

Dynamics and Control of the Smart Maintenance and Inspection Robotic Free-Flyer
(SMIRFF)

by

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To my mother Dr. Sangeeta Godbole, my father Dr. Rajendra Godbole and my
sister Anuja.

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I owe all the good things in my life to my parents, my sister and my family. I thank them for all the sacrifices they have made to make my life happier.

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ABSTRACT

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(SMIRFF)

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The aim of the future space missions would be exploring near earth planets and asteroids. This would mean longer flight durations requiring a space habitat. Longer flight durations would require frequent maintenance and repairs of the space habitat. This would involve the risk of exposing astronauts to the harsh environments while conducting extra-vehicular activities. This risk can be minimised by using a tele-operated free-flying robot. Free-flying robots are a unique class of robots which do not have rigid fixed base and float in space.

A concept of Smart Maintenance Inspection and Robotic Free-flyer (SMIRFF) is presented in this thesis. SMIRFF is a compact free-flyer with two robotic arms. It uses cold-gas thrusters to position and orient itself in space. With an array of on-board sensors, cameras and robotic arms, SMIRFF is capable of performing visual inspection along with minimal maintenance and repair works. SMIRFF weighs roughly around 10 Kg. Since NASA has been exploring the possibilities of using 3-D printing technology, custom made versions of this robot can be additively manufactured for task specific purposes.

The aim of the thesis is to derive the kinematics of SMIRFF using modified Denavit-Hartenberg parameters. Using these kinematic relations, the non-linear dynamical model is constructed using an Iterative Newton-Euler method. A simulation showing the effect of simple manipulator arm motion on the base is demonstrated. The next step involves showing usefulness of the simulation environment, SimMechanics (MATLAB/SIMULINK) in simulating such complex multi-body systems. Finally, a quaternion based feedback controller is used to control the attitude of the floating base by counteracting the reaction moments generated due to the motion of the manipulator arms.

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

The search for existence of life beyond Earth has led mankind to explore beyond the visible horizon for thousands of years. This curiosity led to the manned mission to Moon and long duration space experimentation at International Space Station (ISS). Space agencies around the world are planning manned missions to nearby asteroids and planets like Mars. These exploration missions will require longer flight and operation times. One of the objectives of this missions would be to have a habitat in Cis-lunar orbit to serve as a station for these missions. These missions would possess challenges such as periodic maintenance, inspection and periodic works. The nature of the radiation environment beyond low-Earth orbit (LEO) and the need for long duration flights and missions adds unnecessary risk to conducting multiple extravehicular activities (EVA). Therefore, the need for an asset that can be controlled from within a spacecraft to conduct operations outside of a space structure is critical for the future of manned space flight. Free-flying robots can be the asset which can assist these extra-vehicular activities.

Engineers at NASA Johnson Space Center have previously worked on the development of a free-flyer known as the Miniature Autonomous Extravehicular Robotic Camera (Mini AERCam) [1]. The predecessor to Mini AERCam, called AERCam “Sprint”, was flight tested on STS-87 in December 1997. Simplified Aid for EVA Rescue (SAFER) backpack inspired the development of AERCam Sprint [2]. Sprint was a 14-inch diameter, 35-pound sphere that contained two television cameras, an

avionics system and 12 small nitrogen gas-powered thrusters. Fig 1.2 shows the exploded view of Mini AER cam showing the internal components and the assembly details.



Figure 1.1: AERCam Sprint
(Image Courtesy:
www.start1.jpl.nasa.gov)

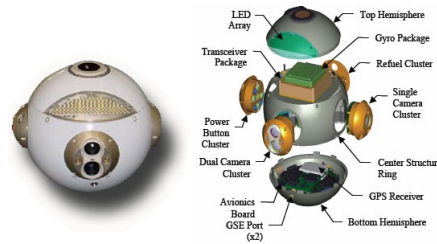


Figure 1.2: Mini AERCam
(Image Courtesy:
www.spaceflight.nasa.gov)

Another free-flyer, Synchronized Position Hold Engage and Reorient Experimental Satellite system (SPHERES), developed by Space Systems Laboratory, MIT was launched in 2006 [3]. It was roughly spherical and about the size of a bowling ball. The frame of the SPHERES is made of aluminium and the components are enclosed in a plastic shell. It has 12 thrusters and the navigation is aided by ultrasound beacons and infra-red transmitters. Fig 1.3 shows the SPHERES in operation on the International Space Station.

All these free-flyers were simply designed as visual inspection free-flyers, i.e, they could just observe without interacting with the surrounding. National Space Development Agency of Japan (NASDA) in November 1997, was successful in launching the Engineering Test Satellite VII (ETS-VII). It had a 2 meter 6 degrees of freedom



Figure 1.3: SPHERES
(Image Courtesy:www.nasa.gov)

manipulator arm mounted on it. It was the first unmanned spacecraft to conduct autonomous rendezvous docking operations successfully. It was designed for an operational period of 1.5 years but ended up being functional for almost 5 years. Fig 1.4 shows the computer graphics simulation for the target capture by the manipulator arm of ETS-VII

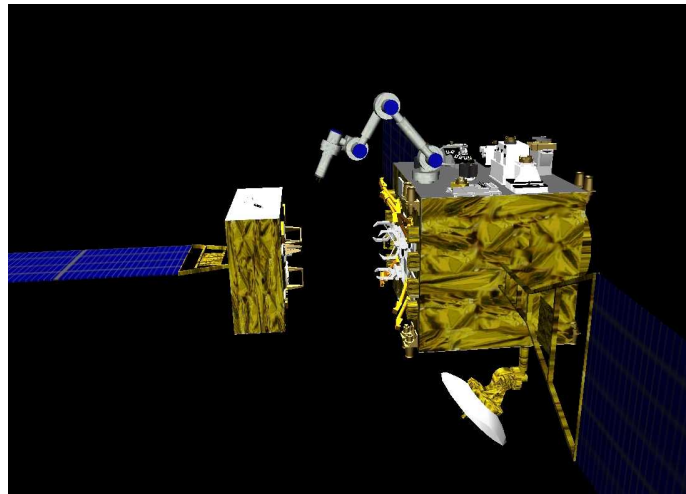


Figure 1.4: ETS VII
(Image Courtesy:www.astro.mech.tohoku.ac.jp)

1.2 SMIRFF: Concept Development

The existing technology previously mentioned was either capable of just visual inspection or was too bulky to serve as an assistant during extra-vehicular activity. Hence implementation of these free-flyers for the minimal repairs or assistance during space walks was difficult. The 2014 Revolutionary Aerospace Systems Concepts and Academic Linkage, a student design competition challenged and provided students to design a tele-operated robot to serve as an assistant or partial replacement for humans for extra-vehicular activity. A team from Aerospace Systems Laboratory from The University of Texas at Arlington developed the concept of a Smart Maintenance Inspection Repair Free-Flyer (SMIRFF) [4]. Fig 1.5 shows the subsystems of the SMIRFF.

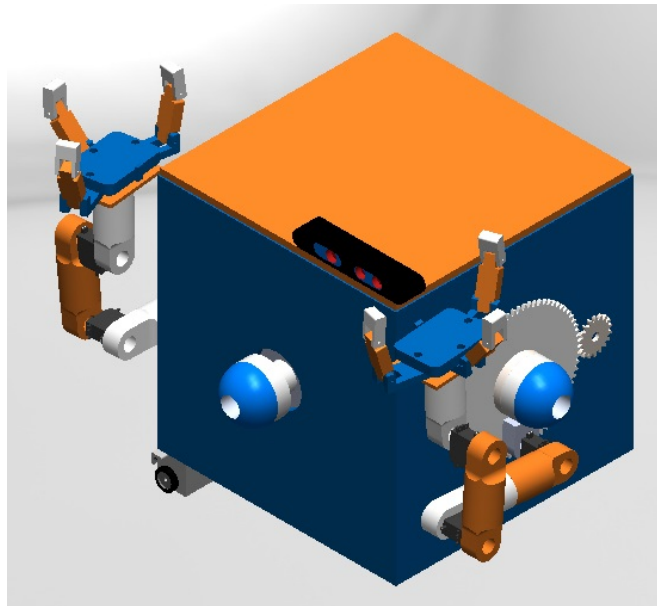


Figure 1.5: SMIRFF

The concept of SMIRFF was based on having a compact, light-weight tele-operated free-flyer. A configuration of 12 cold-gas thrusters was designed to allow control over its flight, positioning and orientation. Use of depth camera, a thermographic camera and and RGB camera was proposed to provide visual aid and have an environmental awareness.

A set of 2 manipulator arms was added to the concept to provide the free-flyer an ability to conduct minimal maintenance and repair works. It was suggested to have the structure and robotic arms 3-D printed. This was in-line with NASA's initiative to have 3-D printing technology in space. The main reason was to reduce the need to resupply the parts. Also, customised end-effectors could be produced using this technology thus enabling the same robot to do multiple operations.

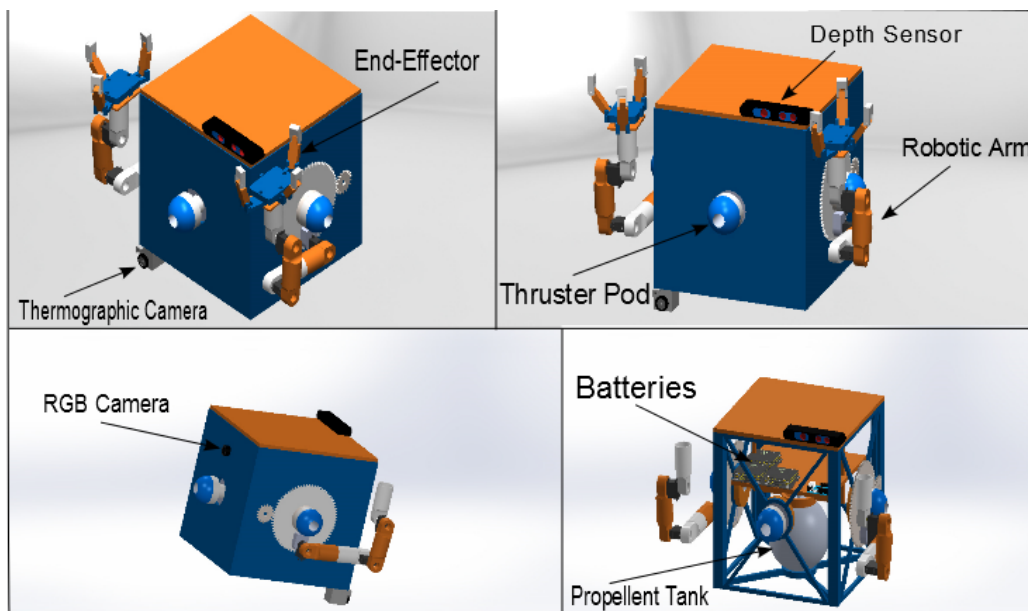


Figure 1.6: SMIRFF Concept

1.3 SMIRFF: Sub-systems

1.3.1 Internal Structure System

The main structure of SMIRFF also known as Base serves the purpose to house the internal components and supporting the external ones. The Base is a cube having each side of 38 cm. A cage-like structure was designed to have a rigid support. The rings on each face serve as a housing to the thrusters and support to the shaft which has bearings to support the robotic arm gear base. The entire Base is covered with a thin aluminium shield which acts as a Faraday cage protecting the internal electronic components.

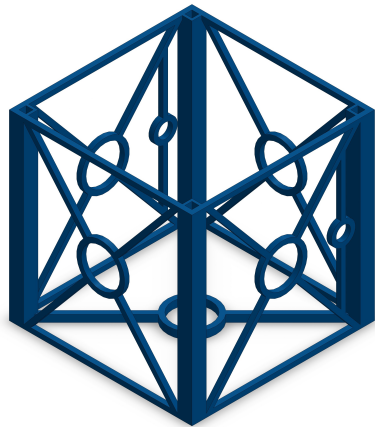


Figure 1.7: Internal Structure

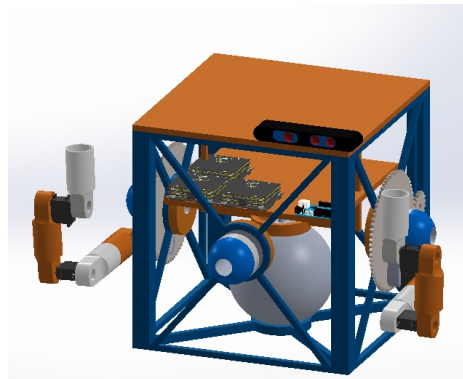


Figure 1.8: Internal Structure Housing the Components

1.3.2 Manipulator Arms

The manipulator system allows the Free-Flyer to interact with its environment. It consists of two 5 degrees of freedom robotic arms. The reason for using two arms is to avoid any reaction force due to asymmetry that would otherwise cause uncontrolled motion if only one arm is used. When maneuvering a heavy payload, one arm can

be used as a grappling arm. Servo motors are used as actuators due to closed loop control on their positions. The manipulator arm when fully stretched has a reach of 50cm from the center of the gear base.

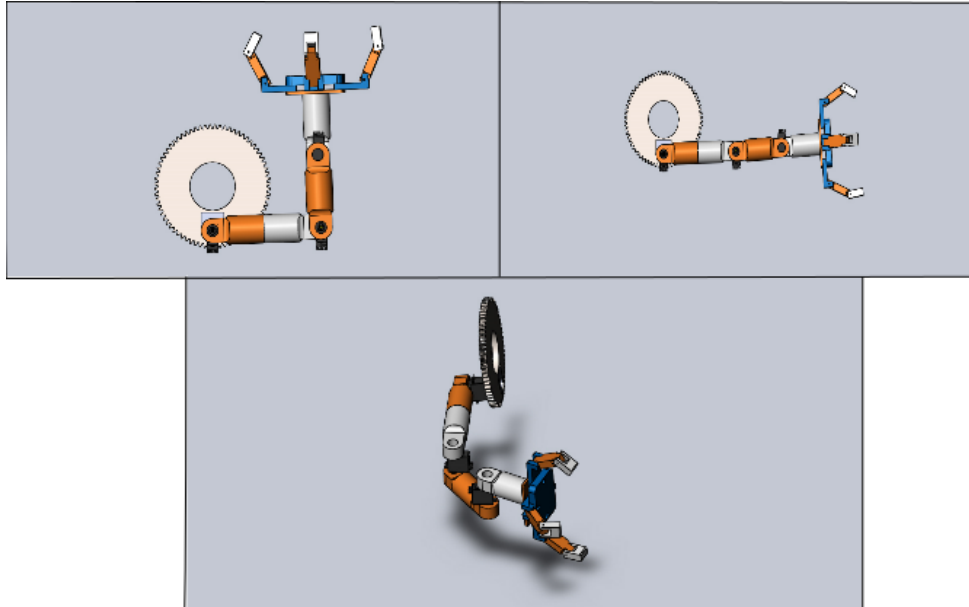


Figure 1.9: Different Robotic Arm Configurations

1.3.3 Propulsion System

To provide controlled locomotion a propulsion system consisting of 12 thrusters was designed. Based on the available propellant types, carbon-dioxide (CO_2) was selected as a propellant for SMIRFF since it provides highest system-specific impulse. System-specific impulse is defined as total impulse delivered to the system divided by the mass of the system.

$$I_{ss} = \frac{I_{total}}{m_{sys}} \quad (1.1)$$

Where, I_{ss} is the system specific impulse,
 I_{total} is the total impulse delivered to the system,
 m_{sys} is the total mass of the system.

Fig 1.10 shows the thruster location and direction in which the individual thruster is fired.

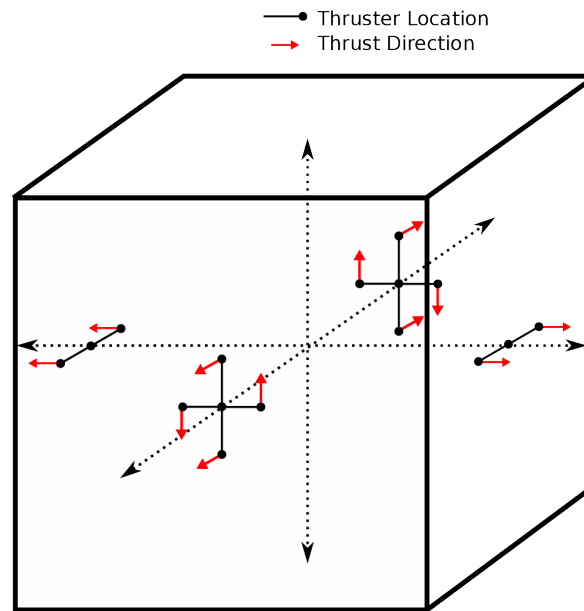


Figure 1.10: Thruster Configuration

1.3.4 Avionics

To have an awareness of the operational environment, it is necessary to equip SMIRFF with an array of sensing equipments. An RGB camera, thermographic camera and depth sensor provide the visual aid and a map of the surroundings. The inertial measurement unit can be used to provide the estimates of the orientation, position and velocity with respect to an inertial coordinate frame. Since SMIRFF is

a part of some space habitat, it is suggested that commands be provided from this habitat. To provide power to the entire system, Lithium-ion batteries are proposed.

CHAPTER 2

KINEMATICS OF SMIRFF

Kinematics is the study of motion of the particles or bodies without considering the forces or torques responsible for this motion. Forward kinematics in the robotics community implies finding the position and orientation of an end-effector of a manipulator in space for given joint values of the manipulator. Likewise, the inverse kinematics problem involves finding the joint values of the manipulator arm when the desired position and orientation of the end-effector are given. To understand the kinematics it is important to study the assignment of the universal and body attached reference frames. This is important since the position and orientation is described with respect to a particular reference frame.

2.1 Reference Frames

A reference frame is a set of co-ordinate axes with respect to which the position or motion or physical laws are defined. The assignment and relation between reference frames is important in field of kinematics and dynamics since a vector quantity in a particular reference frame might be different in the reference frame of our interest. The following assumption that there is a universal frame which is used to reference everything is adopted. All the position, orientation and force vectors are described with respect to this universal frame or with respect to some other frame whose position and orientation with respect to the universal frame is known. The position of a reference frame in 3 dimensional space with respect to a known universal frame is given by a vector of dimension 3×1 . The orientation of a body attached reference frame

is a description of the orientation of the body with respect to a universal reference frame. Unlike position description, the orientation of a body attached reference frame with respect to universal frame of reference in 3 dimensional space is given by a 3×3 matrix.

Throughout this thesis, vectors will be denoted by boldface font and matrices will be expressed in boldface font uppercase letters. The following convention will be used throughout, ${}^A\mathbf{R}_B$ is the rotation matrix which gives the orientation of frame $\{B\}$ with respect to frame $\{A\}$. ${}^A\mathbf{P}_B$ is the position vector which gives the position of origin frame $\{B\}$ in frame $\{A\}$.

2.2 Reference Frame Assignment to SMIRFF

The entire system can be visualised as three parts, the Base and two manipulator arms. Let there be a universal frame of reference, Σ_0 somewhere in space which is used to reference the final position and orientation of the Base and the end-effector of the manipulator arm. Let Σ_1 be the body attached frame of the Base of SMIRFF. The origin of this body attached frame is at the center of mass of the Base. Fig 2.1 shows the universal and body attached frame assignment to the Base.

The frame assignment to each body of the manipulator arm is done using the procedure mentioned in [2]. To describe the location and orientation of each link relative to its neighbours, we define a frame attached to each link. The \hat{Z}_i axis of the frame $\{i\}$, \hat{Z}_i , is coincident with the joint axis. The \hat{X}_i is a common normal to \hat{Z}_i and \hat{Z}_{i+1} and points in a direction from joint i to joint $i+1$. If \hat{Z}_i and \hat{Z}_{i+1} are in the same plane then \hat{X}_i is normal to that plane. \hat{Y}_i is found using the right hand rule to complete the frame $\{i\}$. The Fig 2.2 shows the frame assignment to the manipulator arm on the right face of SMIRFF.

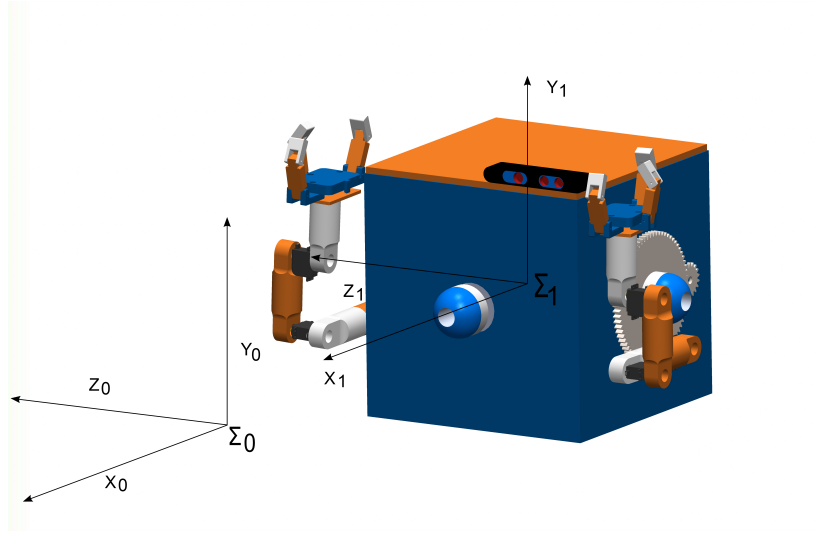


Figure 2.1: Inertial and Base Coordinate Frames of Reference

2.3 Kinematics of Base

The Base has six degrees of freedom (DOF) in the space, three translational $DOFs$ along \hat{X} , \hat{Y} and \hat{Z} axes and three rotational $DOFs$ about \hat{X} , \hat{Y} and \hat{Z} axes. The position of the Base in the space in universal frame of reference is given by vector,

$${}^0\mathbf{P}_1 = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix}^T \quad (2.1)$$

The rotation matrix from inertial frame to body frame is constructed based on the 3-2-1 Euler angle rotation sequence, the matrix elements are shown below

$${}^0_1\mathbf{R} = \begin{bmatrix} c(\theta)c(\psi) & c(\theta)s(\psi) & -s(\theta) \\ -c(\phi)s(\psi) + s(\phi)s(\theta)c(\psi) & c(\phi)c(\psi) + s(\phi)s(\theta)s(\psi) & s(\phi)c(\theta) \\ s(\phi)s(\psi) + c(\phi)s(\theta)c(\psi) & -s(\phi)c(\psi) + c(\phi)s(\theta)s(\psi) & c(\phi)c(\theta) \end{bmatrix} \quad (2.2)$$

Where,

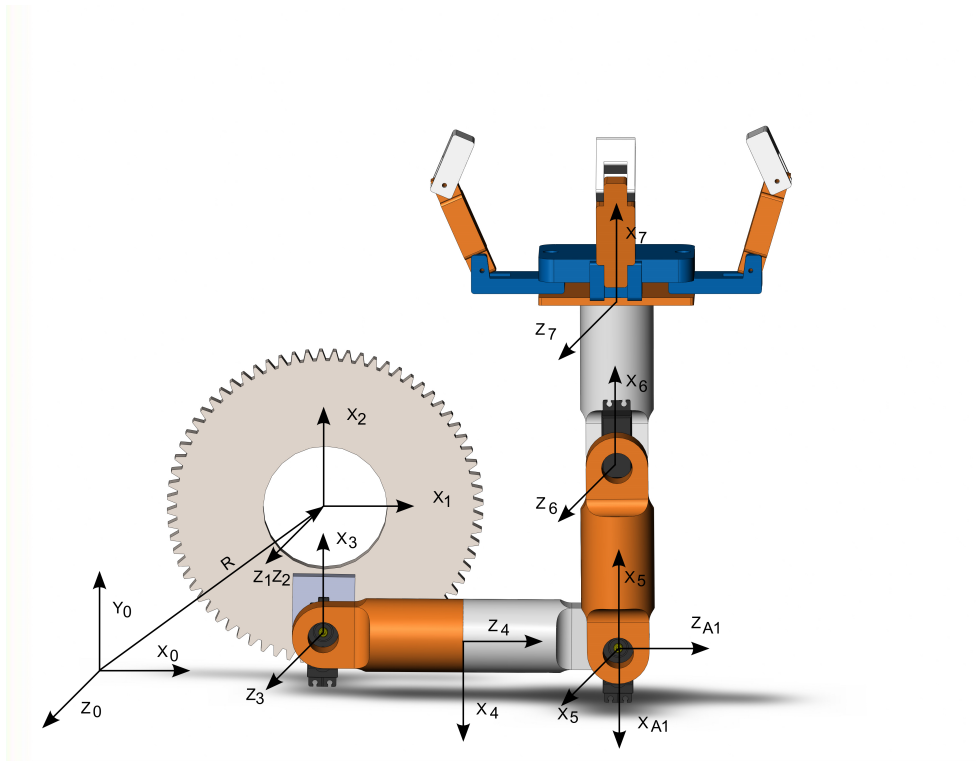


Figure 2.2: Link Co-ordinate Diagram for the Right Manipulator Arm

ϕ = Roll angle,

θ = Pitch angle and

ψ = Yaw Angle

$c(.) = \cos(.)$

$s(.) = \sin(.)$

2.4 Forward Kinematics of Manipulator Arm

Since both the arms are symmetrical to each other, only the procedure to derive the forward kinematics of the right arm is shown here. The procedure for the left arm is similar.

The forward kinematics of the manipulator arm is derived using the modified Denavit-

Hartenberg convention as given in [5]. The relation between any two bodies can be described using four parameters known as Denavit-Hartenberg or D-H parameters.

a_i = Distance from \hat{Z}_i to \hat{Z}_{i+1} measured along \hat{X}_i

α_i = Angle measured from \hat{Z}_i to \hat{Z}_{i+1} about \hat{X}_i

d_i = Distance from \hat{X}_{i-1} to \hat{X}_i measured along \hat{Z}_i

θ_i = Angle measured from \hat{X}_{i-1} to \hat{X}_i about \hat{Z}_i

Fig 2.2 shows the Link Coordinate diagram for the robotic arm. It can be seen that there is an auxiliary frame attached to link 4 to satisfy the rules of the convention.

The D-H parameters for the right manipulator arm are,

Table 2.1: D-H Parameters for the Right Manipulator Arm

Sr. no.	Current frame	Next Frame	a	α	d	θ
1	1	2	0	0°	d_2	θ_2
2	2	3	$-a_3$	0°	d_3	θ_3
3	3	4	0	90°	d_4	θ_4
4	4	Aux ₁	0	0°	d_{a1}	0°
5	Aux ₁	5	0	90°	d_5	θ_5
6	5	6	a_6	0°	$-d_6$	θ_6
7	6	7	a_7	0°	0	0°

The transformation rotation matrix which defines rotation of body [i] relative to body [i - 1] is ${}^i{}_{i-1}\mathbf{R}$, is given by,

$${}^i{}_{i-1}\mathbf{R} = \begin{bmatrix} c(\theta_i) & s(\theta_i) & 0 \\ s(\theta_i)c(\alpha_{i-1}) & c(\theta_i)c(\alpha_{i-1}) & -s(\alpha_{i-1}) \\ s(\theta_i)s(\alpha_{i-1}) & c(\theta_i)s(\alpha_{i-1}) & c(\alpha_{i-1}) \end{bmatrix} \quad (2.3)$$

Where,

$$c(\theta_i) = \cos(\theta_i),$$

$$s(\theta_i) = \sin(\theta_i),$$

$c(\alpha_{i-1}) = \cos(\alpha_{i-1})$ and

$s(\alpha_{i-1}) = \sin(\alpha_{i-1})$

The position of body attached frame of body $[i]$ relative to body attached frame of body $[i - 1]$ is ${}^i{}_{i-1}\mathbf{P}$, is given by

$${}^i{}_{i-1}\mathbf{P}_i = \begin{bmatrix} a_{i-1} \\ -s(\alpha_{i-1})d_i \\ c(\alpha_{i-1})d_i \end{bmatrix} \quad (2.4)$$

The complete transformation matrix which relates body $[i]$ relative to body $[i - 1]$ is given by,

$${}^i{}_{i-1}\mathbf{T} = \begin{bmatrix} {}^i{}_{i-1}\mathbf{R} & {}^i{}_{i-1}\mathbf{P}_i \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix} \quad (2.5)$$

Using the D-H parameters in table 2.1, the individual link-transformation matrices are evaluated. The individual link transformation matrices are then multiplied to find the single transformation that relates Base frame, $\{1\}$ to the end-effector of the right manipulator arm, $\{7\}$.

$${}^1_7\mathbf{T} = {}^1_2\mathbf{T}{}^2_3\mathbf{T}{}^3_4\mathbf{T}{}^4_5\mathbf{T}{}^5_6\mathbf{T}{}^6_7\mathbf{T} \quad (2.6)$$

The final orientation and position of the end-effector of the right manipulator arm in the inertial frame would depend on the orientation and position of the Base.

The transformation matrix ${}^1_7\mathbf{T}$ is a function of the joint variables of the manipulator arm. The Cartesian position and orientation of the end-effector can be computed for a set of joint values from the transformation matrix ${}^1_7\mathbf{T}$.

CHAPTER 3

DERIVATION OF EQUATIONS OF MOTION

3.1 Derivation of Coupled Equations of Motion of the Manipulator Arms with Floating base

The iterative Newton-Euler formulation is used to derive the equations of motion of the manipulator arm with floating Base as described in [6]. The initial conditions required in this formulation are the angular and translational velocity and angular and translational acceleration of the Base. The main difference between dynamics of fixed Base arm and floating Base arm is that the velocities and accelerations of the Base are zero in case of fixed Base condition. Whereas, any joint motion of the manipulator arm produces reaction forces and torques which affects the position and orientation of the floating Base. The effect of change in the state of Base affects the final position and orientation of the end-effector.

The iterative Newton-Euler formulation to derive the equations of motion for the manipulator arm is done in two parts. The first part involves, the outward iteration, from base to end-effector to find the link velocities and accelerations and total force and torque using the initial conditions of the base. The second part involves an inward iteration from end-effector to the base to find the reaction forces and torque at the joints.

The manipulator arm is a chain of multi-bodies which rotate relative to each other. The velocity of link $[i + 1]$ is velocity of link $[i]$ plus velocity component added due to motion of joint $[i + 1]$. Hence, the initial conditions of the Base are required to compute velocities of successive links. ${}^{i+1}\omega_{i+1}$ is the angular velocity of body frame

$[i + 1]$ expressed in the same frame. ${}^{i+1}\mathbf{v}_{i+1}$ is the linear velocity of body frame $[i + 1]$ expressed in the same frame.

