

EFFECTIVE THERMAL CONDUCTIVITY OF POROUS MEDIA BY
INVERSE HEAT TRANSFER ANALYSIS

by

OSCAR JAVIER FABELA II

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Abstract

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Oscar Javier Fabela II, M.S.

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Supervising Professor: Brian H. Dennis

This research describes the application of inverse analysis to estimate the thermal conductivity of a porous material using temperature measurements at the surfaces. Finite volume analysis code is implemented in MATLAB and it is utilized to solve the steady state, axisymmetric heat transfer governing energy equation.

The analysis code is coupled with a numerical optimization method and utilized to predict thermal conductivity using the measurements. An experimental setup was developed to test a porous media composed of a mixture of small pellets and air. In experiments, temperature is measured at outer surface of copper tube. These data, serves as input for the inverse analysis. Inside this copper tube, an electric heater is placed at center along with porous mixture at annular gap. Location of the heater is affirmed centrally by special manufactured fixtures and

by being attaching at both ends. In addition, thermocouples were installed on the surface of the heater to measure the temperature. Validation of the finite volume code was carried out with ANSYS[®] and with experimental data. Here, Semi Analytic Complex Variable Method (SACVM) is used to determine the sensitivity of the predicted material properties on error in temperature measurements. This information is used to determine the locations on the surface of the copper tube to take temperature measurements, so as to reduce the effect of measurement errors on the inversely determined parameter. Temperature data at our equidistant locations on copper tube are used as input to the inverse analysis code. The objective function is minimization of least squares error between measured experimental and predicted temperatures. Conjugate Gradient Method (CGM) is used to solve the resulting problem. Accurate sensitivities for the CGM were computed by SACVM. A heater with diameter of 0.0128 m, and a copper tube with a diameter of 0.0296 m are used for these experiments. Top and bottom of heater and copper tube are insulated to force radial direction heat transfer. Convective heat transfer coefficient is assumed as $8 \text{ W/m}^2 \text{ K}$ and ambient temperature is 295 K.

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Chapter 1

Introduction

1.1 Problem Statement

One of the biggest constraints in the Aerospace field is heating. Because of this some materials cannot withstand the high temperatures required for systems to function more efficiently and some of them to have better performance. It is known that when a new aircraft is designed, its components have to be designed working around the temperature limit of the materials which are going to be used [1]. Such materials have to withstand the high stress at high temperature, as what happened on the tragic accident of the Space Shuttle Columbia in which the thermal protection system, or heat shield, failed causing structural damage to the left wing, leading to losing the entire wing while reentry.

It is because of this and many other reasons that scientist and engineers are developing new materials every day that are stronger, light weighted, and can withstand higher temperatures, to accommodate and overcome these problems. But the physical properties of these new materials are, in most cases, unknown; therefore, more effective ways of determining the new materials properties are needed to be developed.

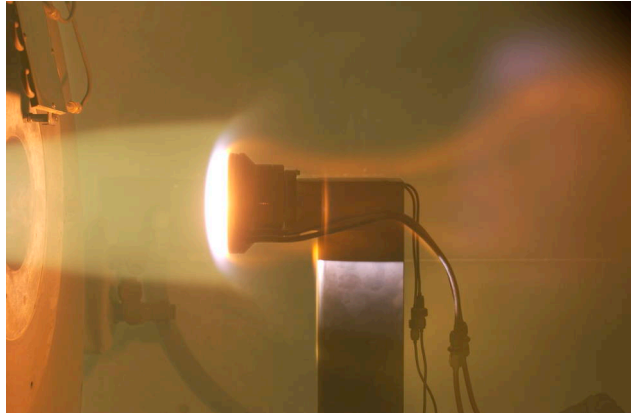


Figure 1-1: Heat Shield Tested on Arc Jet

Many researchers have found methods of finding these unknown properties efficiently, in this study the thermal conductivity of materials is considered; and will be obtained by using inverse analysis, therefore, the focus of this research is to find the thermal conductivity of porous media.

1.2 Conductive Heat Transfer

Heat is considered as the energy exchange from 2 or more particles from a higher temperature to lower temperature. Temperature can be defined as a scalar form of measuring the energy of particles at atomic level, which can be caused from vibrations or electronic states of such atoms [2].

In this case the specific mode of heat exchange is conduction, which is energy transfer that takes place in fluids or solids.

Fourier's law is a relation that governs the interaction between the flow of heat and the temperature field. Given by equation

$$q_{cond} = -k\nabla T \quad (1)$$

Where:

q_{cond} is the heat flux per unit area

k is the thermal conductivity

∇T is the temperature gradient

Heat flux is the rate of heat energy transfer through a given surface. Each material has its own thermal conductivity, which is a property of matter that allows or prevents heat transfer. The temperature gradient is a vector normal to the surface.

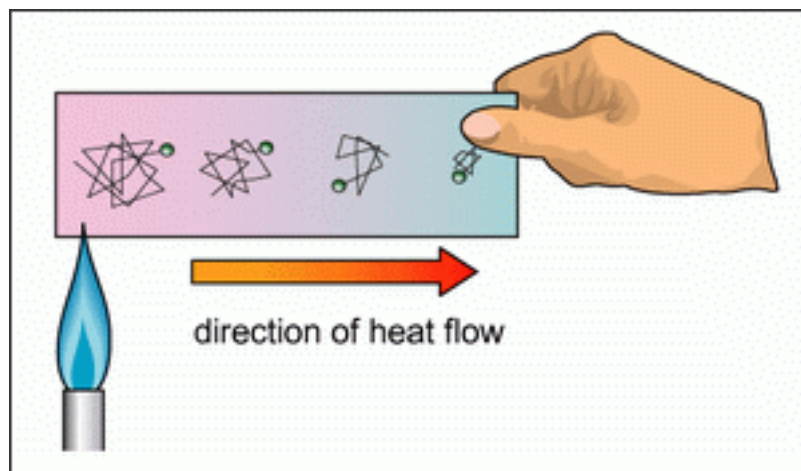


Figure 1-2: Representation of Fourier's law of heat conduction

1.3 Porous Media

Porous media is considered a material or mix of materials, which consist of a solid matrix with an interconnected void, it is assumed that the matrix is rigid or it might be slightly deformed. This voids (pores) allow the flow of one or more fluids, this is called a "single-phase flow" [3].

Porosity or void fraction is a quantity that is determined by three factors; the particle shape that implies that as the particles of the solid phase are rounded, will have a higher porosity than particles that have flat surfaces. The degree of packing also affects the porosity; a tightly packed solid particles will yield a lower porosity. The last factor that affects the porosity is the degree of sorting, this means if the particles are overall the same size will have greater porosity and if they have different shape and size porosity will decrease.

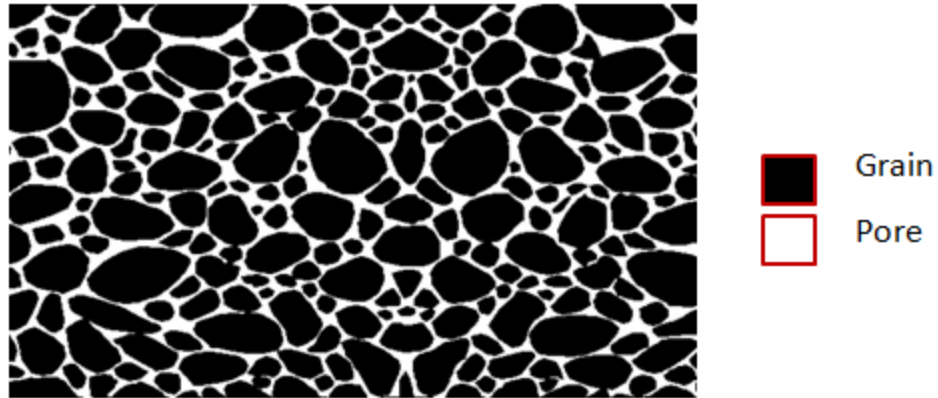


Figure 1-3: Representation of porous media

1.4 Inverse Analysis

A physicist, Viktor Ambartsumian first introduced inverse analysis in 1929 for relation between oscillation frequencies and homogenous string vibration. It was not until later years that his approach was used for other kind of problems. The next 15 years of this paper remained in obscurity and nobody has noticed this paper, then after 1944, many outstanding mathematicians succeeded in obtaining interesting results [4]. One of such results was published by G. Borg, who worked on inverse of Sturm-Liouville Equation in 1946 [5]. Different developments had been taken place over the years in mathematics of inverse problem since then. I. Frank [6] developed least square method for inverse problem. A. Tikhonov [7,8] was well known for introducing regularization methods, D. Specht [9] introduced general regression neural network for

inverse problems. Another recent breakthrough research work was carried out by K. Deb in group in developing elitist multi-objective genetic algorithm [10].

