DESIGN AND STIFFNESS OPTIMIZATION OF QUADRI-DIRECTIONAL COMPOSITE GRID LATTICE STRUCTURES

by

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Abstract

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Composite grid structures are rib stiffeners connected to each other rigidly in a pattern or a lattice such that the whole grid pattern has certain smeared stiffness property and has a certain structural behavior. The main focus of this thesis is to design the geometric pattern of a Quadri-directional composite grid lattice such that it has the prescribed stiffness moduli. The research is divided into three phases, in the first phase, the equation for stiffness moduli, the reduced stiffness matrix [Q] and the laminate stiffness matrices [ABD] of the grid pattern is derived for Quadri-directional grid panels. In the second phase, two methods are developed and discussed to obtain maximum stiffness moduli from the grid structure.

- The explicit stiffness moduli equations of the grid structure is solved to obtain the design variables by giving the required stiffness moduli as the resultant vector.
- Multi-objective genetic algorithm is used to optimize the design variables in the given range such that vital stiffness moduli is maximized.

A MATLAB code is generated for both the methods and the values of the design variables is extracted to be developed as a latticed structure. In the third phase, a Finite Element Model is developed in Hypermesh order to validate the results obtained from the code and analyze the behavior of the grid structure in both the cases.

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1. Introduction

Composite grid structures are lattice structures [1] which consists of composite stiffener blades of usually rectangular cross section. Each stiffener blade is a unidirectional composite structure and collection of several ribs which run in same orientation and placed at regular intervals is a rib family. The standard grid structures used in industries consists of rib families running in 2-4 directions in a repeating pattern. The basic elements of the conventional grid structure is a unit cell, intersection node and rib [4].

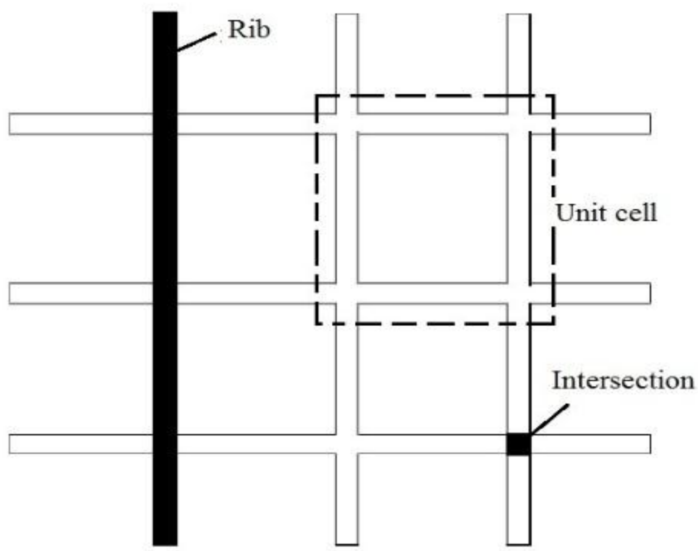


Figure 1-1 Basic elements of a grid structure

Grid structures with ribs running in four directions are referred to as Quadri-directional, grid structures with ribs running in three directions are referred to as tri-directional and while ribs running in only two directions are referred to as angled grid. The conventional grid structures which are widely used in industries such as isogrid, are tri-directional grid structure, whose unit cells form equilateral triangles [1].

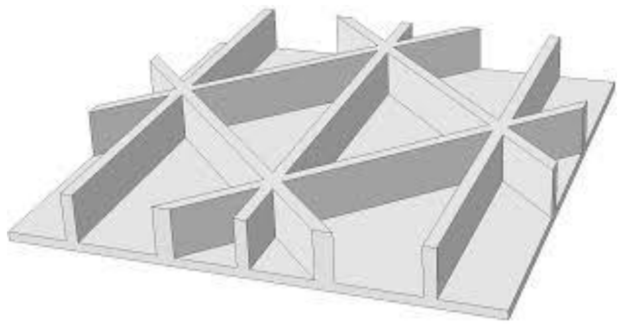
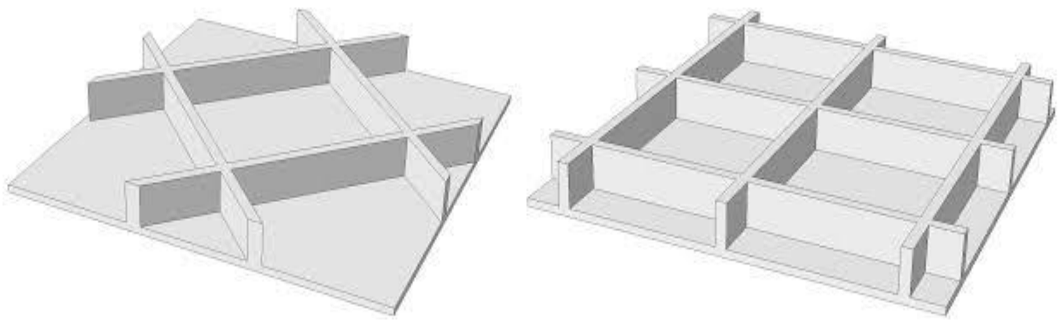


Figure 1-2 Conventional isogrid structure [25]



(a) (b)

Figure 1-3 (a) Angled grid structure and (b) Orthogrid structures [25]

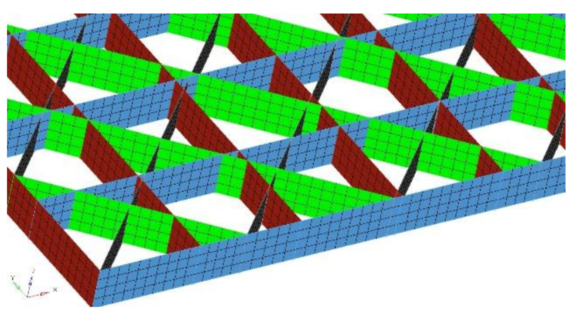


Figure 1-4 Quadridirectional grid structure

The main reason for the grid structures to be developed is to exploit the unidirectional properties of these structures [4]. Each one of these grid structures possess distinct stiffness properties that can be tailored to the requirement same as the composite laminate, but can have very high directional stiffness which makes it an ideal member for reinforcing weak structures. Due to their very high specific modulus to weight ratio, cost and ease of fabrication, it has created a need in industries and extensive studies are carried out on them [3]. The composite grid structures are inherently strong and resilient structure due to their interconnected rib configuration [1]. They have high impact resistance, high stiffness to weight ratio, high load carrying capacity and do not face problems such as material mismatch, delamination, crack propagation which are usually associated with laminates [1]. Although these grid structures have a lot of advantages, they suffer from disadvantages like local/global buckling, crippling failure and very little knowledge of their behavior in failure space hampered their usage in industries [1]. There is a need for extensive research in fabrication and designing techniques [4]. Optimizing the readily available design would reduce the dependency on the new research for better designs and also use of the readily available manufacturing methods.

1.1 Hypothesis

The purpose of the thesis is to provide an insight on how tailorable the grid structures are and ways to design a grid structure to obtain the required structural behavior. The integrated equivalent analytical model of the grid structure developed in [3] has algebraic flexibility which allows the grid structure to be modified according to the requirement and using this model, equations for stiffness moduli and matrices for any grid structure can be developed in terms of design variables which allows customization of grid design. The explicit equations obtained can be solved to obtain the desired variables which is used to design a geometric pattern of the structure of desirable characteristics. The optimization of the grid structures ensures that the maximum stiffness moduli in the required direction is obtained but also helps in retaining certain favorable aspects of the conventional grid structures such as ease of design, use of standard ribs and analysis methods. The design optimization is carried out by the multi-objective genetic algorithm which produces the best design variables after natural evolution of the same from the initial pool of solutions taking in the longitudinal and transverse modulus as the objective functions which are to be maximized and the shear modulus as the stiffness to be minimized. The algorithm used is robust and can be used to maximize any of the stiffness required and curb the effect of other stiffness on the grid structure simultaneously. Since the grid structures are mainly used as reinforcing members, customization of the stiffness required in the vital direction is essential.

1.2 Goals

The main goal is to arrive at a geometric pattern of the Quadri-directional grid structure, such that the structure has requisite stiffness moduli. The goals needed to achieve that are,

- The design variables required for designing geometric pattern of the Quadri-directional lattice structure are determined.
- The explicit equations for stiffness moduli [E_x, E_y, γ_{xy}, G_{xy}], the reduced stiffness matrix [Q] and the stiffness matrices [ABD] for the equivalent laminate model of the Quadri-directional grid structures are derived in terms of design variables.
- The first method is to attain the requisite stiffness which is achieved by solving the stiffness moduli equations to obtain the design variables by giving the objective stiffness moduli as the resultant vector.

- The second method employed is primarily to maximize or minimize stiffness in a given range of design variables rather than obtaining a single desired stiffness. In order to achieve that, transverse and longitudinal stiffness modulus equations of the grid structure is maximized and the shear stiffness modulus is minimized in the given range of design variables using multi-objective genetic algorithm.
- The design variables obtained from both the methods are developed into FEA models and analyzed to validate the results.
- The disparity and significance of the two methods employed to find the design pattern are addressed, the benefits of each method is discussed and further improvements are suggested based on the FEA analysis.

2. Literature review

Orthotropic grid structures and their design is not a new concept, in fact, there has been a lot of research and have been majorly implemented in industries. They are mainly used in aerospace industries to reinforce members with weak buckling strength such as fuselage and launch vehicle fuel tanks [3]. They are also used to strengthen structural members with weak impact resistance and damage tolerance by design and structural engineers [1]. The introduction of new techniques for manufacturing such as continuous filament winding and wet winding processes and advent of new tooling techniques reducing manufacturing difficulties have increased the demand for the grid structures [3]. Several studies have been conducted on grid structures and the following collection of studies and review of literature is done in relevant to the behavior of composite grid structure and their analytical models and multi-objective optimization of design variables, the basis on which this thesis is built upon.

2.1 Grid structure behavior and characteristics

2.1.1 Grid stiffness moduli and matrix

There has been numerous research on the analysis and behavior of grid structure and their characteristics. But they are limited to buckling and crippling analysis as majority of the grid structures used in industries now is hampered by the lack of understanding of their behavior in failure space [1]. The research article [1] deals with the grid structure strengths, grid structure weaknesses, the effects of empty, soft, hard and rigid inclusions, the effects of missing ribs, the effects of nodal offset, the impact of soft and hard repairs to the grid structure lattice, and the impact of joining grid structures together which would be fundamental concept to understand the grid structure characteristics and failure concepts and to analyze the behavior of the final optimized model. The main focus is to develop constitutive stiffness equations for the entire grid structure for different patterns. [2] have presented a paper where they introduce the explicit formulas for the elastic moduli of a helical composite lattice plates and the stiffness matrices for the same is derived from those equations. The method used by [2] is to superpose the individual rib stiffness to obtain the stiffness matrix of the entire grid structure from which the explicit equations of the elastic moduli of the grid structure is attained.

2.1.2 ABD matrix of the grid structure and design variables

In [3] the equivalent analytical model is developed for the grid structure which included the in-plane bending, shear, hygrothermal effect and local buckling of the ribs. As a result, the model is almost accurate and can easily be written in the form of a code which form the major part of this code required for obtaining the design pattern. Earlier studies including [5] introduced a stiffness model for isogrid structures, but did not account for the torsional effect of the ribs. [3] introduces the robust form of the equivalent stiffness model in form of the design variables such as distance between each rib of the same family and other family, orientation of the ribs of different family which would serve as the variables for developing the geometric pattern of the grid. The equivalent stiffness model developed for this thesis does not account for the hygro-thermal stress in order to ease the computation process.

The buckling properties of the grid and optimization methods have been developed to find the buckling and crippling loads from the analytical model of the grid structure [7,8,25].

2.2 Design Optimization

2.2.1 Multi-objective optimization

The process of optimizing systematically and simultaneously a collection of objective functions is called multi-objective optimization [9]. It tries to find the optimal design variables in the given boundary conditions subjected to a vector of objective functions. For example, we want to minimize the cost and material for manufacturing a component and maximize the quality of it, multi-objective optimization is used. There are several criteria to choose the best optimization method for a given problem and the main criteria are the ease of coding and the quality and availability of the algorithm to give the optimal points which solve the criterion. The various methods are discussed in [10, 14] and their efficiency of obtaining the answers were also discussed. Multi-objective optimization of the composite structures design have been carried out on several composite structures, [10] discusses a method of optimizing composite laminates using Vector Evaluated Artificial Bee Colony (VEABC) algorithm and confers the advantages of using it over other multi-objective algorithms. The objective functions in the thesis are nonlinear and using VEABC would result in long computational time, the algorithm is not commercial available and not suited for design optimization, the use of genetic algorithm optimization stated in the paper presented by [10], and the efficiency of the algorithm to handle multiple variables with high accuracy, availability and the amenable approach in obtaining optimal result, culminated for this algorithm to be chosen.

2.2.2 Genetic Algorithm

A genetic algorithm (GA) is a method for solving both constrained and unconstrained optimization problems based on a natural selection process that mimics biological evolution [11]. Multi-objective genetic algorithm solves problems by identifying the pareto front, the set of evenly distributed non-dominated optimal solutions with or without bound or linear constraint [12]. The optimal solutions are observed and the features are extracted which are carried over to the next population of variables. Genetic algorithm has been implemented in multi-disciplinary design optimization (MDO) method to develop in several researches [13, 14].

2.3 Computer Aided Finite element Modeling and Analysis

Finite element modelling is carried out on the grid structure in order to visualize the characteristic behavior and to obtain the directional displacements values of elastic moduli. Since the model to be analyzed is a composite structure, several studies which deal with the composite analysis is surveyed. [17, 27, 28]

3. Parametric laminate properties of Grid structures

To obtain the relation between the stress and strain in grid structure, the laminate stiffness matrices of the entire grid structure, the [ABD] matrices has to be attained. In order to do that, an equivalent analytical laminate model is developed and the laminate stiffness matrix equations is derived from laminated plate theory [3]. According to laminated plate theory, the stress-strain relation is given by,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ k \end{bmatrix} \tag{1}$$

Where,

[N]=Force matrix of the equivalent laminate model.

[M]= Moment matrix of the equivalent laminate model.

[A]= Extensional stiffness matrix of the equivalent laminate model.

[B]= Extensional-bending stiffness matrix of the equivalent laminate model.

[D]= Bending stiffness matrix of the equivalent laminate model.

$[\varepsilon_0]$=Midplane strains of the equivalent laminate model.

[k]= curvature strains of the equivalent laminate model.

To derive the elastic moduli of an equivalent laminate model of the QDG, the reduced stiffness matrix [Q] of each lamina has to be obtained and by laminated plate theory,

$$[\sigma] = [Q][\varepsilon] \tag{2}$$

Where,

$[\sigma]$=stress matrix of equivalent lamina.

$[Q]$=stiffness matrix of equivalent lamina.

