A TWO-STAGE STOCHASTIC PROGRAMMING MODEL FOR ENHANCING SEISMIC RESILIENCE OF WATER PIPE NETWORKS

By

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DISSERTATION

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Dedication

I dedicate this work to Dr. Jay, my Supervising Professor. Without his unending support (inspiration, guidance, tolerance, therapy, friendship) I would never be able to finish my work. In addition, I dedicate this dissertation to my family and friends for their unwavering support.

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ABSTRACT

TWO-STAGE STOCHASTIC MODEL FOR ENHANCING SEISMIC RESILIENCE OF WATER PIPE NETWORKS

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Earthquakes are sudden and inevitable disasters that can cause enormous losses and suffering, and having accessible water is critically important for earthquake victims. To address this challenge, utility managers do preventive procedures on water pipes periodically to withstand future earthquake damage. The existing seismic vulnerability models usually consider simple methods to find the pipes to rehabilitate with highest priority. In this research, we develop an optimization approach to determine which water pipes to rehabilitate subject to a limited budget to achieve highest network serviceability after a disaster. We propose a two-stage stochastic mixed integer nonlinear program (MINLP). The MINLP model cannot be solved by commercial optimization software, like BARON even for problems with a very small number of scenarios. Consequently, we propose piecewise linear functions (PLF) to approximate the nonlinearity in the MINLP. Therefore, we formulate a mixed integer linear program (MILP) to approximate the MINLP. The optimization of the MILP is still challenging to solve, so we introduce a sequential heuristic algorithm to mitigate this computational issue and find bounds for the approximated optimal solution. Consequently, the solution we find using the sequential algorithm is within 2% of optimality.

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CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW

Earthquakes are sudden and inevitable disasters that can cause enormous losses and suffering. Preparing enough utility resources right after the earthquake is one of the most vital actions. Water, as the most important resource for keeping humans alive in these kinds of disasters, plays an important role. In addition, disruption in water supply causes other problems like disturbance in firefighter work. Historical data from past earthquakes show the importance of providing drinkable water right after an earthquake. For addressing this challenge, utility managers do a preventive procedure that repairs some of their water pipes periodically, but the important question is which pipes should be repaired since utility managers have limited budget. Existing seismic vulnerability models just consider simple methods to find pipes with highest priority (Shahandashti and Pudasaini 2019).

1.1. Water Resource Optimization Overview

Water resource optimization problems in the last three decades were reviewed in Tayfur (2017). Most of the reviewed papers notably employed simulation and meta-heuristic methods to optimize decisions. OPTIMA is a software that designs a new water pipe network in a new area by simulation techniques. It considers different criteria like economic ones and shows the final design in a geographical and spatial map (Fedra 2005). Some researchers used mathematical modelling to find an optimal design in new water pipe networks. Sarbu (1997) introduced a linear programming model that optimizes the diameter and length of each pipe in a new or extension of a water pipe network. His model used analysis of flow in water pipe networks with hydraulic equations introduced by Cross (1936). In another study, Yan et al. (2013) evaluated the hydraulic performance of a system by using hydraulic equations to assess the effect of rotational speed of different pumps on pump heads and flow rates in all loops in a water pipe network. Alvisi and Franchini (2010) used hydraulic equations to calibrate the roughness of the pipes in a water pipe network considering grey numbers. In other work, Sarbu and Kalmar (2002) introduced a nonlinear model that considers hydraulic equations, resource consumption, operating costs, along with other factors. They optimized the head loses along with pipe diameters by using a gradient method. Djebedjian, Herrick, and Rayan (2016) introduced a nonlinear model for designing a new water pipe network. They considered hydraulic equations, flow rate, pressure, and cost to evaluate the pipe diameter according to existing pipe diameters. They found a local optimal solution for the model by the sequential unconstrained minimization technique of Fiacco and McCormick (1964). Rayan et al. (2003) used their method in a real extension in an Egyptian water pipe network by considering Newton-Raphson method for the hydraulic analysis. Caballero and Ravagnani (2019) introduced a mixed integer nonlinear problem (MINLP) to design a new network with unknown flow direction and pipe diameters. They ignored pressure and flow speed in their network. They used the commercial solver BARON to solve their problem by calculating a tight bound for their nonlinear and nonconvex equations. Audu and Ovuworie (2010) introduced a Geospatial Information System (GIS) for planning and managing water pipe networks in Nigeria by acquiring data from a Global Positioning System (GPS) and integrating data in a GIS. Bilal and Pant (2020) introduced a new water distribution network for a Hanoi water network that has a complex structure. They optimized their model by introducing a hybrid metaheuristic algorithm along with a simulation software called EPANET. Cassiolato et al. (2020) proposed a mixed integer nonlinear problem (MINLP) for extending an existing water distribution network. They minimized the cost of the water distribution network and solved their model using GAMS (general algebraic modelling system) for four test problems. As we mentioned before, most of the papers in this area used meta-heuristic approaches to optimize the network (Abdel-Gawad 2001, Djebedjian, Yaseen, and Ryan 2008, Yaseen 2007, AbdelBary 2008). As discussed in these papers, they designed a new or an extension of an existing water pipe network. What if the water pipe network already exists, and we want to find an optimal rehabilitation policy in the network?

1.2. Water Pipe Network Rehabilitation Overview

Most of the water resource optimization papers design a new or an extension of an existing water pipe network. However, some researchers studied existing complex infrastructure problems like existing water pipe network rehabilitation problems. In Bernhardt and McNeil (2004), they introduced an agent-based model to simulate civil infrastructure systems. In Al-Khafaji, Mesheb and Jabbar Abrahim (2019), they presented a decision support mathematical model that decides a best maintenance strategy for an existing water pipe network. In Aşchilean et al. (2017a), they introduced the Analytic Hierarchy Process (AHP) method for solving a water pipe network rehabilitation problem. They concluded using a slip-line method is the best method and apply it to Cluj-Napoca, Romania, as their case study. In another study, Aşchilean et al. (2017b) analysed priorities selecting in a water pipe network rehabilitation problem. They concluded that asbestos cement pipes should be rehabilitated first.

1.3. Water Pipe Network Response to Earthquake Overview

Earthquakes, as a sudden and inevitable disaster, can cause enormous losses and suffering. Preparing enough utility resources right after the earthquake is one of the most vital actions. Water, as the most important resource, for keeping humans alive in these types of disasters, plays an important role. Historical data from past earthquakes show the importance of providing drinkable water right after the earthquake (Pudasaini and Shahandashti 2018, Shahandashti and Pudasaini 2019). To address this challenge, utility managers do a preventive procedure and decide to repair some of their water pipes periodically, but the important question is which pipes should be repaired since the underground placement of the pipes imposes an uncertainty on this decision, and the budget source is always limited. Existing seismic vulnerability models consider simple methods to find pipes with the highest priority (Pudasaini and Shahandashti 2018, Shahandashti and Pudasaini 2019). In order to increase reliability of the water pipe network in disasters like an earthquake, Wu and Baker (2017) assessed a water pipe network in each stage of an earthquake to find a best retrofit decision. In their study, they described how earthquake factors affect a water pipe network. Then they estimated the network reliability, and finally they introduced three methods for retrofitting pipes. O'Rourke et al. (2014) analyzed water and wastewater network response to several recent earthquakes in New Zealand. Next, they evaluated repair rate according to topological factors after an earthquake. In another report, Shi and O'Rourke (2006) developed a GIS-based model to simulate performance of a water pipe network in an earthquake. They evaluated their model results by comparing them with the 1994 Northridge earthquake. Trautman et al. (2013) proposed a structure for designing a decision support system for water pipe network planning, engineering, and management. They tested their model by applying it to a San Francisco water pipe network in a hypothetical earthquake. Macaskill, and Guthrie (2018) assessed the role of funding in disaster recovery, and they considered New Zealand earthquakes.