An inverse analysis problem is basically the process of calculating the causal factors from a set of observations. In another words, one has the results of a problem and goes back to figure out what were the reasons, factors or parameters that caused such result. Inverse analysis is very important in mathematics and in science because with this method one can find metrics or parameters that are not easily observed. Another reason is that sometimes it is very difficult or even impossible to use a temperature probe or a strain gauge in some body geometries, or even the environment of where measurements are needed to be performed. Inverse analysis is widely used in many fields that go from oceanography, weather prediction, to structural and chemical analysis.

1.5 Inverse heat transfer problems

Inverse Heat Transfer Problems (IHTP) are widely used to find and estimate unknown properties or quantities in the analysis of a specific problem in thermal science, it relies on temperature measurements or heat fluxes to find the causes of the phenomena. The need of IHTP has risen in recent years due to the fact that as

technology advances, more heat problem arise that goes from metallurgy to space flight. Which in many instances those systems are operating at the thermal limit of materials, and even sometimes the thermal experiments can become very costly in some occasions limited to only one try, these are the cases in which IHTP are very important and even sometimes the only option.

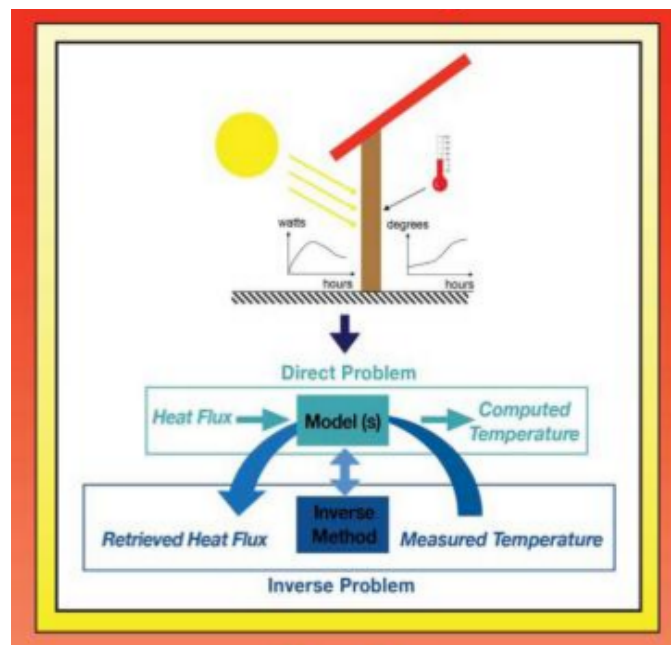


Figure 1-4: Inverse Problem Representation
 (H.R.B. Orlande, O. Fudym, D. Maillet, R.M. Cotta, 'Thermal Measurements and Inverse Techniques', CRC Press, Taylor & Francis Group)

In this study, is presented the implementation of an experimental analysis to obtain data that will be utilized as input for an inverse analysis numerical formulation to

estimate the thermal conductivity on porous media. This will be accomplished by carrying out experiments with a copper tube that will contain in the center a smaller heating rod, and in the annular gap, a mixture of porous material will be enclosed, insulating the top and bottom of this tube to enforce radial heat transfer. Temperature measurements will be taken at the surfaces of the heating rod and on the outer surface of the copper tube, experimental arrangement can be viewed in Figure 1-5. A finite volume code was developed in order to simulate temperature measurements on a cylindrical configuration; these values are to be compared with the experimentally obtained measurements. The temperature values are then utilized for the inverse problem formulation and they are analyzed using Complex Variable Semi Analytical Method to compute the sensitivities of the calculated temperature measurements, and a Conjugate Gradient Method is utilized to find the solution in this study due to its low computational time.

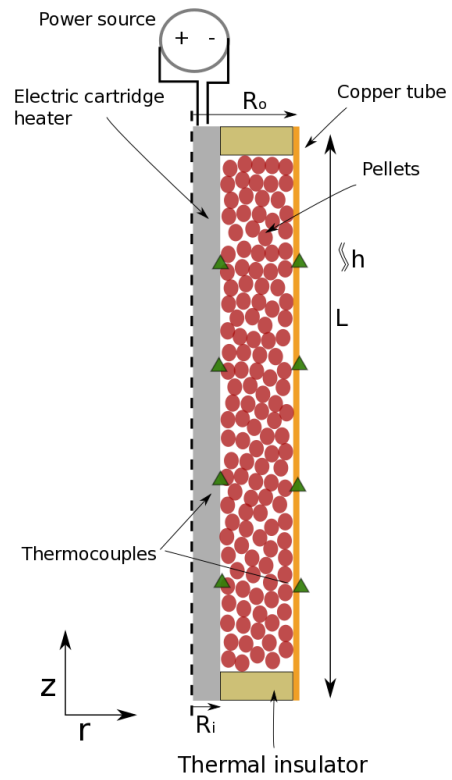


Figure 1-5: Schematic of Axisymmetric Experimental Arrangement

Chapter 2

Literature Review

Inverse analysis is widely utilized to find and estimate unknown properties, using different known properties such as temperature or heat fluxes, which are then used as boundary conditions for the problem formulation. Inverse heat transfer problem is primarily used to determine such unknowns, e.g. thermal properties, boundary conditions even geometry of the body.

Many researchers have used IHTP to find the solution for their problems, most of the problems can be considered in two phases, which are the analysis and the process of optimization. The analysis process some of the parameters are assumed and then the results are now obtained by numerical methods, e.g. finite element method, finite volume method. Now in the process of optimization, which is the phase that will calculate the parameters that are going to be estimated, are calculated by many methods, e.g. least-squares method, conjugate gradient, that will guide the solution to the for the new unknown parameters. This is an iterative procedure that will continue until certain stop criterion exists.

Even though usually all the inverse analysis problems require some degree of iteration, Lin et al [11] were able to solve a one-dimension heat conduction problem by replacing the non-linear least-squares method with a linear inverse model in which the

thermal conductivity is represented explicitly; so this way the iterative calculations and the analysis and optimization could be eliminated.

Huang et al [12] demonstrated that using conjugate gradient method is a valid and accurate way to obtain the thermal conductivity and the heat capacitance by solving an inverse problem. Yang et al [13] instead of using the known linearization methods, they investigated a new method of linearize the nonlinear problem, they used a linear least-squares error method to estimate the thermal conductivity, which in their research was temperature dependent. Skorek et al [14] were able to use inverse transfer analysis but in their research, they were able to find the boundary conditions of a transient temperature field with heat flux did. As well as Yang did, they took a different approach from the usual, they used a method that they called Enhanced Least Squares Adjustment Method which yields a higher stability for the ill posed problem, by using a priori information from the problem, they modified the covariance matrix which is the matrix containing the initially estimated unknowns by making its elements smaller, giving a higher accuracy.

Some other researchers have done their analysis on finding the thermal conductivity of materials on different ways, Nan et al [15], were able to find the thermal conductivity of composite materials by using effective medium approximation. Another researcher, S.M. Marcus [16] was able to obtain the thermal conductivity of polymers, glasses and ceramics, by using modulated Differential Scanning Calorimetry, which is a thermal analysis technique that measures heat flow into or out of a material as a function of temperature of time. Temperature is measured from a reference material

and a sample material that are placed in a thermos-electric disk at a set temperature and the heat flux is compared, as shown in Figure 2-1.

