FINITE ELEMENT MODELING AND STRESS DISTRIBUTION OF UNIDIRECTIONAL COMPOSITE MATERIALS UNDER TRANSVERSAL LOADING

by

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Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

DECEMBER 2016

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DEDICATION

This thesis is dedicated to my family. I also dedicate this thesis to my teachers from whom I have learned so much.

ACKNOWLEDGEMENTS

I would like to use this opportunity to thank my thesis guide Dr. Andrey Beyle, for his inspiration and continuous guidance throughout my work. It has been an incredible journey and a great experience working with him on research projects. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my thesis study. I would also like to thank him for his encouragement to help me cross all the obstacles.

I would also like to thank Dr. Wen S Chan and Dr. Endel Iarve for serving on my committee and providing numerous learning opportunities by their insightful comments and hard questions.

I would like to extend a special appreciation to Miss. Amruta Rupa for not only being a good friend but also helping me throughout my thesis and academics. I want to thank Mr. Gopal Gulwani and all other people I met during my research for their support and assistance. You all have been wonderful.

Finally, I would like to dedicate my thesis to my parents Mr. Vishnu Agarwal and Mrs. Rita Agarwal for sending me so far from home and having faith in me and my work.

November 29, 2016

ABSTRACT

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The University of Texas at Arlington, 2016

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Micromechanics of Composites analyze stresses inside any heterogeneous material. These stresses can not only be used for calculation of effective stiffness or compliance, but also for predicting strength and failure modes for these materials. This thesis is devoted to the stress analysis of unidirectional composites by finite element method. The key distinction from other finite element method modeling of the unidirectional composite was that the load on the cell was not prescribed, but was to be calculated taking into account the influence of the closest neighbors of the cell. Transversal unidirectional tension/compression and transversally symmetrical biaxial tension/compression were analyzed.

In this project, two kinds of fiber materials were mainly focused upon namely; Carbon and E-glass. Here single cell and multi-cell models for cylindrical, square and hexagonal geometries were considered. The entire work was primarily

focused on the cylindrical model since it constitutes the basic model in any mechanical industry. The models were experimented by taking different fiber volumes and applying relative pressure/loading to each. Stresses on the boundary were analyzed between the interface of fiber and matrix. Same was done with the multi-cell models, and analytical results were determined. Produced practical data was compared with analytical solutions for single cell and infinitely big regular array of inclusions in the matrix.

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Chapter 1

INTRODUCTION AND OBJECTIVE

Micromechanics of Composites development has started from the early sixties of the 20th century, and these composites achieved some success in prediction of effective stiffness and compliance tensor, effective linear thermal expansion dyadic, and some effective physical characteristics for material with known properties of fibers and matrix and known architecture of material. However, prediction of the strength properties was not so successful. The difference in predicted and experimentally determined strength is usually explained by ignoring many types of technological imperfections in idealized models. Another source of the mismatch is the use of simplified approaches in the stress analysis giving the distribution of micro-stresses (stresses in the scale significantly below the diameter of fiber). This work is devoted to the most precise analysis of micro-stresses made mostly by FEM modeling. The primary attention is put on transversal stresses, where the strength prediction is much worse than in prediction of longitudinal strength.

There are several models elaborated for prediction of the effective properties of unidirectional composites. Some models are dealing with a single cell, consisting of one fiber inside some volume of the matrix. The ratio of volumes fiber to matrix is the same as in the total composite. The stresses acting on the cell were prescribed going from the stress-state of the whole composite. In some models three phases were taken into account: the cell was "embedded" into quasi-uniform composite with unknown effective properties. The model developed by Van Fo Fy is considered double periodic lattice of the identical fibers inside infinite volume of the matrix. This model is the most accurate model because the stresses on the boundary of a single cell are calculated but not prescribed. Unfortunately, this model is based on very complicated mathematical apparatus of double periodic bi harmonic functions presented in the form of series over Weierstrass' elliptic functions.

Our FEM numerical analysis takes into account the only interaction of a selected cell with its closest neighbors. The results of some calculations were compared with theoretical prediction by single cell model in its most primitive coaxial cylinder form ignoring the type of lattice created by fibers and double periodic model.

In unidirectional fiber reinforced composites, the stiffness of the materials is very high in the direction of the fiber and moderately low in the direction perpendicular to the fiber. This characteristic is because of the high axial stiffness of the fibers. Due to this, a unidirectional fiber reinforced composite is not isotropic in nature. The stiffness properties are nearly the same in two directions, but it varies in the direction of the fiber.

1.1 Role of Composite Materials

Composite materials are the combination of two or more materials, which result in high strength and stiffness properties of fibers that are realized in monolithic materials with the help of binding matrix. Each material in composite retains its separate physical, chemical, and mechanical properties. Most of the composites are highly anisotropic in nature and are characterized by several strength and stiffness parameters. In composites, fibers are stronger and stiffer and act as a load-bearing constituent. Whereas matrix is soft and weak, but it transmits the load from fiber to fiber, protects fibers from outside environment and helps fibers from breakage [2].

In nature, the mechanical properties of anisotropic materials are usually exploited within the structure. For instance, wood has excellent strength in the direction of the fiber, then in the transversal direction [1].

Selection of fiber architecture can significantly change composites, and composite material can be designed individually for each particular application. They are composed of two or more phases and can be formulated to meet the needs of a specific application with considerable ease.

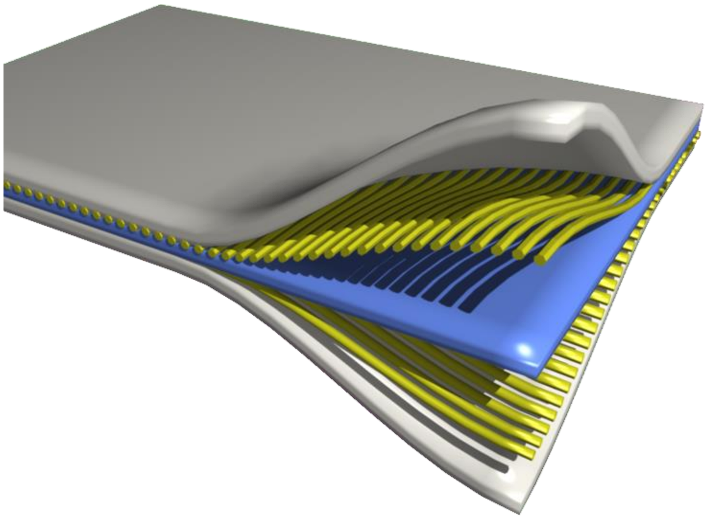


Figure 1.1 Image of a composite [18]

1.2 Classification of Composites

Composites are classified into two distinct phases:

The **first phase** is based upon the matrix constituent. The role of a matrix is to transfer the stresses between the fibers, protect the surface from abrasion, provide a barrier against adverse environments, and to provide lateral support. Matrix materials are classified into Organic matrix composites, Metal matrix composites, and Ceramic matrix composites.

Organic matrix composites are assumed to have two classes of composites, which are polymers and carbon matrix composites. Polymers are the substances that have large number of similar unit of molecules bonded together. Polymers are of various kinds, namely Rubbers, Thermoplastics, and Thermosets.

- Rubbers are the cross-linked polymers which have a semi crystalline state below the room temperature.
- Thermoplastics are the resins or plastics which can be repeatedly softened on heating and hardened on cooling. Example: nylon, polyethylene, polystyrene, polyamides, and polypropylene.
- Thermosets are the resins which are in chemically reached until almost all the molecules are irreversible cross-linked in a 3-D network. Example: epoxy, vinyl esters, phenolic, polyimides resins, and polyester.

Metal Matrix Composites offer high strength, fracture resistance, stiffness, and toughness to the composites than those offered by polymers. Most of the metals and alloys can be used as matrices with the reinforcement materials, which need to stand over a wide range of temperature and non-reactive too. Aluminum, magnesium, and titanium are the most popular matrix metals materials used because of their strength to weight ratios resulting in high strength and stiffness parameters. These metals are highly used for aerospace applications.

Ceramic Matrix Materials exhibit adamant ionic and covalent bonding. These materials have high melting points, high compression strength, excellent corrosion resistance, and can withstand high temperatures. Some of the commonly used ceramic materials are aluminum oxide, silicon carbide, and silicon nitride.

The **second phase** of classification is reinforcement form. Fibers are the principle constituents in the reinforcement. The various kinds of reinforcements are fiber reinforced composites, laminar composites, and particulate composites.

- **Fiber Reinforced Composites** are further divided into continuous and discontinuous fibers. Continuous fibers are the one where the properties of reinforcement are same throughout the length of the fiber. Here the elastic modulus of the composite does not change with the increase/decrease in the length of the fiber. On the other hand, discontinuous fiber or short fiber composites are the one where the properties of reinforcement vary with the fiber length. Fibers are short in diameter and bend easily when pushed axially, although they have very good tensile properties. Therefore, fibers must be supported to avoid them from bending and buckling.

Fibers are usually circular or near to circular in cross-section. In a composite, fibers occupy more volume than matrix and these are the major load carrying component. They are used as reinforcement in composites to provide strength and stiffness to the materials. They also provide heat resistance and protect the composite from corrosion. Performance of fiber depends upon its length, shape, orientation, and composition of the fibers. Anisotropic fibers have more strength in longitudinal direction when compared to transversal direction.

- **Laminar Composites** are formed by stacking number of layers of materials together. Each layer has its own strength, stiffness, and modulus in metallic, ceramic, or polymeric matrix material. Coupling may occur between the layers of laminates depending upon the sequence of stacking. The most commonly used materials in laminar composites are carbon, glass, boron, and silicon carbide for fiber, and epoxies, polyether ether ketone, aluminum, and titanium for matrix. Layers having different materials can be bounded, resulting in hybrid laminate.

- **Particulate Composites** refers to the material where reinforcement is embedded in the matrix. One best example of particulate composite is concrete, here rock or gravel acts as the reinforcement and is integrated with the cement which is a matrix. In particulate composites, fibers are of various shapes like triangle, square, or round but the dimensions of their sides are found to be nearly equal. These composites can be very small, less than 0.25 microns, chopped glass fibers, carbon Nano-tubes, or hollow spheres. Each material provides different properties and is embedded in the matrix.

1.3 Unidirectional Reinforced Composites

These are the composites having single fiber orientation in a layer, and for different layers in a laminate it has different single fiber orientation in each layer. Fibers are usually distributed uniformly through the matrix and there exists perfect bonding between fiber and matrix material.

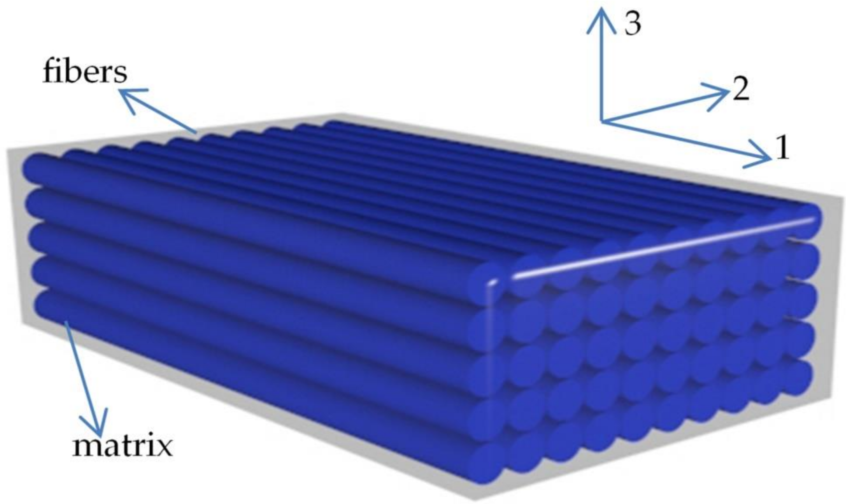


Figure 1.2 Unidirectional reinforced composite

1.4 Volume Concentrations

The most important data needed to model a composite is its content of constituent's phases [5]. During the analysis of composites, it is necessary to quantify the content of each phase (i.e., matrix and fiber) by its volume to the composite volume. A correct concentration of fibers can make composites stronger and stiffer. The concentration of constituents depends upon the geometry of the composite. In this work, it is explained how the concentration of fiber (denoted by cc) affects the strength for different geometries of composites.

For each of the architecture of composite material, there is an upper limit of volume concentration of fibers. For example, for square packing of round fibers it is Pi/4=0.785, but for hexagonal packing, it is Pi/ (2*sqrt3) =0.907. As higher the content of fiber, higher are the longitudinal strength and stiffness of materials. In reality after about 75% of the upper limit of the concentration of fibers, the properties of composites are going down, what is related to technological defects such as voids, zones of fibers contact without a layer of the matrix between them, etc. The transversal stress concentration is also contributing to incomplete use of the maximal potential of fibrous composites. Study of this concentration is important from many aspects.

1.5 Objective

1.5.1 Motivation

Composites were the materials used mostly in the exotic applications like aircraft, sports goods, and aerospace applications. Aluminum and steel were the metals used in most of the applications, but now the total volume of composites produced is approaching the total volume of production of aluminum and steel.

The use of composites has started growing because of several stiffness and strength parameters. Composites can be significantly changed by the selection of fiber architecture, and composite materials can be designed individually for each particular application. They are composed of two or more phases and can be formulated to meet the needs of the specific applications with considerable ease.

1.5.2 Goals and Objective

The primary objective of this work is to find the stress analysis of unidirectional composites by finite element method. As compared to the other finite element method modeling of unidirectional composites, here, the load on the cell is not prescribed but is calculated by taking into account the influence of the closest neighbors of the cell. This analysis is computed in ANSYS 17.0, and the results are compared with the analytical calculations from Mathcad.

Since the transversal strength of the composites is not high, precise prediction of it is important. It can be done by the detailed study of stress distribution inside composite material.

Independently of boundary conditions at infinity (applied stresses), the distribution of stresses in the main array is double periodic one. Only in few cells near the outer boundary, it is intermediate. It is showing that the boundary conditions for the single cell never be arbitrary as it is done in the majority of FEM models of composites. This is the reason why this report is devoted not only to individual cells but also to the small array of cells. It is the intermediate case between single cell analysis and infinite array of cells analysis (Van Fo Fy).

Finite Element Modeling of a composite material is considered for single and multiple cell models of cylindrical, square, and hexagonal shapes by considering different volumes of fibers to determine the stress distributions inside the composite due to compression/tension in the transversal direction.

Chapter 2

LITERATURE REVIEW

Prediction of newly developed composite materials by using available theoretical methods was far from satisfactory in the 1960s. Growth in study of the behavior of composite material was extensive. Survey articles like (Hashin, 1964; Chamis and Sandeckyj, 1968) were obsolete after they published. The elastic behavior of composites also matured and studies were done by (Ashton et al., 1969) as some excellent intro to the subject and some further study is done by (Hashin, 1972) and (Frantsevich and Karpinos,1972 [14]).

2.1 Modeling of Composites

Considering the variety of composite materials, there is an abundance of theoretical treatments possible. But the work under this study is restricted to the linear elastic behavior of composites under static loading conditions.

(Hill, 1963-64) did study on the relationship between the effective elastic moduli of the composite and properties of its constituents. He did some fundamental study in which the effective elastic moduli of the composite and its relation to phase volume fractions and elastic moduli of the constituents. He found out results of the relationship between effective elastic moduli of composite and constituent properties by carefully averaging the stress and strain field within the phases. He also derived exact expressions for the effective elastic moduli by restricting elastic properties of the constituents. He made an assumption that shear moduli of the phases of a composite made from isotropic constituents are equal which lead to an exact expression of bulk modulus. His work was further extended by Yu and Sendeckyj, when they worked on fiber reinforced composites to get analytical solution that doesn't make the assumptions of equal phase shear.

