

FEM ANALYSIS OF COMPOSITE STRINGERS
FOR AN AIRPLANE FUSELAGE

By

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Abstract

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With the advent of technology, materials have advanced many folds; One such technical revelation has been Fiber-reinforced Composite Materials. Composite materials have two major advantages, among many others: Improved strength and stiffness, especially compared to other materials on a unit weight basis. For example, Composite materials have stiffness comparable to the stiffness of steel but with a strength decimal order higher and more than three times lighter. These advantages have led to new aircraft and spacecraft designs that are radical departures from past efforts based on conventional materials. The stringers serve to take up(along with the skin) the compression and tension forces created at the bending of the fuselage, besides the stringers with frames strengthen the skin, thus increasing its critical compression and shearing stress. This paper focusses on a comparative study of Aluminum Alloys and Composite materials for a Stringer. Composite materials such as Glass Fiber reinforced composite, Kevlar Epoxy and Carbon fiber reinforced composite are used for the study. Stringers with various cross-sections such as I-beam, box beam and double I-beam are analyzed for composite materials, along with those made up of Aluminum alloys for the same boundary conditions. It is observed that compared to Aluminum 6061-T6,

composites show lower displacement in Carbon and Kevlar, whereas lower stresses in all for the same boundary and loading conditions.

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Chapter 1

Aircraft and spacecraft industry is moving towards a low-emission aircraft depending upon the consumer requirements and the ever-changing market needs. There is a constant requirement to reduce the cost of operation as the cost of fossil fuels keep on increasing day by day. At the same time, it is of utmost importance to reduce the emissions from the aircraft as some greenhouse gasses keep on increasing in the atmosphere.

A viable solution to this problem would be to reduce the weight of the aircraft, hence reducing the operational cost, which in turn would reduce the fuel consumption. With a decrease in fuel consumption, the emissions will be reduced as well. To achieve this, the first thing that needs to be done is to change the material from the conventionally used materials. Material needs to be used which weighs less, but at the same time possesses the same strength and stiffness characteristics so that the safety of the aircraft is not compromised.

In the recent past, Composites have proven to meet these requirements. Extensive research is being carried out on various types and composites and many composite materials that offer high strength and stiffness characteristics, at the same time being low in weight have been developed.

1.1 What are Stringers?

The main cylindrical body of an aircraft body is called fuselage. The longitudinal stiffeners of a fuselage are termed as stringers. These stringers serve to take up (along with the skin) the compression and tension forces created by bending of the fuselage. They prevent the skin from bending. They also prevent buckling under compression and

shearing stress. Besides, the stringers with frames strengthen the skin, thus increasing its critical and shearing stress.

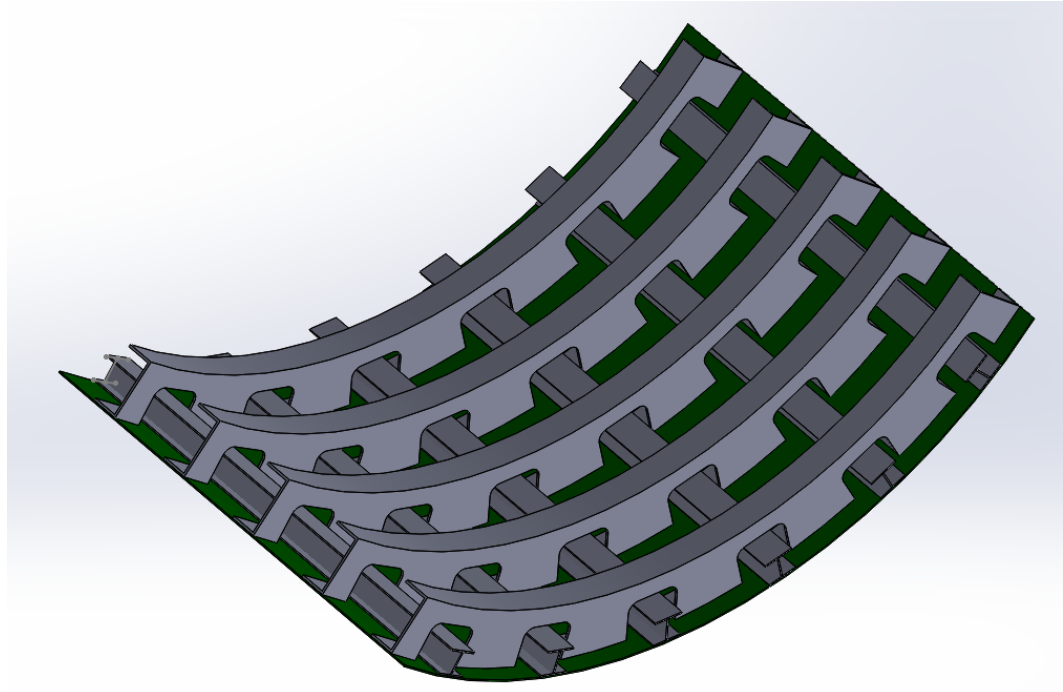


Figure 1-1 Stringer-frame assembly.

The figure 1-1 shows a stringer and frame assembly. Here the longitudinal members are the stringers. Whereas, the circumferential members are called as frames.

Depending upon the requirements for particular cases, the cross sections of both stringers and frames can be varied. A few cross sections which are being in use at present are I-stringer, J-stringer, Z-stringer, box, double I, extruded Y, extruded hat and formed hat. These can be seen in Figure 1-2.

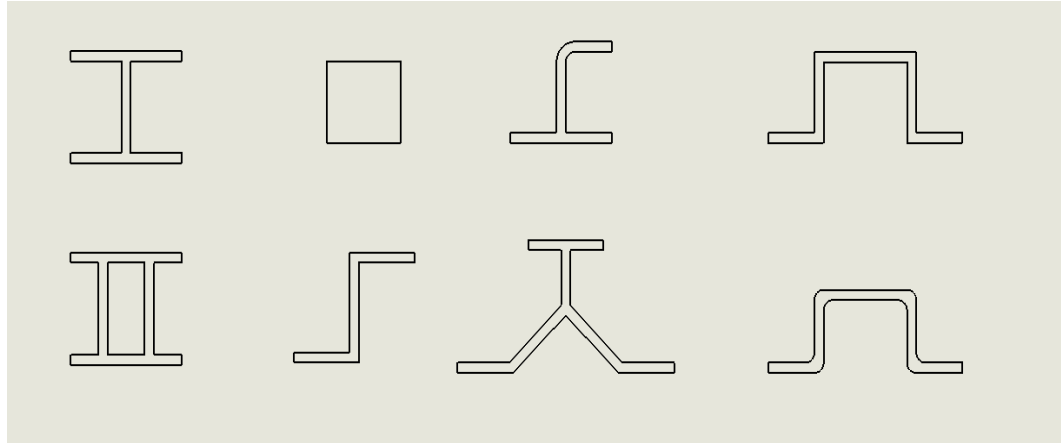


Figure 1-2 Stringer cross-sections

1.2 Research Objectives

Aluminum and aluminum alloys have been used for years now in the manufacturing of aircraft. Also, steel was being used before that. Now, a combination of both is being used as per requirements. But, with the advent of technology, these are being replaced by composites. A great amount of funding is being provided in the development and research of composites, which wasn't the case a few years ago.

In the 1960s, similar investment was made to replace steel with titanium. However, the outcome was not as desired and losses were faced. Since then, investment in new materials has been difficult as investors are skeptical of investing in new materials. But in the recent past, more and more investment has increased for composites as positive outcomes have been seen.

There are varieties of composites available for selection. Selecting one that would be perfect for our requirement is of utmost importance. Therefore, a composite that would be the best for making stringer needs to be selected. Also, out of all the stringer

cross-sections available to us, selecting the one which provides the optimum amount of strength, stiffness and provides lowest stresses is also equally important.

The main aim of the following research work is to compare and correlate results obtained from analysis of various materials and the cross-sections of various stringers. The boundary conditions for all the analyses are kept the same, and a comparison between Al 6061-T6, Glass Fiber Epoxy, Kevlar Epoxy and Carbon Fiber Epoxy is carried out. Standard cross-sections like I-beam, box beam and double I-beam are used for these analyses.

1.3 Composites

Any material consisting of two or more components with different properties and distinct boundaries between the components can be referred to as a composite material [1]. Most composites consist of fibers of one material tightly bound into another material called a matrix. The matrix binds the fibers together somewhat like an adhesive and makes them more resistant to external damage. Whereas the fibers make the matrix stronger and stiffer and help it resist cracks and fractures. Fibers and matrix are usually (but not always) made from different types of materials [2].

The principal constituent in the fiber reinforced composite is the fiber. It is the major load-carrying component. It provides with the stiffness and strength to the composite. Fibers occupy the largest volume of the composite. On an average, it is 67% of the composite.

The other component of the composites is called the matrix. The main function of the matrix is to transfer the stresses between the fibers. Fibers usually do not have great environmental resistance. This is where the matrix helps provide a barrier against

adverse environmental conditions. Matrix also helps protect the fibers from abrasion.

They also help provide lateral support which a reason why they influence the compressive strength of the composite.

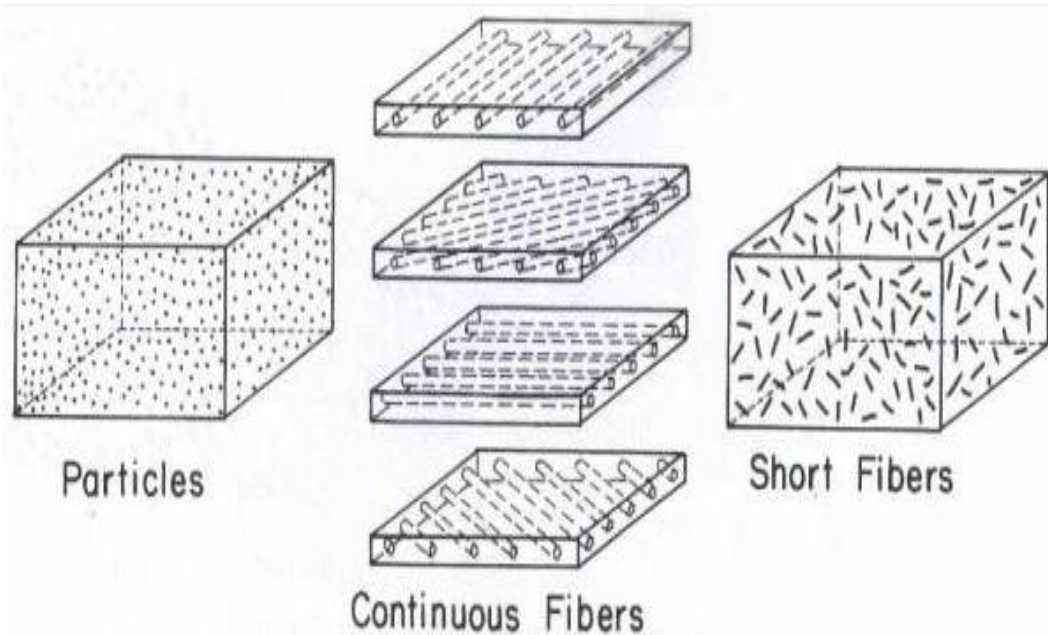


Figure 1-3 Fiber Classification[3]

The advantages of composites are:

- High specific stiffness
- High specific strength
- Low density
- Corrosion resistance
- Design flexibility
- Low thermal expansion
- Cost saving
- Part count reduction

- Easy fabrication

There also a few challenges when it comes to the use of the composites. They are as follows:

- Analysis of composites is a bit complicated
- In-plane loading may produce shear deflection
- In-plane loading may produce out of plane deformation
- Delamination failure is common in composites. It does not occur in metals
- Weakness in out of plane deformation that may cause delamination.

Chapter 2

Geometry

In this chapter, we will study about the various geometries that we have used for the study. A stringer of 1000mm was used for the study. That is the standard distance between two frames. A standard thickness of 3mm was used for Aluminum, Carbon Epoxy, and Kevlar Epoxy. As Glass Epoxy is stiff as compared to other materials, a thickness of 5mm is used as compensation.

The three cross-sections used for the study are I beam, box beam and double I beam.