The angular velocity and translational velocity of the link $[i]$ is computed as,

$${}^{i+1}\boldsymbol{\omega}_{i+1} = {}^i{}^{i+1}\mathbf{R}^i\boldsymbol{\omega}_i + \dot{\theta}_{i+1} {}^{i+1}\hat{Z}_{i+1} \quad (3.1)$$

$${}^{i+1}\mathbf{v}_{i+1} = {}^i{}^{i+1}\mathbf{R} [{}^i\mathbf{v}_i + {}^i\boldsymbol{\omega}_i \times {}^i\mathbf{P}_{i+1}] \quad (3.2)$$

The angular and translational acceleration of the link $[i]$ is computed as,

$${}^{i+1}\dot{\boldsymbol{\omega}}_{i+1} = {}^i{}^{i+1}\mathbf{R}^i\dot{\boldsymbol{\omega}}_i + \left({}^i{}^{i+1}\mathbf{R}^i\boldsymbol{\omega}_i\right) \times \dot{\theta}_{i+1} {}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1} {}^{i+1}\hat{Z}_{i+1} \quad (3.3)$$

$${}^{i+1}\dot{\mathbf{v}}_{i+1} = {}^i{}^{i+1}\mathbf{R} \left[{}^i\dot{\boldsymbol{\omega}}_i \times {}^i\mathbf{P}_{i+1} + {}^i\boldsymbol{\omega}_i \times \left({}^i\boldsymbol{\omega}_i \times {}^i\mathbf{P}_{i+1}\right) + {}^i\dot{\mathbf{v}}_i\right] \quad (3.4)$$

Where, $(i + 1):2 \rightarrow 7$ for the right arm. $i = 1$ requires the initial conditions of the Base.

The Newton-Euler equations are used to compute the inertial force and moment acting on the center of mass of each link. Before computing these forces and moments, the translational acceleration of center of mass of each link is computed. The translational acceleration of the center of mass of link $[i + 1]$ is computed as follows,

$${}^{i+1}\dot{\mathbf{v}}_{\mathbf{C}_{i+1}} = {}^{i+1}\dot{\boldsymbol{\omega}}_{i+1} \times {}^{i+1}\mathbf{P}_{\mathbf{C}_{i+1}} + {}^{i+1}\boldsymbol{\omega}_{i+1} \times \left({}^{i+1}\boldsymbol{\omega}_{i+1} \times {}^{i+1}\mathbf{P}_{\mathbf{C}_{i+1}} \right) + {}^{i+1}\dot{\mathbf{v}}_{i+1} \quad (3.5)$$

Where, ${}^{i+1}\mathbf{P}_{\mathbf{C}_{i+1}}$ is the location of center of mass of link $[i + 1]$ expressed in body frame $\{i + 1\}$.

The inertial force and moment acting on link $[i + 1]$ which causes this motion is computed as,

$${}^{i+1}\mathbf{F}_{i+1} = m_{i+1} {}^{i+1}\dot{\mathbf{v}}_{\mathbf{C}_{i+1}} \quad (3.6)$$

$${}^{i+1}\mathbf{N}_{i+1} = {}^{\mathbf{C}_{i+1}}\mathbf{I}_{i+1} {}^{i+1}\dot{\boldsymbol{\omega}}_{i+1} + {}^{i+1}\boldsymbol{\omega}_{i+1} \times {}^{\mathbf{C}_{i+1}}\mathbf{I}_{i+1} {}^{i+1}\boldsymbol{\omega}_{i+1} \quad (3.7)$$

$\{C_i\}$ has its origin at center of mass of the link and orientation as the frame $\{i\}$. ${}^{\mathbf{C}_{i+1}}\mathbf{I}_{i+1}$ is the moment of inertia of link $[i + 1]$ about its center of mass.

Once the inertial forces and moments are computed, the next step involves finding the joint forces and torques responsible for the motion of each link using inward iterations. In a multi-body system each body or a link experiences reaction forces and torques from its neighbours apart from the inertial forces and moments.

Let,

\mathbf{f}_i = Force exerted on link $[i]$ by link $[i - 1]$,

\mathbf{n}_i = Torque exerted on link $[i]$ by link $[i - 1]$,

Now, the force balance equation would give the relationship between these forces exerted by the neighbouring links and the inertial force as,

$${}^i\mathbf{f}_i = {}^i_{i+1}\mathbf{R} {}^{i+1}\mathbf{f}_{i+1} + {}^i\mathbf{F}_i \quad (3.8)$$

The torque balance equation gives the equation which relates the torque exerted by the neighbouring links on the body and the inertial moment.

$${}^i\mathbf{n}_i = {}^i\mathbf{N}_i + {}^i_{i+1}\mathbf{R}^{i+1}\mathbf{n}_{i+1} + {}^i\mathbf{P}_{C_i} \times {}^i\mathbf{F}_i + {}^i\mathbf{P}_{i+1} \times {}^i_{i+1}\mathbf{R}^{i+1}\mathbf{f}_{i+1} \quad (3.9)$$

The required joint torque is found by taking the \hat{Z} component of the torque applied by a link on its neighbour.

$$\tau_i = {}^i n_i \hat{Z}_i \quad (3.10)$$

The force and torque acting on the end-effector is zero if the manipulator is free to move in space. If the manipulator is interacting with the surrounding, then there is a non-zero force and torque acting on the end-effector.

The inward iteration from the end-effector to the first link can be continued till base to find the total force and torque acting on the Base.

3.2 Vehicle Dynamics

The total force acting on the Base with n-manipulators is given by

$${}^1\mathbf{f}_1 = {}^1\mathbf{F}_1 + \sum_{i=1}^n {}^1_{2_i}\mathbf{R}^{2_i}\mathbf{f}_{2_i} \quad (3.11)$$

Where,

${}^1_{2_i}\mathbf{R}$ = The orientation matrix of the first link of the $[i]^{th}$ manipulator with respect to the Base.

${}^2_i\mathbf{f}_{2_i}$ = The total force acting on the first link of the $[i]^{th}$ manipulator

${}^1\mathbf{f}_1$ is the external force acting on the Base, which is provided by the thrusters.

$\sum_{i=1}^n {}^1_{2_i}\mathbf{R}^{2_i}\mathbf{f}_{2_i}$, is the total reaction force acting on the base due to motion of the first

link of all the n -manipulators.

The reaction force and torque exerted on the base is negative of ${}^2\mathbf{f}_{2_i}$ and ${}^2\mathbf{n}_{2_i}$.

The equivalent force acting on the center of mass of the Base results in the translational dynamics. The equivalent force includes the force applied by the thrusters and the reaction force due to motion of the arm and the force acting on the end effector of the arm.

The inertial force acting on the Base in inertial frame is given by,

$${}^0\mathbf{F}_1 = M_{Base} {}^0\dot{\mathbf{v}}_1 \quad (3.12)$$

M_{Base} is the Mass of the Base which is a state, since the mass of the fuel is reduced when the thrusters are fired.

${}^0\dot{\mathbf{v}}_1$ is the acceleration of the base in inertial frame.

The relation between the inertial force in inertial frame and body frame is given by,

$${}^0\mathbf{F}_1 = {}^0\mathbf{R}^1\mathbf{F}_1 \quad (3.13)$$

The relation between the inertial velocity and the velocity expressed in the body frame is given by,

$${}^0\mathbf{v}_1 = {}^0\mathbf{R}^1\mathbf{v}_1 \quad (3.14)$$

Where,

$${}^1\mathbf{v}_1 = \begin{bmatrix} u & v & w \end{bmatrix}^T \quad (3.15)$$

denotes the velocity vector in body frame.

$${}^0\mathbf{v}_1 = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \quad (3.16)$$

is the velocity vector in inertial frame.

Now,

$${}^0\dot{\mathbf{R}}_1^0\mathbf{R}^T = \mathbf{S}({}^1\boldsymbol{\omega}_1) \quad (3.17)$$

Where, $\mathbf{S}({}^1\boldsymbol{\omega}_1)$ is the skew symmetric matrix which is given by,

$$\mathbf{S}({}^1\boldsymbol{\omega}_1) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \quad (3.18)$$

Where,

$${}^1\boldsymbol{\omega}_1 = \begin{bmatrix} p & q & r \end{bmatrix}^T \quad (3.19)$$

is the angular velocity in the body frame.

From equation 3.12-3.19 the inertial force is given by,

$${}^1\mathbf{F}_1 = M_{base} \left(\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} - \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \right) \quad (3.20)$$

So, the translational Base dynamics can be expressed as,

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \frac{1}{M_{Base}} \begin{bmatrix} F_{bx} \\ F_{by} \\ F_{bz} \end{bmatrix} + \frac{1}{M_{Base}} \sum_{i=1}^n {}^1\mathbf{R}^{2_i} \mathbf{f}_{2_i} \quad (3.21)$$

Where,

$$\begin{bmatrix} F_{bx} & F_{by} & F_{bz} \end{bmatrix}^T \text{ is the thrust force.}$$

Integrating these states would yield the linear velocities in the body frame which can be used to compute the position of the Base in the inertial frame.

The total moment acting on the base with n-manipulators is given by

$${}^1\mathbf{n}_1 = {}^1\mathbf{N}_1 + {}^1\mathbf{P}_{C_1} \times {}^1\mathbf{F}_1 + \sum_{i=1}^n {}^1\mathbf{R}^{2_i} \mathbf{n}_{2_i} + {}^1\mathbf{P}_{2_i} \times {}^1\mathbf{R}^{2_i} \mathbf{f}_{2_i} \quad (3.22)$$

Where,

${}^{2_i}\mathbf{n}_{2_i}$ = The total moment acting on the first link of the $[i]^{th}$ manipulator

${}^1\mathbf{n}_1$ is the external torque acting on the Base, which is provided by the thrusters.

$\sum_{i=1}^n {}^1\mathbf{R}^{2_i} \mathbf{n}_{2_i} + {}^1\mathbf{P}_{2_i} \times {}^1\mathbf{R}^{2_i} \mathbf{f}_{2_i}$, is the total reaction torque due to motion of all n -manipulators.

Now, The body frame of the Base is attached to its center of mass. So,

$${}^1\mathbf{P}_{C_1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad (3.23)$$

Hence, equation 3.23 for the rotational dynamics of the Base is reduced to,

$${}^1\mathbf{n}_1 = {}^1\mathbf{N}_1 + \sum_{i=1}^n {}^1\mathbf{R}^{2_i} \mathbf{n}_{2_i} + {}^1\mathbf{P}_{2_i} \times {}^1\mathbf{R}^{2_i} \mathbf{f}_{2_i} \quad (3.24)$$

The inertial moment acting on the Base in inertial frame is given by,

$${}^0\mathbf{N}_1 = \frac{d}{dt} ({}^0\mathbf{I}_{\text{Base}} {}^0\boldsymbol{\omega}_1) \quad (3.25)$$

Where,

${}^0\mathbf{I}_{\text{Base}}$ is the Moment of Inertia of the Base in inertial frame.

${}^0\boldsymbol{\omega}_1$ is the angular velocity of the Base in Inertial frame.

Now,

$${}^0\mathbf{N}_1 = {}^0\mathbf{R}^1\mathbf{N}_1 \quad (3.26)$$

The moment of inertia of the Base in the body frame is given by,

$${}^1\mathbf{I}_{\text{Base}} = ({}^0\mathbf{R}^T) {}^0\mathbf{I}_{\text{Base}} ({}^0\mathbf{R}) \quad (3.27)$$

The relation between the angular velocity of the Base in Inertial frame and the body frame is given by,

$${}^0\boldsymbol{\omega}_1 = {}^0\mathbf{R}^1\boldsymbol{\omega}_1 \quad (3.28)$$

From 3.18 and 3.25-3.28, the inertial moment is given by,

$${}^1\mathbf{N}_1 = {}^0\mathbf{I}_{\text{Base}} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} {}^0\mathbf{I}_{\text{Base}} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3.29)$$

So, the rotational dynamics due to thrusters and the reaction moment due to the motion of the arm and the moment acting on the end effector of the arm is expressed as,

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = {}^0\mathbf{I}_{\text{Base}}^{-1} \begin{bmatrix} F_{broll} \\ F_{bpitch} \\ F_{bjaw} \end{bmatrix} - {}^0\mathbf{I}_{\text{Base}}^{-1} \left(\begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} {}^0\mathbf{I}_{\text{Base}} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right) + {}^0\mathbf{I}_{\text{Base}}^{-1} \left(\sum_{i=1}^n {}^1_2\mathbf{R}^{2i}\mathbf{n}_{2i} + {}^1\mathbf{P}_{2i} \times {}^1_2\mathbf{R}^{2i}\mathbf{f}_{2i} \right) \quad (3.30)$$

Where,

$\begin{bmatrix} F_{broil} & F_{bpitch} & F_{byaw} \end{bmatrix}^T$ is the external torque provided by the thrusters.

The Rotational kinematics gives the relation between the body axis rate and the orientation of the satellite with respect to the inertial frame.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) \sec(\theta) & \cos(\phi) \sec(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3.31)$$

Integrating the above equation gives the orientation of the Base with respect to the inertial frame.

3.3 Full State Equation of Motion of SMIRFF

The states of the system consists of orientation, angular and linear velocity of the Base in the body frame. It also consists of joint angular displacement and change in the joint angular displacement. The dynamic model of the entire system is represented as,

$$\begin{bmatrix} \mathbf{M}_b & \mathbf{M}_{br} & \mathbf{M}_{bl} \\ \mathbf{M}_{rb} & \mathbf{M}_r & \mathbf{M}_{rl} \\ \mathbf{M}_{lb} & \mathbf{M}_{lr} & \mathbf{M}_l \end{bmatrix} \begin{bmatrix} {}^1\dot{\mathbf{v}}_1 \\ {}^1\boldsymbol{\omega}_1 \\ \ddot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{C}_b & \mathbf{C}_{br} & \mathbf{C}_{bl} \\ \mathbf{C}_{rb} & \mathbf{C}_r & \mathbf{C}_{rl} \\ \mathbf{C}_{lb} & \mathbf{C}_{lr} & \mathbf{C}_l \end{bmatrix} \begin{bmatrix} {}^1\mathbf{v}_1 \\ {}^1\boldsymbol{\omega}_1 \\ \dot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{J}_b^T \\ \mathbf{J}_r^T \\ \mathbf{J}_l^T \end{bmatrix} \mathbf{F}_{\text{ext}} = \begin{bmatrix} {}^1\mathbf{f}_1 \\ {}^1\mathbf{n}_1 \\ \boldsymbol{\tau}_i \end{bmatrix} \quad (3.32)$$

Where, \mathbf{M}_b is a 6×6 inertia matrix of the base.

\mathbf{M}_r and \mathbf{M}_l are 5×5 inertia matrix of the manipulator arms.

\mathbf{M}_{br} and \mathbf{M}_{bl} are 6×5 dynamic coupling matrix between the base and the manipulator arms.

\mathbf{M}_{r1} is a 5×5 dynamic coupling matrix between the manipulator arms.

\mathbf{F}_{ext} is the external force acting on the end-effector when it is interacting with its surrounding.

\mathbf{J}_b^T , \mathbf{J}_r^T and \mathbf{J}_1^T are the Jacobian matrices for the base and the manipulator arms.

$$\begin{bmatrix} \mathbf{C}_b & \mathbf{C}_{br} & \mathbf{C}_{bl} \\ \mathbf{C}_{rb} & \mathbf{C}_r & \mathbf{C}_{rl} \\ \mathbf{C}_{lb} & \mathbf{C}_{lr} & \mathbf{C}_l \end{bmatrix} \begin{bmatrix} {}^1\mathbf{v}_1 \\ {}^1\boldsymbol{\omega}_1 \\ \dot{\boldsymbol{\theta}}_i \end{bmatrix}$$
 contains the non-linear Coriolis and centrifugal force terms.

${}^1\mathbf{f}_1$ and ${}^1\mathbf{n}_1$ are the thrust force and moment acting on the Base.

CHAPTER 4

SIMULATION

The following assumptions were made while formulating the mathematical model of SMIRFF:

- The center of mass of the Base is at the geometric center of the Internal cage structure.
- The center of mass of each link lies at the center of the link along its length.
- The actuators at the joints of the manipulator arm are ideal.

The system is simulated using modelling and simulation software MATLAB. The variable step solver ode45 is used to simulate the system.

4.1 Simulation model

The dynamic model of SMIRFF derived in section 3.3 is given by,

$$\begin{bmatrix} \mathbf{M}_b & \mathbf{M}_{br} & \mathbf{M}_{bl} \\ \mathbf{M}_{rb} & \mathbf{M}_r & \mathbf{M}_{rl} \\ \mathbf{M}_{lb} & \mathbf{M}_{lr} & \mathbf{M}_l \end{bmatrix} \begin{bmatrix} {}^1\dot{\mathbf{v}}_1 \\ {}^1\dot{\boldsymbol{\omega}}_1 \\ \ddot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{C}_b & \mathbf{C}_{br} & \mathbf{C}_{bl} \\ \mathbf{C}_{rb} & \mathbf{C}_r & \mathbf{C}_{rl} \\ \mathbf{C}_{lb} & \mathbf{C}_{lr} & \mathbf{C}_l \end{bmatrix} \begin{bmatrix} {}^1\mathbf{v}_1 \\ {}^1\boldsymbol{\omega}_1 \\ \dot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{J}_b^T \\ \mathbf{J}_r^T \\ \mathbf{J}_l^T \end{bmatrix} \mathbf{F}_{\text{ext}} = \begin{bmatrix} {}^1\mathbf{f}_1 \\ {}^1\mathbf{n}_1 \\ \boldsymbol{\tau}_i \end{bmatrix} \quad (4.1)$$

The system is simulated for a simple case involving unfolding of both the arms by commanding a rotation of joint 4 of both the manipulator arms by -90° . Since the arm is not interacting with its surrounding, the external force acting on the end-effector is zero. The attitude and altitude control due to firing of the thrusters is off. Hence the base is free floating in space. All the states have zero initial conditions.

Integrating these states would yield joint rates, and body rates. The body axis rates p, q and r are the estimates of the components of the angular velocity of the base in the body frame. Using these body axis rates, the orientation of the base in the inertial frame can be estimated. The body translational rates u, v and w are the estimates of the components of the translational velocity of the base in the body frame. Integrating the estimates will yield the inertial position of the base which is used to compute the final position and orientation of the end-effector of the manipulator arm. The joint rates are integrated to compute the joint angles, θ_i .

Due to zero thrust force and zero external force on the end-effector of both the arms, the dynamic model is given by,

$$\begin{bmatrix} \mathbf{M}_b & \mathbf{M}_{br} & \mathbf{M}_{bl} \\ \mathbf{M}_{rb} & \mathbf{M}_r & \mathbf{M}_{rl} \\ \mathbf{M}_{lb} & \mathbf{M}_{lr} & \mathbf{M}_l \end{bmatrix} \begin{bmatrix} {}^1\dot{\mathbf{v}}_1 \\ {}^1\dot{\boldsymbol{\omega}}_1 \\ \ddot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{C}_b & \mathbf{C}_{br} & \mathbf{C}_{bl} \\ \mathbf{C}_{rb} & \mathbf{C}_r & \mathbf{C}_{rl} \\ \mathbf{C}_{lb} & \mathbf{C}_{lr} & \mathbf{C}_l \end{bmatrix} \begin{bmatrix} {}^1\mathbf{v}_1 \\ {}^1\boldsymbol{\omega}_1 \\ \dot{\boldsymbol{\theta}}_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{\tau}_i \end{bmatrix} \quad (4.2)$$

Since both the arms are symmetric, the mass and inertia properties of both the manipulator arms are same. The Mass and inertia properties of both the manipulator arms and Base are summarized in the table 4.1.

Table 4.1: Mass and inertia properties of the manipulator arm

Sr. no.	Component	mass (Kg)	I_{xx} (Kgm^2)	I_{yy} (Kgm^2)	I_{zz} (Kgm^2)
1	Gear	0.3	0.00174692	0.000877221	0.000877221
2	Link 1	0.13	0.000114886	3.73894e-05	0.000118637
3	Link 2	0.18	0.000259367	4.44786e-05	0.000263987
4	Link 3	0.19	0.000291746	4.86811e-05	0.000299249
5	Link 4	0.13	0.00011622	3.70749e-05	0.000119971
6	Base	4.23	0.1551	0.1689	0.1549

The open loop dynamic model of the manipulator arms is given by,

$$\mathbf{M}_{\text{manip}}\ddot{\boldsymbol{\theta}}_i + \mathbf{C}_{\text{manip}}(\phi, \theta, \psi, u, v, w, p, q, r, \boldsymbol{\theta}_i, \dot{\boldsymbol{\theta}}_i) = \boldsymbol{\tau}_i \quad (4.3)$$

Where, $\mathbf{M}_{\text{manip}}$ is the mass matrix for the manipulator arms and $\boldsymbol{\tau}_i$ is the control torque for the manipulator joints.

The model-based portion of the control for the system is proposed as,

$$\boldsymbol{\tau}_i = \boldsymbol{\alpha}\boldsymbol{\tau}'_i + \boldsymbol{\beta} \quad (4.4)$$

$\boldsymbol{\tau}'_i$ is the new input to the system which uses the feedback of the controlled states. Now, choosing $\boldsymbol{\alpha} = \mathbf{M}_{\text{manip}}$ and $\boldsymbol{\beta} = \mathbf{C}_{\text{manip}}(\phi, \theta, \psi, u, v, w, p, q, r, \boldsymbol{\theta}_i, \dot{\boldsymbol{\theta}}_i)$, the feedback can be made independent of the mass properties of the system and will consist of only the states of the system.

The servo portion of the control law which makes use of the feedback is,

$$\boldsymbol{\tau}'_i = \ddot{\boldsymbol{\theta}}_{id} + \mathbf{k}_p\mathbf{e} + \mathbf{k}_v\dot{\mathbf{e}} \quad (4.5)$$

Where, $\boldsymbol{\theta}_{id}$ are the desired manipulator joint angles,

$\mathbf{e} = \boldsymbol{\theta}_{id} - \boldsymbol{\theta}_i$ is the joint angle error,

$\mathbf{k}_p = \text{diag}[k_{p2}, k_{p3}, k_{p4}, k_{p5}, k_{p6}, k_{p8}, k_{p9}, k_{p10}, k_{p11}, k_{p12}]$ is the proportional gain matrix,

$\mathbf{k}_v = \text{diag}[k_{v2}, k_{v3}, k_{v4}, k_{v5}, k_{v6}, k_{v8}, k_{v9}, k_{v10}, k_{v11}, k_{v12}]$ is the differential gain matrix,

Hence the closed loop system dynamics is given by,

$$\ddot{\boldsymbol{\theta}}_i = \ddot{\boldsymbol{\theta}}_{id} + \mathbf{k}_p\mathbf{e} + \mathbf{k}_v\dot{\mathbf{e}} \quad (4.6)$$

4.2 Simulation Results

The desired trajectory for the joint 4 of both the manipulator arms is commanded to rotate by 90° following a quintic polynomial. From fig 4.1 - fig. 4.6 , we can see that the motion of the manipulator arm is purely in the X-Y plane in the

inertial frame. The motion of the arms should produce reaction forces and torques which should disturb the position of the base in the \hat{X} and \hat{Y} direction in the inertial plane. The orientation of the Base should change only about the \hat{Z} axis of the inertial frame.

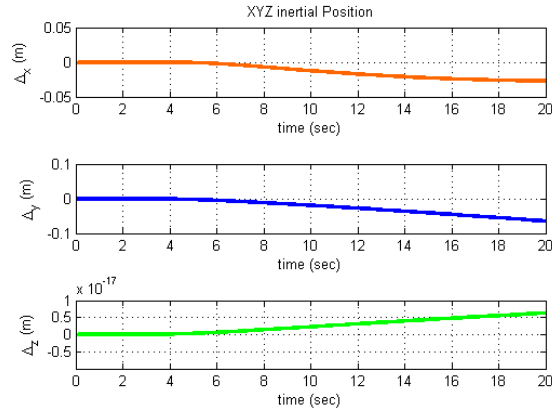


Figure 4.1: Inertial Position components of the Base

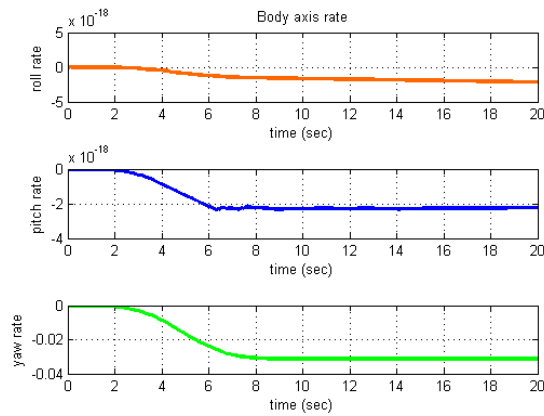


Figure 4.2: Angular Velocity components of the Base in the Body Frame

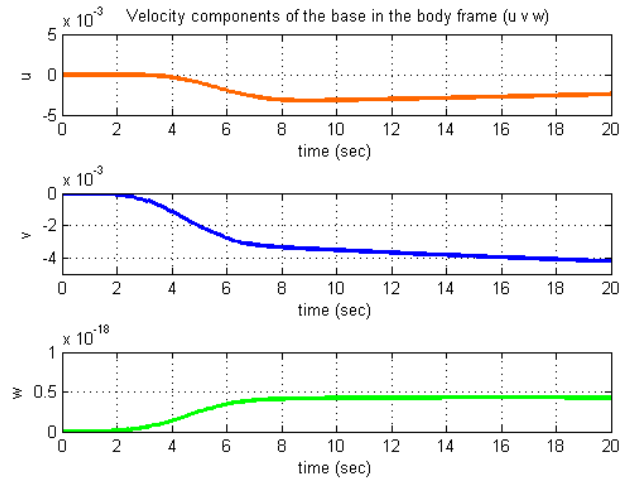


Figure 4.3: Linear Velocity components of the Base in Body Frame

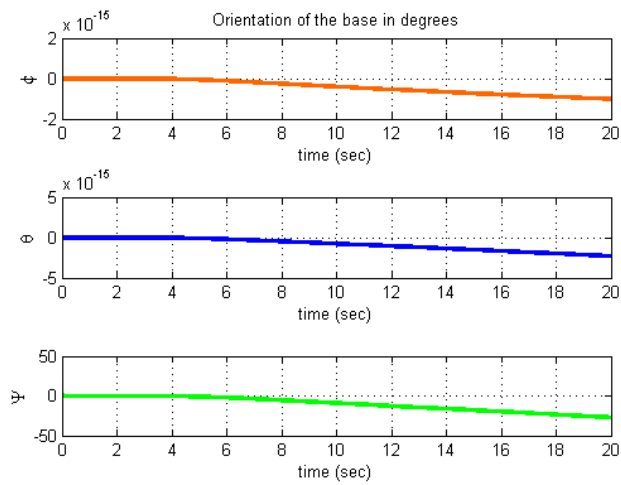


Figure 4.4: Orientation angle of the Base in the Inertial Frame

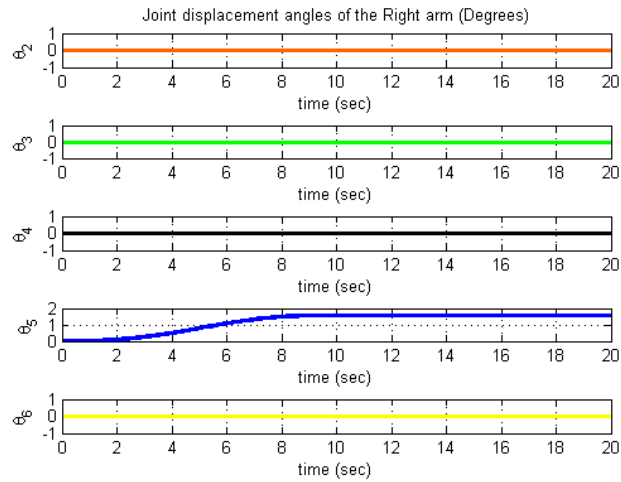


Figure 4.5: Joint Displacement angles of the Right Arm

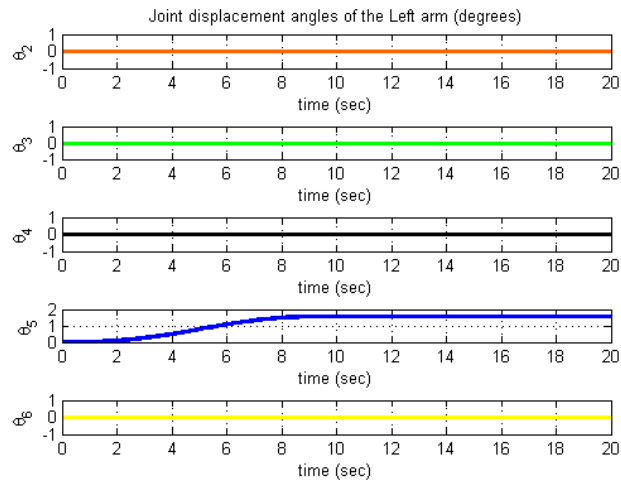


Figure 4.6: Joint Displacement angles of the Left Arm

CHAPTER 5

MODELING IN SimMechanics

To understand the operation of a multi-body system, it is necessary to have a precise kinematic and dynamic model. One is required to have an accurate knowledge of mass and inertia properties while constructing the model. It is easier and less expensive to modify and improve the system when it is in the virtual stage. Thus modelling and simulation play a crucial role in development of any dynamic system. SimMechanics (MATLAB/SIMULINK) is one such tool which offers to simulate interdisciplinary dynamic systems.

5.1 Advantages of using SimMechanics

The dynamic model derived in previous section was highly non-linear. This makes implementation of control laws difficult. A multi-body simulation environment, SimMechanics, was used to model SMIRFF. The main advantage of using SimMechanics is that, the designer is no longer required to derive the equations of motion for a complex multi-body system like SMIRFF. The system can be modelled using blocks representing bodies, joints, force elements and constraints. The mass and inertia properties of any body can be specified in the block representing that particular body. SimMechanics then solves for the equations of motion for the complete system. SimMechanics also contains a variety of sensors to record data, body actuators and joint actuators to facilitate relative motion between two bodies in the system. This simulation environment provides a host of other advantages like,

- Models from CAD software like SolidWorks can be imported in SimMechanics along with the mass, inertia and other assembly properties.
- Electrical, pneumatic, hydraulic and other components can be added using Simscape.
- It provides visualisation and animation possibilities of multi-body dynamics.