(Whitney and Riley, 1965-66; Hermans, 1967) did the study on effective elastic moduli of fiber-reinforced composite. The model they used was concentric cylinder matrix with cylindrical fiber embedded in it, by considering 'Reuss and Voigt' type estimation for elastic moduli to find out the results. This work was followed by Hashin and Rosen. They used the model that consisted of fiber embedded in an unbounded solid possessing the effective moduli of the composite. They were able to formulate the result but they never explicitly solved the problem.

The main object of micromechanics of composites at the first stage of its development was to derive the set of engineering formulae for effective physical and mechanical properties of composites with different structural architecture. Effective characteristics were defined via properties of constituents (fiber, matrix, fillers, layers, fabrics, 3D preforms, etc.), their volumetric concentrations in composites, structural material architecture, interface conditions, etc. Unidirectional composite properties were considered as an important block in building models for other composite materials: orthogonally reinforced, angle-plies, multilayered materials, etc.

The stress distribution analysis was done as an intermediate step, and in many cases, it was distorted by the set of applied simplifying hypotheses.

Among models of unidirectional composites we can mention: coaxial cylindrical model, Hashin-Strikman model, model of diluted suspension of cylindrical inclusions, multiple models coping approaches from the theory of reinforced concrete, etc., self- consistent model, 3-phase model, etc. All these models contain significant simplifications, with distorted stress fields. The only self-non-contradictive model was developed by Van Fo Fy (Frantsevich and Karpinos, 1972) using an exact solution of the double periodic problem of the theory of elasticity for the regular array of parallel fibers included in the continuous matrix phase.

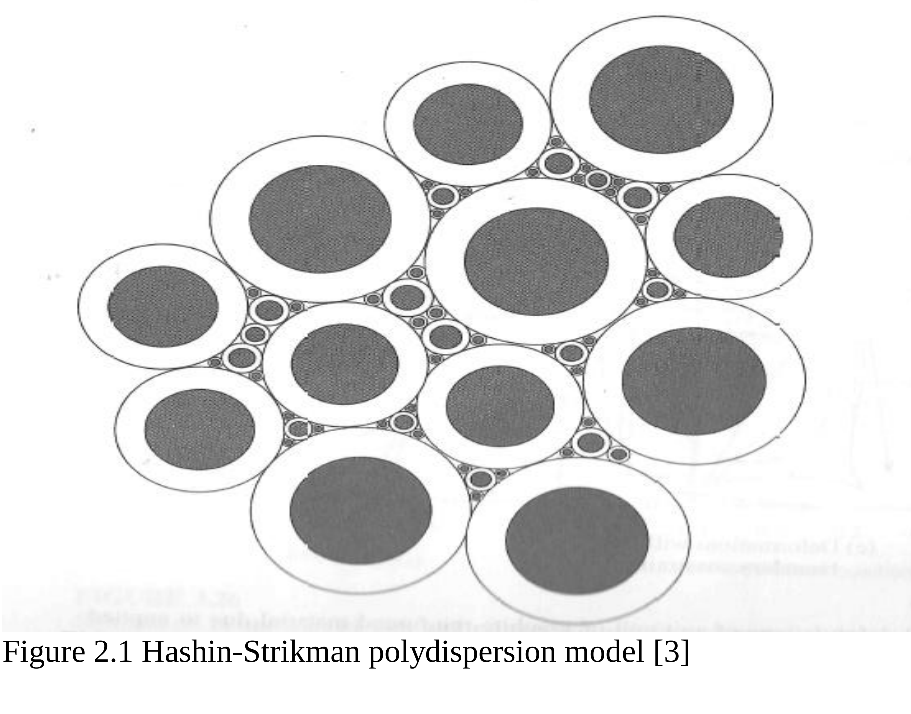


Figure 2.1 Hashin-Strikman polydispersion model [3]

According to R. M. Christensen., after succeeding in the concentric cylindrical models, a different approach was considered by taking a three-phase model [3]. Here all the composite cylinders except one were replaced by equivalent homogeneous media as shown in figure 2.2. Kerner and van der Pol first studied this model, but the results found by them were different from Christensen model, as discussed by Christensen, it was claimed that there were some errors or unjustified assumptions in Kerner and van der Pol approach.

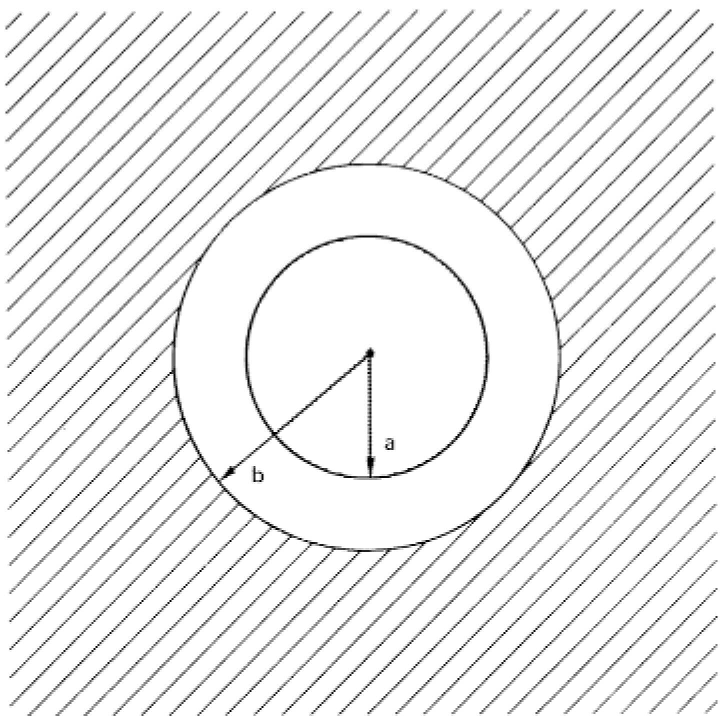


Figure 2.2 Three Phase Model [3]

There was another solution given by Hermans for the same model. However, there were some errors in that work, which was later discussed by Christensen and Lo. In Christensen work, the evaluation of the fiber results acquired was deferred until the behavior of systems involving a suspension of platelets. After this assessment, it is to be noted that composite cylindrical model and three phase model allow full range for the fiber phase. The result discussed by Christensen gives exact solutions for all five effective properties, and the expression for lower and upper bound was compared with the results derived from the spherical model and found to be exact [3]. Thus, it can be observed that these are the best limits that could be determined without indicating the geometry of the phase combinations.

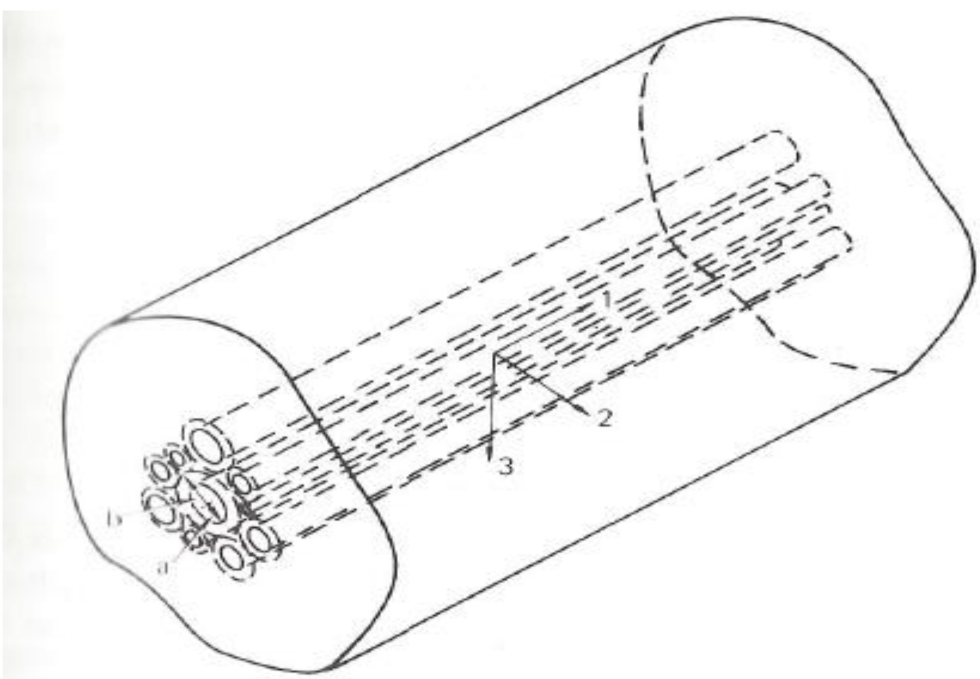


Figure 2.3 Three phase cylindrical model [3]

The most rigorous model of unidirectional fiber reinforced composite was developed by Van Fo Fy in the mid of 1960’s. It is reflected in multiple publications (many of which are translated into English), several books and chapters of books (see, Francevich and Karpinos, for example). His model was reviewed in the Sendetskij chapter in the book Brautman and Crock [13]. Van Fo Fy analyzed regular lattices of cylindrical inclusions in an infinite volume of the matrix. Solution for the fiber loaded by the most general way is known as Michell solution (see, for example, book ‘Theory of Elasticity’ by Timoshenko and Goodier [12]). A solution for the continuous medium having an infinitely huge regular array of parallel cylindrical holes was found by Natanzon in the thirties of 20th century by building biharmonic double periodic functions of complex variables. This solution was used by Fil’shtinsky for analysis of the double periodically perforated plates and shells and by Van Fo Fy to build the model of a unidirectional composite. The solution found by Van Fo Fy has the form of the series over Weierstrass’ double periodic elliptic functions of complex variables. Direct use of Van Fo Fy approach is not very convenient in practice. He derived also simplified formulae keeping only the first terms in Laurent’s series of the rigorous solutions. However, these formulae do not give many different results when compared with the formulae from 3-phase model for effective stiffness of the material. Stress distribution in a matrix can be analyzed only by using non-simplified Van Fo Fy approach.

Chapter 3

MATERIAL CHARACTERIZATION

One of the basic requirements to accurately run a model in ANSYS is the materials and its properties. It is necessary to have all the required material properties to find the stresses and deformation in a composite material. The properties required to run a model in ANSYS are:

- Young’s modulus
- Poison’s ratio
- Density
- Shear modulus
- Coefficient of thermal expansion

Since, composite is made up of two constituents, i.e., matrix and fiber, each of the constituent can be described with different materials and their respective properties.

3.1 MATRIX

A matrix in a composite is a surrounding medium in which fiber is cast or shaped. Its function is to transfer stresses between the fibers and provide a barrier against adverse environments. It also protects the surface from abrasion.

The matrix materials used in this work are:

3.1.1 Epoxy

Epoxies are polymerizable thermosetting resins and are available in a variety of viscosities from liquid to solid. Epoxies are used widely in resins for prepreg materials and structural adhesives [16]. The processing or curing of epoxies is slower than polyester resins. Processing techniques include autoclave molding, filament winding, press molding, vacuum bag molding, resin transfer molding, and pultrusion. Curing temperatures vary from room temperature to approximately 350 °F (180 °C) [16]. The most common cure temperatures range between 250° and 350°F (120–180 °C). According to L. S. Penn and T. T. Chiao., when epoxy resin reacts with a curing agent, it does not release any volatiles or water. Therefore, epoxy does not easily shrink as compared to polyesters or phenolic resins. Moreover, epoxy resins which are cured provide excellent electrical insulation and are resistant to chemicals (1982).

The advantages of epoxies are high strength and modulus, low levels of volatiles, excellent adhesion, low shrinkage, good chemical resistance, and ease of processing. Their major disadvantages are brittleness and the reduction of properties in the presence of moisture.

3.1.2 PEEK (Polyether ether ketone)

Polyether ether ketone, better known as PEEK, is a high-temperature thermoplastic. This aromatic ketone material offers outstanding thermal and combustion characteristics and resistance to a wide range of solvents and proprietary fluids. PEEK can also be reinforced with glass and carbon [16].

Material	**Density (gm/cm^3)**	**Tensile strength(MPa)**	**Young's modulus E (GPa)**	**Poisson's Ratio**
Epoxy	1.17	80	3.4	0.36
PEEk	1.32	96	3.7	0.3779

Table 3.1 Properties of Matrix

3.2 FIBER

Fibers are used to strengthen the composites; they are the principal constituents in fiber reinforced composite materials. Fibers carry majority of the load in composites. They occupy the largest volume fraction of the composite. As the amount of fiber increases the specific strength of the composite increases, since fibers have low weight density. Based upon the material characterization, various kinds of fibers are used.

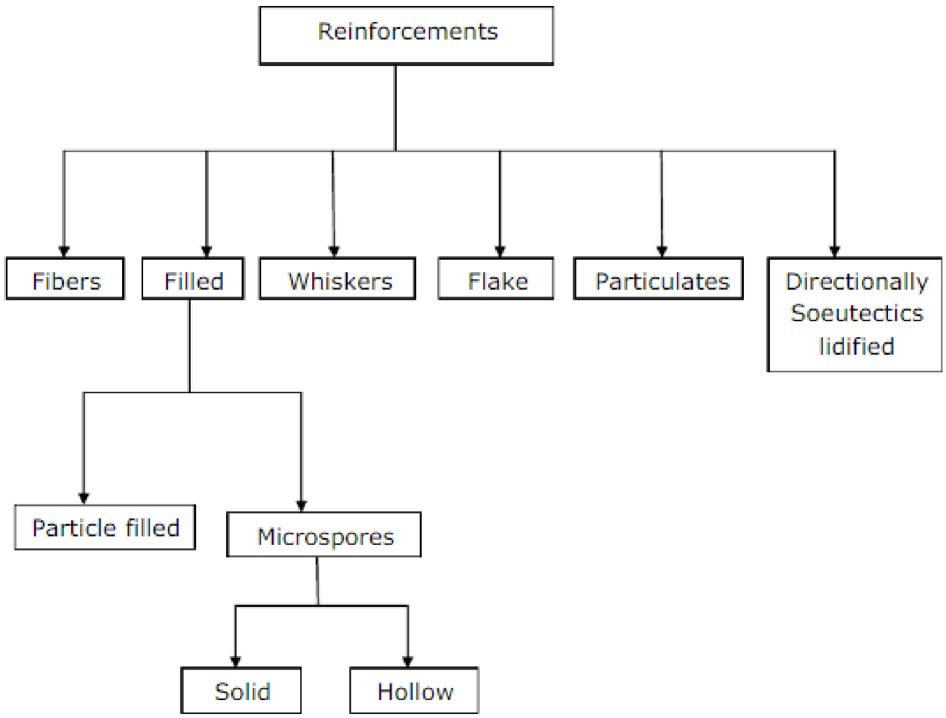


Figure 3.1 Types of fiber reinforcements

3.2.1 Fiberglass

Fiberglass has one of the major applications in the aircraft industry, such as in the manufacturing of fairings, wingtips, and rotor blades of a chopper. There are various kinds of glass fibers used in the aviation industry, such as E-glass or Electrical glass. It is known for its good electrical properties and high resistance to current flow. It is manufactured from borosilicate glass. S-glass and S2-glass, also known as structural glass, has high strength and stiffness. This glass is produced from magnesia-alumina-silicate. The manufacturing cost of this glass is 3 to 4 times than E-glass.

There are several other kinds of glass fibers available, such as C-glass, D-glass, A-glass, and R-glass. But E and S-glass are highly used in industries because of its wide applications and availability.

