OPTIMIZING A SYSTEM OF ELECTRIC VEHICLE CHARGING STATIONS

by

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Abstract

OPTIMIZING A SYSTEM OF ELECTRIC VEHICLE CHARGING STATIONS

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The University of Texas at Arlington, 2018

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There has been a significant increase in the number of electric vehicles (EVs) mainly because of the need to have a greener living. Thus, ease of access to charging facilities is a prerequisite for large scale deployment for EV.

The first component of this dissertation research seeks to formulate a deterministic mixed-integer linear programming (MILP) model to optimize the system of EV charging stations, the locations of the stations and the number of slots to be opened to maximize the profit based on the user-specified cost of opening a station. Despite giving the optimal solution, the drawback of MILP formulation is its extremely high computational time (as much as 5 days). The other limit of this deterministic model is that it does not take uncertainty in to consideration.

The second component of this dissertation is to overcome the first drawback of the MILP model by implementing a two-stage framework developed by (Chawal et al. 2018), which integrates the first-stage system design problem and second-stage control problem of an EV charging stations using a design and analysis of computer experiments (DACE) based system design optimization approach. The first stage specifies the design of the system that maximizes expected profit. Profit incorporates costs for building stations and revenue evaluated by solving a system control problem in the second stage. The results obtained from the DACE based system design optimization approach, when compared to the MILP, provide near optimal solutions. Moreover, the computation time with the DACE approach is significantly lower, making it a more suitable option for practical use.

The third component of this dissertation is to overcome the second drawback of the MILP model by introducing stochasticity in our model. A two-stage framework is developed to address the design of a system of electric vehicle (EV) charging stations. The first stage specifies the design of the system that maximizes expected profit. Profit incorporates costs for building stations and revenue evaluated by solving a system control problem in the second stage. The control problem is formulated as an infinite horizon, continuous-state stochastic dynamic programming problem. To reduce computational demands, a numerical solution is obtained using approximate dynamic programming (ADP) to approximate the optimal value function. To obtain a system design solution using our two-stage framework, we propose an approach based on DACE. DACE is employed in two ways. First, for the control problem, a DACE-based ADP method for continuous-state spaces is used. Second, we introduce a new DACE approach specifically for our two-stage

EV charging stations system design problem. This second version of DACE is the focus of this paper. The “design” part of the DACE approach uses experimental design to organize a set of feasible first-stage system designs. For each of these system designs, the second-stage control problem is executed, and the corresponding expected revenue is obtained. The “analysis” part of the DACE approach uses the expected revenue data to build a metamodel that approximates the expected revenue as a function of the first-stage system design. Finally, this expected revenue approximation is employed in the profit objective of the first stage to enable a more computationally-efficient method to optimize the system design. To our knowledge, this is the only two-stage stochastic problem which uses infinite horizon dynamic programming approach to optimize the second stage dynamic control problem and the first stage system design problem. Moreover, when the designs obtained from our DACE approach and MILP design are solved using DACE-based ADP method (simulation), an improvement of approximately 8% is observed in the simulated profit obtained from ADP design compared to that of MILP design indicating that when uncertainty is considered, DACE ADP design provides the better solution.

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Chapter 1. Introduction

1.1. Research Background

Electric vehicles are becoming important primarily due to environmental factors as the current gasoline vehicular emission is on the rise. There has been a significant increase in the number of electric vehicles mainly because of the need to have a greener living. Though they came into existence in early 19th century the necessity is being felt in the recent times. One of the Environmental assessment of a full Electric Transportation Portfolio by the Electric Power Research Institute and Natural Resource Defense Council has suggested that fueling the transportation using electricity instead of gasoline/petroleum can significantly reduce the Green House gases emissions and other air pollutants. The Context of the study is the climate-projection goals that they are striving to achieve which is reducing 80% of the Green House gas emission levels from 1990 to 2050. As suggested by the study, 60% of the carbon pollution occurs from passenger vehicles emission, if we keep a check on this segment, around 60% of the goal is achieved (He et al. 2015). There has been a significant increase in the adoption of the Electric vehicles in the recent years, this can be partly credited to the increasing global concern for climate change and increasing crude oil prices. Volvo has announced that starting in 2019, all of the new models it produces will be electric or hybrid. Hakan Samuelsson, Volvo president and chief executive, said in a statement. "Volvo Cars has stated that it plans to have sold a total of 1 million electrified cars by 2025. When we said it we meant it. This is how we are going to do it." (https://www.npr.org/sections/thetwo-way/2017/07/05/535596277/all-new-volvo-models-will-be-electric-or-hybrid-starting-in-2019). With this growth in EVs and the fundamental transformation from traditional oil based fleets to electrical power vehicular technologies, concerns have emerged in the Smart City.

One of the key aspects that needs to be considered for EV charging stations is the drive range or the accessibility to charging stations, resulting in restricted adoption of the Electric Vehicle. The driving range varies greatly from manufacturer to manufacturer and also from model to model, as on 2014, the longest EV driving range was of 424 km for Tesla Model S, while the shortest range was of Scion iQ Ev (2013) which was a range of 60 Km. As per U.S Department of Energy (2014), the EV driving range for the vehicle ranges from 100 km- 160 km (Huang et al. 2016). One of the solutions to this problem is installation of many Electric charging stations, but how many to be installed, where they should be installed gives rise to a facility location problem.

This study focuses on one such concern, which is the current and future locations and the number of EV charging stations needed in a particular area. A sufficient number of charging stations with high level of service is required for charging EVs for both today and the future. The need for a sustainable charging station design, with maximum climate benefits, economic profit and social acceptance, is at an all-time high.

1.2. Motivation

One of the most important motivations for this project is that electricity is a clean energy source, and can be generated using multiple renewable energy sources. This is critically important for reducing greenhouse gas emissions. To aid this endeavor the U.S. government pledged to reduce greenhouse gas emissions by approximately 17% by 2020 (http://www.eia.doe.gov/emeu/aer/pdf/pages/sec12_4.pdf, 2010). According to the website of the Energy Information Administration (2016), the transportation sector alone causes up to 36.41% of all energy-related emissions and is the largest producer of carbon dioxide emission in the U.S (https://www.eia.gov/environment/emissions/carbon/). This further increase the need for the transportation sector in the U.S. to find a cleaner alternative. Furthermore, energy power in the transportation sector is derived almost exclusively from fossil fuels, making the U.S. the world's largest consumer of crude oil and petroleum products. Each day, Americans consume approximately 20 million barrels of petroleum, and import approximately 7 million barrels as per the website of the Oak Ridge National Laboratory (http://cta.ornl.gov/data/download36.shtml, 2016). Because of the increasing risk associated with obtaining oil, as well as the growing scarcity, it is threatening both the energy security and the economy. If this trend of heavy reliance on petroleum continues, based on current consumption, the production would not be able to handle it. Eventually, the economy will be unable to afford the high cost of oil dependency.

Environmental pollution is one of the worst effects of our accelerated development rate. An integral part of environment pollution is air pollution, which is caused majorly due to usage of traditional sources of energy such as coal, natural gas, and oil. Finding reliable, renewable, and non-polluting substitute to these is necessary for sustainable development of environment, else there will be serious implications. To combat such high levels of carbon dioxide emissions, we need a major transformation. EVs address that purpose well. With higher number of EVs in market, there will be a substantial reduction in CO2 emissions which would help in sustainable development. The idea of breezing past gas stations without leaving a carbon footprint behind appeals to electric car enthusiasts, but the fear of running out of battery power has been a major barrier in getting people to buy them.

Renewable energy, in the use of EVs are the most effective means of significantly lowering the consumption of oil and controlling the fuel economy. By supporting existing and future technologies of electrical power based vehicular products, it is our hope to shift the transportation sector from oil based designs to a cleaner, more sustainable electric based design. In order to achieve that goal and spread the use of EVs, reliable access to charging stations is required.

In this research, we have proposed a model whose objective is to minimize EV overall system charging cost while still storing an adequate amount of power in the battery to satisfy EV charging demand. This requires many variables such as number of vehicles both now and in the future, cost of each station, monitoring demand, the amount of power generated, and the market price (MP) for energy and trading power with the power grid accordingly. By generating electricity from renewable sources, charging stations may become active participants in the power market.

1.3. System Design Configuration

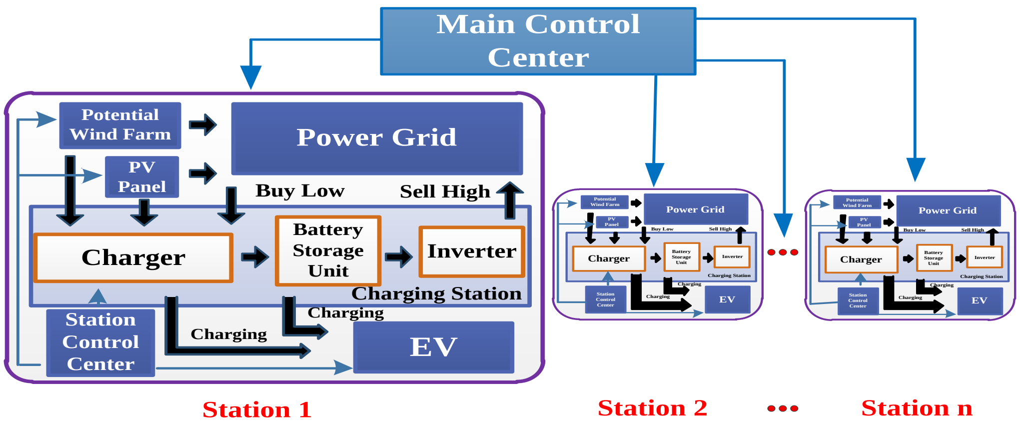

Figure 1.1. Electric Charging Station System (Sarikprueck 2015)

The proposed EV charging station, as illustrated in Figure 1.1, is designed to obtain energy from wind/solar energy generation and the power grid. The energy stored in the charging station is used to charge the EVs. The system can store excessive energy for future demand by storing it in the battery storage unit. The surplus stored energy can be used by the station to satisfy the demand if the energy generated is insufficient. If the energy generated from wind/solar and the battery storage is insufficient to satisfy demand, then the required energy is bought from the power grid. Any excess energy from the storage unit can be sold back to the power grid for added profit.

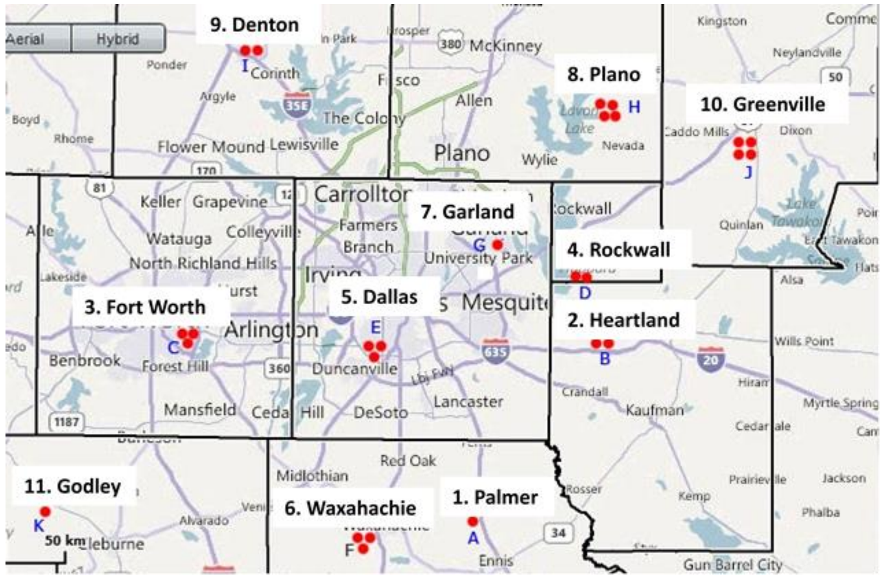

Figure 1.2. The distribution of the station locations

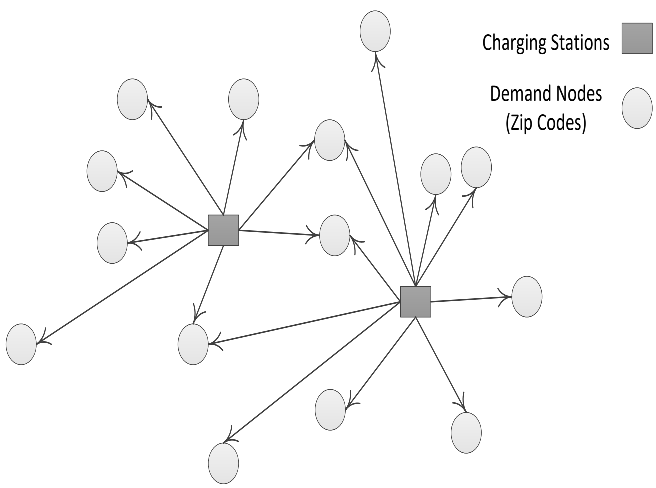

Figure 1.3. Demand Nodes and Charging Stations

Figure 1.2 shows the distribution of the 11 potential station within the Dallas/Fort Worth area in Texas. The 140 hotspots (cities) are spread around these stations. Figure 1.3 shows a visual depiction of the projects primary focus (the distance) and how the demand is determined based on it. The distance between each hotspot and station is pre-determined and is used in the optimization model

The rest of this dissertation is organized as follows: In Chapter 2, the first paper is presented named optimizing a system of electric vehicle charging stations using mixed integer linear programming computer experiments where deterministic models are discussed to optimize the system of EV charging stations. In Chapter 3, the second paper is presented titled a two-stage design and analysis of computer experiments approach for optimizing a system of electric vehicle charging stations where uncertainty is taken into consideration to optimize the system of EV charging stations. Finally, conclusions and future research are presented in Chapter 4.

Reference

He, F., Yin, Y., & Zhou, J. (2015). Deploying public charging stations for electric vehicles on urban road networks. *Transportation Research Part C: Emerging Technologies*, *60*, 227-240.

Huang, K., Kanaroglou, P., & Zhang, X. (2016). The design of electric vehicle charging network. *Transportation Research Part D: Transport and Environment*, *49*, 1-17.

Sarikprueck, P. (2015). *Forecasting of wind, PV generation, and market price for the optimal operations of the regional PEV charging stations*. The University of Texas at Arlington.

Chapter 2. Optimizing a System of Electric Vehicle Charging Stations using Mixed Integer Linear Programming Computer Experiments

Abstract

Due to the economic and environmental concerns associated with fossil fuels and the growing need for sustainability, the use of electric vehicles (EVs) is a viable solution. However, ease of access to charging facilities is a prerequisite for large scale deployment for EV. This paper formulates a mixed-integer linear programming (MILP) model to optimize a system of EV charging stations, locations of EV charging stations, the number of charging slots to be opened, and expected profits. Then, a two-stage framework is considered which integrates the first-stage system design problem and second-stage control problem of an EV charging stations using a design and analysis of computer experiments (DACE) based system design optimization approach. The first stage specifies the design of the system that maximizes expected profit. Profit incorporates costs for building stations and revenue evaluated by solving a system control problem in the second stage. This approach generates a meta-model to predict revenue from the control problem using multivariate adaptive regression splines (MARS), fitted over a binned Latin hypercube (LH) design. Using the revenues from the control problem, the system design problem is then solved. The results obtained from the DACE based system design optimization approach, when compared to the MILP, provide near optimal solutions. Moreover, the computation time with the DACE approach is significantly lower, making it a more suitable option for practical use.

Index Terms **-** Design and analysis of computer experiments based system design optimization approach, electric vehicle charging stations, Latin hypercube sampling design, meta-model, mixed integer linear programming, multivariate adaptive regression splines.

1. Nomenclature

A. Sets:

J	set of potential station locations indexed by j
I	set of demand hot spots indexed by i
$I(j)$	set of demand hot spots within the max-mile radius of the station
T	set of time periods of 15 minutes in a day indexed by t
K	set of basis functions indexed by k

B. Parameters:

m_{ij}	Distance from hotspot i to station j (miles)
φ	Max-mile radius between the hotspots and the station
p_i	Population of EVs at hotspot i
d_t	Demand percentage in time period t
e	Efficiency of the battery
dc	Discharge rate
ϕ	Recapture rate
θ	First time period in a day
ρ	Last time period in a day
r_t	Retail price in time period t
cr	Battery charge capacity

c_j Cost of opening station j

Nc_j Cost of opening a slot at station j

v Minimum battery level

u Maximum battery level

ב The maximum number of slots opened per charging station j

sc Slot capacity

W_t Wind generation (Mwh) in time period t

S_t Solar production (Mwh) in time period t

M_t Market price in time period t

β_0 Y-intercept of MARS function

β_k Least square estimators for basis function k

C. Variables:

$x_j \in \{0,1\}$ Binary variable, if station j is operational

$y_{ij} \in \{0,1\}$ Binary variable, if hotspot i is assigned to station j

$\alpha_{tj} \in \{0,1\}$ Binary variable, if the solar production in time period t is allocated to station j

ω_{tj} Fraction of the total wind generation in time period t allocated to station j

g_{tj}^{+} Electricity bought from grid by station j in time period t

g_{tj}^{-} Electricity sold to grid from direct charge of station j in time period t

B_{tj}^{-} Electricity sold to grid from battery of station j in time period t

D_{tj} Total demand in time period t at charging station j

D^1_{tj} Demand satisfied by direct charge of station j in time period t

D^2_{tj}	Demand satisfied by battery of station *j* in time period *t*
L_{tj}	Battery level of station *j* in time period *t*
Bc_{tj}	Battery charge of station *j* in time period *t*
Ns_j	Number of operational slots at station *j*
Tc_j	Total capacity of slots at station *j*
Nd_{tj}	Nominal demand in time period *t* at station *j*
R_{tj}	Recapture of lost demand of time period $t – 1$ at charging station *j*
a_{tj} , b_{tj} , c_{tj}	Binary decision variables used for piecewise formulation in time period *t* at station *j*
Z_{MILP}	Objective function of MILP formulation
$(\bar{x}, \overline{Ns})$	System station design data points
$Z_{MILP}(\bar{x}, \overline{Ns})$	Objective function of MILP formulation
$Rev\ (\overline{Ns})$	Objective function of MILP formulation without the cost component
BF_k	Basis function *k*

D. Acronyms:

DACE	Design and Analysis of Computer Experiments
EV	Electric Vehicle
LH	Latin Hypercube
MARS	Multivariate Adaptive Regression Splines
MILP	Mixed Integer Linear Programming

2. Introduction

The energy crises during the 1970s created a need for alternative forms of energy for transportation vehicles and thus began the research into EVs. In current times, the need for sustainability has risen exponentially, giving EVs the emphasis they deserve. A recent environmental assessment by the Electric Power Research Institute and Natural Resource Defense Council has suggested that using electricity instead of gasoline/petroleum can significantly reduce greenhouse gas emissions and other air pollutants [1]. To bolster the usage of EVs, governments have taken a variety of initiatives. For example, Norway has given a tax exemption on EVs until 2020 to position itself as the EV leader [2]. By supporting existing and future technologies of electrical power based vehicular products, the transportation sector is expected to shift from an oil based design to a cleaner, more sustainable electric based design. One of the key aspects that needs to be considered for EV charging stations is the drive range or the accessibility to charging stations, resulting in restricted adoption of the EV. One of the solutions to this problem is better installation of EV charging stations, which minimizes operational cost for setting up EV charging stations and maximizes the profit in running the stations. By generating electricity from renewable sources, charging stations may become active participants in the power market. This further increases the need for the transportation sector in the U.S. to find a cleaner alternative.