None of these studies included a solid mathematical optimization model. Simulation optimization techniques are the most used approach in water pipe rehabilitation optimization due to the uncertain nature of earthquakes.

Although this problem is an example of a network flow problem since it is a network, and a knapsack problem since it has budget limit, it is not being considered in these areas of knowledge.

1.4. Knapsack Problem Overview

Knapsack problems have been studied for over a century (Mathews 1896). In this literature review we only studied recent papers. Researchers have used different methods on multi-objective knapsack problems (Lai et al. 2018, Correia, Paquete, and Figueira 2018), meta-heuristic methods are popular in this area (Zouache, Moussaoui, and Abdelaziz 2018, Manicassamy et al. 2018, Arin and Rabadi 2017, Chih 2017, Chen and Hao 2017, Chen and Hao 2016, Haddar et al. 2016, Zhang et al. 2016, Arin and

Rabadi 2016). Song et al. (2018) represented a stochastic quadratic multiple knapsack problem (SQMKP). They proposed a three-step algorithm called Repair-Based Optimization Approach (RBOA) to solve the problem. Furini, Monaci, and Traversi (2018) worked on a special kind of Knapsack Problem with Setup (KPS) where items are categorized upon their fixed cost and capacity. They used three integer linear programming models and compared their results with an improved dynamic programming algorithm. Schulze et al. (2017) considered a bi-dimensional knapsack problem. By relaxing one of the constraints and adding it to the objective function, and they introduced their new model and conducted a sensitivity analysis. At last, by applying dynamic programming they solved their problem. Gao et al. (2017) proposed a new algorithm to solve a multi-dimensional multiple-choice knapsack problem. Their iterative *pseudo-gap* enumeration method uses some more cuts of the non-basic variables. Avci and Topaloglu (2017) developed a multi-start iterated local search algorithm to solve the quadratic multiple knapsack problem. In other work, Meng and Pan (2017) solved the multidimensional knapsack problem by applying a modified harmony search method in an Improved fruit fly optimization algorithm. Christian and Cremaschi (2017) continued their research from 2015 in a Knapsack Decomposition Algorithm (KDA) that decomposed the pipeline management challenges into some knapsack problems in a special time horizon. They showed that the parameters do not affect results, while the time of solving the problem grows linearly by increasing number of items and lingering time horizon.

1.5. Fixed Charge Network Flow (FCNF) Problem Overview

Metaheuristics approaches have also been used to solve a Fixed Charge Network Flow (FCNF) problem (Gendron, Hanafi, and Todosijevic 2018, Calvete et al. 2018). Nicholson and Zhang (2016) introduced a statistical method to assess the impact of different network features on optimal solutions of an FCNF problem. Zhang and Nicholson (2016), in another article, proposed a new probabilistic model to reformulate and relax an FCNF problem and model a linear programming problem. Munguía, et al.

(2017) proposed a local search method for solving a Fixed Charge Capacitated Multi Commodity Network Flow (FCMNF) problem. González et al. (2016) introduced an iterated local search for solving a bi-level Fixed Charge Un-capacitated Multi commodity Network Design problem with User-Optimal Flow (FCMNDP-UOF). Agarwal and Aneja (2017) attempted to choose edges out of a given undirected graph to satisfy given demands in a network with minimum cost. They considered sub-graphs by p-partitioning to decide which one is the best. Angulo and Vyve (2017) used dynamic programming for an FCNF transportation problem. Fakhri and Ghatee (2016) proposed a new partitioning method in Benders decomposition that utilize the branch-and-bound algorithm to solve MINLP problems such as FCMNDP models.

1.6. Contribution

In this research, we develop an optimization model that finds a best rehabilitation policy before an earthquake that maximizes expected service to the people right after the earthquake (Boskabadi, Rosenberger, and Shahandashti 2018, Boskabadi, Rosenberger, and Shahandashti 2019). Figure 1 shows the two-stage stochastic process considered in this research. In stage 1, an initial rehabilitation policy/decision is made subject to the limited budget. Then a hypothetical earthquake occurs and generates a random scenario that determines which pipes are broken. In stage 2, right after the earthquake, a recourse function determines the water flow in the unbroken pipes and maximizes the output flow in generated subgraphs.

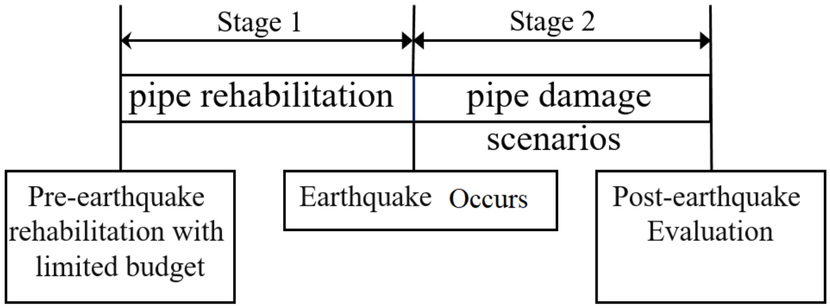


Figure 1 – Two-Stage Stochastic Model

The contribution of this paper is as follows. We formulate the aforementioned optimization model as a two-stage stochastic mixed integer nonlinear program (MINLP). The MINLP model cannot be solved by commercial optimization software, like BARON even for problems with a very small number of scenarios. Consequently, we propose piecewise linear functions (PLF) to approximate the nonlinearity in the MINLP. Therefore, we formulate a mixed integer linear program (MILP) to approximate the MINLP. The optimization of the MILP is still challenging to solve, so we introduce a sequential heuristic algorithm to mitigate this computational issue and find bounds for the approximated optimal solution.

In conclusion, the main contributions of this study can be summarized as follows:

1. We develop an optimization procedure to determine which water pipes to rehabilitate subject to a limited budget to achieve highest network post-disaster serviceability.
2. We formulate a novel optimization model as a two-stage stochastic mixed integer nonlinear programming (MINLP) model.
3. Due to the NP-hard nature of our MINLP model, we propose a MILP-based formulation using PLF.
4. We introduce a sequential heuristic algorithm to mitigate computational issues for finding bounds for the approximated optimal solution.