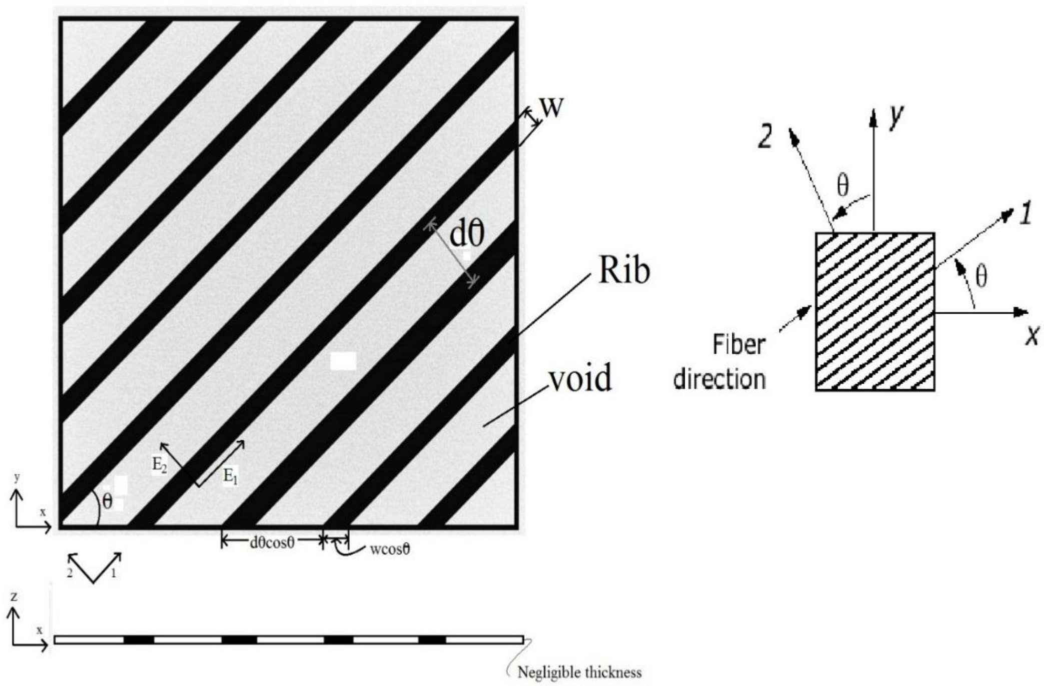
[Ɛ]=strain matrix of equivalent lamina.

According to [2], the rib stiffness matrix of each of the rib family has to be derived separately and then the total stiffness of the grid is then found by the method of super position. Certain assumptions are made to find the stiffness properties of each the grid structure,

- To find the stiffness matrix, each rib family is assumed to be one continuous layer (lamina) with negligible thickness (thin lamina theory), where the rib stiffeners are considered as the unidirectional fibers running in local x direction (direction 1)and the empty space between them is considered as the resin with negligible elasticity [2].
- In light of the super position method, the effect of the other rib family layers are not considered while calculating the stiffness matrix of the one layer and the total stiffness of each layer of the equivalent laminate is found by adding individual stiffness contribution i.e, the summation of the stiffness matrix of each layer of rib family [2].
- Every pattern of grid structure has different mechanical properties and is proportional to its associated lattice design pattern and the rib properties.

$$E_x^{grid}, G_{xy}^{grid}, E_y^{grid}, \vartheta_{xy}^{grid} \quad \propto \quad E_x^{rib}, G_{xy}^{rib}, w, \theta, h, d_\theta$$

- The laminate stiffness matrix is calculated using composite mechanical equations where the height of the ribs as the thickness of the equivalent laminate [3].
- The ribs are one-directional composite material of equal cross-section and elastic properties.
- The ribs have no voids and the hygrothermal effect on the ribs are not considered [3].



(a) (b)

Figure 3-1 (a) Rib family along θ orientation (b) Equivalent lamina model of the rib family along θ orientation

Since the rib family is abridged to an equivalent continuous lamina, the governing theories such as laminated plate theory can be applied to obtain the reduced stiffness matrix. The stiffness constants of the equivalent lamina is obtained by the micromechanical properties of the ribs [2]. The reduced stiffness matrix of a rib family for any given orientation is given by,

$$(Q)_{xy}{}^{\theta deg} = \begin{bmatrix} \frac{Ew}{d_\theta}m^4 & \frac{Ew}{d_\theta}m^2n^2 & \frac{Ew}{d_\theta}m^3n \\ \frac{Ew}{d_\theta}m^2n^2 & \frac{Ew}{d_\theta}n^4 & \frac{Ew}{d_\theta}mn^3 \\ \frac{Ew}{d_\theta}m^3n & \frac{Ew}{d_\theta}mn^3 & \frac{Ew}{d_\theta}m^2n^2 \end{bmatrix} \quad \textbf{(3)}$$

Where,

E=longitudinal modulus of each rib.

w=width of the rib.

θ =orientation of the rib.

dθ =distance between the ribs.

m = cos(θ).

n = sin(θ).

3.1 Quadri-directional grid stiffness properties

Quadri-directional grid structures (QDG) are the lesser known form of the grid structures which have rib families running in four different orientations, usually with one pair of orthogonal rib families (0, 90) and a pair of angled rib families (+θ,-θ) [1]. There are mainly two types of QDG grid structures, conventional and non-conventional form of Quadri-directional grid structures. Conventional form of QDG structures are usually preferred in industries as they have angled ribs running through the intersection of the horizontal and vertical rib family. They are easier to manufacture using present tooling techniques and also high flexural strength and rigidity.

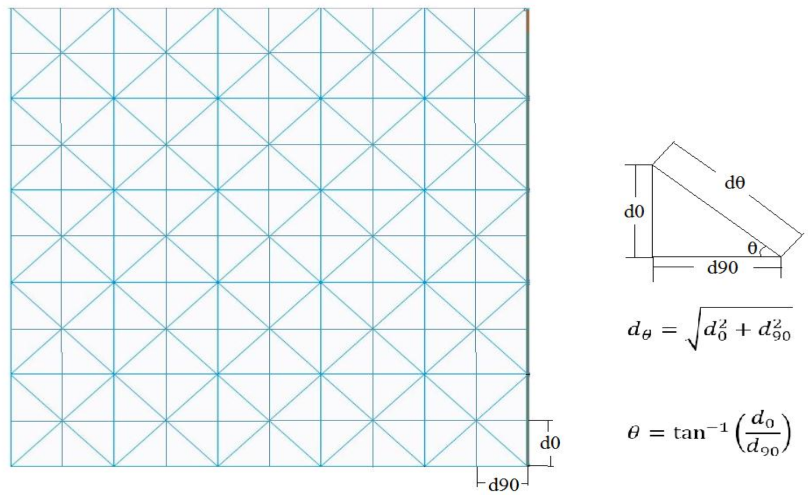


Figure 3-2 Conventional Quadri-directional grid structure

Though the conventional QDG structures are highly preferred, they have limited design flexibility as they are characterized by only two design variables (d_0, d_{90}). This implies that the pattern design for conventional QDG is governed by the distance between the horizontal and vertical rib families. Since the main scope of the thesis is to obtain a generic form of pattern design strategy such that the other grid structures can be devolved from this form, non-conventional grid structure would be an ideal choice as they are characterized by 4 independent design variables (d_0, d_{90}, d_θ, θ) which gives a more control over grid design pattern. In non-conventional grid structures, the angled rib families (+θ, -θ) rib families do not run through the intersection of the 0 and 90 rib families, that is the design variables d_θ and θ are independent of d_0 and d_{90} variables.

Non-conventional are generally avoided by the industries due to their high material usage and manufacturing constraints, but their behavioral properties are analogous to symmetric laminates and the undesirable properties of laminates such as the delamination, crack propagation etc. are reduced and retain desirable properties such as customizability, strength etc.[3] In addition, the properties of other grid structures are a variant of QDG properties, so the optimization can be carried out on other grid structures with minor changes to the QDG structure, decreasing the analysis costs.

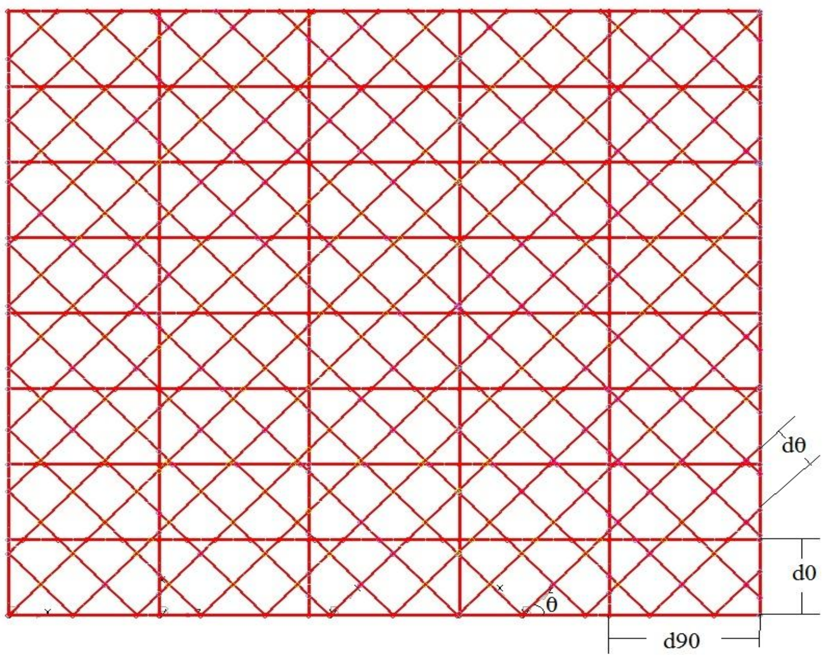


Figure 3-3 Non-conventional Quadri-direction grid structure

Where,

d_0= distance between horizontal ribs.

d_{90}= distance between vertical ribs.

θ= orientation of the angled ribs.

d_θ= distance between the angular ribs.

By varying these variables, any desired pattern of the QDG can be achieved.

3.1.1 Computation of Elastic moduli for QDG

The reduced stiffness matrix of the QDG is the sum of individual stiffness matrices of the rib families in the 0, 90, +θ, -θ orientation. The individual stiffness in each orientation are obtained by substituting the respective values of the orientation angle in equation 3,

$$(Q)_{xy}{}^{0deg} = \begin{bmatrix} \frac{Ew}{d_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \textbf{(4)}$$

$$(Q)_{xy}{}^{90deg} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{Ew}{d_{90}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \textbf{(5)}$$

$$(Q)_{xy}{}^{-\theta deg} = \begin{bmatrix} \frac{Ew}{d_\theta}m^4 & \frac{Ew}{d_\theta}m^2n^2 & -\frac{Ew}{d_\theta}m^3n \\ \frac{Ew}{d_\theta}m^2n^2 & \frac{Ew}{d_\theta}n^4 & -\frac{Ew}{d_\theta}mn^3 \\ -\frac{Ew}{d_\theta}m^3n & -\frac{Ew}{d_\theta}mn^3 & \frac{Ew}{d_\theta}m^2n^2 \end{bmatrix} \quad \textbf{(6)}$$

The total stiffness of the QDG is summation of all the individual matrices of the QDG lattice layer

$$(Q)_{xy}{}^{QDG} = (Q)_{xy}{}^{0} + (Q)_{xy}{}^{90} + (Q)_{xy}{}^{\theta} + (Q)_{xy}{}^{-\theta} \quad \textbf{(7)}$$

So the reduced stiffness matrix of the grid layer is given by,

$$(Q)_{xy}{}^{QDG} = \begin{bmatrix} \frac{Ew}{d_\theta}m^4 + \frac{Ew}{d_0} & \frac{Ew}{d_\theta}m^2n^2 & \frac{Ew}{d_\theta}m^3n \\ \frac{Ew}{d_\theta}m^2n^2 & \frac{Ew}{d_\theta}n^4 + \frac{Ew}{d_{90}} & \frac{Ew}{d_\theta}mn^3 \\ \frac{Ew}{d_\theta}m^3n & \frac{Ew}{d_\theta}mn^3 & \frac{Ew}{d_\theta}m^2n^2 \end{bmatrix} \quad \textbf{(8)}$$

The elastic moduli can be easily obtained from the reduced compliance matrix of the QDG, which can obtained from equation (8).

$$(C)_{xy}{}^{QDG} = \left[(Q)_{xy}{}^{QDG}\right]^{-1} \tag{9}$$

The equations of the moduli can be derived from the stiffness constants in the compliance matrix, and the grid stiffness moduli obtained from the matrix are,

$$E_x{}^{QDG} = \left[\frac{Ew[2d_0m^4+2d_{90}n^4+d_\theta]}{d_0[2d_{90}n^4+d_\theta]}\right] \tag{10}$$

$$E_y{}^{QDG} = \left[\frac{Ew[2d_0m^4+2d_{90}n^4+d_\theta]}{d_{90}[2d_0m^4+d_\theta]}\right] \tag{11}$$

$$\vartheta_{xy}{}^{QDG} = \left[\frac{2d_{90}m^2n^2}{2d_{90}n^4+d_\theta}\right] \tag{12}$$

$$G_{xy}{}^{QDG} = \left[\frac{2Ewm^2n^2}{d_\theta}\right] \tag{13}$$

3.1.2 Computation of Extensional Stiffness matrix (A) and Flexural stiffness matrix (D) for QDG.

The extensional stiffness matrix or [A] matrix gives the total axial stiffness matrix of the QDG for the rib height (h). It can be derived using the [Q] matrix, which is the stiffness matrix of the equivalent lamina of the QDG by the governing composite theories (laminated plate theory) to obtain the total stiffness matrix of the equivalent laminate.

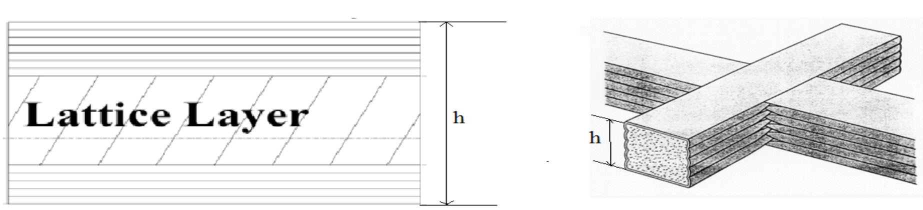


Figure 3-4 Quadri-directional grid laminate [26]

The [A] matrix to be obtained by the laminated plate theory is given by,

$$[A]_{QDG} = \int [Q_{xy}^{QDG}] dh \tag{14}$$

The final matrix obtained is given by [3] which is,

$$[A]_{QDG} = \begin{bmatrix} 2\frac{Ea}{d_\theta}m^4 + \frac{Ea}{d_0} & 2\frac{Ea}{d_\theta}m^2n^2 & 0 \\ 2\frac{Ea}{d_\theta}m^2n^2 & 2\frac{Ea}{d_\theta}n^4 + \frac{Ea}{d_{90}} & 0 \\ 0 & 0 & 2\frac{Ea}{d_\theta}m^2n^2 \end{bmatrix} \tag{15}$$

The flexural stiffness matrix or D matrix gives the total bending stiffness matrix of the QDG for the rib height (h) which is given by,

$$[D]_{QDG} = \int [Q_{xy}^{QDG}] h^2 dh \tag{16}$$

The final matrix can is obtained is given by [3] which is,

$$D_{11} = EI\left[\frac{1}{d_0} + \frac{2m^4}{d_\theta} + \frac{2\tau m^2n^2}{d_\theta}\right] \tag{17}$$

$$D_{22} = EI\left[\frac{1}{d_{90}} + \frac{2n^4}{d_\theta} + \frac{2\tau m^2n^2}{d_\theta}\right] \tag{18}$$

$$D_{21} = D_{12} = EI\left[\frac{2m^2n^2}{d_\theta} - \frac{2\tau m^2n^2}{d_\theta}\right] \tag{19}$$

$$D_{66} = EI\left[\frac{2m^2n^2}{d_\theta} + \frac{\tau}{4d_0} + \frac{\tau}{4d_{90}} + \frac{\tau}{d_\theta}(m^2 - n^2)^2\right] \tag{20}$$

$$[D]_{QDG} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{11} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{21}$$

Where,

a = area of the rib cross-section.