Several papers proposed different approaches and implemented several algorithms for optimizing locations for EV charging stations. The work in [3] - [6] proposed different approaches for finding optimal locations and sizes for charging stations. A robust integer linear optimization and a stochastic programming framework was proposed to solve the strategic optimization problem of determining optimal locations for charging stations of (ad-hoc) electric car-sharing systems, and for considering uncertainty associated with vehicle-to-grid and wind power scenarios, respectively in [7] and [8]. A linear mathematical model to optimize the cost of power trading and auto regressive methods to forecast wind power output and market clearing price for energy was proposed in [9]. A stochastic model was proposed in [10] and [11] for back up flow capturing demand to ensure stability in service coverage and for-profit maximization based on price responsiveness of customers, respectively. The work in [12] - [15] proposed a methodology that ensured little to no queue at the charging station based on customer demand. The work in [16] – [20] proposed a battery swapping methodology as a candidate solution to existing approaches to meet customer demand.

Although the aforementioned papers proposed different approaches and implemented several algorithms for the optimization of locations for EV charging stations, a globally optimal set of stations to be opened, with the corresponding number of slots, has never been found while considering factors such as the customer demand obtained from the city population (hotspots) of EVs, the distances from hotspots to the stations, and available solar energy and wind energy generation. To address these issues, we proposed a deterministic MILP model to obtain a globally optimal set of stations to be opened that maximizes profits. In this research, we considered 11 possible locations for charging stations for 140 demand hotspots in multiple time periods.

The other contribution of this paper is an implemented two-stage framework developed by [28], which addresses the design of a system of EV charging stations using a DACE based system design optimization approach. The first stage specifies the design of the system that maximizes expected profit, which incorporates costs of building stations and revenue from a system control problem in the second stage. The “design” part of the DACE approach uses experimental design to organize a set of feasible system designs of EV charging stations. For each of these system designs, a second-stage control problem is executed, and the corresponding expected revenue is obtained. The “analysis” part of the DACE approach uses the expected revenue data to build a metamodel that approximates the expected revenue as a function of the first-stage system design. Finally, this expected revenue approximation is employed in the profit objective of the first stage to enable a more computationally-efficient method to optimize the system design. The results obtained from the DACE based system design optimization approach, when compared to MILP, provides near optimal solutions with a loss of less than 1% of profit. Furthermore, the DACE approach reduces the computational time from 4 days and 23 hours to less than an hour, making it a much more suitable option for practical use. In addition, our DACE model allows us to generate controllability/revenue functions, which may be used to evaluate different cost scenarios.

The rest of this paper is organized as follows: a System Design Problem Formulation including system design objectives, formulation of the problem using MILP and a DACE based system design optimization approach is presented in detail in Section

3. In Section 4, System Design Experiments are described where the results are discussed. Finally, Conclusions are made in Section 5.

3. System Design Problem Formulation

3.1. System Design Layout

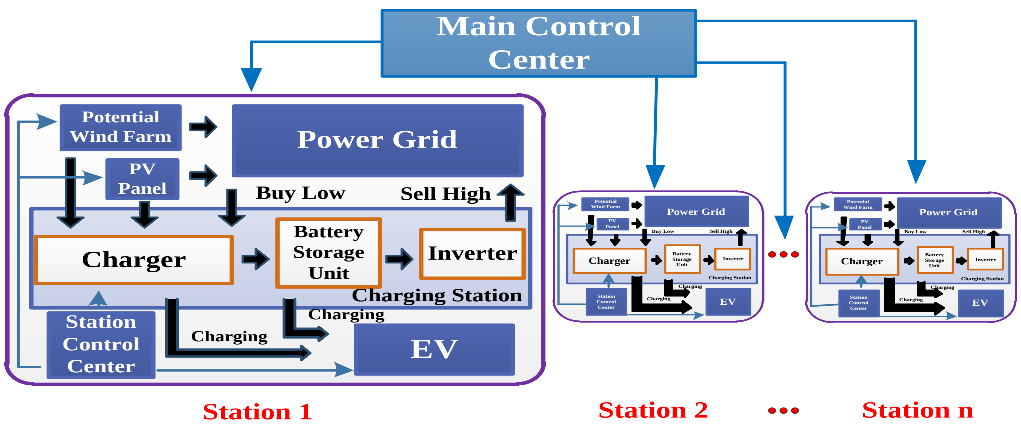

Figure 2.1. EV Charging Station Layout

The proposed EV charging station, as illustrated in Figure 2.1, is designed to obtain energy from wind/solar energy generation and the power grid. The energy stored in the charging station is used to charge the EVs. The system can store excessive energy for future demand by storing it in the battery storage unit. The surplus stored energy can be used by the station to satisfy the demand if the energy generated is insufficient. If the energy generated from wind/solar and the battery storage is insufficient to satisfy demand, then the required energy is bought from the power grid. Any excess energy from the storage unit can be sold back to the power grid for added profit.

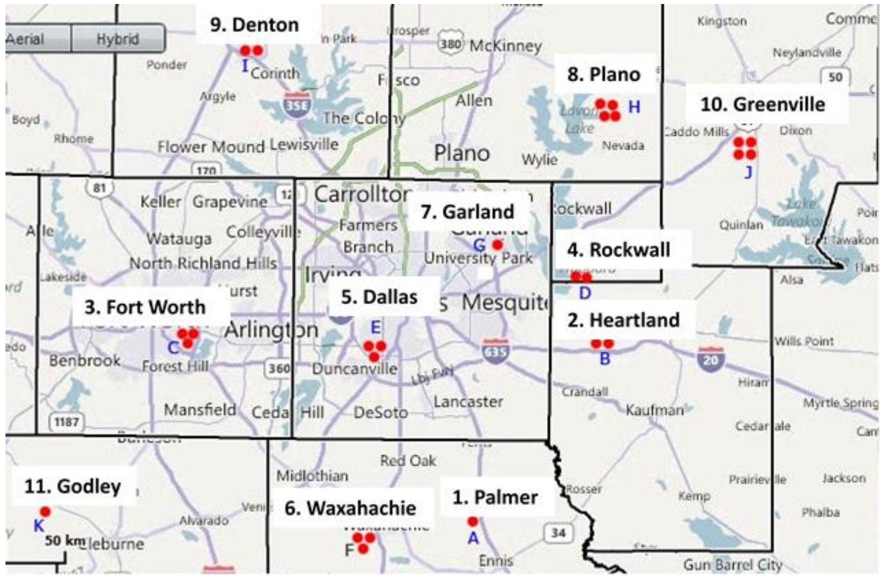

Figure 2.2. The distribution of the station locations

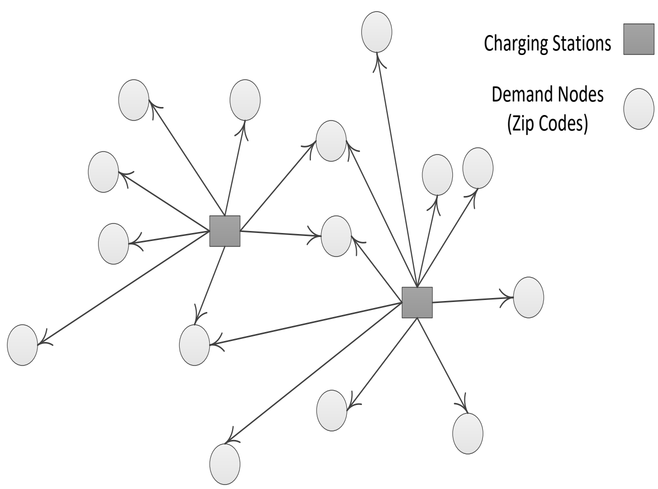

Figure 2.3. Demand Nodes and Charging Stations

Figure 2.2 shows the distribution of the 11 potential station locations within the Dallas/Fort Worth area in Texas. The 140 hotspots (cities) are spread around these stations. Figure 2.3 shows a visual depiction of the projects primary focus (the distance) and how the demand is determined based on it. The distance between each hotspot and station is pre-determined and is used in the optimization model

3.2. Objective

The major objective of this paper is to develop a new model to optimize the number of stations and the number of slots to be opened along with maximizing the overall profits using an MILP formulation. Despite giving the optimal solution, the drawback of the proposed MILP formulation is that it may require extremely high computational time (as much as 5 days) to solve. In general, the formulation can be solved using branch-and-cut, which can consume a lot of time. Hence, we utilized a DACE based optimization approach to determine the system of EV charging stations.

3.3. MILP Formulation

As shown in equation (1) below, the objective is to maximize revenue from selling energy to the grid both from the battery and direct charge across all the stations and the revenue from meeting the demand minus the cost of buying energy from the grid, the cost of constructing an EV station at a potential location and the cost of opening slots.

$$\max \sum_{t\in T}\sum_{j\in J}\left[\left(M_t(g_{tj}^- + B_{tj}^- - g_{tj}^+) + r_t N d_{tj}\right)\right] - \sum_{j\in J}(c_j x_j + Nc_j Ns_j) \tag{1}$$

s.t.

$$\sum_{i\epsilon I(j)} p_i d_t \left[\frac{(\varphi - m_{ij})}{\varphi} y_{ij}\right] = D_{tj} \qquad \forall j \in J, \forall t \in T \tag{2}$$

The constraints in equation (2) ensure that the total demand in time period *t* at charging station *j* is the product of the distance function, which is the percentage of the demand of hotspot *i* assigned to station *j* with the population of hotspot *i* and the general demand percentage in time periods *t.*

$$y_{ij} \leq x_j \qquad \forall i \in I(j), \forall j \in J \tag{3}$$

$$\sum_{j \in J} y_{ij} \leq 1 \qquad \forall i \in I \tag{4}$$

The constraints in equation (3) ensure that demand hotspot *i* cannot be assigned to stations *j*, if stations *j* is not operational. The constraints in equation (4) ensure that each hotspot *i* is served by at most one station.

$$x_j + y_{i\hat{j}} \leq 1 \quad \forall i \in I, \forall j, \hat{j} \in J, m_{ij} < m_{i\hat{j}} \tag{5}$$

The constraints in equation (5) ensure that if *j* is a closer station than $\hat{j}$, then the hotspot will be assigned to the closer one only.

$\forall j \in J, \forall t \in T$, let ε be the upper bound (very large number, + ∞) and a_{tj} be such that

$$a_{tj} = \begin{cases} 1 & if \ D_{tj} + R_{tj} \geq Tc_j \\ 0 & o.w. \end{cases}$$

$$-(1-a_{tj})\varepsilon + Tc_j \leq Nd_{tj} \leq Tc_j + (1-a_{tj})\varepsilon \qquad \forall j \in J, \forall t \in T \tag{6}$$

$$-a_{tj}\varepsilon + D_{tj} + R_{tj} \leq Nd_{tj} \leq D_{tj} + R_{tj} + a_{tj}\varepsilon \qquad \forall j \in J, \forall t \in T \tag{7}$$

The constraints in equation (6) and (7) is a piecewise linear formulation ensuring that if the sum of the total demand and the recapture of the loss demand in time period *t* at station *j* is equal or more than total capacity of the station *j*, then total nominal demand in time period *t* at station *j* is equal to the total capacity of the charging station *j*; otherwise, it is equal to the sum of the total demand and the recapture of the loss demand in time period *t* at station *j*.

When demand for a time period exceeds capacity, we assume 50% of the customers are willing to wait to be served in a subsequent time period (recaptured), while 50% are lost. $\forall j \in J, \forall t \in T$, let b_{tj} be such that

$$b_{tj} = \begin{cases} 1 & if \ \left[D_{(t-1)j} + R_{(t-1)j} - Tc_j\right]\phi \geq 0 \\ 0 & o.w. \end{cases}$$

$$-(1-b_{tj})\varepsilon + [D_{(t-1)j} + R_{(t-1)j} - Tc_j]\phi \leq R_{tj} \leq [D_{(t-1)j} + R_{(t-1)j} - Tc_j]\phi + (1-b_{tj})\varepsilon \quad \forall j \in J, \forall t \in T \ (8)$$

$$-(1-b_{tj})\varepsilon + [D_{\rho j} + R_{\rho j} - Tc_j]\phi \leq R_{\theta j} \leq [D_{\rho j} + R_{\rho j} - Tc_j]\phi + (1-b_{tj})\varepsilon \quad \forall j \in J \ (9)$$

$$-b_{tj}\,\varepsilon \leq R_{tj} \leq b_{tj}\,\varepsilon \quad \forall j \in J, \forall t \in T \ (10)$$

The piecewise constraints in equations (8), (9) and (10) ensure that if 50% of the total demand and recapture from time period *t*-1 at station *j* is equal to or more than the total capacity of the station *j* then recapture in time period *t* at station *j* is 50% of the total demand and recapture of the loss demand in time period *t*-1 at charging station *j* minus the total capacity of the charging station *j*. Otherwise, there will be no recapture. We assume that the recapture of the loss demand at last time period is taken into consideration to calculate the recapture of the loss demand at the first stage.

$\forall j \in J, \forall t \in T$, let c_{tj} be such that

$$c_{tj} = \begin{cases} 1 & if \ [D_{(t-1)j} + R_{(t-1)j} - Tc_j]\phi \geq Tcj \\ 0 & o.w. \end{cases}$$

$$-(1-c_{tj})\varepsilon + Tc_j \leq R_{tj} \leq Tc_j + (1-c_{tj})\varepsilon \quad \forall j \in J, \forall t \in T \ (11)$$

$$-c_{tj}\,\varepsilon + [D_{(t-1)j} + R_{(t-1)j} - Tc_j]\phi \leq R_{tj} \leq [D_{(t-1)j} + R_{(t-1)j} - Tc_j]\phi + c_{tj}\,\varepsilon \quad \forall j \in J, \forall t \in T \ (12)$$

$$-c_{tj}\,\varepsilon + [D_{\rho j} + R_{\rho j} - Tc_j]\phi \leq R_{\theta j} \leq [D_{\rho j} + R_{\rho j} - Tc_j]\phi + c_{tj}\,\varepsilon \quad \forall j \in J \ (13)$$

The piecewise constraints in equations (11), (12) and (13) ensure that if 50% of total demand and recapture of the loss demand in time period *t*-1 at station *j* minus the total capacity of the charging station *j* is equal to or more than the total capacity of the station *j* then recapture of the loss demand in time period *t* at station *j* is equal to the total capacity of the station *j*. Otherwise, it is equal to the sum of the 50% of total demand and recapture of the loss demand in time period *t*-1 at station *j* minus the total capacity of the charging station *j*.

$$Ns_j = \frac{Tc_j}{sc} \qquad \forall j \in J \tag{14}$$

The constraints in equation (14) ensure that the number of slots opened per charging station *j* is the division of total capacity of the charging station *j* by the slot capacity.

$$Nd_{tj} = (D_{tj}^1 + D_{tj}^2) \qquad \forall j \in J, \forall t \in T \tag{15}$$

The constraints in equation (15) ensure that the total nominal demand in time period *t* at charging station *j* is equal to the demand satisfied by the direct charge of station *j* in time period *t* and the demand satisfied by the battery of station *j* in time period *t*.

$\forall j \in J, \forall t \in T$, let α_{tj} be such that

$$\alpha_{tj} = \begin{cases} 1 & \text{if solar production } t \text{ is allocated to station } j \\ 0 & o.w \end{cases}$$

$$L_{tj} = L_{(t-1)j} + Bc_{tj} - e\,{D_{tj}}^2 + eB_{tj}^- \qquad \forall j \in J, \forall t \in T \tag{16}$$

$$L_{\theta j} = L_{\rho j} + Bc_{\theta j} - e\,{D_{\theta j}}^2 + eB_{\theta j}^- \quad \forall j \in J \tag{17}$$

$$Bc_{tj} = W_t\omega_{tj} + S_t\alpha_{tj} + g_{tj}^+ - g_{tj}^- - D_{tj}^1 \quad \forall j \in J, \forall t \in T \tag{18}$$

The set of energy balance constraints include the battery level transition as equation (16), the energy balance for the battery charge as equation (18). Moreover, the constraints in equation (17) ensure the battery level at the first stage is calculated using battery level transition equation and the battery level at the previous stage (96 stage).

$$g_{tj}^{-} \leq (W_t \omega_{tj} + S_t \alpha_{tj} + g_{tj}^{+}) \quad \forall j \in J, \forall t \in T \tag{19}$$

$$g_{tj}^{+} \leq Nd_{tj} \quad \forall j \in J, \forall t \in T \tag{20}$$

The constraints in equation (19) ensure that the electricity sold to the grid from the direct charge of station *j* in time period *t* should be less than or equal to the sum of the total wind purchased by station *j* in time period *t*, the solar production of station *j* in time period *t* and the electricity bought from the grid by station *j* in time period *t*. Similarly, the constraints in equation (20) ensure that the electricity bought from the grid by station *j* in time period *t* should be less than the total nominal demand in time period *t* at charging station *j*.

$$B^{-}{}_{tj} + D_{tj}^{2} \leq dc * e * x_j \quad \forall j \in J, \forall t \in T \tag{21}$$

$$Bc_{tj} \leq cr * x_j \quad \forall j \in J, \forall t \in T \tag{22}$$

$$v * x_j \leq L_{tj} \leq u * x_j \quad \forall j \in J, \forall t \in T \tag{23}$$

The constraints in equation (21) ensures that the sum of the electricity sold back to the grid from the battery of station *j* in time period *t* and the demand satisfied by the battery of station *j* in time period *t* cannot be higher than the product of discharge rate and storage efficiency of station j. Similarly, the constraints in equation (22) ensure that the battery charge of station *j* in time period *t* should be within the battery charge capacity. The constraints in equation (23) ensure that the battery inventory is between the minimum battery level and maximum battery level for each station. Moreover, (21) - (23) are only are considered when station *j* is operational.

$$\sum_{j \in J} \omega_{tj} \leq 1 \qquad \forall t \in T \quad (24)$$

The constraint in equation (24), ensure the fraction of the allocation of the wind generation to all the stations is no more than 1.

$$\omega_{tj} \leq x_j \qquad \forall j \in J, \forall t \in T \quad (25)$$

$$\alpha_{tj} \leq x_j \qquad \forall j \in J, \forall t \in T \quad (26)$$

Constraints in equations (25) and (26) ensure that the total wind purchased by station *j* in time period *t*, and the solar production of station *j* in time period *t* are only possible if the stations are operational.

$$Ns_j \leq \beth\, x_j \qquad \forall j \in J \quad (27)$$

The constraints in equation (27) ensure that the number of slots opened is between zero and the maximum number of slots opened per charging station *j*.

$$Nd_{tj}, D_{tj}, D_{tj}^{1}, D_{tj}^{2}, \omega_{tj}, Tc_j, Ns_j, L_{tj}, g_{tj}^{+}, g_{tj}^{-}, Bc_{tj}, B^{-}_{tj}, R_{tj} \geq 0 \qquad \forall j \in J, \forall t \in T \quad (28)$$

$$x \in \mathbb{B}^{|J|} \quad (29)$$

$$y \in \mathbb{B}^{|I|*|J|} \quad (30)$$

$$\alpha, a, b, c \in \mathbb{B}^{|T|*|J|} \quad (31)$$

The constraints in equations (28)-(31) ensure that that variables are nonnegative, and *x, y* and α, a, b, c are binaries of appropriate dimension. Furthermore, the constraints in equation (4), (5) and (25) create a dependent relationship between the stations and prevent the problem to be separable.