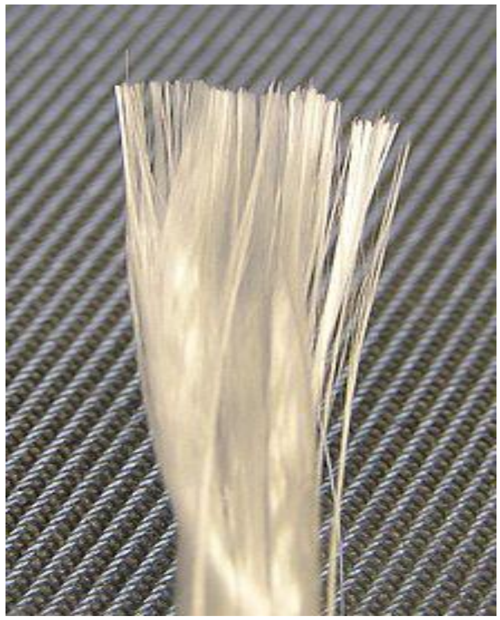


Figure 3.2 Continuous fiber glasses

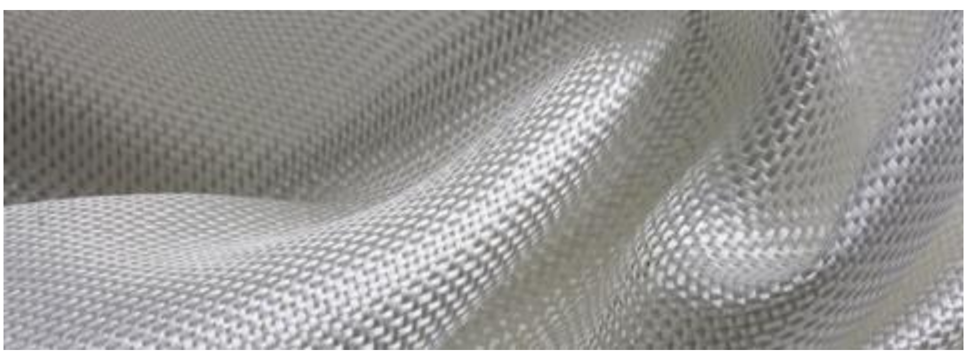


Figure 3.3 E-glass Fabric

3.2.2 Carbon

Carbon fibers are very stiff and firm, 3 to 10 times stiffer than glass fibers. Properties of carbon fibers depend upon the raw materials and the process used for its manufacturing. There are two main raw materials used, which are Poly acrylonitrile (PAN) and Pitch. Pitch fibers are less expensive and lower in strength when compared to PAN fibers.

Most of the fibers are covered with a substance called sizing. This substance acts as the lubricant and antistatic agent between the fibers and helps bundle of fibers stick together as one unit.

Carbon fiber is used for structural aircraft applications, such as floor beams, stabilizers, flight controls, and primary fuselage and wing structure. Advantages include its high strength and corrosion resistance [17].

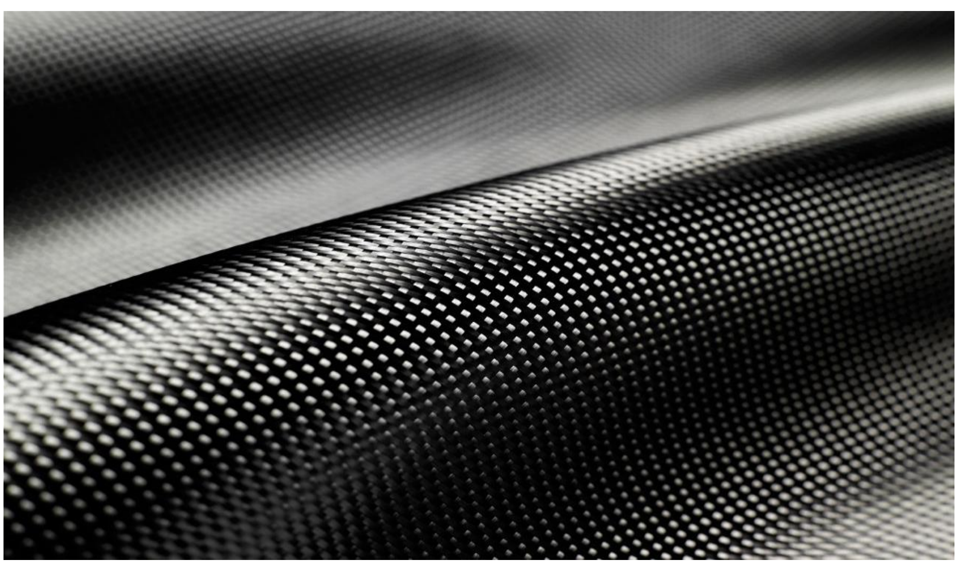


Figure 3.4 Carbon Fabric

Material	Density (g/cm^3)	Tensile strength (GPa)	E1 (GPa)	E2 (GPa)	µ12	µ23	G12 (GPa)	G23 (GPa)
E-glass	2.54	3.45	72.4	72.4	0.21	0.21	29.91	29.91
Low cost carbon	1.80	3.5	300	14	0.3	0.15	8.0	6.087
T-300	1.76	3.53	230	8.0	0.256	0.3	3.08	27.3
P-100	1.78	3.24	796.3	7.23	0.2	0.4	6.89	2.62

Table 3.2 Properties of Fibers

Chapter 4

MODELING AND SIMULATION

4.1 Modeling of unidirectional reinforced composites

This chapter deals with the FEA modeling and simulation techniques used for unidirectional reinforced composites. The procedure used for the analysis is discussed in the following steps:

1. Modeling: Create the geometric model according to the required dimensions in SOLIDWORKS 2016.
2. Preprocessing: Import the model from SolidWorks to ANSYS v16.0, define material properties.
3. Meshing: Creating a refined mesh to provide a better approximation of the solution.
4. Solution process: Applying boundary conditions such as loads and supports to the body, selecting output control, and obtaining the results.
5. Post processing: Review the obtained results and taking the values, plot the graphs.

Few assumptions were made during processing of finite element analysis.

- The fibers are cylinders with circular in cross-section.
- The displacements are continuous across the fiber and matrix interface.
- The temperature is uniform and the material properties do not vary with temperature.

- Normal to the surface and shear stresses are continuous.

4.2 Geometry

Various geometries for unidirectional reinforced composite were considered as shown in figure 4.1. Each model has varying dimensions except the length of the composite. First, a Co-axial cylindrical model was modeled in SolidWorks and simulations were carried out in ANSYS v16.0. Later, a 3-phase model was prepared, and the results of the Co-axial model were compared with this model. The computational results from ANSYS were compared with the analytical calculations to find the percentage error.

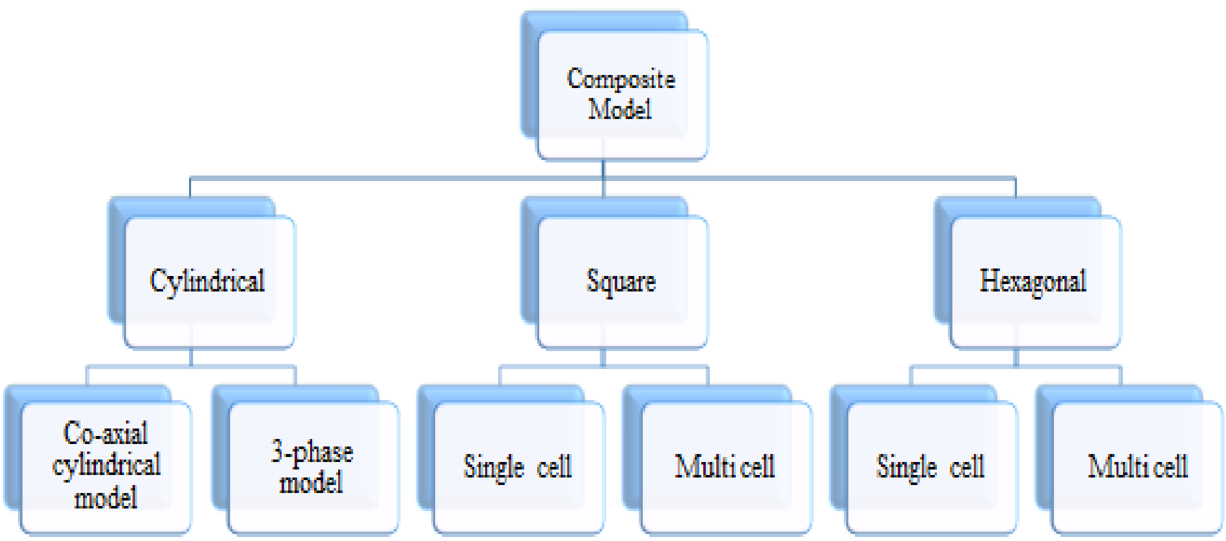


Figure 4.1 Geometry of composite models

This analysis was then carried out for square and hexagonal model for single phase and multi-phase geometries. Results of both single phase and multi-phase models for both the geometries were compared with each other, and these results were also compared with the results of the cylindrical model.

4.3 Cylindrical Model

As discussed above, here two kinds of cylindrical models were considered

1. Co-axial model
2. 3-phase model

4.3.1 Co-axial cylindrical model

In this model two cases were examined by varying the depth of the composite to find the percentage change in stress distribution at the center of the composite. Here, biaxial compression loading is applied as shown in figure 4.4. The body is applied with 6 degrees of freedom to make it rigid, and the load on the body is prescribed as 1MPa. The body was kept at a room temperature of 22°C.

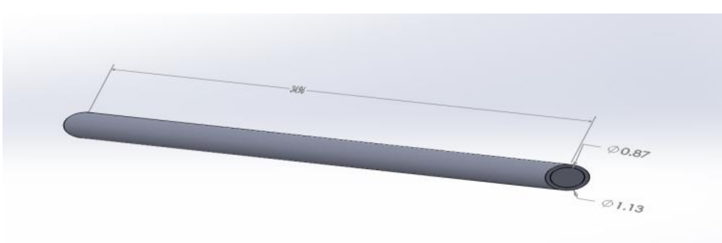


Figure 4.2 Geometry of Co-axial model

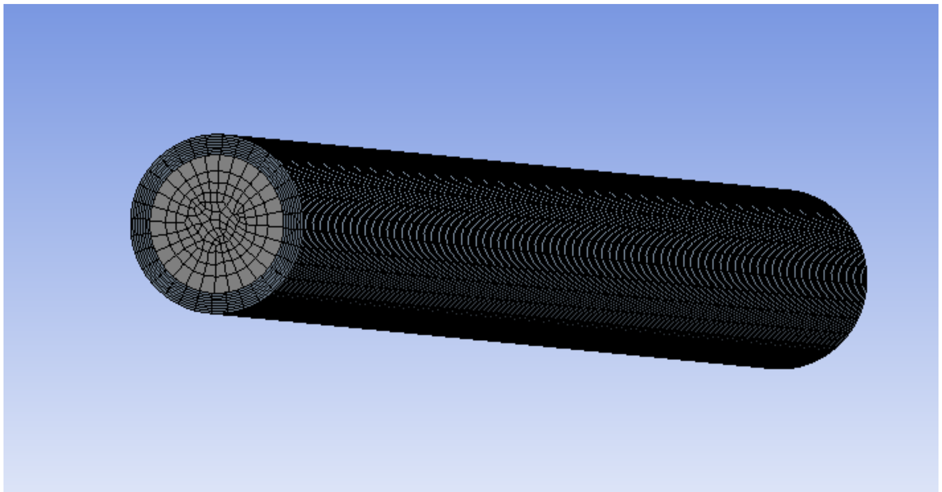


Figure 4.3 Full meshed model of a composite

The model was fully meshed in ANSYS with a body sizing of 15μm and face meshing of size 10μm was done at the interface between matrix and fiber. Since the stresses are calculated at the center of the body, the meshing has to be fine to obtain more accurate results.

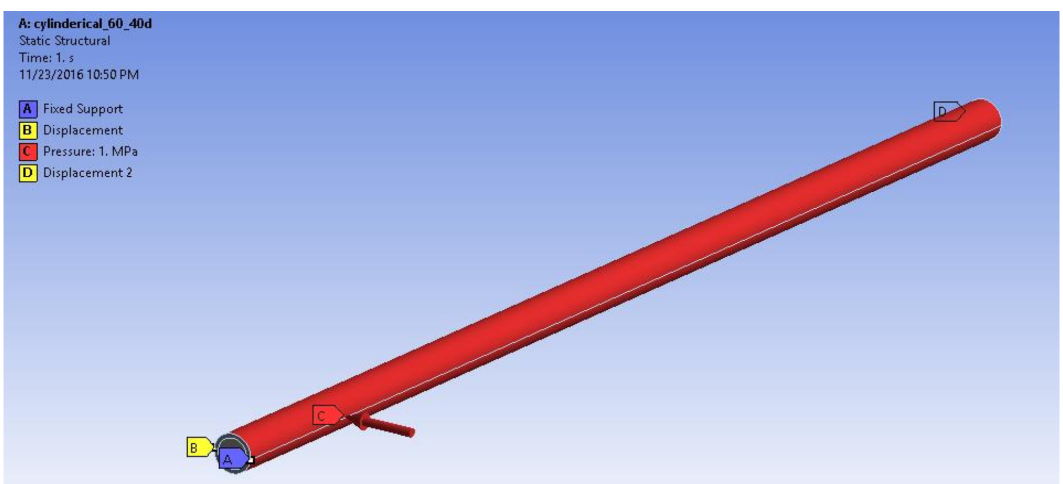


Figure 4.4 Boundary conditions for the co-axial model

Stresses were calculated at the interface of matrix and fiber by fixing nodes at equidistance and probing the values. The values calculated from ANSYS were compared with the analytical values derived from the below derivation.

- **Derivation of single cell model**

Let's we have a set of parallel cylindrical inclusions (fibers) in a binding material (matrix).

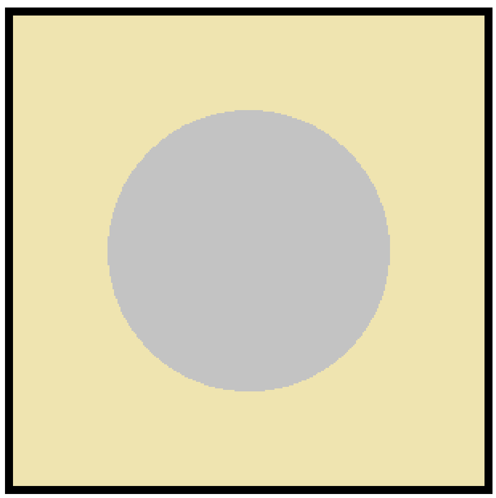


Figure 4.5 Image of real packing of fibers in unidirectional material

a) From constitutive law for fiber.