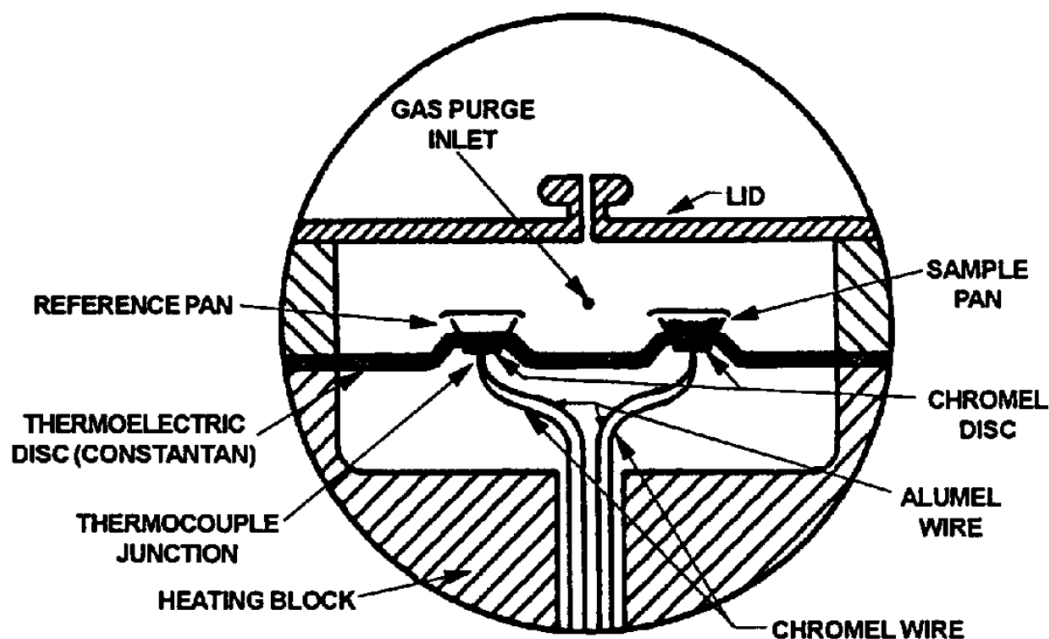


Figure 2-1: Schematic of the heat flux DSC.

It is seen that there are different approaches to obtain the thermal conductivity of different types of materials, now a review of the methods for obtaining the thermal conductivity of porous media will be presented. One very known porous media is sand, which Woodside et al [18] did a study on determined the thermal conductivity of unconsolidated sands by using a transient line heat source which is an experimental

thermal conductivity probe. Kunii [19] performed a theoretical study of the effective thermal conductivity of porous rocks, finding that the contact point of each particle plays a key role in the heat transfer of the porous media. Nozad et al [20] used a transient experimental method along with volume averaging to obtain the effective thermal conductivity of two and three phase systems. Some other researchers, Prasad et al [21], conducted studies on liquid-saturated porous beds and were able to obtain the thermal conductivity of such mixture by carrying out experiments to obtain temperature measurements and that way calculating the thermal conductivity.

As seen from many researchers, inverse heat transfer analysis is a very important field of study, but it is demonstrated that there are other different methods to obtain the thermal conductivity of some materials. And as is present review, some effort has been put into finding the thermal conductivity of porous media.

Chapter 3

Methodology

3.1 Governing Equation

Here and as in any other physical problem, the first law of thermodynamics is applied, which is the conservation of energy takes; this law has to be satisfied at every point of any problem. Which is given by the equation

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E} \quad (2)$$

Rate of energy added + Rate of energy generated – Rate of energy loss = Rate of energy change in the element

In order to obtain the differential form of the energy equation, it must be based on an infinitesimal element. Now the expression has to be rewritten in terms of T, as well some assumptions have to be taken, (1) changes in potential energy are negligible, (2) density is constant, (3) velocity is uniform, (4) pressure is constant, (5) effective thermal conductivity is constant, with these assumptions now it is possible to describe the infinitesimal element, described by Figures 3-1 and 3-1.

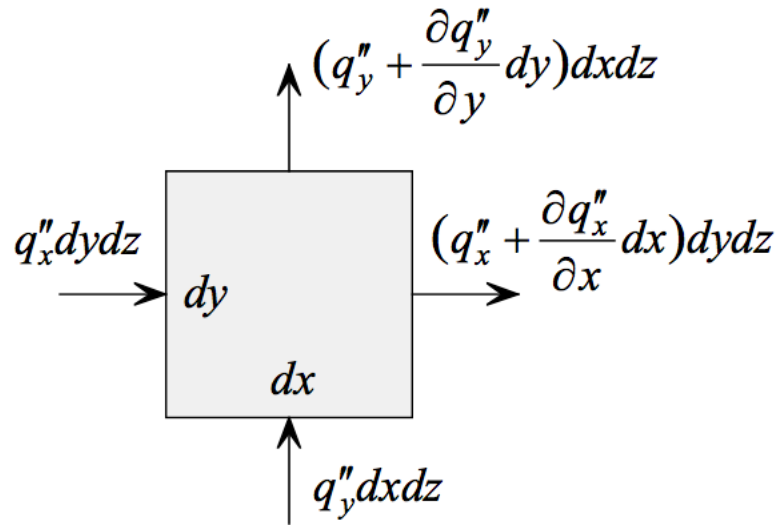


Figure 3-1: Infinitesimal element of conduction

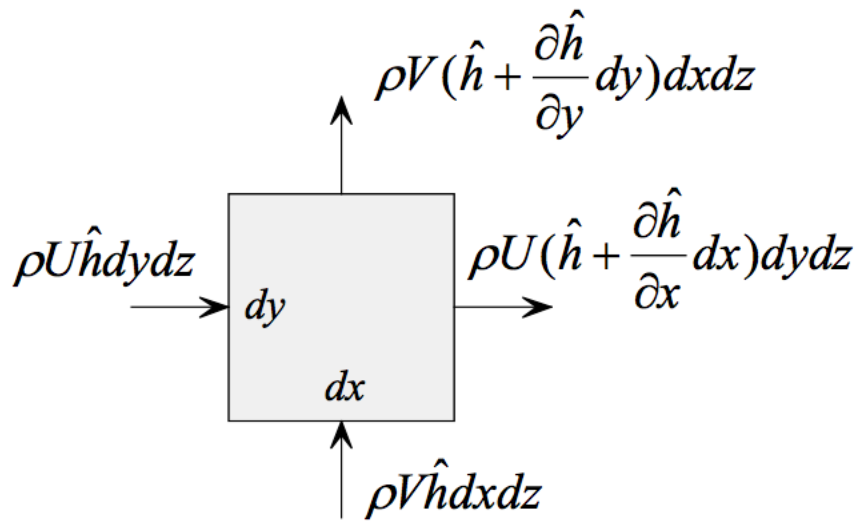


Figure 3-2: Infinitesimal element of mass flow

In the Figures, the z component is not described, but it could be obtained by

taking the x and y components. Here energy is exchanged between the conduction and the mass flow, Energy enters the element by conduction in flux of q'' in the x, y and z direction which is represented by unit area per unit time, and the mass flow rate entering the element, as example the x direction will be given by $\rho U h dy dz$, U is the velocity in the x-direction, h is the enthalpy by unit mass.

