-3 . When bending moment is applied, a load of 1 lb-in is applied at the centroid in a similar way the axial load is applied. The axial and bending moments are applied at centroid because; at centroid an applied axial load do not induces any curvature and the bending moment do not produce ant axial strains. However the torsional load is applied at the shear center of the structure because at the shear center torsion and bending are not coupled at the shear center. Since the centroid of the structure was not in the structure, a mass element was created at the centroid and is connected to the structure using rigid body element connection as shown in figure 5-3.

The boundary condition considered for the analysis is cantilever boundary condition, in which all the degrees of freedom of the nodes at x= 0 is fixed as shown in figure 5-3

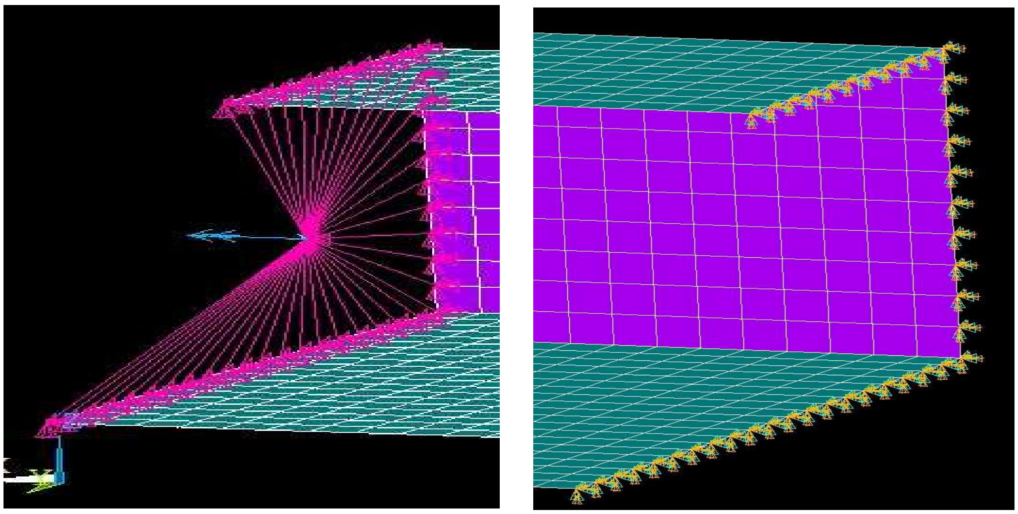

Figure 5-3 Loads and boundary conditions

5.2 Solving

Once the preprocessing is completed, the problem is solved using ANSYS, where we can request the software for the required output results, in this case the ply stresses. The software gives the results as outputs based on the inputs we specified in the preprocessing.

5.3 Post Processing

In post processing, the output can be either obtained as nodal solutions or elemental solution. In this case nodal solutions are used. The outputs results were measured at the mid length of beam. The results from the mid length cross-section is used in order to prevent the inaccuracy of the results due to the effect of boundary conditions and applied loads at the ends of the beam.

5.4 Equivalent Stiffness

5.4.1 Axial Stiffness

The axial stiffness can be calculated by the following equation.

$$\overline{\mathrm{EA}}_x = \frac{\mathrm{F}}{2\left(\mathrm{U}_{x|at\ x=L/2}\right)} \tag{5.1}$$

Where F is applied force along X-direction, L is total length of beam, U is displacement

The results were read at mid-length of the beam to avoid the numerical influence of the location of constraint and the load application.

5.4.2 Bending Stiffness

The bending stiffness's D_x, D_y & D_{xy} for the beam with unsymmetrical C cross section is calculated analytically using the eq 3.20, 3.23 & 3.28 respectively and is verified using the available results [7]. The stiffness of isotropic as well as composite beam with different layup sequences in the sub laminates (top, web and bottom laminates) was calculated and is shown in table 6.3.

Chapter 6

Results for Analysis of Unsymmetrical C-Beam

The solution for all the analysis performed in this research is briefly in the chapter. The results of the parametric study conducted on structural properties such as centroid and shear center for all the cases (case 1 to case 5) can be found in table 6-1 and table 6-2. The solutions of equivalent axial stiffness and equivalent bending stiffness for case 1 to case 5 are discussed in table 6.3 and the analytical and finite element solutions of the ply stresses developed in the composite C beam (Case 2) under axial and bending loads applied at the centroid of the beam can be found in table 6-4 and 6-5. The different cases considered for the parametric study of centroid and shear center are

Case 1: Isotropic beam

- Material used is aluminum
- The width of top flange is 1 inch
- The width of the bottom flange is 2 inch
- The height of the web is 1 inch

Case 2: Composite beam with all laminate layup are [±45°/0°/90°]s

- Material use is AS/3501 graphite/epoxy
- The width of top flange is 1 inch
- The width of the bottom flange is 2 inch
- The height of the web is 1 inch
- The stacking sequence of all sub-laminates are [±45°/0°/90°]s

Case 3: Composite beam with all laminate layup are [±45°/90°/90°]s

- Material use is AS/3501 graphite/epoxy
- The width of top flange is 1 inch
- The width of the bottom flange is 2 inch
- The height width of the web is 1 inch
- The stacking sequence of all sub-laminates are [±45°/90°/90°]s

Case 4: Composite beam with all laminate layup are $[\pm45°/90°/90°]_{2T}$

- Material use is AS/3501 graphite/epoxy
- The width of top flange is 1 inch
- The width of the bottom flange is 2 inch
- The height of the web is 1 inch
- The stacking sequence of all sub-laminates are $[\pm45°/90°/90°]_{2T}$