I = moment of inertia of the rib cross-section.

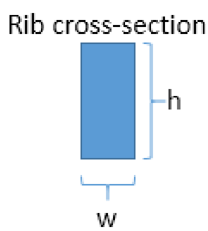
J=torsional stiffness.

G= shear stiffness of the rib.

τ = constant.

The value of the above rib properties depends on the rib cross-sectional shape and rib material properties. The rib stiffeners used for this thesis have a rectangular cross-section as given below and the values of the above constants can be obtained from structural mechanics,

$$a = w \times h \quad \textbf{(22)}$$



$$\tau = \frac{GJ}{Ea} \quad \textbf{(23)}$$

$$I = \frac{wh^3}{12} \quad \textbf{(24)}$$

$$J = \frac{hw^3}{16}\left[\frac{16}{3} - 3.36\frac{w}{h}\left(1 - \frac{w^4}{12h^4}\right)\right] \quad \textbf{(25)}$$

The QDG behaves as a symmetric laminate because of the presence of ribs orientations ±θ and rib orientations along 0 and 90 orientations are neglected as they do not provide any bending or twisting movement. The QDG is equivalent to a balanced un-symmetric laminate and the value of the bending-twisting stiffness matrix [B] is 0.

The total laminate stiffness matrix of the QDG structure is given by,

$$\text{QDG structural stiffness matrix} = \begin{bmatrix} [A_{QDG}] & [zeros(3,3)] \\ [zeros(3,3)] & [D_{QDG}] \end{bmatrix} \quad \textbf{(26)}$$

4. Optimization using Genetic algorithm

4.1 Understanding Genetic Algorithm

Multi-Objective Genetic Algorithm(MOGA) is selected over many calculus based multi-objective optimization algorithm because of its flexibility to handle wide range of parameters at a single time and limitations faced by calculus based methods such as use of derivatives which increase computation time and complexity, Genetic algorithm does not rely on the presence of design variables [9]. Instead it populates the design space and heads towards the optimal solution in that population and “evolves” towards the global optima as shown in Figure 4-1.

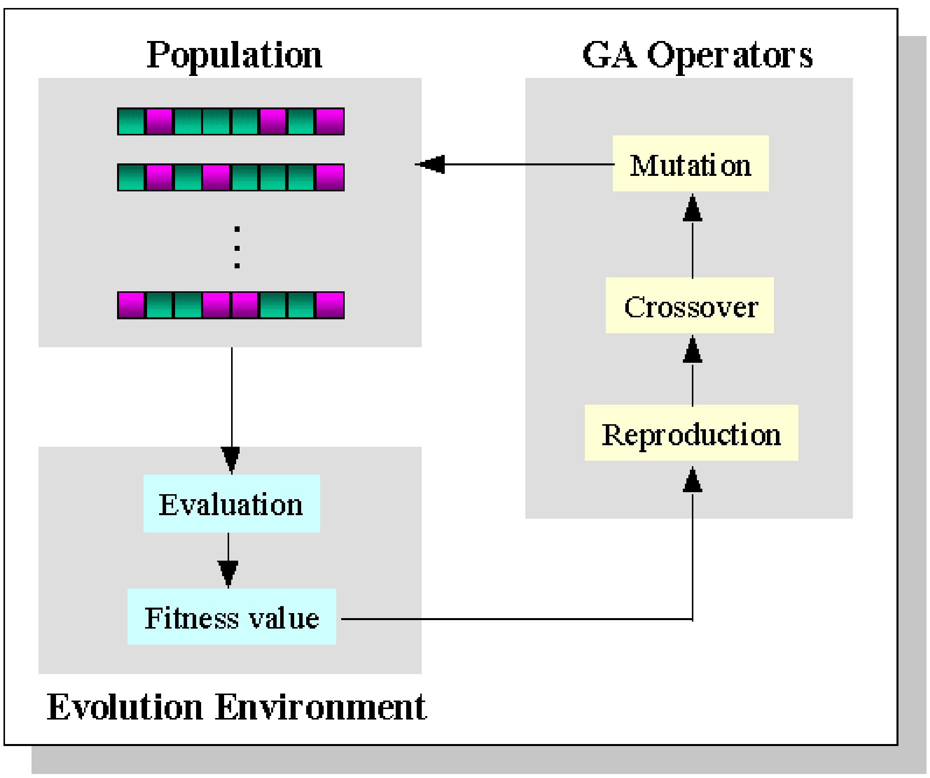


Figure 4-1 Evolution of genetic algorithm to obtain the design variable [15].

4.1.1 Genetic algorithm methodology

It is used for solving both constrained and unconstrained optimization problems based on a natural selection process that mimics biological evolution [9].

- It starts with an initialized set of solutions called population and subsequent solutions from one population are taken and used to form a new population [16].
- The new population consists of all the best chromosomes (vital characteristics of optimal solutions) from the previous population.
- Then best solutions from each population is retained at each iteration.
- The solutions formed in each iterations have the chromosomes of the best solutions from the previous generations so at each iteration they "evolve" towards the optimal solution and they are selected at each iteration according to their fitness and within the constraints [16].
- The process is repeated until termination condition or if the fitness value of previous 5 generations is almost the same.

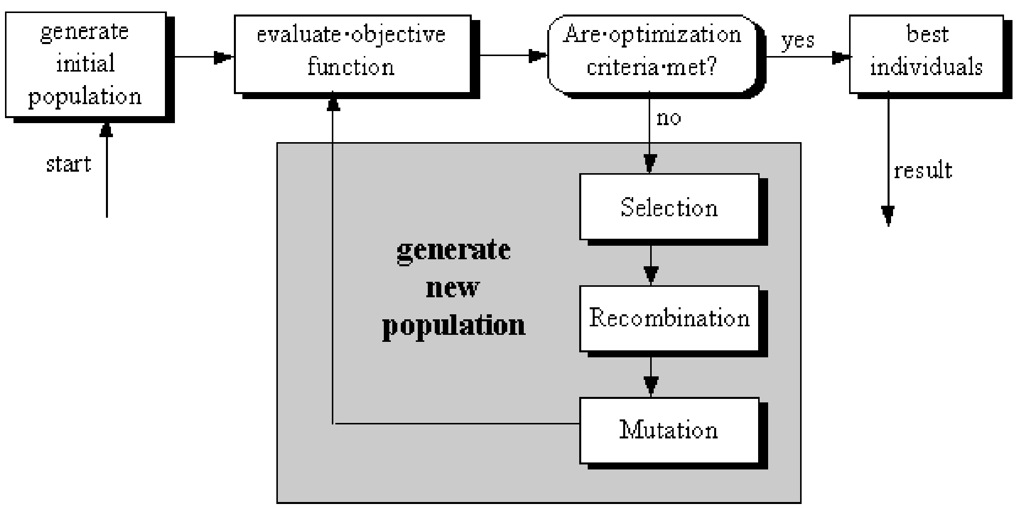


Figure 4-2 Genetic algorithm procedure [18]

4.1.2 Genetic Algorithm terminology

- Parameter/variable: a design variable in the system of interest [17].
- Gene: the encoded form of the parameter which is being optimized [17].
- Chromosome: the complete set of genes which uniquely describes an individual [17].
- Fitness function: The objective function or functions we are trying to maximize or minimize or both simultaneously [17].
- Population: It specifies how many individuals are there in the specified generation.
- Mutation: The operator used to maintain the genetic diversity from one generation of the population to the next [17].
- Cross-over: The operator used to vary the programming of the chromosome or chromosomes from one generation to the next [17].
- Pareto Optimal: A point, $x^* \in X$, is Pareto optimal if there does not exist another point, $x \in X$, such that $F(x) \leq F(x^*)$, and $F_i(x) < F_i(x^*)$ for at least one function [10,22].
- Pareto front: the set evenly distributed non-dominated optimal solutions from each generation [10].

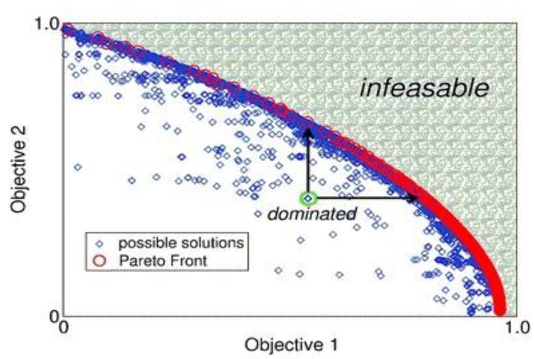


Figure 4-3 Pareto front plot [19]

Since this is a multi-objective optimization, there are more than one optimal point and all the points are not feasible. The Pareto front is a set of optimal points forming a boundary above which the values are infeasible as shown in Figure 4-3. So any point on the Pareto front boundary is an optimal solution which means there are a range of points which can be chosen to create the design based on the requirement. In multi-objective optimization there is a trade-off between objective functions which would mean that it is mathematically impossible to make one individual better without making one of the individuals worse .Pareto front plot can be used to select the optimal points required for design as the amount of trade-off between the first two objective functions can be decided from the plot as shown.

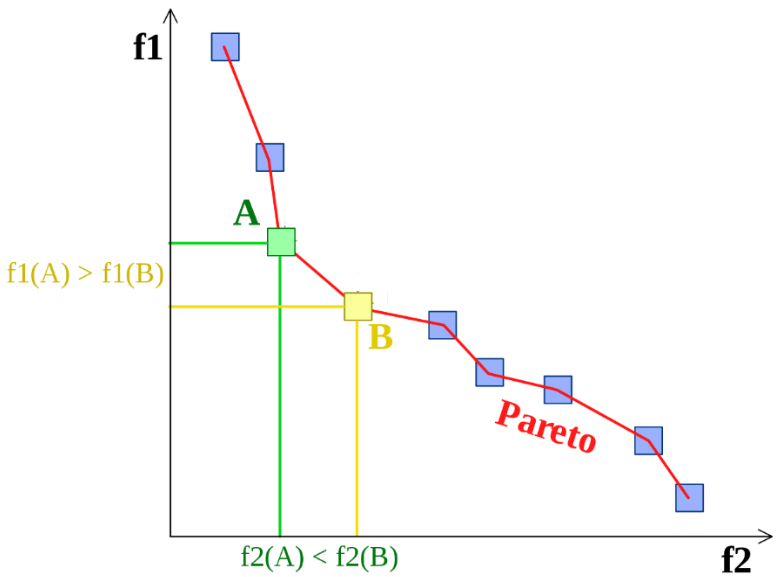


Figure 4-4 Optimal points on Pareto front plot [19].

Here f(A) and f(B) are the optimal solutions and the plot and depending on the amount of desired trade of to be given for the design requirement, one of the points is chosen either A or B.

The main functions of these terminologies and how they affect our result is discussed ahead.

4.1.3 Multi-objective Genetic Algorithm

Multi-objective optimization is concerned with the minimization of multiple objective functions that are subjected to a set of constraints [20]. It is used to solve multi-objective optimization problems by identifying the Pareto front. The main advantage of multi-objective genetic algorithm does not require the functions to be differentiable or continuous [9].The goal of this thesis is to maximize two objective function and to minimize one objective function and all the three are to be done simultaneously as they are defined by the same design variables and since the objective functions are not differentiable functions. Multi-objective GA algorithm is chosen. The program consists of a code written in MATLAB which uses the inbuilt MATLAB module in optimization tool called Multi-objective GA, which is the ideal module to carry optimization of given functions. The syntax for the Multi-Objective GA is given by,

```
X = gamultiobj(FITNESSFCN,NVARS,A,b,Aeq,beq,LB,UB,options)
```

Table 4-1 Inputs of the multi-objective genetic algorithm MATLAB tool

Fields	**Value**
FITNESSFCN	The objective function which is to be maximized or minimized written as a function.
NVARS	Number of input variables
A	Matrix of linear inequality constraints(LHS)
b	Vector of linear in-equality constraints(RHS)
Aeq	Matrix of linear equality constraints(LHS)
beq	Vector of linear equality constraints(RHS)
Options	Options created using gaoptimset

4.2 Establishing of QDG functions required for optimization

4.2.1 Design variables

The basic variables which define a fitness function or a system of interest within a given constraint is called as a design variable [17].

Table 4-2 Design variables for optimizing the QDG structure.

Design variables	**Definition**
d_0	Distance between horizontal ribs.
d_θ	Distance between angled ribs.
d_{90}	Distance between the vertical ribs.
θ	Orientation of the angled ribs.

These are the basic design variables required to design a pattern of QDG structure consisting of ribs with equal cross-sectional area.

The QDG can be further be optimized by choosing the width of the rib(w) and the height of the rib(h) as design variables which would give more flexibility in attaining the accurate objective stiffness, but would face problems such as complexity of programming, complexity of designing the ribs due to manufacturing constraints etc.

4.2.2 Objective functions/Fitness functions

The functions which are used to find the best solution from all the feasible solutions and also how close a design solution is to achieving the set aims are called as fitness functions. In short, the global maxima or minima has to be achieved for fitness functions. The goal here is to achieve a design pattern for QDG such that it has high axial stiffness and low shear stiffness to serve as a reinforcing member for axial loads. Since multiple fitness functions are used we can simultaneously achieve multiple maxima and minima which in this case is maximizing axial stiffness values and minimizing the shear stiffness.

In order to find the maximum, the value (-1) is multiplied to the function which is to be minimized. If g(x,y,z…) gives the minimum of the function, f(x,y,z…)=-g(x,y,z..) gives the maximum [21].

The explicit elastic moduli equations obtained from previous chapter, are taken as the fitness functions. The value of the longitudinal stiffness and lateral stiffness modulus are multiplied with (-1) as they are to be maximized and shear modulus with (1) as it is to be minimized.