So, the \dot{E}_{in} is given by

$$\dot{E}_{in} = q''_x dx dz + q''_y dx dz + q''_z dx dy + \rho U h dy dz + \rho V h dx dz + \rho W h dx dy \quad (3)$$

V and W are the velocity components in y and z directions. All the velocity components are constant.

$$\dot{E}_g = q''' dx dy dz \quad (4)$$

$$\dot{E}_{out} = \left(q''_x + \frac{\partial q''_x}{\partial x} dx \right) dy dz + \left(q''_y + \frac{\partial q''_y}{\partial y} dy \right) dx dz + \left(q''_z + \frac{\partial q''_z}{\partial z} dz \right) dx dy + \rho U \left(h + \frac{\partial h}{\partial x} \right) dy dz + \rho V \left(h + \frac{\partial h}{\partial y} \right) dx dz + \rho W \left(h + \frac{\partial h}{\partial z} \right) dx dy \quad (5)$$

And,

$$\dot{E} = \rho \frac{\partial u}{\partial t} dx dy dz \quad (6)$$

Where u is the internal energy per unit mass. And h can be defined as

$$h = u + \frac{P}{\rho} \quad (7)$$

P is assumed to be constant.

Now substituting the equations into the conservation of energy the following simplified equation can be obtained.

$$-\frac{\partial q_x''}{\partial x} - \frac{\partial q_y''}{\partial y} - \frac{\partial q_z''}{\partial z} + q''' = \rho \left(\frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} + V \frac{\partial h}{\partial y} + W \frac{\partial h}{\partial z} \right) \quad (8)$$

Here the enthalpy change can be defined for constant pressure and the change of temperature.

$$dh = C_p dT \quad (9)$$

Now the Fourier's law can be applied to further simplify the equation.

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q''' = \rho C_p \left(\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} + W \frac{\partial T}{\partial z} \right) \quad (10)$$

As was stated earlier the equations are based on a uniform velocity profile, so this is also to incompressible flow, and for a stationary material with constant thermal conductivity the equation can be further simplified.

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q = \rho C_p \left(\frac{\partial T}{\partial t} \right) \quad (11)$$

In order to be able to analyze the experiment and to find the thermal conductivity, the conduction problem has to be analyzed in the cylindrical coordinate system, which consists on r , z , and θ .

$$k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q = \rho C_p \left(\frac{\partial T}{\partial t} \right) \quad (12)$$

This equation can now be simplified again because in this problem it is assumed that the heat transfer is constant and will have no variations in the θ direction so, it will have rotational symmetry, and can be taken out of the equation. Also the analysis is evaluated at steady state so the time derivate can be eliminated

$$k_m \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) + Q = 0 \quad (13)$$

3.2 Boundary Conditions

From Figure 1-5, the boundary conditions are assumed as follow,

$$k_m \frac{\partial T}{\partial r} = h[T - T_\infty] \text{ at } r = R_0, z \in [0, L] \quad (14)$$

$$k_m \frac{\partial T}{\partial z} = T_m \text{ at } z = 0, r \in [R_i, R_0] \quad (15)$$

$$k_m \frac{\partial T}{\partial z} = 0 \text{ at } z = L, r \in [R_i, R_0] \quad (16)$$

$$T = T_m \text{ at } r = R_i, z \in [0, L] \quad (17)$$

3.3 Finite Volume Formulation

In order to simulate the temperature distribution in the arrangement, a finite volume code was developed in MATLAB, this code will obtain this values by the internal heat generation applied by the boundary conditions that are provided. The setup is discretized on an axisymmetric Cartesian grid, from equation (13) the integration of the finite volume, as explained in [27], is as follows.

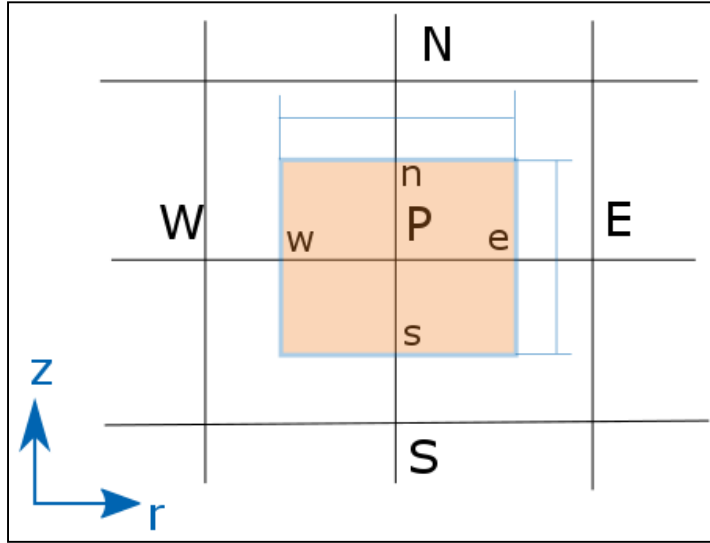


Figure 3-3: Control volume in Cartesian coordinates

$$\int_s^n \int_w^e \left(k_m \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) + Q \right) r \, dr \, dz = 0 \quad (18)$$

Considering only the first term of the left side of the equation for illustration.

$$\int_s^n \int_w^e \left(\frac{1}{r} \frac{\partial}{\partial r} \left(k_m r \frac{\partial T}{\partial r} \right) \right) r \, dr \, dz \quad (19)$$

$$= \left(\left(k_m r \frac{\partial T}{\partial r} \right)_e - \left(k_m r \frac{\partial T}{\partial r} \right)_w \right) \Delta r \, \Delta z \quad (20)$$

$$= \left(\left((k_m r)_e \frac{T_E - T_P}{(\partial r)_e} \right) - \left((k_m r)_w \frac{T_P - T_W}{(\partial r)_w} \right) \right) \Delta r \Delta z \quad (21)$$

$$= \left(\left(\frac{(k_m r)_e}{(\partial r)_e} \right) T_E + \left(\frac{(k_m r)_w}{(\partial r)_w} \right) T_W - \left(\frac{(k_m r)_e}{(\partial r)_e} + \frac{(k_m r)_w}{(\partial r)_w} \right) T_P \right) \Delta r \Delta z \quad (22)$$

With this example of the formulation, a dependent set of algebraic equations can be obtained, given the nature of the obtained matrices, a built-in sparse matrix functions in MATLAB[®] are used to invert the matrices to obtain the solution. Validation of the finite volume code was performed using the commercial software ANSYS[®] APDL [25].

3.4 Inverse Formulation

In order to determine the material properties of the materials, an inverse problem can be formulated as an optimization problem; this is possible if an appropriate objective function is established. By considering the thermal conductivity as an unknown coefficient, it can be considered as a design variable for the optimization problem. The design variable serves as input to the finite volume code that predicts the temperature distribution. The error from the calculated temperatures from the finite

volume code and the actual measured values is minimized by taking the norm of the difference, which is given by equation (23).

$$\text{Objective Function} = S = \sum_{i=1}^n (T_i - T_m)^2 \quad (23)$$

Where n, is the number of measurements, T_i is the computed temperatures from the finite volume code, and T_m is the measured temperature. This objective function is now minimized by using a gradient based optimization method, Conjugate Gradient Method (CGM) is used because of its computational efficiency and convergence rate. A gradient of the objective function to the design variable, in this case thermal conductivity, is often required for this type of algorithm. The temperature measurements have to be taken at optimal location, such location is obtained by using CVSAM. The inverse procedure can be seen on the diagram from Figure 3-4.