3.4. DACE Based System Design Optimization Problem Formulation Approach

DACE as proposed in [26] is used as a statistical basis for designing experiments for efficient prediction. As mentioned in [27], in DACE, an experimental design is used to organize a set of computer experiment runs, to enable fitting of a statistical "meta-model" that approximates a complex system's output from the computer experiment. Specifically, in this paper we use the steps used in [28] to solve the system of EV charging stations using DACE approach.

I. Binned LHS Design

As mentioned in [28], a binned LH design is used to generate 250 training data points and 75 testing data points of the locations of the charging stations and the number of slots at each charging station as shown in Appendix A [(x^1, Ns^1),….. , (x^N, Ns^N)].

II. Second stage control problem

For each system of charging stations $(\bar{x}, \overline{Ns})$ in the design, the corresponding control problem revenue $Rev(\overline{Ns})$ is determined using MILP as shown in equation (32).

$$Rev(\overline{Ns}) = \max \sum_{t \in T} \sum_{j \in J} \left[\left(M_t (g_{tj}^- + B_{tj}^- - g_{tj}^+) + r_t Nd_{tj} \right) \right] \quad (32)$$

s.t. Equations (2) – (31).

III. MARS model

In this paper, MARS, introduced in [29], is fitted to the design obtained by the binned LH and the corresponding revenues generated by solving second stage control problem. The fitted model as illustrated in equation (33), predicts the revenue.

$$\widehat{Rev}(\overline{Ns}) = \beta_0 + \sum_{k=1}^{K} \beta_k BF_k (\overline{\mathrm{Ns}}) \quad (33)$$

The MARS model determined in this research is given in Appendix B. In our paper, we fit two different MARS models, one with basis interaction terms and the other with no interaction.

IV. First stage EV system master problem

The system of charging stations (x^{*}, Ns^{*}) is obtained by maximizing profit using the optimization problem given by (32).

$$\text{where } \hat{Z}_{MILP}(x^{*}, Ns^{*}) = \max \widehat{Rev}(\overline{Ns}) - c_j x_j - Nc_j Ns_j \quad (34)$$

s.t. Equtaions (28) – (31).

To evaluate the quality of (x^{*}, Ns^{*}) , $Z_{MILP}(x^{*}, Ns^{*})$ is then calculated solving equation (1) – (32) using CPLEX, where $x = x^{*}$ and $Ns = Ns^{*}$. The obtained $Z_{MILP}(x^{*}, Ns^{*})$ and (x^{*}, Ns^{*}) are the solutions to the DACE approach.

4. System Design Experiments

4.1. MILP Experiments

From a computational perspective, all our runs are executed on a workstation equipped with Intel Core i7 CPU @3.50 GHz *12 and 32 GB RAM and are solved using [30]. Data on wind generation, solar generation, and market price are from 2012 used in this study [21-23]. Data on demand profiles are from [9]. The average retail price of electricity is 10.17 cents per kilowatt-hour [24]. The maximum and minimum battery capacities are 3.6 MWh and 720 kWh per slot. The charging rate and discharging rates are 600 kW and 75 kW per slot. The capacity of every slot is 0.01875 Mwh. In our study, the storage efficiency is considered to be 79.8% [25]. For convenience, we assumed stations more than a 20-mile radius are not able to fulfill the demand of a hotspot. The cost of the slots opened, are assumed to be 10% of the setup cost of each station. The maximum number of slots to be opened is 10. As market prices change every 15 minutes, our formulation considers a daily control problem consisting of 96 15-minute periods. When demand for a time period exceeds capacity, we assume 50% of the customers are willing to wait to be served in a subsequent time period (recaptured rate), while 50% are lost. Given that the processing time normally requires 5 days for each scenario, and the solution is found before 2 hours, a limit of 6 hours is used in all scenarios, except the scenario in which opening the stations costs $100 per day

Table 2.1. Number of Slots Per Charging Stations vs. Different Cost Scenarios

Cost		Station Location (Individual Profits and # of Slots)											Total	Total Profit	Duration	Status
Station	Slot	Palmer	Heartland	Fort Worth	Rockwall	Dallas	Waxahachie	Garland	Plano	Denton	Greenville	Godly	Open			
$ -	$ -	$160.64	$ 168.80	$1,373.42	$179.46	$ 411.99	$ 167.34	$453.14	$162.96	$ 221.59	$ 163.85	$170.66	11	$ 3,633.83	6 Hrs	Stopped
		1	5	6	1	5	1	3	1	4	1	1	29			
$ 40.00	$ 4.00	$116.64	$ 124.80	$ 845.72	$142.69	$ 971.90	$ 147.54	$219.44	$118.96	$ 174.08	$ 119.85	$126.79	11	$ 3,108.39	6 Hrs	Stopped
		1	1	5	1	5	1	2	1	2	1	1	21			
$ 50.00	$ 5.00	$118.20	$ -	$ 398.59	$151.60	$1,157.51	$ -	$220.31	$424.87	$ 399.27	$ -	$122.38	8	$ 2,992.73	6 Hrs	Stopped
		1	0	5	1	5	0	2	1	1	0	1	17			
$ 60.00	$ 6.00	$ -	$ -	$ 866.69	$355.60	$ 543.56	$ -	$454.75	$ -	$ 684.76	$ -	$ -	5	$ 2,905.34	6 Hrs	Stopped
		0	0	5	1	5	0	2	0	1	0	0	14			
$ 70.00	$ 7.00	$ -	$ -	$ 704.60	$295.34	$ 424.65	$ -	$415.07	$ -	$1,003.91	$ -	$ -	5	$ 2,843.57	6 Hrs	Stopped
		0	0	5	1	4	0	2	0	1	0	0	13			
$ 100.00	$ 10.00	$ -	$ -	$2,072.21	$ -	$ 293.96	$ -	$201.76	$ -	$ 121.45	$ -	$ -	4	$ 2,689.38	4 Days 23 Hrs	Complete
		0	0	5	0	4	0	3	0	1	0	0	13			
$ 200.00	$ 20.00	$ -	$ -	$1,869.73	$ -	$ 405.68	$ -	$ -	$ -	$ -	$ -	$ -	2	$ 2,275.42	6 Hrs	Stopped
		0	0	5	0	4	0	0	0	0	0	0	9			
$ 300.00	$ 30.00	$ -	$ -	$2,067.06	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	1	$ 2,067.06	3 hrs 23 min	Complete
		0	0	5	0	0	0	0	0	0	0	0	5			
$ 400.00	$ 40.00	$ -	$ -	$1,925.83	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	1	$ 1,925.83	5 hrs 33 min	Complete
		0	0	4	0	0	0	0	0	0	0	0	4			
$2,000.00	$200.00	$ -	$ -	$ 51.53	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	1	$ 51.53	6 Hrs	Stopped
		0	0	1	0	0	0	0	0	0	0	0	1			

Table 2.1 illustrates the optimal number of stations to be opened and the number of slots to be opened along with the individual profits generated from each station, at different cost scenarios. Moreover, computational time and the status whether the run is completed or is stopped where a limit of 6 hours is used, is shown in the Table. It is observed that with

no operational cost, all the stations are opened, and the number of slots opened and the total profit generated is higher as compared to the other scenarios. As the cost increases, more stations are left closed and the number of slots decrease within their respective stations. However, when increasing the cost from $70 to $100, the number of slots in Garland increased from 2 to 3, because Garland now captures the demand from Rockwall (closed for $100).

To be as realistic as possible, we realize that it would not be necessary to open 8 or more stations to meet the demand whereas opening under 3 stations will not be sufficient enough to fulfill the demand. As illustrated in the Table above, operational cost of $60, $70 and $100 have 5, 5 and 4 stations being opened respectively. $100 seems to be more convincing cost of operating a charging station as compared to the other two. Hence, a station cost of $100 per day is considered as the base line. Using a cost of $100 per day, it took 4 days and 23 hours of processing time to provide an optimal solution. The objective value of the optimal solution is $2689.38 ($Z_{MILP}$), and the stations to be opened are Forth Worth, Dallas, Garland and Denton and their individual profits for these stations are $2072.21, $293.96, $201.76 and $121.45 per day, respectively. For each of these locations the number of slots opened are 5, 4, 3 and 1, respectively.

It has been observed that the solution (best integer) is normally obtained between 20 minutes to 1 hour 55 minutes, and to prove a solution is the optimal, using the branch and cut algorithm, it takes close to 5 days.

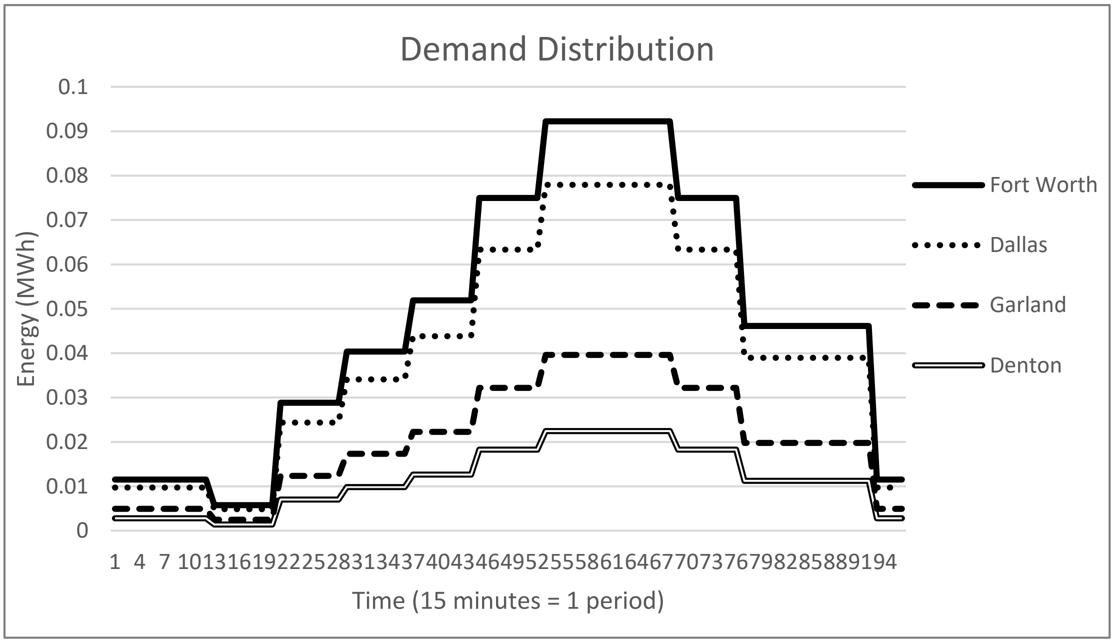

Figure 2.4. Demand distribution at an operational cost of $100

Figure 2.4, illustrates the demand distribution per station in time periods from 1 to 96. It can be observed that the total demand distribution for Fort Worth is the highest, Dallas is the second highest, followed by Garland, and Denton, respectively. The total demand in a day at each station are 4.61, 3.89, 1.97 and 1.12 Mwh, respectively Moreover, the demand is lowest from time period 12 to 20 i.e. from 3 am to 5 am and then it gradually increases and has high demand between time periods 52 to 68 i.e. from 1 pm to 5 pm. Afterward, the demand decreases.

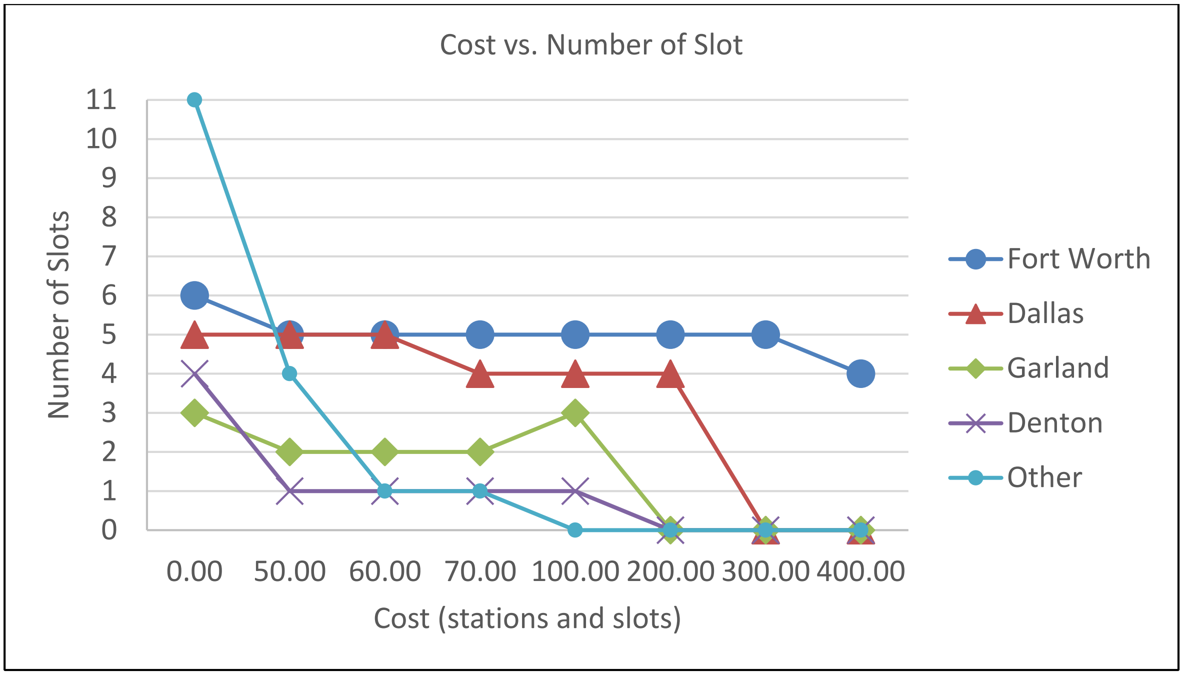

Figure 2.5. Major stations: different cost vs. number of slots

Figure 2.5 indicates that at a cost of $0, all 4 main stations have higher number of slots open. Fort Worth and Dallas have a high slot count of 6 and 5, respectively, whereas Denton and Garland close by with a slot count of 4 and 3, respectively. Fort Worth and Dallas hold a high slot count even though the cost increases, whereas the number of slots at Garland and Denton start decreasing. This plot is a clear depiction of a practical scenario, where at a certain point opening slots with higher costs is not profitable, given the demand. In other words, at a cost $200 per day, the profits generated from Garland and Denton are less than their costs, which leads to their closure.

4.2. DACE Based System Design Optimization Experiments

Table 2.2. Comparisons of the DACE MILP objective solutions

Software for MARS model	Interaction allowed or not	Testing R2	$Z_{MILP}(x^*, Ns^*)$	
			Cplex CP /Couenne (% Diff)	MINOS (% Diff)
ARESLAB	Yes	97.0	2606.8 (3.1)	2566.8 (4.6)
ARESLAB	No	97.7	2670.0 (0.7)	2574.6 (4.3)
SPM	Yes	97.8	2614.0 (2.8)	2555.8 (5.0)
SPM	No	**98.7**	**2678.5 (0.4)**	2629.5 (2.2)

The baseline cost of $100 per station per day is considered in all our DACE based system design optimization experiments. MARS models for revenue are created using 2 software packages, MATLAB 8.6 –ARESLAB toolbox developed by [31] and Salford Predictive Modeler 8.0 (SPM) from [32], one with basis interaction terms and the other with no interaction, creating 4 different models. Once the MARS models are developed, they are optimized using 3 different software programs (CPLEX CP Optimizer [30], AMPL 11.2 developed by [33] using Couenne solver developed by [34] and MINOS solver developed by [35]. The systems of charging stations obtained using CPLEX CP and Couenne are identical, so 8 unique systems were generated using the DACE approach, as shown in Table 2.2. The testing coefficient of determination R-squared for each MARS model is computed as shown in the Table. Moreover, the percentage difference between the objective solutions $Z_{MILP}(x^*, Ns^*)$ and Z_{MILP} is also presented in parenthesis as % Diff in Table 2.2. Based on initial analysis, the DACE approach using the MARS metamodel without interaction from the SPM software performs best, with a percentage difference of $Z_{MILP}(x^*, Ns^*)$ of 0.4%, and has the highest testing R-squared.