Table 4-3 Objective functions for stiffness optimization

$\boldsymbol{f_1(d_0, d_\theta, d_{90}, \theta) = -\left[\frac{Ew[2d_0m^4 + 2d_{90}n^4 + d_\theta]}{d_0[2d_{90}n^4 + d_\theta]}\right]}$
$\boldsymbol{f_2(d_0, d_\theta, d_{90}, \theta) = -\left[\frac{Ew[2d_0m^4 + 2d_{90}n^4 + d_\theta]}{d_{90}[2d_0m^4 + d_\theta]}\right]}$
$\boldsymbol{f_3(d_0, d_\theta, d_{90}, \theta) = \left[\frac{2Ewm^2n^2}{d_\theta}\right]}$

4.2.3 Bounds

The range of design parameters within which the global maxima or minima of the objective function is obtained is called the bounds, specifically lower and upper bounds. It defines the region of design space where the optimal solution has to be searched [9]. Designing QDG structures would have a wide range of the possibilities with fewer constraints and constraints such as volume/weight limit or material usage limit can be included for further optimization, but in order to obtain a generic optimization, only bound constraints are used. Choosing the bounds for the design variables would be a difficult task it would mean a trade-off between obtaining high stiffness and amount of material used in the grid construction.

For example (0.1mm<d_0<∞, 0.1mm<d_{90}<∞, 0.1mm<d_θ<∞, 0.1<θ<∞). By engineering intuition we can say that the maximum stiffness can be obtained in the neighboring region of lowest bound of axial distance variables, but the design is not possible if the lower bound is less than the width of the rib and if the rib width is greater than the rib width the amount of material used to obtain the required stiffness might be high and not practical. Increasing the lower bound very high would also reduce the amount of stiffness obtained. In case of upper bound having high upper bound would result in very high scatter of the initial population which would cause the optimal solution to be missed and the computation of the result somewhat away from the actual maximum or minimum. As a result choosing the bounds for the design can be a horrendous task and requires multiple optimization runs to come to an exact bounds which would give us the optimal result. The bounds should also be given such that they meet the approval of the engineering mechanics, for example in the given example of upper and lower bound of theta lies between 0.1 and infinity which is not possible. From running iterations of the program without constraints it was seen that the bounds have to be less in order to achieve optimal solution because of multiple functions to be optimized.

4.2.4 Constraints, Population and Program termination

4.2.4.1 Constraints

The MOGA is able to handle both linear and non-linear, equal and inequality constraints. The constraints are defined equations or conditions which must be satisfied by the design variables to obtain the optimal solution. The constraints can also be the boundaries of design space within which design variables lie. In this condition the constraints are called as bound constraints and can be defined in the lower and upper bounds. The conditions which the design variables must satisfy for the design of the QDG pattern is,

Table 4-4 Constraints to be solved by the objective functions

Parameters	**Conditions to be satisfied**
d_0	Positive and greater than width of the rib(w)
d_θ	Positive and greater than 2*w*cos(θ)
d_{90}	Positive and greater than width of the rib(w)
θ	$0<\theta<2\pi$

The design optimization is carried as bound constraint optimization which means that there is no equality or inequality constraints to be satisfied by the objective functions and the values of the optimal points have to be within the given bounds.

4.2.4.2 Population

The population defines the number of individual solutions that are generated at each iteration. Having high population results in a closer value to the optimal results but may increase the computation time. If the initial population is set, the successive generation of individuals (children) are generated by the fitness function, elitism, crossover and mutation.

- Elitist children: The individuals which have the best chromosomes and are close to the optimal solution are carried over to the next generation without any changes [23,24].

- Cross-over children: The children which are generated by taking two or three individuals from previous generation with good chromosomes [23].
- Mutated children: The children which are generated by taking one individual from previous generation and introducing random changes to it [23].

The diversity, range and the size of the initial population results in better convergence of the solution to the optimal solution. The initial population by default is (15* number of design variables) but since the design optimization of QDG has only bound constraints and multiple objectives to be satisfies, there is large number of design variables which satisfies the functions and close to optimal solution so the population size at each iteration is increased to 1000 to get close to the optimal solution.

4.2.4.3 Termination

When the optimization code reaches the maximum optimum value, running the code further is time consuming. As a result the termination criteria has to be introduced such that the program terminates after the values start repeating. When the Pareto front optimal solution has a small variation occurring more than 5 times in successive generations the program is terminated.

5. Methodology and Results

5.1 Method 1: Objective stiffness matrix vector as input

5.1.1 Initializations and assumptions

The elastic moduli may be reduced to two design variables (d_0, d_{90}), if conventional design of QDG is used, but since non-conventional QDG design is being considered, 4 variables are extracted (d_0, d_{90}, θ, d_θ) by solving 4 equations of elastic moduli (10, 11, 12 13). In order to obtain the variables some of the properties of the ribs are kept constant such as the height and width if the rib stiffeners owing to manufacturing constraints and the complexity of the design. Some of the assumptions made for this method are as follows,

- The ribs stiffeners are unidirectional composite structures with uniform cross-sectional area and they are made up of same material including the ribs of different families. (E, G, h, w for all the ribs are constant)
- The hygrothermal effect of ribs is not considered.

Table 5-1 Composite material properties used in the MATLAB code [3]

Composite material used	T300/5208 unidirectional carbon-epoxy
Longitudinal modulus(E1)	181e9 pa
Transverse modulus(E2)	10.3e9 pa
Shear modulus(G12)	7.17e9 pa
Poisson's ratio(vxy)	0.28
Height of the rib blade cross-section(h)	20 mm
Width of the rib blade cross section(w)	10 mm
Density of the rib stiffener(ρ)	1.6e-6 kg/mm^3

The equations solved sometimes return the values of distance design variables (d_0, d_{90}, d_θ) and the orientation (θ) <0 and the orientation (θ)>π/2 which cannot be plotted or designed due to geometrical constraints and if the values of the input stiffness is less than the rib stiffener properties the equations may become complex. In order to avoid that, certain assumptions are made before the equations are solved which are given below,

Table 5-2 bounds of the design variables allotted for the design pattern for method 1

Variable	**lower bound**	**Upper bound**
d_0	0	∞
d_{90}	0	∞
d_θ	0	∞
θ	0	π/2
E_x	E_{xQDG}(Objective stiffness)	∞
E_y	E_{yQDG}(Objective stiffness)	∞
v_{xy}	0.27	0.37
G_{xy}	G_{xyQDG}(Objective stiffness)	∞

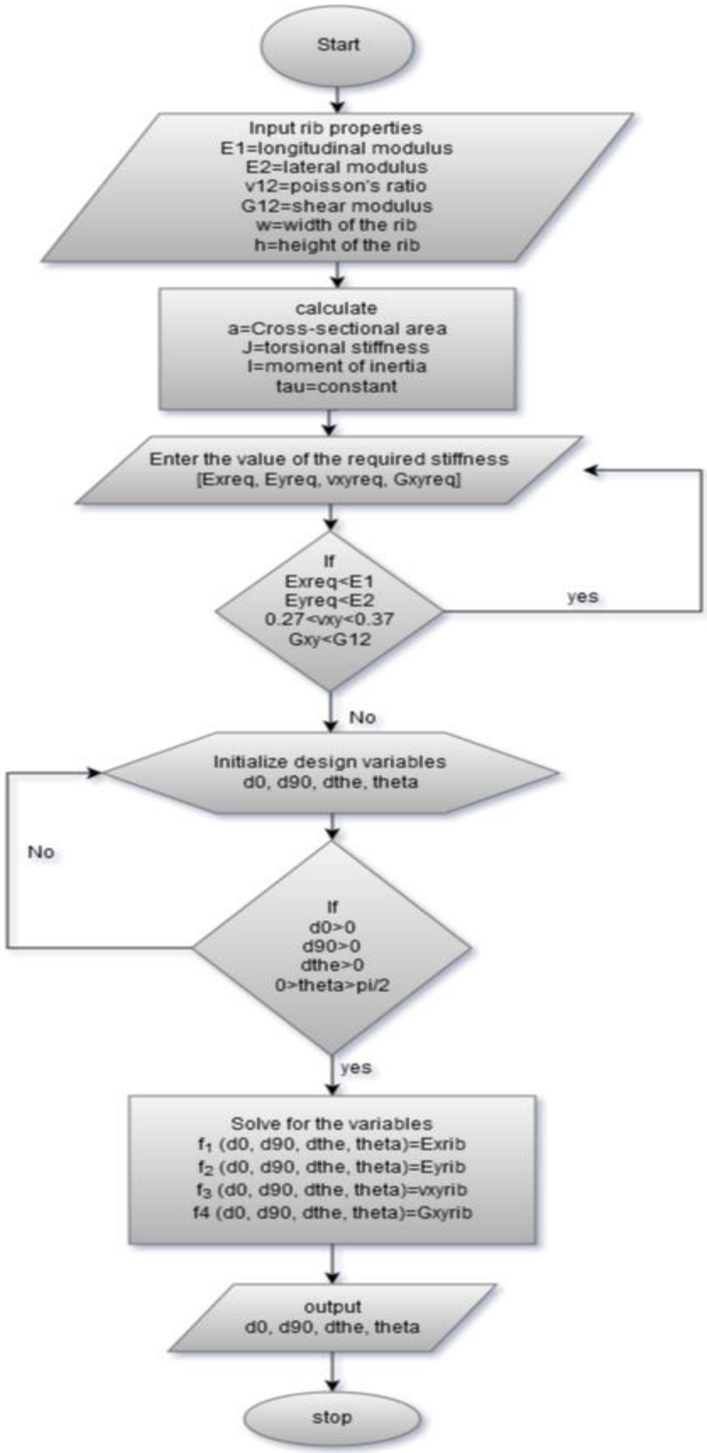
Table 5-3 Objective functions to be solved in terms of design variables by method 1

$E_{xQDG} = f_1(d_0, d_\theta, d_{90}, \theta)$	$\left[\frac{Ew[2d_0 m^4 + 2d_{90} n^4 + d_\theta]}{d_0[2d_{90} n^4 + d_\theta]}\right]$
$E_{xQDG} = f_1(d_0, d_\theta, d_{90}, \theta)$	$\left[\frac{Ew[2d_0 m^4 + 2d_{90} n^4 + d_\theta]}{d_{90}[2d_0 m^4 + d_\theta]}\right]$
$E_{xQDG} = f_1(d_0, d_\theta, d_{90}, \theta)$	$\left[\frac{2d_{90} m^2 n^2}{2d_{90} n^4 + d_\theta}\right]$
$E_{xQDG} = f_1(d_0, d_\theta, d_{90}, \theta)$	$\left[\frac{2Ewm^2 n^2}{d_\theta}\right]$

5.1.2 Process

The intention of this method is to extract the design variables of the QDG, which when modeled should have the stiffness moduli similar or exact as the objective stiffness vector. The objective stiffness vector is defined by the user as the stiffness to be obtained by the QDG. The equations for the stiffness moduli for QDG obtained in the previous chapters are solved for which the resultant vector is the desired stiffness vector. A MATLAB code is generated to solve the equations for the stiffness moduli and the design variables obtained are used to generate the geometric pattern of the QDG model.

5.1.2.1 Flowchart



5.1.2.2 Objective Stiffness Vector

Table 5-4 The objective stiffness vector given to solve the equations in MATLAB

$\mathbf{E_{xQDG}}$	$1*10^{11}$ pa
$\mathbf{E_{yQDG}}$	$0.9*10^{11}$pa
$\mathbf{V_{xyQDG}}$	0.3
$\mathbf{G_{xyQDG}}$	$0.7*10^{11}$pa

5.1.3 Results

5.1.3.1 Design Variables Obtained

The rib stiffness properties (Table 5-1) and the values of objective stiffness vector (Table-5-4) are taken as the inputs in the code and the design variables are obtained by solving the equations (Table 5-3) to obtain the design variables as follows,

Table 5-5 Results obtained from the MATLAB code for method 1

$\mathbf{d_0}$	23.0008 mm
$\mathbf{d_{90}}$	41.0016 mm
$\boldsymbol{\theta}$	47.1018 degrees
$\mathbf{d_\theta}$	12.8591 mm

5.1.3.2 Computer Aided Design (CAD) Model

The resultant values of the design variables obtained from the MATLAB code is used to design the Quadri-directional grid CAD model in Hypermesh. A mid-surface model of the ribs are created and each family of the ribs are created as a separate component and then merged together to form one complete QDG structure. To create a rib family, for example horizontal rib family, surfaces of are created on XY plane with the height of the surface (h) which is the height of the rib, is along Z direction. The ribs are placed at a distance (d_0) apart from each other along the Y axis for a defined width and length as shown in the figure. The thickness of the rib (width=w) is given as the thickness of the element to the mid surface model or in other words the ribs are designed as a single ply running in the orientation in local(1) direction, whose ply thickness is equal to the width of the rib.

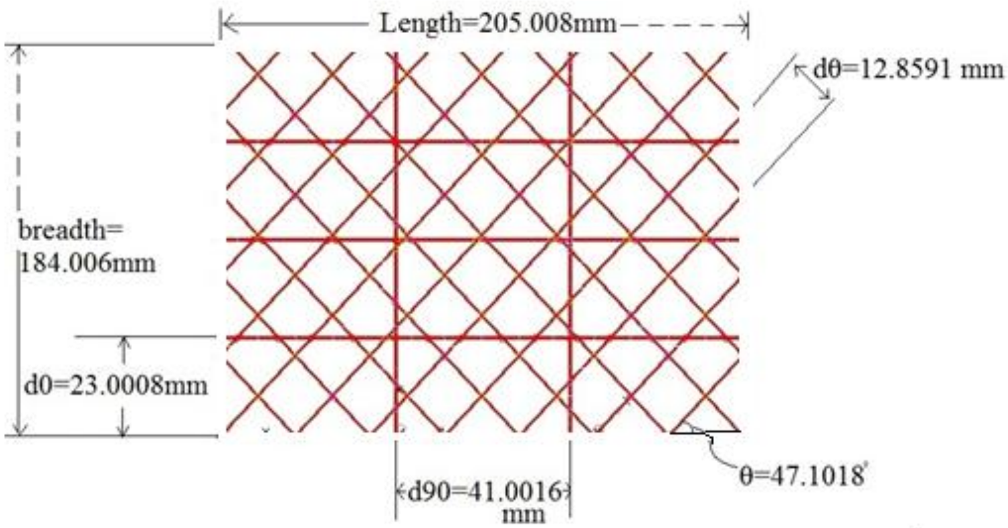


Figure 5-1 Dimensions of the QDG CAD model for Method 1

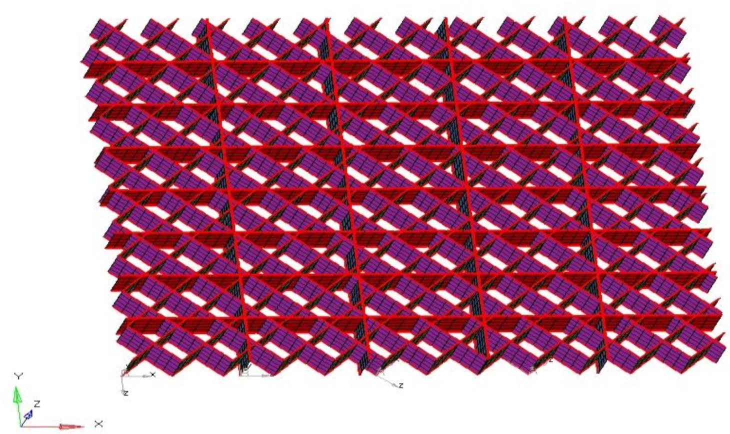


Figure 5-2 CAD/FEA model of the QDG for method

5.1.3.3 Meshing and Static structural analysis

The designed structure is meshed using 2D quad elements. As the model used is not too large, higher refinement is used to get the accurate results and the reduction of speed due to higher refinement is very less and can be obtained without too much of a delay. The property of each rib is defined separately though the card image chosen is the same. The property card used for the rib is PCOMP card. The PCOMP card allows the user to define the number of plies and the orientation in the card edit section. The number of plies for this is taken as 1 and the width of the ply is given as the 10, this creates a single ply of width 10mm and height of 20mm which is the given dimension of the rib. Similar procedure is carried for other rib families and then the components are merged to form the QDG structure.