5.2 Modelling of SMIRFF in SimMechanics

The CAD model of SMIRFF modelled in SolidWorks was imported to SimMechanics using the SimMechanics utility link. The utility link helps us to save the CAD file in .xml form which can be imported in SimMechanics. The .stl files provide the geometry of the bodies in animation.

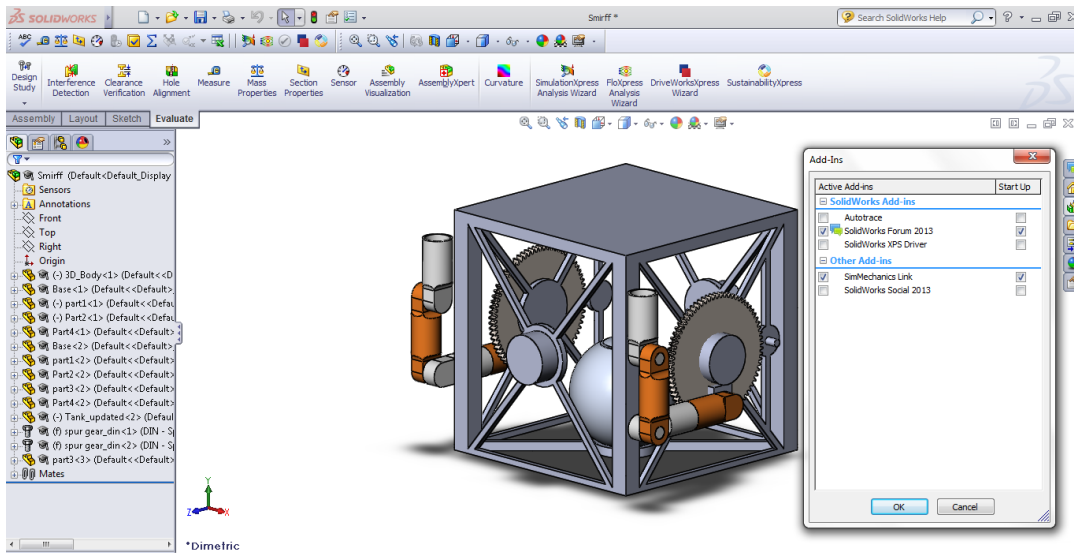


Figure 5.1: Importing the SolidWorks model to SimMechanics

Fig 5.2 shows the block diagram block diagram containing the sub-assemblies of both the manipulator arms and the kinematic relation of the Base with respect to the inertial or ground reference frame. Fig 5.3 depicts the block diagram for the right manipulator arm, with initial joint conditions and desired trajectory blocks. The root or Ground block represents the inertial frame of reference. Since the Base has 6 DOFs, a block representing 6 DOFs is attached as a connection between the Base and the Ground. Fig 5.4 shows the animation window in the SimMechanics environment. The Body Sensors measure various parameters like the position, orientation, velocity and the acceleration of individual bodies.

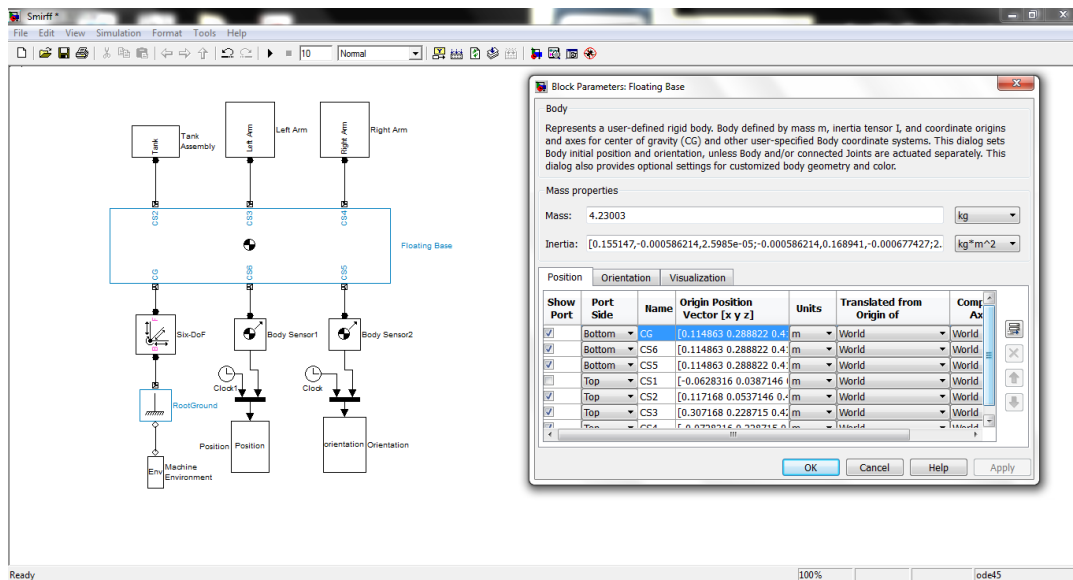


Figure 5.2: Block diagram in SimMechanics Environment

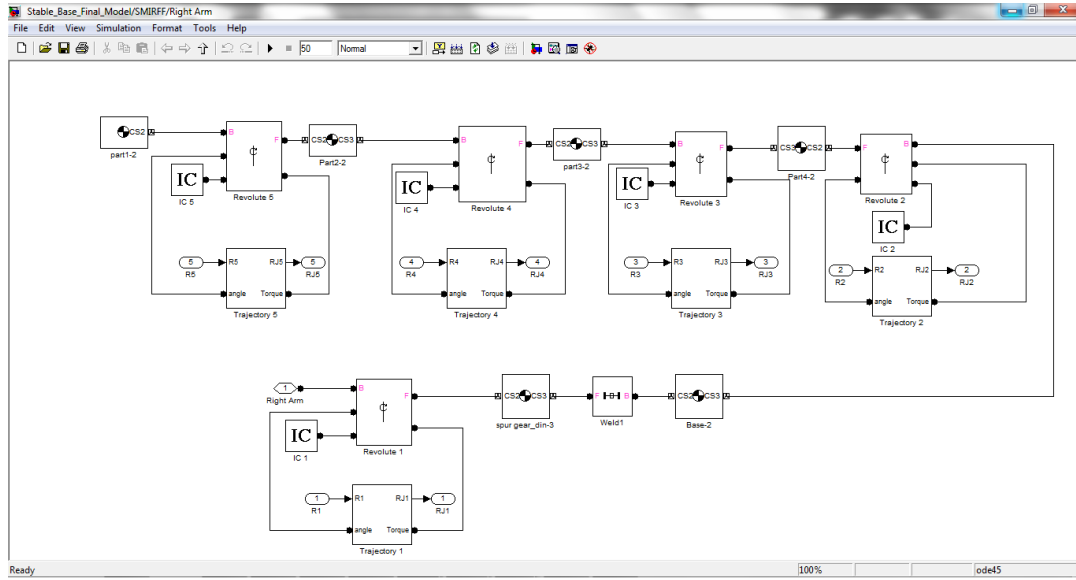


Figure 5.3: Sub-assembly of the right manipulator arm in SimMechanics

5.3 Simulation in SimMechanics

The simulation shown in section 4.1 is repeated in the SimMechanics environment. The joint 4 of the manipulator arm is rotated by 90° in ‘Y-Z’ inertial plane. The reaction forces are generated in \hat{Y} and \hat{Z} directions in the inertial frame. Hence the orientation of the Base changes only about the \hat{X} axis of the inertial frame. Fig 5.5 and 5.6 shows the change in the inertial position and orientation of the Base. Fig 5.7 and 5.8 show the joint displacements of the manipulator arm. The feedback for the rotation of the joint was recorded using the joint sensor block. A PID controller block was used to control the torque on the manipulator joints.

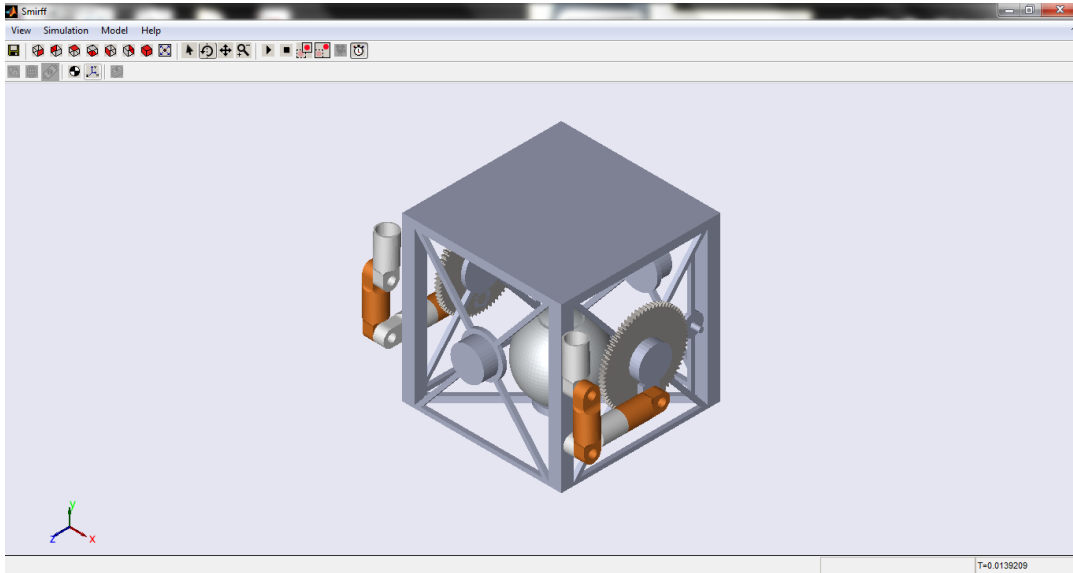


Figure 5.4: Animation in SimMechanics

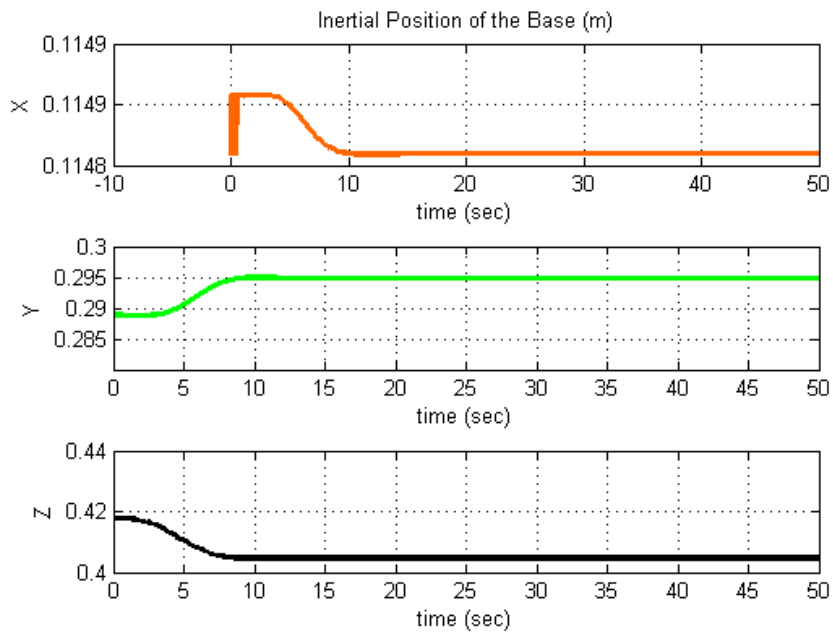


Figure 5.5: Inertial Position of the Base

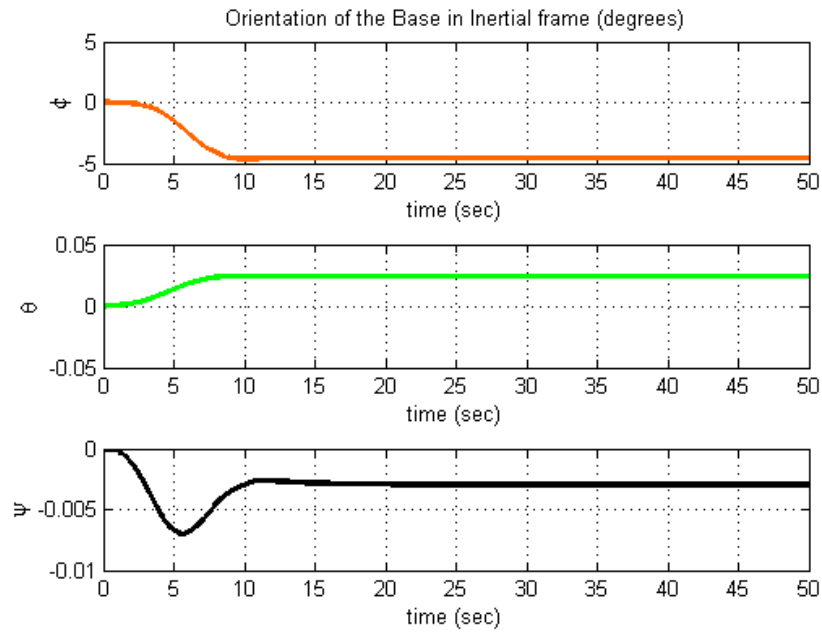


Figure 5.6: Orientation of the Base in the Inertial Frame

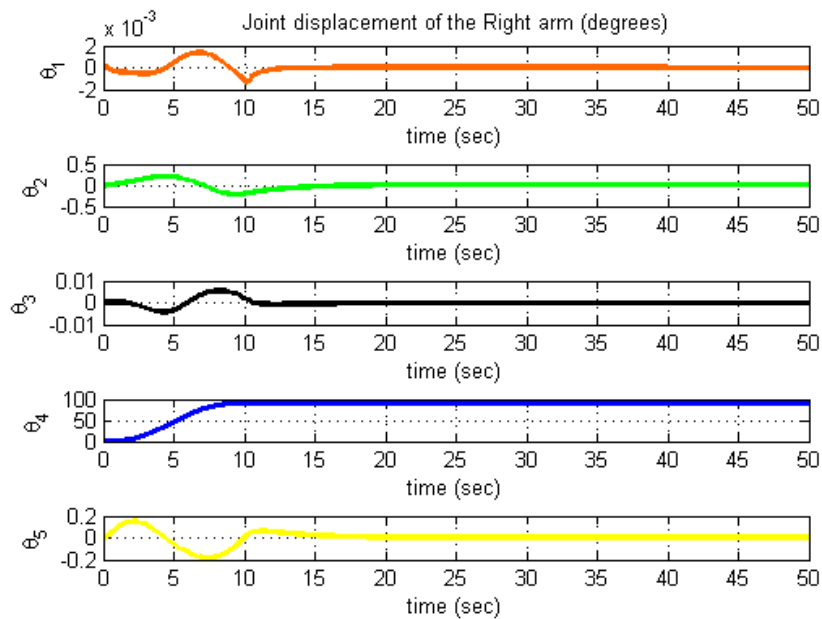


Figure 5.7: Joint Displacement of the Right Arm

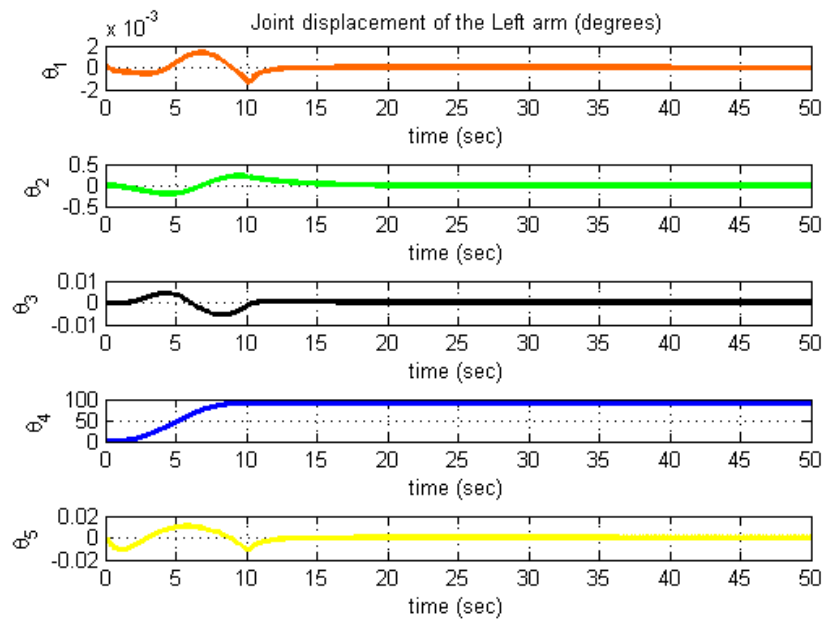


Figure 5.8: Joint Displacement of the Left Arm

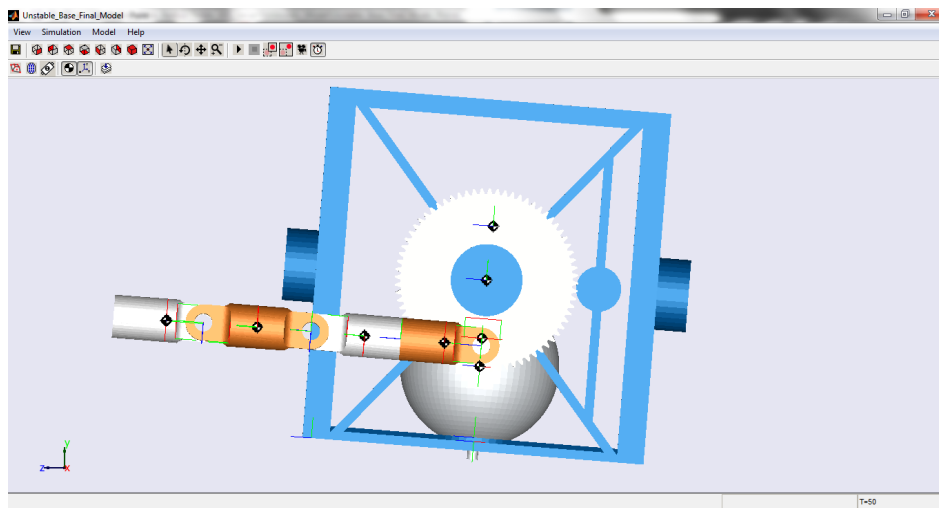


Figure 5.9: Animation of the motion of the Manipulator Arm in SimMechanics

CHAPTER 6

COMPARISON BETWEEN THE DERIVED DYNAMIC MODEL AND SimMechanics MODEL

The simulation results from the SimMechanics environment and the dynamic model derived from the Iterative Newton-Euler method are compared in this section. The inertial frame assignment in both the methods was different. Hence the results from the derived dynamic model are expressed in an inertial frame which is oriented with the inertial frame of the SimMechanics environment. The joint 4 of both the manipulator arms were commanded to rotate by 90° along a trajectory defined by a quintic polynomial.

The SimMechanics model uses the mass and inertia properties of the bodies obtained from the CAD model. These mass and inertia properties were assumed during construction of the dynamic model using Iterative Newton-Euler method. The results from both the simulation shown in Fig 6.1, 6.2, 6.3 and 6.4 helps us to verify the derived dynamic model. The small deviation in the results exists because of the assumptions made in the derived model.

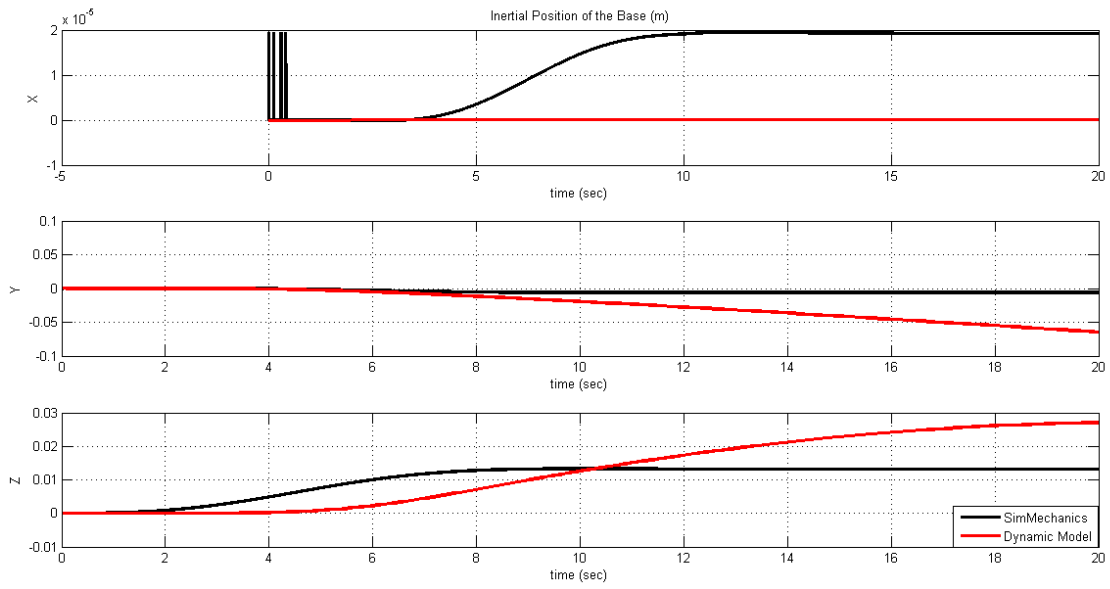


Figure 6.1: Inertial Displacement of the Base

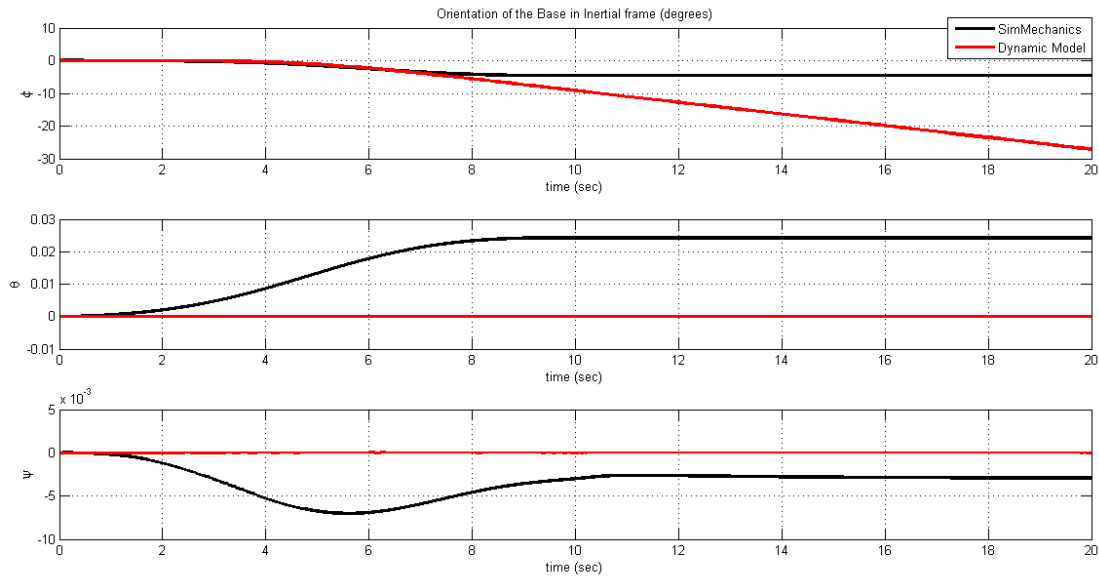


Figure 6.2: Orientation of the Base in the Inertial Frame

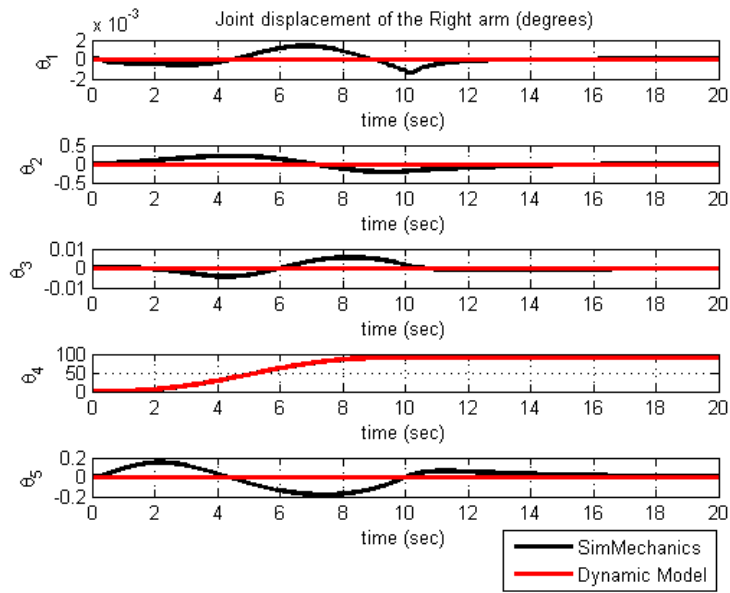


Figure 6.3: Joint Displacement of the Right Arm

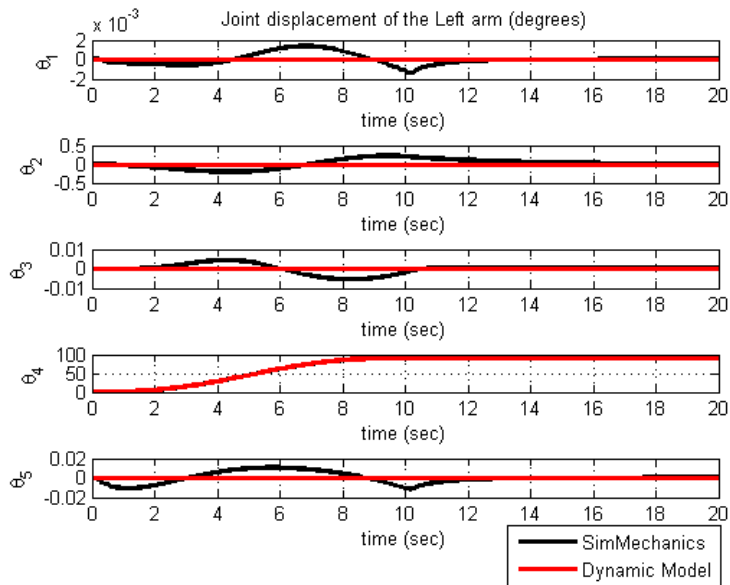


Figure 6.4: Joint Displacement of the Left Arm

CHAPTER 7

CONTROL OF BASE ATTITUDE

7.1 Angular Momentum of the System

The angular momentum of the system can be calculated link by link as,

$$\mathbf{H} = \sum_{i=1}^n (\mathbf{I}_i \boldsymbol{\omega}_i + m_i \mathbf{p}_i \times \dot{\mathbf{p}}_i) \quad (7.1)$$

Where, \mathbf{I}_i , m_i , \mathbf{p}_i and $\boldsymbol{\omega}_i$ are the inertia tensor, mass, position vector and angular velocity of body $[i]$ in the inertial system.

In absence of any external force or torque, the total angular momentum of the system will be zero.

The joint 4 of both the manipulator arms were commanded to rotate by 90° in 10 seconds using a trajectory following a quintic polynomial and the angular momentum was calculated over a period of 100 seconds.

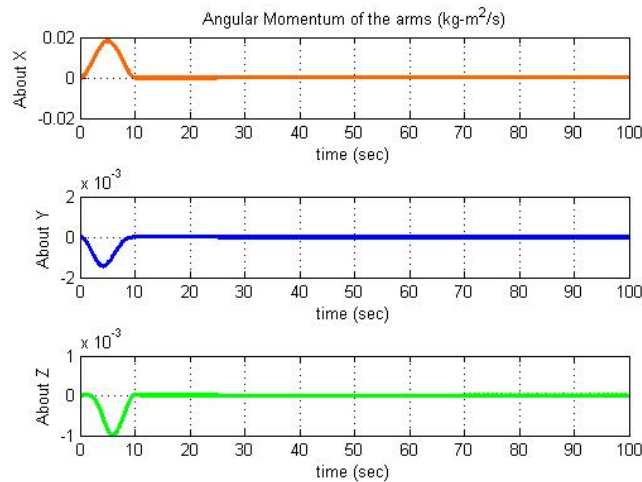


Figure 7.1: Total Angular Momentum of the Manipulator arms

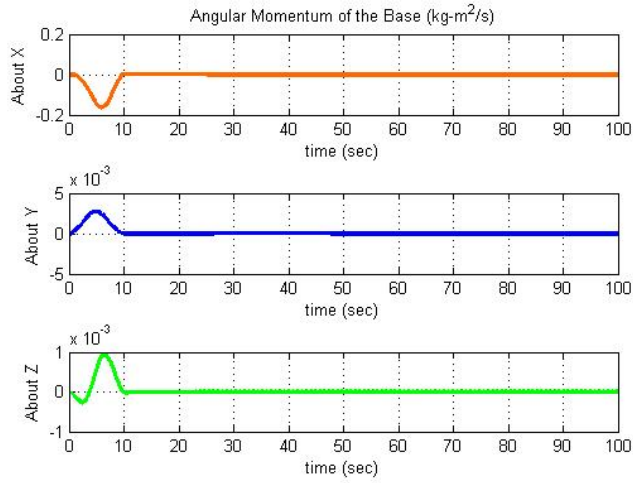


Figure 7.2: Total Angular Momentum of the Base

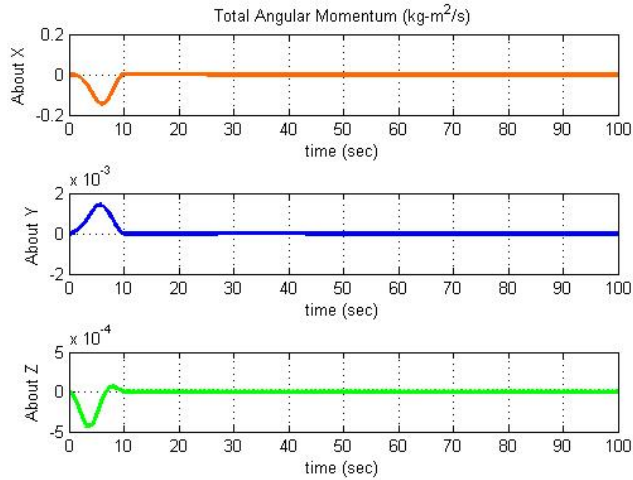


Figure 7.3: Total Angular Momentum of SMIRFF

From Fig 7.1, Fig 7.2, Fig 7.3, it is observed that the change in the angular momentum of the entire system takes place only when the manipulator arms move. Also it can be seen that the sign of the angular momentum of the Base and Ma-

nipulator arms is opposite. To stabilize the Base the excess momentum needs to be counteracted by control moment gyros or reaction wheels. Reaction wheel is a type of flywheel used primarily by spacecraft for attitude control.