Case 5: Composite beam with all laminate layup are different

- Material use is AS/3501 graphite/epoxy
- The width of top flange is 1 inch
- The width of the bottom flange is 2 inch
- The height width of the web is 1 inch
- The stacking sequence of the top flange is $[\pm45°/0°/90°]_s$
- The stacking sequence of the bottom flange is $[\pm45°/0°/0°]_s$
- The stacking sequence of the web laminate is $[\pm45°]_{2s}$

6.1 Results of Centroid of Composite Beam with Unsymmetrical C Cross Section

The axial, bending and are applied at the centroid of the composite C-Beam. The calculation of centroid of composite C beam is different from the isotropic C-Beam. For isotropic material centroid is dependent only on the cross-section of the geometry whereas, for composite structures, the centroid location depends upon the geometry, stacking sequence and ply orientation at some cases and is discussed briefly in this chapter.

The analytic solutions for the composite C-Beam is verified by substituting isotropic properties in the developed solution and verifying the same using finite element analysis

Table 6-1 Results for centroid of C-Beam

case	Centroid	Present Method (Eq. 3.4a & 3.4b) (in)
1 Isotropic	Z_C	0.41
	Y_C	0.63
2 $[\pm 45°/0°/90°]_s$	Z_C	0.41
	Y_C	0.63
3 $[\pm 45°/90°/90°]_s$	Z_C	0.41
	Y_C	0.63
4 $[\pm 45°/90°/90°]_{2s}$ Unsymmetrical layup	Z_C	0.41
	Y_C	0.63
5 Top $[\pm 45°/0°/90°]_s$ Bottom $[\pm 45°/0°/0°]_s$ Web $[\pm 45°]_{2s}$	Z_C	0.307
	Y_C	0.789

6.2 Results of Shear Center of Beam with Unsymmetrical C Cross Section

The developed analytical solution for finding shear center is can be used for any open cross section (symmetrical & unsymmetrical). In the analysis it is found that shear center of a composite structure is a structural properties when the stacking sequences in the flanges and web are same where as it is a material property when the stacking sequence is different in all the flanges and web. The solution is verified by finding shear center by substituting isotropic properties and the verifying it with results from ANSYS Beam tool.

Table 6-2 Results for shear center of C-Beam

Case	Shear Center	Present Method (Eq.4.33 & 4.34) (in)
1 Isotropic	Z_{SC}	0.191
	Y_{SC}	-0.466
2 $[\pm45°/0°/90°]_s$	Z_{SC}	0.191
	Y_{SC}	-0.466
3 $[\pm45°/90°/90°]_s$	Z_{SC}	0.191
	Y_{SC}	-0.466
4 $[\pm45°/90°/90°]_{2s}$ Unsymmetrical layup	Z_{SC}	0.191
	Y_{SC}	-0.466
5 Top $[\pm45°/0°/90°]_s$ Bottom $[\pm45°/0°/0°]_s$ Web $[\pm45°]_{2s}$	Z_{SC}	0.134
	Y_{SC}	-0.473

6.3 Equivalent Stiffness of C-Beam

The equivalent stiffness of the cross-section was derived in chapter 3. The equivalent stiffness is the stiffness of the entire structure that is the combined stiffness of top and bottom flanges and the web, which is a structural property.

Table 6-3 Results of stiffness's for all case

Case		Unit	Present
1 Isotropic	$\overline{EA}$	Lb	1,632,000
	$\overline{D}_x$	Lb-in^2	337,550
	$\overline{D}_y$	Lb-in^2	576,480
	$\overline{D}_{xy}$	Lb-in^2	-184,580
2 [±45°/0°/90°]s	$\overline{EA}$	Lb	1,271,900
	$\overline{D}_x$	Lb-in^2	263,030
	$\overline{D}_y$	Lb-in^2	449,260
	$\overline{D}_{xy}$	Lb-in^2	-143,850
3 $[\pm 45°/90°/90°]_s$	$\overline{EA}$	Lb	558840
	$\overline{D}_x$	Lb-in^2	125590
	$\overline{D}_y$	Lb-in^2	197400
	$\overline{D}_{xy}$	Lb-in^2	-63204
4 $[\pm 45°/90°/90°]_{2s}$ Unsymmetrical layup	$\overline{EA}$	Lb	525000
	$\overline{D}_x$	Lb-in^2	108670
	$\overline{D}_y$	Lb-in^2	185820
	$\overline{D}_{xy}$	Lb-in^2	-59604
5 Top [±45°/0°/90°]s Bottom $[\pm 45°/0°/0°]_s$ Web $[\pm 45°]_{2s}$	$\overline{EA}$	Lb	1402500
	$\overline{D}_x$	Lb-in^2	277390
	$\overline{D}_y$	Lb-in^2	492560
	$\overline{D}_{xy}$	Lb-in^2	-151260

6.4 Ply Stress results

The ply stresses are calculated for case 2 and are compared with finite element solutions. The present method can also be used to calculate ply stresses for all kinds of layup sequences. The results from the finite element solutions are extracted from the nodes at the center of the C-beam as shown by red line in figure 6-1.