To find the elasticity moduli of the QDG along X and Y direction, one end of the rib is constrained and at the other end of the QDG, the load is applied. The degrees of freedom is constrained only in the direction of the applied forced displacement load and the other degrees of freedom is not constrained to avoid the reaction forces at the constraints. Since the model is an open structure the values of stiffness moduli obtained can be largely different as localized displacements may occur if normal loads are applied, so an enforced displacement load is applied to avoid the problems related to local displacements. A displacement of **10mm** is applied on the model. The modulus along x direction is calculated for the FEA model using the following formula.

$$E_x^{model} = \frac{\sigma_x^{avg}}{\epsilon_x} \quad \textbf{(27)}$$

The average stress is calculated from the reaction forces at the constraints by,

$$\sigma_x^{avg} = \frac{Total\ Reaction\ Force\ (Rf_x)}{Constrained\ Area(a)} \quad \textbf{(28)}$$

Where,

Rf_x= total reaction force along x direction at the constraints.

a= total reaction area =$\sum(\text{constraint}) * (\text{the area at that particular constraint})$.

The strains are calculated using the formula,

$$\epsilon_x = \frac{displacement\ along\ x\ direction}{original\ length\ of\ the\ model\ along\ x\ direction} \quad \textbf{(29)}$$

The reaction forces at the constraints can be extracted in hypermesh by selecting the SPCF card in global output request control card.

The transverse modulus is obtained by following similar method along y direction.

Obtaining shear modulus is complicated as there would be reaction forces acting along both x and y direction which would also increase the complexity if the model is an open structure. In order to decrease the error, enforced displacement is applied as the load the reaction forces is taken for calculation. The set-up for shear modulus analysis is as shown in the figure below.

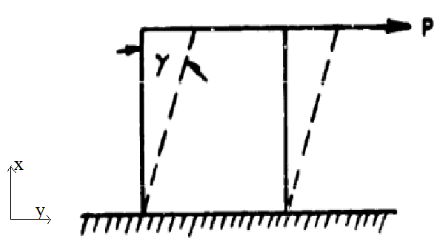


Figure 5-3 Shear modulus analysis setup

Where (p) is the enforced displacement load and the (γ) is the displacement along y direction. A small value of forced displacement (p) of **0.1mm** is applied in the y direction, in order to reduce the reaction forces in x direction to avoid complex calculations. The shear modulus is obtained by formula

$$G_{xy}^{model} = \frac{\tau_{xy}^{avg}}{\gamma_{xy}} \quad \textbf{(30)}$$

Where,

τ_{xy}^{avg} =Average shear stress=$\frac{Total\ Reaction\ Force\ (Rf_y)}{Constrained\ Area(a)}$

a= total reaction area =$\sum(\text{constraint}) * (\text{the area at that particular constraint})$.

The shear strain is calculated using the formula,

$$\gamma_{xy} = \frac{displacement\ along\ y\ direction}{original\ length\ of\ the\ model\ along\ x\ direction} \quad \textbf{(31)}$$

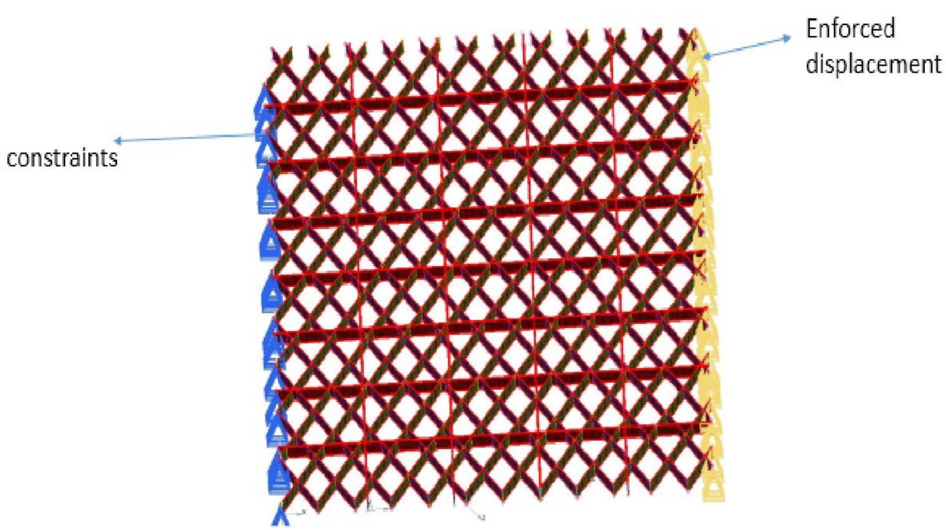


Figure 5-4 FEA analysis setup of QDG structure in Hypermesh

The above figure shows the generic analysis setup in hypermesh for obtaining the moduli, only the direction of applied load and the constraints change depending on the modulus to be obtained.

5.1.3.4 Analysis of results and validation

From the above preliminary analysis setup the analysis is carried out and the results are obtained as follows

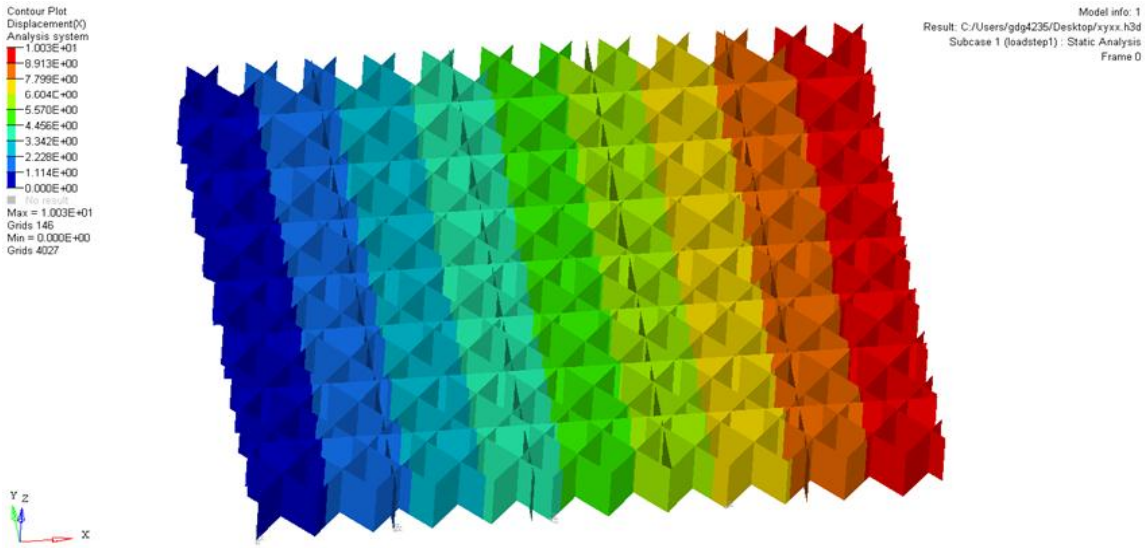


Figure 5-5 Enforced displacement result in x direction to find longitudinal stiffness modulus

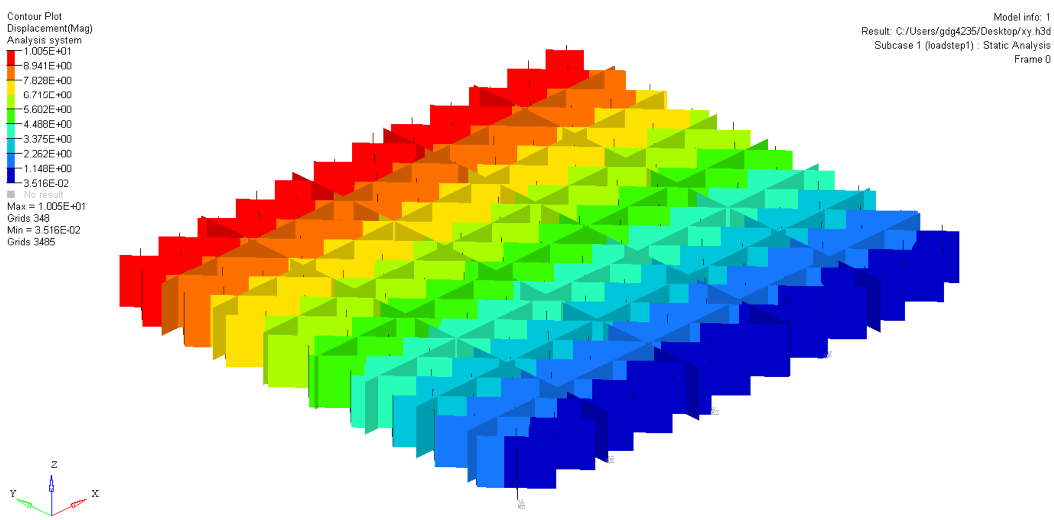


Figure 5-6 Enforced displacement result in y direction to find transverse stiffness modulus

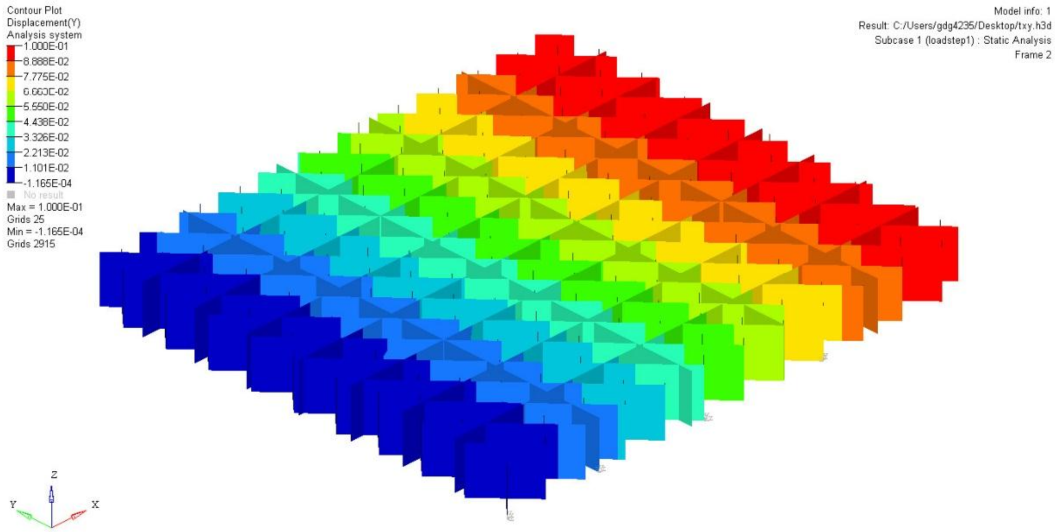


Figure 5-7 Enforced displacement result in y direction to find shear stiffness modulus

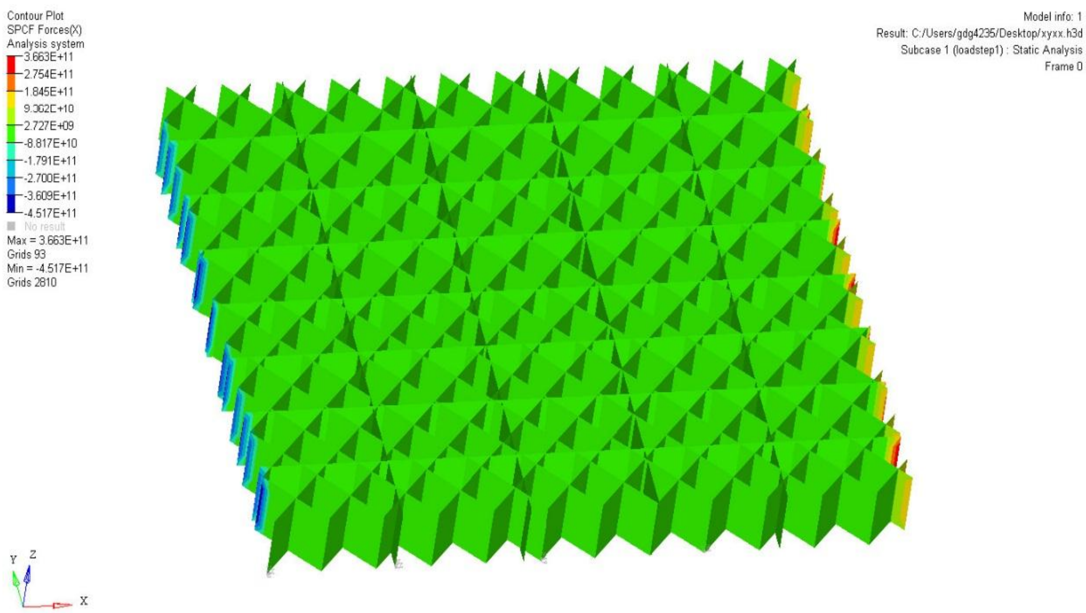


Figure 5-8 Reaction forces along X direction at the constraints (Ex)