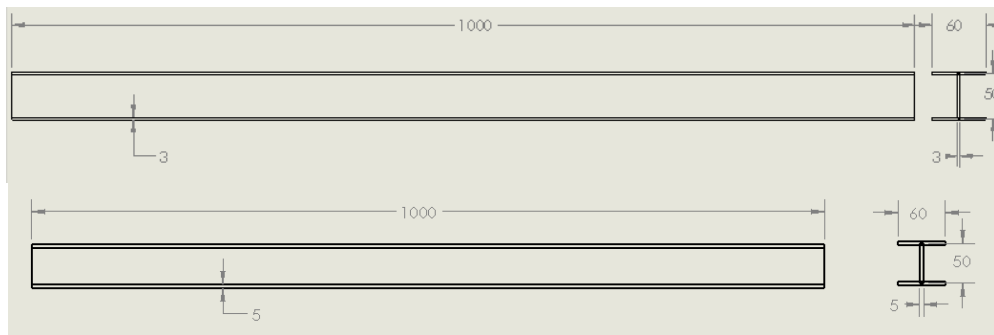


Figure 2-1 I beam cross-section

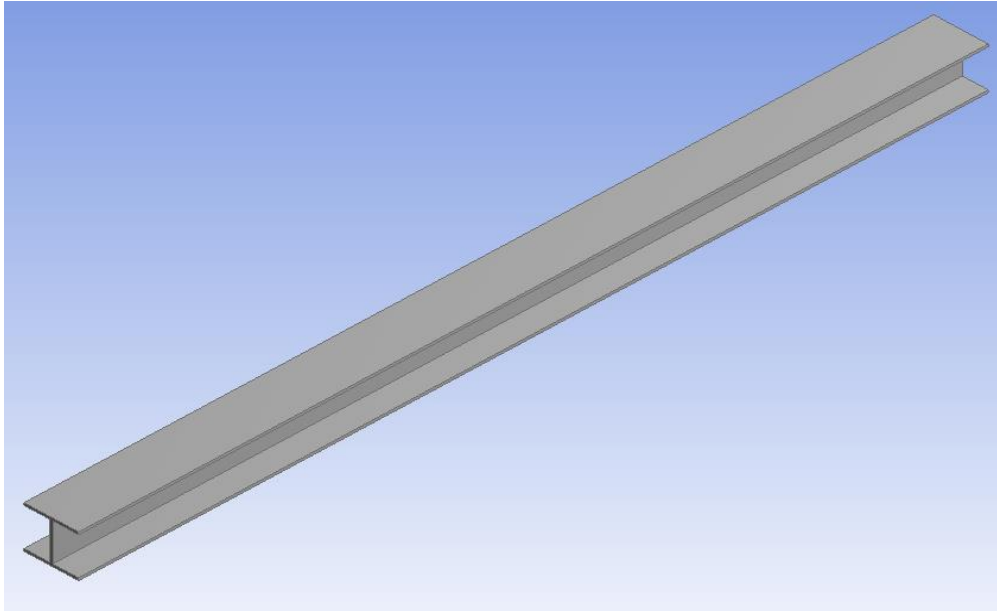


Figure 2-2 I beam geometry

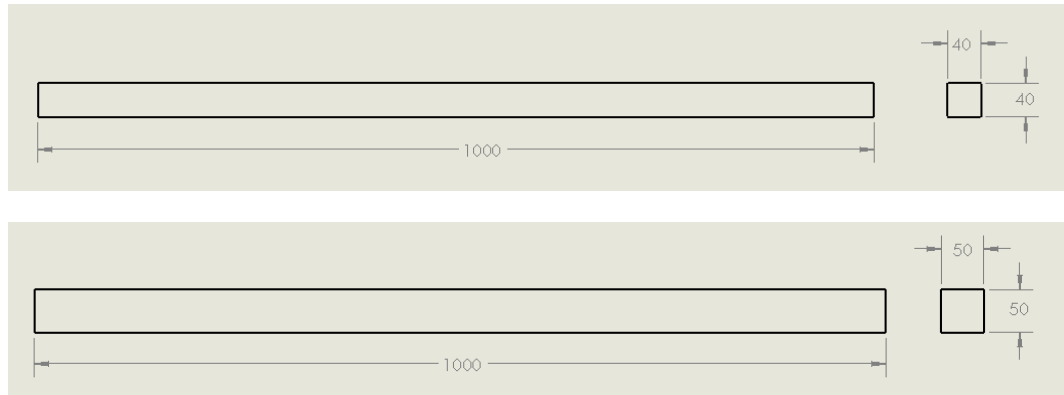


Figure 2-3 Box cross-section

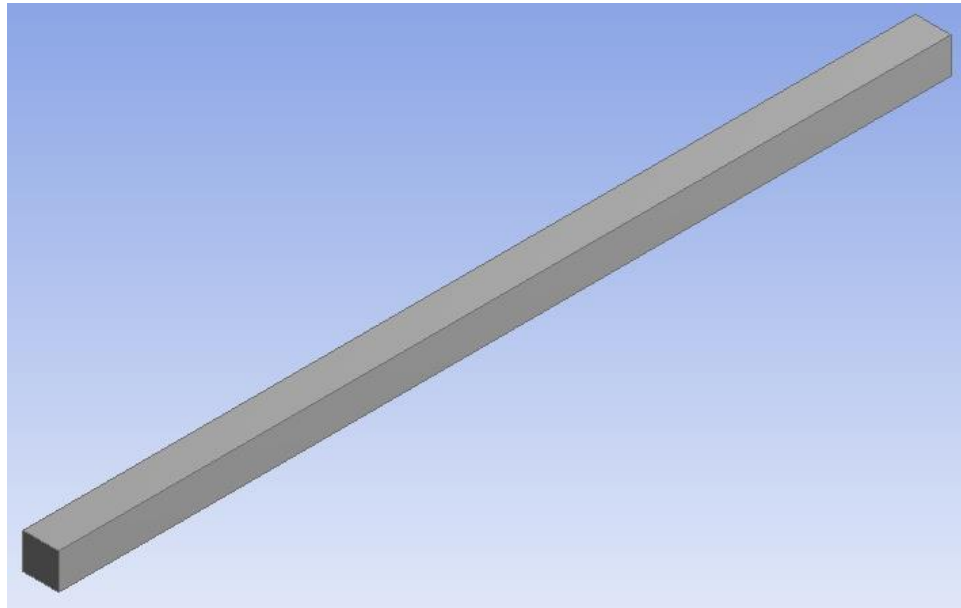


Figure 2-4 Box geometry

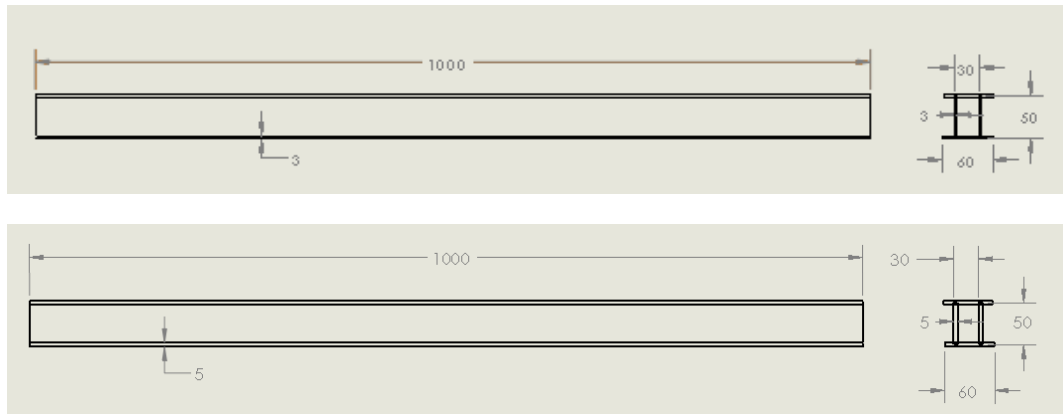


Figure 2-5 Double I dimensions

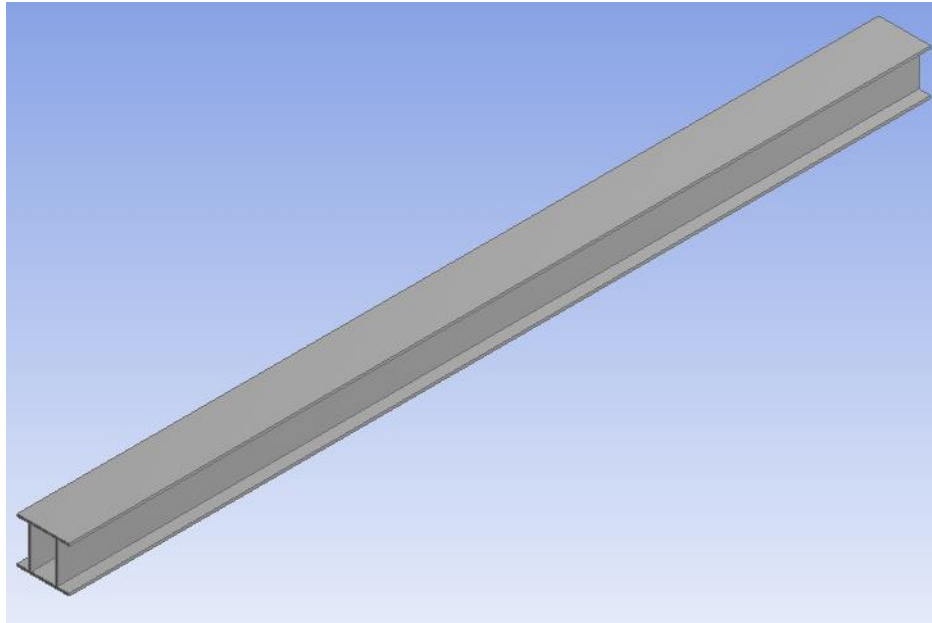


Figure 2-6 Double I geometry

The section properties of the I-beam and Box beam need to be calculated

I-beam is a type of beam often used in trusses in buildings. I-beam is manufactured from structural steels with hot and cold rolling or welding processes. Top and bottom plates of an I-beam are named as flanges and the vertical plate which connect the flanges is named as the web. Different dimensions of I-beam exist in the market and can be supplied by the beam suppliers. Due to its shape, I-beam has a high moment of inertia and stiffness which makes it resistant to bending moments. The web provides resistance against shear forces. These beams are not resistant to torsional loading (twisting), and they shall not be used in the cases where torsion is dominant.[4]

The sectional properties and their formulas for I-beam are as follows:

	Parameter/Condition	Symbol	Equation
1	Cross section area	A	$A = 2Bh + Hb$
2	Area moment of inertia	I_{xx}	$I_{xx} = H^3b/12 + 2[h^3B/12 + hB(H+h)^2/4]$
3	Area moment of inertia	I_{yy}	$I_{yy} = BH^3/12 + 2(B^3h/12)$
4	Section modulus	S_{xx}	$S_{xx} = 2I_{xx}/(H + 2h)$
5	Section modulus	S_{yy}	$S_{yy} = 2I_{yy}/B$
6	Center of gravity	x_{cog}	$x_{cog}=B/2$
7	Center of gravity	y_{cog}	$y_{cog}=H/2 + h$
8	Mass	M	$M = AL\rho$
9	Radius of gyration	r	$r = (I/A)^{0.5}$
10	Polar moment of inertia	J	$J = I_{xx} + I_{yy}$

Table 2-1 Section properties for I-beam

Rectangular bars (including square bar) are solid bars (or flats) with rectangle cross section. They are produced from stainless steel, carbon steel, alloy steel, and aluminum. The manufacturing method for rectangular bars are cold/hot rolling and drawing. Rectangular bars are offered by manufacturers in a variety of sizes.[5]

The sectional properties and their formulas for box beam are as follows:

	Parameter/Condition	Symbol	Equation
1	Cross section area	A	$A = BH$
2	Area moment of inertia	I_{xx}	$I_{xx} = BH^3/12$
3	Area moment of inertia	I_{yy}	$I_{yy} = HB^3/12$
4	Section modulus	S_{xx}	$S_{xx} = I_{xx}/y_{cog}$
5	Section modulus	S_{yy}	$S_{yy} = I_{yy}/x_{cog}$
6	Center of gravity	x_{cog}	$x_{cog} = B/2$
7	Center of gravity	y_{cog}	$y_{cog} = H/2$
8	Mass	M	$M = AL\rho$
9	Radius of gyration	r	$r = (I/A)^{0.5}$
10	Polar moment of inertia	J	$J = I_{xx} + I_{yy}$

Table 2-2 Section properties for box beam

Chapter 3

Boundary Conditions

In this chapter, we consider the types of loads that act upon the stringers during a normal flight condition. Also, we consider the cases where the stringer is between two frames. Hence, we consider three different cases for the stringers. The first one is where we consider it to work as a cantilever beam. This is where we fix one end of the stringer and consider the deflections and stresses on the stringers.