Noise is added to the calculated temperature values, and the experimentally measured values are used as input to the inverse analysis, if the difference in the objective function value from the previous iteration is less than 10^{-6} iterations will come to an end.

$$\text{Stopping Criteria} \leq S = \sum_{i=1}^n (T_i - T_m)^2 \quad (24)$$

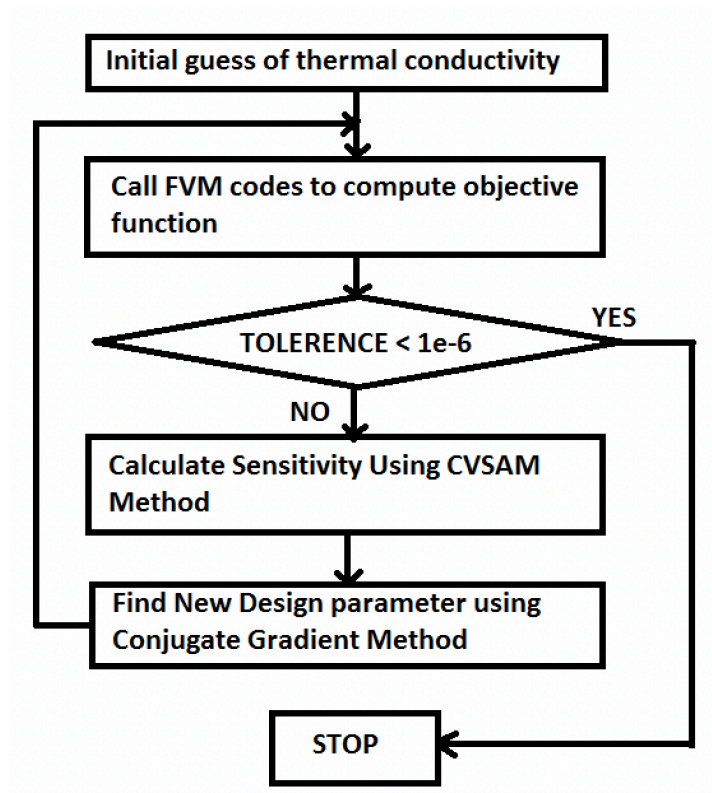


Figure 3-4: Inverse Procedure – Flow Chart

3.5 Sensitivity Analysis

As a gradient based optimization method is being used, the sensitivity of the objective function has to be calculated, this will ensure that the accuracy and efficiency of the method, proving its reliability. This is done by computing the differentiation with respect to the variables are to be obtained. The method used in this study to calculate the sensitivities is the Complex Variable Semi Analytic Method (CVSAM), which is used of its high accuracy and efficiency [26].

On the finite volume formulation, the steady state heat transfer conduction equation is given by

$$[K]\{T\} = \{f\} \quad (25)$$

Where $[K]$ is the stiffness matrix, $\{T\}$ is the temperature and $\{f\}$ is the load vector, input design variables are considered as $\{X\} = \{X_1, X_2, \dots, X_n\}$. The SAM combines the efficiency of the analytical method with the simplicity of the finite difference method. Differentiating the finite volume equation with respect to the design variables on both sides, giving the following equation.

$$\frac{\partial [K]}{\partial \{X\}} \{T\} + \frac{\partial \{T\}}{\partial \{X\}} [K] = \frac{\partial \{f\}}{\partial \{X\}} \quad (26)$$

Here the derivative of the temperature with respect to the design variable $\{X\}$ can be written as,

$$\frac{\partial \{T\}}{\partial \{X\}} = \frac{\{\Delta T\}}{\{\Delta X\}} = [K]^{-1} \left(\frac{\{\Delta f\}}{\{\Delta X\}} - \frac{\{\Delta K\}}{\{\Delta X\}} \{T\} \right) \quad (27)$$

In this equation, the same stiffness matrix is utilized to obtain the temperature and the sensitivity. SAM programming is convenient and computationally efficient. However, using the finite difference method to compute $\frac{\{\Delta K\}}{\{\Delta X\}}$ and $\frac{\{\Delta f\}}{\{\Delta X\}}$, the Semi Analytic Method inherits a drawback from the FDM, the perturbation size $\{\Delta X\}$ has to

be such that is small enough to avoid truncation errors but large enough to avoid subtractive cancelation. This issue can be eliminated if the finite difference is considered in the complex plane. A Taylor series expansion with the perturbation size in the imaginary plane is used to obtain an optimal size with respect to the design variables.

$$f(x + i\Delta x) = f(x) + i\Delta x \Delta f'(x) - \frac{\Delta x^2 f''(x)}{2!} - \frac{i\Delta x^3 f'''(x)}{3!} + \dots \quad (28)$$

Real and imaginary part are separated and grouped together, the first order derivative is as follows.

$$f'(x) = \frac{\text{Imag}(f(x+i\Delta x))}{\Delta x} + O(\Delta x^2) \quad (29)$$

It can be seen in this equation that there is no subtraction of any terms, this means that this approach avoids the subtractive cancellation errors that are a drawback in the FDM. The perturbation size can be small without any loss in accuracy. By combining the equations (27) and (29), the CVSAM is obtained, and the sensitivity of the response is given by the following equation.

$$\frac{\partial \{T\}}{\partial \{X\}} \approx [K]^{-1} \left(\frac{\text{Imag} \{f(X_i + i\Delta X_i)\}}{\{\Delta X_i\}} - \frac{\text{Imag} \{K(X_i + i\Delta X_i)\}}{\{\Delta X_i\}} \{T\} \right) \quad (30)$$

3.6 Conjugate Gradient Method

In order to minimize the objective function (23), a gradient-based optimization method is utilized, And Conjugate Gradient Method is selected in this study due to its high rate of convergence and robustness. A diagram of the process of the method can be seen in Figure 3-5. The design variable for each iteration is minimized by searching along the direction of descent $D^{(n)}$ and search size $\beta^{(n)}$

$$k_m^{n+1} = k_m^n - \beta^{(n)} D^{(n)} \quad (31)$$

Where, D is the direction of descent, β is the step size, and n is the number of iteration. The direction of descent of the current iteration is calculated using linear combination of direction of descent of previous iteration and gradient direction as given by

$$D^{(n)} = \frac{dS^{(n)}}{dk_m} + \gamma^{(n)} D^{(n-1)} \quad (32)$$

Where, $\gamma^{(n)}$ is the Conjugate Coefficient

$$\gamma^{(n)} = \frac{\frac{dS^{(n)}}{dk_m} \frac{dS^{(n)}}{dk_m}}{\frac{dS^{(n-1)}}{dk_m} \frac{dS^{(n-1)}}{dk_m}} \quad (33)$$

The step size can be obtained by minimizing the objective function S with respect to step size $\beta^{(n)}$ as follows

$$\beta^{(n)} = \frac{\frac{dT^n}{dk_m}(T_i - T_m)}{\left(\frac{dT^n}{dk_m}\right)T \frac{dT^n}{dk_m}} \quad (34)$$

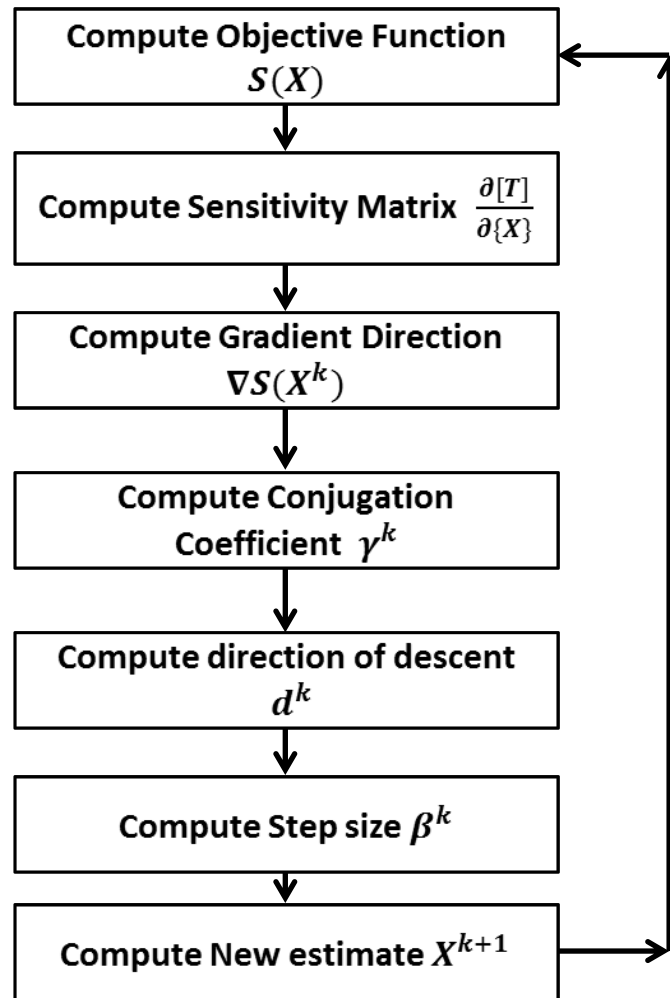


Figure 3-5: Conjugate Gradient Method Diagram

3.7 Algorithm

As seen on Figure 3-4, the algorithm for the Inverse Analysis is as follows.