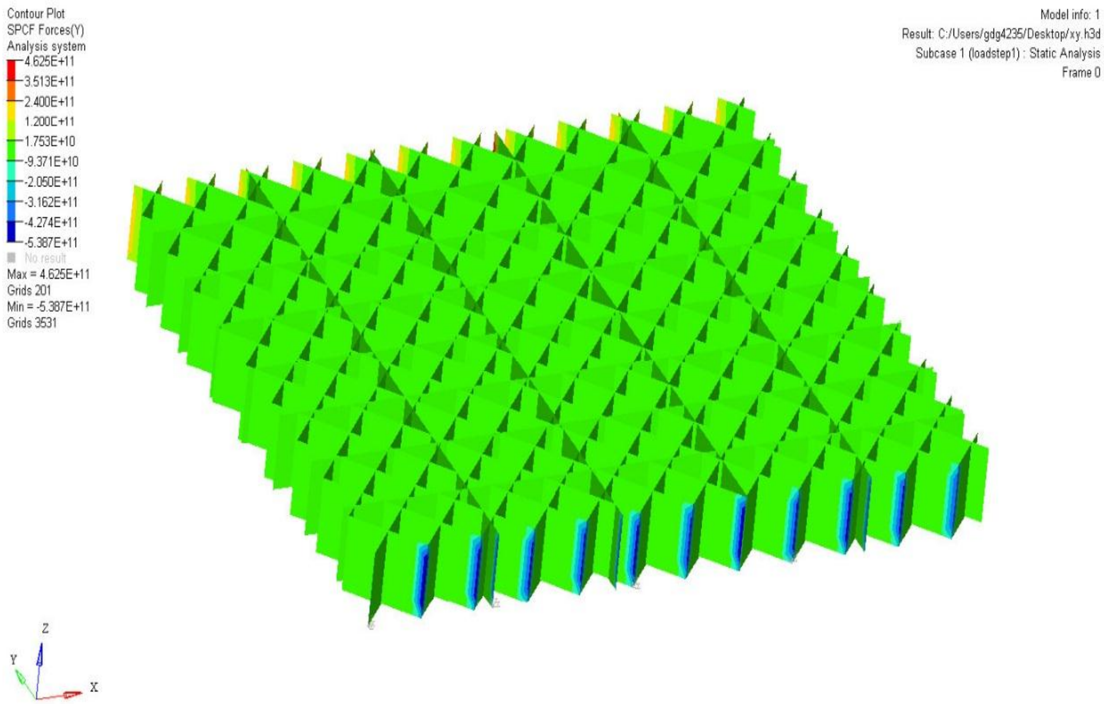


Figure 5-9 Reaction forces along Y direction at the constraints (Ey)

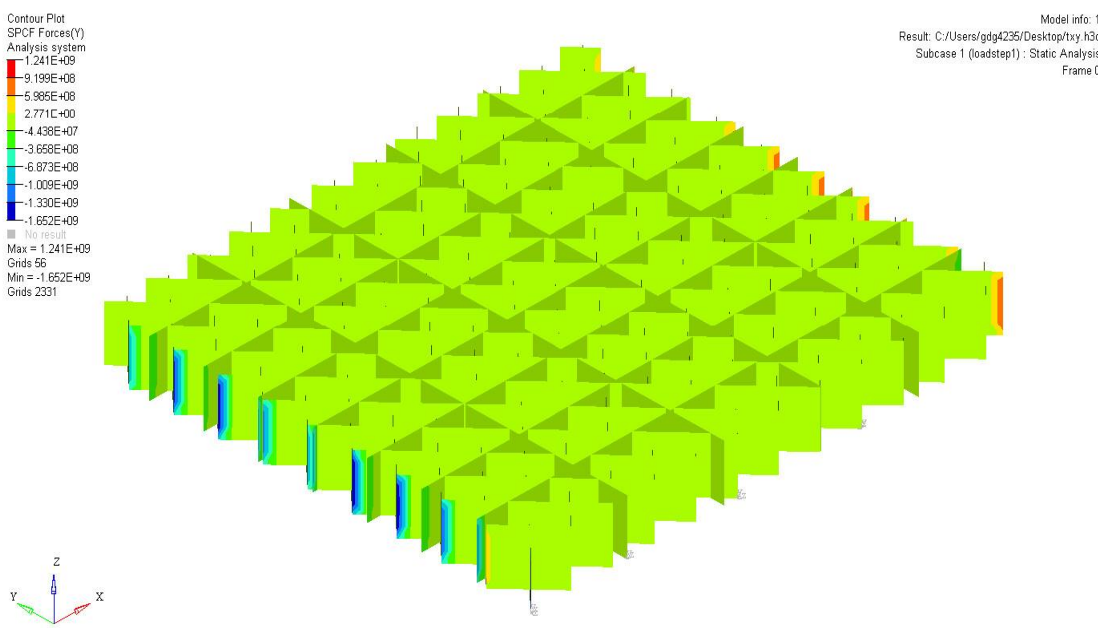


Figure 5-10 Reaction forces along Y direction at the constraints (Gxy)

From the above FE analysis results, the values required for obtaining the elastic moduli are extracted and the equation (27-31) are solved to obtain the elastic moduli.

Table 5-6 Results obtained from FE analysis

	Average stress(N/mm^2) obtained from the FEA	**applied strain on the model**	**Elastic modulus(E)(N/mm^2)**
longitudinal	$4.956977*10^{10}$	0.0487786	$Ex=10.1622*10^{10}$
transverse	$4.773463*10^{10}$	0.05434765	$Ey=8.7832*10^{10}$
shear	$3.35416*10^{7}$	$4.87786*10^{-4}$	$Gxy=6.8763*10^{10}$

The values obtained from the structural analysis is compared with the Objective stiffness vector to see the variation in the results

Table 5-7 Comparison of results

	Objective Stiffness Moduli for MATLAB (N/mm^2)	**Obtained Stiffness Moduli from FE analysis**	**% variation**
Ex	$10*10^{10}$	$10.1622*10^{10}$	1.622
Ey	$9*10^{10}$	$8.7832*10^{10}$	-2.41
Gxy	$7*10^{10}$	$6.8763**10^{10}$	1.767

The results from the FE model, validate that the results obtained from the code and the QDG structure is designed is close to the stiffness to be obtained. Variation of the grid structure results is because of the following reasons,

- The magnitude of design variables have been rounded off after the 3rd decimal point as the modeling software can take only up to the third decimal place this cause the FEA model to be slightly off the exact value.

- The sample model used is small and the larger model would have resulted in a better stress result.
- Some of the constraint areas may have multiple connecting points and the reaction forces at that point may not be the exact value as obtained.
- The values of the material properties are smeared along the ribs.

5.2 Method 2: Multi-Objective Optimization to maximize the elastic moduli

5.2.1 Rib stiffeners property initialization

The Genetic algorithm optimization is able to handle many variables, but some of the properties are kept constant such as the height and width if the rib stiffeners owing to manufacturing constraints and the complexity of the design. Some of the assumptions made for this design optimization are as follows:

- The ribs stiffeners are unidirectional composite structures with uniform cross-sectional area and they are made up of same material including the ribs of different families. (E, G, h, w for all the ribs are constant)
- The hygrothermal effect of ribs is not considered.

Rib properties used is taken from Table 5-1.

5.2.2 Genetic algorithm initialization

The main goal of the Algorithm is to find the Pareto-optimal value which satisfies all the objective functions, constraints and boundary conditions. Mathematically, a single point solution to a multi-objective problem does not exist unless the utopia point happens to be attainable [9]. The objective functions are set such that its goal is to maximize two functions and minimize one function, There may exists many optimal points which can satisfy the equations as they are subjected to only bound constraints. The advantage of using the

Multi-objective GA is that they find the points that are globally Pareto-optimal, which means the points obtained are the points which are globally optimum points. In order to obtain the closest point and the global maxima/minima, several runs of the algorithm have to be made and the range of the design variables have to be reduced close to the vicinity of the optimal points by intuition so that a close answer to the actual maxima/minima is obtained. Since random initial population is generated each time program is run, points obtained after each run may also be different. This can be reduced or curbed by setting initial population, high population size at each generation and by lowering the range of variables. The initial values taken for the optimization are as follows,

Table 5-8 Range of design variables for stiffness optimization

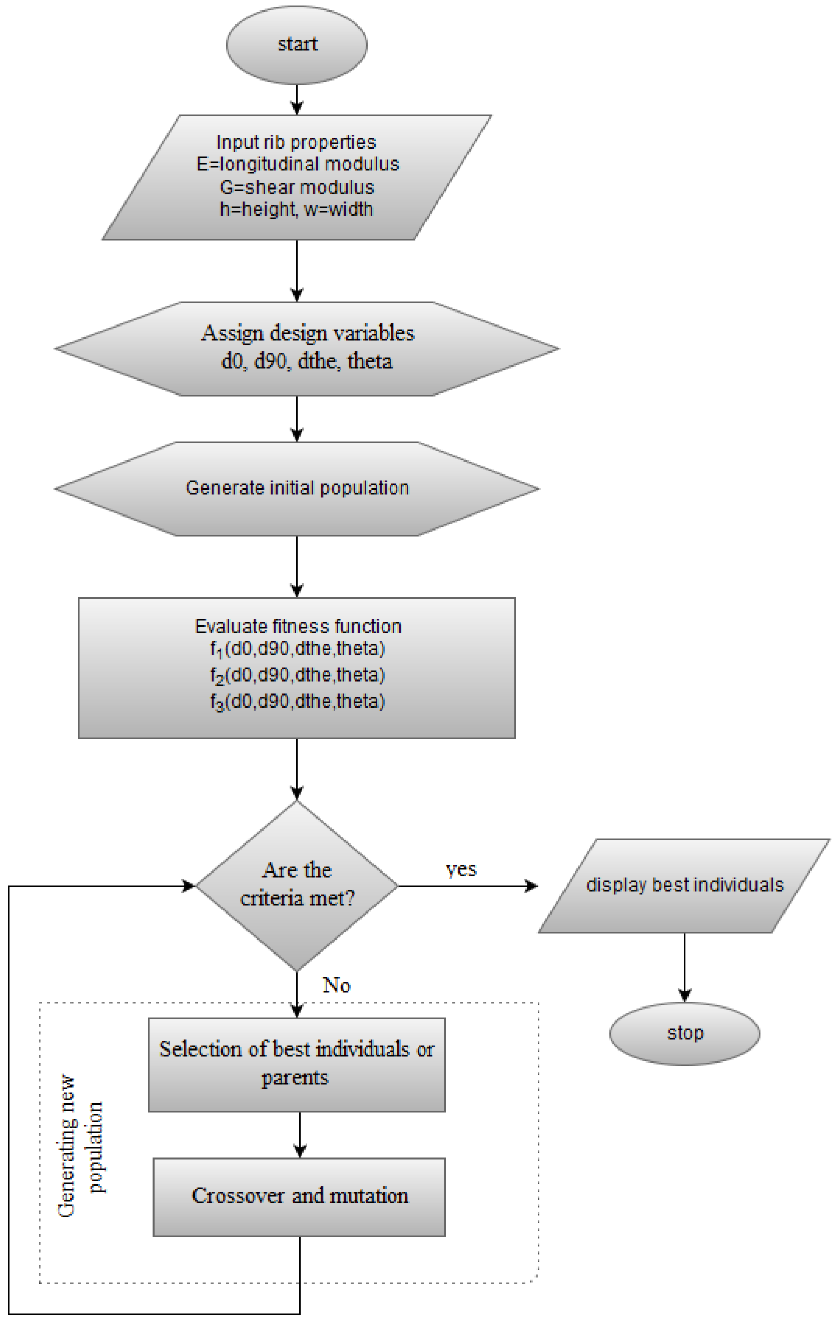
Initial Range of design variables	**d_0(mm)**	**d_{90}(mm)**	**d_θ(mm)**	**θ(radians)**
Lower bound	50	50	50	0
Upper bound	150	150	150	π/2

Table 5-9 Initializations for optimization

Initial population	1000
Initial population range	Lower bound to upper bound
Migration direction	Both
Termination Criteria	If Pareto optimal solution doesn't change for 5 consecutive generations.

The values of above initialization were arrived upon several runs of optimization before finalizing on these values.

5.2.3 Flowchart



5.2.4 Optimization results

The MATLAB tool for optimization (optimtool) can be used to run the program but in order to obtain the required plot, coding it would provide much more flexibility on the problem. Using the values of initialization and the equations discussed in the earlier chapters, the optimization code is written in MATLAB (appendix b). The optimization terminated after 212 generations and one of the optimum values obtain after iteration are as follows.

During the optimization run the plots of the pareto optimal points at each generation is plotted and the average distance between the points in each generation is also plotted.

Pareto front points generated at each iteration gives us all the optimal points at each generation of the algorithm. Any of the points here on the curve (final gen plot) can be chosen for modeling as all of them satisfy the constraints as shown in Figure 4-4. The pareto plot built in the matlab plots just the first and the second objective function which in this case is the longitudinal and transverse stiffness moduli respectively.

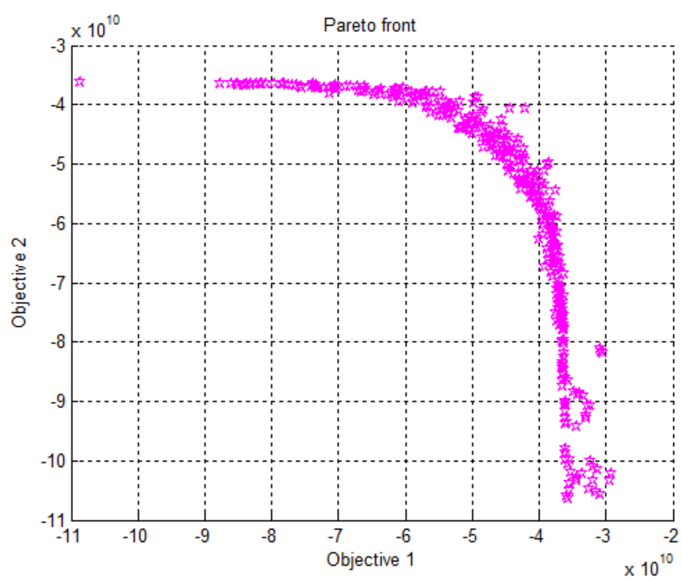


Figure 5-11 Pareto-front plot on first generation

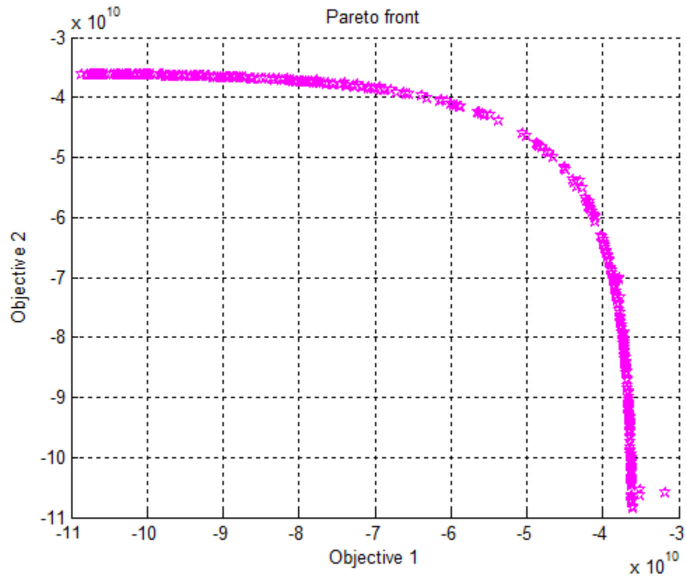


Figure 5-12 Pareto-front plot on 212th (Final) generation

As the values of the optimal points at each generation converges as the Ex and Ey get close to the maxima and the Gxy close to the minima, the values of the design variables also converge to the optimal points which can be shown in the figure 5-11 which gives the average distance between the two solutions in the population in each generation.