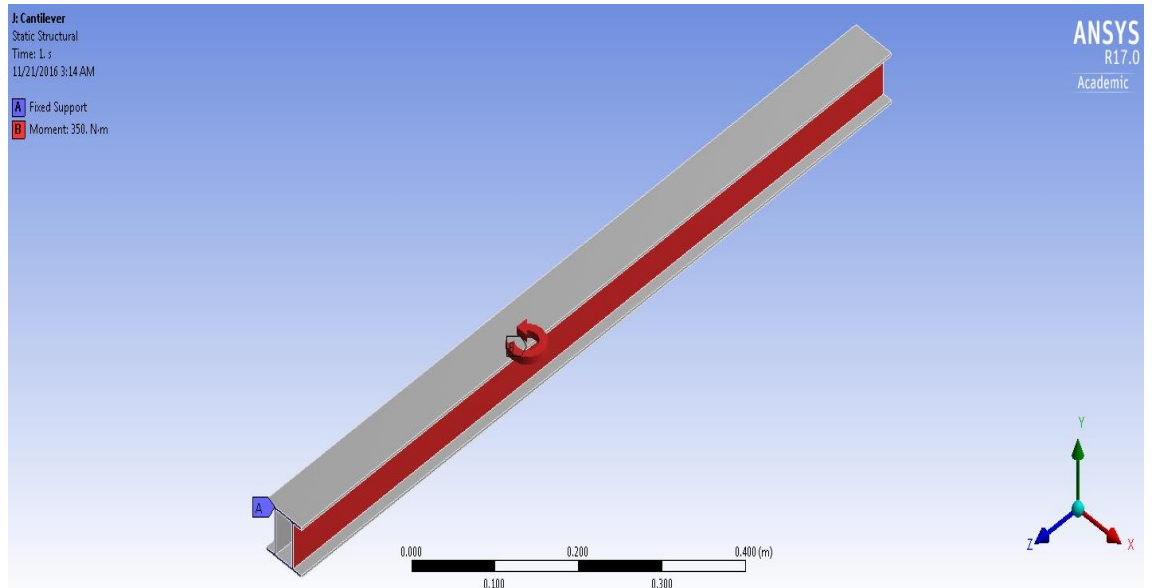


Figure 3-1 Cantilever

The second condition is where we fix the stringer in all the directions. We fix one corner of the stringer in all x, y and z-direction, so it acts as a fixed support. Diagonally across the face of the stringer, we fix z direction. However, the stringer is allowed to

move in x and y-direction. Also, diagonally across the body of stringer from the first point, we fix that point in x and y-direction, while allowing movement in the z direction.

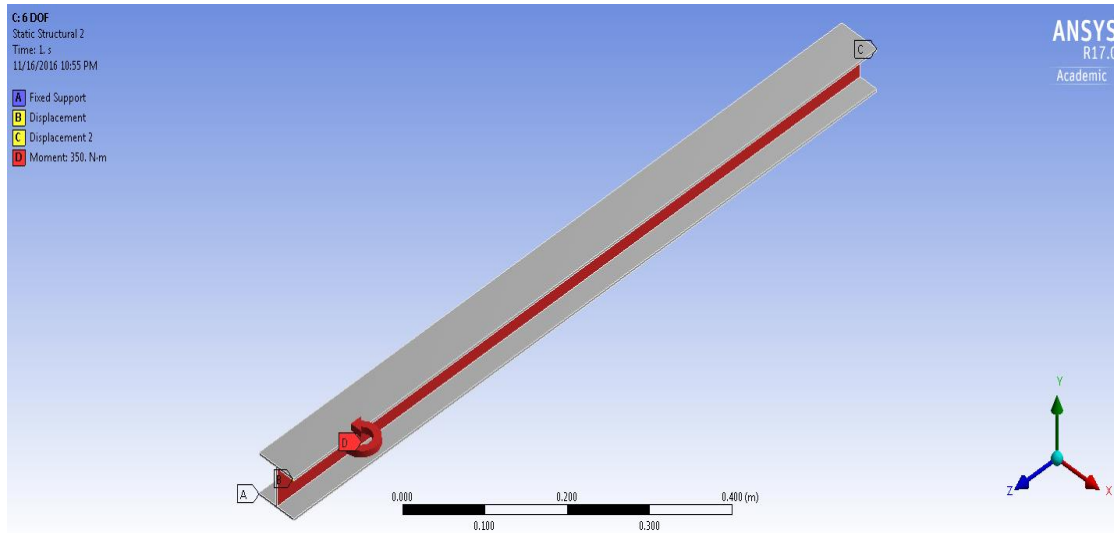


Figure 3-2 6 DOF

The third condition we have considered clamping both the ends of the stringer. Here, both the ends are fixed and not allowed to move. They are fixed between the boundaries.

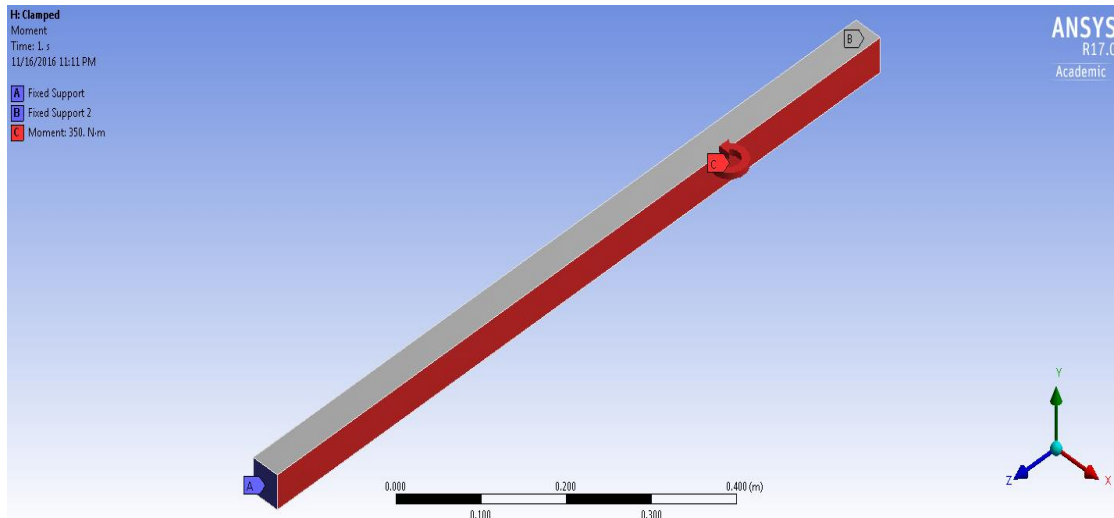


Figure 3-3 Clamped

Along with these boundary conditions, we consider bending of the fuselage. Hence, we consider the bending of the stringer along with that. A bending moment acts on the side face of the stringer. We consider a moment of 350 Nm. The moment is distributed all over the face.

Chapter 4

Loading Conditions

Three loading conditions will be considered in this chapter. The conditions are bending of the stringer, torsion acting on the stringer and buckling of the stringer.

4.1 Bending

The bending moments M_y and M_z cause the fuselage to bend. A part of the fuselage is subject to tensile stresses, while the other part is in compression. The line separating these two regions is called the neutral axis. It is a straight line, and it always goes through the center of gravity of the cross-section. The stresses applied on the fuselage can be derived from the general equation.

$$\sigma_x = \left(\frac{M_z I_{yy} - M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2} \right) y + \left(\frac{M_y I_{zz} - M_z I_{yz}}{I_{yy} I_{zz} - I_{yz}^2} \right) z$$

The fuselage cross-section can have +non-symmetric shapes theoretically; therefore, this general equation is programmed into the model for the calculation of the bending stress.

If the cross-section of the fuselage is symmetric about the y-axis or about the z-axis (or both), then $I_{xy} = 0$. This simplifies the above equation to:

$$\sigma_x = \frac{M_y}{I_{yy}} z + \frac{M_z}{I_{zz}} y$$

The direct stress is calculated for each stringer and neighboring skin panels. In other words, the calculated stress on a location of the stringer is the total stress working on the stringer and the half of the two neighboring skin panels. Since the equation of the bending stress does not specify the specific bending stress in the stringer and skin panel,

the bending stress is converted to force, which applies to this specific part. Whereas later the specific stress in the stringer and skin panel are calculated separately according to the applied force and material for the stringer and skin panel. The applied force distribution on the fuselage can be calculated by, [6]

$$F = \sigma_x \cdot \left(\frac{E_{str}}{E_{skmean}} A_{str} + \frac{E_{skleft}}{E_{skmean}} \frac{1}{2} A_{skleft} + \frac{E_{skright}}{E_{skmean}} \frac{1}{2} A_{skright} \right)$$

4.2 Torsion

The action of twisting or the state of being twisted is termed as torsion. Consider torque acting on one face of the stringer while considering the other end to be fixed. A torsion is created in the stringer.

To calculate the torsional stiffness we use,

$$\theta := \frac{T \cdot l}{G \cdot J}$$

When we consider the composites, it is not easy to calculate the Shear modulus. It varies on the composition of fiber and matrix from composite to composite. The formula we use for that is,

$$G := G_m \cdot \frac{\left(1 - c \cdot \left(\frac{G_m - G_f}{G_m + G_f} \right) \right)}{\left(1 + c \cdot \left(\frac{G_m - G_f}{G_m + G_f} \right) \right)}$$

4.3 Buckling

A force that laterally shortens the beam until the beam can no longer support that load is buckling. It is the most common and catastrophic failure observed in the

Buckling of aluminum depends on thickness, width, stiffness, Poisson's ratio and aspect ratio. The equations to calculate buckling strength are for isotropic materials. The isotropic buckling equations are used to calculate the buckling strength

of an Aluminum Alloy panel as shown in Equation, where E_c is the Elastic Modulus of the material in compression.

$$\sigma_{cr} = C_c \frac{\pi^2 E_c}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 = K_c E_c \left(\frac{t}{b}\right)^2 \text{ where } K_c = f\left(\frac{a}{b}\right)$$

Chapter 5

Materials

The study compares Aluminum 6061-T6 against the composite materials. The composite materials used for the study are Carbon Epoxy, Glass Epoxy, and Kevlar Epoxy.

The material properties required for complete assessment of the results are given below:

- Young's Modulus, E (GPa)
- Shear Modulus, G (GPa)
- Poisson's Ratio, ν

5.1 Aluminum 6061-T6

6061 has various characteristics and uses. It has Excellent joining characteristics, good acceptance of applied coatings. It combines relatively high strength, good workability and high resistance to corrosion. It also has great chipping characteristics. It is being used in wide applications such as Aircraft fittings, hydraulic pistons, bike frames, etc.

Physical Properties:

Ultimate Tensile Strength	310 MPa
Tensile Yield strength	276 MPa
Modulus of Elasticity	68.9 MPa
Poisson's Ratio	0.33
Shear Modulus	26 GPa

5.2 Glass Fiber

It is used for various aircraft structures. Glass fibers have also been used for helicopter rotor blades. There are a few different type of Glass fibers in use. Electrical glass, or E-glass, which is used for electrical applications. It has high resistance to current flow. It is made from borosilicate glass. S-glass is identified as Structural glass. It has higher strength than the E-glass. S-glass is made from magnesia-alumina-silicate. Glass Fibers have a high strength which is maintained even in a humid environment, but degrades with elevated temperatures. Compared to other composites they have low stiffness but also are comparatively cheaper. They have high chemical and biological resistance. It has a white color and is available as a dry fabric or prepreg material.



Figure 5-1 Glass Fabric [7]

5.3 Kevlar

Aramid fibers are also called as Kevlar. They are light in weight, at the same time being strong and tough. Two different kinds of aramid fibers are used in industry. Kevlar 49 and Kevlar 29. Kevlar 49 has high stiffness, and Kevlar 29 has comparatively low stiffness. These fibers have high resistance to impact damage. Which is the reason they are used in applications where the areas are prone to impact damages? One of the

disadvantages of Kevlar Fibers is that they absorb up to 8 % of their weight in water. Hence, they need protection from the environment. They are also difficult to cut and drill. Which is why special tools are required to cut them. These fibers have a yellow color and are available as dry fabric and prepreg material.