The remainder of this study is as follows. The two-stage stochastic programming model and the piecewise linear approximation is described in chapter 2. A sequential heuristic algorithm to mitigate this computational issue and problem instances are provided in chapter 3. Finally, conclusion and future research are provided in chapter 4.

CHAPTER 2: MODEL DESCRIPTION

2.1. General Formulation

A water pipe network is typically represented as a graph $G = (N, A)$, where N is the set of nodes and A is the set of arcs/pipes (figure 2). A node $n \in N$ shows an Origin, Destination, or just a transition node/ a junction of pipes. An arc $a \in A$ shows a pipe in the network connecting two nodes. Each arc $a \in A$ has some features such as flow, velocity, length, and diameter. Since the ultimate goal of a water pipe network is transferring water through the network, we consider flow as the main feature in each arc, and we define each arc with its flow from node i to j (f_{ij}).

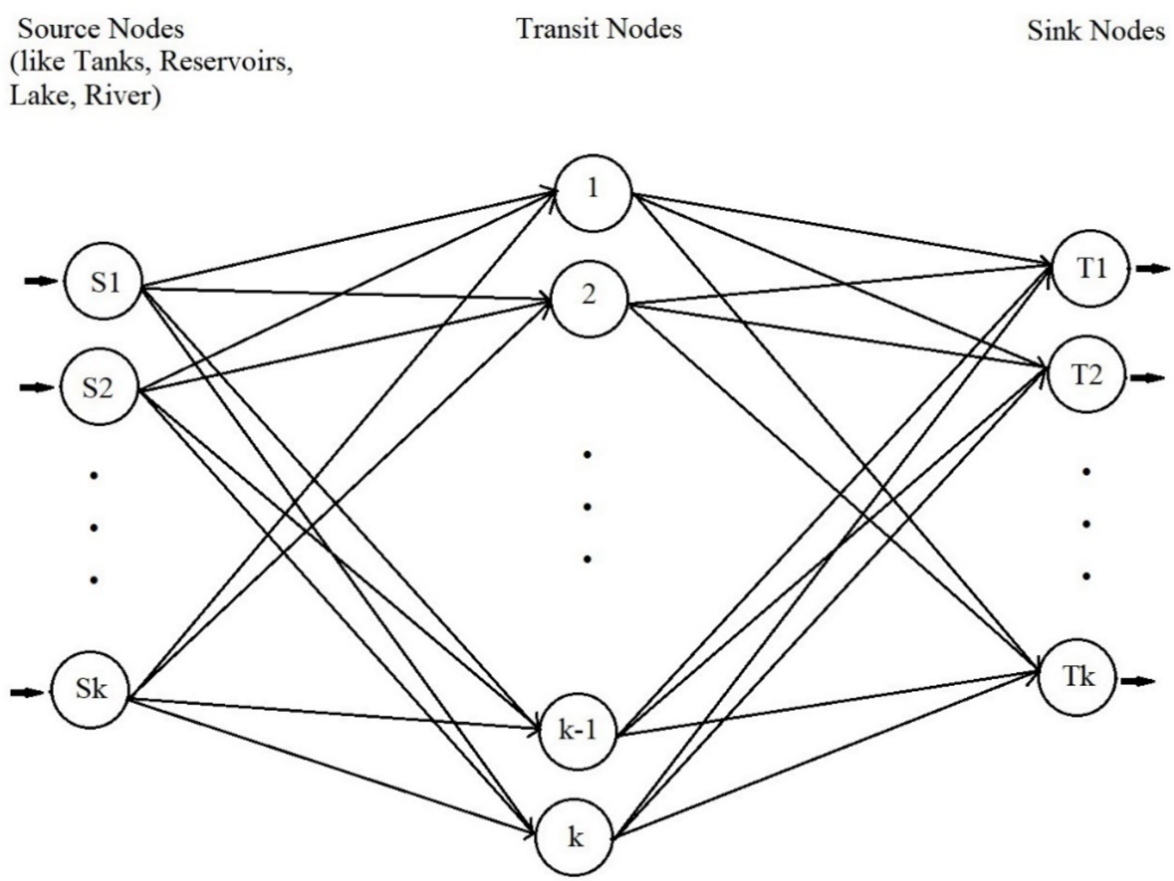


Figure 2 – Network Flow

We consider an existing hydraulic method - originally proposed by professor Hardy Cross (Cross 1936) - that depends on the head balance method in a water pipe network. This method considers closed-loop pipe networks. In this method, we assume the following:

- Water is withdrawn from nodes only, not directly from pipes.
- The water entering the system will have positive value, and the water leaving the system will have negative value.
- Minor losses in long pipes are ignored.
- For any individual pipe in the network, there is flow associated with it.
- The summation of inflow is equal to the summation of outflow at any junction/node.

For clarification, we present the formulation in an example. Let T_{ij} be a constant number for each pipe from node *i* to *j* presenting its physical features. let ρ be an experimental constant, usually equal to 1.852. The Hardy Cross formulation says that summation of T_{ij} times $f_{ij}^{1.852}$ for all pipes in special loop must be equal to zero.

It is worthwhile to note that the direction of the flow in each pipe has a critical role. In this formulation, each closed loop has a predefined direction, and if the direction of the flow in any pipe in that loop is the same, the coefficient T_{ij} is considered to be positive, and negative otherwise.

For simplicity, consider the example in figure 3 from hydraulics lecture notes.[1] We labeled each pipe with a number. In following network, we have three loops, loop 1 consists of pipes 1, 2, and 3; loop 2 consists of pipes 2, 4, and 5; and loop 3 consists of pipes 1, 3, 4, and 5. One of these three loop equations is redundant which means if two of them are satisfied the third one will be satisfied automatically, but we present all three equations here. Figure 3 shows predefined directions for the loops and flow directions in

[1] http://site.iugaza.edu.ps/sghabayen/files/2012/02/ch4-part-2.pdf

the pipes. T_{ij} for pipes 1 to 5 are given as 0.0187, 0.0187, 0.0092, 0.0280, and 0.0023. By using Hardy Cross method, we can guarantee head balance in the network.