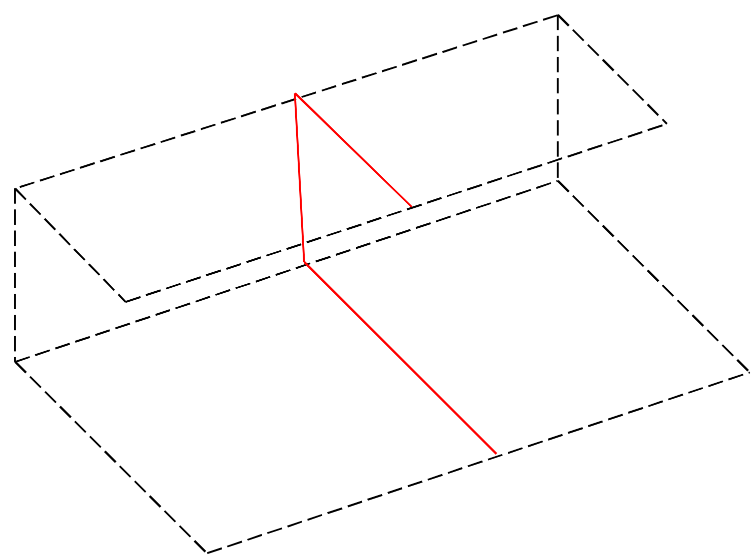

Figure 6-1 Location of nodes from which results are extracted

6.4.1 Composite Beam with all Laminate Layup [±45°/0°/90°]$_s$ (case 2)

The geometry of the unsymmetrical C beam for this case was, the top flange is1 inches and the web is 1 inches and the width of the bottom flanges is 2 inches.

6.4.1.1 Ply stresses in X-Y coordinate under axial load

An axial load of 1 lb is applied at the centroid of the composite C beam. The stresses are found to be uniform distributed in the same angle plies. The maximum stress is developed in the zero degree ply and the 90 degree ply carries the minimum stress. The stresses calculated using the present method shows excellent agreement with FEM results and are shown in the table 6-4. T and B in the layer column represent the top and bottom surfaces of each layer.

Table 6-4 Result of axial stresses under axial load for case 2

Layer		Top Flange (psi)			Bottom Flange (psi)			Web (psi)		
		PM	FEM	%Diff	PM	FEM	%Diff	PM	FEM	%Diff
8 (45)	T	4.16	4.21	-1.18	4.16	4.16	0	4.16	4.10	1.4
	B	4.16	4.21	-1.18	4.16	4.16	0	4.16	4.10	1.4
7 (-45)	T	4.16	4.21	-1.18	4.16	4.17	0.24	4.16	4.10	1.4
	B	4.16	4.21	-1.18	4.16	4.17	0.24	4.16	4.10	1.4
6 (0)	T	15.73	16.01	1.74	15.73	15.76	0.19	15.73	15.86	0.81
	B	15.73	16.01	1.74	15.73	15.76	0.19	15.73	15.86	0.81
5 (90)	T	0.94	0.96	1.92	0.94	0.94	0	0.94	0.94	0
	B	0.94	1.92	-2.34	0.94	0.94	0	0.94	0.94	0
4 (90)	T	0.94	0.96	1.92	0.94	0.94	0	0.94	0.94	0
	B	0.94	1.92	-2.34	0.94	0.94	0	0.94	0.94	0
3 (0)	T	15.73	16.01	1.74	15.73	15.76	0.19	15.73	15.86	0.81
	B	15.73	16.01	1.74	15.73	15.76	0.19	15.73	15.86	0.81
2 (-45)	T	4.16	4.21	-1.18	4.16	4.17	0.24	4.16	4.10	1.4
	B	4.16	4.21	-1.18	4.16	4.17	0.24	4.16	4.10	1.4
1 (45)	T	4.16	4.21	-1.18	4.16	4.16	0	4.16	4.10	1.4
	B	4.16	4.21	-1.18	4.16	4.16	0	4.16	4.10	1.4

6.4.1.2 Ply stresses in X-Y coordinate under bending moment

Table 6-5 lists the stress developed in top, bottom and web laminates when a one pound-inch of bending moment was applied at the centroid of the composite beam with unsymmetrical C cross section. The maximum stress is generated in the 0 degree ply whereas the 90 degree ply carries the minimum stress. T and B in the layer column represent the top and bottom surface of each layer. The stresses calculated using the present method shows excellent agreement with FEM results and are shown in the table 6-5.

Table 6-5 Result of axial stresses under bending moment for case 2

Layer		Top Flange (psi)			Bottom Flange (psi)			Web (psi)		
		PM	FEM	%Diff	PM	FEM	%Diff	PM	FEM	%Diff
8 (45)	T	16.66	16.75	0.53	-12.64	-12.48	1.2	7.87	8.04	2.1
	B	16.58	16.67	0.53	-12.71	-12.59	0.95	7.85	8.07	2.7
7 (-45)	T	16.68	16.68	0	-12.61	-12.58	0.23	7.88	8.08	2.4
	B	16.59	16.59	0	-12.71	-12.68	0.23	7.85	8.11	3.2
6 (0)	T	63.31	62.93	0.61	-47.34	-47.72	0.79	30.01	30.41	1.3
	B	62.85	62.47	0.60	-47.80	-48.18	0.78	29.71	30.6	2.9
5 (90)	T	3.81	3.75	1.6	-2.80	-2.79	0.3	1.79	1.83	2.1
	B	3.79	3.72	1.8	-2.82	-2.84	0.7	1.78	1.84	3.2
4 (90)	T	3.84	3.88	1.03	-2.77	-2.73	1.4	1.80	1.84	2.1
	B	3.81	3.86	1.2	-2.80	-2.76	1.4	1.80	1.85	2.7
3 (0)	T	64.69	64.52	0.26	-46.11	-46.15	0.08	30.29	30.98	2.2
	B	64.23	64.10	0.17	-46.42	-46.61	0.4	30.25	31.13	2.8
2 (-45)	T	17.18	17.04	0.82	-12.11	-12.29	1.4	8.03	8.22	2.3
	B	17.12	17.20	0.46	-12.21	-12.30	1.5	8.02	8.25	2.6
1 (45)	T	17.18	17.0	0.5	-12.11	-12.30	1.5	8.03	8.25	2.6
	B	17.11	16.87	1.4	-12.18	-12.49	2.4	8.01	8.28	3.2