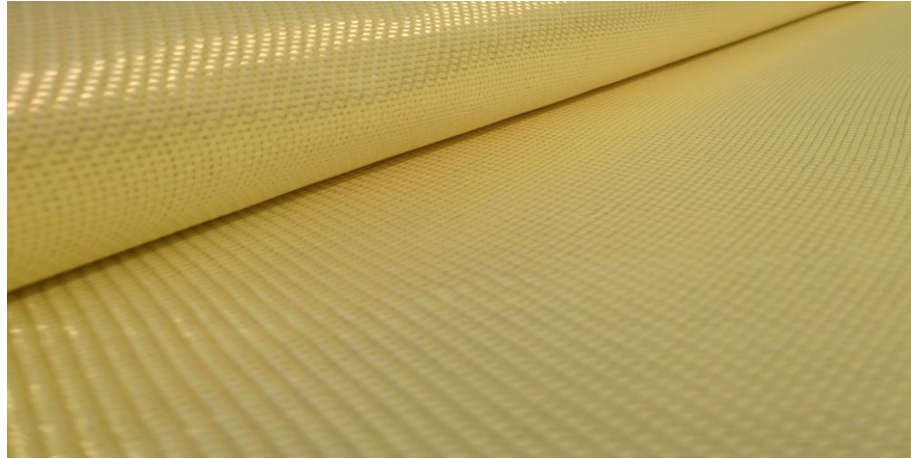


Figure 5-2 Kevlar Fabric [8]

5.3 Carbon Fiber

Carbon and Graphite fibers are based on the graphene layer networks present in carbon. These fibers take a longer time to prepare as compared to other fibers. They also need an extended time of temperature processing. These are the reasons why they are comparatively more expensive than others are. Carbon fibers are very strong and stiff when compared to any other fiber. They are about 3 to 10 times stronger than the glass fibers. These have a modulus, which is about four times higher than the modulus of steel, whereas the density is lower by the same factor. These fibers also have higher resistance to corrosion. Their disadvantage is that they have lower conductivity than aluminum. Thus, need a protection coating for parts that are prone to lightning strikes. They are usually black in color and are available as dry fabric and prepreg material.

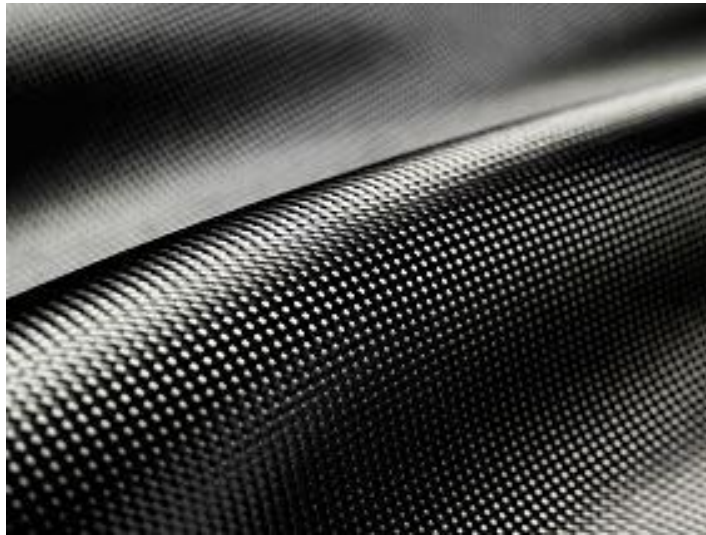


Figure 5-3 Carbon Fabric [9]

5.4 Epoxy Resin

These are polymerizable thermosetting resins and are available in a variety of viscosities from liquid to solid. Specific guidelines and instructions need to be followed while selecting these for a particular application. They are widely used as resins and structural adhesives. They have high strength and modulus, low shrinkage, brilliant adhesive properties, great chemical resistance and are also very easy to process. [10]

The properties of these materials are as follows:

Properties	Glass Fiber	Kevlar Fiber	Carbon Fiber	Epoxy Matrix
E1 (GPA)	85.5	151.17	300	3
E2 (GPA)	85.5	4.1	14	3
v12	0.23	0.35	0.3	0.3
v23	0.23	0.15	0.15	0.3
G12 (GPA)	35	2.9	8	1.11
G23 (GPA)	35	1.782	6.087	1.154

Table 5-1 Fiber and Matrix Properties

These are the properties of fibers and matrix. We need to find combined properties. For which we use a Mathcad program that we have created. The basic flow of the program is explained below.

Block1.

Enter data:

Five elastic properties of monotropic fibers, two elastic properties of polymer matrix;

Two coefficients of linear thermal expansion of monotropic fibers, one coefficient of linear thermal expansion of polymer matrix.

Block 2.

Calculation of the other properties using conditions of the symmetry of material, the condition of elastic potential, and the connection between shear modulus, Young modulus and Poisson coefficient for the planes of isotropy.

For orthotropic materials (including isotropic and monotropic ones) only 12 components of the compliance matrix are non-zero

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$$

or using engineering constants for notation

$$\begin{pmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{pmatrix}$$

Condition of elastic potential requires that matrix of compliance has to be symmetrical one

$$C_{ij} = C_{ji}$$

From this condition, it is follows

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}; \quad \frac{\nu_{13}}{E_1} = \frac{\nu_{31}}{E_3}; \quad \frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}$$

and only nine constant of an orthotropic material is independent ones.

For Monotropic/ (transversal isotropic) material with axis of monotheropy x_1 (or plane of isotropy $x_2 - x_3$) we have (equal constants are denoted by the same symbols)

$$\begin{pmatrix} C_{11} & \square & \square & 0 & 0 & 0 \\ \square & \square & \square & 0 & 0 & 0 \\ \square & \square & \square & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \diamond & 0 \\ 0 & 0 & 0 & 0 & 0 & \diamond \end{pmatrix}$$

Thus, we have six different constants: $C_{11} = 1/E_1$, $C_{66} = 1/G_{12}$, square ($C_{22} = C_{33} = 1/E_2$), triangle ($C_{12} = C_{13} = C_{21} = C_{31} = -\nu_{12}/E_1$), parallelogram ($C_{23} = C_{32} = -\nu_{23}/E_2$), and rhombus ($C_{44} = C_{55} = 1/G_{23}$). This material is still not monotropic, but square orthotropic one (geometrical image is a prism, with sizes l_1, l_2, l_3 with square cross-section $l_2 = l_3$, but $l_1 \neq l_2$). For such material, if we apply stress $\sigma_2 = \sigma$ in direction, we obtain the strain $\varepsilon_2 = \varepsilon$, and if we apply equal stress $\sigma_3 = \sigma$ in direction, we obtain the equal strain $\varepsilon_3 = \varepsilon$ due to the symmetry of the compliance matrix written, but in general case if we apply stress σ in the diagonal direction, the strains in the diagonal direction will be different $\varepsilon_d \neq \varepsilon$. The condition of isotropy in the plane of isotropy requires that the strain in a diagonal direction ε_d from the same stress $\sigma_d = \sigma$ must be the same $\varepsilon_d = \varepsilon$. This requirement gives additional relationship

$$C_{44} = 2(C_{22} - C_{23})$$

Or

$$G_{23} = \frac{E_2}{2(1 + \nu_{23})}$$

Thus, only five independent elastic constants are remaining.

Geometrical image of such anisotropy is a cylinder.

For isotropic material, we have the matrix of compliances

$$\begin{pmatrix} \square & \square & \square & 0 & 0 & 0 \\ \square & \square & \square & 0 & 0 & 0 \\ \square & \square & \square & 0 & 0 & 0 \\ 0 & 0 & 0 & \diamond & 0 & 0 \\ 0 & 0 & 0 & 0 & \diamond & 0 \\ 0 & 0 & 0 & 0 & 0 & \diamond \end{pmatrix}$$

Here only three different characteristics are present, denoted by square ($1/E$), by triangle ($-\nu/E$), and by rhombus ($1/G$). However, this material is still not isotropic, but cubically orthotropic. Its geometrical image is a cube. Application of additional requirement of isotropy

$$G = \frac{E}{2(1+\nu)}$$

Gives isotropic material, which has only two independent elastic constants.

Geometrical image of such material is a sphere.

Coefficients of linear thermal expansion of anisotropic material can be represented in the matrix form

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

The orthotropic material has only three non-zero components.

$$\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$$

Monotropic material with axis of monotheropy x_1 (or plane of isotropy $x_2 - x_3$)

we have two equal coefficients $\alpha_{22} = \alpha_{33}$ and matrix is written as

$$\begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_2 \end{pmatrix}$$

Here single index system is used. For isotropic material all three coefficients are equal

$$\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix}$$

Comments:

There are different notations used in literature about composites. The sequence of indices in Poisson's coefficients may be opposite one. To prevent confusion: biggest Poisson's coefficient is divided by biggest Young modulus, and smallest Poisson's coefficient is divided by smallest Young modulus.

There are different definitions of shear modulus are used in literature, connecting shear stress with simple shear strain or with pure shear strain. They are twice different. It is necessary to check, which definition is used and not confuse them.

There are different notations for the compliance matrix and matrix of stiffness.

Compliance matrix: $C_{ij}, S_{ij}, a_{ij}, c_{ij}, \dots$

Stiffness matrix: $C_{ij}, S_{ij}, A_{ij}, d_{ij}, \dots$

We were using in lectures two notations for compliance matrix C_{ij}, a_{ij} and two notations for stiffness matrix S_{ij}, A_{ij} . In the first pair of notations, symbols correspond

names of matrices (**C**ompliance, **S**tiffness). What logic was used by some American authors (Robert Jones, Richard Christensen) denoted compliance matrix by letter **S** and stiffness matrix by letter **C**, is not clear.

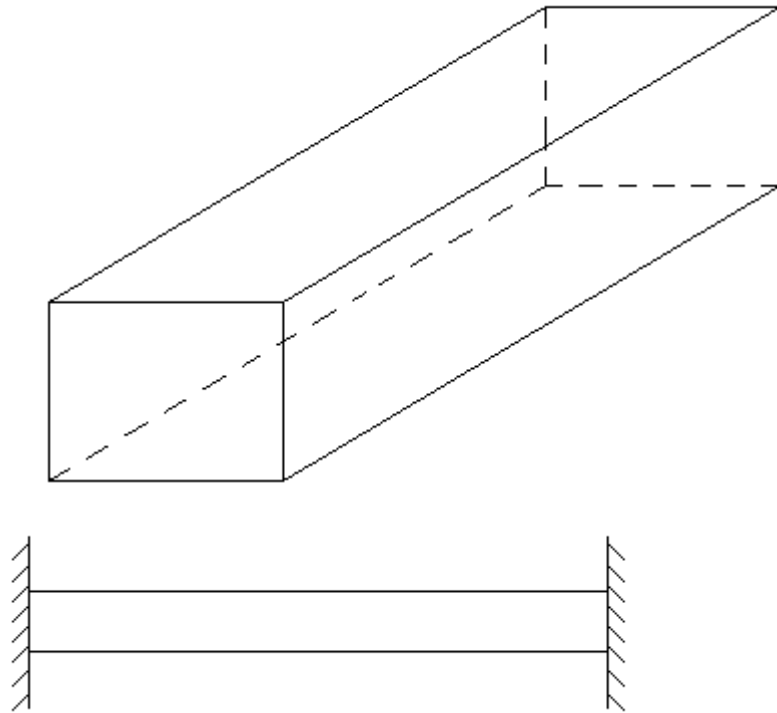
Block 3.

Calculation of the matrix of compliances a_{ij} (or C_{ij}) for fibers and matrix.

Block 4.

Calculation of the two elements of the matrix of compliances b_{ij} for the plane strain state for fibers and matrix. Calculations of the plane bulk moduli K_{23} for fiber and matrix. Calculation of the constant η of thermal expansion.