N= {1, 2, 3, 4}; *A*= {1, 2, 3, 4, 5}; T_{ij} = {0.0187, 0.0187, 0.0092, 0.0280, 0.0023}

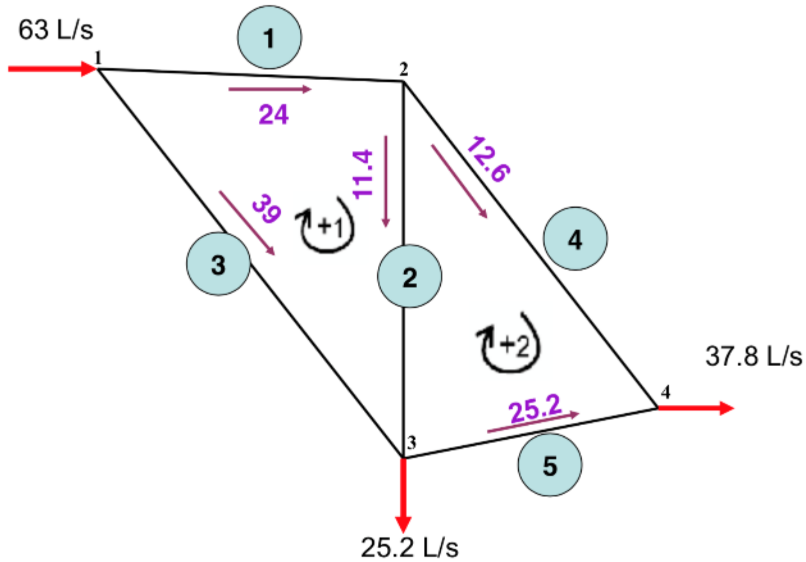


Figure 3 – Water Pipe Network Example

Loop 1: $T_{12}f_{12}^{1.852} + T_{23}f_{23}^{1.852} - T_{13}f_{13}^{1.852} = 0$

$0.0187 * 24^{1.852} + 0.0187 * 11.4^{1.852} - 0.0092 * 39^{1.852} = 0$

Loop 2: $T_{24}f_{24}^{1.852} - T_{23}f_{23}^{1.852} - T_{34}f_{34}^{1.852} = 0$

$0.0280 * 12.6^{1.852} - 0.0187 * 11.4^{1.852} - 0.0023 * 25.2^{1.852} = 0$

Loop 3: $T_{12}f_{12}^{1.852} + T_{24}f_{24}^{1.852} - T_{13}f_{13}^{1.852} - T_{34}f_{34}^{1.852} = 0$

$0.0187 * 24^{1.852} + 0.0280 * 12.6^{1.852} - 0.0092 * 39^{1.852} - 0.0023 * 25.2^{1.852} = 0$

2.2. Stochastic programming model for water pipe rehabilitation

We introduce a stochastic programming model for the water pipe rehabilitation problem with a recourse flow function to maximize the output flow after the earthquake.

Let the water pipe network be represented as a graph $G = (N, A)$, where N is a set of nodes, and A is a set of arcs/pipes as mentioned before. For each arc $(i,j) \in A$, let rehabilitation decision variable

$$x_{ij} = \begin{cases} 1; & \text{if pipe from node } i \text{ to } j \text{ is rehabilitated} \\ 0; & \text{otherwise} \end{cases}$$

In addition, let Ξ be a set of random scenarios in which each determines which pipes break according to a Monte Carlo simulation. For each scenario $\xi \in \Xi$, let P^{ξ} be the probability that scenario ξ occurs. We can calculate the P^{ξ} as follows:

$$P^{\xi} = \frac{\text{frequency of special scenario } \xi \text{ happens}}{\text{all scenarios}} \tag{1}$$

Let a *loop* be a sequence of connected pipes that begins and ends with the same node. In network literature, a loop is usually referred to as a cycle, but in this research, we have elected to use the term loop to be consistent with the hydraulic literature. Before the earthquake, let K be the set of all loops in the network G. For each loop $k \in K$, let the loop variable O_k^{ξ} be

$$O_k^{\xi} = \begin{cases} 1; & \text{if all pipes in loop k are unbroken in scenario } \xi \\ 0; & \text{if at least one of the pipes in loop } k \text{ is broken in scenario } \xi \end{cases}$$

We consider A_k as a subset of A which contains pipes in special closed loop k in each scenario.

We define binary parameter r_{ij}^{ξ} that is 1 when the pipe from node i to j breaks in scenario $\xi \in \Xi$ if unrehabilitated and 0 otherwise. Therefore, a network with $|A|$ pipes creates $2^{|A|}$ rehabilitation policies. For each of rehabilitation policy, there are $2^{|A|''}$scenarios where $|A|''$ is the subset of A containing unrehabilitated pipes. Each scenario corresponds to a sub-graph $g \in G$. Therefore, the model becomes huge even for a small-sized water pipe network.

For each arc $(i,j) \in A$ and scenario $\xi \in \Xi$, let f_{ij}^{ξ} be the flows from node i to j in scenario $\xi \in \Xi$. In addition, we define NF_i^{ξ} to be the net flow (inflow/outflow) of node i in scenario $\xi \in \Xi$. Let N_t be the subset of N consisting of demand nodes and N_s be the subset of N that contains source nodes. Therefore,

$NF_i^{\xi} = 0 \; if \; i \in N \backslash N_t \cup N_s, NF_i^{\xi} \geq 0 \; if \; i \in N_t,$ and $NF_i^{\xi} \leq 0 \; if \; i \in N_s$.

In addition, let l_{ij} be the cost of rehabilitating the pipe from node *i* to *j*, and let *L* be the rehabilitation budget. Moreover, let T_{ij} be a certain coefficient for each pipe that depends on the physical features of the pipe like its material and diameter, let ρ be an experimental constant, usually equal to 1.852, the hydraulic literature often defines the *pressure* in a pipe (*i*, *j*) to be $T_{ij} f_{ij}^{\xi \; \rho}$ (Cross, H. 1936). In addition, let U_{ij} be the maximum possible flow in each pipe (*i*, *j*) ∈ *A*. Therefore, the *extensive form* of the stochastic programming model for the water pipe rehabilitation problem is formulated as:

$$\text{Max } ESSI_x = \sum_{\xi \in \Xi} (P^{\xi}) \left(\frac{\sum_{i \epsilon N_t} NF_i^{\xi}}{\text{pre earthquake outflow}} \right) \quad (2)$$

$$\sum_{(i\ j) \in A} l_{ij} \, x_{ij} \leq L \quad (3)$$

$$\sum_{j \epsilon N:\,(i\ j) \epsilon A} f_{ij}^{\xi} - \sum_{j \epsilon N:(j\ i) \epsilon A} f_{ji}^{\xi} = NF_i^{\xi} \qquad \forall i \epsilon N, \; \forall \xi \in \Xi \quad (4)$$

$$O_k^{\xi} = \prod_{(i\ j) \epsilon A_k} (1 - (1 - x_{ij}) r_{ij}^{\xi}) \qquad \forall k \in K, \; \forall \xi \in \Xi \quad (5)$$

$$O_k^{\xi} (\textstyle\sum_{(i\ j) \epsilon A_k} T_{ij} f_{ij}^{\xi \; \rho}) = 0 \qquad \forall k \in K, \; \forall \xi \in \Xi \quad (6)$$

$$0 \leq f_{ij}^{\xi} \leq (1 - (1 - x_{ij}) r_{ij}^{\xi}) \, U_{ij} \qquad \forall (i,j) \epsilon A, \forall \xi \in \Xi \quad (7)$$

$$f_{ij}^{\xi} f_{ji}^{\xi} = 0 \qquad \forall (i,j) \epsilon A, \; \forall \xi \in \Xi \quad (8)$$

$$x_{ij} \in \{0, 1\} \qquad \forall (i,j) \epsilon A \quad (9)$$

$$O_k^{\xi} \in \{0, 1\} \qquad \forall k \in K, \forall \xi \in \Xi \quad (10)$$