7.2 Quaternion Feedback Controller For Stabilizing Orientation of the Base in the Inertial Frame of Reference

The Base attitude can be controlled by small reaction wheels when the reactive force and moment generated by the manipulator arm are not too large. Whereas when the reaction forces and moments are too large the thrusters are used to control the attitude. For the simulations demonstrated in the previous sections, the Base can be controlled by simple reaction wheels. This will help to conserve fuel.

A quaternion feedback controller to control the attitude of the Base is developed in this section using the procedure given in [7].

For simplicity the dynamic equations for the rotational motion of the are approximated as follows neglecting the effect of the manipulator arms. Hence the Euler's equation for the rotational motion of the Base about body axis is given by,

$$\mathbf{I}_{\text{Base}}\dot{\boldsymbol{\omega}} = \boldsymbol{\Omega}\mathbf{I}_{\text{Base}}\boldsymbol{\omega} + \mathbf{u} \quad (7.2)$$

Where, $\boldsymbol{\omega} = \begin{bmatrix} p & q & r \end{bmatrix}^T$ is the angular velocity in the body frame,

\mathbf{I}_{Base} is the moment of inertia of the base in the body frame,

$\boldsymbol{\Omega}$ is a skew symmetric matrix given by,

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \quad (7.3)$$

The Euler axis is defined as an imaginary axis such that rotation about that axis changes the orientation of a rigid body from one configuration to another in a single

rotation ϕ_{ori} . A quaternion is used to define this rotation. The quaternions are defined as,

$$q_i = c_i \sin\left(\frac{\phi_{ori}}{2}\right) \quad (7.4)$$

where $i=1,2,3$

$$q_4 = \cos\left(\frac{\phi_{ori}}{2}\right) \quad (7.5)$$

c_i are the direction cosines of Euler axis with respect to inertial frame. q_1, q_2 and q_3 indicate the direction of Euler axis. Last element, q_4 , gives the rotation angle about the Euler axis.

The quaternion kinematic differential equations are given by,

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}\mathbf{q} + \frac{1}{2}q_4\boldsymbol{\omega} \quad (7.6)$$

$$\dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^T\mathbf{q} \quad (7.7)$$

Where, $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T$

The quaternions satisfies the following relation,

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \quad (7.8)$$

The quaternion error in simplified form is given by

$$\begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \\ q_{4e} \end{bmatrix} = \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad (7.9)$$

Where, q_{ie} is the quaternion error,

q_{ic} is the commanded quaternion,

q_i is the actual quaternion. The control law proposed in [7] consists of a linear

quaternion feedback error, linear body rate error and non-linear body rate feedback term.

$$\mathbf{u} = -\Omega \mathbf{I}_{\text{base}} \boldsymbol{\omega} - \mathbf{D} \boldsymbol{\omega} - \mathbf{K} \mathbf{q}_e \quad (7.10)$$

Where, \mathbf{D} and \mathbf{K} are the constant gain matrices,

$$\mathbf{q}_e = \begin{bmatrix} q_{1e} & q_{2e} & q_{3e} \end{bmatrix}^T$$

Choosing gain matrices as

$$\mathbf{D} = \mathbf{I}_{\text{Base}} \text{diag} [d_{11}, d_{22}, d_{33}] \quad (7.11)$$

$$\mathbf{K} = \mathbf{I}_{\text{Base}} \text{diag} [k_{11}, k_{22}, k_{33}] \quad (7.12)$$

If $d_{11} = d_{22} = d_{33} = d$ and $k_{11} = k_{22} = k_{33} = k$ then,

$$\mathbf{D} = d \mathbf{I}_{\text{Base}} \quad (7.13)$$

$$\mathbf{K} = k \mathbf{I}_{\text{Base}} \quad (7.14)$$

Where d and k are constants and \mathbf{I} is the inertia tensor matrix.

Now, the closed loop equations are reduced to

$$\dot{\boldsymbol{\omega}} = -d \boldsymbol{\omega} - k \mathbf{q}_e \quad (7.15)$$

$$\dot{\mathbf{q}} = \frac{1}{2} \Omega \mathbf{q} + \frac{1}{2} q_4 \boldsymbol{\omega} \quad (7.16)$$

$$\dot{q}_4 = -\frac{1}{2} \boldsymbol{\omega}^T \mathbf{q} \quad (7.17)$$

7.3 Selection of Gains for the Controller

Let $\boldsymbol{\lambda}$ be a unit vector along Euler axis. Hence,

$$\mathbf{q} = \sin \left(\frac{\phi_{ori}}{2} \right) \boldsymbol{\lambda} \quad (7.18)$$

Using small angle assumption,

$$\boldsymbol{\omega} = \dot{\phi}_{ori} \boldsymbol{\lambda} \quad (7.19)$$

the equation of motion for the angular motion of the Base is reduced to,

$$\ddot{\phi}_{ori} + d\dot{\phi}_{ori} + k\frac{\phi_{ori}}{2} = 0 \quad (7.20)$$

This is a second order linear differential equation. if ζ is the damping ratio and ω_n is the natural frequency,

$$d = 2\zeta\omega_n \quad (7.21)$$

$$\frac{k}{2} = \omega_n^2 \quad (7.22)$$

For a critically damped system, $\zeta=1$. Choosing the appropriate values of d and k , we can obtain a desired response of the system.

7.4 Simulation Results After Implementing the Quaternion Based Feedback Controller

The joint 4 of both the manipulator arms were commanded to rotate by 90° along a trajectory defined by a quintic polynomial. The control law derived in section 7.2 was implemented to control the Base attitude. The Base was commanded to be oriented with the inertial frame. Fig 7.4 - 7.8, show that the proposed control law is implemented successfully even for the approximated the rotational dynamics of the base. The orientation of the Base was controlled such that it was continuously oriented with the inertial frame.

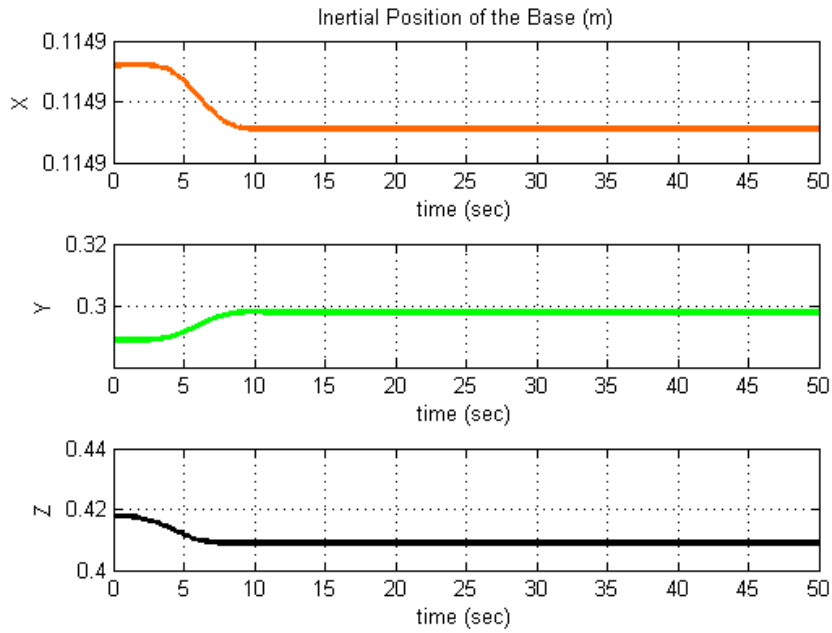


Figure 7.4: Inertial Position of the Base After Implementation of the Attitude Controller

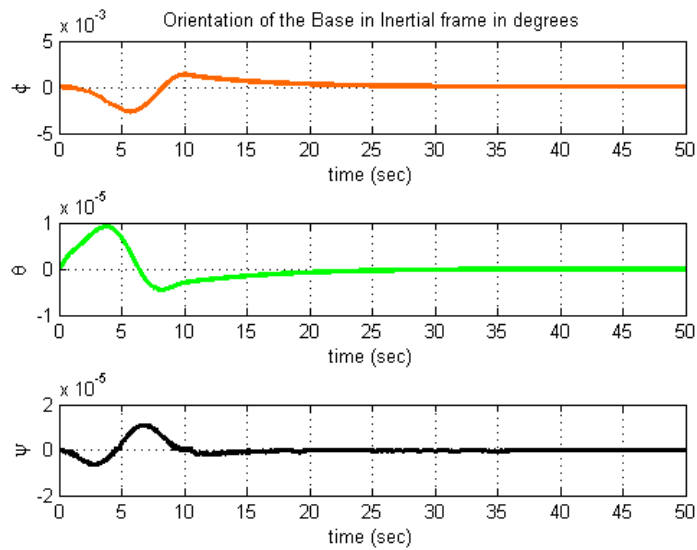


Figure 7.5: Orientation of the Base in the Inertial Frame After Implementation of the Attitude Controller

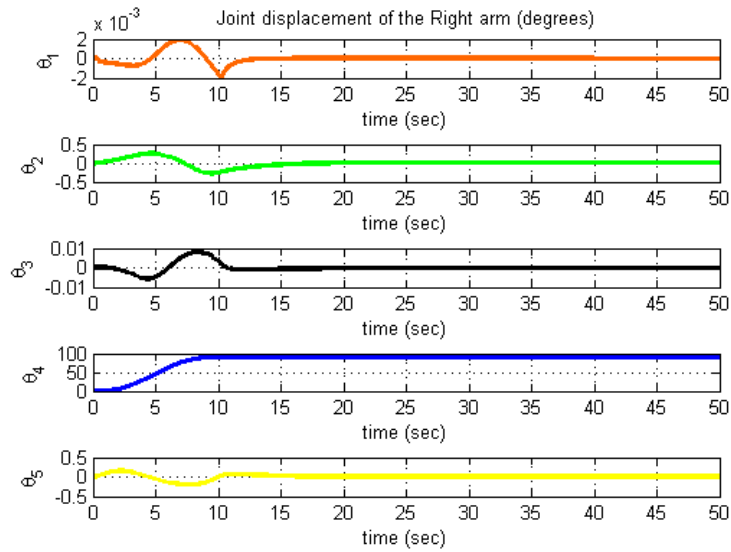


Figure 7.6: Joint Displacement of the Right Arm After Implementation of the Attitude Controller

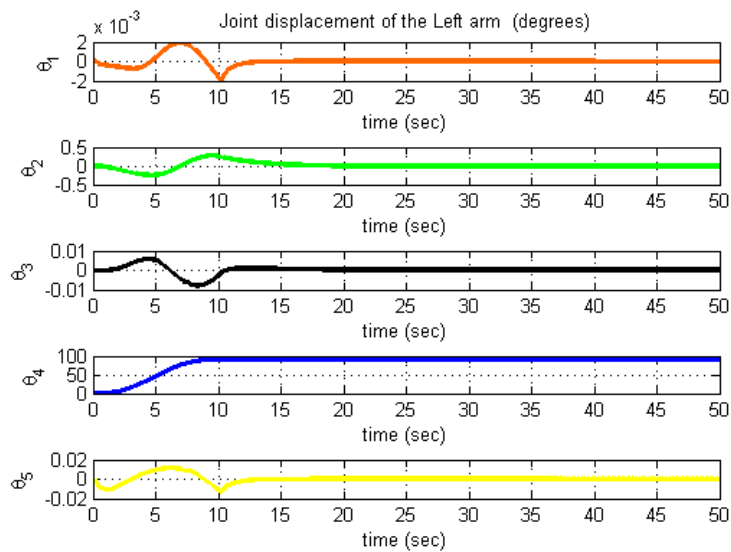


Figure 7.7: Joint Displacement of the Left Arm After Implementation of the Attitude Controller

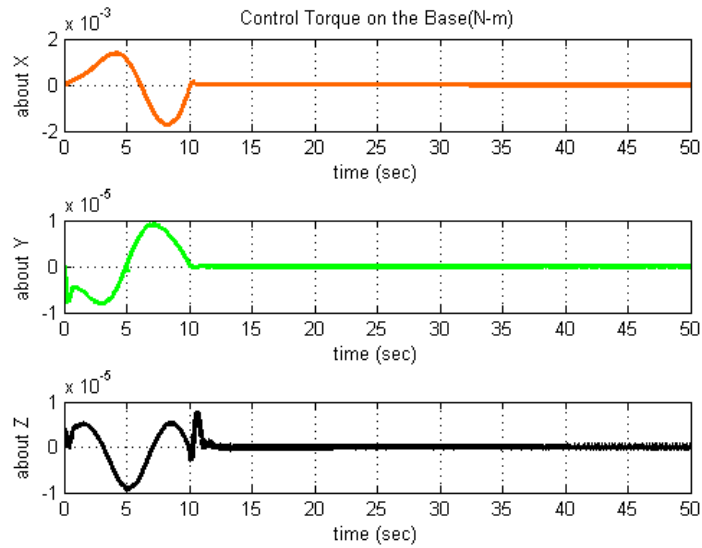


Figure 7.8: Control Torque Applied to the Base

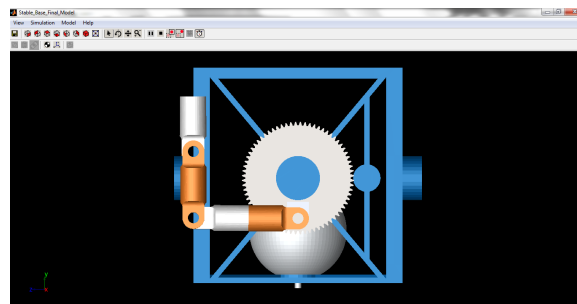


Figure 7.9: Animation Window Before the Rotation of the Arms

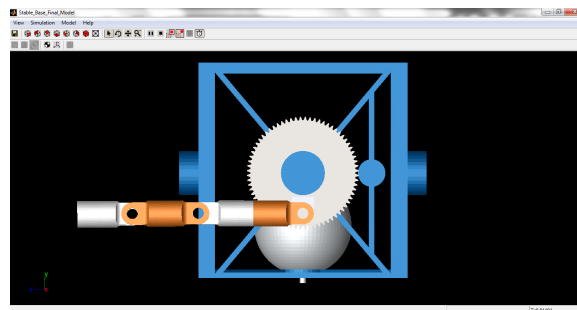


Figure 7.10: Animation Window After the Rotation of the Arms

CHAPTER 8

LARGE ANGLE MANEUVERS OF THE BASE USING QUATERNION FEEDBACK REGULATOR

8.1 Large Angle Maneuvers of the Base Using Quaternion Feedback Regulator

It is necessary to have the complete knowledge of the surrounding the SMIRFF is working in. Sometimes the camera or other visual sensors might not be able to look around the complete area around it. Hence it is required to change the orientation of the base to have a full map of the surrounding. Two cases of large angle maneuvers were simulated and the motion of the base was controlled using reaction wheel torque. The quaternion feedback regulator developed in the section 7.2 is used to perform these large angle maneuvers of the base. The commanded quaternions are updated using the commanded Euler angles as follows,

$$q_{1c} = \sin(\phi_c) \cos(\theta_c) \cos(\psi_c) + \cos(\phi_c) \sin(\theta_c) \sin(\psi_c) \quad (8.1)$$

$$q_{2c} = -\sin(\phi_c) \cos(\theta_c) \sin(\psi_c) + \cos(\phi_c) \sin(\theta_c) \cos(\psi_c) \quad (8.2)$$

$$q_{3c} = \sin(\phi_c) \sin(\theta_c) \cos(\psi_c) + \cos(\phi_c) \cos(\theta_c) \sin(\psi_c) \quad (8.3)$$

$$q_{4c} = -\sin(\phi_c) \sin(\theta_c) \sin(\psi_c) + \cos(\phi_c) \cos(\theta_c) \cos(\psi_c) \quad (8.4)$$

Where, q_{1c} , q_{2c} , q_{3c} , q_{4c} are the commanded quaternions.

ϕ_c , θ_c and ψ_c are the commanded Euler angles.

Using these commanded quaternions the quaternion error is computed which is then used to compute the control torque.

8.1.1 Roll Maneuver of the Base Without Changing the Orientation of the Manipulator arms in the Base Frame

The base was commanded to perform a roll maneuver without changing the orientation of the manipulator arms in the base frame. The roll angle ϕ , was commanded to change from 0° to 90° in 180 sec in the inertial frame following a linear trajectory.

Fig 8.2 shows the orientation history of the base with respect to the inertial frame. Fig 8.6 and Fig 8.7 show the joint displacement history of the manipulator arms. It can be seen that the orientation of the manipulator arm doesn't change in the base frame as the roll angle of the base changes. Fig 8.3 shows the applied torque in Nm over the simulation period to perform the roll maneuver. Fig 8.4 and Fig 8.5 show the screenshot of the animation window before and after the maneuver. From the Fig 8.2, 8.6 and 8.7, it can be seen that the control law used to control the attitude of the base can be used to perform this large angle maneuver.

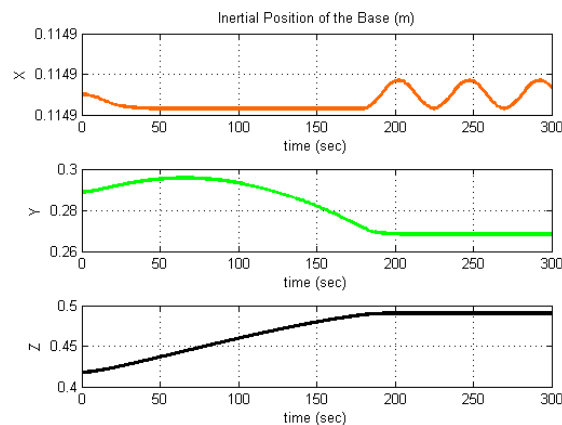


Figure 8.1: Inertial Position of the Base After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Base Frame

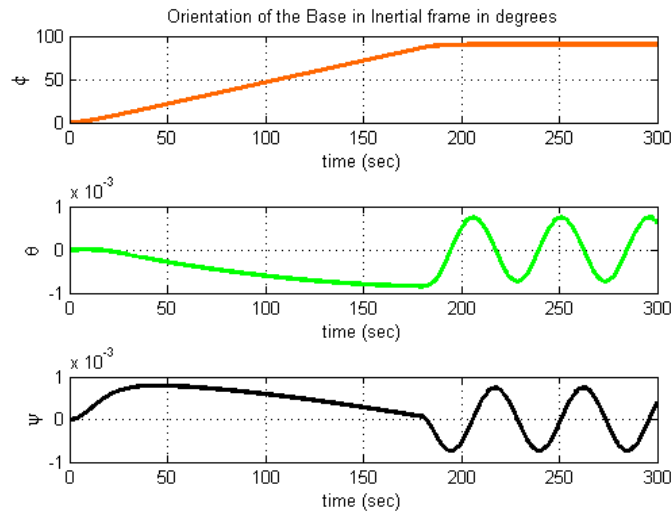


Figure 8.2: Orientation of the Base in the Inertial Frame After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Base Frame

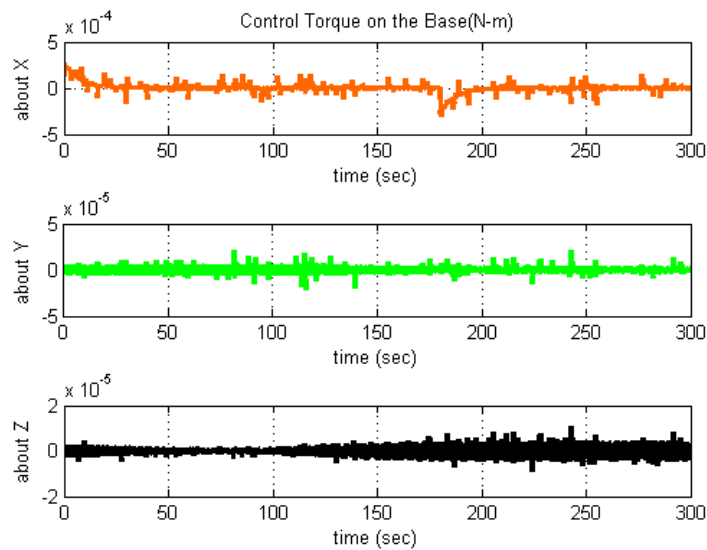


Figure 8.3: Control Torque Applied to the Base to Perform the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Base Frame

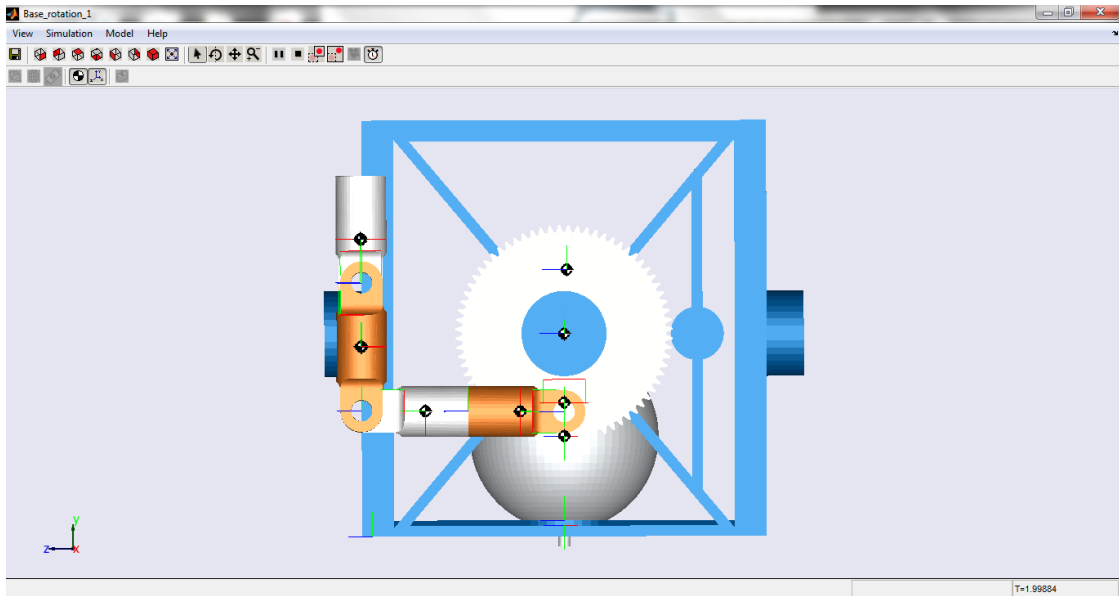


Figure 8.4: Animation Window Before Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Base Frame

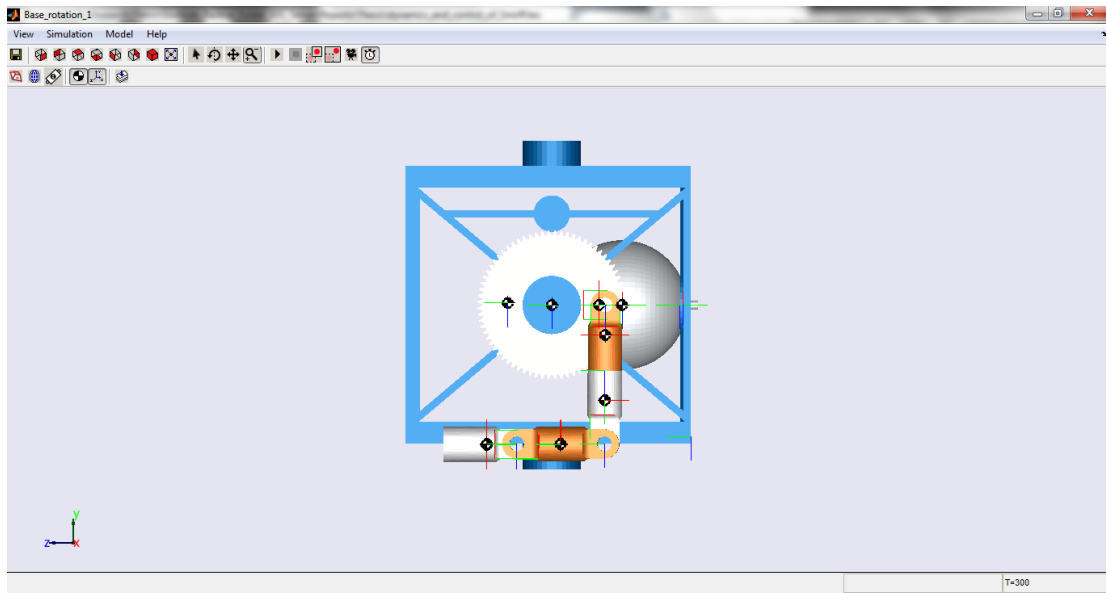


Figure 8.5: Animation Window After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Base Frame

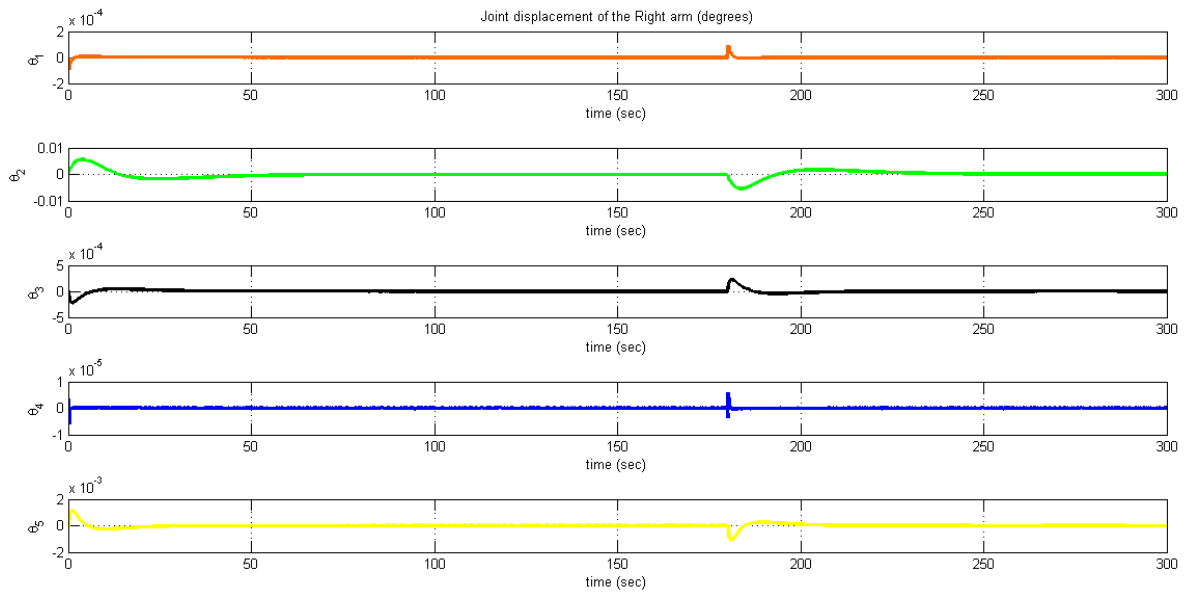


Figure 8.6: Joint Displacement of the Right Arm After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Base Frame

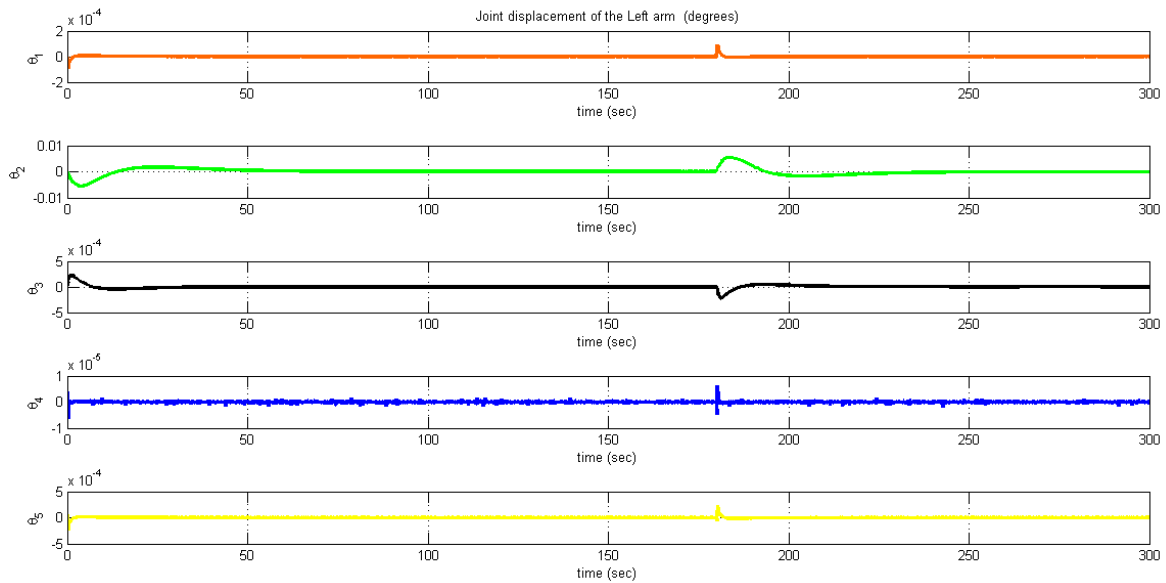


Figure 8.7: Joint Displacement of the Left Manipulator Arm After Performing the Roll Maneuver Without Changing its Orientation in the Base Frame

8.1.2 Roll Maneuver of the Base Without Changing the Orientation of the Manipulator arms in the Inertial Frame

The roll angle of the base is commanded to change by 90° without changing the orientation of the manipulator arms with respect to the inertial frame. Hence the gear on which the manipulator arm is mounted has to rotate with the same rate as the base rotates but in the opposite direction. The roll angle of the base is used as a feedback to control the joint angle of the gear.

Fig 8.9 shows the orientation history of the base with respect to the inertial frame. Fig 8.13 and Fig 8.14 show the joint displacement history of the manipulator arms. It can be seen that the orientation of the manipulator arm doesn't change in the inertial frame as the roll angle of the base changes. Fig 8.10 shows the applied torque in Nm over the simulation period to perform the roll maneuver without changing the orientation of the manipulator arms. Fig 8.11 and Fig 8.12 show the screenshot of the animation window before and after the maneuver. Fig 8.9, 8.13 and 8.14 demonstrate the roll maneuver of the base without changing the orientation of the manipulator arms in the inertial frame.