Chapter 7

Conclusion and Future Work

A closed-form analytical solution is developed based on classical lamination theory and narrow beam theory for analyzing laminated composite beam with unsymmetrical C cross-section. The developed solutions can be used to determine the ply stresses and the cross-section properties such as centroid, shear center, axial and bending stiffness. The solution validated by substituting isotropic material properties and considering all plies zero-degree plies. The ply stresses developed in the composite C beam under axial force and bending moment were calculated using the developed analytical solutions and the results are verified using ANSYS

The ply stress results obtained from present method exhibits excellent agreement with FEM results. It is concluded that the present method can be used as an effective tool calculate ply stresses, centroid and shear center with high accuracy.

From the parametric study, it is found that

- Like isotropic structures, the location of the centroid of a composite structure is dependent of its structural configuration if all of the flanges and web of the beam have same family laminates with symmetrical and balanced, unbalanced or unsymmetrical layup regardless the ply orientation and the stacking sequence of laminate.
- But the centroid location of a composite C-beam structure is dependent on the laminate material properties and stacking sequence besides its structural configuration the flange and web laminates are of different family.
- The location of shear center is also found to be similar structural response to the centroid of the structures. For the case with all laminate layup are $[\pm45°/0°/90°]_s$,

the present method shows excellent agreement with FEM for both axial force case and bending moment case. For axial force, the stresses are uniform on the same ply orientation of the cross-section.

- The present analytic method for finding shear center is applicable for the symmetrical as well as unsymmetrical cross sections

In future studies, the present method can be extended to analyze for an uneven composite C-beam under torsion. Torsional stiffness and warping stiffness of this beam can be obtained by similar to the approach developed in this thesis. Extension of this analysis can be easily extended to the beam under hygrothermal environmental condition. The interlaminar shear stress of this beam under transverse load can be also easily obtained.