Let's imagine that the ends of the prism are fixed, and we apply stress to the lateral side



Of the prism. Due to axial strains are forbidden by fixing, some axial stresses are appearing, because Poisson's effect never be realized in the axial direction. Let's analyze this situation. Let's the strains in the direction x_1 are forbidden by such constraint. Let's write the constituent law for orthotropic material

$$\varepsilon_1 = \alpha_1 \Delta T + C_{11} p_1 + C_{12} p_2 + C_{13} p_3 = 0$$

$$\varepsilon_2 = \alpha_2 \Delta T + C_{21} p_1 + C_{22} p_2 + C_{23} p_3$$

$$\varepsilon_3 = \alpha_3 \Delta T + C_{31} p_1 + C_{32} p_2 + C_{33} p_3$$

We can express axial stress p_1 from the first equation

$$p_1 = -\frac{1}{C_{11}} (\alpha_1 \Delta T + C_{12} p_2 + C_{13} p_3)$$

And substitute it into the second two equations. We receive

$$\varepsilon_2 = \left(\alpha_2 - \alpha_1 \frac{C_{21}}{C_{11}} \right) \Delta T + \left(C_{22} - \frac{C_{21}C_{12}}{C_{11}} \right) p_2 + \left(C_{23} - \frac{C_{21}C_{13}}{C_{11}} \right) p_3$$

$$\varepsilon_3 = \left(\alpha_3 - \alpha_1 \frac{C_{31}}{C_{11}} \right) \Delta T + \left(C_{32} - \frac{C_{31}C_{12}}{C_{11}} \right) p_2 + \left(C_{33} - \frac{C_{31}C_{13}}{C_{11}} \right) p_3$$

We see that we have received two-dimensional analog of constitutive law

$$\varepsilon_2 = \eta_2 \Delta T + b_{22} p_2 + b_{23} p_3$$

$$\varepsilon_3 = \eta_3 \Delta T + b_{32} p_2 + b_{33} p_3$$

Here for monotropic material

$$\eta_2 = \eta_3 = \eta = \left(\alpha_2 - \alpha_1 \frac{C_{21}}{C_{11}} \right)$$

$$b_{22} = b_{33} = \left(C_{22} - \frac{C_{12}^2}{C_{11}} \right)$$

$$b_{23} = b_{32} = \left(C_{23} - \frac{C_{12}C_{13}}{C_{11}} \right)$$

From these formulae we see that the difference of these characteristics is connected with shear modulus by the same way, as difference of components of usual compliance matrix

$$b_{22} - b_{23} = C_{22} - C_{23} = \frac{1}{2} C_{44} = \frac{1}{2} \cdot \frac{1}{G_{23}}$$

The sum of these elements

$$b_{22} + b_{23} = C_{22} + C_{23} - 2 \frac{C_{12}^2}{C_{11}} = \frac{1}{2K_{23}}$$

Has remarkable meaning: K_{23} - is a two-dimensional analog of the bulk modulus. K_{23} is equal to the ratio of the pressure applied to the lateral surfaces of the constrained prism to the volume strain of the prism. We can define the “elastic modulus” \tilde{E} at the plane strain state and “Poisson’s coefficient” $\tilde{\nu}$ at the plane state by the way similar to the definition of usual characteristics:

$$\tilde{E}_2 = 1/b_{22} = E_2 / (1 - \nu_{21}\nu_{12}); \quad \tilde{\nu}_{23} = -b_{23}/b_{22} = \frac{\nu_{23} + \nu_{12}\nu_{21}}{1 - \nu_{12}\nu_{21}}$$

Comments.

Formulae derived for the effective modulus K_{23} of composite are relatively simple and more precision than formulae derived for the effective transversal Young modulus E_2 of composite. It is reasonable to find K_{23} and G_{23} , and after that to find E_2 from the condition of isotropy.

In the literature sometimes is used another definition of K_{23} , which is two times bigger.

Block 5.

Calculation of the effective properties of the unidirectional composite.

1. Longitudinal Young modulus

$$\langle E_1 \rangle = E_1^F c + E^M (1 - c) + \frac{2c(1 - c)(\nu_{12}^F - \nu^M)^2}{b_{22}^M (1 + c) + (b_{22}^F + b_{23}^F - b_{23}^M)(1 - c)} =$$

$$\begin{aligned}
&= E_1^F c + E^M (1-c) + \frac{4c(1-c)(\nu_{12}^F - \nu^M)^2 G^M}{1 + 2(1-c)G^M (b_{22}^F + b_{23}^F) + 2cG^M (b_{22}^M + b_{23}^M)} = \\
&= E_1^F c + E^M (1-c) + \frac{4c(1-c)(\nu_{12}^F - \nu^M)^2 G^M}{1 + (1-c)G^M / K_{23}^F + cG^M / K_{23}^M} = \\
&= E_1^F c + E^M (1-c) + \frac{4c(1-c)(\nu_{12}^F - \nu^M)^2 G^M}{1 + (1-c)G^M / K_{23}^F + c(1-2\nu^M)}
\end{aligned}$$

You can use any of these formulae. The last term is usually small. Compare results of calculations with a simplified formula for the “mixture rule.”

$$\langle E_1 \rangle = E_1^F c + E^M (1-c)$$

(here (1-c) –is volume part of the matrix; in your projects c=0.7, (1-c)=0.3) and

with formula

$$\langle E_1 \rangle = E_1^F c$$

which is used when fibers are significantly stiffer than the matrix.

2. Poisson’s coefficient in LT-plane (longitudinal-transversal).

$$\langle \nu_{12} \rangle = \nu^M + \frac{2c}{1 + c + \left[\tilde{\nu}^M + (b_{22}^F + b_{23}^F) / b_{22}^M \right] (1-c)} (\nu_{12}^F - \nu^M)$$

Compare the result with the result of the application of the “mixture rule.”

$$\langle \nu_{12} \rangle = \nu_{12}^F \cdot c + \nu^M \cdot (1-c)$$

3. Longitudinal-transversal shear modulus (simply called also longitudinal modulus).

$$\langle G_{12} \rangle = G^M \left[1 + \frac{2c \frac{G_{12}^F - G^M}{G_{12}^F + G^M}}{1 - c \frac{G_{12}^F - G^M}{G_{12}^F + G^M}} \right]$$

or

$$\langle G_{12} \rangle = G^M + \frac{2c(G_{12}^F - G^M)}{1 + c + (1 - c) \frac{G_{12}^F}{G^M}}$$

Compare results of calculations with result of application of the “mixture rule” for

the direct characteristic

$$\langle G_{12} \rangle = G_{12}^F c + G^M (1 - c)$$

and inversed characteristic

$$\langle 1/G_{12} \rangle = (c/G_{12}^F) + [(1 - c)/G^M]$$

or

$$\langle G_{12} \rangle = \frac{1}{\frac{c}{G_{12}^F} + \frac{(1 - c)}{G^M}}$$

Shear modulus in transversal (T-T) plane.

First of all, calculate value

$$\kappa^M = 3 - 4\nu_{23}$$

Now, we can calculate the shear modulus

$$\langle G_{23} \rangle = G^M \frac{\kappa^M + c + (1-c)G^M / G_{23}^F}{(1-c)\kappa^M + (1+c\kappa^M)G^M / G_{23}^F}$$

or

$$\langle G_{23} \rangle = G^M + \frac{4c(G_{23}^F - G^M)(1-\nu^M)}{\frac{1}{\kappa^M} + c + (1-c)\frac{G_{23}^F}{G^M}} \cdot \frac{1}{\kappa^M}$$

Compare it with calculations based on the direct and inversed "mixture rule."

$$\langle G_{23} \rangle = G_{23}^F \cdot c + G^M \cdot (1-c)$$

and

$$\langle G_{23} \rangle = \frac{1}{\frac{c}{G_{23}^F} + \frac{(1-c)}{G^M}}$$

Respectively.

5. Modulus of biaxial transversal compression \tilde{K}_{23} under condition of zero axial

strain

$$\frac{1}{\langle K_{23} \rangle} = \frac{1}{K_{23}^M} + \frac{c \left(\frac{1}{K_{23}^M} + \frac{1}{G^M} \right) \left(\frac{1}{K_{23}^F} - \frac{1}{K_{23}^M} \right)}{\frac{c}{K_{23}^M} + \frac{1-c}{K_{23}^F} + \frac{1}{G^M}}$$

or

$$\langle K_{23} \rangle = K_{23}^M + \frac{c}{\frac{1}{(K_{23}^F - K_{23}^M)} + \frac{(1-c)}{(G^M + K_{23}^M)}}$$

or

$$\langle\langle b_{22} \rangle\rangle + \langle\langle b_{23} \rangle\rangle = (b_{22}^M + b_{23}^M) + \frac{2c \left[(b_{22}^F + b_{23}^F) - (b_{22}^M + b_{23}^M) \right]}{1 + c + \left[\tilde{\nu}^M + (b_{22}^F + b_{23}^F) / b_{22}^M \right] (1 - c)}$$

Calculation of the transversal Young modulus.

Let's use now the formulae written above

$$b_{22} - b_{23} = \frac{1}{2G_{23}}$$

The sum of these elements

$$b_{22} + b_{23} = \frac{1}{2K_{23}}$$

Solving system of these two equations, we can find

$$\langle b_{22} \rangle = \frac{1}{\langle \tilde{E}_2 \rangle} = \frac{1}{4} \left(\frac{1}{\langle G_{23} \rangle} + \frac{1}{\langle K_{23} \rangle} \right)$$

$$\langle b_{23} \rangle = -\frac{\langle \tilde{\nu}_{23} \rangle}{\langle \tilde{E}_2 \rangle} = \frac{1}{4} \left(\frac{1}{\langle K_{23} \rangle} - \frac{1}{\langle G_{23} \rangle} \right)$$

From these expressions, we can find $\langle \tilde{E}_2 \rangle$ and $\langle \tilde{\nu}_{23} \rangle$ (for the plane strain

state). From these values, we can find $\langle E_2 \rangle$ and $\langle \nu_{23} \rangle$. Really

$$\langle \tilde{E}_2 \rangle = 1 / \langle b_{22} \rangle = \langle E_2 \rangle / (1 - \langle \nu_{21} \rangle \langle \nu_{12} \rangle);$$

$$\langle \tilde{\nu}_{23} \rangle = -\langle b_{23} \rangle / \langle b_{22} \rangle = \frac{\langle \nu_{23} \rangle + \langle \nu_{12} \rangle \langle \nu_{21} \rangle}{1 - \langle \nu_{12} \rangle \langle \nu_{21} \rangle}$$

or

$$\frac{\langle E_2 \rangle}{\langle \tilde{E}_2 \rangle} = 1 - \langle \nu_{21} \rangle \langle \nu_{12} \rangle$$

or after dividing the first equation by $\langle E_2 \rangle$:

$$\frac{1}{\langle \tilde{E}_2 \rangle} = \frac{1}{\langle E_2 \rangle} - \frac{\langle \nu_{21} \rangle}{\langle E_2 \rangle} \langle \nu_{12} \rangle = \frac{1}{\langle E_2 \rangle} - \frac{\langle \nu_{12} \rangle^2}{\langle E_1 \rangle}$$

or finally we have a formula for calculating the transversal Young modulus:

$$\frac{1}{\langle E_2 \rangle} = \frac{1}{\langle \tilde{E}_2 \rangle} + \frac{\langle \nu_{12} \rangle^2}{\langle E_1 \rangle}$$

Now we can calculate $\langle \nu_{21} \rangle = \langle \nu_{12} \rangle \frac{\langle E_2 \rangle}{\langle E_1 \rangle}$

and use it in the formula for $\langle \nu_{23} \rangle$:

$$\langle \nu_{23} \rangle = \langle \tilde{\nu}_{23} \rangle (1 - \langle \nu_{12} \rangle \langle \nu_{21} \rangle) - \langle \nu_{12} \rangle \langle \nu_{21} \rangle$$

We can check the results obtained by using formula for the plane of isotropy

$$\langle G_{23} \rangle = \frac{\langle E_{23} \rangle}{2(1 + \langle \nu_{23} \rangle)}$$

Calculation of the coefficients of the linear thermal expansion.

$$\langle \alpha_1 \rangle = \frac{1}{\langle E_1 \rangle} \left\{ c \alpha_1^F E_1^F + (1-c) \alpha^M E^M + \frac{2c(1-c)(\nu_{12}^F - \nu^M)(\eta_2^F - \eta^M)}{1+c + [\tilde{\nu}^M + (b_{22}^F + b_{23}^F)/b_{22}^M](1-c)} \right\}$$

$$\langle E_1 \rangle = E_1^F c + E^M (1-c) + \frac{2c(1-c)(\nu_{12}^F - \nu^M)^2}{b_{22}^M(1+c) + (b_{22}^F + b_{23}^F - b_{23}^M)(1-c)} =$$

$$\langle \eta_2 \rangle = \eta^M + \frac{2c(\eta_2^F - \eta^M)}{1+c + \left[\tilde{\nu}^M + (b_{22}^F + b_{23}^F)/b_{22}^M \right] (1-c)}$$

Here $\eta_2 = \alpha_2 + \alpha_1 \nu_{12} = \alpha_2 - \alpha_1 C_{21} / C_{11}$. From effective, $\langle \eta_2 \rangle$ it is easy to calculate $\langle \alpha_2 \rangle$ using formula written above.

Calculations of properties of the layers in the rotated system of coordinates.

Make cycle for angles φ from 0° until 90° .