In this MINLP, the objective function (2) maximizes the Expected System Serviceability Index (ESSI). Constraint (3) is a knapsack constraint that restricts the cost of rehabilitation to be less than a predetermined budget. Constraint set (4) ensures that the difference between input and output flow at each node is equal to the supply or demand of that node, also referred to as *flow conservation constraints*. Constraint set (5) ensures that each loop exists if and only if all of its pipes are unbroken after the earthquake. We discuss these constraints in more detail in Section 2.3. Constraint set (6) ensures that each remaining loop, satisfies *head balance* with $\rho = 1.852$. Constraint set (7) defines the relationship among *x*, *r*, and *f*. It guarantees that if a pipe is rehabilitated before the earthquake (*x* = 1), the earthquake does not break it. On the other hand, if a pipe is not rehabilitated before the earthquake (*x* = 0), it is broken after the earthquake if $r^{\xi} = 1$. Consequently, the flow *f* on an arc in scenario ξ can be nonzero if *x* = 1 or $r^{\xi} = 0$. Constraint set (8) makes sure that each flow is just in one direction in each pipe. Constraint sets (9) and (10) are integer restrictions.

The *deterministic equivalent (DE) model* for water pipe rehabilitation problem is:

$$Max\ Q(x_{ij}) \tag{12}$$

$$\text{s.t.: } \sum_{(i\ j)\in A} l_{ij}\, x_{ij} \le L \tag{3}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j)\epsilon A \tag{9}$$

Where $Q(x_{ij})$ is the *expected second-stage recourse function* defined as:

$$Q\left(x_{ij}\right) = E_{\xi} Q\left(x_{ij},\xi\right), \qquad \text{and} \tag{13}$$

$$Q\left(x_{ij},\xi\right) = max \sum_{i\epsilon N_t} NF_i^{\xi} \tag{14}$$

s.t.:

$$\sum_{j\epsilon N:\,(i\ j)\epsilon A} f_{ij}^{\xi} - \sum_{j\epsilon N:(j\ i)\epsilon A} f_{ji}^{\xi} = NF_i^{\xi} \qquad \forall i\epsilon N,\ \forall \xi \in \Xi \tag{4}$$

$$O_k^{\xi} = \prod_{(i\ j)\epsilon A_k} (1-(1-x_{ij})r_{ij}^{\xi}) \qquad \forall k \in K,\ \forall \xi \in \Xi \tag{5}$$

$$O_k^{\xi} (\sum_{(i\ j)\epsilon A_k} T_{ij} f_{ij}^{\xi\ \rho}) = 0 \qquad \forall k \in K,\ \forall \xi \in \Xi \tag{6}$$

$$0 \le f_{ij}^{\xi} \le (1-(1-x_{ij})r_{ij}^{\xi})\, U_{ij} \qquad \forall (i,j)\epsilon A, \forall \xi \in \Xi \tag{7}$$

$$f_{ij}^{\xi} f_{ji}^{\xi} = 0 \qquad \forall (i,j)\epsilon A,\ \forall \xi \in \Xi \tag{8}$$

Given a rehabilitation decision variable $x \in X$, the constraints in (4-8) can be decomposed by each rehabilitation decision variable and scenario resulting in $|X| \times |\Xi|$ recourse sub-problem.

2.3. Challenges with MINLP

Challenges in this MINLP are as follow:

1. Constraint (7) is not tight enough, which can be slow down Branch and Bound.
2. The number of loops is exponentially large with respect to the number of pipes in the network, and we need to detect all minimum loops in the network. This challenge affects the number of constraints in constraint sets (5) and (6).
3. Constraint sets (5), (6), and (8) are nonlinear and nonconvex.

2.3.1. Tightening Flow Upper Bound

To address first challenge, we compute the maximum possible flow in each pipe $(i, j) \in A$ by solving a network flow problem as follow:

$$U_{ij} = Max\ f_{ij} \qquad (11)$$

$$s.t.: \sum_{j \epsilon N:\,(i\ j)\epsilon A} f_{ij} - \sum_{j \epsilon N:(j\ i)\epsilon A} f_{ji} = NF_i \qquad \forall i \epsilon N,\ \forall \xi \in \Xi \qquad (4)$$

$$f_{ij} \geq 0 \qquad \forall (i,\ j) \in A \qquad (7a)$$

In this model, the objective function (11) maximizes the flow in a given pipe, while the model considers the flow conservation constraint set that we described before.

2.3.2. Minimum Loop Detection

The second challenge we encounter in this model is the detection of all loops in the network. To address this challenge, we consider a *minimum loop* to be one with no internal loops. For example, Loops 1 and 2 in figure 3 are minimum loops, but Loop 3 is not minimum. If a pipe breaks, the set of minimum loops may change in the created subgraph. let K^{ξ} be the set of all minimum closed loops in scenario ξ. In this study, we detect all minimum loops for subgraphs dynamically in each scenario. The detection of minimum loops in a graph can be done by different methods (Sedgewick 1983, Tucker 2006). Topological sorting algorithms and Depth-First Search are the prevalent algorithms in loop detection. The GrTheory toolbox for MATLAB, used in this research, adds one edge at a time to the spanning tree to find loops. On the other hand, the number of minimum loops changes in each scenario, therefore the number of constraints in constraint sets (5) and (6) is changes in our model dynamically with respect to the scenario. We revisit this challenge in the sequential algorithm.

2.3.3. Linear Approximation Models

Since the previously given stochastic MINLP model is computationally intractable, even for a small number of scenarios, we formulate an approximation as an MILP. We have three nonlinear constraint sets, (5), (6) and (8). For linearization of constraint set (5), we introduce two new constraint sets that can substitute for constraint set (5):

$$O_k^\xi \leq 1 - (1 - x_{ij}) r_{ij}^\xi \qquad \forall k \in K, \forall (i,j) \epsilon A_k,\ \forall \xi \in \Xi \quad (5a)$$

$$O_k^\xi \geq \sum_{\forall (i\ j) \epsilon A_k} 1 - (1 - x_{ij}) r_{ij}^\xi - (A_k - 1) \qquad \forall k \in K, \forall (i,j) \epsilon A_k,\ \forall \xi \in \Xi \quad (5b)$$

It guarantees if all the pipes in a special loop *k* is rehabilitated before earthquake, ($x_{ij} = 1$), the loop will exist in the scenario ξ∈ Ξ, ($O_k^\xi = 1$). On the other hand, if at least one of the pipes in loop *k* breaks, $\left(r_{ij}^\xi = 1\right)$, the loop is gone in scenario ξ∈ Ξ, ($O_k^\xi = 0$).