Appendix A

MATLAB Code for Analytical Solution

The MATLAB codes requires ABD matrix as input

```
%ABD Matrix for Top Flange sub laminate

format shortE

 Af1= [345903.023, 98295.539,          0       ;

       98295.539,     345903.023,        0       ;

        0.000,            0.000,       123803.742];

Bf1= [ 0,                  0,                     0;

        0,                  0,                     0;

        0,                  0,                     0];

Df1 = [42.563,          21.366,      7.054;

         21.366,          33.158,      7.054;

          7.054,           7.054,      24.768];

 ABDf1= [Af1, Bf1;

             Bf1, Df1];

 abbdtf1 = inv(ABDf1);

% ABD Matrix for Web sub laminate

 Aw= [345903.023, 98295.539,          0       ;

       98295.539,     345903.023,        0       ;
```

```
        0.000,            0.000,       123803.742];

Bw= [0,                 0,                  0;

        0,                 0,                  0;

        0,                 0,                  0];

 Dw= [42.563,          21.366,      7.054;

         21.366,          33.158,      7.054;

          7.054,           7.054,      24.768];

 ABDw= [Aw, Bw;

             Bw, Dw];

 abbdtw = inv(ABDw);

% ABD matrix for Bottom Flange sub lamiate

Af2= [345903.023, 98295.539,           0       ;

       98295.539,     345903.023,        0       ;

        0.000,            0.000,       123803.742];

Bf2= [0,                 0,                  0;

        0,                 0,                  0;

        0,                 0,                  0];
```

```
Df2= [42.563,          21.366,      7.054;

          21.366,          33.158,      7.054;

           7.054,           7.054,      24.768];


ABDf2=    [Af2, Bf2;

                 Bf2, Df2];

abbdtf2 = inv(ABDf2);

 %Global ABD matrix FOR C BEAM

 ABDs=ABDf1+ABDf2+ABDw;

 % Axial, coupling and bending stiffness for top flange

 A1sf1=1/ ((abbdtf1 (1, 1))-((abbdtf1 (1, 4)*abbdtf1(1,4))/abbdtf1(4,4)));

B1sf1= 1/ ((abbdtf1 (1, 4))-((abbdtf1 (1, 1)*abbdtf1(4,4))/abbdtf1(1,4)));

D1sf1= 1/ ((abbdtf1 (4, 4))-((abbdtf1 (1, 4)*abbdtf1(1,4))/abbdtf1(1,1)));

 % Axial, coupling and bending stiffness for WEB

 A1sw=1/((abbdtw(1,1))-((abbdtw(1,4)*abbdtw(1,4))/abbdtw(4,4)));

B1sw= 1/((abbdtw(1,4))-((abbdtw(1,1)*abbdtw(4,4))/abbdtw(1,4)));

Dsw= 1/((abbdtw(4,4))-((abbdtw(1,4)*abbdtw(1,4))/abbdtw(1,1)));

% Axial, coupling and bending stiffness for bottom flange
```

```
A1sf2=1/ ((abbdtf2(1,1))-((abbdtf2(1,4)*abbdtf2(1,4))/abbdtf2(4,4)));

B1sf2= 1/ ((abbdtf2(1,4))-((abbdtf2(1,1)*abbdtf2(4,4))/abbdtf2(1,4)));

D1sf2= 1/ ((abbdtf2(4,4))-((abbdtf2(1,4)*abbdtf2(1,4))/abbdtf2(1,1)));

 %Geometrical configuration of C beam

bf1=1;

bf2=2;

hw=1;

% Distance from Centroid/mid-section of each sub laminate

zw=0.54;        %0.04+ 1/2 =0.54

zf1=1.06;       % 0.04+1+0.02

zf2=0.02;       % 0.004/2= 0.02

yf1=0.5;

yf2=1;

yw =0.02;

tw =0.04;

% Centroid calculation formula for composite structures

zc= (hw*A1sw*zw + bf1*A1sf1*zf1 + bf2*A1sf2*zf2)/(hw*A1sw+bf1*A1sf1+bf2*A1sf2);

Yc= (hw*A1sw*yw + bf1*A1sf1*yf1 + bf2*A1sf2*yf2)/(hw*A1sw+bf1*A1sf1+bf2*A1sf2);
```

```
%distance from centroid to mid layer of flange 1

z1c= zf1-zc;

y1c= - (Yc-yf1);

% distance from the centroid t0 mid layer of flange 2

z2c= - (zc-zf2);

y2c= - (Yc-yf2);

 %distance from the center of the web to the centroid

z3c= zw-zc;

y3c= -(Yc-yw);

 %Equivalent stiffness calculation

 EAb= (bf1*A1sf1 + hw*A1sw + bf2*A1sf2 );

 Dxb=((bf1*(A1sf1*z1c*z1c + 2*B1sf1*z1c+D1sf1))+(bf2*(A1sf2*z2c*z2c +
2*B1sf2*z2c+D1sf2))+ (A1sw*(((1/12)*hw*hw*hw)+hw*z3c*z3c)));

 Dyb= (A1sf1*(((bf1^3)/12)+bf1*y1c*y1c)+
A1sf2*(((bf2^3)/12)+bf2*y2c*y2c)+(A1sw*y3c*y3c+ 2*B1sw*y3c +Dsw));

 Dxyb=((A1sf1*z1c + B1sf1)*bf1*y1c)+ ((A1sf2*z2c + B1sf2)*bf2*y2c)+((A1sw*y3c
+B1sw)*hw*z3c);

%%Stress calculation

P =1;                               % P is axial force applied at centroid
```

```
Mxb = 0;                              % Mxb is moment applied in the x direction

Mzb =0;

% strains at centroid

exc = P/EAb;

kxc = (Mxb*Dyb-Mzb*Dxyb)/(Dxb*Dyb-(Dxyb*Dxyb));

kzc = (Mzb*Dxb-Mxb*Dxyb)/(Dxb*Dyb-(Dxyb*Dxyb));