In this study, systems of charging stations from DACE approaches with metamodels without interaction terms are more accurate than those with interaction terms, suggesting that there is not much of a demand shift because of the stations are far apart from each other. In other words, demand distribution (e.g., equations 4 and 5) and allocating wind across the different stations (e.g., equations 25) has little influence on the solution in this case study.

Table 2.3. Number of Slots (MILP vs. DACE)

	Number of slots per opened Stations (cost $100)				
	Fort Worth	Dallas	Garland	Denton	Total
MILP	5	4	3	1	13
Cplex CP/Couenne	4	5	2	2	13
MINOS	3	5	2	2	12

The system design build (x^*, Ns^*) obtained from our best model is further analyzed and compared to that from solving the MILP in using branch-and-cut in Table 2.3. Observe that all the system design builds have the same open stations, which are at locations Fort Worth, Dallas, Garland and Denton. Also, the total number of slots opened using MILP, CPLEX CP, AMPL – Couenne are 13, whereas AMPL – MINOS are 12.

4.3. Separable Case

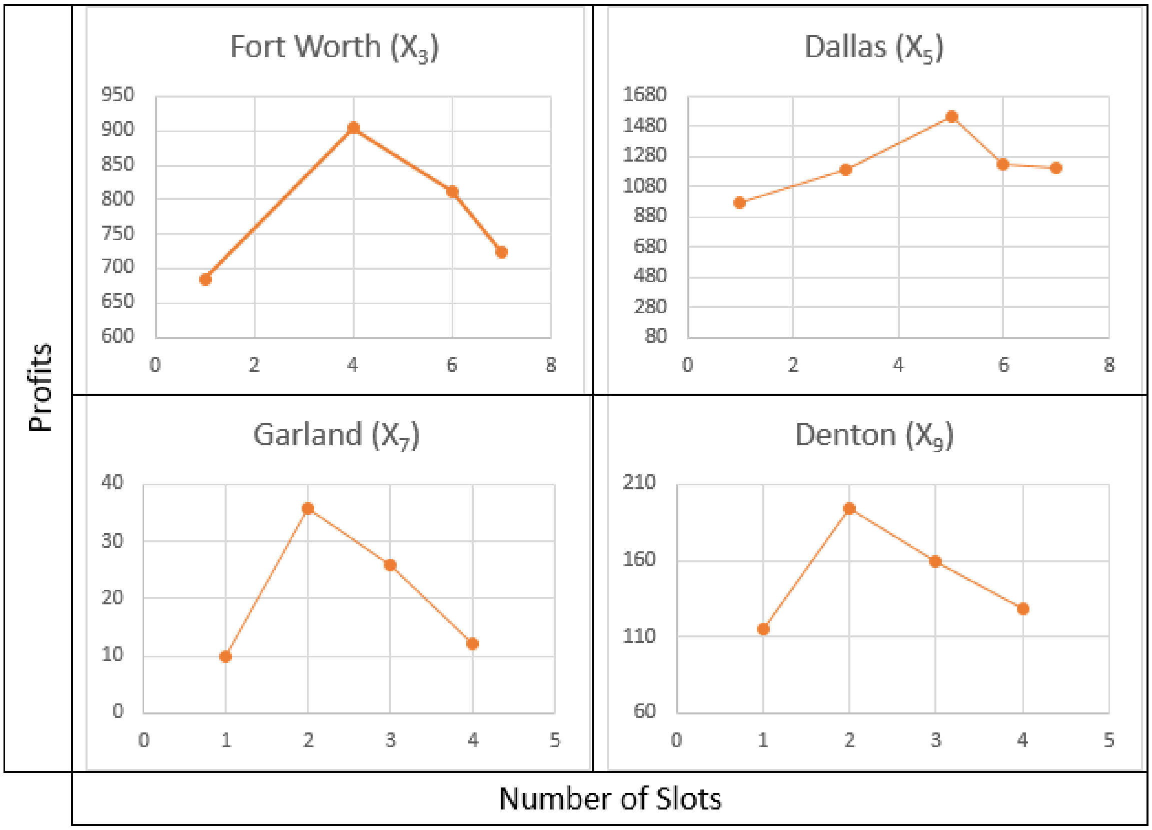

Figure 2.6. Separable Case

The DACE approach allow us to analyze the marginal profits as a function of the number of slots opened at each station. Plots of the marginal profits to open slots are given in Figure 2.6. The significant basis functions are associated with Fort Worth, Dallas, Garland, and Denton. All of the other basis functions associated with other stations have zero coefficients in the estimated revenue function, meaning that MARS determined that the marginal profits of these stations are insignificant, and consequently, the DACE optimization step kept them closed. It can be observed that for the Fort Worth station, 4 slots is the optimal solution, whereas for the Dallas station, 5 slots is the optimal solution. Similarly, for Garland and Denton, 2 slots per station is the optimal solution, which resembles the results (x^*, Ns^*) obtained from CPLEX CP and AMPL – Couenne. Moreover, the profit obtained by Fort Worth with the optimal number of slots is $904.55. Similarly, for Dallas, Garland and Denton, it is $1543.95, $35.67 and $194.71 respectively adding up to be $2678.88, identical to the $Z_{MILP}(x^*, Ns^*)$ obtained from MARS model without interaction using SPM software, as illustrated in Table 2.3.

4.4. CPU Time Comparisons

Table 2.4. CPU Time Comparisons (MILP vs. DACE)

Task	**Time**
Binned LHS Design	1 sec
Revenue Function (250 Training and 75 Testing - 10 sec average)	54 min

Interaction		No interaction	
MARS	*Times*	*MARS*	*Times*
SPM Cplex CP	12 sec 1 min 20 sec	SPM Cplex CP	6 sec 34 sec
DACE - Total	**55 min 32 sec**	**DACE - Total**	**54 min 40 sec**

Original MILP (CPLEX) - 4 days 23 hours

To justify the use of the DACE based system design optimization approach, the process run time is calculated for the 2 different models (interaction and non-interaction) as shown in Table 2.4. Once a binned LHS design is generated using MATLAB, the revenue values are collected using CPLEX. Although it took 54 minutes for this process, it is only required to be completed once for all further processes as it is independent of the cost scenarios. After the revenues are collected, it is used to fit the MARS model using the SPM software. Finally, the MARS model is optimized on CPLEX CP and the results are collected. The best process time is 54 minutes and 40 seconds, for the non-interaction model. Given that this model is the revenue model (cost not considered), it is flexible to handle different cost scenarios, without having to collect responses. This, in comparison to the original computation time of 4 days and 23 hours, is more practical.

5. Conclusion

A mixed-integer linear programming (MILP) model is formulated to optimize the locations of the EV Charging stations, the number of slots to be opened at each station and the overall profit. Based on the results obtained from solving the MILP, Fort Worth has the most slots to be opened, followed by Dallas, Garland, and Denton. The drawback of this approach, despite giving an optimal solution, is its extremely high computational time (as much as 5 days). Hence, we utilized a two-stage framework and a DACE based system design optimization approach to solve the system of EV charging stations. In this study, systems of charging stations from DACE approaches with metamodels without interaction terms are better than those with interaction terms, suggesting that there is not much of a demand shift because of the stations being far apart from each other. Moreover, the DACE approach allow us to analyze the marginal profits as a function of the number of slots opened at each station. The significant basis functions are associated with Fort Worth, Dallas, Garland, and Denton. Instead of 4 days and 23 hours using branch and cut to solve the MILP, the DACE approach requires roughly 1 hour to find a solution within 1% of optimal.

For future work, solving the problem with stochastic input variables for wind and solar power generation and market price will be investigated.

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Chapter 3. A Two-Stage Design and Analysis of Computer Experiments Approach for Optimizing a System of Electric Vehicle Charging Stations

Abstract

A two-stage framework is developed to address the design of a system of electric vehicle (EV) charging stations. The first stage specifies the design of the system that maximizes expected profit. Profit incorporates costs for building stations and revenue evaluated by solving a system control problem in the second stage. The control problem is formulated as an infinite horizon, continuous-state stochastic dynamic programming problem. To reduce computational demands, a numerical solution is obtained using approximate dynamic programming (ADP) to approximate the optimal value function. To obtain a system design solution using our two-stage framework, we propose an approach based on design and analysis of computer experiments (DACE). DACE is employed in two ways. First, for the control problem, a DACE-based ADP method for continuous-state spaces is used. Second, we introduce a new DACE approach specifically for our two-stage EV charging stations system design problem. This second version of DACE is the focus of this paper. The "design" part of the DACE approach uses experimental design to organize a set of feasible first-stage system designs. For each of these system designs, the second-stage control problem is executed, and the corresponding expected revenue is obtained. The "analysis" part of the DACE approach uses the expected revenue data to build a metamodel that approximates the expected revenue as a function of the first-stage system design. Finally, this expected revenue approximation is employed in the profit objective of the first stage to enable a more computationally-efficient method to optimize

the system design. To our knowledge, this is the only two-stage stochastic problem which uses infinite horizon dynamic programming approach to optimize the second stage dynamic control problem and the first stage system design problem.

1. Introduction

Alternative energy sources are critically important for curbing greenhouse gas emissions and creating a more independent energy economy. According to the website of the Energy Information Administration (2016), the transportation sector alone causes up to 36.41% of all energy-related emissions and is the largest producer of carbon dioxide emission in the U.S. This presents an urgent need for the transportation sector in the U.S. to act on emissions abatement. Furthermore, energy power in the transportation sector is derived almost exclusively from fossil fuels, making the U.S. the world's largest consumer of crude oil and petroleum products. Each day, Americans consume approximately 20 million barrels of petroleum, and import approximately 7 million barrels as per the website of the Oak Ridge National Laboratory (2016). Not only is available oil harder and more dangerous to attain, but the volatility of oil prices is threatening both energy security and the economy. If this trend of heavy reliance on petroleum continues as projected, the gap between oil consumption and production is going to become even wider. Eventually, the cost of oil dependence to both national security and the economy will be too high to afford. The average price of electricity is relatively stable compared to oil prices, and the electric power grid is fully constructed throughout the country, making electricity more attractive than other alternative substitutions for petroleum, such as biomass and hydrogen.

Environmental pollution is one of the worst effects of our accelerated development rate. An integral part of environment pollution is air pollution, which is caused primarily by usage of traditional sources of energy such as coal, natural gas, and oil. Finding reliable, renewable, and non-polluting substitutes to these is necessary for sustainable preservation of natural environment. To combat such high levels of air pollutants, we need a major transformation. With higher number of EVs in market, there will be a substantial reduction in CO_2 emissions which would help in sustainable development. The idea of breezing past gas stations without leaving a carbon footprint behind appeals to electric car enthusiasts, but the fear of running out of battery power has been a major barrier in getting people to buy EVs.

Just like traditional vehicles, EVs will need "re-fueling." In our research, we study the design of system of EV charging stations to support the growing population of EVs. An appropriate design must hedge against the volatility of the market price of electricity. We propose a method for assessing system controllability that reduces computational costs and allows more extensive design exploration. Moreover, our method allows us to consider uncertainty in both systems input and in the demand for the output. When dealing with high-dimensional infinite horizon stochastic dynamic programming data set with continuous state space, the limit called 'curse of dimensionality' obstructs the solution as the size of the state space grows exponentially, making it computationally intractable to solve using traditional methods.[1] To resolve this limit, Chen et al. (2017a) proposed design

[1] Kulvanitchaiyanunt, Asama. "A Design And Analysis Of Computer Experiments-based Approach To Approximate Infinite Horizon Dynamic Programming With Continuous State Spaces." (2014).

and analysis of computer experiments (DACE) based infinite horizon approximate dynamic programming (ADP) algorithm to sample the state space with design of experiment and to approximate the value function via statistical modeling methods. The control problem is formulated as an infinite horizon, continuous-state stochastic dynamic programming problem. ADP approach is based on DACE approach, to optimize this high-dimensional, large-scale, EV charging stations system design problem and control problem over continuous spaces.

This paper focuses on the DACE approach for the two-stage stochastic optimization where a binned Latin hypercube (LH) experimental design is derived, and a multivariate adaptive regression splines (MARS) model is applied for analyzing computer experiment data. The first stage focuses on the system design problem, while the second stage focuses on the system control problem. Requirements and parameters for cost and performance are handled by the first stage. For a specific system design, simulated information on system behavior and uncertainties is used by the second-stage control optimization to provide a performance objective on controllability back to the first stage. The expected revenueis returned from the second stage as part of the objective for the first-stage system design master problem, which is then solved using the computer program. Uncertainties due to the variation in wind power, solar power, the market price of electricity, and customer demand will be incorporated by our EV system simulation, and since these uncertainties are not well understood, they will be modeled flexibly to enable exploration of various levels of uncertainty. Historical data will be employed fully possible.

1.1 Literature review

In the existing papers, different approaches and algorithms had been proposed and implemented such as the design of optimal EV charging profiles, optimization of the locations for EV charging stations, DC fast charging system, battery swapping methodology, economic operation of a microgrid-like EV parking deck to ease drivers' anxiety about the driving range with the increase the number of EVs on road.

1.1.1 General approaches

Ashtari et al. (2012) recorded vehicle usage and used it to predict PEV charging profiles and electrical range reliability. The effect of charging scenario on electrical range reliability is more significant where battery sizes are smaller. Steen et al. (2012) proposed an approach to control PEVs charging based on the charging behavior estimated from the demographical statistical data, which leads to a well-developed public charging infrastructure that could reduce the stress on the residential distribution systems since part of the charging can be done in commercial areas. Xie et al. (2016) formulated the fair energy scheduling problem as an infinite-horizon MDP and employs ADP methodology to maximize long-term fairness and flatten the peak load in the distribution network. Zhu et al. (2012) provided a dynamic game theoretic optimization framework based on stochastic mean field game theory, the optimization will provide an optimal charging strategy for the EVs to proactively control their charging speed in order to minimize the cost of charging. To improve the penetration of sustainable energy in the charging systems, Badawy and Sozer (2017), Khodayar et al. (2012), Marano and Rizzoni (2008), and Guo et al. (2014) had employed wind or solar generation as a type of energy resources to supply electricity to the charging stations. Yao et al. (2017), considering demand response, formulated a binary optimization problem to simultaneously maximize the number of EVs for charging and minimize the monetary expenses. Sarikprueck et al. (2017) proposed a novel regional EV DC fast charging system equipped with renewable resources, such as wind and solar energy, to serve EV demand. However, the decision-making procedure is deterministic, and the system does not consider the future state but only the current state while making the decisions.

1.1.2 Approaches to optimize location of EV charging stations

The work in Tang et al. (2013), Lin and Hua (2015) and Arslan and Karasan (2016) proposed different approaches for finding optimal locations and sizes for charging stations. Chawal et al. (2018) proposed a deterministic mixed integer linear programming (MILP) model to obtain the optimal number of stations to be opened and the corresponding number of slots, along with profits for each station and overall profits based on the user-specified cost of opening a station. A robust integer linear optimization and a stochastic programming framework was proposed to solve the strategic optimization problem of determining optimal locations for charging stations of (ad-hoc) electric car-sharing systems, and for considering the uncertainty factors associated with vehicle-to-grid and wind power scenarios, respectively in Brandstatter et al. (2017) and Battistelli et al. (2012). Kulvanitchaiyanunt et al. (2015) utilized the system as mentioned in Sarikprueck et al. (2017), to make a decision for each stage through linear programming (LP) without considering the uncertainty.

Chen et al. (1999) proposed a finite horizon approach to approximate the value function, aiming to overcome the "curse of dimensionality". Orthogonal array (OA) was used to sample the state space and MARS to approximate the value function. Later, Chen et al. (2006) summarized this approach as DACE based ADP. ADP is a modeling framework that offers several strategies for tackling the curses of dimensionality in large, multi-period, stochastic optimization problems (Powell 2011). Chen et al. (2017) applied this DACE-based ADP approach to solve a control of a system of EV charging stations by solving large-scale, high-dimensional, dynamic control system (expected revenue) so that the decision-making procedure considers the uncertainty. As mentioned in the abstract, DACE is employed in two ways. First, for the control problem, a DACE-based ADP method for continuous-state spaces is used. Second, we introduce a new DACE approach specifically for our two-stage EV charging stations system design problem. The first version of DACE is adopted from Chen et al. (2017). In our paper, we employed this expected revenue in the profit objective of the first stage to optimize the system design.

1.1.3 Multi-stage stochastic programming framework

Several papers utilized a two-stage and a multi-stage stochastic programming framework to solve a finite-horizon stochastic programming problem. For instance, Pilla et al. (2008) utilized a two-stage stochastic programming framework to assign crew-compatible aircraft in the first stage, and to enhance the demand capturing potential of swapping in the second stage. He implemented design and analysis of computer experiments to reduce the computation involved in solving the problem. Pan et al. (2010) developed a two-stage stochastic program to optimally locate the stations prior to the realization of battery demands, loads, and generation capacity of renewable power sources. The overall objective function combines the first-stage cost and the expected cost of recourse actions over all scenarios. In the first stage, the location and size of exchange stations are decided. The expectation term in the objective represents the second-stage recourse cost of satisfying PHEV demands and meeting demands for power over a set of scenarios. Guo et al. (2016) addresses a two-stage framework for the economic operation of a microgrid-like electric vehicle (EV) parking deck. In this paper, the first stage provides the parking deck operators with a stochastic approach for dealing with the uncertainty of solar energy. The second stage introduces a model predictive control-based operation strategy of EV charging dealing with the uncertainty of parking behaviors within the real-time operation. Lulli and Sen (2004) presented a branch-and-price method to solve special structured multistage stochastic integer programming problem. This method is then specialized to a batch-sizing problem under uncertainty. They consider a finite-horizon sequential decision process under uncertainty.

1.2 Contribution

The aforementioned papers proposed different approaches and implemented several algorithms such as the design of optimal EV charging profiles, optimizing the locations for EV charging stations, DC fast charging system, battery swapping methodology, economic operation of a microgrid-like EV parking deck. By contrast the primary goal of our research is to optimize the EV system problem which includes both the system design problem and the dynamic control problem.

Pilla et al. (2008), Pan et al. (2010), Guo et al. (2016) and Lulli & Sen (2004) utilized a two-stage and a multi-stage stochastic programming framework to solve a finite-horizon stochastic programming problem whereas we develop a two-stage DACE framework to solve an infinite horizon stochastic dynamic problem.

As compared to Chawal et al. (2018) and Kulvanitchaiyanunt et al. (2015), where MILP and LP are used respectively to solve the optimization problem, where demand, market price, wind generation and solar generation are deterministic. In this paper, we have introduced stochasticity in our model by varying those parameters and solve it using infinite horizon ADP.