Since constraint (6) is a nonlinear equality constraint; it is not convex, nor concave. Linearization of binary variable $O_k^\xi, k \in \{1, \dots, K\}$ interacts with continuous variables $f_{ij}^{\xi\ \ \rho}, i, j \in \{1, \dots, N\}$ in each scenario $\xi \in \Xi$ is as follows (McCormick 1983), where *M* is a big positive constant:

$$-MO_k^\xi \leq \sum_{(i\ j) \epsilon A_k : r_{ij} = 0} T_{ij} f_{ij}^{\xi\ \rho} - \sum_{(i\ j) \epsilon A_k : r_{ij} = 0} T_{ij} f_{ji}^{\xi\ \rho} \leq MO_k^\xi \quad \forall k \in K,\ \forall \xi \in \Xi \qquad (6a)$$

These constraints can be written as two constraints:

$$-MO_k^\xi \leq \sum_{(i\ j) \epsilon A_k : r_{ij} = 0} T_{ij} f_{ij}^{\xi\ \rho} - \sum_{(i\ j) \epsilon A_k : r_{ij} = 0} T_{ij} f_{ji}^{\xi\ \rho} \quad \forall k \in K,\ \forall \xi \in \Xi \qquad (6b)$$

$$\sum_{(i\ j)\epsilon A_k:r_{ij}=0} T_{ij} f_{ij}^{\xi\ \rho} - \sum_{(i\ j)\epsilon A_k:r_{ij}=0} T_{ij} f_{ji}^{\xi\ \rho} \leq MO_k^{\xi} \qquad \forall k \in K,\ \forall \xi \in \Xi \tag{6c}$$

However, we still have the term $f_{ji}^{\xi\ \rho}$ that is nonlinear. By using Piecewise Linear Functions (PLF) (Ahuja, Magnanti and Orlin 1988), constraint sets (6b) and (6c) can be approximated and linearized. The term $f_{ij}^{\xi\ \rho}$ can be estimated by a PLF with *S* linear pieces. For each linear piece *s* = 1, …, *S*, let m_s be the slope, c_s be the intercept, (a'_s, a'_{s+1}) be the domain, and $w_{ij\ s}$ be a binary variable that indicates the flow of pipe (*i*, *j*) is in the domain of *s*. Figure 3 shows an example for 3 linear pieces in PLF.

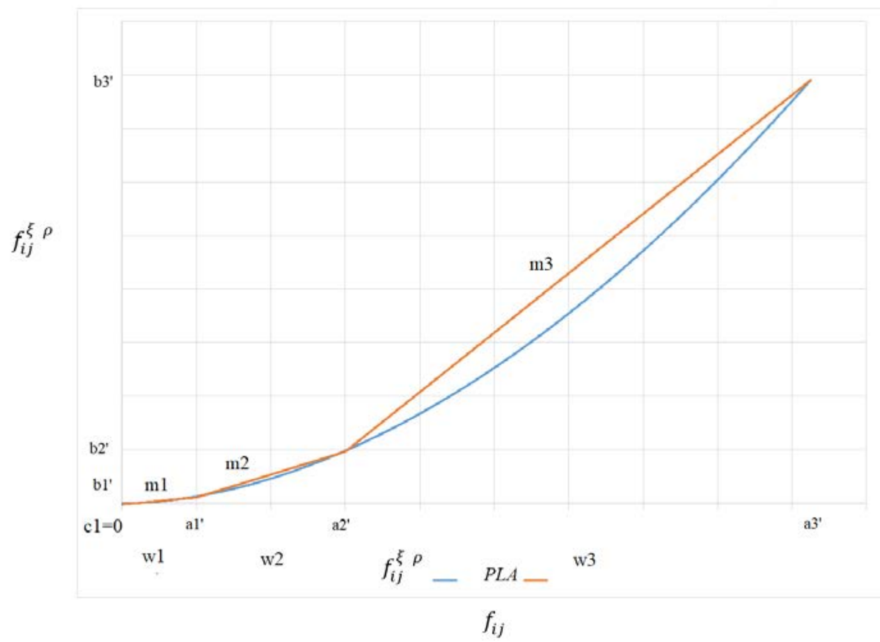


Figure 4 – Piecewise Linear Approximation (PLA)

As it can be seen in figure 4, $f_{ij}^{\xi\ \rho}$ can be estimated by the summation of f functions (3 functions in the example shown in figure 4) in the form of $f_{ij\ s}^{\xi}$ that satisfy the following condition:

$$f_{ij}^{\xi\ \rho} \approx \left(m_1 f_{ij\ 1}^{\xi} + c_1 w_{ij\ 1}\right) + \left(m_2 f_{ij\ 2}^{\xi} + c_2 w_{ij\ 2}\right) + \cdots + \left(m_s f_{ij\ s}^{\xi} + c_s w_{ij\ s}\right) \tag{15}$$

$$\text{s.t.:} \quad 0 \leq f_{ij\ 1}^{\xi} \leq a'_1 w_{ij\ 1} \qquad \forall (i,j)\epsilon A, \forall \xi \in \Xi \tag{16}$$

$$a'_{s-1} w_{ij\ s} \leq f_{ij\ s}^{\xi} \leq a'_s w_{ij\ s} \qquad \forall (i,j)\epsilon A, \forall \xi \in \Xi;\ \forall s \epsilon \{2, 3, \dots, S\} \tag{17}$$

$$w_{ij\,1} + w_{ij\,2} + \cdots + w_{ij\,s} = 1 \qquad \forall (i,j) \epsilon A \qquad (18)$$

$$w_{ij\,1}, w_{ij\,2}, \ldots, w_{ij\,s} \in \{0,\ 1\} \qquad \forall (i,j) \epsilon A \qquad (19)$$

Therefore, constraints (6b) and (6c) can be approximated as follows:

$$-MO_k^{\xi} \le \sum_{(i\,j)\epsilon A_k} T_{ij}\left(\sum_{s=1}^{S} m_s\, f_{ij\,s}^{\xi} + c_s\, w_{ij\,s}\right) - \sum_{(i\,j)\epsilon A_k} T_{ij}\left(\sum_{s=1}^{S} m_s\, f_{ij\,s}^{\xi} + c_s\, w_{ij\,s}\right)$$

$$\forall k \in K,\ \forall \xi \in \Xi \qquad (6d)$$

$$\sum_{(i\,j)\epsilon A_k} T_{ij}\left(\sum_{s=1}^{S} m_s\, f_{ij\,s}^{\xi} + c_s\, w_{ij\,s}\right) - \sum_{(i\,j)\epsilon A_k} T_{ij}\left(\sum_{s=1}^{S} m_s\, f_{ji\,s}^{\xi} + c_s\, w_{ij\,s}\right) \le MO_k^{\xi}$$

$$\forall k \in K,\ \forall \xi \in \Xi \qquad (6e)$$

The linearization method for constraint set (8) when multiplication of two continuous variables must be equal to zero can be formulated as below[1]. We use a set of binary variables g_{ij}^{ξ}.

$$f_{ij}^{\xi} \le g_{ij}^{\xi} U_{ij} \qquad \forall (i,j) \epsilon A,\ \forall \xi \in \Xi \qquad (8a)$$

$$f_{ji}^{\xi} \le (1 - g_{ij}^{\xi}) U_{ij} \qquad \forall (i,j) \epsilon A,\ \forall \xi \in \Xi \qquad (8b)$$

$$g_{ij}^{\xi} \in \{0, 1\} \qquad \forall (i,j) \epsilon A,\ \forall \xi \in \Xi \qquad (8c)$$

On the other hand, when two continuous variables cannot take positive values at the same time and at least one of them must be zero, we can use a set of auxiliary binary variables g_{ij}^{ξ} and some large positive numbers like U_{ij} to transform the complementary condition to linear constraints.