 %TOP FLANGE (SUB LAMINATE -1)

y1=Yc-(bf1/2);    %distance from centroid to any point of the top flange

 % Mid plane strain and curvature at top flange

e0xf1= exc+z1c*kxc+y1*kzc;

kxf1 =kxc;

%Equivalent forces acting at top flange

Nxf1 = A1sf1*e0xf1 + B1sf1*kxc;

Mxf1 = B1sf1*e0xf1 + D1sf1*kxc;

Mxyf1= - ((abbdtf1(1,6)*Nxf1 + abbdtf1(1,6)*Mxf1)/abbdtf1(6,6));

 abbdtf1r = [abbdtf1(1,1), abbdtf1(1,4),abbdtf1(1,6);

                 abbdtf1(1,2), abbdtf1(2,4),abbdtf1(2,6);

                  abbdtf1(1,3), abbdtf1(3,4),abbdtf1(3,6);
```

```
                    abbdtf1(1,4), abbdtf1(4,4),abbdtf1(4,6);

                    abbdtf1(1,5), abbdtf1(5,4),abbdtf1(5,6);

                    abbdtf1(1,6), abbdtf1(6,4),abbdtf1(6,6)];

% e0kf1 mid plane stain and curvature of flange 1

e0kf1=   abbdtf1r* [Nxf1;

                               Mxf1;

                               Mxyf1];

  ef1= [e0kf1(1,1);

           e0kf1(2,1);

           e0kf1(3,1)];

   kf1= [e0kf1(4,1);

         e0kf1(5,1);

         e0kf1(6,1)];

%BOTTOM FLANGE (SUBLAMINATE -2)

y2=Yc-(bf2/2);    %distance from centroid to anypoint of the top flange

e0xf2= exc+z2c*kxc+y2*kzc;

kxf2 =kxc;

 Nxf2 = (A1sf2*e0xf2 + B1sf2*kxc);
```

```
Mxf2 = (B1sf2*e0xf2 + D1sf2*kxc);

Mxyf2=-((abbdtf2(1,6)*Nxf2 + abbdtf2(1,6)*Mxf2)/abbdtf2(6,6));

 abbdtf2r = [abbdtf2(1,1), abbdtf2(1,4),abbdtf2(1,6);

                  abbdtf2(1,2), abbdtf2(2,4),abbdtf2(2,6);

                   abbdtf2(1,3), abbdtf2(3,4),abbdtf2(3,6);

                   abbdtf2(1,4), abbdtf2(4,4),abbdtf2(4,6);

                   abbdtf2(1,5), abbdtf2(5,4),abbdtf2(5,6);

                    abbdtf2(1,6), abbdtf2(6,4),abbdtf2(6,6)];

 e0kf2= abbdtf2r* [Nxf2;

                              Mxf2;

                              Mxyf2];

 ef2=[ e0kf2(1,1);

        e0kf2(2,1);

        e0kf2(3,1)]

  kf2=[ e0kf2(4,1);

          e0kf2(5,1);

          e0kf2(6,1)]

 % WEB (SUBLAMINATE)
```

```
y3 = Yc-(tw/2);

e0xw = exc+z3c*kxc+y3*kzc;

kxw = kzc;

Nxw =A1sw*e0xw + B1sw *kzc;

Mxw =B1sw*e0xw + Dsw *kzc;

Mxyw=-((abbdtw(1,6)*Nxw + abbdtw(1,6)*Mxw)/abbdtw(6,6));

Nxb= Nxf1*bf1+Nxf2*bf2+Nxw*hw;

abbdtwr=   [abbdtw(1,1), abbdtw(1,4),abbdtw(1,6);

             abbdtw(1,2), abbdtw(2,4),abbdtw(2,6);

             abbdtw(1,3), abbdtw(3,4),abbdtw(3,6);

             abbdtw(1,4), abbdtw(4,4),abbdtw(4,6);

             abbdtw(1,5), abbdtw(5,4),abbdtw(5,6);

             abbdtw(1,6), abbdtw(6,4),abbdtw(6,6)];

 e0kw=   abbdtwr* [Nxw;

                    Mxw;

                    Mxyw];

ew=[  e0kw(1,1);

       e0kw(2,1);
```

```
        e0kw(3,1)];
kw=[  e0kw(4,1);
        e0kw(5,1);
        e0kw(6,1)]
 Q0 = 1.0e+07 *[2.0118,    0.0392,         0;
                 0.0392,    0.1308,         0;
                 0,         0,          0.1030];
  Q45 = 1.0e+06 *[6.5825,   4.5225,    4.7025;
                   4.5225,   6.5825,    4.7025;
                   4.7025 ,   4.7025 ,   5.1602];
Qm45= 1.0e+06 *[6.5825,    4.5225,   -4.7025;
                 4.5225,   6.5825,   -4.7025;
                 -4.7025,  -4.7025,    5.1602];
  Q90= 1.0e+07 *[ 0.1308 ,   0.0392 ,   0.0000;
                   0.0392,   2.0118,   0.0000;
                   0.0000,   0.0000,   0.1030];
% Stress transformation matrix
```

```
T0= [1  ,   0 ,    0;
    0  ,   1 ,    0;
    0  ,   0 ,    1];
T45= [5.0000e-001 , 5.0000e-001,  1.0000e+000;
         5.0000e-001, 5.0000e-001, -1.0000e+000;
        -5.0000e-001, 5.0000e-001, 2.2204e-016];
Tm45= [5.0000e-001,  5.0000e-001, -1.0000e+000;
            5.0000e-001,  5.0000e-001 , 1.0000e+000;
            5.0000e-001 ,-5.0000e-001 , 2.2204e-016];
T90 = [0.0000,    1.0000,    0.0000;
           1.0000,    0.0000 ,  -0.0000;
          -0.0000,    0.0000,   -1.0000];
d=0.005
%TOP FLANGE "t"is the distance from mid layer of top flange to corresponding layers under consideration
t1= 4*d;
t2= 3*d;
t3= 2*d;
```

```
t4= 1*d;

t5= 0*d;

t6= -1*d;

t7= -2*d;

t8= -3*d;

t9= -4*d;

%Bottom FLANGE "b"is the distance from midlayer of bottom flange to corresponding

 %layers under consideration

b1 = 4*d;

b2 = 3*d;

b3 = 2*d;

b4 = 1*d;

b5 = 0*d;

b6 = -1*d;

b7 = -2*d;

b8 = -3*d;

b9 = -4*d;