1. Specify physical domain, governing equation, boundary conditions and surface temperature of both the heater and the Copper tube. For given case, with the known value of thermal conductivity forward problem is solved to get surface temperature value of the Copper tube.

2. Solve entire problem with initial guess of thermal conductivity. (thermal conductivity will be updated after each iteration).

3. Compute objective function as given in Eqn. (23).

4. Check for the stopping criteria for given objective function. If it is less than 10^{-6} then stop calculation otherwise follow step 5.

5. Calculate sensitivity matrix using equation (30).

6. Calculate gradient direction using equation (32).

7. Calculate Conjugation coefficient using equation (33).

8. Calculate the direction of descend from equation (32).

9. Calculate step size from equation (34).

10. Calculate new thermal conductivity k_m and go to next iteration i.e. (n+1).

11. Go to step 2.

3.8 Porosity

The porosity of the experiment is calculated as follows.

$$\epsilon = \frac{V_f}{V_f + V_s} = \frac{V_f}{V} \quad (35)$$

Where V_f and V_s being the volume of the fluid and the volume of the solid, V the volume of the annular gap in the arrangement. [29, 30]. For this study air is used as the fluid, and it is assumed that its thermal conductivity is 0.0257 W/mK, and it is constant throughout all the experiments.

3.9 Modeling

The arrangement is modeled as a hollow cylinder in 2 dimensions, and is an axisymmetric model, and it is used for the Finite Volume code to compute the temperatures required for the analysis. It can be seen in Figure 3-6. It is assumed that heat exchanges with air occur by natural convection on the right side of the model with a heat transfer coefficient of $8 \text{ W/m}^2\text{K}$, the right side of the model is subjected to Neumann boundary conditions, which is obtained by parametrizing the measured temperatures with the ambient temperature. The left side of the model is subjected to Dirichlet boundary conditions, these temperatures values are obtained by experimentation and parametrized to be used on the finite volume code. The top and the

bottom of the model are assumed to be adiabatic. The model parameters can be seen on Table 3-1.

Properties	Dimensions
Heater Radius (m)	0.00635
Copper Tube Radius (m)	0.0148
Length (m)	0.01905
Thermal Conductivity (W/mK)	k_m
Heat Transfer Coefficient(W/m ² K)	8

Table 3-1: Model Parameters

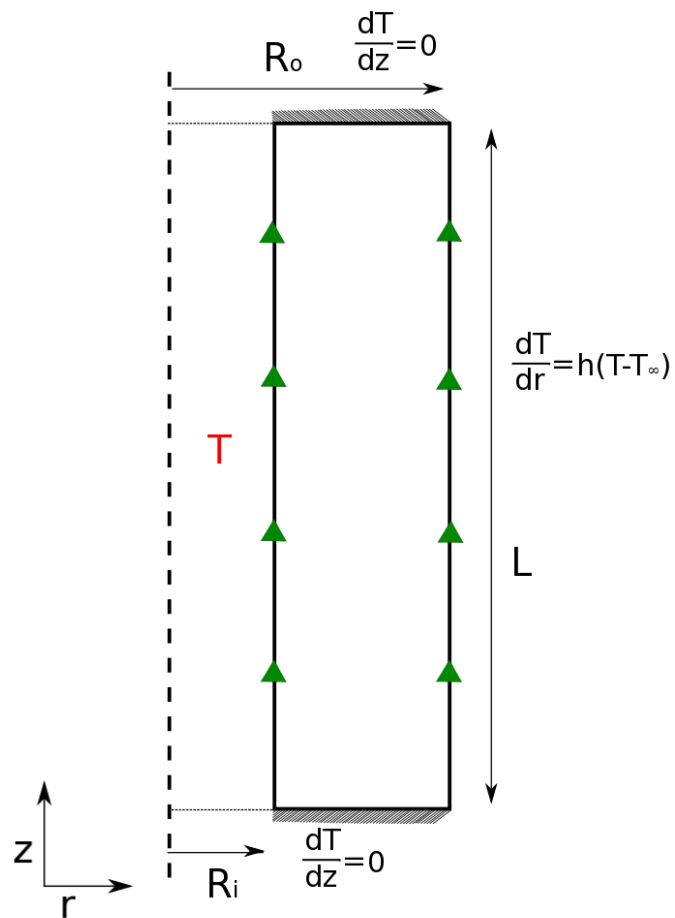


Figure 3-6: Model used for Finite volume code

3.10 Experimentation

In order to obtain the proper boundary conditions for the Inverse Analysis method, a series of experiments were carried out in a closed plastic 2x1x1 ft. plastic box to allow natural convection. In this box a copper tube of length 0.1905 m, 0.0148 m outer radius, and a wall thickness of 0.0008 m was placed in a horizontal configuration. Inside this tube, a Tutco 750 W electrical cartridge heater with diameter of 0.0128 m is placed in the center. In order to ensure that the heater was centered inside the copper tube, special fixtures were manufactured, this will create an annular gap where the porous media will be placed. The top and bottom of the copper tube are insulated with Asbestos to force radial heat transfer.

A set of K-type thermocouples are placed at the surfaces of the cartridge heater and another set is placed in copper tube surface. The sensors output is measured by National Instruments NI-921110-channel thermocouple input module which is placed inside a NI cDAQ 9174 4-slot USB. The channels are configured to write data in the micro voltage range. This equipment is used alongside with a NI LabVIEW® for data acquisition from the thermocouple readings, program can be seen in Figure 3-7.

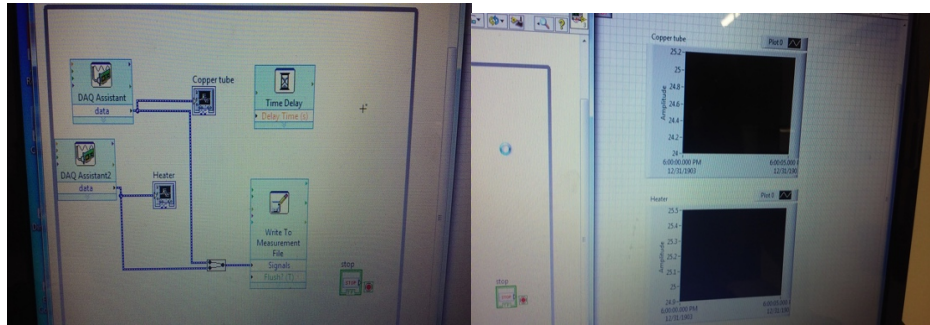


Figure 3-7: LabVIEW Program

Temperature measurements are recorded at every second until steady state heat transfer is reached, these measurements were collected at four equidistant points on the surface of the cartridge heater as well as on the copper tube surface. In Figure 3-8 the copper tube and the thermocouples can be appreciated.



Figure 3-8: Copper tube setup

The Tutco cartridge heater was calibrated using a calorimeter as seen on Figure 3-9. The heater was outputting the same power as it was supplied, so it was properly set to be used in the experiments. The power to the heater in the experiments was supplied by using a BK Precision, Model VSP12010 programmable PFC DC 120V/10A power supply, shown in Figure 3-10.