Fiber is considered as monotropic with an axis of monotropy along the axis of the cylinder.

$$\varepsilon_{rr} = \alpha_T \Delta T + \frac{1}{E_T}\sigma_{rr} - \frac{\nu_{TT}}{E_T}\sigma_{\theta\theta} - \frac{\nu_{LT}}{E_L}\sigma_{zz};$$

$$\varepsilon_{\theta\theta} = \alpha_T \Delta T - \frac{\nu_{TT}}{E_T}\sigma_{rr} + \frac{1}{E_T}\sigma_{\theta\theta} - \frac{\nu_{LT}}{E_L}\sigma_{zz};$$

$$\varepsilon_{zz} = \alpha_L \Delta T - \frac{\nu_{TL}}{E_T}\sigma_{rr} - \frac{\nu_{TL}}{E_T}\sigma_{\theta\theta} + \frac{1}{E_L}\sigma_{zz};$$

$$\varepsilon_{r\theta} = \frac{1}{G_{TT}}\sigma_{r\theta};$$

$$\varepsilon_{rz} = \frac{1}{G_{LT}}\sigma_{rz};$$

$$\varepsilon_{\theta z} = \frac{1}{G_{LT}}\sigma_{\theta z};$$

$$\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}; G_{TT} = \frac{E_T}{2\left(1+\nu_{TT}\right)}$$

Deriving the above equation

$$\varepsilon_{zz} = \alpha_L \Delta T - \frac{\nu_{TL}}{E_T}\sigma_{rr} - \frac{\nu_{TL}}{E_T}\sigma_{\theta\theta} + \frac{1}{E_L}\sigma_{zz} = \Lambda;$$

$$\sigma_{zz} = \Lambda E_L + \frac{\nu_{TL}}{E_T}E_L\left(\sigma_{rr} + \sigma_{\theta\theta}\right) - \alpha_L \Delta T E_L;$$

$$\varepsilon_{\theta\theta} = \alpha_T \Delta T - \frac{\nu_{TT}}{E_T}\sigma_{rr} + \frac{1}{E_T}\sigma_{\theta\theta} - \frac{\nu_{LT}}{E_L}\left(\Lambda E_L + \frac{\nu_{TL}}{E_T}E_L\left(\sigma_{rr} + \sigma_{\theta\theta}\right) - \alpha_L \Delta T E_L\right);$$

$$\varepsilon_{zz} = \alpha_L \Delta T - \frac{\nu_{TL}}{E_T}\sigma_{rr} - \frac{\nu_{TL}}{E_T}\sigma_{\theta\theta} + \frac{1}{E_L}\left(\Lambda E_L + \frac{\nu_{TL}}{E_T}E_L\left(\sigma_{rr} + \sigma_{\theta\theta}\right) - \alpha_L \Delta T E_L\right);$$

$$\varepsilon_{\theta\theta} = \left(\alpha_T + \nu_{LT}\alpha_L\right)\Delta T - \frac{\nu_{TT} + \nu_{TL}\nu_{TT}}{E_T}\sigma_{rr} + \frac{1-\nu_{TL}\nu_{TT}}{E_T}\sigma_{\theta\theta} - \nu_{LT}\Lambda;$$

$$\varepsilon_{zz} = \alpha_L \Delta T - \frac{\nu_{TL}}{E_T}\sigma_{rr} - \frac{\nu_{TL}}{E_T}\sigma_{\theta\theta} + \frac{1}{E_L}\left(\Lambda E_L + \frac{\nu_{TL}}{E_T}E_L\left(\sigma_{rr} + \sigma_{\theta\theta}\right) - \alpha_L \Delta T E_L\right);$$

Three versions of this law:

a1) Plane stress state $\sigma_{zz} = 0$ reflects case of monotropic washer imbedded into a plate;

a2) Plane strain state with zero axial strain $\varepsilon_z = 0$;

a3) Plane strain state with axial strain constant $\varepsilon_z \neq 0; \varepsilon_z = const \equiv \Lambda$; The constant is found from the condition of zero axial force acting on the whole system fibers + matrix.

$$\varepsilon_{rr} = \left(\alpha_T + \nu_{LT}\alpha_L\right)\Delta T + \frac{1-\nu_{LT}\nu_{TL}}{E_T}\sigma_{rr} - \frac{\nu_{TT}+\nu_{LT}\nu_{TL}}{E_T}\sigma_{\theta\theta} - \nu_{LT}\Lambda;$$

$$\varepsilon_{\theta\theta} = \left(\alpha_T + \nu_{LT}\alpha_L\right)\Delta T - \frac{\nu_{TT}+\nu_{LT}\nu_{TL}}{E_T}\sigma_{rr} + \frac{1-\nu_{LT}\nu_{TL}}{E_T}\sigma_{\theta\theta} - \nu_{LT}\Lambda;$$

$$\sigma_{zz} = \Lambda E_L - \alpha_L E_L \Delta T + \frac{\nu_{TL}E_L}{E_T}\sigma_{rr} + \frac{\nu_{TL}E_L}{E_T}\sigma_{\theta\theta}$$

$$\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}; G_{LT} = \frac{E_T}{2\left(1+\nu_{TT}\right)}; \alpha_T^{\square} = \alpha_T + \nu_{LT}\alpha_L; E_T^{\square} = \frac{E_T}{1-\nu_{LT}\nu_{TL}}; \frac{\nu_T^{\square}}{E_T^{\square}} = \frac{\nu_{TT}+\nu_{LT}\nu_{TL}}{E_T};$$

$$\nu_T^{\square} = \left(\nu_{TT}+\nu_{LT}\nu_{TL}\right)\left(1-\nu_{LT}\nu_{TL}\right);$$

b) From constitutive law for isotropic matrix

$$\varepsilon_{rr} = \alpha^{\square}\Delta T + \frac{1}{E^{\square}}\sigma_{rr} - \frac{\nu^{\square}}{E^{\square}}\sigma_{\theta\theta} - \nu\ \Lambda;$$

$$\varepsilon_{\theta\theta} = \alpha^{\square}\Delta T - \frac{\nu^{\square}}{E^{\square}}\sigma_{rr} + \frac{1}{E^{\square}}\sigma_{\theta\theta} - \nu\ \Lambda;$$

$$\sigma_{zz} = \Lambda E\ - \alpha\ E\ \Delta T + \frac{\nu\ E}{E}\sigma_{rr} + \frac{\nu\ E_L}{E}\sigma_{\theta\theta}$$

$$\varepsilon_{r\theta} = \frac{1}{G}\sigma_{r\theta};$$

$$\varepsilon_{rz} = \frac{1}{G}\sigma_{rz};$$

$$\varepsilon_{\theta z} = \frac{1}{G}\sigma_{\theta z};$$

$$G\ = \frac{E}{2\left(1+\nu\ \right)} = \frac{E^{\square}}{2\left(1+\nu^{\square}\right)}$$

c) From equations of equilibrium

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{1}{r}(\sigma_{rr}-\sigma_{\theta\theta})=0;$$
$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{2}{r}\sigma_{r\theta}=0;$$
$$\frac{\partial\sigma_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta}+\frac{1}{r}\sigma_{rz}=0;$$

d) From Cauchy relationships

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r}\,;\ \varepsilon_{\theta\theta}=\frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta}+u_r\right);\ \varepsilon_{zz}=\frac{\partial u_z}{\partial z};$$
$$\varepsilon_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right);\ \varepsilon_{\theta z}=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\theta}\right);\ \varepsilon_{zr}=\frac{1}{2}\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right);$$

This is a problem of symmetric biaxial transversal compression, and the problem is axially symmetrical.

Practically it is special case of Lamé problem. The equation of equilibrium is:

$$\sigma_\theta=\sigma_r+r\frac{d\sigma_r}{dr}$$

Axial stress is not uniform but the averaged stress depends on the selected version of the plane strain state problem; in all cases:

$$\langle\sigma_z\rangle=\frac{2\pi\int_{r_f}^{r_c}\sigma_z^M r\,dr+2\pi\int_0^{r_f}\sigma_z^I r\,dr}{\pi r_c^2}$$

Radial and circumferential strains are connected by equation of strain compatibility

$$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr}$$

From the Cauchy equations, connecting strains and displacements in axi-symmetrical problem, and Substituting constitutive law into equation of strain compatibility gives

$$\alpha_T^{\square} E_T^{\square} r\frac{d\Delta T}{dr} + 3r\frac{d\sigma_{rr}}{dr} + r^2\frac{d^2\sigma_{rr}}{dr^2} = 0;$$

For uniform temperature field

$$3r\frac{d\sigma_{rr}}{dr} + r^2\frac{d^2\sigma_{rr}}{dr^2} = 0;$$
$$\sigma_{rr} = C_1 + C_2 r^{-2};$$

Circumferential stresses are

$$\sigma_{\theta\theta} = \sigma_{rr} + r\frac{d\sigma_{rr}}{dr} = \frac{d}{dr}\left(r\sigma_{rr}\right) = \frac{d}{dr}\left(C_1 r + \frac{C_2}{r}\right) = C_1 - C_2 r^{-2};$$

Boundary conditions in fiber:

$C_2^I = 0;$ due to finite stress at $r = 0;$

Boundary condition in matrix is

$$\sigma_{rr}\Big|_{r=r_{cell}} = -p;$$
$$C_1^M + C_2^M r_{cell}^{-2} = -p;$$
$$C_1^M = -p - C_2^M r_{cell}^{-2}$$

Two boundary conditions between fiber and matrix are:

$$\sigma_{rr}^{I}\Big|_{r=r_I} = \sigma_{rr}^{M}\Big|_{r=r_I};$$

$$u_r^{I}\Big|_{r=r_I} = u_r^{M}\Big|_{r=r_I} \Leftrightarrow \varepsilon_{\theta\theta}^{I}\Big|_{r=r_I} = \varepsilon_{\theta\theta}^{M}\Big|_{r=r_I}$$

or

$$C_1^I = C_1^M + C_2^M r_I^{-2} = -p - C_2^M r_{cell}^{-2} + C_2^M r_I^{-2};$$

$$u_r^{I}\Big|_{r=r_I} = u_r^{M}\Big|_{r=r_I} \Leftrightarrow \varepsilon_{\theta\theta}^{I}\Big|_{r=r_I} = \varepsilon_{\theta\theta}^{M}\Big|_{r=r_I};$$

$$\alpha^{M\square}\Delta T - \frac{\nu^{M\square}}{E^{M\square}}\left(-p - C_2^M r_{cell}^{-2} + C_2^M r_I^{-2}\right) + \frac{1}{E^{M\square}}\left(-p - C_2^M r_{cell}^{-2} - C_2^M r_I^{-2}\right) - \nu^M \Lambda =$$

$$\alpha_T^{\square}\Delta T - \frac{\nu_{TT}^{\square}}{E_T^{\square}}\left(-p - C_2^M r_{cell}^{-2} + C_2^M r_I^{-2}\right) + \frac{1}{E_T^{\square}}\left(-p - C_2^M r_{cell}^{-2} + C_2^M r_I^{-2}\right) - \nu_{LT}\Lambda;$$

$$C_2^M\left[\frac{\nu^{M\square}}{E^{M\square}}(c-1) - \frac{1}{E^{M\square}}(1+c) - \frac{\nu_{TT}^{\square}}{E_T^{\square}}(c-1) + \frac{1}{E_T^{\square}}(c-1)\right] =$$

$$\left[\left(\alpha_T^{\square} - \alpha^{M\square}\right)\Delta T - \left(\nu_{LT} - \nu^M\right)\Lambda - p\left(\frac{1}{E_T^{\square}} - \frac{1}{E^{M\square}} - \frac{\nu_{TT}^{\square}}{E_T^{\square}} + \frac{\nu^{M\square}}{E^{M\square}}\right)\right] r_I^2$$

Now, finding C_1^M ;

$$C_1^M = -p - C_2^M r_{cell}^{-2}$$

And

$$\sigma_{rr}^{M}\Big|_{r=r_I} = C_1^M + C_2^M r_I^{-2} = -p - C_2^M r_{cell}^{-2} + C_2^M r_I^{-2} =$$

$$-p + C_2^M\left(1-c\right) r_I^{-2} =$$

$$-p + \left(1-c\right) p \frac{\left[\frac{\left(\alpha_T^{\square} - \alpha^{M\square}\right)\Delta T - \left(\nu_{LT} - \nu^M\right)\Lambda}{p} - \left(\frac{1}{E_T^{\square}} - \frac{1}{E^{M\square}} - \frac{\nu_{TT}^{\square}}{E_T^{\square}} + \frac{\nu^{M\square}}{E^{M\square}}\right)\right]}{\left[\frac{\nu^{M\square}}{E^{M\square}}(c-1) - \frac{1}{E^{M\square}}(1+c) - \frac{\nu_{TT}^{\square}}{E_T^{\square}}(c-1) + \frac{1}{E_T^{\square}}(c-1)\right]} =$$

$$-p\left\{1 - \left(1-c\right)\frac{\left[\frac{\left(\alpha_T^{\square} - \alpha^{M\square}\right)\Delta T - \left(\nu_{LT} - \nu^M\right)\Lambda}{p} - \left(\frac{1}{E_T^{\square}} - \frac{1}{E^{M\square}} - \frac{\nu_{TT}^{\square}}{E_T^{\square}} + \frac{\nu^{M\square}}{E^{M\square}}\right)\right]}{\left[\frac{\nu^{M\square}}{E^{M\square}}(c-1) - \frac{1}{E^{M\square}}(1+c) - \frac{\nu_{TT}^{\square}}{E_T^{\square}}(c-1) + \frac{1}{E_T^{\square}}(c-1)\right]}\right\}$$

4.3.2 3-Phase Model

Coaxial cylindrical model is not taking effect of surrounding cells at all. The most primitive way to take this effect into account is realized in 3-phase model, in which the elementary cell (cylindrical, which is contradictive itself, because continuum medium never be built from cylinders) is put into the composite medium with “spread characteristics”. It is not taken into account the effect of neighbor fibers but it is better than the absence of surrounding medium at all.

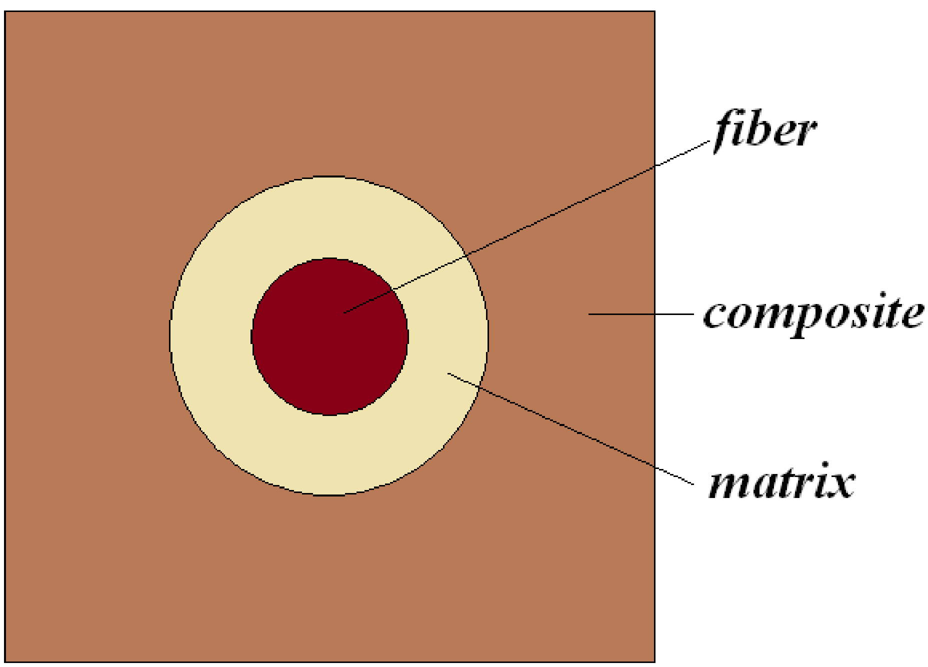


Figure 4.6 3-phase model with cylindrical cell

In this model unidirectional glass fiber has been used as reinforcing material, epoxy resin has been used as matrix material, while a composite of glass and epoxy is used as the outer body. A uniaxial transverse load of 1 MPa is applied to the two parallel surfaces of the composite at room temperature. A mesh is generated, which is not too fine but acceptable. Same boundary conditions of co-axial cylindrical cell model are applied in this case and analysis is carried out.

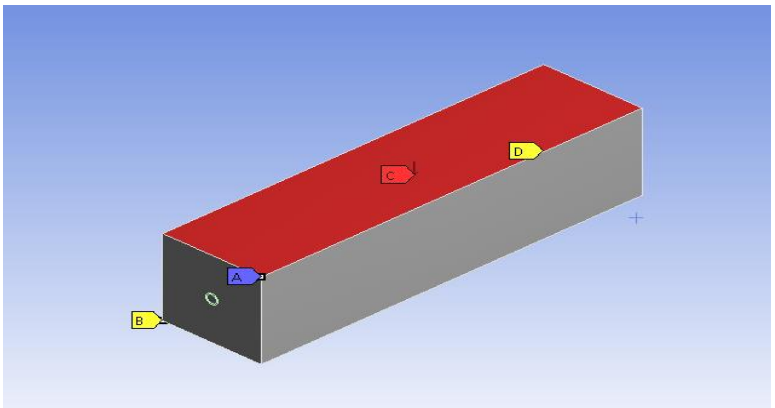


Figure 4.7 Boundary Conditions

4.4 Square and Hexagonal Model

This work is carried out on various symmetries of composites. Single and multi-cell models of square and hexagonal geometries of the matrix were considered for analysis. A numerical investigation of the composite material utilized as a part of the analysis was performed in ANSYS v16.0. Modeling a composite material using homogenization is common, where fibers introduce anisotropy. In this work, material properties depend upon the non-linear behavior of fibers. As a result, it is necessary to study micromechanics of materials.