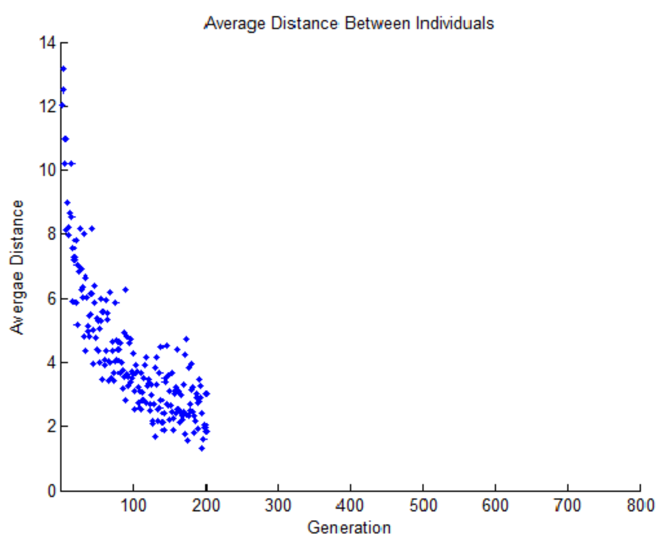


Figure 5-13 Average distance in each generation

After the optimization run the values of the design variables are obtained are as follows,

Table 5-10 Design variables obtained from stiffness optimization

d0	50.2276
d90	50.0441
dθ	50.0449
θ	43.1455
Ex	5.0297e+10
Ey	4.6252e+10
vxy	0.3634
Gxy	1.8008e+10

5.2.5 Design and Analysis

The above values are modeled in similar method as the in the previous chapter

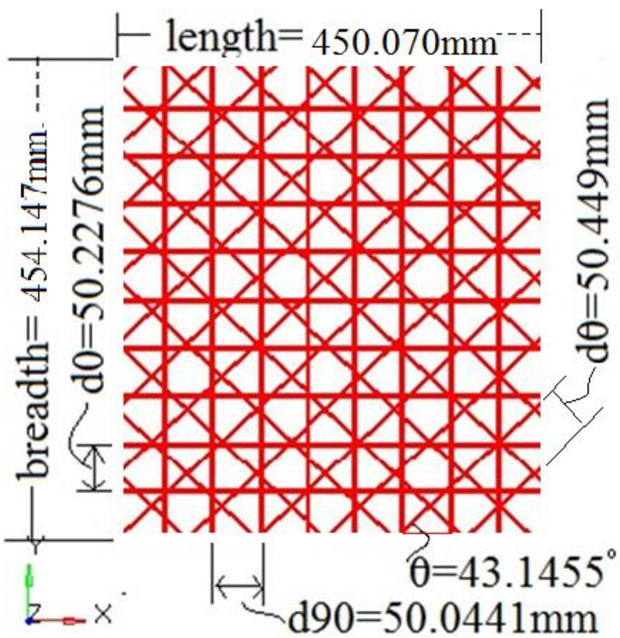


Figure 5-14 Dimensions of the QDG CAD model for Method 2

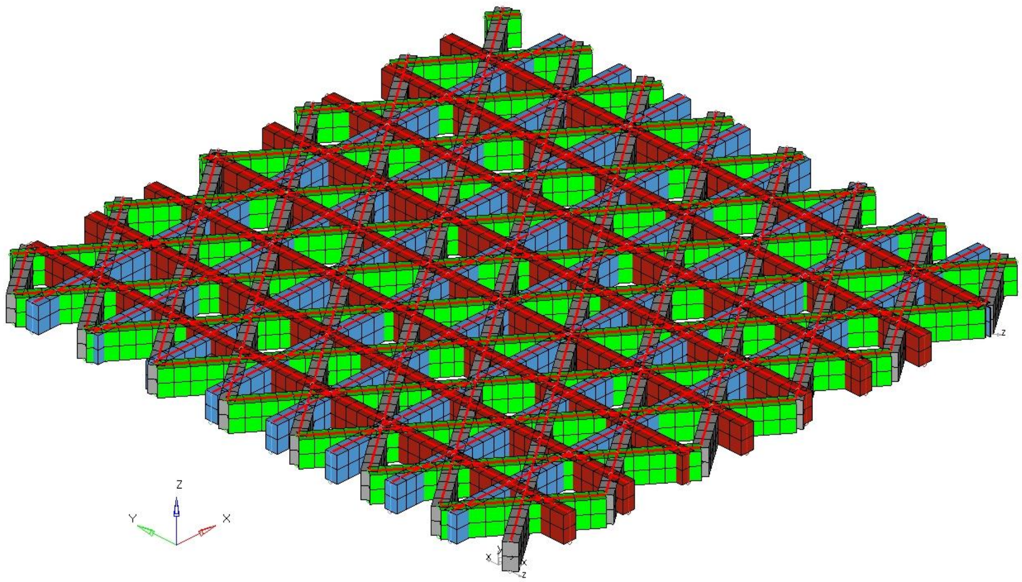


Figure 5-15 CAD/FEA model of the QDG for method 2

The FE analysis for the above model is carried out similar to the previous method and the plots obtained are as follows

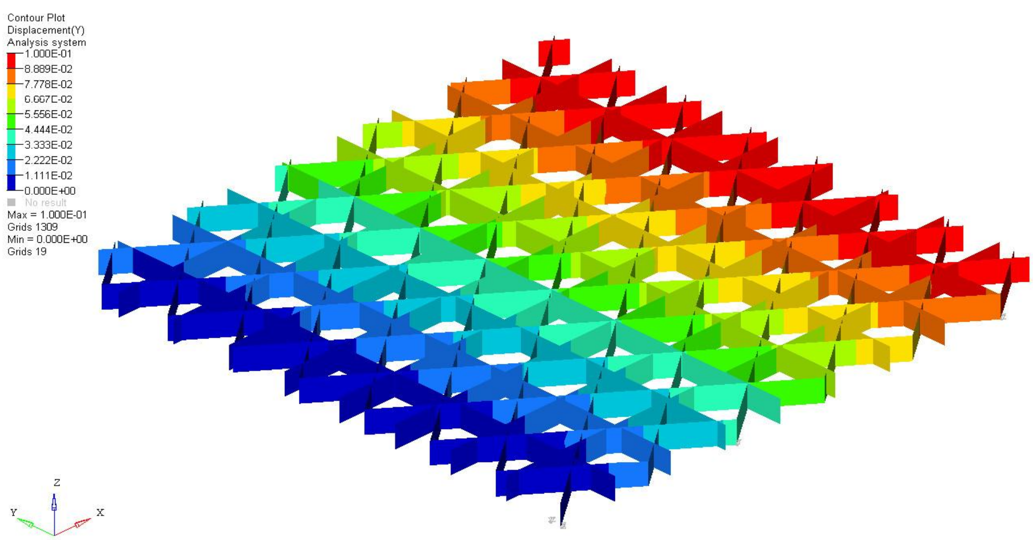


Figure 5-16 Enforced displacement result in x direction to find longitudinal stiffness modulus

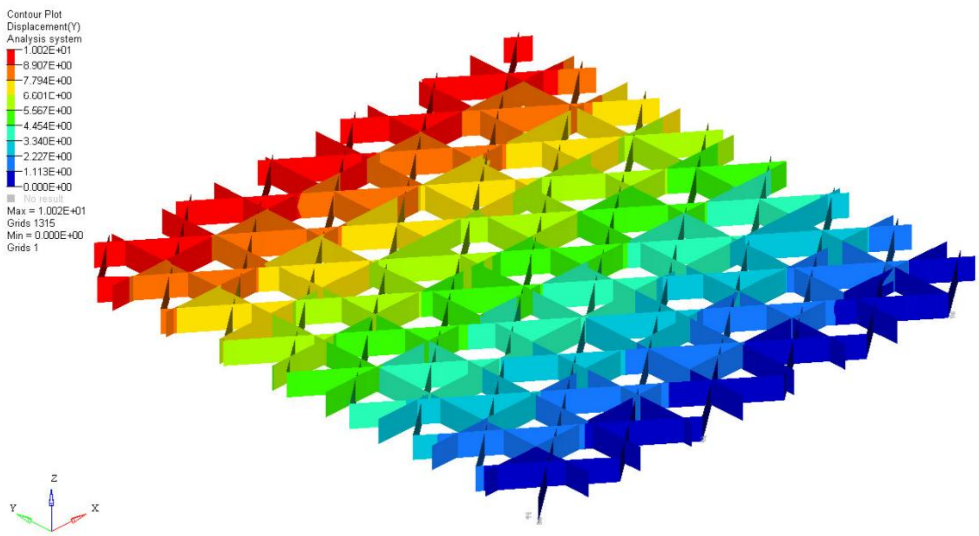


Figure 5-17 Enforced displacement result in Y direction to find transverse stiffness modulus

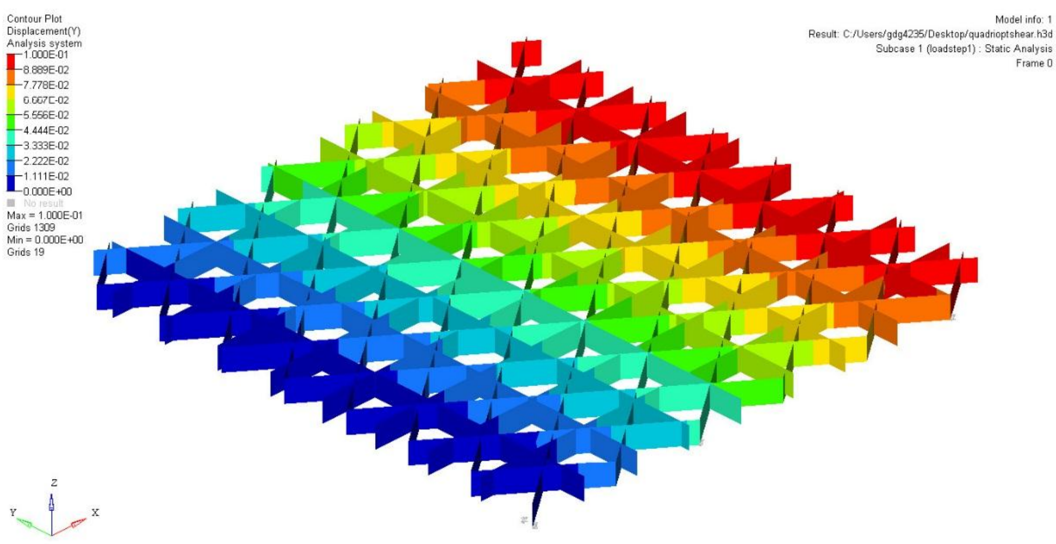


Figure 5-18 Enforced displacement result in Y direction to find shear stiffness modulus

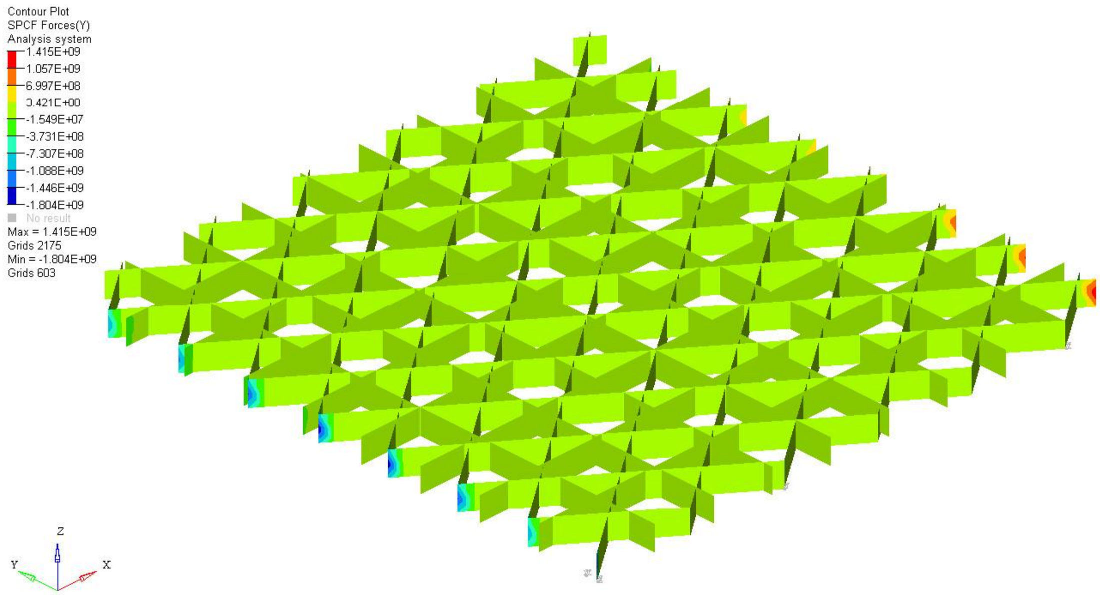


Figure 5-19 Reaction forces along X direction at the constraints (Ex)

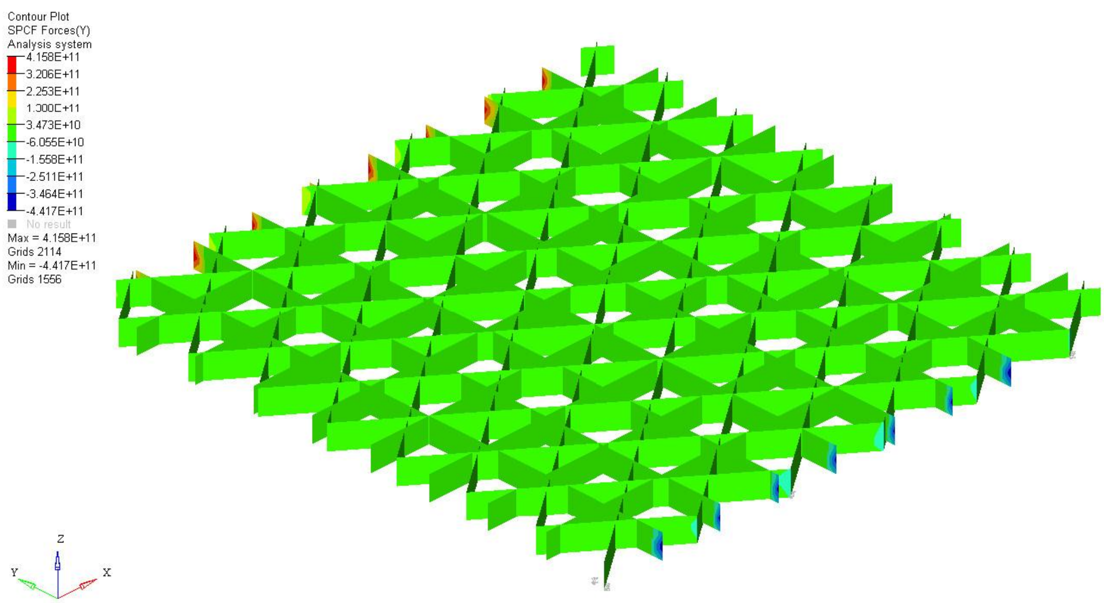


Figure 5-20 Reaction forces along Y direction at the constraints (Ey)