In network simplex it is known that even when we consider flow in two directions, just one of them will get a number (Ahuja, Magnanti and Orlin 1988). Therefore, we relax constraint (8).

2.4. Revised Two-Stage Stochastic Programming Formulation

The final approximate MILP consists of objective function (2), and constraints (3), (4), (5a), (5b), (6d), (6e), (7), (9), (10), (16), (17), (18), and (19):

[1] https://www.leandro-coelho.com/linearization-product-variables/

$$\text{Max } ESSI_x = \sum_{\xi \in \Xi} (P^{\xi}) \left(\frac{\sum_{i \epsilon N_t} NF_i^{\xi}}{\text{pre earthquake outflow}} \right) \quad (2)$$

$$\sum_{(i\ j) \in A} l_{ij}\, x_{ij} \leq L \quad (3)$$

$$\sum_{j \epsilon N:\,(i\ j) \epsilon A} f_{ij}^{\xi} - \sum_{j \epsilon N:(j\ i) \epsilon A} f_{ji}^{\xi} = NF_i^{\xi} \qquad \forall i \epsilon N,\ \forall \xi \in \Xi \quad (4)$$

$$O_k^{\xi} \leq 1 - \left(1 - x_{ij}\right) r_{ij}^{\xi} \qquad \forall k \in K^{\xi}, \forall (i,j) \epsilon A_k,\ \forall \xi \in \Xi \quad (5a)$$

$$O_k^{\xi} \geq \sum_{\forall (i\ j) \epsilon A_k} 1 - \left(1 - x_{ij}\right) r_{ij}^{\xi} - (A_k - 1) \qquad \forall k \in K^{\xi}, \forall (i,j) \epsilon A_k,\ \forall \xi \in \Xi \quad (5b)$$

$$-MO_k^{\xi} \leq \sum_{(i\ j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s\, f_{ij\,s}^{\xi} + c_s\, w_{ij\,s}\right) - \sum_{(i\ j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s\, f_{ij\,s}^{\xi} + c_s\, w_{ij\,s}\right)$$

$$\forall k \in K^{\xi},\ \forall \xi \in \Xi \quad (6d)$$

$$\sum_{(i\ j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s\, f_{ij\,s}^{\xi} + c_s\, w_{ij\,s}\right) - \sum_{(i\ j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s\, f_{ji\,s}^{\xi} + c_s\, w_{ij\,s}\right) \leq MO_k^{\xi}$$

$$\forall k \in K^{\xi},\ \forall \xi \in \Xi \quad (6e)$$

$$0 \leq f_{ij}^{\xi} \leq \left(1 - (1 - x_{ij}) r_{ij}^{\xi}\right) U_{ij} \qquad \forall (i,j) \epsilon A, \forall \xi \in \Xi \quad (7)$$

$$x_{ij} \in \{0, 1\} \qquad \forall (i,j) \epsilon A \quad (9)$$

$$O_k^{\xi} \in \{0, 1\} \qquad \forall k \in K^{\xi}, \forall \xi \in \Xi \quad (10)$$

$$0 \leq f_{ij\,1}^{\xi} \leq a'_1 w_{ij\,1} \qquad \forall (i,j) \epsilon A, \forall \xi \in \Xi \quad (16)$$

$$a'_{s-1} w_{ij\,s} \leq f_{ij\,s}^{\xi} \leq a'_s w_{ij\,s} \qquad \forall (i,j) \epsilon A, \forall \xi \in \Xi;\ \forall s \epsilon \{2, 3, \dots, S\} \quad (17)$$

$$w_{ij\,1} + w_{ij\,2} + \cdots + w_{ij\,s} = 1 \qquad \forall (i,j) \epsilon A \quad (18)$$

$$w_{ij\,1}, w_{ij\,2}, \dots, w_{ij\,s} \in \{0,\ 1\} \qquad \forall (i,j) \epsilon A \quad (19)$$

Moreover, the expected MILP second-stage recourse function consists of objective function (14), and constraints (4), (5a), (5b), (6d), (6e), (7), (9), (10), (16), (17), (18), and (19).

$$Q\left(x_{ij}, \xi\right) = max \sum_{i \epsilon N_t} NF_i^{\xi} \quad (14)$$

$$\sum_{j\epsilon N:\,(i\ j)\epsilon A} f_{ij}^{\xi} - \sum_{j\epsilon N:(j\ i)\epsilon A} f_{ji}^{\xi} = NF_i^{\xi} \qquad \forall i\epsilon N,\ \forall\xi \in \Xi \qquad (4)$$

$$O_k^{\xi} \leq 1 - (1 - x_{ij})r_{ij}^{\xi} \qquad \forall k \in K^{\xi}, \forall(i,j)\epsilon A_k,\ \forall\xi \in \Xi \ (5a)$$

$$O_k^{\xi} \geq \sum_{\forall(i\ j)\epsilon A_k} 1 - (1 - x_{ij})r_{ij}^{\xi} - (A_k - 1) \qquad \forall k \in K^{\xi}, \forall(i,j)\epsilon A_k,\ \forall\xi \in \Xi \ (5b)$$

$$-MO_k^{\xi} \leq \sum_{(i\ j)\epsilon A_k} T_{ij}(\sum_{s=1}^{S} m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s}) - \sum_{(i\ j)\epsilon A_k} T_{ij}(\sum_{s=1}^{S} m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s})$$

$$\forall k \in K^{\xi},\ \forall\xi \in \Xi \qquad (6d)$$

$$\sum_{(i\ j)\epsilon A_k} T_{ij}(\sum_{s=1}^{S} m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s}) - \sum_{(i\ j)\epsilon A_k} T_{ij}(\sum_{s=1}^{S} m_s f_{ji\,s}^{\xi} + c_s w_{ij\,s}) \leq MO_k^{\xi}$$

$$\forall k \in K^{\xi},\ \forall\xi \in \Xi \qquad (6e)$$

$$0 \leq f_{ij}^{\xi} \leq (1 - (1 - x_{ij})r_{ij}^{\xi})\ U_{ij} \qquad \forall(i,j)\epsilon A, \forall\xi \in \Xi \qquad (7)$$

$$x_{ij} \in \{0, 1\} \qquad \forall(i,j)\epsilon A \qquad (9)$$

$$O_k^{\xi} \in \{0, 1\} \qquad \forall k \in K^{\xi}, \forall\xi \in \Xi \qquad (10)$$

$$0 \leq f_{ij\,1}^{\xi} \leq a'_1 w_{ij\,1} \qquad \forall(i,j)\epsilon A, \forall\xi \in \Xi \qquad (16)$$

$$a'_{s-1} w_{ij\,s} \leq f_{ij\,s}^{\xi} \leq a'_s w_{ij\,s} \qquad \forall(i,j)\epsilon A, \forall\xi \in \Xi;\ \forall s\epsilon\{2, 3, \dots, S\} \qquad (17)$$

$$w_{ij\,1} + w_{ij\,2} + \cdots + w_{ij\,s} = 1 \qquad \forall(i,j)\epsilon A \qquad (18)$$

$$w_{ij\,1}, w_{ij\,2}, \dots, w_{ij\,s} \in \{0,\ 1\} \qquad \forall(i,j)\epsilon A \qquad (19)$$