As compared to Chen et. al (2017), which considered only one randomly selected system design, we optimize the system design problem. Design of experiments is used to generate a binned LH design which is solved by DACE-based ADP approach to generate expected revenues. A MARS model is then fitted over the revenues obtained from the control problem and is solved by the computer program to deliver the best-known system design problem, making it more realistic and a robust model.

Moreover, when the designs obtained from our DACE approach and MILP design from Chawal et al. (2018) are solved using DACE-based ADP approach (simulation), an improvement of approximately 8% is observed in the simulated profit obtained from ADP design compared to that of MILP design indicating that when uncertainty is considered, DACE-based ADP design provides the better solution. To our knowledge, currently there exists no tractable approaches that can globally optimize a general high dimensional infinite horizon stochastic dynamic problem. Hence, to evaluate the quality of our methodology we employ the two-stage DACE framework to optimize the problem mentioned in Chawal et al. (2018). The results gained are near optimal with less than 1% loss in the solution generated by the obtained design (DACE MILP) as compared to the global optimal solution obtained using MILP method. These findings justify the effectiveness of the developed methodology and its widespread practical applications.

2. Methodology

In this section, we will describe about two–stage framework, infinite horizon SDP and their general formulation.

2.1 Two-Stage Stochastic Design and Control Framework

Our two-stage structure models the design problem as the first stage master problem and the dynamic control problem as the second stage sub problem. In general, there are two types of decision variables, the first stage system design variables and the second stage dynamic control variables. Like two-stage stochastic programming as mentioned by Birge and Louveaux (2011) and Sen and Sherali (2006), the first stage objective includes the second-stage cost objective. In classical stochastic programming, the second stage considers many future scenarios and solves the second-stage optimization for each of these scenarios. In our two-stage formulation, the second stage is a dynamic control problem that explores the future states of the system under uncertainty. For a specific system design, simulated information on system behavior and uncertainties is used by the second-stage control optimization to provide a performance objective on controllability back to the first stage. In theory, the expected revenueis returned from the second stage as part of the objective for the first-stage system design master problem.

2.1.1 First Stage

The first stage master problem is the system design function where the objective consists of costs on the design parameters and an expected cost $E_s\,[V\,(s, x)]$ from the second - stage optimal value function over possible initial states.[2] The calculation of the expected cost $E_s\,[V\,(s, x)]$ is illustrated in the section 3.2 below. As mentioned earlier, the system design problem variables consist of the locations of the charging stations and the number of slots at each station. The first stage master design problem can be formulated as:

$$c(x) + E_s\,[V\,(s, x)] \tag{1}$$

$$\text{s.t.} \quad x \in \Gamma_D \tag{2}$$

where;

- x are the system design problem variables.
- $c(x)$ is the "cost" objective in each time period of the second stage control problem.
- s is the initial state of the control problem.
- Γ_D is the constraint set for the system design variables (parameters).
- $V\,(s, x)$ is the optimal value function for the second stage dynamic control problem.

2.1.2 Second Stage

The main objective of this stage is to solve the dynamic control sub-problem using the DACE approach for the ADP as mentioned before. The second stage dynamic control infinite horizon sub - problem can be formulated mathematically as:

$$V(s_t; x) = \min E_\varepsilon\,[g(s_t, u, \varepsilon; x) + \gamma V(s_{t+1}; x)] \tag{3}$$

[2] Kulvanitchaiyanunt, Asama. "A Design And Analysis Of Computer Experiments-based Approach To Approximate Infinite Horizon Dynamic Programming With Continuous State Spaces." (2014).

$$\text{s.t. } u_t \in \Gamma_c(x), \tag{4}$$

$$s_{t+1} = h(s_t, u, \varepsilon; x) \tag{5}$$

The following notation is for the second stage dynamic control problem, given the set of system design variables *x*:

- s_t is the state of the system at the beginning of time period *t*.
- u is the second stage control vector.
- ε represents the uncertainty in system state dynamics.
- $g(s_t, u, \varepsilon; x)$ is the "cost" objective in each time period of the second stage control problem, given the system is in state s_t.
- $h(s_t, u, \varepsilon; x)$ is the state transition equation from time period *t* to *t*+1.
- $V(s_t; x)$ is the optimal value function for the second stage dynamic control problem.
- γ is the discount factor on future values.
- $\Gamma_c(x)$ is the constraint set for the control variables.

Finding an exact FVF is intractable for medium-sized problems. To reduce computational demands, numerical ADP, solution methods are needed to approximate the optimal value function. Following is a generalized approximate FVF (aFVF) ($\hat{V}$):

$$\hat{V}(s_t; x) = \min E_\varepsilon \left[g(s_t, u, \varepsilon; x) + \gamma \hat{V}(s_{t+1}; x) \right] \tag{6}$$

where $\hat{V}(s_t; x)$ is an approximation of the value function at time *t*.

3. Case Study: System of Electric Vehicle Charging Station

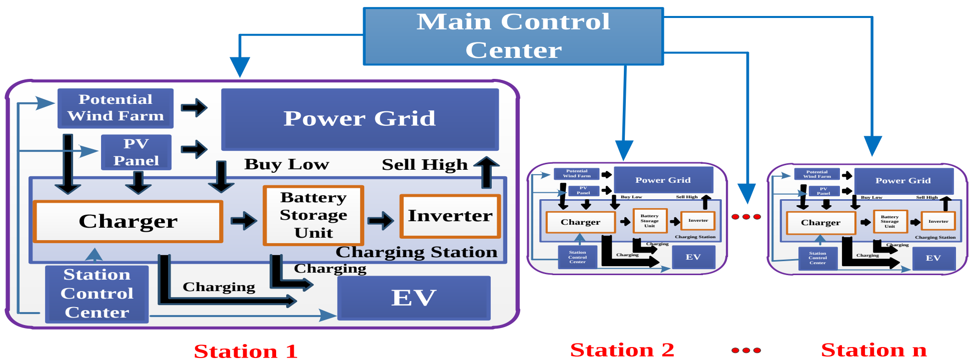

Figure 3.1. EV Charging Station Layout

The proposed EV charging station (Figure 3.1) is designed to get charged by wind/solar energy generation and electricity from the power grid. The wind/solar energy can provide energy to the grid. The energy stored in the charge station is used to charge the EVs. The system can store excessive wind/solar energy for future demand by storing it in the battery storage unit. The station can use the surplus stored energy to satisfy the demand, if the energy generated is not sufficient to satisfy the demand. Any excess energy from the storage unit will be sold back to the power grid for added profit.

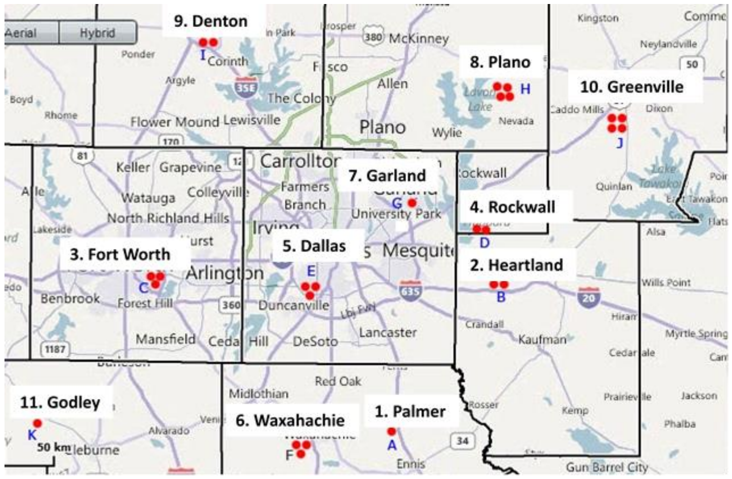

Figure 3.2. The locations of the station

Figure 3.2 above shows the distribution of the 11 potential station within the Dallas/Fort Worth area in Texas. The 140 hotspots (cities) we are focusing on are spread around these stations. In the following section, we discuss in detail our problem formulation as well as a step by step approach to solve our problem. We base our ADP approach on DACE, to solve this two–stage high-dimensional, large-scale, infinite-horizon, EV charging station system design problem and control problem over continuous spaces.

3.1 DACE Approach

DACE as proposed in Sacks et al. (1989) is used as a statistical basis for designing experiments for efficient prediction. DACE uses design of experiments (DoE) to sample the state space and statistical modeling to efficiently represent performance measures from a computer model. In our case study, we generate binned LH design to obtain a set of feasible first-stage system designs. For each of these design build, a corresponding revenue is obtained by solving the second stage control problem. We then use this expected revenue data to build a MARS model that approximates the expected revenue as a function of the first-stage system design. Finally, this expected revenue approximation is employed in the profit objective of the first stage to enable a more computationally-efficient method to optimize the system design. Our EV system design problem has taken into consideration uncertainties due the variation in wind power, solar power, the market price of electricity, and customer demand.

The following notation is used:

- N is the total set of design points.
- J is set of potential stations locations indexed by j.
- c_j is the cost of opening station j.
- Ns_j is the number of slots at station j.
- Nc_j is the cost of opening slots at station j.
- $x_j \in \{0,1\}$ is a binary variable, if station j is operational or not.
- $(\bar{x}, \overline{Ns})$ represents system station design data points.
- $Y(\overline{Ns})$ is the objective function of control problem (revenues).
- $Z(\bar{x}, \overline{Ns})$ is objective function of master problem (profits).
- BF_k is k-th basis function.
- β_0 is Y-intercept for MARS function.
- β_k represents least square estimators for k-th basis function.

The procedure for the DACE approach is as follows:

1. Create an experimental design (e.g., binned LH design) to sample points in the system design space. Each point corresponds to a set of parameters for specific system build.
2. For each system design point, solve the control problem (Chen et al. 2017). These solution runs correspond to the computer experiment. Save the approximate future value function from each run.
3. For each approximate future value function, calculate the expected value over the initial state space using a numerical integration approach that samples initial states and then averages these responses to obtain the expected revenues. These expected revenues correspond to the responses for each experimental design point specifying a system design.
4. Fit a statistical model (e.g., MARS) to the design obtained by the experimental design (e.g., binned LH) in step 1 and the corresponding expected revenues generated by step 3.
5. Use the obtained statistical model from step 4 in the first stage to identify the system design that maximizes profit, where profit is calculated by subtracting the cost component from expected revenue.
6. Calculate the true expected profit using the optimized system designs from step 5, where the expected revenue component is determined by simulating the ADP policy.

3.1.1 Binned LH Design

To be realistic, we considered how many stations would need to be open in order to meet the estimated demand. A range of about 4 to 7 stations was seemingly adequate. While we wanted to explore a range from 2-10 stations open, we desired an experimental design with more instances in the 4-7 range, or about half the stations closed.

As an initial experimental design, we used the function lhs design (number of design points were taken to be 100 and 75 for training and testing data points respectively, and the number of predictor variables were taken to be 11) in MATLAB, which may be downloaded from https://www.mathworks.com/. A partial output from the MATLAB function is shown in Table 3.1. We can then convert the fractional values in Table 3.1 to values between 0 – 10 that represent the number of slots; however, a direct conversion would yield instances that have all or most stations open, we instead created 19 bins that are equally-spaced over the [0,1] range. The first 9 bins, i.e., almost half of the bins, are assigned to zero slots, and the remaining 10 bins are equally distributed from 1 to 10 slots as specified in the Table 3.2 below. In other words, given the station is open, the number of slots that an open station will have, is a discrete uniform distribution with values between 1 and 10.

Table 3.1. 20 points using MATLAB lhsdesign

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
0.738	0.282	0.451	0.165	0.448	0.310	0.783	0.642	0.235	0.507	0.497
0.839	0.024	0.657	0.703	0.320	0.616	0.528	0.793	0.757	0.520	0.610
0.243	0.092	0.186	0.462	0.103	0.286	0.421	0.050	0.886	0.678	0.002
0.515	0.757	0.532	0.629	0.106	0.824	0.074	0.854	0.853	0.908	0.583
0.456	0.670	0.627	0.960	0.862	0.818	0.237	0.420	0.627	0.936	0.757
0.329	0.736	0.029	0.317	0.541	0.988	0.585	0.427	0.717	0.884	0.665
0.832	0.575	0.474	0.072	0.964	0.466	0.442	0.344	0.929	0.583	0.517
0.986	0.778	0.895	0.789	0.994	0.489	0.905	0.574	0.456	0.900	0.828
0.592	0.043	0.143	0.377	0.411	0.611	0.057	0.312	0.299	0.116	0.970
0.182	0.375	0.443	0.558	0.364	0.864	0.484	0.395	0.690	0.338	0.300
0.119	0.219	0.669	0.191	0.221	0.041	0.572	0.362	0.304	0.029	0.786
0.568	0.442	0.323	0.078	0.459	0.195	0.148	0.677	0.682	0.505	0.261
0.305	0.842	0.959	0.142	0.265	0.974	0.582	0.714	0.058	0.720	0.027
0.254	0.259	0.695	0.842	0.481	0.980	0.874	0.994	0.994	0.185	0.096
0.544	0.557	0.225	0.305	0.439	0.477	0.215	0.955	0.197	0.424	0.318
0.179	0.020	0.562	0.404	0.261	0.840	0.168	0.367	0.659	0.085	0.293
0.661	0.150	0.885	0.622	0.095	0.956	0.070	0.162	0.374	0.298	0.034
0.058	0.805	0.565	0.825	0.422	0.700	0.715	0.943	0.745	0.800	0.171
0.032	0.178	0.772	0.737	0.518	0.396	0.017	0.296	0.023	0.493	0.440
0.269	0.792	0.900	0.381	0.333	0.439	0.295	0.565	0.096	0.785	0.745

Table 3.2. Equally Spaced 19 bins

0.05263158	0
0.10526316	0
0.15789474	0
0.21052632	0
0.26315789	0
0.31578947	0
0.36842105	0
0.42105263	0
0.47368421	0
0.52631579	1
0.57894737	2
0.63157895	3
0.68421053	4
0.73684211	5
0.78947368	6
0.84210526	7
0.89473684	8
0.94736842	9
1	10

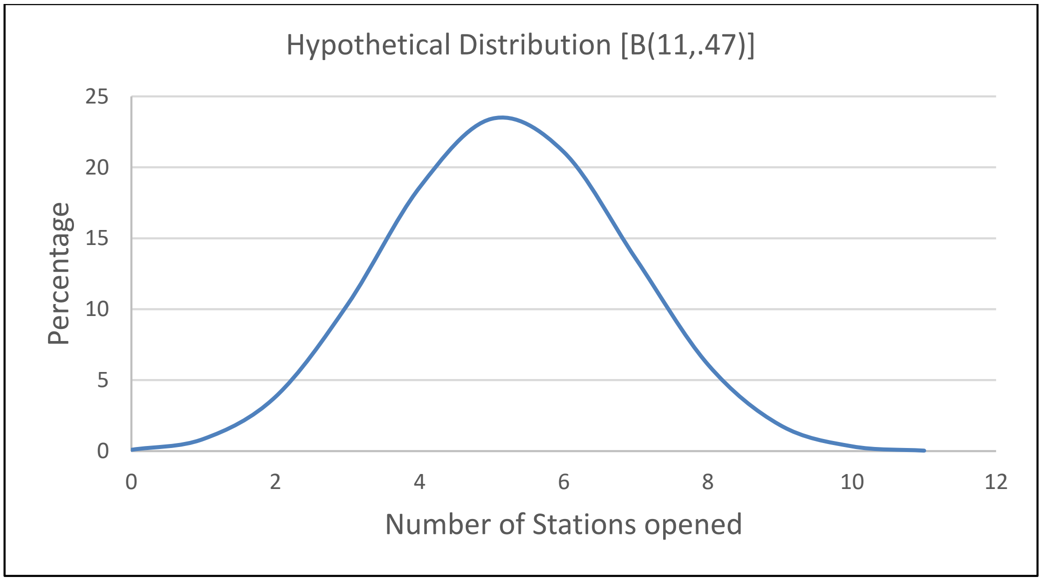

Figure 3.3. Hypothetical distribution

Hypothetically, we can state that the number of stations opened is a binomial random variable where n = 11 and p = .47368421 [B(11,.47)] as shown in Figure 3.3. The step function in Figure 3.4 illustrates how the bins are overlaid on the fractional values from Table 3.1. We refer to this experimental design as a binned LH design. Using this approach, we generated 100 training data points and 75 testing data points over the first-stage decision space, consisting of the locations of the charging stations and the number of slots at each charging station. We denote the pairs of location and number of slots from each experimental design point by ((x^1, Ns^1), ….. , (x^N, Ns^N)), where N (175) represents the total number of sample design points. The entire training and testing binned LH designs are included in the Appendix A. As observed in Figure 3.5, we can clearly observe that our binned LH design (observed distribution) closely resembles the desired hypothetical distribution, thus matching our requirement. About 80% of our instances have between 4-7 stations open.