Moreover, it is necessary to study the effects of radial and hoop stresses, and also the effects of buckling at the interface of matrix and fiber.

This section explains the finite element modeling of square and hexagonal composites for both single and multi-cell geometries. A step by step procedure is followed from modeling geometry and then to applying boundary conditions. Various geometries used in this work are shown in the figures below.

Later, in this section, the impact of the parameters used to develop the models is discussed, for example, the meshing size of the composites, and the materials used for matrix and fibers are detailed. Boundary conditions are applied later, and for each model the boundary conditions are kept same.

First, consider the square block with 1*1 mm2 in an area of a matrix and inserting a fiber as cylindrical rods with total volume of 60%. Fixing the composite such that it has 6 degrees of freedom, it is made to be rigid such that it does not get displaced on the application of load. The body is compressed with a pressure of 1MPa from all the four sides. The results were obtained on a standard mesh, later a more uniform mesh is generated throughout the matrix and fiber, and a finer mesh is created at the interface between fiber and matrix, displaying very good agreement, both in macroscopic and microscopic features.

To study the mesh convergence it is important to analyze the growth of stresses at a given point with respect to the size of the mesh. In this work, the results obtained use meshes in the range of 60,000 to 90,000 nodes, based on single cell model and multi-cell model.

In a single cell, a unidirectional fiber is merged with the matrix and biaxial loading is applied. The body is fixed with 6 DOF such that, it is rigid and does not move on the application of loads.

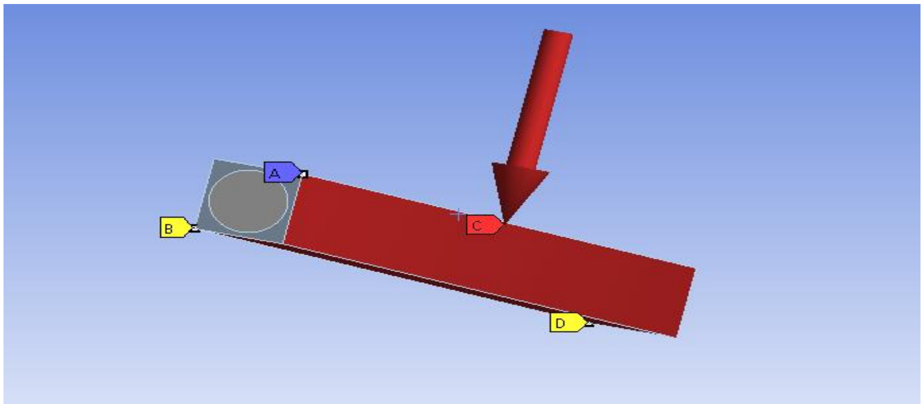


Figure 4.8 Single cell square model

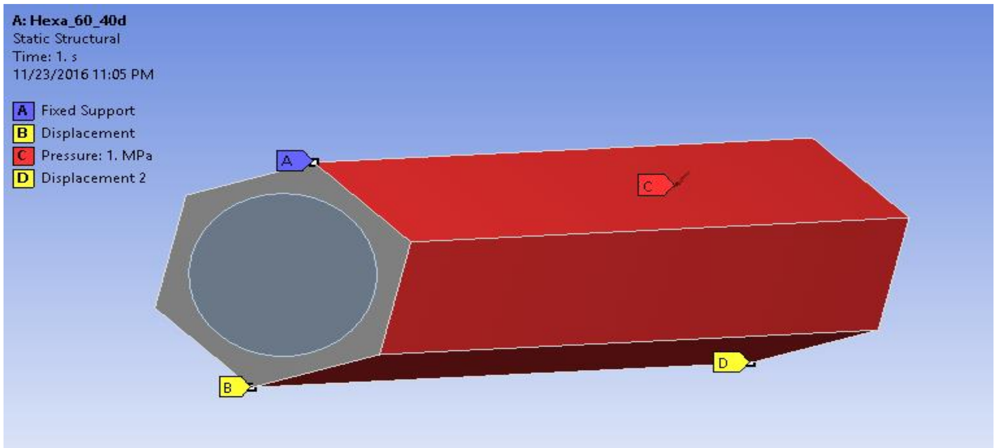


Figure 4.9 Single cell hexagon model

4.4.1 Multi-cell models

The most rigorous model of the regular lattice of fibers in matrix never be modeled (numerically verified) by FEM because of infinite numbers of fibers. Taking into account the nearest neighbor fibers only has to show what is lost in 3-phase model, where the heterogeneous surrounding structure of the material was replaced by a quasi-homogeneous composite medium with spread effective characteristics.

In this square model, nine homogenous fibers are used as reinforcement and the middle fiber is surrounded by all other eight fibers as shown in the figure 4.10. The stresses on the middle cell are calculated by taking into account the effect of closest neighboring fibers. The same methodology is used for a hexagon model, but here the arrangement of fibers is in hexagonal lattice as shown in figure 4.11. The results obtained were compared with the results of single cell model and are discussed in next section.

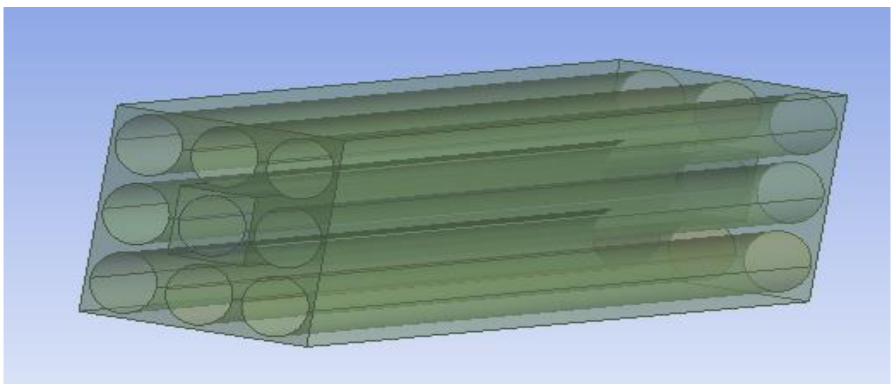


Figure 4.10 Multi-cell square model

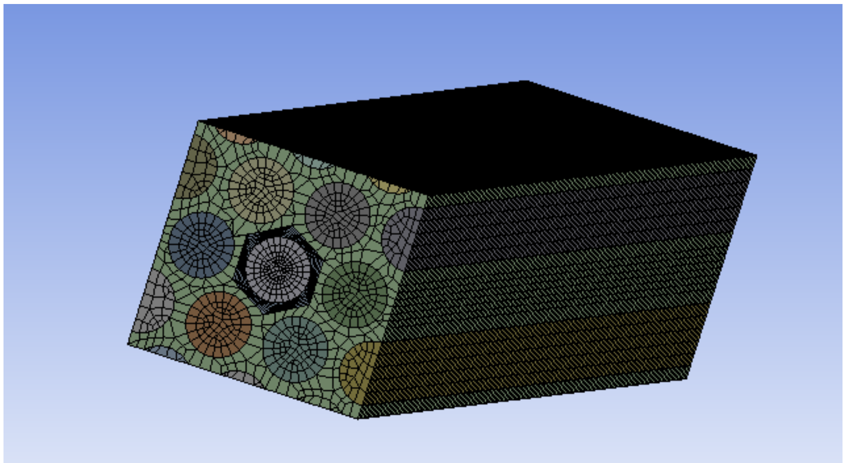


Figure 4.11 Meshed multi-cell hexagon model

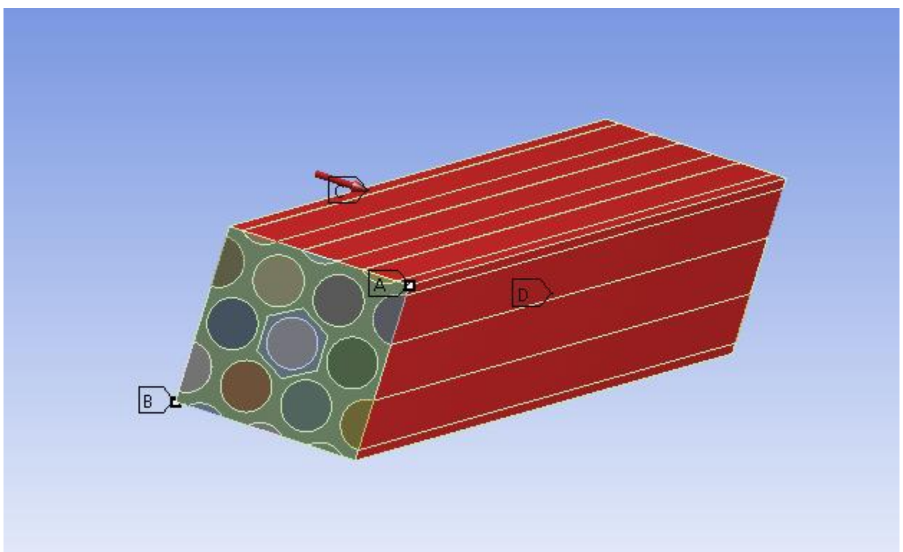


Figure 4.12 Applied boundary conditions

Chapter 5

RESULTS AND DISCUSSION

5.1 Co-axial cylindrical model

Simulations from ANSYS

Distribution of all stresses is close to all theoretical ones, and small deviations are seen due to insufficiently fine cells. These results show validity for all other models. Maximum deviation was found to be less than 4%.

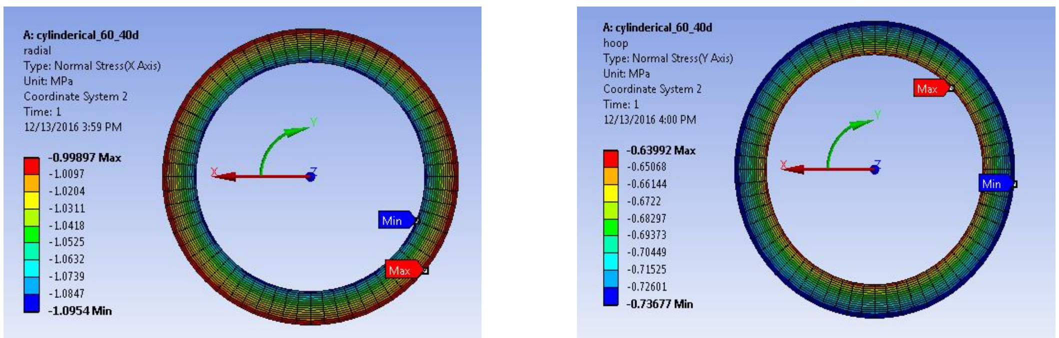


Figure 5.1 Radial and Circumferential Stresses for cylindrical model

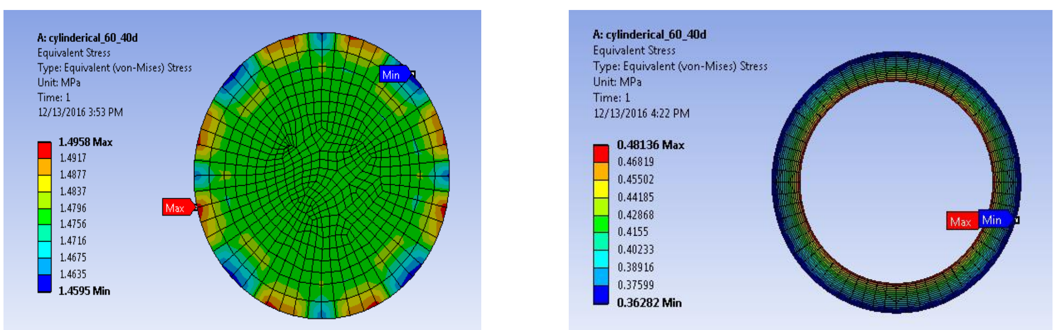
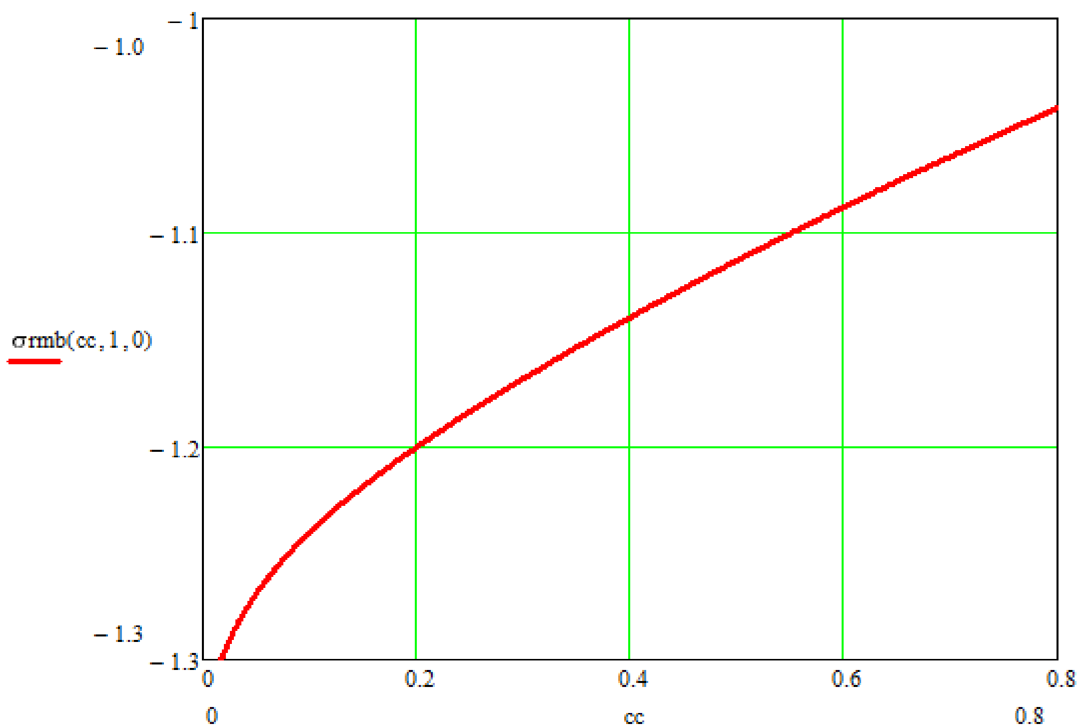


Figure 5.2 Von-misses for cylindrical model

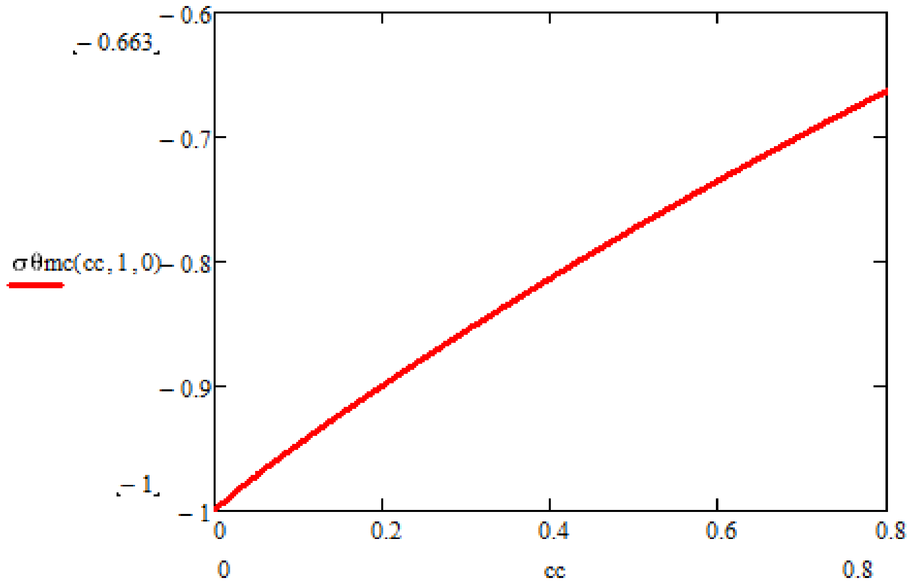
5.1.1 Analytical Results:

With the increase in the concentration of fiber, the stress concentration factor is decreasing for both radial stress and circumferential stresses. Stresses are represented in parts of applied outer pressure.