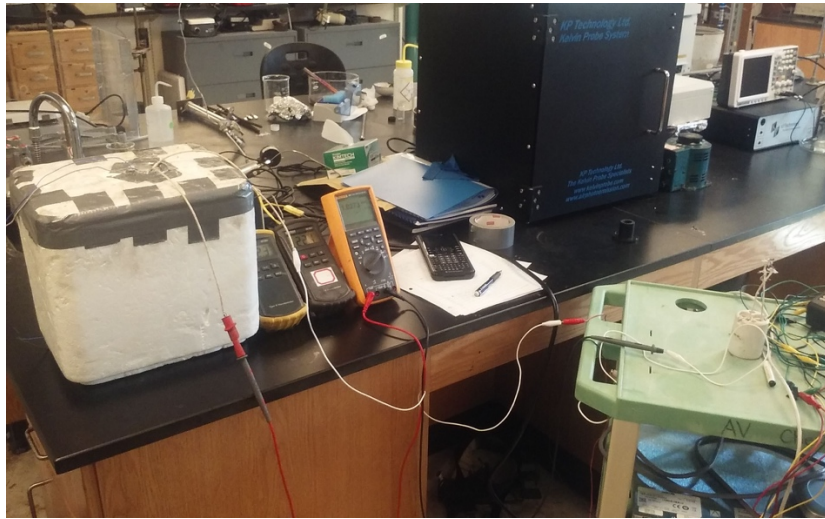


Figure 3-9: Cartridge Heater Calibration



Figure 3-10: BK Power Supply

In order to verify the voltage and the current that was supplied to the experiment, an Extech instruments, model EX330 multimeter was connected in series to measure the current, a Fluke 287 True-RMS multimeter was connected in parallel to measure the voltage. The power supplied to the heater were 1W, 2W and 3W, in order to determine if there is a difference in the conductivities. A K-type thermocouple measures the ambient temperature inside the box at each experiment run. The experimental arraignment can be seen in Figure 3-11.



Figure 3-11: Experimental Setup

The materials that were used to obtain the overall thermal conductivity are presented in Table 3-2.

Number	Material	Dimensions	Type
1	Glass	3mm diameter	bead
2	Silica Catalyst	3mm long by 2 mm diameter	cylinder
3	Aluminum unpolished	1/4 in	pellet
4	Aluminum polished finish	3/8 in	pellet
5	Aluminum unpolished	3/8 in	pellet

Table 3-2: Materials Used

Chapter 4

Results and Discussion

The obtained results from the inverse heat transfer analysis code are presented in Tables 4-1,4-2,4-3. In Figure 4-1 is shown a representation on the readings for the case of Silica Catalyst at the three-different power supplied to the system, and how did the system got to steady state.

Material	k_m (W/mK)	ε
1	0.0877	0.5
2	0.0357	0.4968
3	0.1103	0.4057
4	0.1418	0.3868
5	0.1000	0.4528

Table 4-1: Mean Thermal conductivity at 1 Watt

Material	k_m (W/mK)	ϵ
1	0.1018	0.5
2	0.0420	0.4968
3	0.1330	0.4057
4	0.1600	0.3868
5	0.1290	0.4528

Table 4-2: Mean Thermal conductivity at 2 Watt

Material	k_m (W/mK)	ϵ
1	0.1020	0.5
2	0.0430	0.4968
3	0.1390	0.4057
4	0.1660	0.3868
5	0.1340	0.4528

Table 4-3: Mean Thermal conductivity at 3 Watt

The results obtained from the Inverse Analysis code show a trend that implies that by increasing the power supplied to the system, the thermal conductivity increases, which was expected. Results can be seen in Figure 4-2. These results show agreement with previous work done by [16, 21, 28]. It is well known that the thermal conductivity of glass is about 0.8 W/mK, this can also be seen on Marcus [16], they found the

thermal conductivity of glass to be 0.76 W/mK. In order to obtain the thermal conductivity of the solid phase it is needed to be calculated using the following equation that takes on account the porosity of the media.

$$k_m = k_f^\epsilon k_s^{(1-\epsilon)} \quad (36)$$

where k_f is the thermal conductivity of the fluid, in this case air being 0.0257 W/mK, and k_s is the thermal conductivity of the solid, which was found to be 0.7977 W/mK for the 3 Watt case. Showing agreement with previous work done.

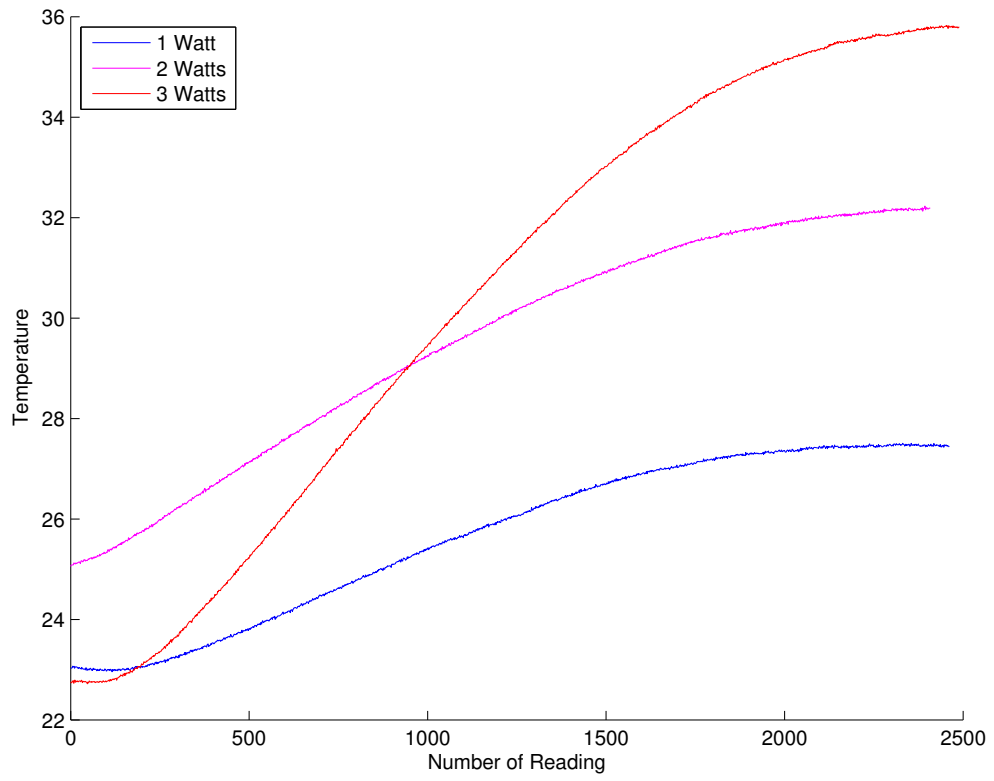


Figure 4-1: Temperature at Copper Tube Surface – Silica Catalyst

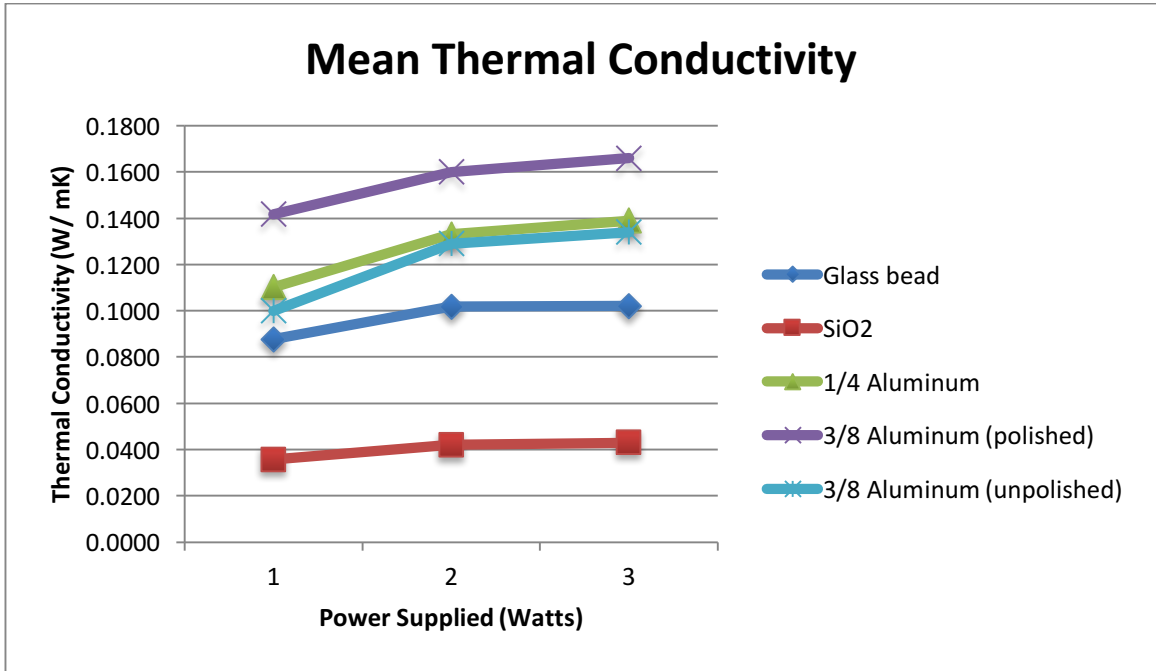


Figure 4-2: Experimental Results

In order to check the robustness of the analysis, random noise was added to the simulated temperature measurements, to verify how this affects the solution, a MATLAB Random noise array of the normal distribution was used.