Use formulae written on the lecture for the ply under $+\varphi$ and $-\varphi$ degrees, note previously, which properties are independent of the sign of the angle, and which are anti-symmetrical ones.

$$5C'_{11} = C_{11} \cos^4 \varphi + \frac{1}{2} C_{12} \sin^2 2\varphi + C_{22} \sin^4 \varphi + \frac{1}{4} C_{66} \sin^2 2\varphi$$

$$C'_{22} = C_{11} \sin^4 \varphi + \frac{1}{2} C_{12} \sin^2 2\varphi + C_{22} \cos^4 \varphi + \frac{1}{4} C_{66} \sin^2 2\varphi$$

$$C'_{33} = C_{33}$$

$$C'_{44} = C_{44} \cos^2 \varphi + C_{55} \sin^2 \varphi$$

$$C'_{55} = C_{44} \sin^2 \varphi + C_{55} \cos^2 \varphi$$

$$C'_{66} = (C_{11} - 2C_{12} + C_{22}) \sin^2 2\varphi + C_{66} \cos^2 2\varphi$$

$$C'_{12} = \frac{1}{4} (C_{11} + C_{22} - C_{66}) \sin^2 2\varphi + C_{12} (\cos^4 \varphi + \sin^4 \varphi)$$

$$C'_{13} = C_{13} \cos^2 \varphi + C_{23} \sin^2 \varphi$$

$$C'_{23} = C_{13} \sin^2 \varphi + C_{23} \cos^2 \varphi$$

$$C'_{16} = \left[C_{22} \sin^2 \varphi - C_{11} \cos^2 \varphi + \left(C_{12} + \frac{1}{2} C_{66} \right) \cos 2\varphi \right] \sin 2\varphi$$

$$C'_{26} = \left[C_{22} \cos^2 \varphi - C_{11} \sin^2 \varphi - \left(C_{12} + \frac{1}{2} C_{66} \right) \cos 2\varphi \right] \sin 2\varphi$$

$$C'_{36} = (C_{23} - C_{13}) \sin 2\varphi$$

$$C'_{45} = \frac{1}{2} (C_{44} - C_{55}) \sin 2\varphi$$

$$C'_{14} = C'_{15} = C'_{24} = C'_{25} = C'_{34} = C'_{35} = C'_{46} = C'_{56} = 0$$

These formulae show that if we cut samples under different angles on respect to the natural axes of symmetry and we test them, we must receive polar diagrams of properties such as Young modulus not arbitrary ones, but corresponding this law.

The same matrix can be used for calculation of components of tensors of the second rank, such as tensor of thermal conductivity λ_{ij} or tensor of linear thermal expansion α_{ij} in rotated system of coordinates; for example, for an orthotropic material having three independent components after rotating around axis x_3 :

$$\alpha'_1 = \alpha_1 \cos^2 \varphi + \alpha_2 \sin^2 \varphi$$

$$\alpha'_2 = \alpha_1 \sin^2 \varphi + \alpha_2 \cos^2 \varphi$$

$$\alpha'_3 = \alpha_3$$

$$\alpha'_6 = (\alpha_2 - \alpha_1) \sin 2\phi$$

For the team 2 using these formulae not needed: if the property of the ply A contains index one then the corresponding property of the ply B will be the same as the property of the ply A, which has index two at the same place, where was index 1, for example $C_{11}^B = C_{22}^A$.

For the team three, the same as for the team two plus you need to find properties of diagonal pairs of layers. It can be done by the same way as for the team 1, but you have a special case $\pm 45^\circ$. Or it can be done from the properties of the orthogonal reinforced material 1:1 by a rotating system of coordinates by $\pm 45^\circ$. [11]

Effective composite properties calculated via properties of matrix and fibers for 70% of fibers per volume.

The combined properties obtained after running the program are obtained,

Properties	Glass/Epoxy	Kevlar/Epoxy	Carbon/Epoxy
E1(GPa)	61.401	108.276	210.9
E2(GPa)	13.454	3.814	7.744
v12	0.259	0.34	0.3
v23	0.436	0.246	0.3
G12(GPa)	5.362	2.119	3.608
G23(GPa)	4.685	1.531	2.978
Density(g/cc)	2.6	1.3	1.55

Table 5-2 combined fiber matrix properties

Chapter 6

Simulations

The whole ANSYS procedure can be divided into three steps. They are:

- Preprocessing: The model is created, and elements and mesh are assigned.

The material properties are also given as input.

- Solution: The loads and the boundary conditions are given. All the control like step size, solver, etc. are given in this step. The solution is obtained.

- Post-processing: The results are reviewed, and the desired results are evaluated and mapped.

ANSYS Composite PrepPost is used for analyzing composite set up layer by layer. Engineering layered composites involves complex definitions that include numerous layers, materials, thicknesses and orientations.[12] A positive or negative angle can be given to the layers or layer at 90 degrees can be set as per our requirement. Multiple layers of different orientation can be set one below the other for a required thickness.

The engineering challenge is to predict how well the finished product will perform under real-world working conditions. Simulation is ideal for this when considering stresses and deformations as well as a range of failure criteria. ANSYS Composite PrepPost software provides all necessary functionalities for finite element analysis of layered composite structures.

You can choose to work with either shell theory (thin-composites) or move to modeling solid composites in the case of thicker parts [13]. In addition, fabrics can be assigned to the layers. An example of the layer setup is shown below,

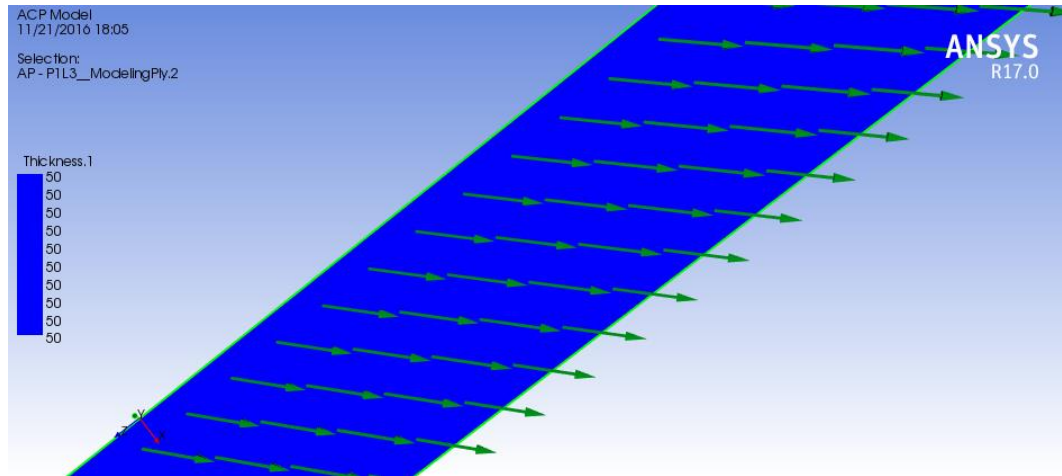


Figure 6-1 Box Beam 45 degrees layup

Figure 6-1 shows a box beam which is at 45 degrees. Similarly, set up can be done for any angle we require. The layer set up direction is fixed. It can be done for almost all geometries. Though doing it for the box beam is the easiest. We can also decide the direction in which layers should be set so that we have the required thickness. It is set as shown in figure 6-2.

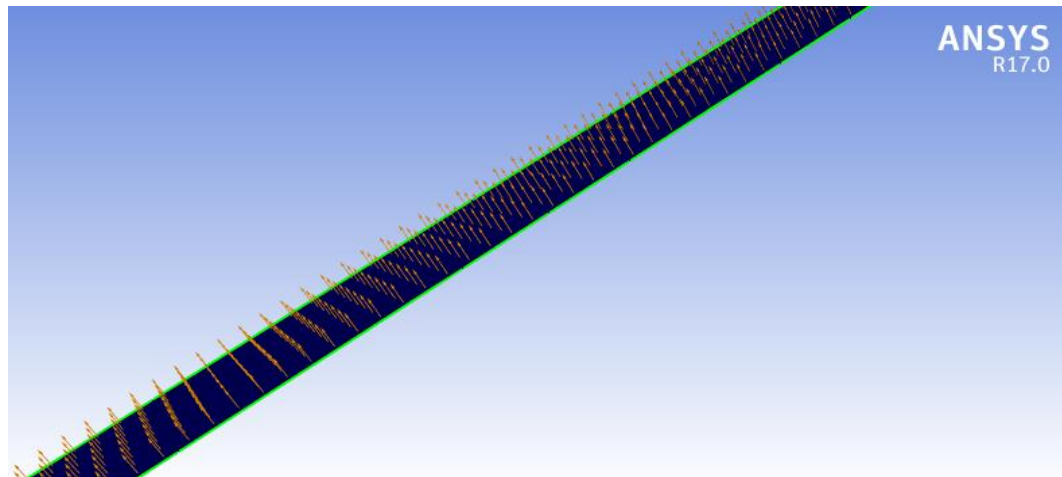


Figure 6-2 Lay-up direction

When it comes to I, beam a normal lay-up cannot be done. A straightforward layup of layers there will not be any edges for bonding or adhesion. In metals, straightforward edges can be joined by welding or by riveting. However, when it comes to composites, layers need to be laid one by one to bond. Hence, when making an I-beam with the help of the composites, the process is different. Here, two C-sections are made on opposite sides. Hence, two opposite C's meet each other and form a rib for the I-beam. Moreover, the top and bottom faces are laid to bond everything and complete the I-beam.

The layers are set according to the method explained above even in ANSYS PrepPost. Figure 6-3 shows the lay-up directions of the I-beam that we have done.

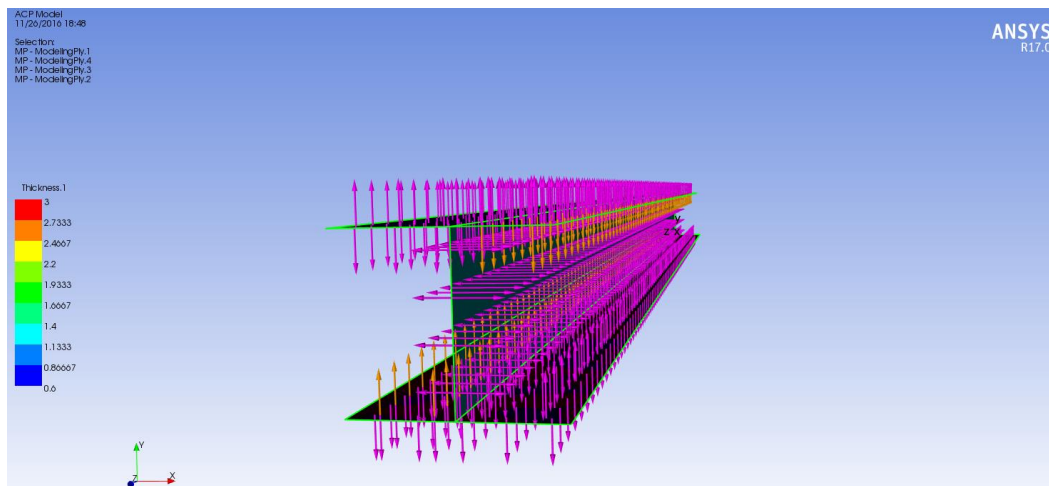


Figure 6-3 I-beam lay-up

This is done for the composite materials of Carbon and Glass fibers. Similarly, simulations are done in ANSYS Workbench with boundary conditions explained in earlier chapters for Aluminum, Carbon Epoxy, Glass Epoxy and Kevlar Epoxy.

Chapter 7

Results

The simulations are run for all the Aluminum, Carbon Epoxy, Kevlar Epoxy and Glass Epoxy as mentioned in the previous chapters. In these simulations, the boundary conditions specified earlier are used. Also, all the materials and boundary conditions are used on all the three beams under study viz. I-beam, box beam and double I-beam.

The results obtained from the simulations are discussed below:

7.1 Deformation:

Deformations for all the cases are found out, and comparisons are found out.