The graph below shows the dependence of radial stress in the matrix on the interface matrix-fiber on the volumetric concentration of fiber.



This graph below shows the dependence of hoop stress in a matrix on the outer surface of the cell on the volumetric concentration of fibers.



Similarly, the graph for dependence of hoop stress in matrix on the interface matrix-fiber on volumetric concentration of fibers is shown below.

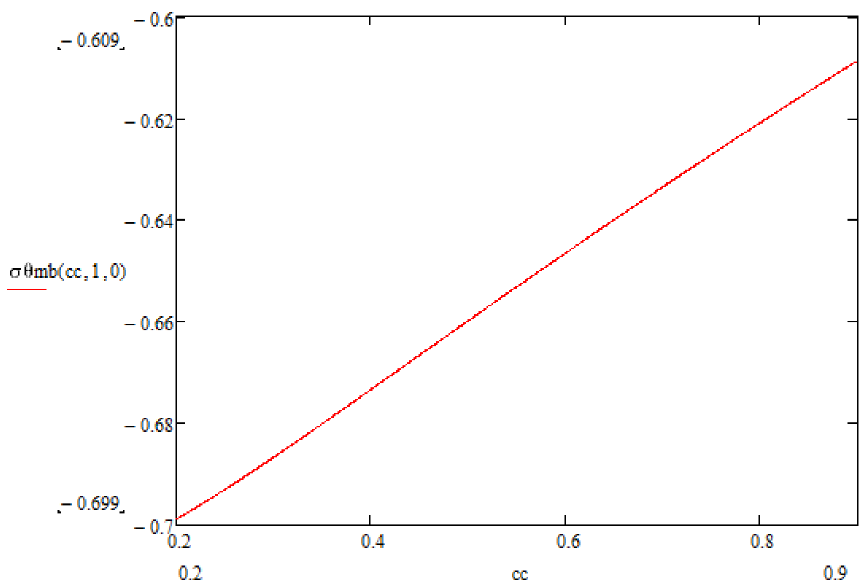


Table 5.1 Cylindrical single cell model for Epoxy and E-glass

Epoxy & E-glass	Fiber Vol.	Radial Stress (B)		Hoop Stress (B)		Hoop Stress (C)	
Analytical Plain Strain	50	-1.113		-0.66		-0.773	
	60	-1.088		-0.647		-0.735	
	70	-1.065		-0.634		-0.698	
Analytical Plain Stress	50	-1.177		-0.468		-0.536	
	60	-1.137		-0.452		-0.589	
	70	-1.099		-0.437		-0.645	
			% Error		% Error		% Error
Cylinder 20d	50	-1.116	2.38%	-0.647	2.0%	-0.785	1.52%
	60	-1.089	1.68%	-0.633	1.4%	-0.744	1.2%
	70	-1.086	3.23%	-0.626	1.27%	-0.713	2.1%
Cylinder 40d (Computational)	50	-1.115	2.29%	-0.651	1.38%	-0.785	1.52%
	60	-1.099	2.61%	-0.637	1.56%	-0.745	1.34%
	70	-1.09	3.61%	-0.629	0.79%	-0.714	2.24%

* All values are in MPa

Table 5.2 Cylindrical single cell model for Epoxy and carbon

Epoxy & Carbon	fiber Vol.	Radial Stress (B)		Hoop Stress (B)		Hoop Stress (C)	
Analytical Plain Strain	50	-1.054		-0.839		-0.893	
	60	-1.042		-0.830		-0.873	
	70	-1.031		-0.822		-0.853	
Analytical Plain Stress	50	1.122		-0.635		-0.757	
	60	1.095		-0.620		-0.715	
	70	1.07		-0.606		-0.675	
Cylinder 40d (Computational)			% Error		% Error		% Error
	50	-1.0846	2.9%	-0.822	2%	-0.892	0.1%
	60	-1.0628	2%	-0.811	2.3%	-0.874	0.1%
	70	-1.0704	3.8%	-0.803	2.3	-0.858	0.6%

*All values are in MPa

• The analytical and computational results from above table gives a maximum deviation in stresses below 4%, for 70% concentration of fibers.

• Below 3%, for 60% concentration of fibers.

• Below 2.5%, for 50% concentration of fibers.

• Since carbon has low stiffness in nature, it has low radial stresses when compared to E-glass. In the axial direction, carbon has much higher young's modus than E-glass, but it has lower young's modulus in the transversal direction.

• These deviations from exact solution are the results of insufficiently fine mesh. Still, these deviations are acceptable. Therefore, this density of mesh can be used for other models.

5.2 Square and Hexagonal Model with Single Cell

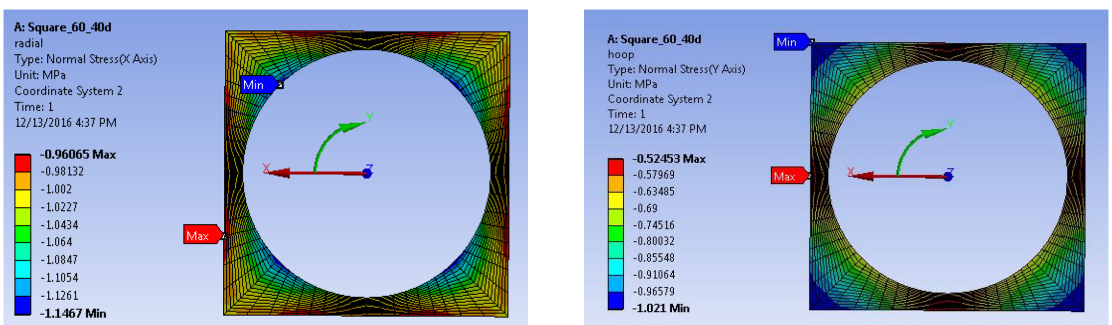


Figure 5.3 Radial and Hoop stresses for square model

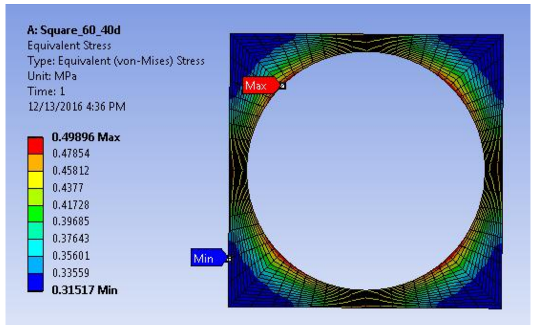


Figure 5.4 Von-misses for square model

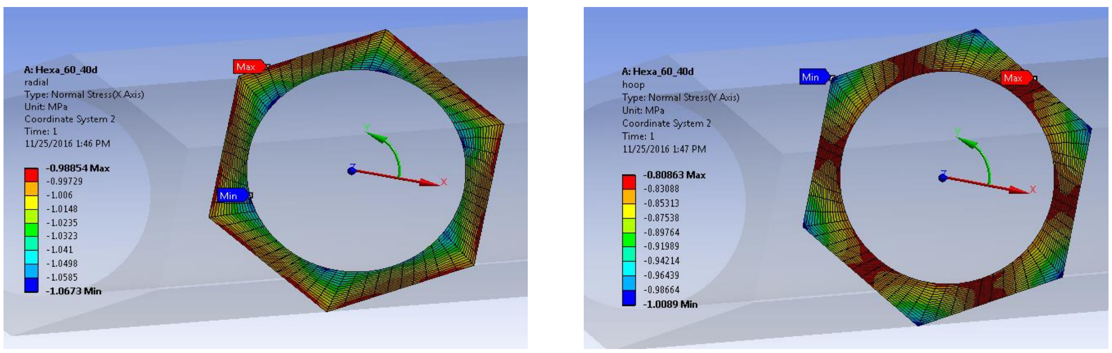


Figure 5.5 Radial and Hoop stresses for Hexagon model

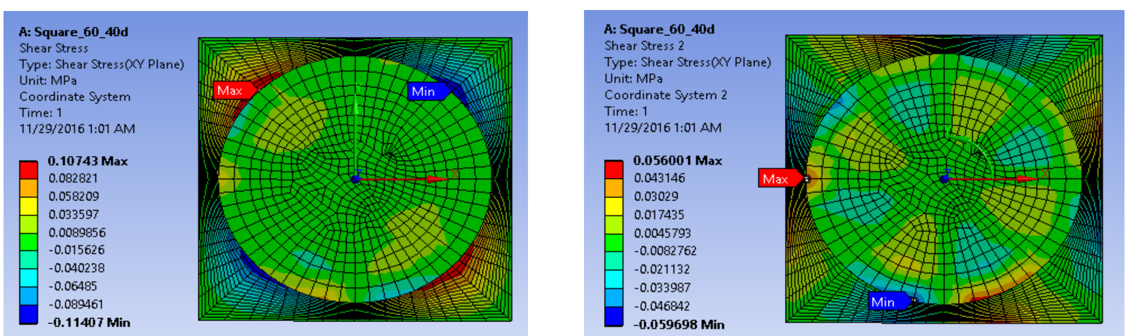


Figure 5.6 Shear stresses for Square model

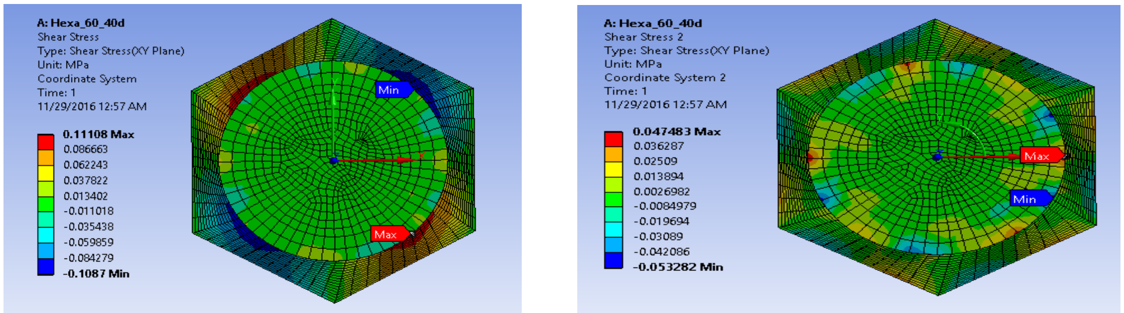


Figure 5.7 Shear stresses for Hexagon model

5.3 Square and Hexagon model with multi-cell

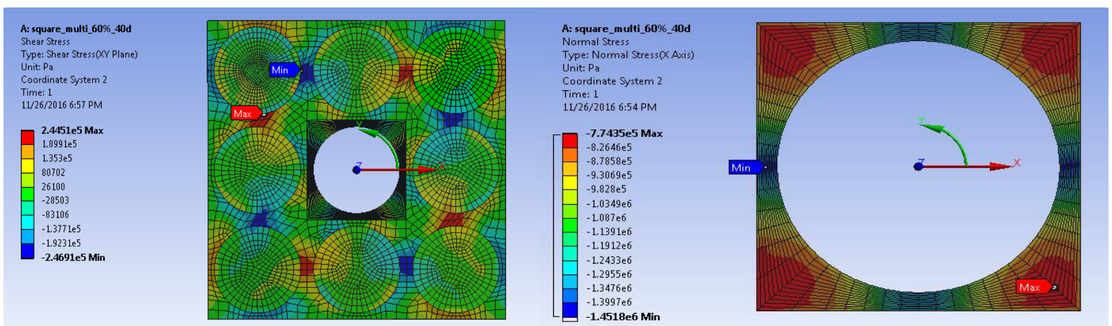


Figure 5.8 Radial Stress of square model with 60% volume of fiber

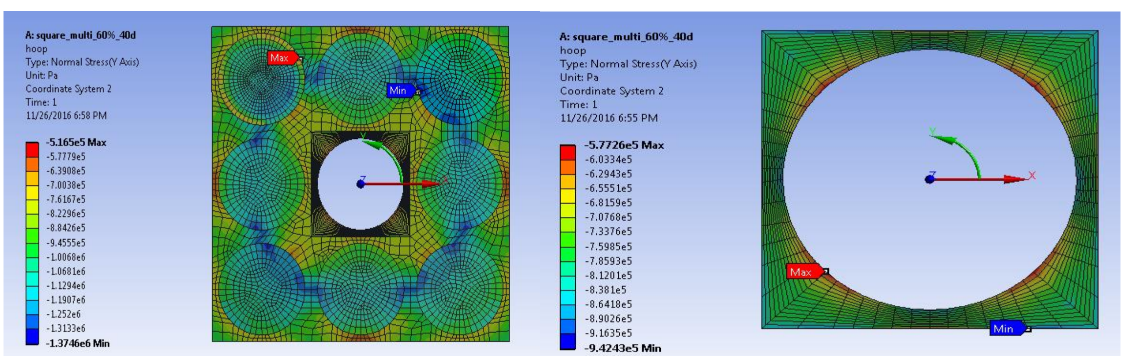


Figure 5.9 Circumferential Stress of square model with 60% volume of fiber

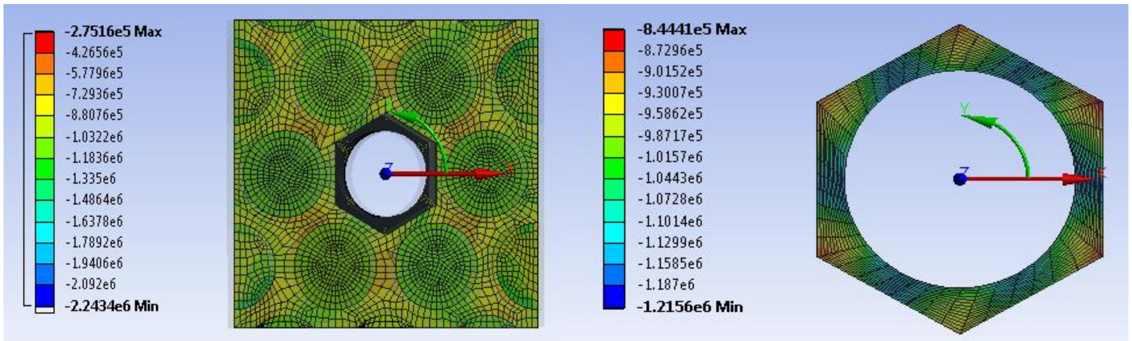


Figure 5.10 Radial Stress for hexagon model with 60% volume of fiber

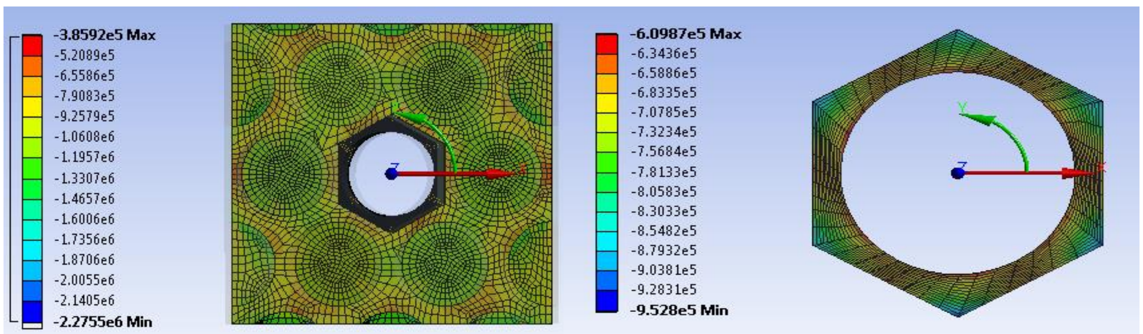


Figure 5.11 Circumferential Stress for hexagon model with 60% vol. of fiber.