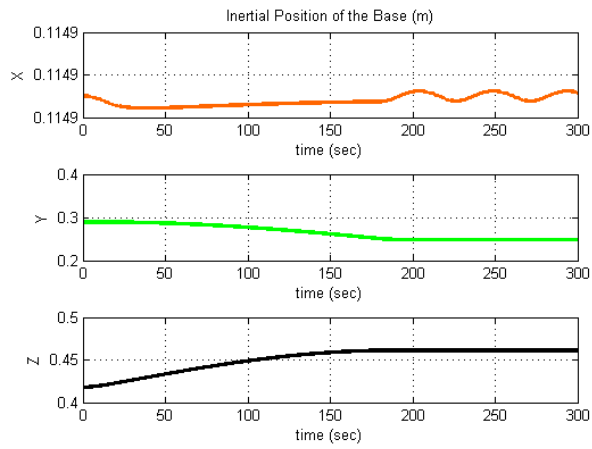


Figure 8.8: Inertial Position of the Base After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Inertial Frame of Reference

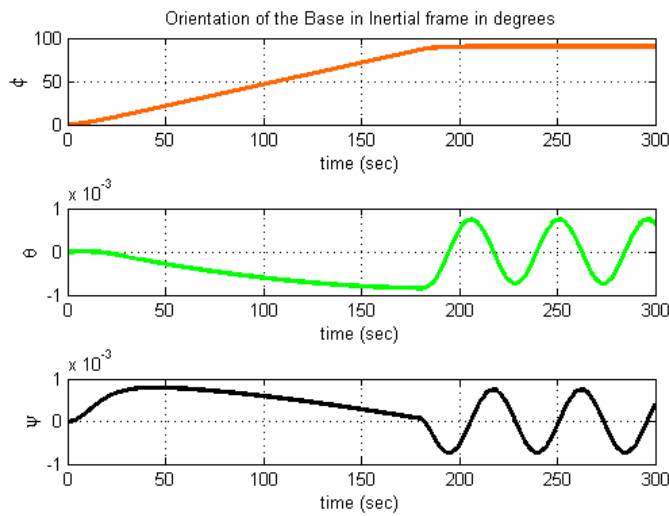


Figure 8.9: Orientation of the Base in the Inertial Frame After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Inertial Frame

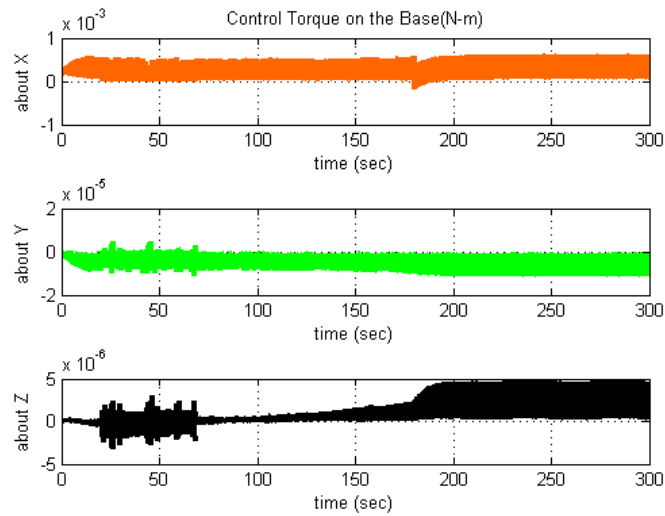


Figure 8.10: Control Torque Required to Perform the Roll Maneuver of the Base Without Changing the Orientation of the Robotic arms in the Inertial Frame

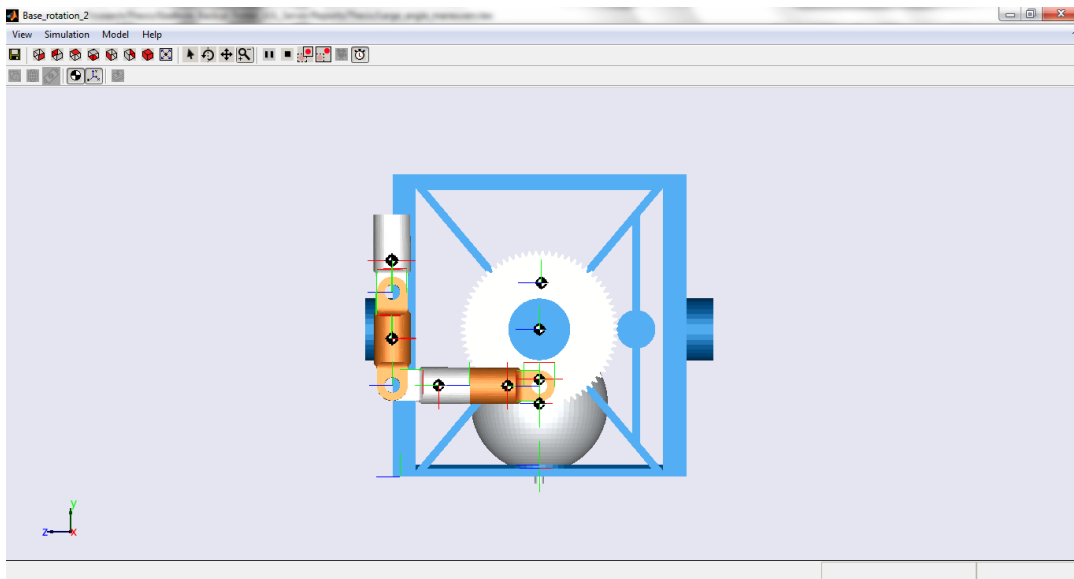


Figure 8.11: Animation Window Before Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Inertial Frame

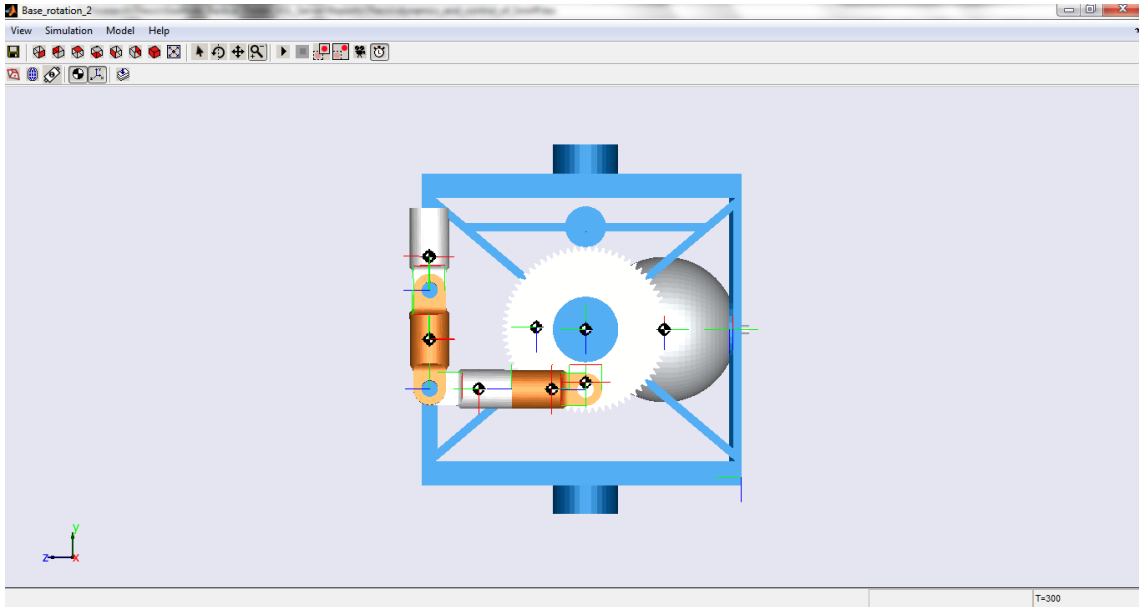


Figure 8.12: Animation Window After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Inertial Frame

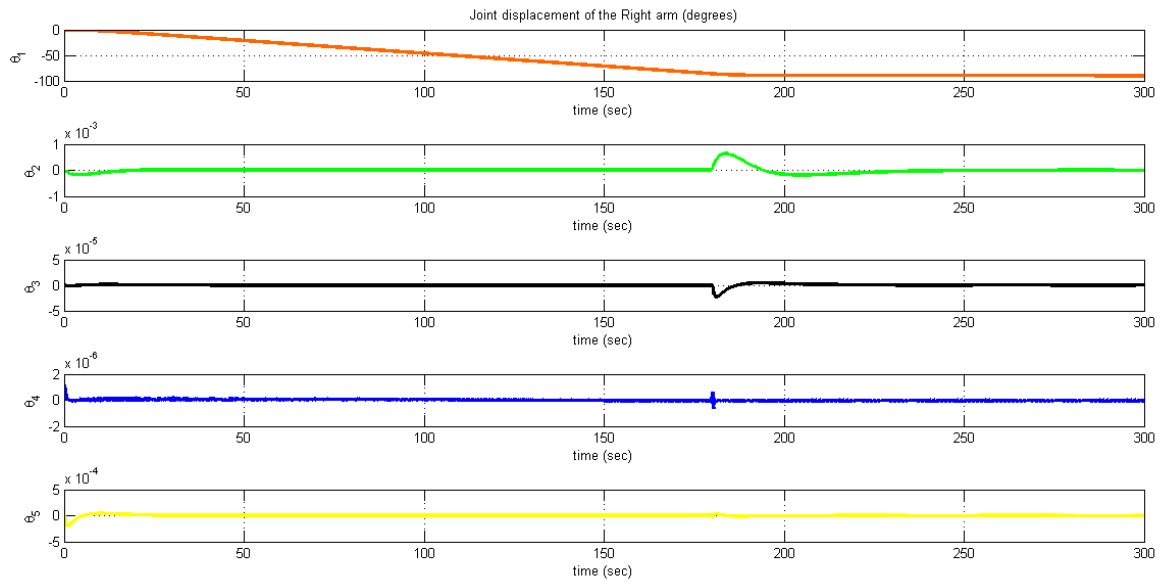


Figure 8.13: Joint Displacement of the Right Manipulator Arm After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Inertial Frame

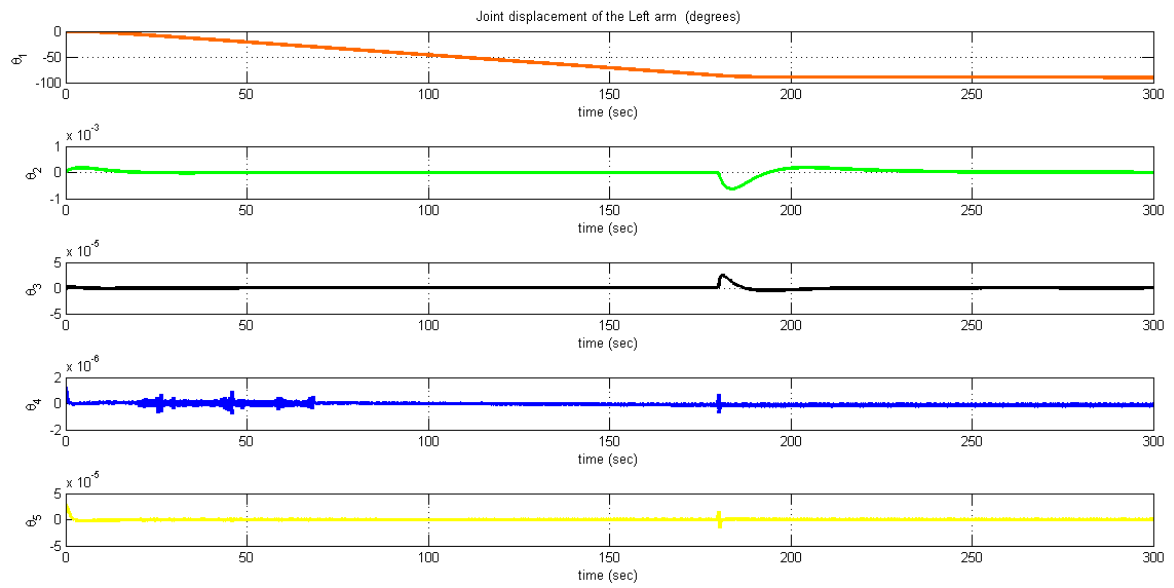


Figure 8.14: Joint Displacement of the Left Arm After Performing the Roll Maneuver Without Changing the Orientation of the Robotic arms in the Inertial Frame

CHAPTER 9

CONCLUDING REMARKS AND FUTURE WORK

9.1 Thesis Contribution

A compact design for a free-flying robot with two manipulator arms was presented. With longer duration of future space missions, this technology will prove to be an important asset for performing extra-vehicular activities. The major focus of the work was on

- Developing the concept of SMIRFF
- Constructing a non-linear mathematical model for SMIRFF
- Simulating the system with simple manipulator arm maneuvers to verify the mathematical model
- Showing usefulness of simulation environment like SimMechanics for performing complex maneuvers

The recursive Newton-Euler method was used to derive the mathematical model of SMIRFF. The kinematic model for the manipulator arms was based on the modified Denavit-Hartenberg Parameters. It was shown that the attitude control law designed for bigger satellites can be used to control the Base by approximating the dynamics.

9.2 Future Work

The work presented gives a foundation to the feasibility of the concept. The simulation environment developed for the SMIRFF can be used to test various Guidance Navigation and Control laws. Some future studies that can be done are,

- Workspace analysis

- Study of effect of joint rates on the position and orientation of the Base
- Coordinated arm control to move payload in space
- Study the case where one arm is holding a rigid structure while other arm moves the payload

9.3 Final Words

The work presented in this thesis and the work done by the researchers in the past show that this concept can play a crucial role in reducing the risks to the human life involved in the extra-vehicular activities. The concept of SMIRFF can be an ideal companion to the man in the future deep space explorations.

APPENDIX A

LINK LENGTHS AND MASS PROPERTIES

A.1 Mass and Inertia Properties

Table A.1: Mass and inertia properties of the manipulator arm

Sr. no.	Component	mass (<i>Kg</i>)	I_{xx} (<i>Kg.m</i> ²)	I_{yy} (<i>Kg.m</i> ²)	I_{zz} (<i>Kg.m</i> ²)
1	Gear	0.3	0.00174692	0.000877221	0.000877221
2	Link 1	0.13	0.000114886	3.73894e-05	0.000118637
3	Link 2	0.18	0.000259367	4.44786e-05	0.000263987
4	Link 3	0.19	0.000291746	4.86811e-05	0.000299249
5	Link 4	0.13	0.00011622	3.70749e-05	0.000119971
6	Base	4.23	0.1551	0.1689	0.1549

A.2 Part Dimensions

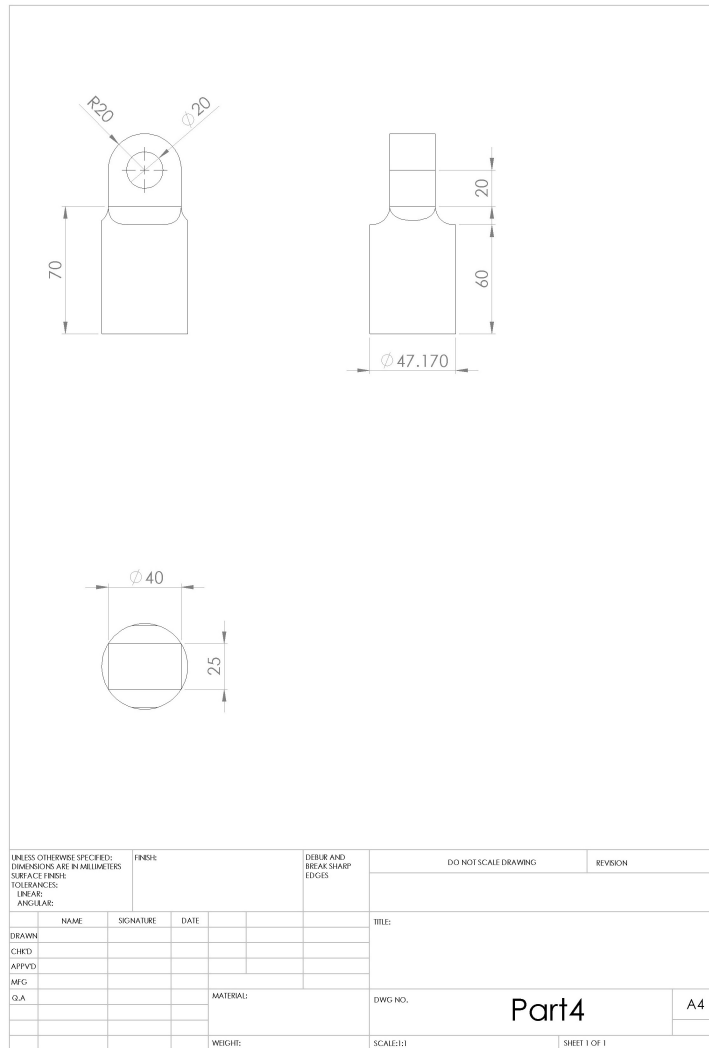


Figure A.1: Link 1

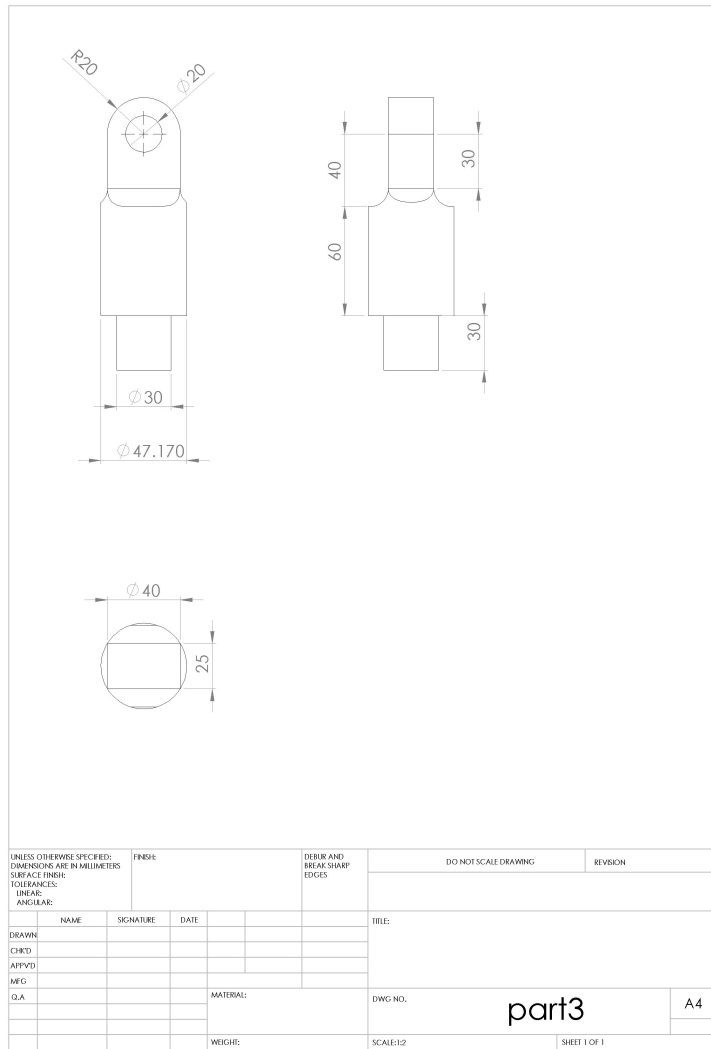


Figure A.2: Link 2

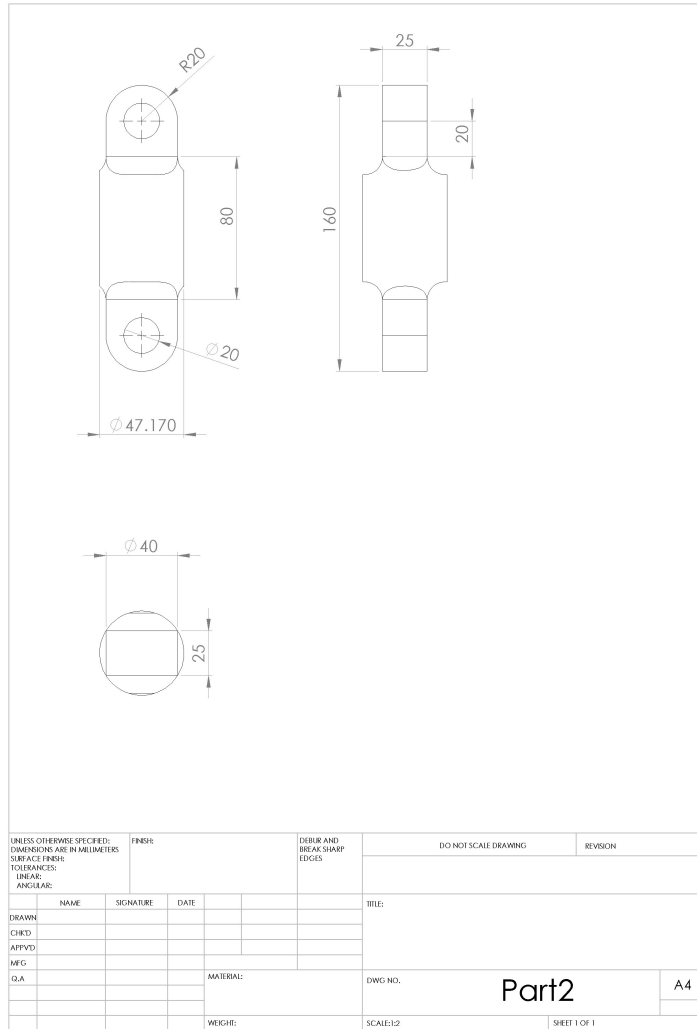


Figure A.3: Link 3

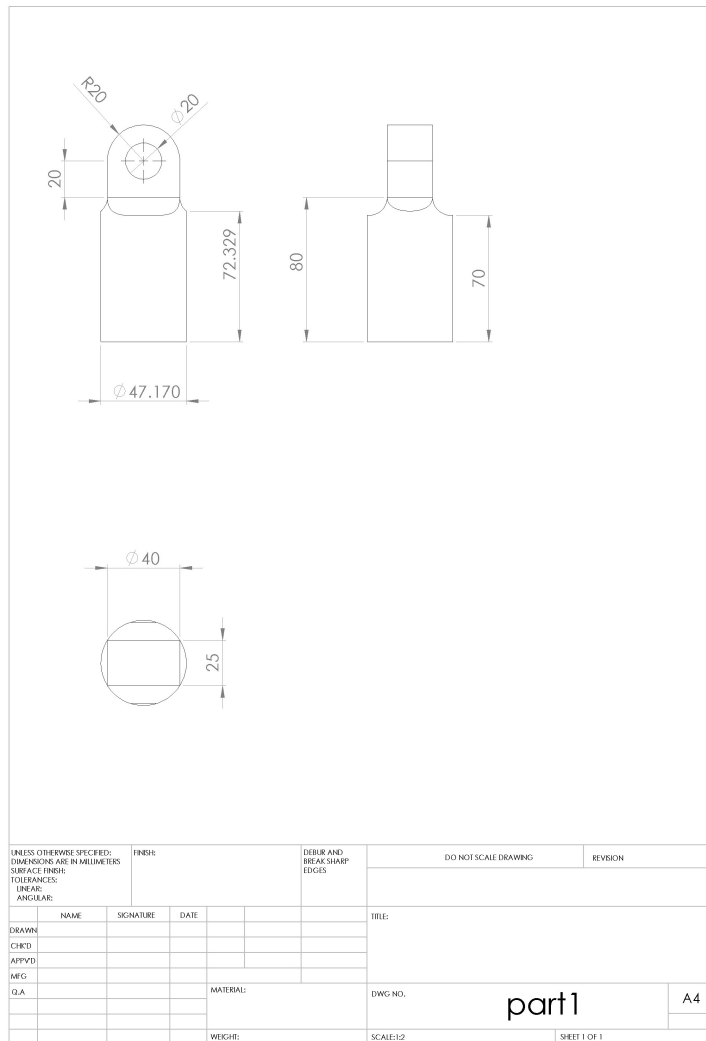


Figure A.4: Link 4

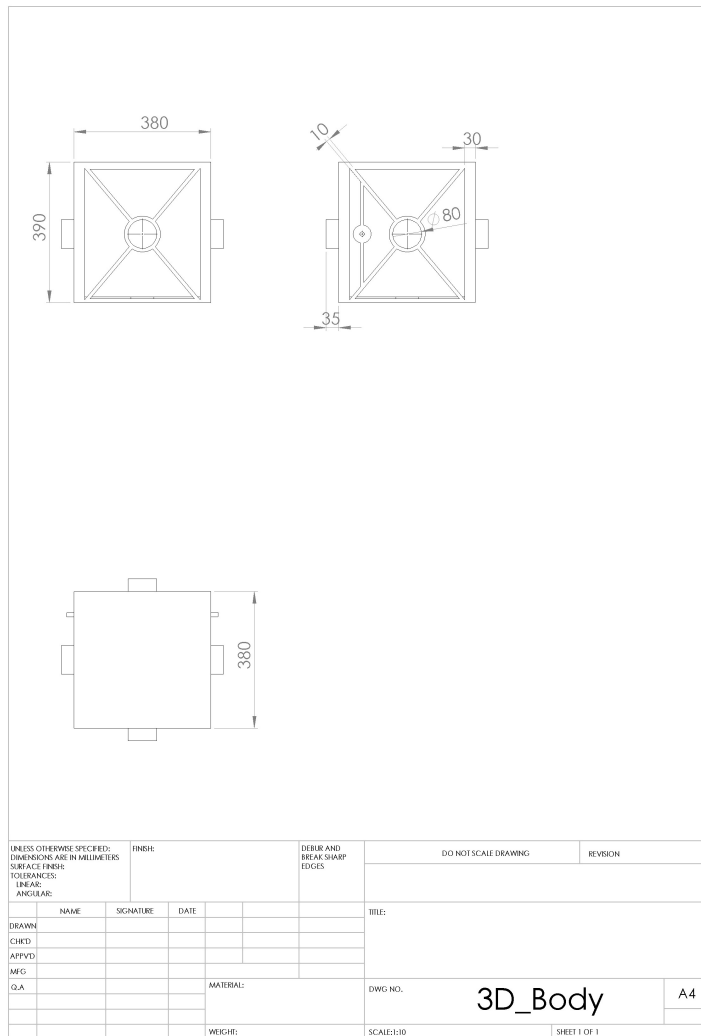


Figure A.5: Base

APPENDIX B
EQUATIONS OF MOTION

B.1 Forward Kinematics of the Manipulator Arm

The modified D-H parameters for both the manipulator arm are given in the following tables,

Table B.1: Modified D-H Parameters for the Right Manipulator Arm

Sr. no.	Current frame	Next Frame	a	α	d	θ
1	1	2	0	0°	d_2	θ_2
2	2	3	$-a_3$	0°	d_3	θ_3
3	3	4	0	90°	d_4	θ_4
4	4	Aux ₁	0	0°	d_{a1}	0°
5	Aux ₁	5	0	90°	d_5	θ_5
6	5	6	a_6	0°	$-d_6$	θ_6
7	6	7	a_7	0°	0	0°

Table B.2: Modified D-H Parameters for the Left Manipulator Arm

Sr. no.	Current frame	Next Frame	a	α	d	θ
1	1	8	0	0°	$-d_8$	θ_8
2	8	9	$-a_9$	0°	$-d_9$	θ_9
3	9	10	0	90°	d_{10}	θ_{10}
4	10	Aux ₂	0	0°	d_{a2}	0°
5	Aux ₂	11	0	90°	$-d_{11}$	θ_{11}
6	11	12	a_{12}	0°	d_{12}	θ_{12}
7	12	13	a_{13}	0°	0	0°

θ_2 - θ_6 are the joint angles for the right manipulator arm.

θ_8 - θ_{12} are the joint angles for the left manipulator arm.

Using the modified D-H parameters the forward kinematics relation is derived. The forward kinematics gives the position and orientation of the end-effector for a

given set of joint angles. The transformation matrix which relate the body attached frame of the consecutive links of the manipulator arm is given by,

$${}_{i-1}^i\mathbf{T} = \begin{bmatrix} c(\theta_i) & s(\theta_i) & 0 & a_{i-1} \\ s(\theta_i)c(\alpha_{i-1}) & c(\theta_i)c(\alpha_{i-1}) & -s(\alpha_{i-1}) & -s(\alpha_{i-1})d_i \\ s(\theta_i)s(\alpha_{i-1}) & c(\theta_i)s(\alpha_{i-1}) & c(\alpha_{i-1}) & c(\alpha_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{B.1})$$

Where,

$$c(\theta_i) = \cos(\theta_i),$$

$$s(\theta_i) = \sin(\theta_i),$$

$$c(\alpha_{i-1}) = \cos(\alpha_{i-1}) \text{ and}$$

$$s(\alpha_{i-1}) = \sin(\alpha_{i-1})$$

The individual link transformation matrices are then multiplied to find the single transformation that relates Base frame, $\{1\}$ to the end-effector of the manipulator arm.

$${}^1_7\mathbf{T} = {}^1_2\mathbf{T}{}^2_3\mathbf{T}{}^3_4\mathbf{T}{}^4_5\mathbf{T}{}^5_6\mathbf{T}{}^6_7\mathbf{T} \quad (\text{B.2})$$

$${}^1_{13}\mathbf{T} = {}^1_8\mathbf{T}{}^8_9\mathbf{T}{}^9_{10}\mathbf{T}{}^{10}_{11}\mathbf{T}{}^{11}_{12}\mathbf{T}{}^{12}_{13}\mathbf{T} \quad (\text{B.3})$$

Thus, the position and orientation of the end-effector in the Base frame can be calculated using these transformation matrices. The inertial position and orientation of the end-effector depends on the position and orientation of the Base with respect to the inertial frame of reference.