The deformations for Aluminum are as shown below,

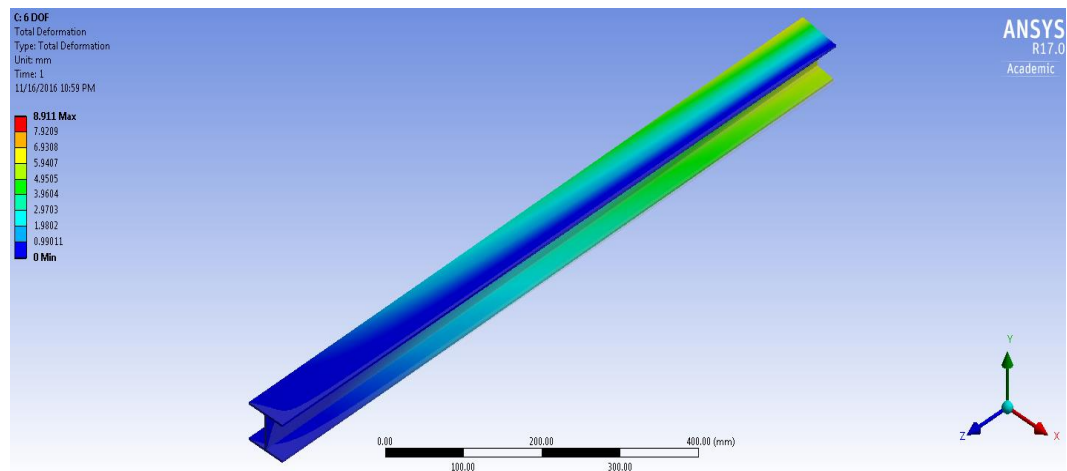


Figure 7-1 Deformation in Aluminum I-beam 6 DOF

7.2 Stresses:

Similar to total deformation, equivalent stresses are also calculated for all the cases. They can be shown as below,

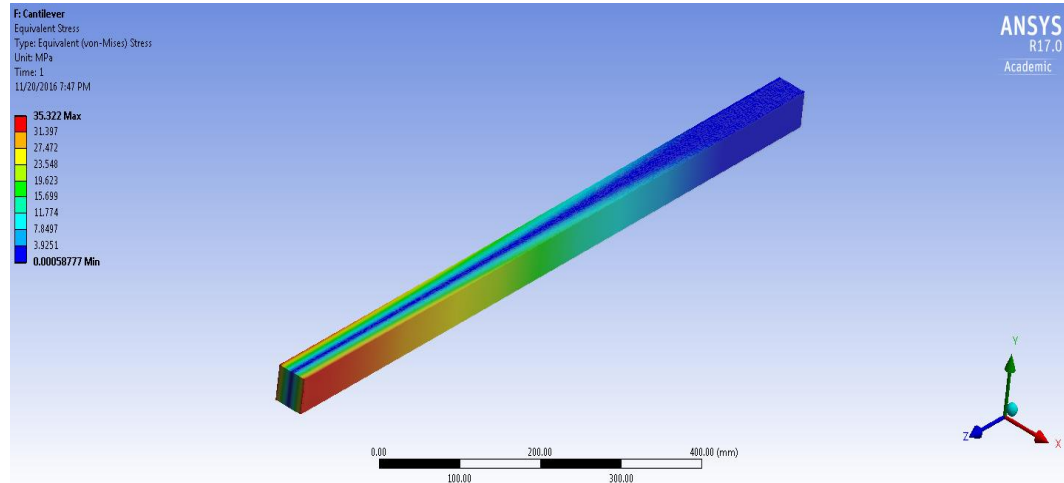


Figure 7-2 Stresses in Carbon-Epoxy box beam cantilever

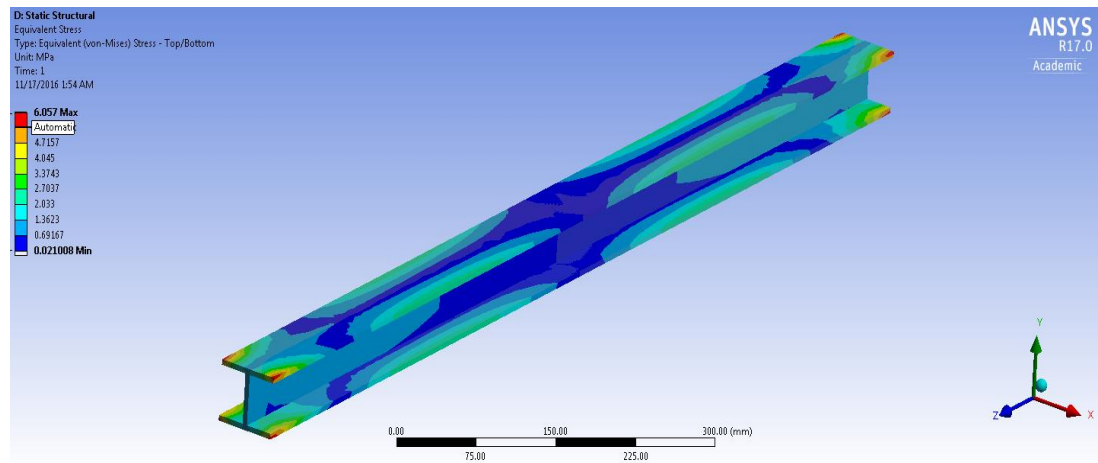


Figure 7-3 Stresses in Glass Laminates I-beam clamped

7.3 Torsional Results:

Similar to the stresses and deformations in Workbench, the torsional analysis was done on ANSYS APDL. The final outputs were as shown below,

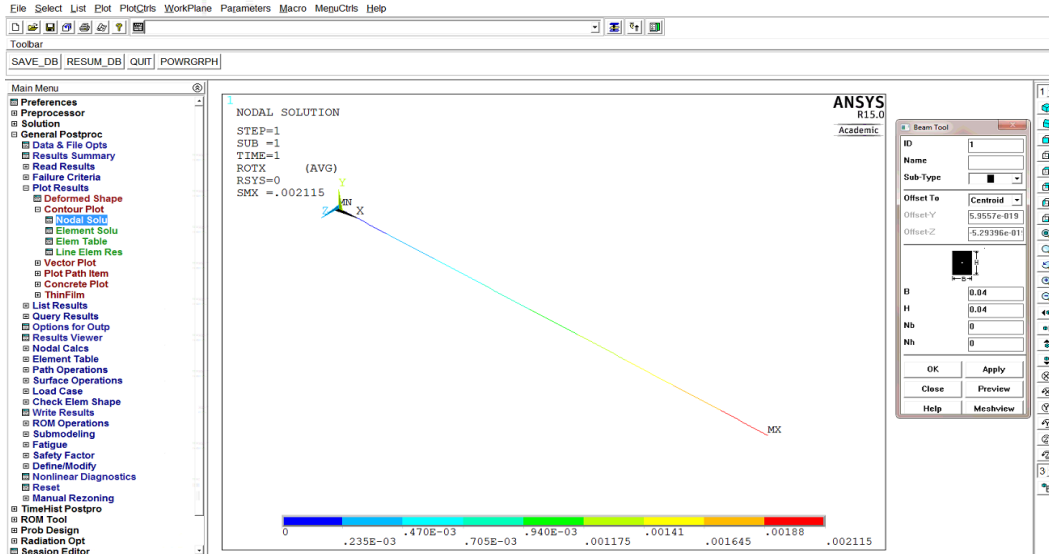


Figure 7-4 Torsion in Aluminum Cantilever case

Analytical, as well as the APDL results, were found out. The difference between the two was very low. Both the results are shown below,

Material	Shear Modulus (GPa)	Angular displacement (Radians)	Angular displacement (Degree)	Shear Modulus (GPa)	Angular Displacement (Radians)	Angular Displacement (Degree)
Carbon Epoxy	3.608	0.014	0.809	3.608	0.013	0.744
Kevlar Epoxy	2.119	0.024	1.378	2.119	0.022	1.267
Glass Epoxy	5.362	0.005	0.311	5.362	0.0044	0.252
Aluminum	26	0.00196	0.1123	26	0.0018	0.1033
Glass Epoxy	5.362	0.0057	0.3265	5.362	0.0047	0.2692
Aluminum	26	0.00201	0.1152	26	0.002	0.11459

Table 7-1 Results of torsional analysis

7.4 Buckling Results:

Buckling analysis was also done for the beams. The load factors were calculated for each case.

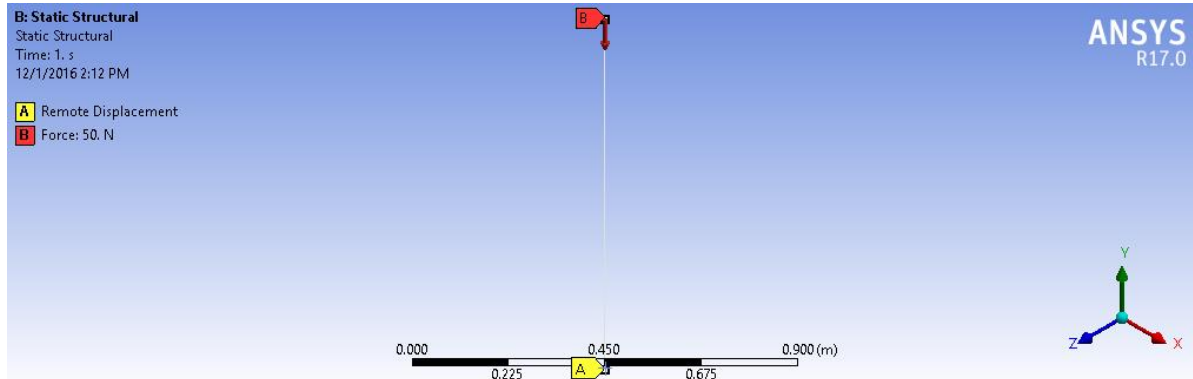


Figure 7-5 Buckling

Material	I beam		Double I-Beam		Box Beam	
	X direction	Z direction	X direction	Z direction	X direction	Z direction
Aluminum	183.38	421.99	313.89	469.89	362.3	362.3
Carbon Epoxy	65.347	65.347	924.41	1341.2	1080.2	1085.3
Kevlar Epoxy	33.595	33.595	476.88	688.64	554.62	558.79
Glass Epoxy	102.8	102.8	277.99	414.15	321.56	321.76

Table 7-2 Load multipliers

7.5 Deformation and Stress Result Tables:

We have obtained results for all deformation and stresses for each case. The results are as shown below:

I-Beam							
	Geometry	Aluminum 6061 T6	Glass Epoxy	Kevlar Epoxy	Carbon Epoxy	Glass Laminates	Carbon Laminates
Displacement(mm)	Cantilever	16.485	50.601	11.455	5.8557	9.3952	5.9161
	6 DOF	8.911	10.624	7.203	4.129	2.4236	1.2127
	Clamped	0.054348	0.14856	0.37457	0.19054	0.027557	0.21401
Stresses(MPa)	Cantilever	126.5	64.663	121.53	120.85	58.827	117.49
	6 DOF	4290.2	3307.8	10761	10906	67.126	4482.2
	Clamped	16.454	8.7662	36.633	37.117	6.057	28.922

Table 7-3 I beam Deformation and Stresses

Box Beam							
	Geometry	Aluminum 6061 T6	Glass Epoxy	Kevlar Epoxy	Carbon Epoxy	Glass Laminates	Carbon Laminates
Displacement(mm)	Cantilever	8.3194	17.445	5.5338	2.8113	3.2449	2.8036
	6 DOF	0.10587	0.22215	0.083327	0.042294	x	1.5879
	Clamped	0.022194	0.04757	0.031919	0.016322	0.0089821	0.015519
Stresses (MPa)	Cantilever	52.445	28.592	35.87	35.322	16.92	32.984
	6 DOF	19.031	13.821	19.907	19.989	x	18.115
	Clamped	5.6461	3.1448	0.031919	6.5729	1.5622	3.4229

Table 7-4 Box beam Deformation and Stresses

Double I-Beam							
	Geometry	Aluminum 6061 T6	Glass Epoxy	Kevlar Epoxy	Carbon Epoxy	Glass Laminates	Carbon Laminates
Displacement(mm)	Cantilever	10.399	32.351	7.5413	3.8767	5.7911	3.14
	6 DOF	19.255	11.926	6.787	2.959	0.15508	2.173
	Clamped	0.035672	0.10016	0.18259	0.095116	0.018101	0.1504
Stresses (MPa)	Cantilever	83.147	48	78.509	78.483	36.854	76.331
	6 DOF	9579.6	3816.8	11375	11488	20.107	3773.3
	Clamped	12.6	6.7337	28.39	28.446	4.2602	19.989

Table 7-5 Double I-beam Deformation and Stresses

7.6 Deformation and Stress Result Graphs:

These results can be better interpreted by showing them in graphs. Following are the graphs for both deformation and stress cases:

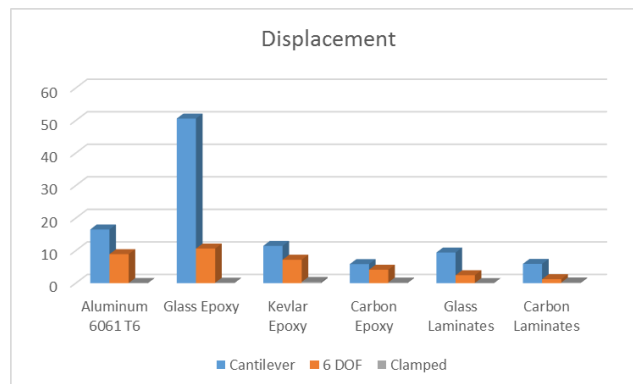


Figure 7-6 Deformation in I-beam

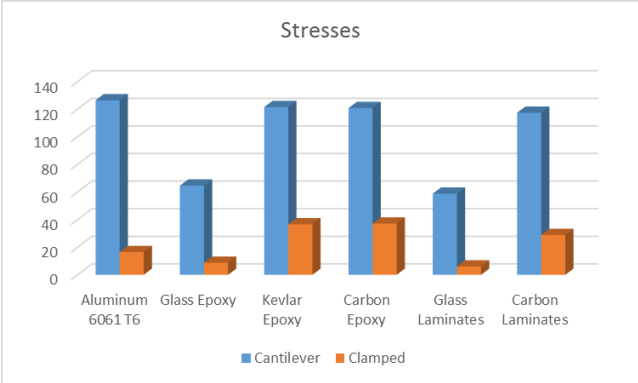


Figure 7-7 Stresses in I-beam

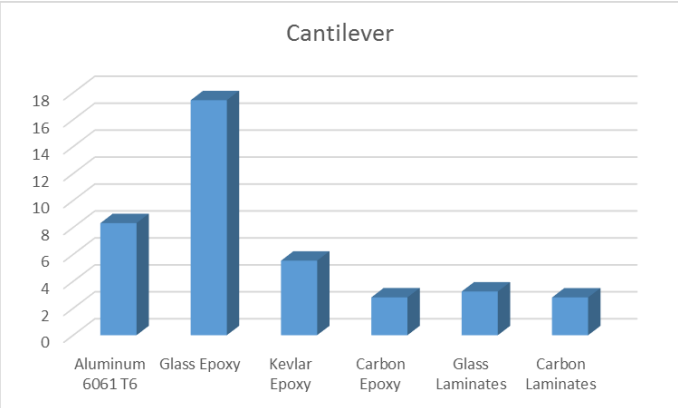


Figure 7-8 Deformation in box beam (Cantilever)

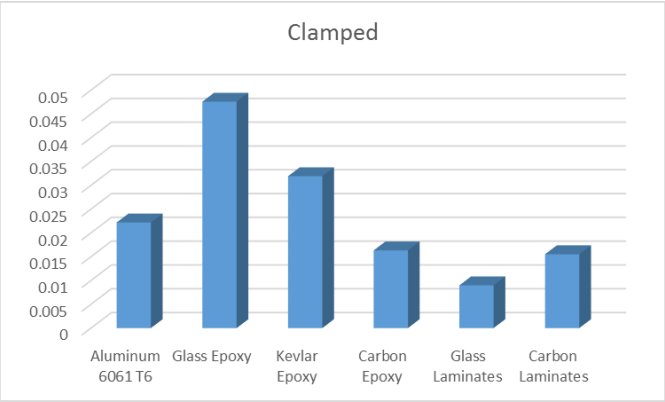


Figure 7-9 Deformation in box beam (Clamped)

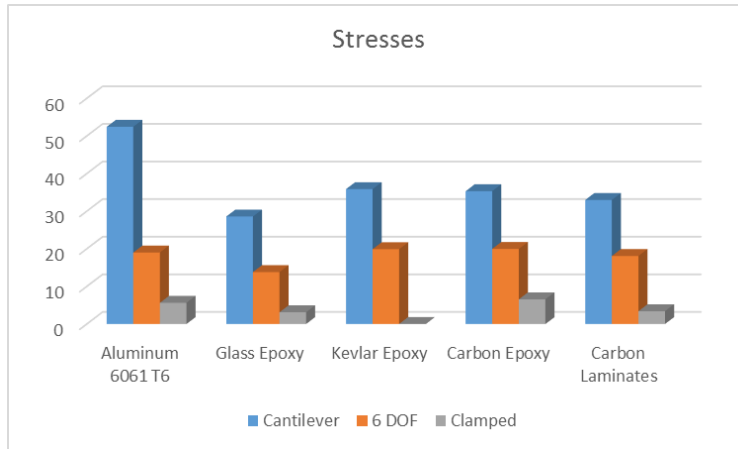


Figure 7-10 stresses in box beam

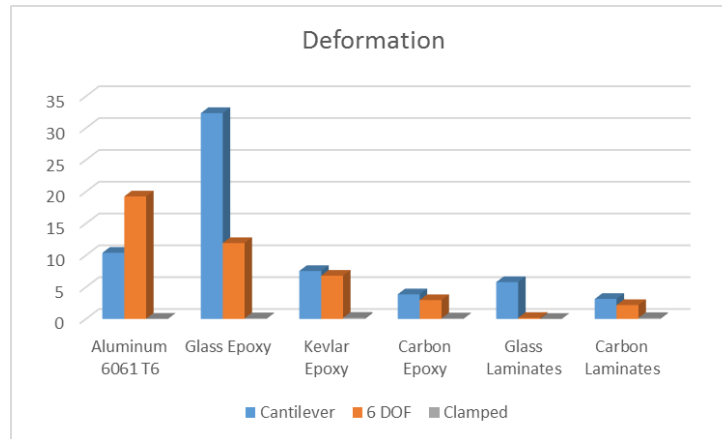


Figure 7-11 Deformation in Double I beam

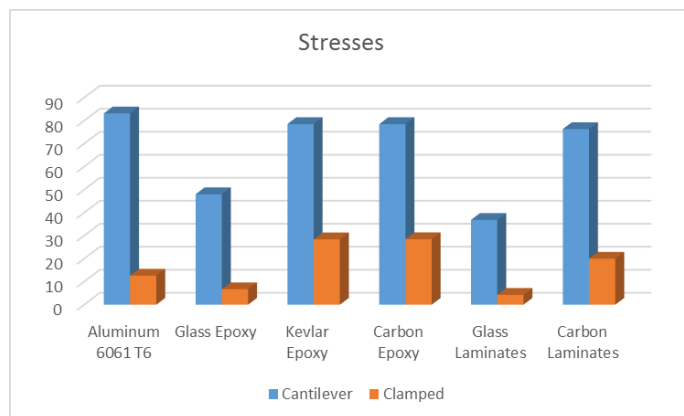


Figure 7-12 stresses in Double I beam

7.7 Weight difference:

There is a huge difference between all the materials used for the comparison. They have a different composition, different densities, and hence different weight. A weight comparison of all the materials along with an equivalent graph is shown below:

Cross Section	Aluminum 6061-T6	Glass Fiber Epoxy	Carbon Fiber Epoxy	Kevlar Epoxy
I beam	1.3284	1.696	0.7626	0.6396
Box Beam	4.32	5.3	2.48	2.08
Double I beam	1.7334	2.26	0.9951	0.8346

All weights are in kg*

Table 7-6 Weight difference

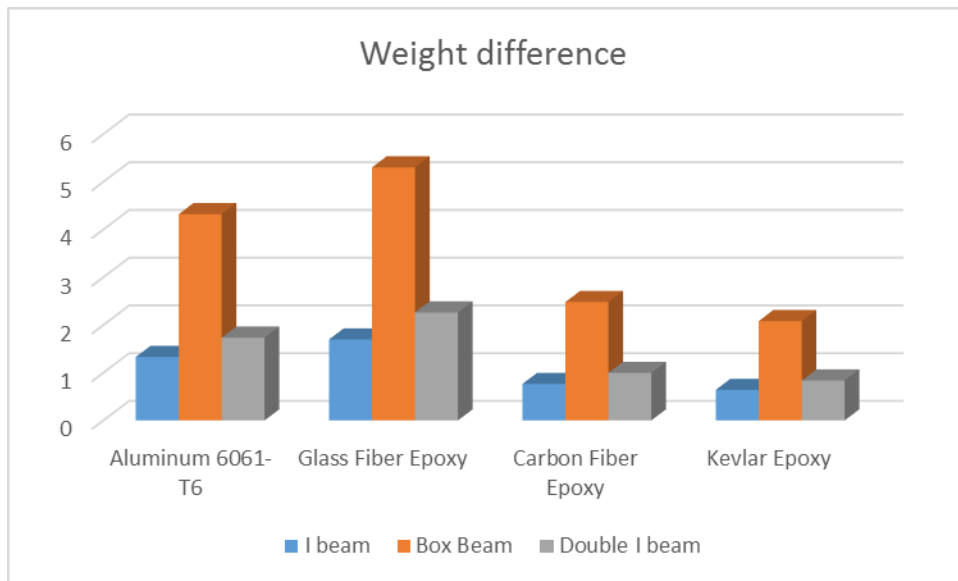


Figure 7-13 Weight difference of beams

Chapter 8

Conclusion

From the results obtained, it is clear that from all the materials used for the study, Carbon Epoxy and Kevlar Epoxy provide the highest strength and stiffness as compared to other materials. These two have low densities and hence are the lightest options. It is observed that Glass Epoxy is not strong or stiff as others. It is also heavier. However, it provides us with the lowest stresses. Which means it can handle more stresses than any other material in our study.

When it comes to the cross sections, box beam shows the lowest deformation and stresses, but they are the heaviest. I beam the lightest option but has the lowest strength and stiffness. Whereas, double I-beam provides good strength and stiffness along with moderate weight. Hence, for our application, double I-beam seems like the best cross section to work with.

Buckling and torsional analysis help us validate our case even more. Buckling analysis shows us that Carbon and Kevlar have the highest load multipliers, hence are better materials for our study.

Aluminum I beam profile better resists to torque. To make composite profiles competitive, it is necessary either to increase percentage of diagonal layers (what will decrease longitudinal and bending stiffness and strength) or to replace open contour profile by close contour profile or to combine these methods together. Box beam and double I composite beams have to have torsional stiffness comparable with aluminum I-beam. However, they have to be lighter than aluminum I beam and provide much higher bending strength and stiffness.

According to the findings, we can rank the materials as shown below,

Property	Glass Fiber	Kevlar Fiber	Carbon Epoxy
Stiffness	Lowest	Moderate	Highest
Cost	Lowest	Moderate	Highest
Weight	Highest	Lowest	Moderate
Stresses	Highest	Moderate	Lowest

Table 8-1 Material Ranking

Future Work

A few suggestions can be made to further the study of this topic,

- Analysis of stringer along with skin can be done with the same loads, hence giving us better boundary conditions.
- Stringer-Frame interaction can be studied as a whole. Also, intricate design parameters can be found out. Also, a complete stringer-frame-skin assembly can be studied.
- Analysis can be done on various other stringer cross-sections like J-stringer, hat stringer, formed hat and Z-stringer can be done along with the results obtained from the current cross-sections.
- Fiber-Matrix concentration has been taken as 0.7 in our case. It can be varied to 0.5, 0.6, 0.65 and the results can be compared.

References

- [1] Nikesh Samarth | Institute of Chemical Technology, Mumbai (n.d.). Retrieved from <http://ictmumbai.academia.edu/NikeshSamarth>
- [2] Prajwal Ambekar - Google+. (n.d.). Retrieved from <https://plus.google.com/104962146215261396286>
- [3] <http://www.aid-n.com/what-are-composite-materials-and-the-function/composite-materials-development/>
- [4] I-Beam - Sectional Properties Calculator. (n.d.). Retrieved from <http://www.amesweb.info/SectionalPropertiesTabs/SectionalPropertiesIbeam.aspx>
- [5] Solid Rectangular Bar - Sectional Properties Calculator. (n.d.). Retrieved from <http://www.amesweb.info/SectionalPropertiesTabs/SectionalPropertiesSolidRectangl>
- [6] Ilhan Sen, Aircraft Fuselage Study, 2010
- [7] <http://interiorflat.com/paintable-wallpaper-which-is-better/>
- [8] https://www.aliexpress.com/price/nomex-fiber_price.html
- [9] <http://www.sailingscuttlebutt.com/2013/10/17/addressing-environmental-impact-carbon-fiber/>
- [10] Bending, Shear and Torsion - aerostudents.com. (n.d.). Retrieved from <http://www.aerostudents.com/files/aircraftStructuralAnalysis/bendingShearAndTors>
- [11] Dr. Andrey Beyle, Personal Conversation.
- [12] Tutorial 86: Full ANSYS ACP tutorial - GrabCAD. (n.d.). Retrieved from <https://grabcad.com/tutorials/tutorial-86-full-ansys-acp-tutorial>
- [13] ANSYS Composite PrepPost: Composites Simulation. (n.d.). Retrieved from <http://ansys.com/Products/Structures/ANSYS-Composite-PrepPost>
- [14] Composite properties from S.Peters (Ed.) Handbook of Composites

[15] Mechanics-of-Composite-Materials-R-Jones

[16] Aircraft Structures – Djpeery

[17] http://www.performancecomposites.com/carbonfibre/mechanicalproperties_2.asp

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