Table 5.3 Square and Hexagon model for Epoxy and E-glass

Epoxy & E-glass		Radial Stress (B)		Hoop Stress (B)		Hoop Stress (C)		Von-misses	
Single Cell		Min	Max	Min	Max	Min	Max	Min	max
Square 40d	50	-1.1498	-1.0428	-0.68172	-0.62439	-1.0465	-0.6076	0.461	0.505
	60	-1.2086	-1.0207	-0.71147	-0.59791	-1.0091	-0.53961	0.452	0.495
	70	-1.1433	-0.98873	-0.6526	-0.2345	-1.0172	-0.5375	0.445	0.476
Hexagon 40d	50	-1.1404	-1.0699	-0.6856	-0.6514	-0.99616	-0.67406	0.470	0.507
	60	-1.1483	-1.304	-0.67228	-0.62563	-0.97421	-0.61346	0.455	0.493
	70	-1.1165	-1.0172	-0.6643	-0.6092	-0.9717	-0.5695	0.443	0.484
multi-cell									
Square 40d	50	-1.2848	0.9455	-0.7190	-0.6098	-0.8726	-0.6555	0.315	0.676
	60	-1.4641	-0.8422	-0.7870	-0.5570	-0.8043	-0.5416	0.3063	0.6482
	70	-1.125	-0.3525	-0.5936	-0.2712	-0.6815	-0.1184	0.1044	0.5390
Hexagon 40d	50	-1.172	-1.062	-0.6848	-0.6280	-0.9347	-0.6923	0.4654	0.5244
	60	-1.167	-1.038	-0.6884	-0.6098	-0.9244	-0.6491	0.4277	0.5297
	70	-1.2144	-0.9504	-0.5799	-0.7074	-0.9131	-0.6097	0.4088	0.5376

*All values are in MPa

Table 5.4 Square and Hexagon model for Epoxy and Carbon

Epoxy & Carbon		Radial Stress (B)		Hoop Stress (B)		Hoop Stress (C)		Von-misses	
Single Cell		Min	Max	Min	Max	Min	Max	Min	max
Square 40d	50	-1.088	-1.010	-0.8543	-0.8097	-1.0094	-0.7956	0.322	0.351
	60	-1.1	-0.9946	-0.8712	-0.7768	-1.0039	-0.7619	0.320	0.348
	70	-1.057	-1.000	-0.8528	-0.7737	-1.0049	-0.7402	0.319	0.327
Hexagon 40d	50	-1.1	-1.029	-0.8615	-0.8254	-1.008	-0.8387	0.326	0.353
	60	-1.067	-1.0215	-0.8449	-0.8192	-1.009	-0.8086	0.321	0.345
	70	-1.055	-1.010	-0.8383	-0.7984	-1.009	-0.7747	0.317	0.33
multi-cell									
Square 40d	50	-1.1064	-0.9825	-0.8521	-0.8156	-1.1404	-0.7491	0.301	0.345
	60	-1.1704	-0.9720	-0.8557	-0.8054	-1.0159	-0.7402	0.298	0.348
Hexagon 40d	50	-1.1025	-0.9546	-0.8703	-0.8145	-1.0208	-0.8414	0.3013	0.3542
	60	-1.0865	-0.9938	-0.8581	-0.8166	-1.0215	-0.8154	0.2966	0.3614

*All values are in MPa

Collective results for all models are shown in the table below.

Table 5.5 Comparison of values for all the models

Epoxy & E-glass		Radial Stress (B)		Hoop Stress (B)		Hoop Stress (C)	
Cylinder 40d	50	-1.115		-0.658		-0.785	
	60	-1.099		-0.637		-0.745	
	70	-1.09		-0.635		-0.703	
Single Cell		Min	Max	Min	Max	Min	Max
Square 40d	50	-1.1498	-1.0428	-0.68172	-0.62439	-1.0465	-0.6076
	60	-1.2086	-1.0207	-0.71147	-0.59791	-1.0091	-0.53961
	70	-1.1433	-0.98873	-0.6526	-0.2345	-1.0172	-0.5375
Hexagon 40d	50	-1.1404	-1.0699	-0.6856	-0.6514	-0.99616	-0.67406
	60	-1.1483	-1.304	-0.67228	-0.62563	-0.97421	-0.61346
	70	-1.1165	-1.0172	-0.6643	-0.6092	-0.9717	-0.5695
multi-cell							
Square 40d	50	-1.2848	0.9455	-0.7190	-0.6098	-0.8726	-0.6555
	60	-1.4641	-0.8422	-0.7870	-0.5570	-0.8043	-0.5416
	70	-1.125	-0.3525	-0.5936	-0.2712	-0.6815	-0.1184
Hexagon 40d	50	-1.172	-1.062	-0.6848	-0.6280	-0.9347	-0.6923
	60	-1.167	-1.038	-0.6884	-0.6098	-0.9244	-0.6491
	70	-1.2144	-0.9504	-0.5799	-0.7074	-0.9131	-0.6097

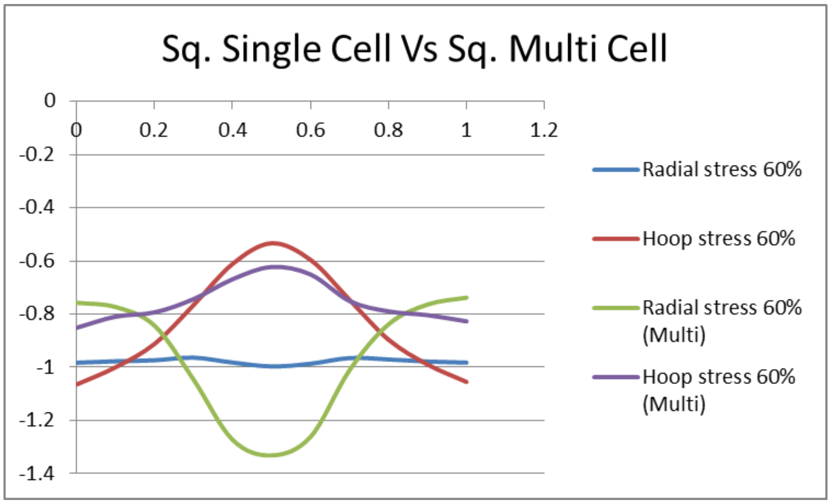
*All values are in MPa

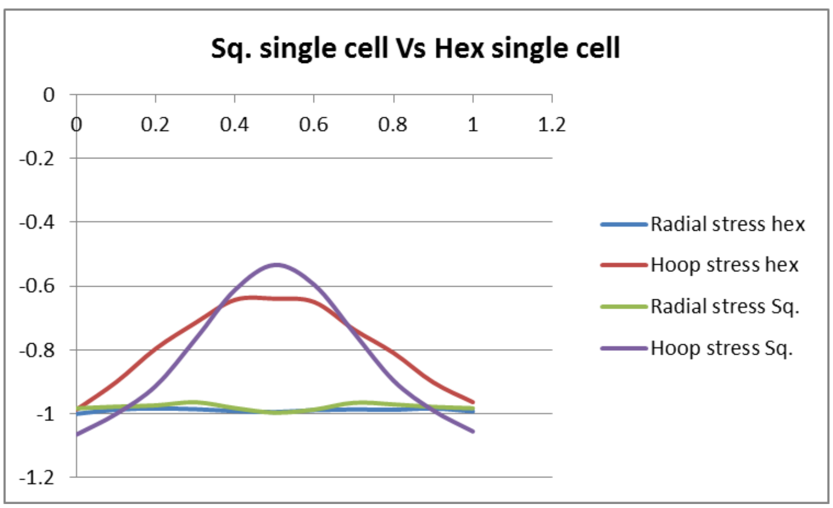
Stress concentration factor significantly depends on the model used for its calculation. For example, for 60% of fibers per volume, the cylindrical model gives absolute value of radial stress on the boundary of fiber-matrix as 1.1*p* (where *p* is applied stress on outer surface of the cell) but in framework of multi-cell model for the square packing of the fibers the stress concentration factor is 1.46p, not 1.1p. This difference is appeared due to stress interaction of neighbor fibers taking matrix into account instead of spreading such interaction along the circumference. Cylindrical models are used for prediction of effective elastic properties of unidirectional fiber reinforced materials. However, as it is following from the obtained results, the use of cylindrical models for predicting of failure and effective strength of composites will produce wrong results. Taking only geometry of the cell into account (stress concentration factor for the discussed case is 1.21p) as it is done for the square cell model giving better results than the cylindrical model, but it is still far from the results of the multi-cell model. Proposed multi-cell FEM model can be used instead of a periodical model (Van Fo Fy), which requires the application of complicated mathematical technique.

From the results, we can observe that hexagonal cell is showing behavior between cylindrical and square cells. Hexagonal is much better than other type of packing, since it has more concentration of fiber and it shows less stress concentration than square cell.

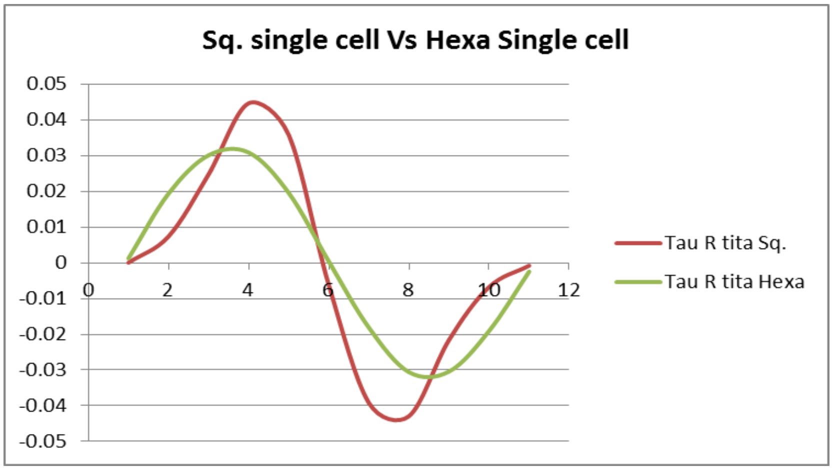
5.4 Graphs

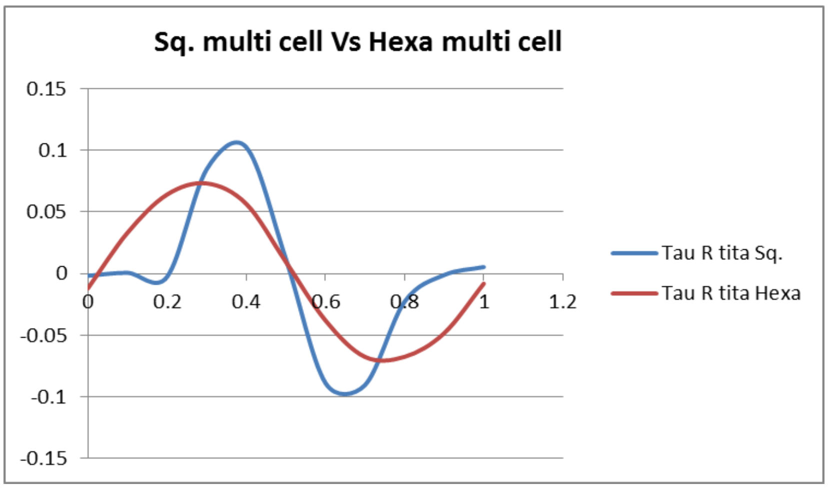
5.4.1 Comparison of stress based on various geometry's for E-Glass and Epoxy



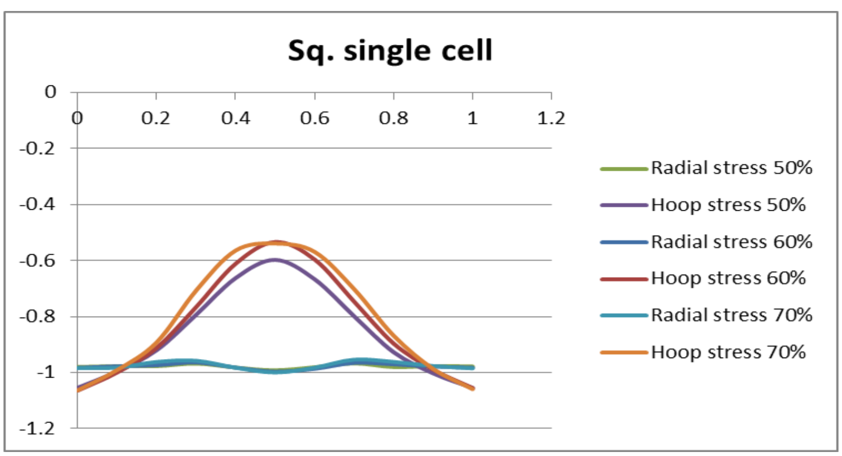


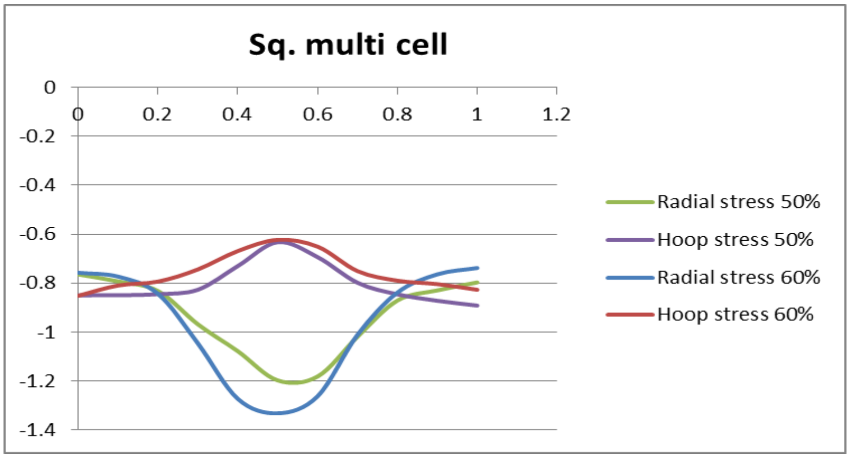
Here stress distribution is more uniform in simple single cell model when compared to complex geometries of multi-cell models. Since non uniform stresses are acting due to the effect of neighboring fibers in multi-cell models.