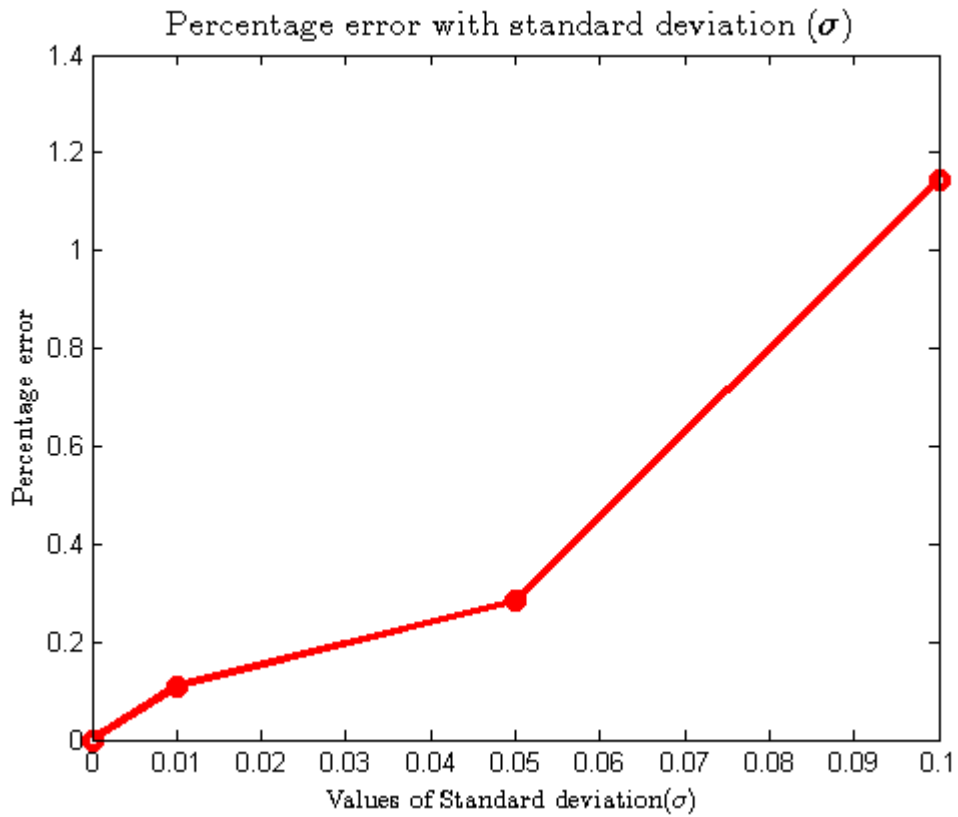


Figure 4-3: Percentage Error Vs. Noise Standard Deviation

Chapter 5

Conclusion and Future Work

5.1 Conclusions

The proposed methodology to formulate the solution of the Inverse Heat Transfer Problem (IHTP) was presented. This method was determined by using inverse analysis, with a developed finite volume code that uses Semi-Analytical Complex Variable Method (SACVM) to obtain the sensitivity of the objective function and Conjugate Gradient Method as an optimization method to minimize the function. Experiments were performed to obtain the surface temperatures at the cartridge heater and the copper tube that were used as boundary conditions for the inverse problem, using different materials as porous media under different power output conditions. The presented method is a viable way to determine the thermal conductivity of the materials of which this property is unknown. Experimental data and analysis show agreement with previous investigations.

5.1 Future Work

A new method is proposed with a different approach to obtain the thermal properties of different materials, conducting a new set of experiments in a new arrangement that consists in a 15-Gal water tank which will be heated to a set temperature and will have a copper tube across two of its surfaces and will have in the center a smaller aluminum tube which will have a flow of N_2 inside this tube. The arrangement is center using rubber stoppers. In the annular gap of between the two tubes, a porous material will be placed.

Water on the outside of the copper tube will be heated at a set temperature, this will create heat transfer to the porous material and therefore to the N_2 that will flow in the inside the aluminum tube. Thermocouples will read the temperature change from the inlet to the outlet of the flow of N_2 across the experiment, therefore, with this temperature difference, the mass flow rate of the N_2 and the bath temperature, the thermal conductivity of the porous media can be calculated using the Inverse Heat Transfer Analysis. This will be achieved by modifying the boundary conditions and by modifying the geometry in the finite volume code. The proposed arrangement can be appreciated in Figure 5-1.

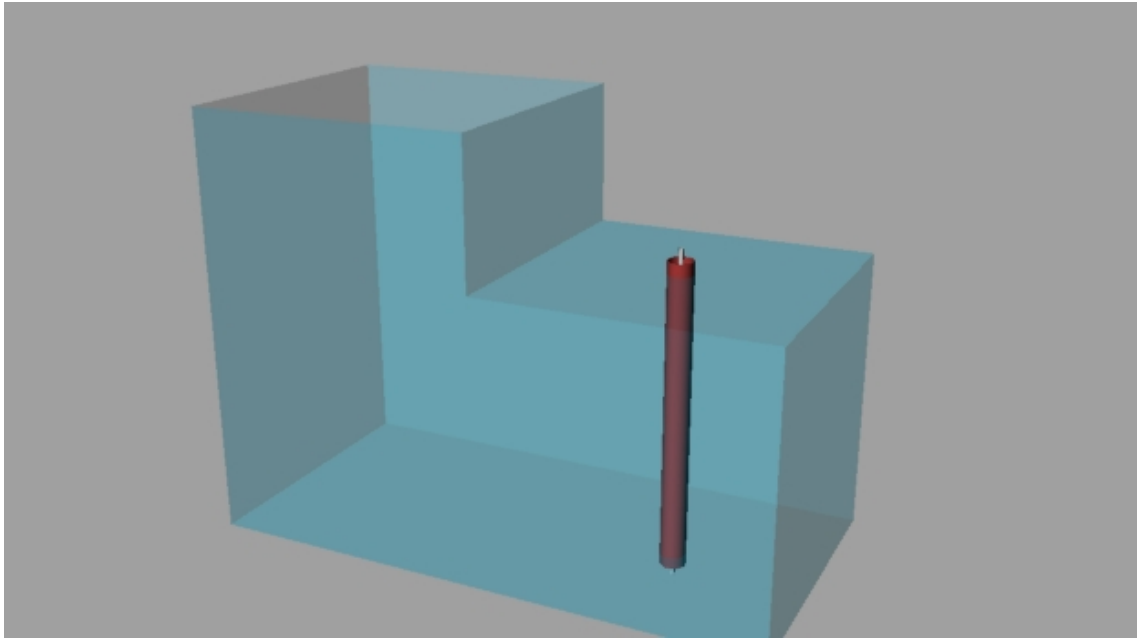


Figure 5-1: New method for finding thermal conductivity

Chapter 6

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Chapter 7

Biographical Information

Oscar Javier Fabela II obtained his Master's of Science Degree in Aerospace Engineering on December of 2017, from The University of Texas at Arlington. Oscar joined the university right after obtaining his Bachelors of Science in Aerospace Engineering and in Mechanical Engineering from New Mexico State University in May of 2016.