Let ${}^0\mathbf{P}_1$ be the position of the Base in the inertial frame. Let ${}^0_1\mathbf{R}$ be the rotation matrix which gives the orientation of the Base with respect to the inertial frame. The

rotation matrix from inertial frame to body frame is constructed based on the 3-2-1 Euler angle rotation sequence, the matrix elements are shown below

$${}^0_1\mathbf{R} = \begin{bmatrix} c(\theta)c(\psi) & c(\theta)s(\psi) & -s(\theta) \\ -c(\phi)s(\psi) + s(\phi)s(\theta)c(\psi) & c(\phi)c(\psi) + s(\phi)s(\theta)s(\psi) & s(\phi)c(\theta) \\ s(\phi)s(\psi) + c(\phi)s(\theta)c(\psi) & -s(\phi)c(\psi) + c(\phi)s(\theta)s(\psi) & c(\phi)c(\theta) \end{bmatrix} \quad (\text{B.4})$$

Where,

ϕ = Roll angle,

θ = Pitch angle and

ψ = Yaw Angle

$c(\cdot) = \cos(\cdot)$

$s(\cdot) = \sin(\cdot)$

The Position of the end-effectors in the inertial frame are given by,

$${}^0\mathbf{P}_7 = {}^0\mathbf{P}_1 + {}^0_1\mathbf{R}^1\mathbf{P}_7 \quad (\text{B.5})$$

$${}^0\mathbf{P}_{13} = {}^0\mathbf{P}_1 + {}^0_1\mathbf{R}^1\mathbf{P}_{13} \quad (\text{B.6})$$

B.2 Mathematical Modelling Using Iterative Newton-Euler Method

The summary of the outward iteration to compute the individual link velocities, accelerations and the inertial force and moment acting on the link is given by,

$${}^{i+1}\boldsymbol{\omega}_{i+1} = {}^{i+1}_i\mathbf{R}^i\boldsymbol{\omega}_i + \dot{\theta}_{i+1} {}^{i+1}\hat{Z}_{i+1} \quad (\text{B.7})$$

$${}^{i+1}\mathbf{v}_{i+1} = {}^{i+1}_i\mathbf{R} [{}^i\mathbf{v}_i + {}^i\boldsymbol{\omega}_i \times {}^i\mathbf{P}_{i+1}] \quad (\text{B.8})$$

$${}^{i+1}\dot{\boldsymbol{\omega}}_{i+1} = {}^i{}^{i+1}\mathbf{R}^i\dot{\boldsymbol{\omega}}_i + {}^i{}^{i+1}\mathbf{R}^i\boldsymbol{\omega}_i \times \dot{\theta}_{i+1} {}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1} \cdot {}^{i+1}\hat{Z}_{i+1} \quad (\text{B.9})$$

$${}^{i+1}\dot{\mathbf{v}}_{i+1} = {}^i{}^{i+1}\mathbf{R} \left[{}^i\dot{\boldsymbol{\omega}}_i \times {}^i\mathbf{P}_{i+1} + {}^i\boldsymbol{\omega}_i \times \left({}^i\boldsymbol{\omega}_i \times {}^i\mathbf{P}_{i+1} \right) + {}^i\dot{\mathbf{v}}_i \right] \quad (\text{B.10})$$

Where, $(i+1):2 \rightarrow 7$ for the right arm and $(i+1):8 \rightarrow 13$ for the left arm. The initial velocity and acceleration conditions of the Base are required to compute the velocity and acceleration of the first link of each of the manipulator arm.

The Newton-Euler equations are used to compute the inertial force and moment acting on the center of mass of each link. Before computing these forces and moments, the translational acceleration of center of mass of each link is computed. The translational acceleration of the center of mass of link $[i+1]$ is computed as follows,

$${}^{i+1}\dot{\mathbf{v}}_{\mathbf{C}_{i+1}} = {}^{i+1}\dot{\boldsymbol{\omega}}_{i+1} \times {}^{i+1}\mathbf{P}_{\mathbf{C}_{i+1}} + {}^{i+1}\boldsymbol{\omega}_{i+1} \times \left({}^{i+1}\boldsymbol{\omega}_{i+1} \times {}^{i+1}\mathbf{P}_{\mathbf{C}_{i+1}} \right) + {}^{i+1}\dot{\mathbf{v}}_{i+1} \quad (\text{B.11})$$

Where, ${}^{i+1}\mathbf{P}_{\mathbf{C}_{i+1}}$ is the location of center of mass of link $[i+1]$ expressed in body frame $\{i+1\}$.

According to Newton and Euler's equation, inertial force and moment acting on link $[i+1]$ which causes this motion is computed as,

$${}^{i+1}\mathbf{F}_{i+1} = m_{i+1} {}^{i+1}\dot{\mathbf{v}}_{\mathbf{C}_{i+1}} \quad (\text{B.12})$$

$${}^{i+1}\mathbf{N}_{i+1} = {}^{\mathbf{C}_{i+1}}\mathbf{I}_{i+1} {}^{i+1}\dot{\boldsymbol{\omega}}_{i+1} + {}^{i+1}\boldsymbol{\omega}_{i+1} \times {}^{\mathbf{C}_{i+1}}\mathbf{I}_{i+1} {}^{i+1}\boldsymbol{\omega}_{i+1} \quad (\text{B.13})$$

$[C_i]$ has its origin at center of mass of the link and orientation as the frame $\{i\}$.

${}^{C_{i+1}}\mathbf{I}_{i+1}$ is the moment of inertia of link $[i + 1]$ about its center of mass.

Once the inertial forces and moments are computed, the next step involves finding the joint forces and torques responsible for the motion of each link using inward iterations. In a multi-body system each body or a link experiences reaction forces and torques from its neighbours apart from the inertial forces and moments.

Let,

$\mathbf{f}_i =$ Force exerted on link $[i]$ by link $[i - 1]$,

$\mathbf{n}_i =$ Torque exerted on link $[i]$ by link $[i - 1]$,

Now, the force balance equation would give the relationship between these forces exerted by the neighbouring links and the inertial force as,

$${}^i\mathbf{f}_i = {}^i_{i+1}\mathbf{R}^{i+1}\mathbf{f}_{i+1} + {}^i\mathbf{F}_i \quad (\text{B.14})$$

The torque balance equation gives the equation which relates the torque exerted by the neighbouring links on the body and the inertial moment.

$${}^i\mathbf{n}_i = {}^i\mathbf{N}_i + {}^i_{i+1}\mathbf{R}^{i+1}\mathbf{n}_{i+1} + {}^i\mathbf{P}_{C_i} \times {}^i\mathbf{F}_i + {}^i\mathbf{P}_{i+1} \times {}^i_{i+1}\mathbf{R}^{i+1}\mathbf{f}_{i+1} \quad (\text{B.15})$$

The required joint torque is found by taking the \hat{Z} component of the torque applied by a link on its neighbour.

$$\tau_i = {}^i n_i \cdot {}^i \hat{Z}_i \quad (\text{B.16})$$

The details of Base dynamics are given in section 3.2

The dynamic model of SMIRFF is given by,

$$\begin{bmatrix} \mathbf{M}_b & \mathbf{M}_{br} & \mathbf{M}_{bl} \\ \mathbf{M}_{rb} & \mathbf{M}_r & \mathbf{M}_{rl} \\ \mathbf{M}_{lb} & \mathbf{M}_{lr} & \mathbf{M}_l \end{bmatrix} \begin{bmatrix} {}^1\dot{\mathbf{v}}_1 \\ {}^1\dot{\boldsymbol{\omega}}_1 \\ \ddot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{C}_b & \mathbf{C}_{br} & \mathbf{C}_{bl} \\ \mathbf{C}_{rb} & \mathbf{C}_r & \mathbf{C}_{rl} \\ \mathbf{C}_{lb} & \mathbf{C}_{lr} & \mathbf{C}_l \end{bmatrix} \begin{bmatrix} {}^1\mathbf{v}_1 \\ {}^1\boldsymbol{\omega}_1 \\ \dot{\boldsymbol{\theta}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{J}_b^T \\ \mathbf{J}_r^T \\ \mathbf{J}_l^T \end{bmatrix} \mathbf{F}_{\text{ext}} = \begin{bmatrix} {}^1\mathbf{f}_1 \\ {}^1\mathbf{n}_1 \\ \boldsymbol{\tau}_i \end{bmatrix} \quad (\text{B.17})$$

The inertia, mass and link length values are substituted in the equations to simplify the mass matrix. The Mass matrix of the system is given by,

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_b & \mathbf{M}_{br} & \mathbf{M}_{bl} \\ \mathbf{M}_{rb} & \mathbf{M}_r & \mathbf{M}_{rl} \\ \mathbf{M}_{lb} & \mathbf{M}_{lr} & \mathbf{M}_l \end{bmatrix} \quad (\text{B.18})$$

The elements of the inertia matrix of the base \mathbf{M}_b is given by,

$$\mathbf{M}_b(1, 1) = 6.1$$

$$\mathbf{M}_b(1, 2) = 0$$

$$\mathbf{M}_b(1, 3) = 0$$

$$\mathbf{M}_b(1, 4) = 0$$

$$\begin{aligned} \mathbf{M}_b(1, 5) &= (5.9 \times 10^{-3}) \cos(\theta_{10}) - (5.9 \times 10^{-3}) \cos(\theta_4) + (2.8 \times 10^{-2}) \cos(\theta_5) \sin(\theta_4) \\ &+ (2.8 \times 10^{-2}) \cos(\theta_{11}) \sin(\theta_{10}) + (6.7 \times 10^{-3}) \cos(\theta_5) \cos(\theta_6) \sin(\theta_4) \\ &+ (6.7 \times 10^{-3}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{10}) - (6.7 \times 10^{-3}) \sin(\theta_4) \sin(\theta_5) \sin(\theta_6) \\ &- (6.7 \times 10^{-3}) \sin(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) \end{aligned}$$

$$\begin{aligned}
\mathbf{M}_b(\mathbf{1}, \mathbf{6}) = & -\left(9.5 \times 10^{-2}\right) \sin(\theta_2) \sin(\theta_3) - \left(2.8 \times 10^{-2}\right) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \\
& + \left(2.8 \times 10^{-2}\right) \cos(\theta_2) \cos(\theta_3) \sin(\theta_5) + \left(2.8 \times 10^{-2}\right) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{11}) \\
& - \left(5.9 \times 10^{-3}\right) \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) - \left(5.9 \times 10^{-3}\right) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \\
& + \left(5.9 \times 10^{-3}\right) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{10}) + \left(5.9 \times 10^{-3}\right) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{10}) \\
& - \left(2.8 \times 10^{-2}\right) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - \left(2.8 \times 10^{-2}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \\
& - \left(2.8 \times 10^{-2}\right) \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_6) \\
& - \left(2.8 \times 10^{-2}\right) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_9) - \left(9.5 \times 10^{-2}\right) \sin(\theta_8) \sin(\theta_9) \\
& - \left(2.8 \times 10^{-2}\right) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) + \left(5.2 \times 10^{-2}\right) \sin(\theta_2) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_{12}) + \left(5.2 \times 10^{-2}\right) \sin(\theta_8) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{11}) + \left(9.5 \times 10^{-2}\right) \cos(\theta_2) \cos(\theta_3) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_6) + \left(9.5 \times 10^{-2}\right) \cos(\theta_2) \cos(\theta_3) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) + \left(9.5 \times 10^{-2}\right) \cos(\theta_8) \cos(\theta_9) \\
& - \left(6.7 \times 10^{-3}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11} + \theta_{12}) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_3) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_9) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) \sin(\theta_6) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_5)
\end{aligned}$$

$$\mathbf{M}_b(\mathbf{2}, \mathbf{2}) = 6.1$$

$$\mathbf{M}_b(\mathbf{2}, \mathbf{3}) = 0$$

$$\begin{aligned} \mathbf{M}_b(\mathbf{2}, \mathbf{4}) &= (5.9 \times 10^{-3}) \cos(\theta_4) - (5.9 \times 10^{-3}) \cos(\theta_{10}) - (2.8 \times 10^{-2}) \cos(\theta_5) \sin(\theta_4) \\ &\quad - (2.8 \times 10^{-2}) \cos(\theta_{11}) \sin(\theta_{10}) - (6.7 \times 10^{-3}) \cos(\theta_5) \cos(\theta_6) \sin(\theta_4) \\ &\quad - (6.7 \times 10^{-3}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{10}) + (6.7 \times 10^{-3}) \sin(\theta_4) \sin(\theta_5) \sin(\theta_6) \\ &\quad + (6.7 \times 10^{-3}) \sin(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) \end{aligned}$$

$$\mathbf{M}_b(\mathbf{2}, \mathbf{5}) = 0$$

$$\begin{aligned} \mathbf{M}_b(\mathbf{2}, \mathbf{6}) &= (9.5 \times 10^{-2}) \sin(\theta_2 + \theta_3) - (5.2 \times 10^{-2}) \cos(\theta_8) - (5.2 \times 10^{-2}) \cos(\theta_2) \\ &\quad + (9.5 \times 10^{-2}) \sin(\theta_8 + \theta_9) + (6.7 \times 10^{-3}) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) \\ &\quad + (5.9 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - (5.9 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) \\ &\quad + (2.8 \times 10^{-2}) \sin(\theta_2 + \theta_3) \sin(\theta_5) + (2.8 \times 10^{-2}) \sin(\theta_8 + \theta_9) \sin(\theta_{11}) \\ &\quad - (5.9 \times 10^{-3}) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + (5.9 \times 10^{-3}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \\ &\quad + (2.8 \times 10^{-2}) \cos(\theta_2 + \theta_3) \cos(\theta_4) \cos(\theta_5) + (2.8 \times 10^{-2}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \\ &\quad + (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) + (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) \\ &\quad + (6.7 \times 10^{-3}) \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_6) + (6.7 \times 10^{-3}) \cos(\theta_3) \cos(\theta_6) \sin(\theta_2) \sin(\theta_5) \\ &\quad - (2.8 \times 10^{-2}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) + (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12}) \\ &\quad + (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) + (6.7 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_{12}) \\ &\quad + (6.7 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_{11}) + (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \\ &\quad + (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) \sin(\theta_6) \\ &\quad - (6.7 \times 10^{-3}) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) - (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) \\ &\quad - (6.7 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) + (6.7 \times 10^{-3}) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) \end{aligned}$$

$$\mathbf{M}_b(\mathbf{3}, \mathbf{3}) = 6.1$$

$$\begin{aligned} \mathbf{M}_b(\mathbf{3}, \mathbf{4}) = & -0.028 \cos(\theta_8) \cos(\theta_9) \sin(\theta_{11}) + (5.9 \times 10^{-3}) \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \\ & + (5.9 \times 10^{-3}) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - (5.9 \times 10^{-3}) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \\ & - (5.9 \times 10^{-3}) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{10}) + 0.028 \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \\ & + 0.028 \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) + 0.028 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \\ & + 0.028 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + 0.095 \sin(\theta_2) \sin(\theta_3) \\ & - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_6) - 0.052 \sin(\theta_8) \\ & - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_5) - 0.095 \cos(\theta_2) \cos(\theta_3) \\ & + 0.028 \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_9) - 0.095 \cos(\theta_8) \cos(\theta_9) \\ & + 0.028 \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) - 0.052 \sin(\theta_2) \\ & - (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_{12}) - 0.028 \cos(\theta_2) \cos(\theta_3) \sin(\theta_5) \\ & - (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{11}) + 0.095 \sin(\theta_8) \sin(\theta_9) \\ & + (6.7 \times 10^{-3}) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_6) + (6.7 \times 10^{-3}) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \\ & + (6.7 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{12}) + (6.7 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \\ & + (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_3) + (6.7 \times 10^{-3}) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \\ & + (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_9) + (6.7 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \\ & - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) - (6.7 \times 10^{-3}) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) \sin(\theta_6) \\ & - (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_{10}) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) - (6.7 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_{11}) \sin(\theta_{12}) \end{aligned}$$

$$\begin{aligned} \mathbf{M}_b(\mathbf{3}, \mathbf{5}) = & -0.095 \sin(\theta_8 + \theta_9) - (5.9 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \\ & + (5.9 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) - 0.028 \cos(\theta_2) \sin(\theta_3) \sin(\theta_5) \\ & - 0.028 \cos(\theta_3) \sin(\theta_2) \sin(\theta_5) - 0.028 \cos(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \\ & - 0.028 \cos(\theta_9) \sin(\theta_8) \sin(\theta_{11}) + (5.9 \times 10^{-3}) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \\ & - (5.9 \times 10^{-3}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) - 0.028 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \\ & - 0.028 \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) + 0.028 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \\ & - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) + 0.052 \cos(\theta_2) \\ & - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) + 0.052 \cos(\theta_8) \\ & - (6.7 \times 10^{-3}) \cos(\theta_3) \sin(\theta_2) \sin(\theta_5 + \theta_6) - 0.095 \cos(\theta_2) \sin(\theta_3) \\ & + 0.028 \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) - 0.095 \cos(\theta_3) \sin(\theta_2) \\ & - (6.7 \times 10^{-3}) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{11} + \theta_{12}) - (6.7 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_{12}) \\ & - (6.7 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_{11}) - (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \\ & - (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) + (6.7 \times 10^{-3}) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) \sin(\theta_6) \\ & + (6.7 \times 10^{-3}) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) + (6.7 \times 10^{-3}) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) \\ & + (6.7 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) - (6.7 \times 10^{-3}) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) \\ & - (6.7 \times 10^{-3}) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) \end{aligned}$$

$$\mathbf{M}_b(\mathbf{3}, \mathbf{6}) = 0$$

$$\begin{aligned}
\mathbf{M}_b(\mathbf{6}, \mathbf{6}) = & (1.6 \times 10^{-3}) \cos(\theta_6) + (1.6 \times 10^{-3}) \cos(\theta_{12}) - (1.5 \times 10^{-2}) \sin(\theta_3) + (1.1 \times 10^{-2}) \sin(\theta_5) \\
& - (1.5 \times 10^{-2}) \sin(\theta_9) + (1.1 \times 10^{-2}) \sin(\theta_{11}) - (9.5 \times 10^{-4}) \cos(\theta_3) \sin(\theta_4) + 2.0 \times 10^{-1} \\
& + (2.5 \times 10^{-3}) \cos(\theta_5) \sin(\theta_6) + (2.5 \times 10^{-3}) \cos(\theta_6) \sin(\theta_5) + (9.5 \times 10^{-4}) \cos(\theta_9) \sin(\theta_{10}) \\
& + (2.5 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{12}) + (2.5 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_{11}) - (4.5 \times 10^{-3}) \sin(\theta_3) \sin(\theta_5) \\
& - (4.5 \times 10^{-3}) \sin(\theta_9) \sin(\theta_{11}) + (1.9 \times 10^{-4}) \cos(\theta_4)^2 - (2.1 \times 10^{-3}) \cos(\theta_5)^2 + (2.5 \times 10^{-4}) \cos(\theta_6)^2 \\
& + (1.9 \times 10^{-4}) \cos(\theta_{10})^2 - (2.1 \times 10^{-3}) \cos(\theta_{11})^2 + (2.5 \times 10^{-4}) \cos(\theta_{12})^2 - (1.6 \times 10^{-3}) \cos(\theta_5)^2 \cos(\theta_6) \\
& - (1.6 \times 10^{-3}) \cos(\theta_{11})^2 \cos(\theta_{12}) + (2.1 \times 10^{-3}) \cos(\theta_4)^2 \cos(\theta_5)^2 - (2.5 \times 10^{-4}) \cos(\theta_4)^2 \cos(\theta_6)^2 \\
& - (5.1 \times 10^{-4}) \cos(\theta_5)^2 \cos(\theta_6)^2 + (2.1 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 - (2.5 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{12})^2 \\
& - (5.1 \times 10^{-4}) \cos(\theta_{11})^2 \cos(\theta_{12})^2 + (7.1 \times 10^{-4}) \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) - (7.1 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_{10}) \\
& + (5.1 \times 10^{-4}) \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6)^2 + (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12})^2 - (1.1 \times 10^{-3}) \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) \\
& + (1.6 \times 10^{-3}) \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) - (1.1 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12}) - (1.1 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) \\
& + (1.6 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) + (1.6 \times 10^{-3}) \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6) - (1.1 \times 10^{-3}) \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) \\
& + (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12}) - (4.5 \times 10^{-3}) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - (4.5 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \\
& - (1.6 \times 10^{-3}) \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) - (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) \\
& - (1.1 \times 10^{-3}) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) + (5.1 \times 10^{-4}) \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) \\
& - (1.1 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) + (1.1 \times 10^{-3}) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) \sin(\theta_6) \\
& + (1.1 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) + (5.1 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) \\
& - (5.1 \times 10^{-4}) \cos(\theta_4)^2 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) - (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12})
\end{aligned}$$

The elements of the inertia matrix for the right arm \mathbf{M}_r is given by,

$$\begin{aligned}
\mathbf{M}_r(\mathbf{1}, \mathbf{1}) = & 0.001601 \cos(\theta_6) - 0.01525 \sin(\theta_3) + 0.0106 \sin(\theta_5) - 0.00095 \cos(\theta_3) \sin(\theta_4) \\
& + 0.002535 \cos(\theta_5) \sin(\theta_6) + 0.002535 \cos(\theta_6) \sin(\theta_5) - 0.004461 \sin(\theta_3) \sin(\theta_5) \\
& + (4.8 \times 10^{-8}) \cos(\theta_3)^2 - 0.001601 \cos(\theta_5)^2 \cos(\theta_6) + 0.002136 \cos(\theta_4)^2 \cos(\theta_5)^2 \\
& + 0.0001953 \cos(\theta_4)^2 - 0.002136 \cos(\theta_5)^2 + 0.0002544 \cos(\theta_6)^2 \\
& - (2.88 \times 10^{-7}) \cos(\theta_4)^2 \sin(\theta_3) + (8.0 \times 10^{-8}) \cos(\theta_4)^2 \sin(\theta_5) - (6.4 \times 10^{-8}) \cos(\theta_3)^2 \cos(\theta_4)^2 \\
& + (7.105 \times 10^{-15}) \cos(\theta_3)^2 \cos(\theta_5)^2 - (8.0 \times 10^{-20}) \cos(\theta_3)^2 \cos(\theta_6)^2 \\
& - 0.0002544 \cos(\theta_4)^2 \cos(\theta_6)^2 - 0.0005087 \cos(\theta_5)^2 \cos(\theta_6)^2 + 0.0007124 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \\
& - (7.105 \times 10^{-15}) \cos(\theta_3)^2 \cos(\theta_5)^2 \cos(\theta_6)^2 \\
& + 0.0005087 \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6)^2 + 0.001601 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) \\
& - 0.001067 \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) - 0.001067 \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) \\
& - (3.2 \times 10^{-8}) \cos(\theta_4)^2 \sin(\theta_3) \sin(\theta_5) + 0.001601 \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6) + 0.02433 \\
& - 0.004461 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.001601 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) \\
& + 0.001067 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) \sin(\theta_6) + 0.0005087 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) \\
& - 0.001067 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) - 0.0005087 \cos(\theta_4)^2 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{1}, \mathbf{2}) &= 0.001601 \cos(\theta_6) - 0.007623 \sin(\theta_3) + 0.0106 \sin(\theta_5) - 0.000475 \cos(\theta_3) \sin(\theta_4) \\
&+ 0.002535 \cos(\theta_5) \sin(\theta_6) + 0.002535 \cos(\theta_6) \sin(\theta_5) - 0.002231 \sin(\theta_3) \sin(\theta_5) \\
&- 0.002136 \cos(\theta_5)^2 + 0.0002544 \cos(\theta_6)^2 - 0.001601 \cos(\theta_5)^2 \cos(\theta_6) \\
&+ 0.002136 \cos(\theta_4)^2 \cos(\theta_5)^2 - 0.0002544 \cos(\theta_4)^2 \cos(\theta_6)^2 - 0.0005087 \cos(\theta_5)^2 \cos(\theta_6)^2 \\
&+ 0.0007124 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) + 0.0005087 \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6)^2 \\
&- 0.0005336 \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) - 0.0005336 \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) \\
&+ 0.001601 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) - (2.0 \times 10^{-8}) \cos(\theta_4)^2 \sin(\theta_3) \sin(\theta_5) \\
&+ 0.001601 \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6) - 0.002231 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \\
&- 0.001601 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) - 0.0005336 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \\
&+ (8.0 \times 10^{-9}) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_5) + 0.0005336 \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) \sin(\theta_6) \\
&+ 0.0005087 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) - 0.0005087 \cos(\theta_4)^2 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) \\
&+ 0.01928 + 0.0001951 \cos(\theta_4)^2 - (3.0 \times 10^{-7}) \cos(\theta_4)^2 \sin(\theta_5)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{1}, \mathbf{3}) &= 0.001128 \cos(\theta_4) - 0.000475 \cos(\theta_4) \sin(\theta_3) + 0.0003562 \cos(\theta_4) \sin(\theta_5) \\
&- 0.005298 \cos(\theta_5) \sin(\theta_4) + 0.0008004 \sin(\theta_4) \sin(\theta_6) - 0.001267 \cos(\theta_5) \cos(\theta_6) \sin(\theta_4) \\
&+ 0.002231 \cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - 0.002136 \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) \\
&+ 0.0002544 \cos(\theta_6) \sin(\theta_4) \sin(\theta_6) + 0.001267 \sin(\theta_4) \sin(\theta_5) \sin(\theta_6) \\
&- 0.001601 \cos(\theta_5)^2 \sin(\theta_4) \sin(\theta_6) - 0.0005336 \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) \sin(\theta_6) \\
&- 0.0005087 \cos(\theta_5) \cos(\theta_6)^2 \sin(\theta_4) \sin(\theta_5) - 0.0005087 \cos(\theta_5)^2 \cos(\theta_6) \sin(\theta_4) \sin(\theta_6) \\
&+ 0.0005336 \cos(\theta_5) \cos(\theta_6) \sin(\theta_3) \sin(\theta_4) - 0.001601 \cos(\theta_5) \cos(\theta_6) \sin(\theta_4) \sin(\theta_5)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{1}, \mathbf{4}) &= 0.002231 \cos(\theta_3) \cos(\theta_5) - 0.003386 \cos(\theta_4) - 0.001601 \cos(\theta_4) \cos(\theta_6) \\
&- 0.0003562 \cos(\theta_5) \sin(\theta_4) - 0.001267 \cos(\theta_4) \cos(\theta_5) \sin(\theta_6) - 0.005298 \cos(\theta_4) \sin(\theta_5) \\
&- 0.001267 \cos(\theta_4) \cos(\theta_6) \sin(\theta_5) + 0.002231 \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) \\
&- 0.0005336 \cos(\theta_3) \sin(\theta_5) \sin(\theta_6) + 0.0005336 \cos(\theta_3) \cos(\theta_5) \cos(\theta_6) \\
&+ 0.0005336 \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) + 0.0005336 \cos(\theta_4) \cos(\theta_6) \sin(\theta_3) \sin(\theta_5)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{1}, \mathbf{5}) &= 0.0002668 \sin(\theta_5 + \theta_6) \sin(\theta_3 - 1.0 \theta_4) - 0.0004535 \cos(\theta_4) - 0.0008004 \cos(\theta_4) \cos(\theta_6) \\
&+ 0.0002668 \sin(\theta_3 + \theta_4) \sin(\theta_5 + \theta_6) + 0.0005336 \cos(\theta_5 + \theta_6) \cos(\theta_3) \\
&- 0.001267 \sin(\theta_5 + \theta_6) \cos(\theta_4)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{2}, \mathbf{2}) &= 0.001601 \cos(\theta_6) + 0.0106 \sin(\theta_5) + 0.002535 \cos(\theta_5) \sin(\theta_6) + 0.002535 \cos(\theta_6) \sin(\theta_5) \\
&+ 0.0001948 \cos(\theta_4)^2 - 0.002136 \cos(\theta_5)^2 + 0.0002544 \cos(\theta_6)^2 \\
&+ \left(8.0 \times 10^{-8}\right) \cos(\theta_4)^2 \sin(\theta_5) + 0.002136 \cos(\theta_4)^2 \cos(\theta_5)^2 \\
&- 0.0002544 \cos(\theta_4)^2 \cos(\theta_6)^2 + 0.0005087 \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6)^2 \\
&- 0.0005087 \cos(\theta_5)^2 \cos(\theta_6)^2 + 0.0007124 \cos(\theta_4) \cos(\theta_5) \sin(\theta_4) \\
&+ 0.001601 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) - \left(2.4 \times 10^{-8}\right) \cos(\theta_4)^2 \cos(\theta_6) \sin(\theta_5) \\
&+ 0.001601 \cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_6) - 0.001601 \cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_5) \sin(\theta_6) \\
&+ 0.0005087 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) - 0.001601 \cos(\theta_5)^2 \cos(\theta_6) \\
&- 0.0005087 \cos(\theta_4)^2 \cos(\theta_5) \cos(\theta_6) \sin(\theta_5) \sin(\theta_6) + 0.01928
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{2}, \mathbf{3}) &= 0.001128 \cos(\theta_4) + 0.0003562 \cos(\theta_4) \sin(\theta_5) - 0.005298 \cos(\theta_5) \sin(\theta_4) \\
&+ 0.0008004 \sin(\theta_4) \sin(\theta_6) - 0.001267 \cos(\theta_5) \cos(\theta_6) \sin(\theta_4) - 0.002136 \cos(\theta_5) \sin(\theta_4) \sin(\theta_5) \\
&+ 0.0002544 \cos(\theta_6) \sin(\theta_4) \sin(\theta_6) + 0.001267 \sin(\theta_4) \sin(\theta_5) \sin(\theta_6) \\
&- 0.001601 \cos(\theta_5)^2 \sin(\theta_4) \sin(\theta_6) - 0.0005087 \cos(\theta_5) \cos(\theta_6)^2 \sin(\theta_4) \sin(\theta_5) \\
&- 0.001601 \cos(\theta_5) \cos(\theta_6) \sin(\theta_4) \sin(\theta_5) - 0.0005087 \cos(\theta_5)^2 \cos(\theta_6) \sin(\theta_4) \sin(\theta_6)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{2}, \mathbf{4}) &= -0.003386 \cos(\theta_4) - 0.001601 \cos(\theta_4) \cos(\theta_6) - 0.005298 \cos(\theta_4) \sin(\theta_5) \\
&- 0.0003562 \cos(\theta_5) \sin(\theta_4) - 0.001267 \cos(\theta_4) \cos(\theta_5) \sin(\theta_6) - 0.001267 \cos(\theta_4) \cos(\theta_6) \sin(\theta_5)
\end{aligned}$$

$$\mathbf{M}_r(\mathbf{2}, \mathbf{5}) = -\cos(\theta_4) (0.001267 \sin(\theta_5 + \theta_6) + 0.0008004 \cos(\theta_6) + 0.0004535)$$

$$\begin{aligned}
\mathbf{M}_r(\mathbf{3}, \mathbf{3}) &= -\left(4.441 \times 10^{-16}\right) \cos(\theta_5 - \theta_6)^2 + 0.001601 \cos\left(\theta_5 + \frac{\theta_6}{2}\right)^2 \\
&+ 0.001601 \cos(0.5 \theta_6)^2 + 0.0002544 \cos(\theta_5 + \theta_6)^2 + 0.00239 \cos(\theta_5)^2 - 0.0007508
\end{aligned}$$