%WEB "w"is the distance from midlayer of web to corresponding
```

```
%layers under consideration

w1=  4*d

w2=  3*d;

w3=  2*d;

w4=  1*d;

w5=  0*d;

w6= -1*d;

w7= -2*d;

w8= -3*d;

w9= -4*d;

%Top flange global stress

Sf1k1xyU45   = (Q45* (ef1 + (t1*kf1)));

Sf1k1xyL45    = (Q45* (ef1 + (t2*kf1)));

Sf1k2xyUm45 = (Qm45*(ef1 + (t2*kf1)));

Sf1k2xyLm45 = (Qm45*(ef1 + (t3*kf1)));

Sf1k3xyU0     = (Q0* (ef1 + (t3*kf1)));

Sf1k3xyL0      = (Q0* (ef1 + (t4*kf1)));

Sf1k4xyU90    = (Q90* (ef1 + (t4*kf1)));
```

```
Sf1k4xyL90     = (Q90* (ef1 + (t5*kf1)));

Sf1k5xyU90     = (Q90* (ef1 + (t5*kf1)));

Sf1k5xyL90   = (Q90* (ef1 + (t6*kf1)));

Sf1k6xyU0    = (Q0* (ef1 + (t6*kf1)));

Sf1k6xyL0     = (Q0* (ef1 + (t7*kf1)));

Sf1k7xyUm45= (Qm45*(ef1 + (t7*kf1)));

Sf1k7xyLm45 = (Qm45*(ef1 + (t8*kf1)));

Sf1k8xyU45   = (Q45* (ef1 + (t8*kf1)));

Sf1k8xyL45    = (Q45* (ef1 + (t9*kf1)));

 %Top flange local stresses

Sf1k112U45   = T45 *Sf1k1xyU45;

Sf1k112L45   = T45 *Sf1k1xyL45;

Sf1k212Um45 = Tm45*Sf1k2xyUm45;

Sf1k212Lm45 = Tm45*Sf1k2xyLm45;

Sf1k312U0     = T0*Sf1k3xyU0;

Sf1k312L0      = T0*Sf1k3xyL0;

Sf1k412U90    = T90*Sf1k4xyU90;

Sf1k412L90    = T90*Sf1k4xyL90;
```

```
Sf1k512U90    = T90*Sf1k5xyU90;

Sf1k512L90    = T90*Sf1k5xyL90;

Sf1k612U0    = T0 *Sf1k6xyU0;

Sf1k612L0       = T0 *Sf1k6xyL0;

Sf1k712Um45 = Tm45*Sf1k7xyUm45;

Sf1k712Lm45  = Tm45*Sf1k7xyLm45;

Sf1k812U45    = T45 *Sf1k8xyU45;

Sf1k812L45     = T45 *Sf1k8xyL45;

%Bottom flange global ply stresses

Sf2k1xyU45  = (Q45* (ef2 + (b1*kf2)));

Sf2k1xyL45  =  (Q45* (ef2 + (b2*kf2)));

Sf2k2xyUm45 = (Qm45*(ef2 + (b2*kf2)));

Sf2k2xyLm45 = (Qm45*(ef2 + (b3*kf2)));

Sf2k3xyU0     = (Q0* (ef2 + (b3*kf2)));

Sf2k3xyL0      = (Q0* (ef2 + (b4*kf2)));

Sf2k4xyU90    = (Q90* (ef2 + (b4*kf2)));

Sf2k4xyL90     = (Q90* (ef2 + (b5*kf2)));

Sf2k5xyU90    = (Q90* (ef2 + (b5*kf2)));
```

```
Sf2k5xyL90     = (Q90* (ef2 + (b6*kf2)));

Sf2k6xyU0      = (Q0* (ef2 + (b6*kf2)));

Sf2k6xyL0       = (Q0* (ef2 + (b7*kf2)));

Sf2k7xyUm45 = (Qm45*(ef2 + (b7*kf2)));

Sf2k7xyLm45 = (Qm45*(ef2 + (b8*kf2)));

Sf2k8xyU45   = (Q45* (ef2 + (b8*kf2)));

Sf2k8xyL45  = (Q45* (ef2 + (b9*kf2)))

%Bottom flange local stresses

 Sf2k112U45  = T45 *Sf2k1xyU45;

Sf2k112L45    = T45 *Sf2k1xyL45;

Sf2k212Um45 = Tm45*Sf2k2xyUm45;

Sf2k212Lm45  = Tm45*Sf2k2xyLm45;

Sf2k312U0      = T0 *Sf2k3xyU0;

Sf2k312L0       = T0 *Sf2k3xyL0;

Sf2k412U90    = T90 *Sf2k4xyU90;

Sf2k412L90     = T90 *Sf2k4xyL90;

Sf2k512U90    = T90 *Sf2k5xyU90;

Sf2k512L90     = T90 *Sf2k5xyL90;
```

```
Sf2k612U0      = T0 *Sf2k6xyU0;

Sf2k612L0       = T0 *Sf2k6xyL0;

Sf2k712Um45  = Tm45*Sf2k7xyUm45;

Sf2k712Lm45  = Tm45*Sf2k7xyLm45;

Sf2k812U45    = T45 *Sf2k8xyU45;

Sf2k812L45    = T45 *Sf2k8xyL45;

%Web global stresses

Swk1xyU45    = (Q45* (ew + (b1*kw)))

Swk1xyL45     = (Q45* (ew + (b2*kw)))

Swk2xyUm45  = (Qm45*(ew + (b2*kw)))

Swk2xyLm45  = (Qm45*(ew + (b3*kw)))

Swk3xyU0       = (Q0* (ew + (b3*kw)))

Swk3xyL0       = (Q0* (ew + (b4*kw)))

Swk4xyU90     = (Q90*(ew + (b4*kw)))

Swk4xyL90     = (Q90* (ew + (b5*kw)))

Swk5xyU90    = (Q90* (ew + (b5*kw)))

Swk5xyL90    = (Q90* (ew + (b6*kw)))

Swk6xyU0     = (Q0* (ew + (b6*kw)))
```

```
Swk6xyL0      = (Q0* (ew + (b7*kw)))

Swk7xyUm45 = (Qm45*(ew + (b7*kw)))

Swk7xyLm45  = ( Qm45*(ew + (b8*kw)))

Swk8xyU45    = (Q45* (ew + (b8*kw)))

Swk8xyL45    = (Q45* (ew + (b9*kw)))

 %Web local stresses

Swk112U45   = T45 *Swk1xyU45;

Swk112L45    = T45 *Swk1xyL45;

Swk212Um45 = Tm45*Swk2xyUm45;

Swk212Lm45  = Tm45*Swk2xyLm45;

Swk312U0      = T0 *Swk3xyU0;

Swk312L0      = T0 *Swk3xyL0;

Swk412U90   = T90 *Swk4xyU90;

Swk412L90    = T90 *Swk4xyL90;

Swk512U90   = T90 *Swk5xyU90;

Swk512L90   = T90 *Swk5xyL90;

Swk612U0    = T0 *Swk6xyU0;

Swk612L0   = T0 *Swk6xyL0;
```

```
Swk712Um45 = Tm45*Swk7xyUm45;

Swk712Lm45 = Tm45*Swk7xyLm45;

Swk812U45  = T45 *Swk8xyU45;

Swk812L45  = T45 *Swk8xyL45 ;

 Ky= Dxb/((Dxb*Dyb)-Dxyb^2);

Kz= Dyb/((Dxb*Dyb)-Dxyb^2);

Kyz= Dxy/((Dxb*Dyb)-Dxyb^2)

a=Ky*A1sf1
b=Kyz*A1sf1
c=Kz*A1sf1
 s=0.98
Fy= -Ky*A1sf1*((((bf1-Yc)*s^2)/2)-(s^3/6)) + Kyz*A1sf1*z1c *(s^2/2)
Zsc= (-Fy*zs)+0.02
Fz=Kyz*A1sf1*((((bf1-Yc)*s^2)/2)-(s^3/6))-Kz*A1sf1*z1c *(s^2/2)
Ysc=(Fz*zs)+0.02
```