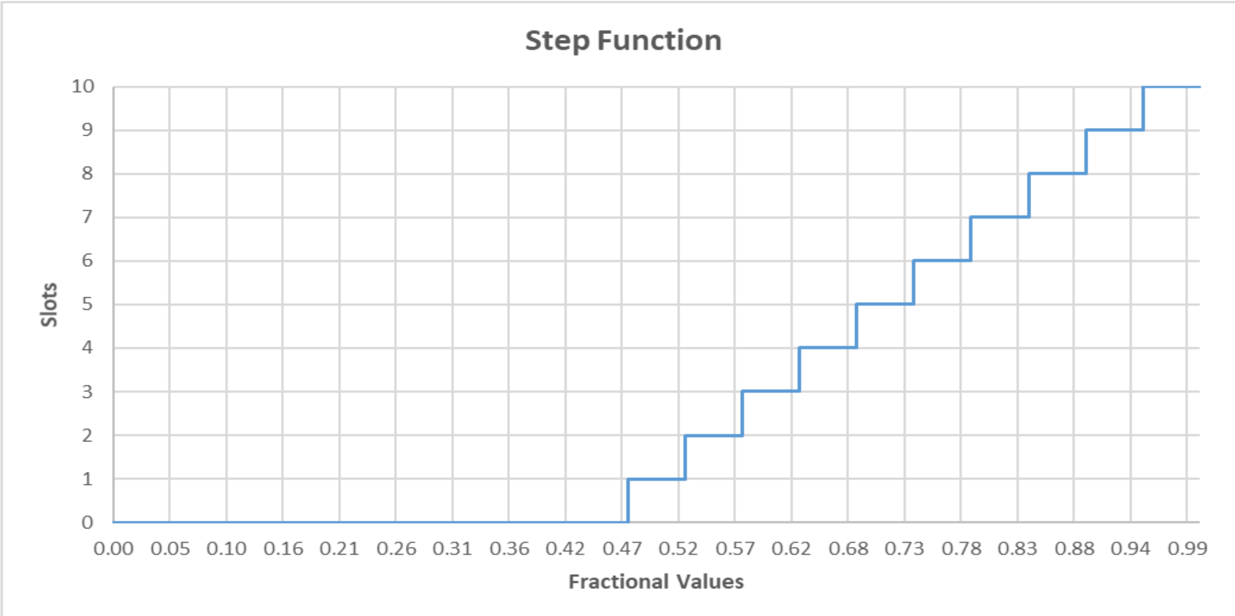

Figure 3.4. Step function

Table 3.3. 20 points from the Binned LH Design (Partial)

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
6	0	0	0	0	0	6	4	0	1	1
7	0	4	5	0	3	2	7	6	1	3
0	0	0	0	0	0	0	0	8	4	0
1	6	2	3	0	7	0	8	8	9	3
0	4	3	10	8	7	0	0	3	9	6
0	5	0	0	2	10	3	0	5	8	4
7	2	1	0	10	0	0	0	9	3	1
10	6	9	6	10	1	9	2	0	9	7
3	0	0	0	0	3	0	0	0	0	10
0	0	0	2	0	8	1	0	5	0	0
0	0	4	0	0	0	2	0	0	0	6
2	0	0	0	0	0	0	4	4	1	0
0	7	10	0	0	10	3	5	0	5	0
0	0	5	7	1	10	8	10	10	0	0
2	2	0	0	0	1	0	10	0	0	0
0	0	2	0	0	7	0	0	4	0	0
4	0	8	3	0	10	0	0	0	0	0
0	7	2	7	0	5	5	9	6	7	0
0	0	6	6	1	0	0	0	0	1	0
0	7	9	0	0	0	0	2	0	6	6

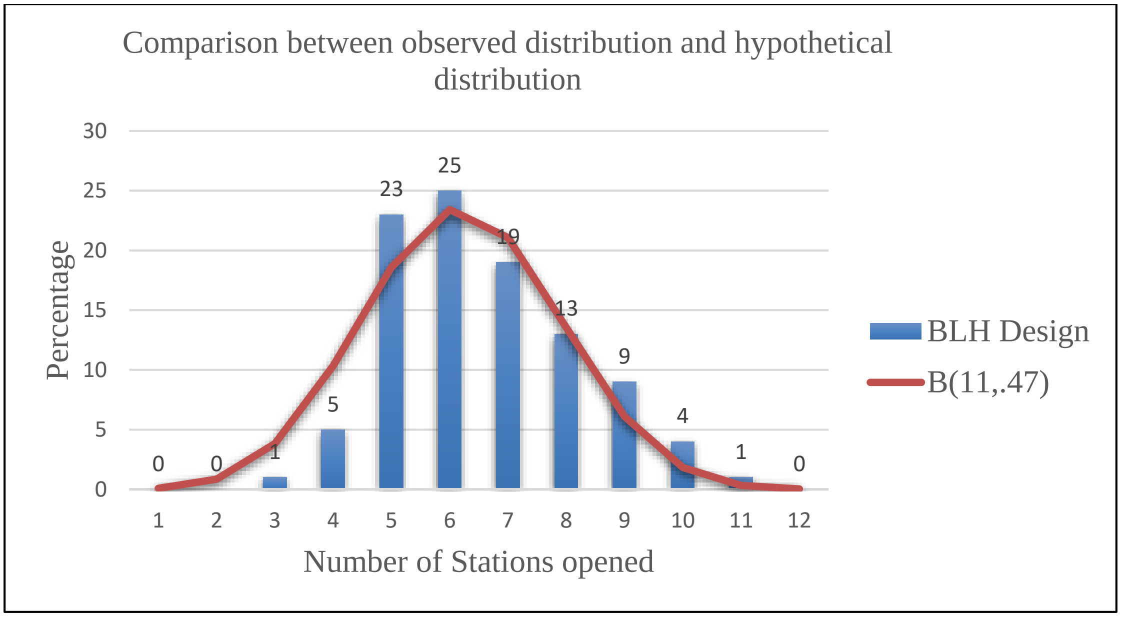

Figure 3.5. Comparison between observed distribution and hypothetical distribution

3.1.2 Second-stage control problem

For each system design point, we solved the control problem using DACE-based ADP approach as mentioned in Chen et al. (2017). These solution runs correspond to the computer experiment. Save the approximate future value function from each run.

For each approximate future value function, calculate the expected value over the initial state space using a numerical integration approach that samples initial states and then averages these responses to obtain the expected revenues. These expected revenues correspond to the responses for each experimental design point specifying a system design. In this case study, we sampled 117 initial states using Sobol sequence and then averaged 117 simulated revenues to obtain the expected revenue. The obtained revenues $(Y_{(x^1, Ns^1)}, \ldots, Y_{(x^N, Ns^N)})$ is the solution to our second-stage control problem, and represent $E_s[V(s; x)]$ component of the equation(1). This will be referred to as Y_{DA}.

For our system design, simulated information on system behavior and uncertainties is used by the simulation to provide a performance objective on controllability to return back to the first stage. The simulated ADP policy takes into consideration uncertainties due the variation in wind power, solar power, the market price of electricity, and customer demand. As shown in Figure 3.6, each row is fed as an input to the second-stage control problem. The DACE-based ADP solution policy (Chen et al. 2017) for the control problem is then simulated, as described above, to obtain the expected revenues. In the DACE perspective for the two-stage framework, the "computer model" is the combination of the DACE-based ADP solution approach followed by the simulation of the ADP policy. The expected revenues are the response values corresponding to the binned LH experimental design points over the system design space. The next section uses these data to fit a metamodel for the DACE approach.

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
6	0	0	0	0	0	6	4	0	1	1
7	0	4	5	0	3	2	7	6	1	3
0	0	0	0	0	0	0	0	8	4	0
1	6	2	3	0	7	0	8	8	9	3
0	4	3	10	8	7	0	0	3	9	6
0	5	0	0	2	10	3	0	5	8	4
7	2	1	0	10	0	0	0	9	3	1
10	6	9	6	10	1	9	2	0	9	7
3	0	0	0	0	3	0	0	0	0	10
0	0	0	2	0	8	1	0	5	0	0
0	0	4	0	0	0	2	0	0	0	6
2	0	0	0	0	0	0	4	4	1	0
0	7	10	0	0	10	3	5	0	5	0
0	0	5	7	1	10	8	10	10	0	0
2	2	0	0	0	1	0	10	0	0	0
0	0	2	0	0	7	0	0	4	0	0
4	0	8	3	0	10	0	0	0	0	0
0	7	2	7	0	5	5	9	6	7	0
0	0	6	6	1	0	0	0	0	1	0
0	7	9	0	0	0	0	2	0	6	6

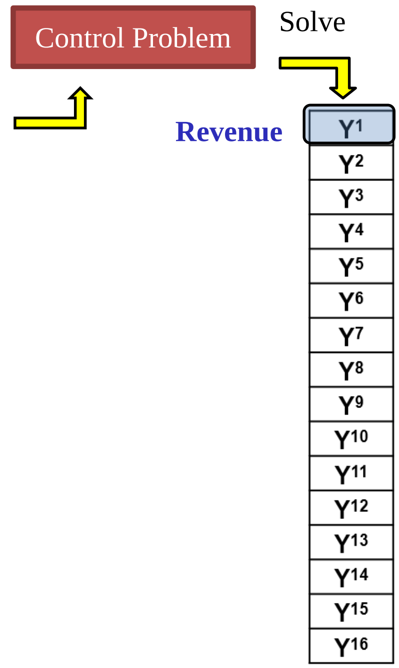

Figure 3.6. Computing revenues solving control problem

3.1.3 MARS model

Because curvature is apparent in the scatter plots, we employed the flexible MARS modeling method (Friedman 1991, Tsai and Chen 2005) to identify piecewise-linear basis functions. MARS is fitted to the expected revenue data from the previous section. The fitted model predicts the revenue as:

$$\hat{Y}_{DA} = \beta_0 + \sum_{k=1}^{K} \beta_k BF_k(\overline{\text{Ns}}) \tag{7}$$

The MARS function is created using Salford Predictive Modeler 8.0 (SPM) downloaded from https://www.salford-systems.com/products/mars. The obtained MARS model is as follows:

$$\hat{Y}_{DA} = 1766.01 - 96.3572 * BF_3 + 78.3721 * BF_4 - 69.8796 * BF_5 - 91.8993 * BF_8 + 753.933 * BF_9 - 61.2331 * BF_{10} + 77.2286 * BF_{11} + 69.5352 * BF_{12} + 61.6513 * BF_{14} \quad (8)$$

s.t

$BF_9 = \max(0, 1 – Ns_2);$
$BF_5 = \max(0, 5 – Ns_3);$
$BF_{10} = \max(0, 5 - Ns_5);$
$BF_{14} = \max(0, 1 - Ns_6);$
$BF_3 = \max(0, 4 – Ns_7);$
$BF_{12} = \max(0, 1 - Ns_8);$
$BF_8 = \max(0, 2 - Ns_9);$
$BF_{11} = \max(0, 1 - Ns_{10});$
$BF_4 = \max(0, 1 – Ns_{11});$

As shown in the equations above, the MARS forward stepwise algorithm selected 14 basis functions while the MARS backward procedure pruned 5 basis functions indicating 9 basis functions should be the best size of the fitted model. In this study, we also observed that the MARS model without interactions performed better on the test data set than the one with interactions. We surmise this to be reasonable because the stations are fairly far from each other, leading to little shifting of demand. Further, the power trading component in the control problem dominates revenue generation compared to the allocation of wind power across the different stations. For our best MARS model shown above, the testing R^2 is 94.4%.

3.1.4 First stage system design master problem

As mentioned earlier, the expected revenue is returned from the second stage as part of the objective for the first-stage system design master problem. Instead of directly solving the second-stage control problem within the iterations of the first-stage optimization, the MARS model in equation (8) is used to represent the expected revenue from the second stage. The first stage is optimized to identify the system design that maximizes profit, where profit is calculated by subtracting by subtracting the cost component from the MARS model to obtain an estimate for the expected profit ($\hat{Z}_{DA}$) and to obtain best system design points (x_{DA}^*, Ns_{DA}^*), as shown in the following equation:

$$x_{DA}^*, Ns_{DA}^* \in arg\ max\ \hat{Y}_{DA}(Ns) - cx - NcNs \tag{9}$$

$$s.t.\ (x, Ns, B, g, Nd) \in \Gamma_D \tag{10}$$

Here, Γ_D represents the set of constraints for the system design variables.

The optimization was implemented in Cplex Cp, using IBM ILOG CPLEX 12.6.3 (downloaded from https://www.ibm.com/us-en/marketplace/ibm-ilog-cplex). The obtained system design variables are then solved using step 2 and step 3 our DACE approach to generate the true revenue ($Y_{DA}(x_{DA}^*, Ns_{DA}^*)$) for the given system design. Once we have the true revenue, we calculate the true profit by subtracting the cost component, as shown in the following equation:

$$Z_{DA}(x_{DA}^*, Ns_{DA}^*) = Y_{DA}(Ns_{DA}^*) - cx_{DA}^* - NcNs_{DA}^* \tag{11}$$

The obtained Z_{DA} and (x_{DA}^*, Ns_{DA}^*) are the solutions to our first stage master problem.

3.2 Case Study: System Design Experiments

3.2.1 ADP Results

From a computational perspective, all our simulations are executed on a workstation equipped with Intel Core i7 CPU @3.50 GHz *12 and 32 GB RAM. Table 3.4 illustrates the best-known number of stations to be opened and the number of slots to be opened along with the individual profits generated from each station. Taking practicality into consideration, a station cost of $100 per day is considered as the base line. It took 4 days and 3 hours of processing time to provide the best solution. The obtained best known solution is $2,132.14, and the stations to be opened are Forth Worth, Dallas, Garland and

Denton, with their individual profits being $916.94, $1,013.91, $124.17 and $77.12, respectively. For each of these locations the numbers of slots being opened are 5, 5, 4 and 2, respectively.

Table 3.4. Numbers of Slot and profit generated per stations

	Fort Worth	Dallas	Garland	Denton	Total
Slots	5	5	4	2	16
Profit	$916.94	$1,013.91	$124.17	$77.12	$2,132.14

Figure 3.7 illustrates the station wise profit distribution. The profit distribution between Dallas, Fort Worth, Garland and Denton are 47%, 43%, 6% and 4%, respectively, indicating Dallas to be the most profitable station, followed by Fort Worth, Garland, and Denton.

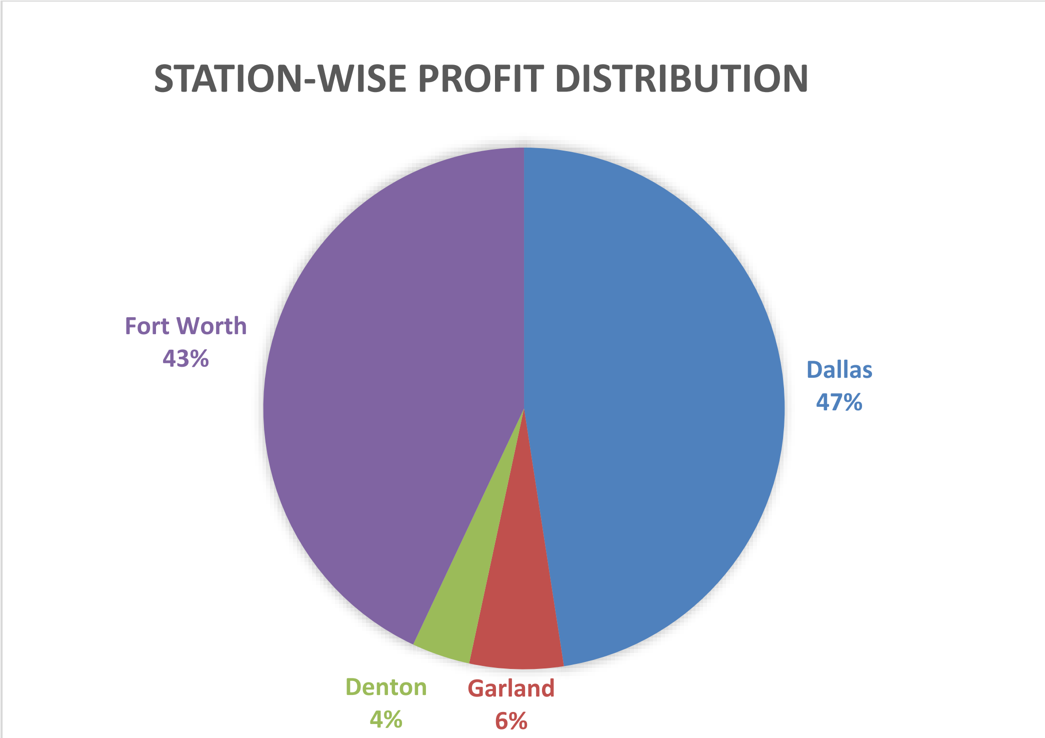

Figure 3.7. Station-wise profit distribution

3.2.2 Separable Case

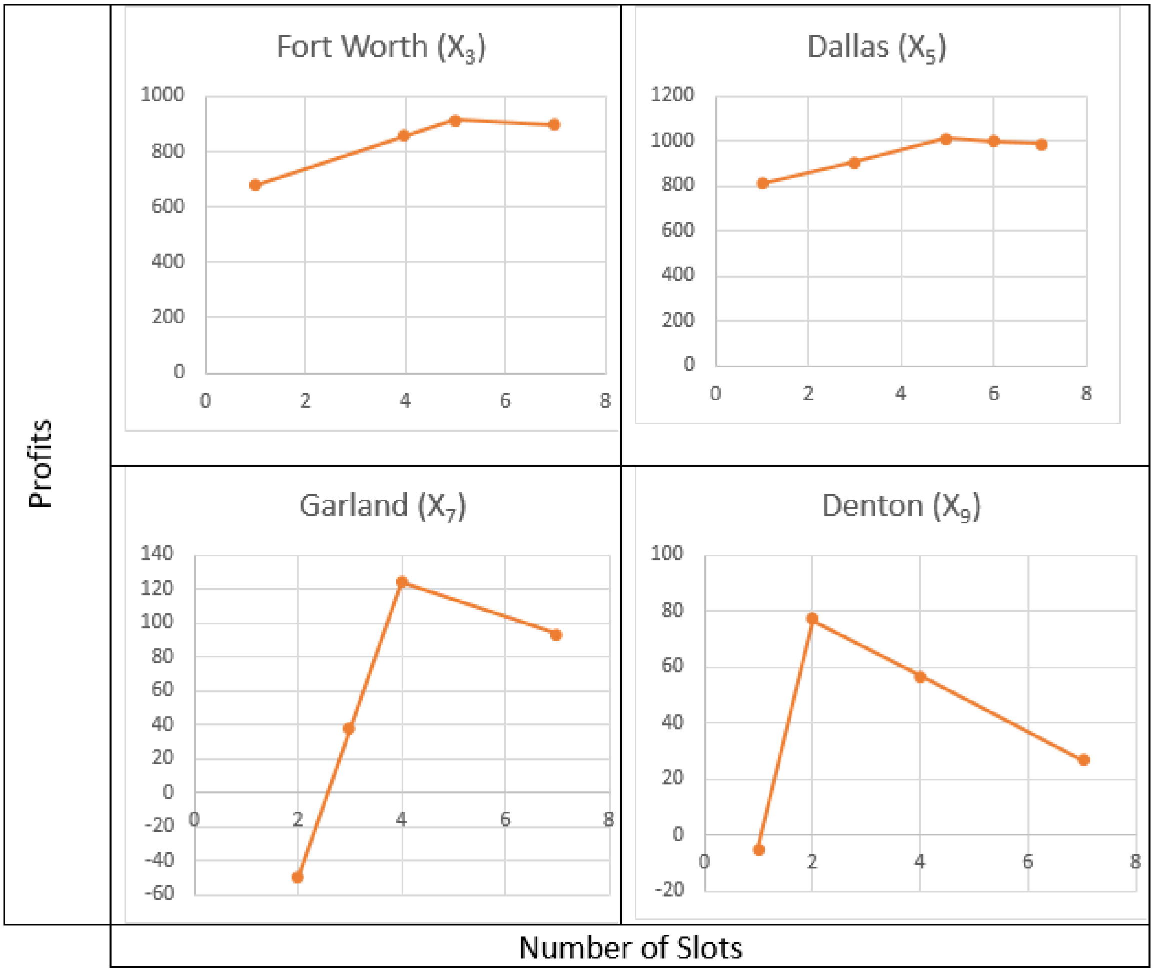

Figure 3.8. Separable Case

The DACE approach can give more insightful descriptions to the first stage that help us analyze the nature of individual stations. Figure 3.8 uses separable curves to show the relationship between each station and number of slots to be opened per station, based on the MARS model. The significant basis functions are associated with Dallas, Fort Worth, Garland, and Denton. All the other basis functions associated with other stations are zero, meaning the remaining seven stations are closed. It can be observed in Figure 3.8 that the optimal number of slots for Fort Worth, Dallas, Garland and Denton are 5, 5 ,4 and 2, respectively, identical to the slots obtained in Table 3.4.