$$\mathbf{M}_r(\mathbf{3}, \mathbf{4}) = -0.0003562 \sin(\theta_5)$$

$$\mathbf{M}_r(\mathbf{3}, \mathbf{5}) = 0$$

$$\mathbf{M}_r(\mathbf{4}, \mathbf{4}) = -\left(2.0 \times 10^{-8}\right) \cos(\theta_6)^2 + 0.001601 \cos(\theta_6) + 0.003386$$

$$\mathbf{M}_r(\mathbf{4}, \mathbf{5}) = 0.0008004 \cos(\theta_6) + 0.0004535$$

$$\mathbf{M}_r(\mathbf{5}, \mathbf{5}) = 0.0004535$$

The elements of the inertia matrix of the left arm \mathbf{M}_1 is given by,

$$\begin{aligned}
\mathbf{M}_1(\mathbf{1}, \mathbf{1}) = & (1.6 \times 10^{-3}) \cos(\theta_{12}) - (1.5 \times 10^{-2}) \sin(\theta_9) + (1.1 \times 10^{-2}) \sin(\theta_{11}) \\
& + (9.5 \times 10^{-4}) \cos(\theta_9) \sin(\theta_{10}) + (2.5 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{12}) + (2.5 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_{11}) \\
& - (4.5 \times 10^{-3}) \sin(\theta_9) \sin(\theta_{11}) + (1.9 \times 10^{-4}) \cos(\theta_{10})^2 - (2.1 \times 10^{-3}) \cos(\theta_{11})^2 \\
& + (2.5 \times 10^{-4}) \cos(\theta_{12})^2 - (1.6 \times 10^{-3}) \cos(\theta_{11})^2 \cos(\theta_{12}) + (2.1 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \\
& - (2.5 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{12})^2 - (5.1 \times 10^{-4}) \cos(\theta_{11})^2 \cos(\theta_{12})^2 - (7.1 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_{10}) \\
& + (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12})^2 - (1.1 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12}) \\
& - (1.1 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) + (1.6 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) + (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12}) \\
& - (4.5 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) - (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) \\
& - (1.1 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) + (1.1 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + (5.1 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) - (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) + 2.4 \times 10^{-2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_1(\mathbf{1}, \mathbf{2}) = & (1.6 \times 10^{-3}) \cos(\theta_{12}) - (7.6 \times 10^{-3}) \sin(\theta_9) + (1.1 \times 10^{-2}) \sin(\theta_{11}) + (4.7 \times 10^{-4}) \cos(\theta_9) \sin(\theta_{10}) \\
& + (2.5 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{12}) + (2.5 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_{11}) - (2.2 \times 10^{-3}) \sin(\theta_9) \sin(\theta_{11}) \\
& + (1.9 \times 10^{-4}) \cos(\theta_{10})^2 - (2.1 \times 10^{-3}) \cos(\theta_{11})^2 + (2.5 \times 10^{-4}) \cos(\theta_{12})^2 - (1.6 \times 10^{-3}) \cos(\theta_{11})^2 \cos(\theta_{12}) \\
& + (2.1 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 - (2.5 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{12})^2 - (5.1 \times 10^{-4}) \cos(\theta_{11})^2 \cos(\theta_{12})^2 \\
& - (7.1 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_{10}) + (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12})^2 - (5.3 \times 10^{-4}) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12}) \\
& - (5.3 \times 10^{-4}) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) + (1.6 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) + (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12}) \\
& - (2.2 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) - (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) \\
& - (5.3 \times 10^{-4}) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) + (5.3 \times 10^{-4}) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + (5.1 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) - (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) + 1.9 \times 10^{-2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_1(\mathbf{1}, \mathbf{3}) = & (4.7 \times 10^{-4}) \cos(\theta_{10}) \sin(\theta_9) - (1.1 \times 10^{-3}) \cos(\theta_{10}) - (3.6 \times 10^{-4}) \cos(\theta_{10}) \sin(\theta_{11}) \\
& - (5.3 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{10}) + (8.0 \times 10^{-4}) \sin(\theta_{10}) \sin(\theta_{12}) \\
& - (1.3 \times 10^{-3}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{10}) + (2.2 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{10}) \\
& - (2.1 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{10}) \sin(\theta_{11}) + (2.5 \times 10^{-4}) \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{12}) \\
& + (1.3 \times 10^{-3}) \sin(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) - (1.6 \times 10^{-3}) \cos(\theta_{11})^2 \sin(\theta_{10}) \sin(\theta_{12}) \\
& - (5.3 \times 10^{-4}) \sin(\theta_9) \sin(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) - (5.1 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12})^2 \sin(\theta_{10}) \sin(\theta_{11}) \\
& - (5.1 \times 10^{-4}) \cos(\theta_{11})^2 \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{12}) + (5.3 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{10}) \\
& - (1.6 \times 10^{-3}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{11})
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_1(\mathbf{1}, \mathbf{4}) = & (2.2 \times 10^{-3}) \cos(\theta_9) \cos(\theta_{11}) - (3.4 \times 10^{-3}) \cos(\theta_{10}) - (1.6 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{12}) \\
& - (5.3 \times 10^{-3}) \cos(\theta_{10}) \sin(\theta_{11}) + (3.6 \times 10^{-4}) \cos(\theta_{11}) \sin(\theta_{10}) + (5.3 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) \\
& - (1.3 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_{12}) - (1.3 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{12}) \sin(\theta_{11}) \\
& + (2.2 \times 10^{-3}) \cos(\theta_{10}) \sin(\theta_9) \sin(\theta_{11}) - (5.3 \times 10^{-4}) \cos(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + (5.3 \times 10^{-4}) \cos(\theta_9) \cos(\theta_{11}) \cos(\theta_{12}) + (5.3 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12})
\end{aligned}$$

$$\begin{aligned} \mathbf{M}_1(1, 5) &= (2.7 \times 10^{-4}) \sin(\theta_{11} + \theta_{12}) \sin(\theta_9 - (1.0 \times 10^0) \theta_{10}) - (4.5 \times 10^{-4}) \cos(\theta_{10}) \\ &\quad - (8.0 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{12}) + (2.7 \times 10^{-4}) \sin(\theta_9 + \theta_{10}) \sin(\theta_{11} + \theta_{12}) \\ &\quad + (5.3 \times 10^{-4}) \cos(\theta_{11} + \theta_{12}) \cos(\theta_9) - (1.3 \times 10^{-3}) \sin(\theta_{11} + \theta_{12}) \cos(\theta_{10}) \end{aligned}$$

$$\begin{aligned} \mathbf{M}_1(2, 2) &= (1.6 \times 10^{-3}) \cos(\theta_{12}) + (1.1 \times 10^{-2}) \sin(\theta_{11}) + (2.5 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{12}) \\ &\quad + (2.5 \times 10^{-3}) \cos(\theta_{12}) \sin(\theta_{11}) + (1.9 \times 10^{-4}) \cos(\theta_{10})^2 - (2.1 \times 10^{-3}) \cos(\theta_{11})^2 + (2.5 \times 10^{-4}) \cos(\theta_{12})^2 \\ &\quad - (1.6 \times 10^{-3}) \cos(\theta_{11})^2 \cos(\theta_{12}) + (2.1 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 - (2.5 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{12})^2 \\ &\quad - (5.1 \times 10^{-4}) \cos(\theta_{11})^2 \cos(\theta_{12})^2 - (7.1 \times 10^{-4}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_{10}) \\ &\quad + (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12})^2 + (1.6 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) \\ &\quad + (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11})^2 \cos(\theta_{12}) - (1.6 \times 10^{-3}) \cos(\theta_{10})^2 \cos(\theta_{11}) \sin(\theta_{11}) \sin(\theta_{12}) \\ &\quad + (5.1 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) - (5.1 \times 10^{-4}) \cos(\theta_{10})^2 \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{11}) \sin(\theta_{12}) + 1.9 \times 10^{-2} \end{aligned}$$

$$\begin{aligned} \mathbf{M}_1(2, 3) &= (8.0 \times 10^{-4}) \sin(\theta_{10}) \sin(\theta_{12}) - (3.6 \times 10^{-4}) \cos(\theta_{10}) \sin(\theta_{11}) \\ &\quad - (5.3 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{10}) - (1.1 \times 10^{-3}) \cos(\theta_{10}) - (1.3 \times 10^{-3}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{10}) \\ &\quad - (2.1 \times 10^{-3}) \cos(\theta_{11}) \sin(\theta_{10}) \sin(\theta_{11}) + (2.5 \times 10^{-4}) \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{12}) \\ &\quad + (1.3 \times 10^{-3}) \sin(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) - (1.6 \times 10^{-3}) \cos(\theta_{11})^2 \sin(\theta_{10}) \sin(\theta_{12}) \\ &\quad - (5.1 \times 10^{-4}) \cos(\theta_{11}) \cos(\theta_{12})^2 \sin(\theta_{10}) \sin(\theta_{11}) - (5.1 \times 10^{-4}) \cos(\theta_{11})^2 \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{12}) \\ &\quad - (1.6 \times 10^{-3}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{11}) \end{aligned}$$

$$\begin{aligned} \mathbf{M}_1(2, 4) &= (3.6 \times 10^{-4}) \cos(\theta_{11}) \sin(\theta_{10}) - (1.6 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{12}) \\ &\quad - (5.3 \times 10^{-3}) \cos(\theta_{10}) \sin(\theta_{11}) - (3.4 \times 10^{-3}) \cos(\theta_{10}) - (1.3 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_{12}) \\ &\quad - (1.3 \times 10^{-3}) \cos(\theta_{10}) \cos(\theta_{12}) \sin(\theta_{11}) \end{aligned}$$

$$\mathbf{M}_1(2, 5) = - (1.0 \times 10^0) \cos(\theta_{10}) \left((1.3 \times 10^{-3}) \sin(\theta_{11} + \theta_{12}) + (8.0 \times 10^{-4}) \cos(\theta_{12}) + 4.5 \times 10^{-4} \right)$$

$$\begin{aligned} \mathbf{M}_1(3, 3) &= (1.6 \times 10^{-3}) \cos(\theta_{11} + (5.0 \times 10^{-1}) \theta_{12})^2 + (1.6 \times 10^{-3}) \cos((5.0 \times 10^{-1}) \theta_{12})^2 \\ &\quad + (2.5 \times 10^{-4}) \cos(\theta_{11} + \theta_{12})^2 + (2.4 \times 10^{-3}) \cos(\theta_{11})^2 - 7.5 \times 10^{-4} \end{aligned}$$

$$\mathbf{M}_1(3, 4) = (3.6 \times 10^{-4}) \sin(\theta_{11})$$

$$\mathbf{M}_1(3, 5) = 0$$

$$\mathbf{M}_1(4, 4) = (1.6 \times 10^{-3}) \cos(\theta_{12}) + 3.4 \times 10^{-3}$$

$$\mathbf{M}_1(4, 5) = (8.0 \times 10^{-4}) \cos(\theta_{12}) + 4.5 \times 10^{-4}$$

$$\mathbf{M}_1(5, 5) = 4.5 \times 10^{-4}$$

The elements of the dynamic coupling matrix between the base and the right manipulator arm are given by,

$$\begin{aligned}
\mathbf{M}_{\mathbf{br}}(1, 1) &= 0.052 \sin(x_2) + 0.095 \cos(\theta_2) \cos(\theta_3) - 0.095 \sin(\theta_2) \sin(\theta_3) + 0.028 \cos(\theta_2) \cos(\theta_3) \sin(\theta_5) \\
&\quad - \left(5.9 \times 10^{-3}\right) \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) - \left(5.9 \times 10^{-3}\right) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \\
&\quad - 0.028 \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - 0.028 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - 0.028 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_6) + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_5) \\
&\quad - \left(6.7 \times 10^{-3}\right) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_6) - \left(6.7 \times 10^{-3}\right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \\
&\quad - \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_3) - \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) + \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) \sin(\theta_6)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{br}}(1, 2) &= 0.095 \cos(\theta_2) \cos(\theta_3) - 0.095 \sin(\theta_2) \sin(\theta_3) + 0.028 \cos(\theta_2) \cos(\theta_3) \sin(\theta_5) + \left(5.9 \times 10^{-3}\right) \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \\
&\quad + \left(5.9 \times 10^{-3}\right) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) - 0.028 \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - 0.028 \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \\
&\quad - 0.028 \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_6) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_5) - \left(6.7 \times 10^{-3}\right) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_6) \\
&\quad - \left(6.7 \times 10^{-3}\right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) - \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_3) \\
&\quad - \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) \sin(\theta_6)
\end{aligned}$$

$$\mathbf{M}_{\mathbf{br}}(1, 3) = -1.0 \cos(\theta_2 + \theta_3) \left(\left(5.9 \times 10^{-3}\right) \cos(\theta_4) + \sin(\theta_4) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_5 + \theta_6) + 0.028 \cos(\theta_5) \right) \right)$$

$$\mathbf{M}_{\mathbf{br}}(1, 5) = 6.7 \times 10^{-3} \cos(\theta_5 + \theta_6) \sin(\theta_2 + \theta_3) + 6.7 \times 10^{-3} \sin(\theta_5 + \theta_6) (\sin(\theta_3) - \cos(\theta_3))$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{br}}(2, 2) &= 0.095 \cos(\theta_2) \sin(\theta_3) + 0.095 \cos(\theta_3) \sin(\theta_2) - \left(5.9 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \\
&\quad + 0.028 \cos(\theta_2) \sin(\theta_3) \sin(\theta_5) + 0.028 \cos(\theta_3) \sin(\theta_2) \sin(\theta_5) + \left(5.9 \times 10^{-3}\right) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \\
&\quad + 0.028 \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) - 0.028 \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) + \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_5) \sin(\theta_2) \sin(\theta_6) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_3) \cos(\theta_6) \sin(\theta_2) \sin(\theta_5) + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \\
&\quad - \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \sin(\theta_5) \sin(\theta_6) - \left(6.7 \times 10^{-3}\right) \cos(\theta_4) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \\
&\quad + \left(6.7 \times 10^{-3}\right) \cos(\theta_4) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) + \left(6.7 \times 10^{-3}\right) \cos(\theta_2) \cos(\theta_5) \sin(\theta_3) \sin(\theta_6)
\end{aligned}$$

$$\mathbf{M}_{\mathbf{br}}(2, 3) = -1.0 \sin(\theta_2 + \theta_3) \left(\left(5.9 \times 10^{-3}\right) \cos(\theta_4) + \sin(\theta_4) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_5 + \theta_6) + 0.028 \cos(\theta_5) \right) \right)$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{br}}(2, 5) &= \sin(\theta_2) \left(\sin(\theta_3) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_5 + \theta_6) \right) \right) - \sin(\theta_2) \left(1.0 \cos(\theta_3) \cos(\theta_4) \left(\left(6.7 \times 10^{-3}\right) \sin(\theta_5 + \theta_6) \right) \right) \\
&\quad - 1.0 \cos(\theta_2) \left(\cos(\theta_3) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_5 + \theta_6) \right) + \cos(\theta_4) \sin(\theta_3) \left(\left(6.7 \times 10^{-3}\right) \sin(\theta_6 + \theta_5) \right) \right)
\end{aligned}$$

$$\begin{aligned} \mathbf{M}_{\mathbf{br}}(\mathbf{3}, \mathbf{3}) &= \cos(\theta_4) \left(0.028 \cos(\theta_5) + \left(6.7 \times 10^{-3} \right) \cos(\theta_5) \cos(\theta_6) - \left(6.7 \times 10^{-3} \right) \sin(\theta_5) \sin(\theta_6) \right) \\ &\quad - 1.0 \sin(\theta_4) \left(\left(5.9 \times 10^{-3} \right) \cos(\theta_5)^2 + \left(5.9 \times 10^{-3} \right) \sin(\theta_5)^2 \right) \end{aligned}$$

$$\mathbf{M}_{\mathbf{br}}(\mathbf{3}, \mathbf{4}) = -1.0 \sin(\theta_4) \left(\left(6.7 \times 10^{-3} \right) \sin(\theta_5 + \theta_6) + 0.028 \sin(\theta_5) \right)$$

$$\mathbf{M}_{\mathbf{br}}(\mathbf{3}, \mathbf{5}) = -1.0 \sin(\theta_4) \left(\left(6.7 \times 10^{-3} \right) \cos(\theta_5) \sin(\theta_6) + \left(6.7 \times 10^{-3} \right) \cos(\theta_6) \sin(\theta_5) \right)$$

$$\begin{aligned} \mathbf{M}_{\mathbf{br}}(\mathbf{4}, \mathbf{4}) &= \left(3.4 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \left(3.6 \times 10^{-4} \right) \cos(\theta_2) \sin(\theta_3) \sin(\theta_5) + \left(3.6 \times 10^{-4} \right) \cos(\theta_3) \sin(\theta_2) \sin(\theta_5) \\ &\quad + \left(6.1 \times 10^{-3} \right) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - \left(3.4 \times 10^{-3} \right) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \left(2.2 \times 10^{-3} \right) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) \\ &\quad - \left(6.1 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) - \left(5.3 \times 10^{-3} \right) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) \\ &\quad - \left(1.5 \times 10^{-3} \right) \sin(\theta_2) \sin(\theta_3) \sin(\theta_5) \sin(\theta_6) + \left(3.6 \times 10^{-4} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \\ &\quad - \left(1.5 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \cos(\theta_6) + \left(1.6 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_4) \\ &\quad + \left(5.3 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) \sin(\theta_5) - \left(6.1 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) \sin(\theta_5) \\ &\quad - \left(6.1 \times 10^{-3} \right) \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) \sin(\theta_5) - \left(3.6 \times 10^{-4} \right) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \\ &\quad + \left(1.5 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \sin(\theta_5) \sin(\theta_6) + \left(1.5 \times 10^{-3} \right) \cos(\theta_5) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \\ &\quad - \left(1.6 \times 10^{-3} \right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + \left(5.3 \times 10^{-4} \right) \cos(\theta_5) \sin(\theta_2) \sin(\theta_4) \sin(\theta_6) \\ &\quad + \left(5.3 \times 10^{-4} \right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_4) \sin(\theta_5) + \left(1.3 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_5) \sin(\theta_4) \sin(\theta_6) \\ &\quad + \left(1.3 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_4) \sin(\theta_5) - \left(1.5 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_4) \cos(\theta_5) \sin(\theta_3) \sin(\theta_6) \\ &\quad - \left(1.5 \times 10^{-3} \right) \cos(\theta_2) \cos(\theta_4) \cos(\theta_6) \sin(\theta_3) \sin(\theta_5) - \left(1.5 \times 10^{-3} \right) \cos(\theta_3) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \sin(\theta_6) \\ &\quad - \left(1.5 \times 10^{-3} \right) \cos(\theta_3) \cos(\theta_4) \cos(\theta_6) \sin(\theta_2) \sin(\theta_5) - \left(1.3 \times 10^{-3} \right) \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_6) \\ &\quad - \left(1.3 \times 10^{-3} \right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \sin(\theta_5) \end{aligned}$$

$$\begin{aligned} \mathbf{M}_{\mathbf{br}}(\mathbf{4}, \mathbf{5}) &= \left(4.5 \times 10^{-4} \right) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - \left(4.5 \times 10^{-4} \right) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \\ &\quad - \left(7.3 \times 10^{-4} \right) \cos(\theta_3 + \theta_4) \sin(\theta_5 + \theta_6) \sin(\theta_2) - \left(7.3 \times 10^{-4} \right) \sin(\theta_3 + \theta_4) \sin(\theta_5 + \theta_6) \cos(\theta_2) \\ &\quad - \left(1.5 \times 10^{-3} \right) \cos(\theta_5 + \theta_6) \cos(\theta_2) \cos(\theta_3) \\ &\quad - \left(7.3 \times 10^{-4} \right) \sin(\theta_5 + \theta_6) \cos(\theta_3 - 1.0 \theta_4) \sin(\theta_2) - \left(7.3 \times 10^{-4} \right) \sin(\theta_5 + \theta_6) \cos(\theta_2) \sin(\theta_3 - 1.0 \theta_4) \\ &\quad + \left(1.5 \times 10^{-3} \right) \cos(\theta_5 + \theta_6) \sin(\theta_2) \sin(\theta_3) + \left(5.3 \times 10^{-4} \right) \sin(\theta_5 + \theta_6) \sin(\theta_2) \sin(\theta_4) \\ &\quad + \left(1.3 \times 10^{-3} \right) \sin(\theta_5 + \theta_6) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) + \left(8.0 \times 10^{-4} \right) \cos(\theta_2) \cos(\theta_3) \cos(\theta_6) \sin(\theta_4) \\ &\quad - \left(1.3 \times 10^{-3} \right) \sin(\theta_5 + \theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) - \left(8.0 \times 10^{-4} \right) \cos(\theta_6) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) \end{aligned}$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{br}}(\mathbf{5}, \mathbf{5}) = & \left(4.5 \times 10^{-4}\right) \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) + \left(4.5 \times 10^{-4}\right) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) \\
& + \left(7.3 \times 10^{-4}\right) \sin(\theta_5 + \theta_6) \cos(\theta_3 - 1.0 \theta_4) \cos(\theta_2) - \left(7.3 \times 10^{-4}\right) \sin(\theta_3 + \theta_4) \sin(\theta_5 + \theta_6) \sin(\theta_2) \\
& - \left(1.5 \times 10^{-3}\right) \cos(\theta_5 + \theta_6) \cos(\theta_2) \sin(\theta_3) - \left(1.5 \times 10^{-3}\right) \cos(\theta_5 + \theta_6) \cos(\theta_3) \sin(\theta_2) \\
& - \left(7.3 \times 10^{-4}\right) \sin(\theta_5 + \theta_6) \sin(\theta_3 - 1.0 \theta_4) \sin(\theta_2) + \left(1.3 \times 10^{-3}\right) \sin(\theta_5 + \theta_6) \cos(\theta_2) \sin(\theta_3) \sin(\theta_4) \\
& + \left(1.3 \times 10^{-3}\right) \sin(\theta_5 + \theta_6) \cos(\theta_3) \sin(\theta_2) \sin(\theta_4) + \left(8.0 \times 10^{-4}\right) \cos(\theta_2) \cos(\theta_6) \sin(\theta_3) \sin(\theta_4) \\
& + \left(8.0 \times 10^{-4}\right) \cos(\theta_3) \cos(\theta_6) \sin(\theta_2) \sin(\theta_4) - \left(5.3 \times 10^{-4}\right) \sin(\theta_5 + \theta_6) \cos(\theta_2) \sin(\theta_4) \\
& \left(7.3 \times 10^{-4}\right) \cos(\theta_3 + \theta_4) \sin(\theta_5 + \theta_6) \cos(\theta_2)
\end{aligned}$$

The elements of the dynamic coupling matrix between the base and the left manipulator arm are given by,

$$\begin{aligned}
\mathbf{M}_{\mathbf{b1}}(\mathbf{1}, \mathbf{1}) = & 0.052 \sin(\theta_8) + 0.095 \cos(\theta_8) \cos(\theta_9) - 0.095 \sin(\theta_8) \sin(\theta_9) + 0.028 \cos(\theta_8) \cos(\theta_9) \sin(\theta_{11}) \\
& + \left(5.9 \times 10^{-3}\right) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{10}) - 0.028 \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) - 0.028 \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_9) \\
& - 0.028 \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_{12}) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{12}) - \left(6.7 \times 10^{-3}\right) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_9) - \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + \left(5.9 \times 10^{-3}\right) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{10}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{11})
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{b1}}(\mathbf{1}, \mathbf{2}) = & 0.095 \cos(\theta_8) \cos(\theta_9) - 0.095 \sin(\theta_8) \sin(\theta_9) + 0.028 \cos(\theta_8) \cos(\theta_9) \sin(\theta_{11}) + \left(5.9 \times 10^{-3}\right) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \\
& + \left(5.9 \times 10^{-3}\right) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{10}) - 0.028 \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) - 0.028 \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_9) \\
& - 0.028 \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_{12}) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{11}) - \left(6.7 \times 10^{-3}\right) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{12}) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) - \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_9) \\
& - \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) \\
& + \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_{11}) \sin(\theta_{12})
\end{aligned}$$

$$\mathbf{M}_{\mathbf{b1}}(\mathbf{1}, \mathbf{3}) = -1.0 \cos(\theta_8 + \theta_9) \left(\left(5.9 \times 10^{-3}\right) \cos(\theta_{10}) + \sin(\theta_{10}) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) + 0.028 \cos(\theta_{11}) \right) \right)$$

$$\mathbf{M}_{\mathbf{b1}}(\mathbf{1}, \mathbf{5}) = 6.7 \times 10^{-3} \cos(\theta_{11} + \theta_{12}) \sin(\theta_8 + \theta_9) + 6.7 \times 10^{-3} \sin(\theta_{11} + \theta_{12}) (\sin(\theta_9) - \cos(\theta_9))$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{b1}}(\mathbf{2}, \mathbf{2}) &= 0.095 \cos(\theta_8) \sin(\theta_9) + 0.095 \cos(\theta_9) \sin(\theta_8) - \left(5.9 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) \\
&+ 0.028 \cos(\theta_8) \sin(\theta_9) \sin(\theta_{11}) + 0.028 \cos(\theta_9) \sin(\theta_8) \sin(\theta_{11}) + \left(5.9 \times 10^{-3}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \\
&+ 0.028 \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) - 0.028 \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) \\
&+ \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_{12}) \\
&+ \left(6.7 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_{11}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \\
&- \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_{11}) \sin(\theta_{12}) - \left(6.7 \times 10^{-3}\right) \cos(\theta_{10}) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \\
&+ \left(6.7 \times 10^{-3}\right) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12})
\end{aligned}$$

$$\mathbf{M}_{\mathbf{b1}}(\mathbf{2}, \mathbf{3}) = -1.0 \sin(\theta_8 + \theta_9) \left(\left(5.9 \times 10^{-3}\right) \cos(\theta_{10}) + \sin(\theta_{10}) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) + 0.028 \cos(\theta_{11}) \right) \right)$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{b1}}(\mathbf{2}, \mathbf{5}) &= \sin(\theta_8) \left(\sin(\theta_9) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) \right) \right) - \sin(\theta_8) \left(1.0 \cos(\theta_9) \cos(\theta_{10}) \left(\left(6.7 \times 10^{-3}\right) \sin(\theta_{11} + \theta_{12}) \right) \right) \\
&- 1.0 \cos(\theta_8) \left(\cos(\theta_9) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) \right) + \cos(\theta_{10}) \sin(\theta_9) \left(\left(6.7 \times 10^{-3}\right) \sin(\theta_{12} + \theta_{11}) \right) \right)
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{b1}}(\mathbf{3}, \mathbf{3}) &= \cos(\theta_{10}) \left(0.028 \cos(\theta_{11}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_{11}) \cos(\theta_{12}) - \left(6.7 \times 10^{-3}\right) \sin(\theta_{11}) \sin(\theta_{12}) \right) \\
&- 1.0 \sin(\theta_{10}) \left(\left(5.9 \times 10^{-3}\right) \cos(\theta_{11})^2 + \left(5.9 \times 10^{-3}\right) \sin(\theta_{11})^2 \right)
\end{aligned}$$

$$\mathbf{M}_{\mathbf{b1}}(\mathbf{3}, \mathbf{4}) = -1.0 \sin(\theta_{10}) \left(\left(6.7 \times 10^{-3}\right) \sin(\theta_{11} + \theta_{12}) + 0.028 \sin(\theta_{11}) \right)$$

$$\mathbf{M}_{\mathbf{b1}}(\mathbf{3}, \mathbf{5}) = -1.0 \sin(\theta_{10}) \left(\left(6.7 \times 10^{-3}\right) \cos(\theta_{11}) \sin(\theta_{12}) + \left(6.7 \times 10^{-3}\right) \cos(\theta_{12}) \sin(\theta_{11}) \right)$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{b1}}(\mathbf{4}, \mathbf{4}) &= \left(3.4 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) + \left(3.6 \times 10^{-4}\right) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{11}) + \left(3.6 \times 10^{-4}\right) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{11}) \\
&+ \left(6.1 \times 10^{-3}\right) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) - \left(3.4 \times 10^{-3}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) + \left(2.2 \times 10^{-3}\right) \sin(\theta_8) \sin(\theta_{10}) \sin(\theta_{11}) \\
&- \left(6.1 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) - \left(5.3 \times 10^{-3}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \sin(\theta_{11}) \\
&- \left(1.5 \times 10^{-3}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) + \left(3.6 \times 10^{-4}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \\
&- \left(1.5 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) \cos(\theta_{12}) + \left(1.6 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{10}) \\
&+ \left(5.3 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) \sin(\theta_{11}) - \left(6.1 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \sin(\theta_9) \sin(\theta_{11}) \\
&- \left(6.1 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \sin(\theta_8) \sin(\theta_{11}) - \left(3.6 \times 10^{-4}\right) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) \\
&+ \left(1.5 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{11}) \sin(\theta_{12}) + \left(1.5 \times 10^{-3}\right) \cos(\theta_{11}) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \\
&- \left(1.6 \times 10^{-3}\right) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) + \left(5.3 \times 10^{-4}\right) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_{10}) \sin(\theta_{12}) \\
&+ \left(5.3 \times 10^{-4}\right) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_{10}) \sin(\theta_{11}) + \left(1.3 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{11}) \sin(\theta_{10}) \sin(\theta_{12}) \\
&+ \left(1.3 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{10}) \sin(\theta_{11}) - \left(1.5 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_9) \sin(\theta_{12}) \\
&- \left(1.5 \times 10^{-3}\right) \cos(\theta_8) \cos(\theta_{10}) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{11}) - \left(1.5 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_{12}) \\
&- \left(1.5 \times 10^{-3}\right) \cos(\theta_9) \cos(\theta_{10}) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_{11}) - \left(1.3 \times 10^{-3}\right) \cos(\theta_{11}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \sin(\theta_{12}) \\
&- \left(1.3 \times 10^{-3}\right) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \sin(\theta_{11})
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{bl}}(4, 5) = & \left(4.5 \times 10^{-4}\right) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) - \left(4.5 \times 10^{-4}\right) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \\
& - \left(7.3 \times 10^{-4}\right) \cos(\theta_9 + \theta_{10}) \sin(\theta_{11} + \theta_{12}) \sin(\theta_8) - \left(7.3 \times 10^{-4}\right) \sin(\theta_9 + \theta_{10}) \sin(\theta_{11} + \theta_{12}) \cos(\theta_8) \\
& - \left(1.5 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) \cos(\theta_8) \cos(\theta_9) \\
& - \left(7.3 \times 10^{-4}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_9 - 1.0 \theta_{10}) \sin(\theta_8) - \left(7.3 \times 10^{-4}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_8) \sin(\theta_9 - 1.0 \theta_{10}) \\
& + \left(1.5 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) \sin(\theta_8) \sin(\theta_9) + \left(5.3 \times 10^{-4}\right) \sin(\theta_{11} + \theta_{12}) \sin(\theta_8) \sin(\theta_{10}) \\
& + \left(1.3 \times 10^{-3}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_8) \cos(\theta_9) \sin(\theta_{10}) + \left(8.0 \times 10^{-4}\right) \cos(\theta_8) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_{10}) \\
& - \left(1.3 \times 10^{-3}\right) \sin(\theta_{11} + \theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10}) - \left(8.0 \times 10^{-4}\right) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_9) \sin(\theta_{10})
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_{\mathbf{bl}}(5, 5) = & \left(4.5 \times 10^{-4}\right) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{10}) + \left(4.5 \times 10^{-4}\right) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{10}) \\
& + \left(7.3 \times 10^{-4}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_9 - 1.0 \theta_{10}) \cos(\theta_8) - \left(7.3 \times 10^{-4}\right) \sin(\theta_9 + \theta_{10}) \sin(\theta_{11} + \theta_{12}) \sin(\theta_8) \\
& - \left(1.5 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) \cos(\theta_8) \sin(\theta_9) - \left(1.5 \times 10^{-3}\right) \cos(\theta_{11} + \theta_{12}) \cos(\theta_9) \sin(\theta_8) \\
& - \left(7.3 \times 10^{-4}\right) \sin(\theta_{11} + \theta_{12}) \sin(\theta_9 - 1.0 \theta_{10}) \sin(\theta_8) + \left(1.3 \times 10^{-3}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_8) \sin(\theta_9) \sin(\theta_{10}) \\
& + \left(1.3 \times 10^{-3}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_9) \sin(\theta_8) \sin(\theta_{10}) + \left(8.0 \times 10^{-4}\right) \cos(\theta_8) \cos(\theta_{12}) \sin(\theta_9) \sin(\theta_{10}) \\
& + \left(8.0 \times 10^{-4}\right) \cos(\theta_9) \cos(\theta_{12}) \sin(\theta_8) \sin(\theta_{10}) - \left(5.3 \times 10^{-4}\right) \sin(\theta_{11} + \theta_{12}) \cos(\theta_8) \sin(\theta_{10}) \\
& \left(7.3 \times 10^{-4}\right) \cos(\theta_9 + \theta_{10}) \sin(\theta_{11} + \theta_{12}) \cos(\theta_8)
\end{aligned}$$