APPENDIX B

ANSYS 15 Code for Finite Element Analysis

```
/UNITS,BIN

/PREP7

/TRIAD,LBOT

! Define Parameter

L=10

bf1=1

bf2=2

w=1

! Define Key point

K,1,0,0,0

K,2,L,0,0

K,3,L,bf2,0

K,4,0,bf2,0

K,5,0,0,w

K,6,L,0,w

K,7,L,bf1,w

K,8,0,bf1,w

! Dummy point at centroid
```

```
K,9,L,0.63,0.41

!Define Area

A,5,6,7,8

A,1,2,3,4

A,1,2,6,5

AGLUE,ALL

/PNUM,AREA,1

! Define New Working plane

WPROTA,,-90

CSWPLA,11,0

WPROTA,,90

! Define Material Properties

ET,1,SHELL181

KEYOPT,1,3,2

KEYOPT,1,8,2

ET,2,MASS21

R,1

MP,EX,1,20.0e6
```

```
MP,EY,1,1.3e6

MP,EZ,1,1.3e6

MP,PRXY,1,0.30

MP,PRYZ,1,0.49

MP,PRXZ,1,0.30

MP,GXY,1,1.03e6

MP,GYZ,1,0.90e6

MP,GXZ,1,1.03e6

MP,CTEX,1,1.0e-6

MP,CTEY,1,30e-6

MP,CTEZ,1,30e-6

! Top flange

SECTYPE,1,SHELL,,TFlange

SECDATA,0.005,1,45,3

SECDATA,0.005,1,-45,3

SECDATA,0.005,1,0,3

SECDATA,0.005,1,90,3

SECDATA,0.005,1,90,3
```

```
SECDATA,0.005,1,0,3

SECDATA,0.005,1,-45,3

SECDATA,0.005,1,45,3

SECOFFSET,BOTTOM

! Bottom flange

SECTYPE,2,SHELL,,BFlange

SECDATA,0.005,1,45,3

SECDATA,0.005,1,-45,3

SECDATA,0.005,1,0,3

SECDATA,0.005,1,90,3

SECDATA,0.005,1,90,3

SECDATA,0.005,1,0,3

SECDATA,0.005,1,-45,3

SECDATA,0.005,1,45,3

SECOFFSET,TOP

! Web

SECTYPE,3,SHELL,,Web

SECDATA,0.005,1,45,3
```

```
SECDATA,0.005,1,-45,3

SECDATA,0.005,1,0,3

SECDATA,0.005,1,90,3

SECDATA,0.005,1,90,3

SECDATA,0.005,1,0,3

SECDATA,0.005,1,-45,3

SECDATA,0.005,1,45,3

SECOFFSET,TOP

! Mesh Attribute & SIZE CONTROL & MESH

ASEL,S,AREA,,1

AATT,1,,1,0,1

ASEL,S,AREA,,2

AATT,1,,1,0,2

ASEL,S,AREA,,3

AATT,1,,1,11,3

LSEL,S,LENGTH,,L

LESIZE,ALL,,,L*10,1

LSEL,S,LINE,,2
```

```
LSEL,A,LINE,,4

LESIZE,ALL,,,bf1*10,1

LSEL,S,LINE,,6

LSEL,A,LINE,,8

LESIZE,ALL,,,bf2*10,1

LSEL,S,LINE,,9

LSEL,A,LINE,,10

LESIZE,ALL,,,w*10,1

ALLSEL

AMESH,ALL

CSYS,0

KSEL,S,KP,,9

KATT,1,1,2,0

KSEL,S,KP,,9

KMESH,ALL

NSEL,S,LOC,X,L

CERIG,4142,ALL,ALL,,,

!Apply Constrain
```

```
NSEL,S,LOC,X,0

D,ALL,ALL,0

ALLSEL

!Apply Force

F,4142,MX,1

/SOLU

ANTYPE,STATIC

SOLVE
```

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Biographical Information

Martin M Kora received his B.E degree in Aeronautical Engineering from Anna University, India. He worked in National Aerospace Laboratories as a Project engineer and as a junior design engineer in Jetwings Technologies. In 2013, he joined the University of Texas at Arlington in August 2010 for Master’s program in Aerospace Engineering and graduated in May 16, 2015.

His academic interest lies in structural analysis and engineering design. He further plans to work as a structural analysis engineer.