3.2.3 Comparison between DACE based ADP and MILP

Table 3.5. Comparison between ADP Design and MILP Design

	Fort Worth	Dallas	Garland	Denton	Z_{DA}	Z_M
ADP Design	5	5	4	2	$2132.14	$2669.06
MILP Design	5	4	3	1	$1973.35	$2689.38
DACE MILP Design	4	5	2	2	$2014.41	$2678.53

In Table 3.5, Z_{DA} is the solution obtained using DACE based ADP approach, whereas Z_M is the solution obtained from MILP from Chawal et al. (2018). As mentioned earlier, the best known ADP system design (x_{DA}^*, Ns_{DA}^*) opens stations at Fort Worth, Dallas, Garland and Denton with their numbers of slots as 5, 5, 4 and 2, respectively. By comparison, from Chawal et al. (2018), the best known MILP system design (x_M^*, Ns_M^*) also opens stations at Fort Worth, Dallas, Garland and Denton, but with slightly different numbers of slots, specifically, 5, 4, 3 and 1, respectively. When the ADP system design and MILP system design are both evaluated using the ADP approach to the second-stage control problem (Chen et al. 2017), the simulated expected profits obtained are $2132.14 and $1973.35, respectively. Hence, the expected profit generated by ADP system design is estimated to exceed that generated by the MILP system design by $158.79. The difference in the expected profit can be attributed to the fact that ADP solves the problem taking stochasticity into consideration. Hence, we can see an improvement of 8.04% as shown below.

$$\frac{Z_{DA}(x^*_{DA},Ns^*_{DA}) - Z_{DA}(x^*_M,Ns^*_M)}{Z_{DA}(x^*_M,Ns^*_M)} = 8.06\%$$

By contrast, when evaluating both using the MILP approach to the second-stage control problem (Chawal et al. 2018), the MILP system design generates an expected profit that greater than that of the ADP system design by $20.32, indicating that when uncertainty is not taken into account, the deterministic MILP system design provides the better solution.

As a final comparison, we sought to assess the concept of a DACE approach by utilizing a DACE approach with the MILP version of the second-stage control problem from Chawal et al. (2018). To our knowledge, there does not exist attractable approach to globally optimize a general high-dimensional, infinite-horizon stochastic dynamic programming problem. By evaluating the DACE approach in the deterministic environment of the MILP version, we can directly compare the performance of DACE to the globally optimized MILP solution. If DACE performs similarly to the globally optimal solution in the deterministic environment, then we can be more assured that the DACE approach for the stochastic environment is yielding a quality solution.

Our two-stage DACE framework yielded nearly optimal results with only 0.41% loss in profit generated by the obtained DACE MILP system design (x^*_{DM}, Ns^*_{DM}), as compared to the globally optimal profit ($2689.38) obtained using the MILP method as shown in the calculation below. This verifies the effectiveness of the developed methodology using DACE.

$$\frac{Z_M(x^*_M,Ns^*_M) - Z_M(x^*_{DM},Ns^*_{DM})}{Z_M(x^*_M,Ns^*_M)} = 0.41\%$$

4. Conclusion

We presented a two-stage stochastic optimization problem that seeks to optimize a system design problem in the first-stage, while solving an infinite-horizon stochastic dynamic programming problem in the second stage. We introduced the first computationally-tractable approach to solve such a problem. Our approach utilized the DACE-based ADP work of Chen et al. (2017) to optimize the second-stage stochastic dynamic programming problem, and developed a DACE approach to incorporate the result of the second-stage into the first-stage system design problem. Computational results for the DFW Metroplex EV charging stations case study demonstrated the efficacy of the DACE approach, yielding 8% higher expected profit than an MILP-based solution evaluated in the stochastic environment, and yielding less than 1% loss in profit comparing a DACE approach to the globally optimal MILP solution in the deterministic environment. Moreover, the DACE approach using a MARS metamodel provides insight that uncovers the underlying nature of the first-stage design variables.

For future work, alternatives to ADP to solve infinite horizon stochastic problem can be explored. Instead of MARS, other statistical models that best approximates the outputs as functions of the input variables of the system can be investigated. Alternative design of experiment methods as compared to LH can also be employed and tested.

Acknowledgement

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Chapter 4. General Conclusion

In this dissertation, due to the reason that there has been a significant increase in the number of EVs, our primary focus is to optimize the system of Electric Vehicle charging station which includes the locations of the stations and the number of slots to be opened to maximize the profit based on the user-specified cost of opening a station since. There are mainly three components in this dissertation research. The first component mainly seeks to formulate a deterministic MILP model to optimize the system of EV charging stations. Based on the overall results obtained from the MILP approach in regard to the number of slots to be opened, Fort Worth followed by Dallas, Garland, and Denton and so on. The drawbacks of this approach, despite giving the optimal solution, is its extremely high computational time (on average 5 days) and it does not take uncertainty in to consideration. Hence, we utilized a two-stage framework and used DACE based system design optimization approach to solve the system of EV charging stations, which is the second component of this dissertation. Both of these components are presented in Chapter 2 (first paper). Instead of 4 days and 23 hours using MILP, DACE based system design optimization approach took roughly 1 hour to get the near optimal solution with a loss of less than 1% accuracy indicating it is the most desirable approach, in terms of time saving and resource saving. Based on the overall result obtained from this approach in regard to the number of slots to be opened, Dallas has the more number of slots to be opened, followed by Fort Worth, Garland, and Denton and so on indicating there are equally optimal solutions obtained from different system design build. In all cases, the model without interaction is providing us the better solutions than the model with interaction. In addition, it helps us to gain understanding about input and output relationship using separable curve, where individual station natures can be studied to show the relationship between each station and number of slots to be opened per station to generate the maximum profit. Moreover, it helps us to generate the controllability function/revenue function (cost not considered), hence is flexible to handle different cost scenarios. The third component of this dissertation, presented in chapter 3 (second paper), is to consider uncertainty in our model for which we apply DACE based infinite horizon ADP algorithm to solve a large-scale, high-dimensional, infinite horizon, EV charging station control problem over a continuous state and decision space to optimize the system of the EV Charging stations. Based on the overall result in regard to the number of slots to be opened, Dallas has the more number of slots to be opened, followed by Fort Worth, Garland, and Denton and so on. Moreover, when the designs obtained from our DACE ADP design and MILP design are solved using ADP approach (simulation), we observed an improvement of approximately 8% in the simulated solution obtained from ADP design indicating that when uncertainty is considered, DACE ADP design provides the better solution. On employing the two-stage DACE framework to optimize the problem mentioned in DACE MILP, the results gained are near optimal with less than 1% loss in the solution verifying the effectiveness of the developed methodology and its widespread practical applications. As mentioned earlier, to the best of our knowledge, this is the only two-stage stochastic problem which uses infinite horizon dynamic programming approach to optimize the second stage dynamic control problem and the first stage system design problem.

For future work, alternatives to MILP to solve the deterministic EV problem can be investigated which can solve the problem quicker. Another design of experiment methods as compared to binned LH can also be employed and tested. Instead of MARS, other statistical models that best approximates the outputs as functions of the input variables of the system can be investigated. Lastly, substitute to ADP to solve infinite horizon stochastic problem can be explored. Moreover, this methodology can be utilized to work on the smart city application.

Appendix A

Binned Latin Hypercube Sampling Design[3]

[3] Kulvanitchaiyanunt, Asama. "A Design And Analysis Of Computer Experiments-based Approach To Approximate Infinite Horizon Dynamic Programming With Continuous State Spaces." (2014).

Table A.1. 250 points Binned Latin Hypercube Design Training Data Set

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
0	0	0	2	4	8	6	1	6	4	6
0	8	4	0	0	3	0	0	0	6	1
4	8	0	6	0	0	9	1	6	0	0
0	4	0	1	1	3	0	6	0	5	9
0	5	3	10	0	0	4	8	8	0	0
0	0	5	0	4	0	3	0	1	0	0
0	0	3	7	0	0	0	0	0	6	9
1	0	10	10	3	0	2	2	0	4	0
0	10	0	0	3	0	2	9	9	0	0
6	9	7	8	6	0	0	0	4	0	5
4	1	0	0	0	4	1	0	0	2	0
2	0	1	0	9	9	8	5	9	9	6
0	2	0	6	2	0	0	1	7	0	0
10	0	7	2	7	1	1	0	0	0	3
0	0	0	0	8	4	6	9	0	9	6
0	0	3	1	10	4	0	2	0	0	4
9	5	0	4	0	3	0	0	0	3	1
6	0	0	8	0	0	0	6	8	5	6
10	0	0	7	0	0	1	10	0	0	0
0	0	4	9	0	6	0	0	0	0	0
0	0	9	3	0	0	4	7	0	0	0
0	4	0	0	0	10	0	0	0	0	1
2	0	8	5	8	10	0	0	0	0	0
0	9	0	0	6	3	8	0	0	10	8
2	8	0	5	9	5	4	6	6	9	0
0	2	0	5	9	5	0	0	0	6	9
9	0	6	0	5	1	5	2	0	4	1
8	0	0	0	0	2	0	1	0	7	10
0	0	0	9	0	0	10	0	3	10	9
0	0	5	8	0	0	0	8	0	5	7
5	1	8	8	7	0	3	7	0	1	6
7	6	4	0	0	2	0	0	10	0	10
9	9	0	7	3	0	4	0	6	8	0
10	1	0	0	6	3	0	0	0	1	0
7	2	7	5	0	0	10	0	6	0	0
4	2	0	0	0	10	2	1	4	3	0

7	2	0	7	0	5	0	0	0	2	3
0	0	6	0	0	0	10	0	1	0	6
5	10	0	7	0	3	0	4	0	0	0
0	7	5	0	7	0	7	10	0	9	0
0	4	2	0	0	0	8	1	2	0	3
8	9	3	5	0	5	5	5	0	8	5
1	10	0	0	0	2	0	0	1	5	10
0	4	0	0	4	0	0	9	10	3	0
9	0	0	0	9	0	2	1	0	0	0
0	0	9	0	0	0	6	7	0	0	0
0	4	0	5	7	9	0	8	0	8	0
6	2	3	2	7	3	5	0	0	0	7
4	0	0	0	0	5	5	7	0	8	2
4	0	0	4	1	0	7	1	3	4	2
0	0	2	7	0	5	5	0	6	0	1
0	3	8	0	1	5	0	4	0	0	0
3	8	0	0	0	8	10	4	0	0	0
5	3	0	0	9	0	9	10	0	0	0
1	2	8	2	4	0	10	0	3	0	5
1	5	9	0	2	10	5	0	0	0	0
0	0	0	0	7	0	0	0	1	0	2
2	10	0	6	3	10	0	6	10	0	4
0	6	0	6	1	0	1	8	3	0	6
1	0	3	2	0	3	0	2	6	5	2
0	7	0	0	7	0	10	1	0	0	9
1	5	6	6	0	8	7	0	0	1	7
1	3	1	0	0	2	6	0	0	9	6
0	8	4	8	0	0	8	2	0	0	6
0	0	0	0	2	0	1	8	0	0	4
0	9	0	2	0	0	6	0	0	6	0
8	0	2	0	3	8	5	0	0	7	0
9	0	0	0	10	0	0	10	3	0	0
1	0	0	0	0	0	7	3	0	0	2
0	3	3	9	0	5	0	8	0	0	3
0	0	6	2	3	0	0	5	9	0	0
8	5	9	0	0	0	7	6	0	0	0
7	0	6	10	4	0	5	2	6	2	3
4	0	4	6	0	7	5	0	0	6	0
0	6	2	0	0	6	0	2	6	0	0

3	1	0	8	8	0	0	0	0	6	0
2	0	0	0	0	6	0	7	0	0	4
4	0	7	0	0	0	6	7	0	7	2
8	0	0	0	2	0	1	0	0	6	8
6	5	0	3	0	9	0	0	0	4	0
0	0	0	10	0	0	0	8	9	0	0
3	9	6	0	7	1	1	0	4	5	2
4	0	0	3	2	0	2	2	4	7	0
0	5	2	7	0	0	0	6	0	0	0
0	0	6	2	0	7	7	9	3	0	4
3	7	0	0	5	0	0	9	0	7	7
9	6	0	0	0	0	0	0	10	8	4
1	5	1	0	6	6	8	6	6	0	3
0	7	0	0	5	8	0	0	0	0	0
0	0	0	0	0	7	0	0	4	1	4
8	8	0	0	0	1	4	0	4	5	0
0	6	0	2	5	8	0	0	0	4	0
7	7	0	0	9	6	3	0	0	0	0
10	2	0	0	0	2	7	8	10	10	0
7	0	0	0	0	2	0	6	6	8	6
7	0	4	2	0	10	0	6	6	6	0
0	0	6	0	0	7	7	10	0	4	10
9	6	3	7	0	0	2	9	0	0	8
7	8	7	8	0	5	4	0	2	0	10
0	10	7	0	1	0	0	5	0	6	10
0	0	2	5	9	0	6	10	0	0	0
7	10	3	0	0	0	0	6	9	10	7
0	0	0	10	10	7	4	6	8	7	8
3	0	7	2	4	0	4	7	1	1	0
0	9	0	9	0	4	0	0	0	0	4
0	0	0	4	0	0	8	2	0	0	10
2	2	3	0	0	0	0	3	7	0	0
10	0	0	0	8	5	0	7	0	0	0
2	0	1	0	5	0	3	10	0	0	0
0	0	0	8	1	0	4	8	0	0	2
0	0	0	7	0	0	0	0	8	8	0
0	0	1	6	0	0	2	5	0	0	0
6	0	9	0	0	6	0	0	8	7	8
0	1	8	3	8	1	7	1	0	3	10

3	0	0	5	0	0	7	10	0	8	0
0	5	0	0	1	5	3	0	0	0	0
0	7	0	5	0	9	3	7	0	0	0
1	5	5	5	2	0	7	0	10	0	0
0	1	6	0	4	0	0	0	0	4	3
0	1	6	0	2	1	4	2	2	4	0
2	2	10	3	0	7	0	7	2	1	5
6	7	6	5	0	0	2	0	8	0	7
6	9	0	0	8	0	0	0	0	0	9
0	0	0	1	9	6	0	0	9	0	8
0	7	0	4	0	5	9	8	5	9	2
9	0	0	0	8	0	0	0	7	0	1
0	2	0	0	0	0	8	9	4	0	0
0	0	9	0	3	0	4	0	3	4	5
6	2	3	1	6	0	0	0	2	2	2
10	0	0	0	5	7	3	4	0	8	0
0	4	0	3	0	0	0	7	5	1	0
9	0	2	1	0	0	0	2	5	0	0
0	0	0	4	0	1	4	0	10	5	0
4	0	10	0	7	9	4	0	5	0	3
0	2	0	5	4	0	0	3	2	0	0
1	1	6	2	0	8	0	0	8	3	0
5	0	0	1	0	3	0	3	0	0	0
1	0	0	0	0	1	9	10	6	0	5
6	0	0	8	0	6	10	0	1	5	0
4	0	5	0	9	1	5	1	5	1	0
0	3	7	0	0	0	0	10	0	0	2
7	0	0	0	0	0	0	10	1	6	0
0	7	9	0	5	0	0	7	4	3	0
0	3	0	10	5	0	1	3	10	4	9
5	5	3	0	0	0	0	1	1	0	0
8	0	0	0	4	3	7	0	9	0	4
0	0	0	8	2	0	0	0	7	9	0
0	0	0	3	0	9	0	0	0	2	3
0	0	5	10	0	9	2	5	7	0	3
5	3	6	0	2	6	6	0	0	4	4
1	2	0	10	2	0	0	9	0	4	0
9	1	4	4	8	0	0	0	0	0	0
5	4	0	0	8	8	0	4	1	5	0

0	4	0	4	4	4	9	7	4	10	7
0	6	2	0	6	0	0	4	0	8	0
9	9	10	10	3	0	3	0	2	0	0
0	0	0	6	1	1	9	5	9	1	1
0	9	0	0	0	2	10	1	0	0	0
7	0	7	8	0	0	1	0	9	7	0
3	0	0	7	10	0	3	2	10	0	0
2	0	1	7	8	0	0	0	3	0	0
0	5	5	0	0	0	0	0	7	6	0
0	10	3	0	9	9	0	0	4	0	0
4	0	3	5	0	0	4	0	0	1	0
0	0	0	0	8	4	0	0	5	3	0
4	9	8	0	10	2	2	0	5	0	5
0	0	0	0	0	1	4	9	2	3	4
4	7	0	0	0	3	6	3	3	0	1
0	2	6	0	8	8	5	2	0	0	0
0	0	5	0	0	3	6	0	0	0	0
0	3	8	0	1	1	0	0	7	3	3
0	3	5	0	1	0	0	0	9	0	0
10	0	0	0	0	0	0	0	1	10	10
0	0	9	0	0	0	6	7	0	7	0
0	0	9	10	1	5	4	5	4	3	8
9	6	0	3	6	0	5	0	10	0	9
8	0	0	0	0	3	7	4	0	0	0
0	0	0	4	7	5	2	3	7	3	9
6	0	6	10	0	0	2	9	0	0	0
0	0	0	6	8	4	1	8	3	0	4
0	0	0	0	0	0	0	0	0	6	9
0	7	0	5	0	10	9	0	0	5	3
0	3	10	5	0	0	6	0	1	1	6
0	0	4	1	1	0	8	3	5	8	8
0	10	0	0	2	0	0	6	3	0	0
0	0	2	9	0	6	0	0	0	0	0
0	0	0	4	3	7	0	0	1	0	2
0	0	5	0	4	9	3	0	0	2	0
0	9	8	0	9	0	4	0	3	0	9
0	0	5	5	6	0	0	4	0	0	5
5	8	8	0	0	5	10	0	5	0	10
10	0	8	0	10	8	0	7	5	0	8