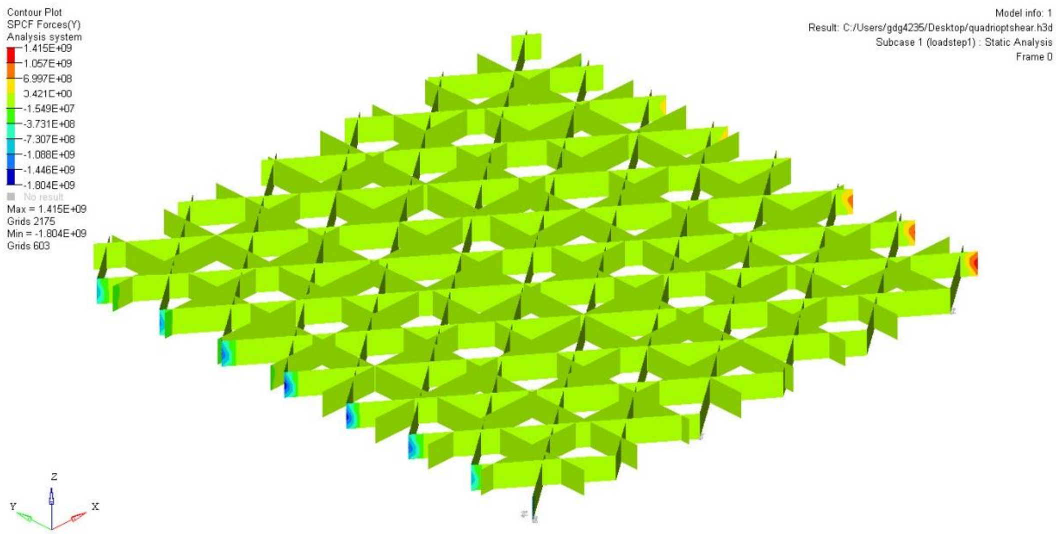


Figure 5-21 Reaction forces along Y direction at the constraints (Gxy)

From the above FE analysis results, the values required for obtaining the elastic moduli are extracted and the equation (27-31) are solved to obtain the elastic moduli.

Table 5-11 Results obtained from FE analysis

	Average stress(N/mm^2) obtained from the FEA	**applied strain on the model**	**Elastic modulus(E)(N/mm^2)**
longitudinal	$1.0942*10^{9}$	0.0222187	$Ex=4.9247*10^{10}$
transverse	$1.023017*10^{9}$	0.0220193	$Ey=4.646*10^{10}$
shear	4146898.168	$2.22187*10^{-4}$	$Gxy=1.8664*10^{10}$

The values obtained from the structural analysis is compared with the optimized values obtained from genetic algorithm to see the variation in the results

Table 5-12 Comparison of results

	Stiffness Moduli obtained from optimization (MATLAB) (N/mm^2)	**Obtained Stiffness Moduli from FEA analysis**	**% variation**
Ex	5.0297e+10	$4.9247*10^{10}$	1.622
Ey	4.6252e+10	$4.646*10^{10}$	-2.41
Gxy	1.8008e+10	$1.8664*10^{10}$	3.64

The results from the FE model, validate that the results obtained from the optimization and the error percentage obtained is as shown. The reason for the exact result not to be obtained are as follows.

- The magnitude of design variables have been rounded off after the 3rd decimal point as the modeling software can take only up to the third decimal place this cause the FEA model to be slightly off the exact value.
- The sample model used is small and the larger model would have resulted in a better displacement result.
- The values of the material properties are smeared along the ribs.

5.2.6 Laminate stiffness matrices obtained from both the methods

Table 5-13 Laminate stiffness matrices

Multi-objective optimization method	$10^{14}\begin{bmatrix} \begin{matrix} 0.0113 & 0.0036 & 0 \\ 0.0036 & 0.0104 & 0 \\ 0 & 0 & 0.0036 \end{matrix} & zeros(3,3) \\ zeros(3,3) & \begin{matrix} 0.3802 & 0.1168 & 0 \\ 0.1168 & 0.3499 & 0 \\ 0 & 0 & 8.3821 \end{matrix} \end{bmatrix}$
Objective stiffness method	$10^{14}\begin{bmatrix} \begin{matrix} 0.072 & 0.000 & 0 \\ 0.000 & 0.0088 & 0 \\ 0 & 0 & 0.000 \end{matrix} & zeros(3,3) \\ zeros(3,3) & \begin{matrix} 2.3996 & 0.0009 & 0 \\ 0.0009 & 2.943 & 0 \\ 0 & 0 & 7.3587 \end{matrix} \end{bmatrix}$

6. Difference and scope of both the methods

The main advantages and disadvantages of each method used here is discussed

Method1 Objective stiffness method:

Advantages

- Desired stiffness from the model can be obtained.
- Highly efficient model and can also provide high axial apart from intended buckling stiffness

Disadvantages

- Less control over the design variables and can obtain variables which cannot be used due to manufacturing constraints.
- Can have high volume due to more number of rib families increasing the weight of the overall structure

Method 2 Multi-objective optimization method

Advantages

- Manufacturing constraints can be included in the code to obtain.
- Multiple stiffness can be maximized or minimized simultaneously

Disadvantages

- Obtained variables in each optimization run can be different because of multiple objective functions and no equality constraints.
- The optimization results can be close to the lower bound and increase the volume of the structure

7. Future work

More study is needed to make this thesis mature and this can be applied to many different situations.

- The main goal of the thesis was to provide a generalization of the designing methods of grid structures so that it can be devolved and used to design conventional QDG, tri-directional and angular grid structure.
- The methods used here can also be combined to provide a better closer solution or can be used to optimize design variables by taking in a different objective function. For example, weight of the grid structure can be reduced by taking in the weight of the grid structure as the main objective function to be minimized and the other reduction stiffness method can be used further by providing the elastic moduli equations as the equality or inequality constraints to be achieved by the grid structure.
- The main purpose of the grid structures are they are used as primary reinforcement members for weak structures. [3] Provides an insight on how to integrate them into the skin members and design of the integrated grid structure can be used to design the grid structure.
- The methods used here can be expanded to other grid structures such as isogrid structures and orthogrid structures

Appendix A

MATLAB code for Method 1

```
%the properties of the ribs in the structure%
disp('enter the material property of the ribs');
disp('1-carbon epoxy(T300/5208)');
disp('2-other materials');
matnum=input('enter the number of the property to be used>');
if matnum==2
  E=input('enter the linear E modulus of the rib in pa>');
  G=input('enter the shear modulus of the rib in pa>');
  disp('enter the dimensions of the ribs in mm as shown');
  w=input('width of the rib cross section>');
  h=input('height of the rib cross section>');
elseif matnum==1
    load T300.txt;
    E=T300(1,1);
    G=T300(1,2);
    w=T300(1,3);
    h=T300(1,4);
end

%area,moment of inertia and the torsional stiffness of the rib%
A=w*h;
I=(w*h^3)/12;
J=((h*w^3)/16)*((16/3)-3.36*(w/h)*(1-(w^4)/(12*h^4)));
tau=(G*J)/(E*I);
x=(5/6);

%objective stiffness of the rib%
rst=input('enter the required stiffness from the grid in pa in the
brackets as following [ex ey vxy gxy]>');
syms d0 d90 dthe theta positive;
assume(theta>0 & theta<pi/2)
assume(d0>0 & d90>0)
m=cos(theta);
n=sin(theta);

%stiffness moduli equations to be solved%
Exrib=((E*w*(2*d0*(m^4)+2*d90*(n^4)+dthe))/(d0*(2*d90*(n^4)+dthe)));
Eyrib=((E*w*(2*d0*(m^4)+2*d90*(n^4)+dthe))/(d90*(2*d0*(m^4)+dthe)));
vxyrib=((2*(m^2)*d90*(n^4))/(2*d90*(n^4)+dthe));
gxyrib=(E*w*2*(m^2)*(n^2))/dthe;

%solving the equations to find d0, d90, theta and -theta
    eqn=sym(zeros(4,1));%initializing%

   eqn(1,1)=Exrib==rst(1,1);
   eqn(2,1)=Eyrib==rst(1,2);
   eqn(3,1)=vxyrib==rst(1,3);
   eqn(4,1)=gxyrib==rst(1,4);

   for i=1:1:4
     eqn(i,1)=rewrite(eqn(i,1),'sin');
   end
```

```
ribdim=solve(eqn(1,1),eqn(2,1),eqn(3,1),eqn(4,1),d0,d90,dthe,theta,'Ig
noreAnalyticConstraints',1);

%obtained values of variables%
d0ob= double(ribdim.d0);
d90ob= double(ribdim.d90);
dtheob= double(ribdim.dthe);
thetaob= double(ribdim.theta)*180/pi;

mm=cosd(thetaob);nn=sind(thetaob);

Exribob=((E*w*(2*d0ob*(mm^4)+2*d90ob*(nn^4)+dtheob))/(d0ob*(2*d90ob*(n
n^4)+dtheob)));
Eyribob=((E*w*(2*d0ob*(mm^4)+2*d90ob*(nn^4)+dtheob))/(d90ob*(2*d0ob*(m
m^4)+dtheob)));
vxyribob=((2*(mm^2)*d90ob*(nn^4))/(2*d90ob*(nn^4)+dtheob));
gxyribob=(E*w*2*(mm^2)*(nn^2))/dtheob;



%the laminate stiffness matrix of the grid structure%
  mm=cos(thetaob);nn=sin(thetaob);
  Arib=(E*A)*[((1/d0ob)+(2*(mm^4)/dtheob))   2*(mm^2)*(nn^2)/dtheob
0;
                2*(mm^2)*(nn^2)/dtheob
((1/d90ob)+(2*(nn^4)/dtheob))      0;
                      0                          0
2*(mm^2)*(nn^2)/dtheob];

  D11=(E*I)*((1/d0ob)+(2*(mm^4)/dtheob)+(2*tau*(mm^2)*(nn^2)/dtheob));

D22=(E*I)*((1/d90ob)+(2*(nn^4)/dtheob)+(2*tau*(mm^2)*(nn^2)/dtheob));
  D12=(E*I)*((2*(mm^2)*(nn^2)/dtheob)-(2*tau*(mm^2)*(nn^2)/dtheob));

D66=(E*I)*((2*(mm^2)*(nn^2)/dtheob)+(tau/4*d0ob)+(tau/4*d90ob)+(tau/2*
dtheob)*(((mm^2)-(nn^2))^2));

  Drib=[D11    D12     0;
        D12    D22     0;
         0      0     D66];

  ABD=[Arib zeros(3,3);
         zeros(3,3) Drib];
  abd=inv(ABD);
  disp(ABD);
```

Appendix B

MATLAB code for Method 2

```
%the properties of the ribs in the structure%
disp('enter the material property of the ribs');
disp('1-carbon epoxy(T300/5208)');
disp('2-other materials');
matnum=input('enter the number of the property to be used>');
if matnum==2
  E=input('enter the linear E modulus of the rib in pa>');
  G=input('enter the shear modulus of the rib in pa>');
  disp('enter the dimensions of the ribs in mm as shown');
  w=input('width of the rib cross section>');
  h=input('height of the rib cross section>');
elseif matnum==1
    load T300.txt;
    E=T300(1,1);
    G=T300(1,2);
    w=T300(1,3);
    h=T300(1,4);
end

  %area,moment of inertia and the torsional stiffness of the rib%
  A=w*h;
  I=(w*h^3)/12;
  J=((h*w^3)/16)*((16/3)-3.36*(w/h)*(1-(w^4)/(12*h^4)));
  tau=(G*J)/(E*I);


  %optimization of the quadri-directional grid structure%
  fit=@desvar; %calling the fitness function%

  %the lower and upper bound of the design variables within which the
optimization is carried out%
  l=[50;50;50;0];u=[150;150;150;pi/2];
  nvar=4; %number of design variables%

  options=gaoptimset('PopulationSize',1000,'PopInitRange',[50 50 50
0;70 70 70 pi/2],'MigrationDirection', 'both','CrossoverFcn', {
@crossoverintermediate [] },'Display', 'off','PlotFcns',
{@gaplotdistance});


  %genetic algorithm optimization of the grid structure%
  [vari,moduli] = gamultiobj(fit,nvar,[],[],[],[],l,u,options);

  %final obtained value after optimization%
  %design variables%
  [rw,cm]=size(vari);
  disp('distance between two horizontal ribs(d0)>');
  disp(vari(rw,1));d0=vari(rw,1);
  disp('distance between two vertical ribs(d90)>');
  disp(vari(rw,2));d90=vari(rw,2);
```

```
  disp('orientation of the angled family ribs(theta)>');
  disp((vari(rw,4)*180/pi));theta=vari(rw,4);
  disp('distance between two angled ribs of opposite family(the and -
the)>');
  disp(vari(rw,3));dthe=vari(rw,3);

  %optimized value of stiffness%

  [rw,cm]=size(moduli);
  disp('optimized longitudinal modulus(Ex)>');
  disp(abs(moduli(rw,1)));Exrib=moduli(rw,1);
  disp('optimized lateral modulus(Ex)>');
  disp(abs(moduli(rw,2)));Eyrib=moduli(rw,2);
  %disp('optimized poissions ratio(vxy)>');
  disp(moduli(rw,3));vxy=moduli(rw,3);
  disp('optimized shear modulus(Gxy)');
  %disp(moduli(rw,4));Gxy=moduli(rw,4);

  %the optimized stiffness matrix of the grid structure%
  m=cos(theta);n=sin(theta);
  Arib=(E*A)*[((1/d0)+(2*(m^4)/dthe))  2*(m^2)*(n^2)/dthe          0;
              2*(m^2)*(n^2)/dthe    ((1/d90)+(2*(n^4)/dthe))      0;
                       0                 0       2*(m^2)*(n^2)/dthe];

  D11=(E*I)*((1/d0)+(2*(m^4)/dthe)+(2*tau*(m^2)*(n^2)/dthe));
  D22=(E*I)*((1/d90)+(2*(n^4)/dthe)+(2*tau*(m^2)*(n^2)/dthe));
  D12=(E*I)*((2*(m^2)*(n^2)/dthe)-(2*tau*(m^2)*(n^2)/dthe));

D66=(E*I)*((2*(m^2)*(n^2)/dthe)+(tau/4*d0)+(tau/4*d90)+(tau/2*dthe)*((
(m^2)-(n^2))^2));

  Drib=[D11    D12     0;
        D12    D22     0;
         0      0     D66];

  ABD=[Arib zeros(3,3);
         zeros(3,3) Drib];
  abd=inv(ABD);
  disp(ABD);
```

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