The inertia coupling matrix for the right and the left manipulator arm is given by,

$$\mathbf{M}_{rl} = \mathbf{M}_{lr} = \mathbf{0}_{5 \times 5}$$

APPENDIX C
MATLAB CODE LISTING FOR SMIRFF

C.1 MATLAB Code Listing for SMIRFF

```
% -----  
% Equations of motion for free-flying Robot with 2 5DOF arms mounted on it  
% Author: Ameya R. Godbole  
% Advisor: Dr. Kamesh Subbarao  
% Aerospace Systems Laboratory. The University Of Texas At Arlington  
% -----  
%%  
clear; clc;  
%% Right arm parameters in meters  
%d2 = 0.19;  
%a3 = 0.08;  
%d3 = 0.03;  
%d4 = 0.09;  
%da1 = 0.1;  
%d5 = 0.03;  
%a6 = 0.12;  
%d6 = 0.03;  
%a7 = 0.1;  
syms d2 a3 d3 d4 da1 d5 a6 d6 a7 real  
%%  
% mass properties in kilograms  
%m1 = 4.23;  
%m2 = 0.3;  
%m3 = 0.132358;  
%m4 = 0.188438;  
%m5 = 0.197906;  
%m6 = 0.133408;  
syms m1 m2 m3 m4 m5 m6 real  
%%  
% Inertia matrix wrt to COM of the body kilograms-m^2  
% I_c22=[0.00174692,0,0;0,0.000877221,0;0,0,0.000877221];  
% I_c33=[0.000114886,0,0;0,3.73894e-05,0;0,0,0.000118637];  
% I_c44=[0.000259367,0,0;0,4.44786e-05,0;0,0,0.000263987];  
% I_c55=[0.000291746,0,0;0,4.86811e-05,0;0,0,0.000299249];  
% I_c66=[0.00011622,0,0;0,3.70749e-05,0;0,0,0.000119971];  
%%  
% Location of COM of each link  
% P_c22 = [0;0;0];  
% P_c33 = [0;-0.045;0];  
% P_c44 = [0;0;0.05];  
% P_c55 = [0.06;0;0];  
% P_c66 = [0.05;0;0];  
%% Left arm parameters in meters  
% d8 = 0.19;  
% a9 = 0.08;  
% d9 = 0.03;  
% d10 = 0.09;  
% da2 = 0.1;  
% d11 = 0.03;  
% a12 = 0.12;  
% d12 = 0.03;
```

```

% a13 = 0.1;
syms d8 a9 d9 d10 da2 d11 a12 d12 a13 real
%%
% mass properties in kilograms
% m8 = 0.3;
% m9 = 0.132358;
% m10 = 0.188438;
% m11 = 0.197906;
% m12 = 0.133408;
syms m8 m9 m10 m11 m12 real
%Inertia matrix wrt to COM of the body kilograms-m^2
% I_c88 = [0.00174692,0,0;0,0.000877221,0;0,0,0.000877221];
% I_c99 = [0.000114886,0,0;0,3.73894e-05,0;0,0,0.000118637];
% I_c1010 = [0.000259367,0,0;0,4.44786e-05,0;0,0,0.000263987];
% I_c1111 = [0.000291746,0,0;0,4.86811e-05,0;0,0,0.000299249];
% I_c1212 = [0.00011622,0,0;0,3.70749e-05,0;0,0,0.000119971];
%
% %Location of COM of each link
% P_c88 = [0;0;0];
% P_c99 = [0;-0.045;0];
% P_c1010 = [0;0;0.05];
% P_c1111 = [0.06;0;0];
% P_c1212 = [0.05;0;0];
%% State/Joint Variables and their derivatives
% Vehicle Pose acceleration and velocity(angular and linear)
syms x y z dx dy dz ddx ddy ddd phi th psi dphi dth dps dddphi dddth ddpsi real
syms p q r u v w dp dq dr du dv dw real
% Right Arm
syms x2 x3 x4 x5 x6 dx2 dx3 dx4 dx5 dx6 ddx2 ddx3 ddx4 ddx5 ddx6 real
% Left Arm
syms x8 x9 x10 x11 x12 dx8 dx9 dx10 dx11 dx12 ddx8 ddx9 ddx10 ddx11 ddx12 real
%%
% Parameters

%%
% Thruster Force and Torque
syms Fbx Fby Fbz Fbpitch Fbyaw Fbroll real
%%Inertia matric wrt to COM of the body grams-m^2
syms I_c11 I_c22 I_c33 I_c44 I_c55 I_c66 I_c77 I_c88 I_c99 I_c1010 I_c1111 I_c1212 real
syms j1_11 j1_22 j1_33 real
syms j2_11 j2_22 j2_33 real
syms j3_11 j3_22 j3_33 real
syms j4_11 j4_22 j4_33 real
syms j5_11 j5_22 j5_33 real
syms j6_11 j6_22 j6_33 real

%I_c11=[0.1551 0 0; 0 0.1689 0; 0 0 0.1549];
%I_c22=[0.00174692,0,0; 0,0.000877221,0; 0,0,0.000877221];
%I_c33=[0.000114886,0,0; 0,3.73894e-05,0; 0,0,0.000118637];
%I_c44=[0.000259367,0,0; 0,4.44786e-05,0; 0,0,0.000263987];
%I_c55=[0.000291746,0,0; 0,4.86811e-05,0; 0,0,0.000299249];
%I_c66=[0.00011622,0,0; 0,3.70749e-05,0; 0,0,0.000119971];

```

```

I_c11 = [j1_11 0 0; 0 j1_22 0; 0 0 j1_33];
I_c22 = [j2_11 0 0; 0 j2_22 0; 0 0 j2_33];
I_c33 = [j3_11 0 0; 0 j3_22 0; 0 0 j3_33];
I_c44 = [j4_11 0 0; 0 j4_22 0; 0 0 j4_33];
I_c55 = [j5_11 0 0; 0 j5_22 0; 0 0 j5_33];
I_c66 = [j6_11 0 0; 0 j6_22 0; 0 0 j6_33];

%I_c88 = [0.00174692,0,0; 0,0.000877221,0; 0,0,0.000877221];
%I_c99 = [0.000114886,0,0; 0,3.73894e-05,0; 0,0,0.000118637];
%I_c1010 = [0.000259367,0,0; 0,4.44786e-05,0; 0,0,0.000263987];
%I_c1111 = [0.000291746,0,0; 0,4.86811e-05,0; 0,0,0.000299249];
%I_c1212 = [0.00011622,0,0; 0,3.70749e-05,0; 0,0,0.000119971];

I_c88 = I_c22;
I_c99 = I_c33;
I_c1010 = I_c44;
I_c1111 = I_c55;
I_c1212 = I_c66;

%%
%%Forward Kinematics
%%
% Rotation Matrices
R01=[cos(th)*cos(psi) cos(th)*sin(psi) -sin(th);...
     -cos(phi)*sin(psi)+sin(phi)*sin(th)*cos(psi) cos(phi)*cos(psi)+sin(phi)*sin(th)*sin(psi) sin(phi)*cos(th);...
     sin(phi)*sin(psi)+cos(phi)*sin(th)*cos(psi) -sin(phi)*cos(psi)+cos(phi)*sin(th)*sin(psi) cos(phi)*cos(th)];
R12=[cos(x2) -sin(x2) 0; sin(x2) cos(x2) 0; 0 0 1];
R23=[cos(x3) -sin(x3) 0; sin(x3) cos(x3) 0; 0 0 1];
R34=[cos(x4) -sin(x4) 0; 0 0 -1; sin(x4) cos(x4) 0];
R4Aux1=[1 0 0; 0 1 0; 0 0 1];
RAux15=[cos(x5) -sin(x5) 0; 0 0 -1; sin(x5) cos(x5) 0];
R45=simplify(R4Aux1*RAux15);
R56=[cos(x6) -sin(x6) 0; sin(x6) cos(x6) 0; 0 0 1];
R67=[1 0 0; 0 1 0; 0 0 1];
R18=[cos(x8) -sin(x8) 0; sin(x8) cos(x8) 0; 0 0 1];
R89=[cos(x9) -sin(x9) 0; sin(x9) cos(x9) 0; 0 0 1];
R910=[cos(x10) -sin(x10) 0; 0 0 -1; sin(x10) cos(x10) 0];
R10Aux2=[1 0 0; 0 1 0; 0 0 1];
RAux211=[cos(x11) -sin(x11) 0; 0 0 -1; sin(x11) cos(x11) 0];
R1011=simplify(R10Aux2*RAux211);
R1112=[cos(x12) -sin(x12) 0; sin(x12) cos(x12) 0; 0 0 1];
R1213=[1 0 0; 0 1 0; 0 0 1];

%%
% Position Vectors
%%
% Position of COM of each Body in the body attached frame
syms P_c11 P_c22 P_c33 P_c44 P_c55 P_c66 P_c88 P_c99 P_c1010 P_c1111 P_c1212 real
%
P_c11=[0;0;0];

P_c22=[0;0;0];
P_c33=[0;-d4/2;0];

```

```

P_c44=[0;0;da1/2];
P_c55=[a6/2;0;0];
P_c66=[a7/2;0;0];

P_c88 = [0;0;0];
P_c99 = [0;-d10/2;0];
P_c1010 = [0;0;da2/2];
P_c1111 = [a12/2;0;0];
P_c1212 = [a13/2;0;0];
%
syms P12 P23 P34 P4Aux1 PAux15 P45 P56 P67 P18 P89 P910 P10Aux2 PAux211 P1112 P1213 real;
%
P12 = [0;0;d2];
P23 = [-a3;0;d3];
P34 = [0;-d4;0];
P4Aux1 = [0;0;da1];
PAux15 = [0;-d5;0];
P45 = [0;-d5;da1];
P56 = [a6;0;-d6];
P67 = [a7;0;0];

P18 = [0;0;-d8];
P89 = [-a9;0;-d9];
P910 = [0;-d10;0];
P10Aux2 = [0;0;da2];
PAux211 = [0;d11;0];

P1011 = simplify(P10Aux2 + R10Aux2*PAux211);

P1112 = [a12;0;d12];
P1213 = [a13;0;0];

%%
%Angular Velocity
Omega11=[p;q;r]; %Base velocity is same as the vehicle velocity

Omega22 = simplify(R12'*Omega11 + [0;0;dx2]);
Omega33 = simplify(R23'*Omega22 + [0;0;dx3]);
Omega44 = simplify(R34'*Omega33 + [0;0;dx4]);
OmegaAux1Aux1 = Omega44;

Omega55 = simplify(R45'*OmegaAux1Aux1 + [0;0;dx5]);
Omega66 = simplify(R56'*Omega55 + [0;0;dx6]);

Omega88 = simplify(R18'*Omega11 + [0;0;dx8]);
Omega99 = simplify(R89'*Omega88 + [0;0;dx9]);
Omega1010 = simplify(R910'*Omega99 + [0;0;dx10]);
OmegaAux2Aux2 = Omega1010;

Omega1111 = simplify(R1011'*OmegaAux2Aux2 + [0;0;dx11]);
Omega1212 = simplify(R1112'*Omega1111 + [0;0;dx12]);

%%

```

```

%Linear Velocity
V11 = [u;v;w]; %Base velocity is same as the vehicle velocity

V22 = simplify(R12'*(V11+cross(Omega11,P12)));
V33 = simplify(R23'*(V22+cross(Omega22,P23)));
V44 = simplify(R34'*(V33+cross(Omega33,P34)));
VAux1Aux1 = simplify(R4Aux1'*(V44+cross(Omega44,P4Aux1)));

V55 = simplify(RAux15'*(VAux1Aux1+cross(OmegaAux1Aux1,PAux15)));
V66 = simplify(R56'*(V55+cross(Omega55,P56)));
V77 = simplify(R67'*(V66+cross(Omega66,P67)));

V88 = simplify(R18'*(V11+cross(Omega11,P18)));
V99 = simplify(R89'*(V88+cross(Omega88,P89)));
V1010 = simplify(R910'*(V99+cross(Omega99,P910)));
VAux2Aux2 = simplify(R10Aux2'*(V1010+cross(Omega1010,P10Aux2)));
V1111 = simplify(RAux211'*(VAux2Aux2+cross(OmegaAux2Aux2,PAux211)));
V1212 = simplify(R1112'*(V1111+cross(Omega1111,P1112)));
V1313 = simplify(R1213'*(V1212+cross(Omega1212,P1213)));

%%
% Angular Acceleration
Omega11_dot=[dp;dq;dr]; %Base acceleration is same as the vehicle acceleration

Omega22_dot = simplify(R12'*Omega11_dot + cross(R12'*Omega11, [0;0;dx2]) + [0;0;ddx2]);
Omega33_dot = simplify(R23'*Omega22_dot + cross(R23'*Omega22, [0;0;dx3]) + [0;0;ddx3]);
Omega44_dot = simplify(R34'*Omega33_dot + cross(R34'*Omega33, [0;0;dx4]) + [0;0;ddx4]);
OmegaAux1Aux1_dot = simplify(R4Aux1'*Omega44_dot + cross(R4Aux1'*Omega44, [0;0;0]) + [0;0;0]);
Omega55_dot = simplify(RAux15'*OmegaAux1Aux1_dot + cross(RAux15'*OmegaAux1Aux1, [0;0;dx5]) + [0;0;ddx5]);
Omega66_dot = simplify(R56'*Omega55_dot + cross(R56'*Omega55, [0;0;dx6]) + [0;0;ddx6]);
Omega77_dot = simplify(R67'*Omega66_dot + cross(R67'*Omega66, [0;0;0]) + [0;0;0]);

Omega88_dot = simplify(R18'*Omega11_dot + cross(R18'*Omega11, [0;0;dx8])+[0;0;ddx8]);
Omega99_dot = simplify(R89'*Omega88_dot + cross(R89'*Omega88, [0;0;dx9])+[0;0;ddx9]);
Omega1010_dot = simplify(R910'*Omega99_dot + cross(R910'*Omega99, [0;0;dx10])+[0;0;ddx10]);
OmegaAux2Aux2_dot = simplify(R10Aux2'*Omega1010_dot + cross(R10Aux2'*Omega1010, [0;0;0])+[0;0;0]);
Omega1111_dot = simplify(RAux211'*OmegaAux2Aux2_dot + cross(RAux211'*OmegaAux2Aux2, [0;0;dx11])+[0;0;ddx11]);
Omega1212_dot = simplify(R1112'*Omega1111_dot + cross(R1112'*Omega1111, [0;0;dx12])+[0;0;ddx12]);
Omega1313_dot = simplify(R1213'*Omega1212_dot + cross(R1213'*Omega1212, [0;0;0])+[0;0;0]);

%%
% Linear Acceleration
V11_dot=[du;dv;dw]; %Base acceleration is same as the vehicle acceleration

V22_dot=simplify(R12'*(cross(Omega11_dot,P12)+cross(Omega11,cross(Omega11,P12))+V11_dot));
V33_dot=simplify(R23'*(cross(Omega22_dot,P23)+cross(Omega22,cross(Omega22,P23))+V22_dot));
V44_dot=simplify(R34'*(cross(Omega33_dot,P34)+cross(Omega33,cross(Omega33,P34))+V33_dot));
VAux1Aux1_dot= simplify(R4Aux1'*(cross(Omega44_dot,P4Aux1)+cross(Omega44,cross(Omega44,P4Aux1))+V44_dot));
V55_dot=simplify(RAux15'*(cross(OmegaAux1Aux1_dot,PAux15)+cross(OmegaAux1Aux1,cross(OmegaAux1Aux1,PAux15))+VAux1Aux1_dot));
V66_dot=simplify(R56'*(cross(Omega55_dot,P56)+cross(Omega55,cross(Omega55,P56))+V55_dot));

V88_dot = simplify(R18'*(cross(Omega11_dot,P18)+cross(Omega11,cross(Omega11,P18))+V11_dot));
V99_dot = simplify(R89'*(cross(Omega88_dot,P89)+cross(Omega88,cross(Omega88,P89))+V88_dot));

```

```

V1010_dot = simplify(R910'*(cross(Omega99_dot,P910)+cross(Omega99,cross(Omega99,P910))+V99_dot));
VAux2Aux2_dot = simplify(R10Aux2'*(cross(Omega1010_dot,P10Aux2)+cross(Omega1010,cross(Omega1010,P10Aux2))+V1010_dot));
V1111_dot = simplify(RAux211'*(cross(OmegaAux2Aux2_dot,PAux211)+cross(OmegaAux2Aux2,cross(OmegaAux2Aux2,PAux211))+VAux2Aux2_dot));
V1212_dot = simplify(R1112'*(cross(Omega1111_dot,P1112)+cross(Omega1111,cross(Omega1111,P1112))+V1111_dot));

%%
% Linear Acceleration of the COM
V_c11 = cross(Omega11_dot,P_c11)+cross(Omega11,cross(Omega11,P_c11))+V11_dot;

V_c22 = cross(Omega22_dot,P_c22)+cross(Omega22,cross(Omega22,P_c22))+V22_dot;
V_c33 = cross(Omega33_dot,P_c33)+cross(Omega33,cross(Omega33,P_c33))+V33_dot;
V_c44 = cross(Omega44_dot,P_c44)+cross(Omega44,cross(Omega44,P_c44))+V44_dot;
V_c55 = cross(Omega55_dot,P_c55)+cross(Omega55,cross(Omega55,P_c55))+V55_dot;
V_c66 = cross(Omega66_dot,P_c66)+cross(Omega66,cross(Omega66,P_c66))+V66_dot;

V_c88 = cross(Omega88_dot,P_c88)+cross(Omega88,cross(Omega88,P_c88))+V88_dot;
V_c99 = cross(Omega99_dot,P_c99)+cross(Omega99,cross(Omega99,P_c99))+V99_dot;
V_c1010 = cross(Omega1010_dot,P_c1010)+cross(Omega1010,cross(Omega1010,P_c1010))+V1010_dot;
V_c1111 = cross(Omega1111_dot,P_c1111)+cross(Omega1111,cross(Omega1111,P_c1111))+V1111_dot;
V_c1212 = cross(Omega1212_dot,P_c1212)+cross(Omega1212,cross(Omega1212,P_c1212))+V1212_dot;

%%
% Forces and Moments
F_11 = m1*(V_c11 + cross(Omega11,V11));

F_22 = m2*V_c22;
F_33 = m3*V_c33;
F_44 = m4*V_c44;
F_55 = m5*V_c55;
F_66 = m6*V_c66;

F_88 = m8*V_c88;
F_99 = m9*V_c99;
F_1010 = m10*V_c1010;
F_1111 = m11*V_c1111;
F_1212 = m12*V_c1212;

N_11 = I_c11*Omega11_dot - cross(Omega11,I_c11*Omega11);

N_22 = I_c22*Omega22_dot+cross(Omega22,(I_c22*Omega22));
N_33 = I_c33*Omega33_dot+cross(Omega33,(I_c33*Omega33));
N_44 = I_c44*Omega44_dot+cross(Omega44,(I_c44*Omega44));
N_55 = I_c55*Omega55_dot+cross(Omega55,(I_c55*Omega55));
N_66 = I_c66*Omega66_dot+cross(Omega66,(I_c66*Omega66));

N_88 = I_c88*Omega88_dot+cross(Omega88,(I_c88*Omega88));
N_99 = I_c99*Omega99_dot+cross(Omega99,(I_c99*Omega99));
N_1010 = I_c1010*Omega1010_dot+cross(Omega1010,(I_c1010*Omega1010));
N_1111 = I_c1111*Omega1111_dot+cross(Omega1111,(I_c1111*Omega1111));
N_1212 = I_c1212*Omega1212_dot+cross(Omega1212,(I_c1212*Omega1212));

%%
% Inward Iterations Assuming there is no force acting on the end effector

```

```

f_77 = [0;0;0];
f_66 = R67*f_77+F_66;
f_55 = R56*f_66+F_55;
f_44 = R45*f_55+F_44;
f_33 = R34*f_44+F_33;
f_22 = R23*f_33+F_22;

f_1313 = [0;0;0];
f_1212 = R1213*f_1313+F_1212;
f_1111 = R1112*f_1212+F_1111;
f_1010 = R1011*f_1111+F_1010;
f_99 = R910*f_1010+F_99;
f_88 = R89*f_99+F_88;

f_11 = (R12*f_22+ R18*f_88 +F_11);

%%
n_77 = [0;0;0];
n_66 = N_66+R67*n_77+cross(P_c66,F_66)+cross(P67,R67*f_77);
n_55 = N_55+R56*n_66+cross(P_c55,F_55)+cross(P56,R56*f_66);
n_44 = N_44+R45*n_55+cross(P_c44,F_44)+cross(P45,R45*f_55);
n_33 = N_33+R34*n_44+cross(P_c33,F_33)+cross(P34,R34*f_44);
n_22 = N_22+R23*n_33+cross(P_c22,F_22)+cross(P23,R23*f_33);

n_1313 = [0;0;0];
n_1212 = N_1212+R1213*n_1313+cross(P_c1212,F_1212)+cross(P1213,R1213*f_1313);
n_1111 = N_1111+R1112*n_1212+cross(P_c1111,F_1111)+cross(P1112,R1112*f_1212);
n_1010 = N_1010+R1011*n_1111+cross(P_c1010,F_1010)+cross(P1011,R1011*f_1111);
n_99 = N_99+R910*n_1010+cross(P_c99,F_99)+cross(P910,R910*f_1010);
n_88 = N_88+R89*n_99+cross(P_c88,F_88)+cross(P89,R89*f_99);

n_11 = (N_11+cross(P_c11,F_11)+R12*n_22+cross(P12,R12*f_22)+R18*n_88+cross(P18,R18*f_88));

%%
% Joint Torque
tau2 = (n_22(3));
tau3 = (n_33(3));
tau4 = (n_44(3));
tau5 = (n_55(3));
tau6 = (n_66(3));

tau8 = (n_88(3));
tau9 = (n_99(3));
tau10 = (n_1010(3));
tau11 = (n_1111(3));
tau12 = (n_1212(3));

tau = ([f_11;n_11;tau2;tau3;tau4;tau5;tau6;tau8;tau9;tau10;tau11;tau12]);

%%
% Mass Matrix
M = (jacobian([f_11;n_11;tau2;tau3;tau4;tau5;tau6;tau8;tau9;tau10;tau11;tau12],...
[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]));

```

```

%%
for i = 1:size(M,1)
    for j = 1:size(M,2)
        M(i,j) = simplify(M(i,j));
    end
end
end
% Components of the Mass Matrix
M_b = (jacobian(tau(1:6,1),[du dv dw dp dq dr]));
for i = 1:size(M_b,1)
    for j = 1:size(M_b,2)
        M_b(i,j) = simplify(M_b(i,j));
    end
end
end
M_br = (jacobian(tau(1:6,1),[ddx2 ddx3 ddx4 ddx5 ddx6]));
for i = 1:size(M_br,1)
    for j = 1:size(M_br,2)
        M_br(i,j) = simplify(M_br(i,j));
    end
end
end
M_bl = (jacobian(tau(1:6,1),[ddx8 ddx9 ddx10 ddx11 ddx12]));
for i = 1:size(M_bl,1)
    for j = 1:size(M_bl,2)
        M_bl(i,j) = simplify(M_bl(i,j));
    end
end
end
M_r = (jacobian(tau(7:11,1),[ddx2 ddx3 ddx4 ddx5 ddx6]));
for i = 1:size(M_r,1)
    for j = 1:size(M_r,2)
        M_r(i,j) = simplify(M_r(i,j));
    end
end
end
M_rl = (jacobian(tau(7:11,1),[ddx8 ddx9 ddx10 ddx11 ddx12]));
for i = 1:size(M_rl,1)
    for j = 1:size(M_rl,2)
        M_rl(i,j) = simplify(M_rl(i,j));
    end
end
end
M_l = (jacobian(tau(12:16,1),[ddx8 ddx9 ddx10 ddx11 ddx12]));
for i = 1:size(M_l,1)
    for j = 1:size(M_l,2)
        M_l(i,j) = simplify(M_l(i,j));
    end
end
end
%%
C = tau-M*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
% Components of the Centrifugal/Coriolis Terms
C1 = tau(1,1) - M(1,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C1 = simplify(C1);
C2 = tau(2,1) - M(2,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C2 = simplify(C2);
C3 = tau(3,1) - M(3,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C3 = simplify(C3);
C4 = tau(4,1) - M(4,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';

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C4 = simplify(C4);
C5 = tau(5,1) - M(5,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C5 = simplify(C5);
C6 = tau(6,1) - M(6,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C6 = simplify(C6);

C7 = tau(7,1) - M(7,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C7 = simplify(C7);
C8 = tau(8,1) - M(8,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C8 = simplify(C8);
C9 = tau(9,1) - M(9,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C9 = simplify(C9);
C10 = tau(10,1) - M(10,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C10 = simplify(C10);
C11 = tau(11,1) - M(11,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C11 = simplify(C11);

C12 = tau(12,1) - M(12,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C12 = simplify(C12);
C13 = tau(13,1) - M(13,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C13 = simplify(C13);
C14 = tau(14,1) - M(14,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C14 = simplify(C14);
C15 = tau(15,1) - M(15,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C15 = simplify(C15);
C16 = tau(16,1) - M(16,:)*[du dv dw dp dq dr ddx2 ddx3 ddx4 ddx5 ddx6 ddx8 ddx9 ddx10 ddx11 ddx12]';
C16 = simplify(C16);

%%
vars = {d2,a3,d3,d4,da1,d5,a6,d6,a7,m1,m2,m3,m4,m5,m6,d8,a9,d9,d10,da2,d11,a12,d12,a13,...
m8,m9,m10,m11,m12,j1_11,j1_22,j1_33,j2_11,j2_22,j2_33,j3_11,j3_22,j3_33,j4_11,j4_22,j4_33,...
j5_11,j5_22,j5_33,j6_11,j6_22,j6_33};

vals = {0.19,0.08,0.03,0.09,0.1,0.03,0.12,0.03,0.1,4.23,0.3,0.132358,0.188438,0.197906,0.133408,0.19,0.08,0.03,...
0.09,0.1,0.03,0.12,0.03,0.1,0.3,0.132358,0.188438,0.197906,0.133408,0.1551,0.1689,0.1549,...
0.00174692,0.000877221,0.000877221,0.000114886,3.73894e-05,0.000118637,0.000259367,4.44786e-05,0.000263987,...
0.000291746,4.86811e-05,0.000299249,0.00011622,3.70749e-05,0.000119971};

digits(2);
C1 = vpa(subs(C1,vars,vals),2);
C2 = vpa(subs(C2,vars,vals),2);
C3 = vpa(subs(C3,vars,vals),2);
C4 = vpa(subs(C4,vars,vals),2);
C5 = vpa(subs(C5,vars,vals),2);
C6 = vpa(subs(C6,vars,vals),2);

C7 = vpa(subs(C7,vars,vals),2);
C8 = vpa(subs(C8,vars,vals),2);
C9 = vpa(subs(C9,vars,vals),2);
C10 = vpa(subs(C10,vars,vals),2);
C11 = vpa(subs(C11,vars,vals),2);
C12 = vpa(subs(C12,vars,vals),2);

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```
C13 = vpa(subs(C13,vars,vals),2);
C14 = vpa(subs(C14,vars,vals),2);
C15 = vpa(subs(C15,vars,vals),2);
C16 = vpa(subs(C16,vars,vals),2);

for i = 1:size(M,1)
    for j = 1:size(M,2)
        M(i,j) = vpa(subs(M(i,j), vars, vals),2);
    end
end
%%
save sym_data_ks
%%
% End of Listing
```

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BIOGRAPHICAL STATEMENT

Ameya R. Godbole was born in Kalyan, India, in 1992. He received his B.E. degree from Mumbai University, India, in 2013 in Mechanical Engineering. While pursuing B.E., he was part of a national level robotics competition, ROBOCON, from 2011-2013. He joined the University of Texas at Arlington in Fall 2013 to pursue an M.S. program in Mechanical Engineering.

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