0	10	0	2	4	4	0	10	6	0	0
0	0	0	10	3	10	10	0	7	0	0
5	0	0	1	0	6	0	4	10	0	3
0	0	0	10	0	0	0	0	8	0	10
0	0	0	1	0	0	0	0	0	0	9
3	0	0	0	0	3	0	3	8	0	10
0	0	0	0	8	7	2	8	6	8	2
10	0	0	0	0	2	0	1	2	0	5
5	0	2	1	0	0	3	0	0	10	6
6	0	0	0	3	0	0	9	0	0	5
9	7	2	1	9	9	0	2	10	3	8
5	0	6	3	5	0	0	0	0	0	8
0	8	0	4	10	0	5	3	1	7	0
7	9	7	7	0	0	0	0	4	0	8
10	8	2	4	0	0	0	0	5	0	9
3	10	4	6	3	0	0	4	3	0	6
3	0	10	0	0	2	5	0	2	0	0
8	2	0	0	0	8	7	7	0	0	5
0	0	0	0	9	9	0	7	0	0	0
0	10	4	2	6	6	9	5	10	0	5
3	0	0	0	0	0	9	5	7	0	5
2	0	0	0	0	0	0	4	0	0	0
0	0	9	0	0	9	3	1	10	7	1
0	5	0	0	9	3	0	0	0	0	0
2	8	7	3	6	8	1	5	2	1	0
8	0	0	0	1	0	7	0	8	9	2
0	0	9	0	0	0	6	3	5	7	4
2	0	5	6	3	3	7	9	0	7	1
0	5	3	7	0	0	0	0	0	2	0
8	0	4	0	0	0	0	10	5	0	0
0	1	9	1	4	0	0	0	4	0	0
4	9	4	0	10	0	5	0	0	0	0
10	8	0	0	0	0	10	0	8	10	6
0	0	0	9	0	0	0	0	0	10	6
0	0	3	0	7	7	0	0	4	2	7
0	6	0	0	9	0	0	2	0	3	1
9	0	0	2	0	0	3	0	0	0	10
4	7	9	5	0	7	0	0	8	0	0
2	3	0	6	0	0	7	3	9	9	0

3	0	0	8	2	2	1	0	7	7	0
6	0	1	0	0	0	8	7	0	0	0
3	8	0	0	0	0	0	8	7	0	3
3	8	10	8	7	0	0	0	3	0	1
0	7	1	1	7	0	0	0	0	0	1
8	0	10	0	0	0	0	0	9	4	0
0	4	4	0	0	5	0	2	0	0	0
5	0	7	6	9	0	10	0	0	0	4
0	0	0	0	10	0	9	0	0	5	10
10	8	0	0	0	0	0	4	0	5	10
8	0	0	4	4	9	7	9	3	0	3
0	10	0	7	0	0	0	8	7	0	0
0	8	0	0	4	0	2	10	10	9	0
0	2	0	0	0	0	8	0	5	10	0
0	1	10	0	6	0	10	7	8	10	0
6	1	1	9	0	5	1	0	4	2	0
0	9	0	0	6	3	1	0	0	5	0
5	4	9	2	0	0	9	0	2	4	0
4	0	0	0	0	0	0	0	9	0	0

Table A.2. 75 points Binned Latin Hypercube Design Testing Data Set

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
0	0	0	0	9	7	0	7	0	0	9
9	1	2	0	0	0	1	1	7	0	7
0	0	8	10	7	0	9	0	0	0	0
0	9	0	10	3	0	9	8	8	0	8
9	0	7	10	0	0	0	0	0	0	9
0	3	0	0	4	0	2	9	3	10	4
9	4	9	0	10	2	10	6	0	7	0
0	0	0	8	5	2	0	0	0	0	6
4	0	0	3	0	0	2	7	2	0	2
0	0	0	0	4	0	8	2	0	6	3
7	8	3	7	0	0	10	5	0	0	0
0	9	0	3	6	0	9	9	1	9	5
0	0	8	0	0	0	0	8	0	0	4
0	2	6	0	10	0	6	3	1	0	6
0	0	5	9	6	0	0	0	0	7	2
0	6	4	0	0	5	0	3	9	7	7
8	0	5	1	5	0	10	7	4	0	0
0	0	9	7	0	0	0	0	0	0	0
0	4	0	1	0	8	0	9	0	0	0
7	4	0	0	8	5	6	0	0	5	0
1	3	0	1	0	3	0	0	3	0	6
10	0	5	6	0	5	0	2	0	0	7
8	0	0	1	1	4	6	0	10	5	1
10	3	9	0	8	0	0	10	0	0	3
0	0	6	0	0	0	8	9	9	0	0
9	0	0	0	4	2	6	0	0	0	4
2	0	0	0	0	0	0	0	0	0	5
7	3	1	5	7	0	0	0	7	0	3
10	1	0	6	3	3	2	7	0	4	0
0	0	5	9	0	0	0	0	0	1	4
0	0	9	4	7	0	0	10	2	6	0
3	0	3	3	0	6	6	1	0	0	0
8	0	0	6	8	0	7	0	9	0	1
1	0	2	6	0	0	0	0	0	10	1
0	0	4	0	2	0	0	7	2	2	0
8	0	0	4	10	4	0	10	10	10	0

0	3	3	8	7	7	3	0	0	0	2
0	3	7	0	0	2	0	0	5	0	0
3	0	0	0	9	0	1	0	1	4	10
0	0	7	0	6	0	7	0	0	0	7
0	7	2	7	0	1	8	0	6	6	0
0	0	0	6	0	2	0	10	0	5	2
3	0	4	0	4	0	0	0	1	1	0
10	8	3	5	0	1	4	0	0	10	0
5	0	5	0	3	4	0	0	0	10	0
6	9	9	10	0	0	5	6	10	0	0
0	3	0	1	0	8	0	8	4	1	3
0	0	0	8	8	0	5	0	6	4	4
3	0	3	3	9	10	9	0	6	3	5
0	4	0	0	6	0	0	6	0	6	0
0	4	7	0	0	0	1	0	2	10	0
8	0	5	10	3	4	0	1	5	9	0
1	0	0	0	0	9	0	5	0	5	0
5	9	10	0	0	4	0	7	5	3	10
1	7	4	4	10	0	4	10	0	0	5
8	7	2	2	0	1	4	0	0	8	0
6	1	8	8	6	0	0	0	0	7	6
10	5	0	9	10	0	6	5	1	8	10
0	0	0	5	10	7	2	0	0	0	0
8	9	3	0	2	10	7	2	4	0	0
4	2	0	8	9	9	9	4	0	0	0
0	7	10	0	3	10	0	0	0	0	7
0	5	8	0	7	5	3	3	4	5	0
0	0	0	0	0	5	0	0	3	6	0
0	6	1	0	8	9	0	1	0	0	0
0	0	0	9	0	7	8	4	0	8	2
6	8	0	0	0	1	0	9	7	7	0
0	0	8	0	5	0	0	0	0	5	0
2	8	2	0	0	0	3	6	0	4	0
5	0	10	0	1	5	10	5	2	0	3
5	0	3	6	2	0	2	0	6	0	2
0	4	8	0	4	0	0	0	6	2	0
7	10	5	2	0	2	0	0	0	1	0
0	6	0	0	3	9	0	5	4	0	0
6	0	0	4	0	0	3	3	7	3	3

Appendix B

MARS MODELS form DACE Based System Design Optimization Approach

MARS MODEL 1 - SPM (No Interaction) – Testing R^2 = 0.972

```
Y = -4509.48 - 167.66 * BF1 + 174.994 * BF2 - 108.953 * BF3
+ 131.244 * BF4 - 55.6662 * BF5 + 123.232 * BF6
- 72.8197 * BF7 + 105.474 * BF8 + 100.507 * BF10
+ 64.3329 * BF14 + 62.3178 * BF16 + 64.7182 * BF18
+ 34.6174 * BF20 + 55.2994 * BF22 + 54.0012 * BF24
+ 104.854 * BF25 + 80.465 * BF27 + 30.6401 * BF31
+ 74.7128 * BF33 + 64.2414 * BF35 + 52.3529 * BF43;

BF1 = max( 0, X3 - 6);
BF2 = max( 0, 6 - X3);
BF3 = max( 0, X5 - 3);
BF4 = max( 0, 3 - X5);
BF5 = max( 0, X7 - 1);
BF6 = max( 0, 1 - X7);
BF7 = max( 0, X9 - 2);
BF8 = max( 0, 2 - X9);
BF10 = max( 0, 1 - X4);
BF14 = max( 0, 1 - X6);
BF16 = max( 0, 1 - X10);
BF18 = max( 0, 1 - X8);
BF20 = max( 0, 2 - X2);
BF22 = max( 0, 1 - X11);
BF24 = max( 0, 1 - X1);
BF25 = max( 0, X3 - 1);
BF27 = max( 0, X5 - 1);
BF31 = max( 0, X5 - 5);
BF33 = max( 0, X9 - 1);
BF35 = max( 0, X3 - 4);
BF43 = max( 0, X7 - 2);
```

MARS MODEL 2 - SPM (Interaction) – Testing R^2 = 0.9878

Y = -4274.67 - 137.876 * BF1 + 159.346 * BF2 + 37.6086 * BF4- 2.87167 * BF5 - 50.802 * BF7 + 91.4667 * BF8 + 27.3196 * BF10 + 61.1549 * BF11 + 1.54567 * BF13 + 68.3091 * BF14 + 33.7468 * BF16 + 59.0458 * BF18 + 56.6548 * BF20 + 59.4317 * BF24 + 25.7099 * BF26 + 2.01206 * BF27 + 14.7328 * BF28 - 1.68534 * BF29 + 15.6408 * BF30 + 8.02824 * BF32 + 1.18542 * BF34 + 51.7449 * BF35 - 12.1581 * BF37 + 234.666 * BF39 - 106.75 * BF41 - 86.7827 * BF43 + 3.97921 * BF44 + 4.3145 * BF46 + 7.06953 * BF50 + 2.76066 * BF51 - 7.91724 * BF52 + 0.770057 * BF53 + 3.83917 * BF54 + 4.70893 * BF55 + 17.6747 * BF57 - 35.9797 * BF59 + 20.0687 * BF60 + 74.9466 * BF61 - 12.218 * BF63 + 6.54636 * BF65 + 28.8268 * BF66 + 26.3259 * BF69 + 38.925 * BF72 - 0.222453 * BF78 + 0.742764 * BF79 + 3.29446 * BF80 + 87.4843 * BF81 - 4.15123 * BF82 - 7.82351 * BF83 + 2.30638 * BF86 + 4.82729 * BF88 + 12.1456 * BF90 + 1.34152 * BF96 - 29.5314 * BF98;

BF1 = max(0, X3 - 6);
BF2 = max(0, 6 - X3);
BF3 = max(0, X5 - 3);
BF4 = max(0, 3 - X5);
BF5 = max(0, X7 - 1);
BF6 = max(0, 1 - X7);
BF7 = max(0, X9 - 2);
BF8 = max(0, 2 - X9);
BF9 = max(0, X4 - 1);
BF10 = max(0, 1 - X4);
BF11 = max(0, X3 - 3);
BF12 = max(0, 3 - X3);
BF13 = max(0, X6 - 1);
BF14 = max(0, 1 - X6);
BF16 = max(0, 1 - X2) * BF6;
BF18 = max(0, 1 - X10);
BF19 = max(0, X8 - 1);
BF20 = max(0, 1 - X8);
BF21 = max(0, X1 - 1);
BF22 = max(0, 1 - X1);
BF24 = max(0, 1 - X11);
BF26 = max(0, 2 - X7) * BF10;
BF27 = max(0, X2 - 3);
BF28 = max(0, 3 - X2);
BF29 = max(0, X5 - 1) * BF2;
BF30 = max(0, 1 - X5) * BF2;
BF32 = max(0, 3 - X7) * BF4;
BF34 = max(0, 4 - X3) * BF28;
BF35 = max(0, X9 - 1);
BF36 = max(0, 1 - X9);
BF37 = max(0, X8 - 9) * BF21;
BF39 = max(0, X8 - 9) * BF12;
BF41 = max(0, X8 - 9) * BF2;
BF43 = max(0, X8 - 9) * BF35;
BF44 = max(0, 9 - X8) * BF35;
BF46 = max(0, 6 - X5) * BF10;
BF50 = max(0, 4 - X8) * BF22;
BF51 = max(0, X3 - 1) * BF4;
BF52 = max(0, 1 - X3) * BF4;
BF53 = max(0, X11 - 2) * BF12;
BF54 = max(0, 2 - X11) * BF12;
BF55 = max(0, X8 - 8) * BF9;
BF57 = max(0, X8 - 9) * BF3;
BF59 = max(0, X4 - 6) * BF6;
BF60 = max(0, 6 - X4) * BF6;
BF61 = max(0, X3 - 1);
BF62 = max(0, 1 - X3);
BF63 = max(0, X8 - 7) * BF62;
BF65 = max(0, X5 - 4) * BF22;
BF66 = max(0, 4 - X5) * BF22;
BF69 = max(0, X4 - 3) * BF6;
BF72 = max(0, 1 - X2) * BF10;
BF78 = max(0, 9 - X3) * BF21;
BF79 = max(0, X1 - 3) * BF19;
BF80 = max(0, 3 - X1) * BF19;

BF81 = max(0, X8 - 9) * BF7;
BF82 = max(0, 9 - X8) * BF7;
BF83 = max(0, X1 - 9) * BF13;
BF86 = max(0, 5 - X7) * BF18;
BF88 = max(0, 8 - X8) * BF36;
BF90 = max(0, 1 - X5) * BF14;
BF96 = max(0, 3 - X8) * BF4;
BF98 = max(0, 1 - X1) * BF4;

MARS MODEL 3 - (No Interaction) – Testing R^2 = 0.9699

Y = -3581.256753661 +94.7403655219315*BF1 +103.541107822435*BF2 +66.4329627180085*BF3 -3.35380708902338*BF4 +51.9033150935411*BF5 +58.1910724607076*BF6 +55.6807595648654*BF7 +99.8673985216016*BF8 -38.6632742926808*BF9 +140.229183389692*BF10 +34.5074132018551*BF11 +42.7389448125185*BF12 +131.464261776945*BF13 +73.3503894915566*BF14 +19.8822939402428*BF15 +69.8465813755455*BF16

BF1 = max(0,1 -x7)
BF2 = max(0,1 -x4)
BF3 = max(0,1 -x6)
BF4 = max(0, x8 -1)
BF5 = max(0,1 -x8)
BF6 = max(0,1 -x11)
BF7 = max(0,1 -x1)
BF8 = max(0,1 -x3)
BF9 = max(0, x5 -1)
BF10 = max(0,1 -x5)
BF11 = max(0,3 -x7)
BF12 = max(0, x5 -5)
BF13 = max(0,1 -x9)
BF14 = max(0,4 -x3)
BF15 = max(0,3 -x10)
BF16 = max(0,1 -x2)

MARS MODEL 4 - MATLAB (No Interaction) – Testing R^2 = 0.9766

Y = -3653.90120407391 +36.0098994231938*BF1 +95.9745009657192*BF2 +66.9555178928153*BF3 +11.495255046659*BF4 +62.6996021293746*BF5 +22.7617927840183*BF6 +64.1023466143187*BF7 +64.0329787759911*BF8 +74.5725491908506*BF9 - 2.23217149635259*BF10 +6.1705988886301*BF11 +5.35436363174631*BF12 +25.1887030742643*BF13 +21.9891466505736*BF14 - 23.9553201919529*BF15 +134.651814407374*BF16 +34.2250765085187*BF17 -22.1134531312855*BF18 +36.7540271328674*BF19 +98.5640890412154*BF20 -31.0385261576136*BF21 +17.9157544972686*BF22 +6.50242735909251*BF23 -3.04944251370636*BF24 +2.3694902512374*BF25 +5.85599474514156*BF26 +10.512643380109*BF27 +65.2527381800415*BF28 +4.55268876144292*BF29 +4.67815763749728*BF30 +0.594362959109518*BF31 +47.3049405432497*BF32 -0.857846460039789*BF33 -3.9681467988764*BF34 +19.8701944444153*BF35 -46.359747982478*BF36

BF1 = max(0,3 -x5)
BF2 = max(0,1 -x7)
BF3 = max(0,1 -x6)
BF4 = BF2 * max(0,2 -x2)
BF5 = max(0,1 -x10)
BF6 = max(0, x8 -1)
BF7 = max(0,1 -x8)
BF8 = max(0,1 -x1)
BF9 = max(0,1 -x11)
BF10 = max(0,3 -x3) * max(0, x2 -1)
BF11 = BF1 * max(0,4 -x7)
BF12 = max(0, x2 -1)
BF13 = BF6 * max(0, x1 -8)
BF14 = BF6 * max(0,8 -x1)
BF15 = BF6 * max(0, x1 -7)
BF16 = max(0,1 -x9)
BF17 = max(0,3 -x7) * max(0,1 -x4)
BF18 = BF6 * max(0,9 -x1)
BF19 = max(0,4 -x3)
BF20 = max(0,2 -x3)
BF21 = max(0, x6 -1) * max(0, x1 -8)

Biographical Information

Ukesh Chawal is currently a PhD candidate in Industrial Engineering program at the University of Texas at Arlington. He received his B.S and M.S in Industrial Engineering from the University of Texas at Arlington. He worked at The University of Texas at Arlington Research Institute (UTARI) as a Technology Commercialization Marketing Intern in 2015 and a PhD Student Intern in PROCESS DATA TECHNOLOGY GROUP - R&D at Air Products in 2017. He is a member of the Center on Stochastic Modeling Optimization and Statistics (COSMOS) at the University of Texas at Arlington and an active member of INFORMS, Tau Beta Pi Engineering Honors Society at UTA and a Certified Six Sigma Green Belt. His research interests include formulating and solving optimization problems using mathematical programming and statistical modeling.