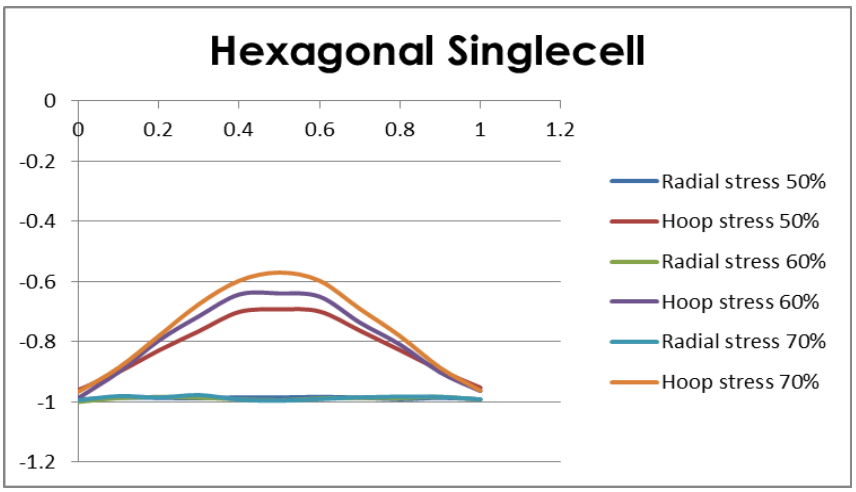


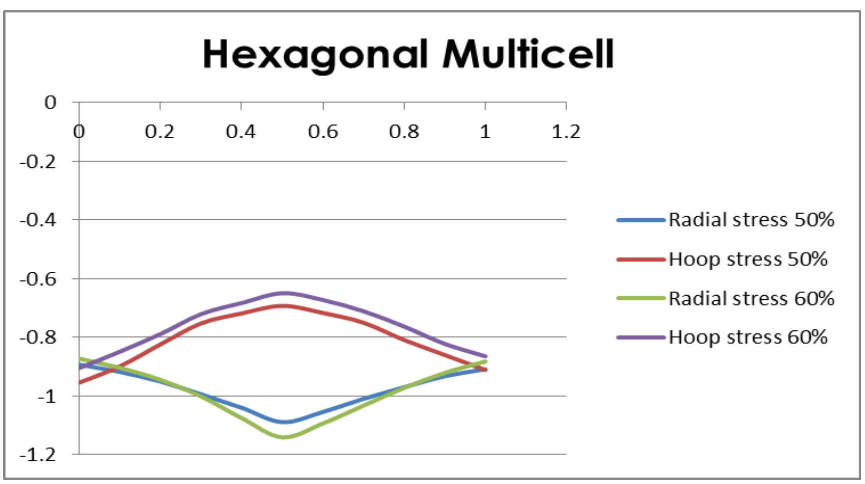


5.4.2 Comparison of stress based on various volumes of fiber



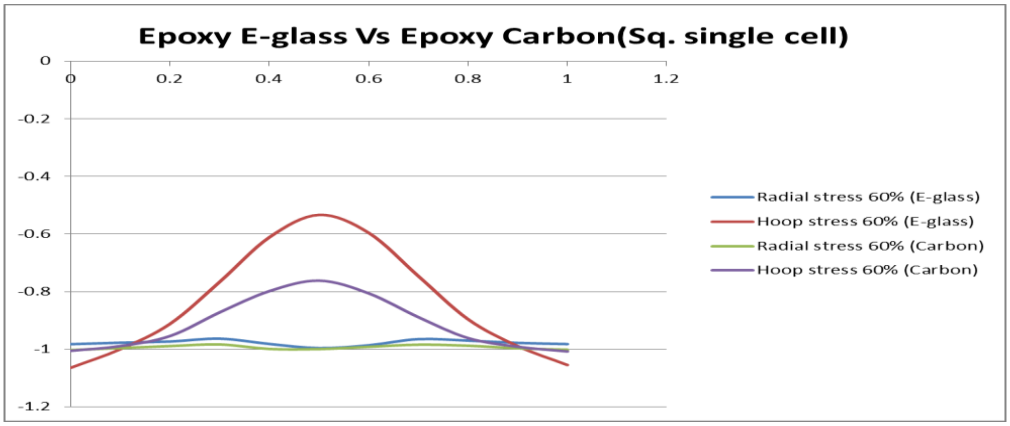

Sq. multi cell
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
0
0.2
0.4
0.6
0.8
1
1.2
Radial stress 50%
Hoop stress 50%
Radial stress 60%
Hoop stress 60%

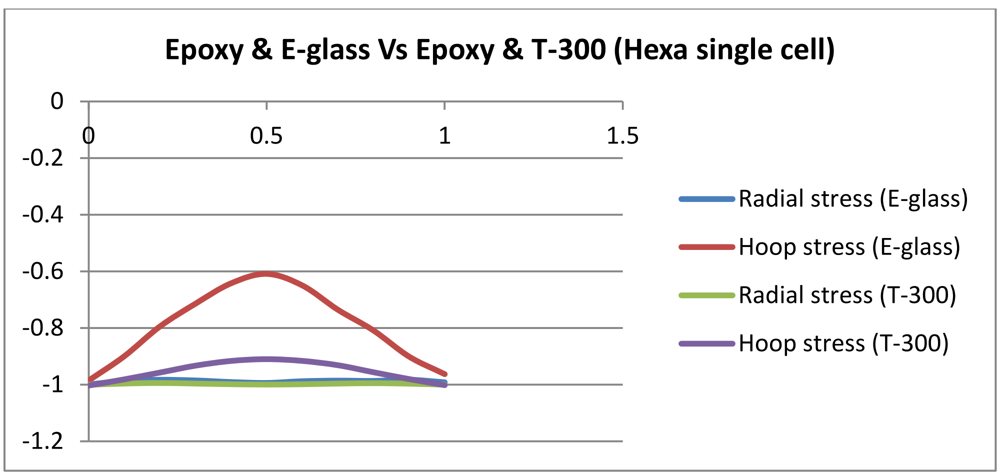

Hexagonal Singlecell
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.2
0.4
0.6
0.8
1
1.2
Radial stress 50%
Hoop stress 50%
Radial stress 60%
Hoop stress 60%
Radial stress 70%
Hoop stress 70%

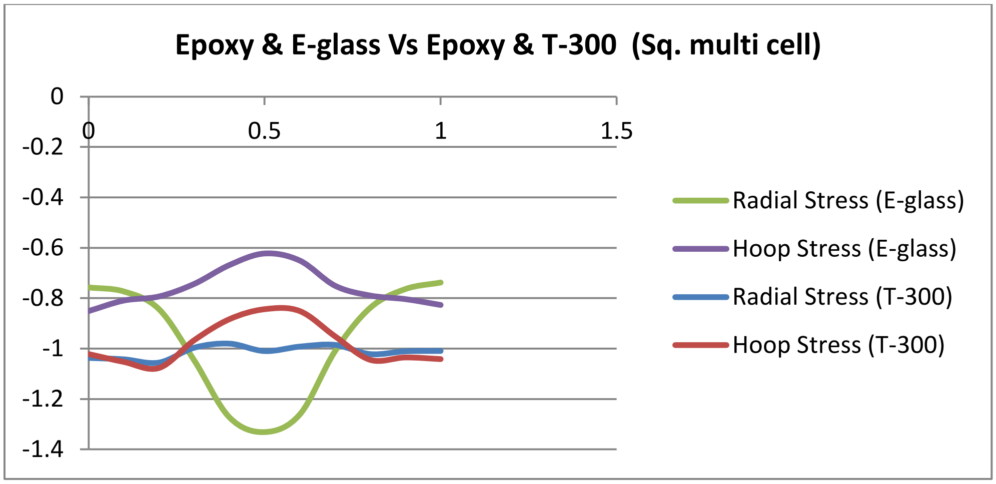

Hexagonal Multicell
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.2
0.4
0.6
0.8
1
1.2
Radial stress 50%
Hoop stress 50%
Radial stress 60%
Hoop stress 60%

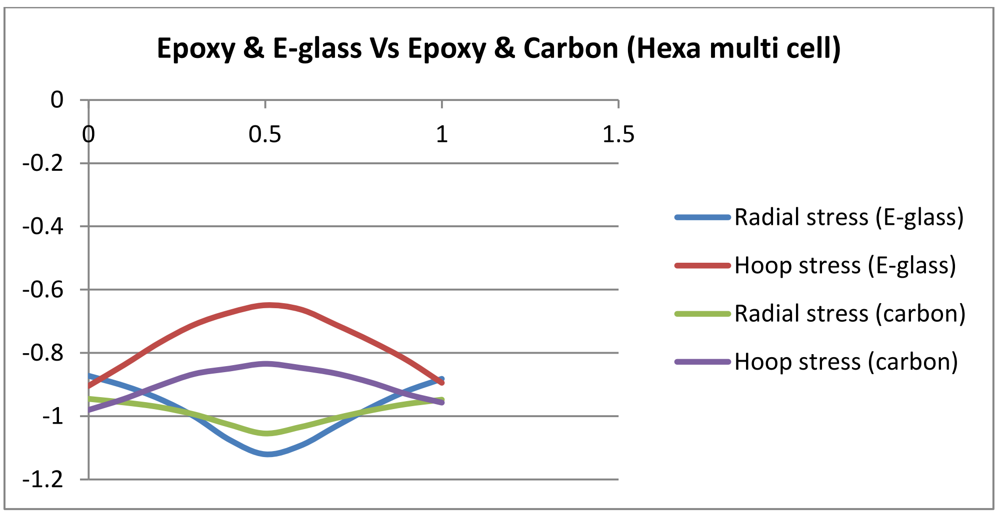
All stresses are going up in absolute value with the increase in concentration of fibers. Multi-cell model is giving higher stresses in absolute values than single cell module due to the effect of neighboring cells.

5.4.3 Comparison of stresses based on various material properties.

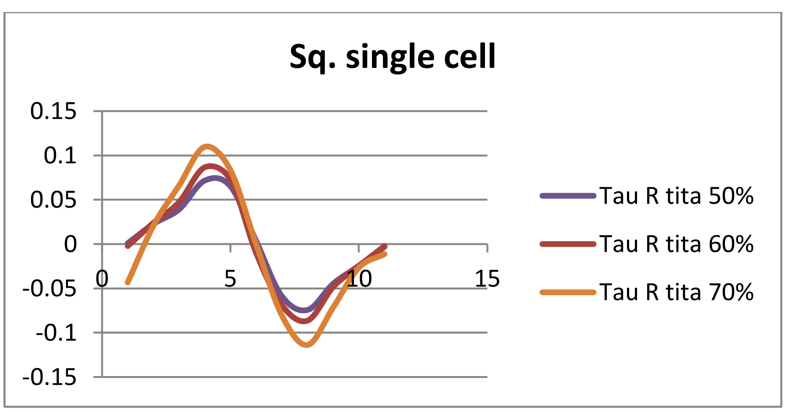


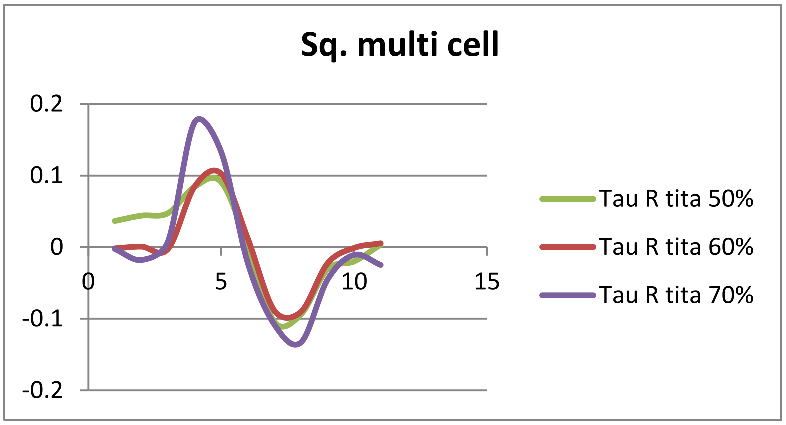


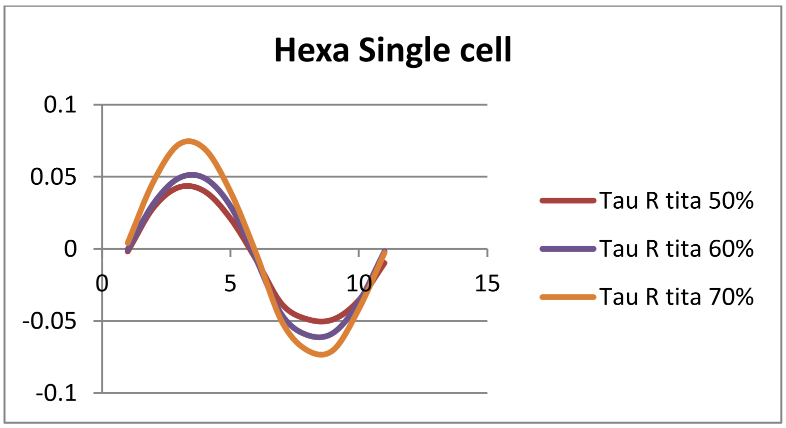


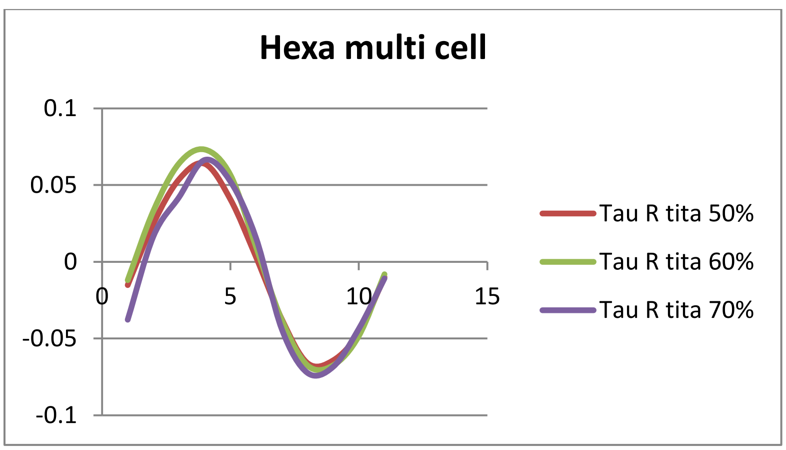


5.4.4 *Shear stress for various volumes of fiber*




Sq. multi cell
0.2
0.1
0
-0.1
-0.2
0
5
10
15
Tau R tita 50%
Tau R tita 60%
Tau R tita 70%


Hexa Single cell
0.1
0.05
0
-0.05
-0.1
0
5
10
15
Tau R tita 50%
Tau R tita 60%
Tau R tita 70%


Hexa multi cell
0.1
0.05
0
-0.05
-0.1
0
5
10
15
Tau R tita 50%
Tau R tita 60%
Tau R tita 70%

5.5 Discussion of the results

When the uniform pressure is applied on the outer surfaces of outer cells, it is not replicated to the internal cells. As the bigger number of cells is more precisely repeated in the array of cells, the stress state is developed in the internal cells. Stress distribution on the outer surfaces of the internal cells is significantly non-uniform, which reflects interactions of neighbor cells. This non-uniformity is important for the failure prediction of the composites and prognosis of the effective strength of unidirectional fiber reinforced materials. The used model is taking into account, where the interaction of the nearest neighbor cells is considered by ignoring the long-distance interactions. Probably, it is enough for the practical applications.

Chapter 6

CONCLUSION

6.1 Summary and Conclusion

- Multi-cell array models have different stresses distributions than single cell models. This uneven stress distribution is because of the effect of surrounding neighboring cell.
- Stress concentration factor is decreasing with the growth of fiber concentration.
- A hexagonal array of fibers is characterized by lower stress concentration factors than compared to the quadratic array of fibers.
- Stress concentration factor for E-glass fiber reinforced composite for biaxial compression is about 15%
- In uniaxial compression, stress concentration factor is about 30%
- For carbon fiber reinforced plastic, the stresses in transversal direction are lower than that in E-glass.

6.2 Future Work

- The aim of this study was to find the stress analysis of unidirectional composites by finite element method for various geometries of composite materials under transversal loading. This study can be further expanded by using Hybrid Composites (multiple fibers).
- Stress analysis on Nanoparticles Reinforced Composites can be conducted.
- FE Analysis on Composite with the fiber orientation of 0°, 90°, 45°, -45°.
- Effect of Matrix/Fiber debonding can be studied by applying tensile loading in transversal direction.
- Stress analysis of unidirectional composites under thermal loading can be performed.

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BIOGRAPHICAL STATEMENT

Pavan Agarwal received his Bachelor's degree in Mechanical Engineering from Jawaharlal Nehru Technological University, Hyderabad in the year 2014. He then decided to pursue his masters in mechanical engineering from the University of Texas at Arlington, USA in spring 2015 and received his degree in fall 2016.

Pavan has always been interested in FE Modeling and Machine designing. He has worked on a project for design of the automotive steering mechanism in his under-graduation. He is always excited to learn something new in every project.

Coming from a business background, Pavan was always ambitious to be an entrepreneur and start his industry in the field of mechanical engineering. Pavan would like to use his skills and experience to develop technology for the comfort of humankind by keeping environmental factors in mind.