2.5. Evaluation Procedure

As we mentioned before, in our study, the problem finds a best rehabilitation policy X before the earthquake that maximizes the expected SSI right after the earthquake. We find bounds for the approximated optimal solution by employing the solution method described in (Mak, Morton, and Wood 1999). Consequently, we solve the MILP defined above for a limited budget or limited rehabilitation length by using the MATLAB and Gurobi. Since the optimization of the MILP is still challenging to solve, we introduce a sequential revised two-stage stochastic solution to find an optimality gap for the MILP optimal solution.

2.5.1. Accuracy of MILP Recourse Function

The evaluation of the MILP and the MINLP has been done in a case when there is no break in the network. We use fmincon function in MATLAB to find an optimal solution for MINLP. Hence, we evaluate how well our MILP approximates the MINLP.

2.5.2. Sequential Revised Two-Stage Stochastic Algorithm

The optimization of the MILP is still challenging to solve. To mitigate this computational issue and find bounds for the approximated optimal solution, we introduce a sequential revised two-stage stochastic solution in Algorithm 1 to find an optimality gap for the MILP optimal solution (Mak, Morton, and Wood 1999). We used MATLAB and Gurobi. Programming code was written in MATLAB to generate the minimum loops and formulate the MILP. GUROBI was used to optimize the MILP.

Algorithm 1. Sequential Revised Two-Stage Stochastic Solution

---

Sort scenarios by their descending probabilities.

Divide them into groups of size *G*, therefore we have $|\Xi|/_G$ groups.

Consider a predefined *L* for length of rehabilitation.

Initialize $i \leftarrow 0$, predefine flow direction, predefine loop direction.

While $i \leq |\Xi|/_G$ **AND** remaining rehab length *L*>0 **do**:

- Solve the MILP (2-4, 5(a), 5(b), 6(d), 6(e), 7, 16-19, 9,10) over Ξ^i to obtain $\bar{x}^i$ and $\text{ESSI}_{\bar{x}^i}$ (MATLAB – GUROBI).

- Let $L \leftarrow L - \sum_j l_j \bar{x}^i{}_j$.
- Let $\acute{x}_j = 1,$ if $\bar{x}^i{}_j = 1$.
- Let $r_j^{\xi} = 0,$ if $\bar{x}^i{}_j = 1,\ \forall \xi \in \Xi^{i+1,\ldots,|\Xi|}/_G$.
- i=i+1.

End while.

Calculate $\mathrm{ESSI}_{\bar{x}}{}^{\mathrm{Seq}} = \sum_{i=1}^{|\Xi|/_G} \sum_{\xi \in \Xi^i} p^{\xi}\ \mathrm{ESSI}_{\bar{x}^i}$.

Report $\bar{x}^{\mathrm{Seq}},\ \mathrm{ESSI}_{\bar{x}}{}^{\mathrm{Seq}}$.

Algorithm 1 sorts all scenarios by their descending probabilities; then they are divided into groups of size *G*. Let *L* be the predefined rehabilitation length. On each iteration, we solve the MILP model over each group of scenarios to find a best-known decision policy and its ESSI. With the pipes in the best-known decision policy, we force them to be rehabilitated in subsequent groups of scenarios. Then we reduce the remaining rehabilitation length by length of these pipes. Algorithm 1 terminates when there are no more groups of scenarios or remaining rehabilitation length. Finally, we calculate expected system serviceability index which is a Lower Bound (LB) for the optimal solution by multiplying each of calculated ESSIs by its probability. Note that if we do not update the rehabilitation length and next groups of scenarios by the pipes in the best-known decision policy in algorithm 1, the expected ESSI is an Upper Bound (UB) for our model.

2.5.3. Discussion

Mak, Morton, and Wood (1999) proved that for an maximization problem with continuous random variables, we can sample a huge number of scenarios, divide these scenarios into smaller subsets of scenarios, solve the maximization problem over each of these group of scenarios, and then take the average of the objective values to determine an expected UB on the objective value of the original maximization problem, since the expected value of maximums is greater than or equal to the maximum of the expected values. Similarly, we can use a modified version algorithm 1 to determine an upper bound in this research. In this particular algorithm (algorithm 1), by not adjusting the length, we essentially solve the original problem within each group of scenarios. Consequently, this creates an expected value of the maximums that is greater than the objective value of an optimal solution and is an UB for our problem.

The other component is the LB. By definition, any feasible first stage solution is less than or equal to an optimal solution. Consequently, any rehabilitation plan we find that is feasible, by definition is a LB to our problem. In this particular algorithm (algorithm 1), by continually reducing the remaining length and solving this problem under the remaining groups of scenarios, we find a feasible rehabilitation plan, Therefore, algorithm 1 will form a bound in expectation on our problem. Note that Mak, Morton, Wood 1999 do not just create an expected bound, but they also create 95% Confident Intervals (CIs), which was not done here but a topic of future research.

It is also worthwhile to note that when we are creating these bounds, we are doing this subject to all of the approximations we explained in section 2.3.3. Consequently, our $\mathrm{ESSI}_{\bar{x}}^{\mathrm{Seq}}$ in algorithm 1 yields approximated expected bounds.

CHAPTER 3: COMPUTATIONAL STUDY

We consider two networks, networks 1 and 2. Network 1 consists of 117 pipes, 92 nodes, and 22 loops before the earthquake (figure 5). The water pipe network length is 65,749 meters. Figure 6 shows network 2, the Modena network. Network 2 consists of 317 pipes, 272 nodes, and 46 loops before the earthquake. The water pipe network length is 71806 meters. The maximum expected system serviceability before earthquake is 387.67 (liter/second). For network 2, we generated 3000 random scenarios using Monte Carlo Simulation from a hypothetical earthquake (Shahandashti and Pudasaini 2019). 308 pipes out of the 317 pipes break at least once in the 3000 scenarios. Consequently, most of the pipes in the network breaks at least in one of the scenarios, and finding a best policy for rehabilitation plays an important role in making decision about which pipe is rehabilitated. However, many of the scenarios within the 3000 are repeated, and we note that there are in fact only 1505 unique scenarios. The probability of each unique scenario has been calculated by (1). We used MATLAB and GUROBI to solve the MILP model. Programming code was written in MATLAB to generate the minimum loops and formulate the MILP. GUROBI was used to optimize the MILP. The model was solved on a workstation with the following device specification: Processor, Intel® Xeon® CPU E3-1285 v6 @ 4.10 GHz., and Installed RAM, 32.0 GB.

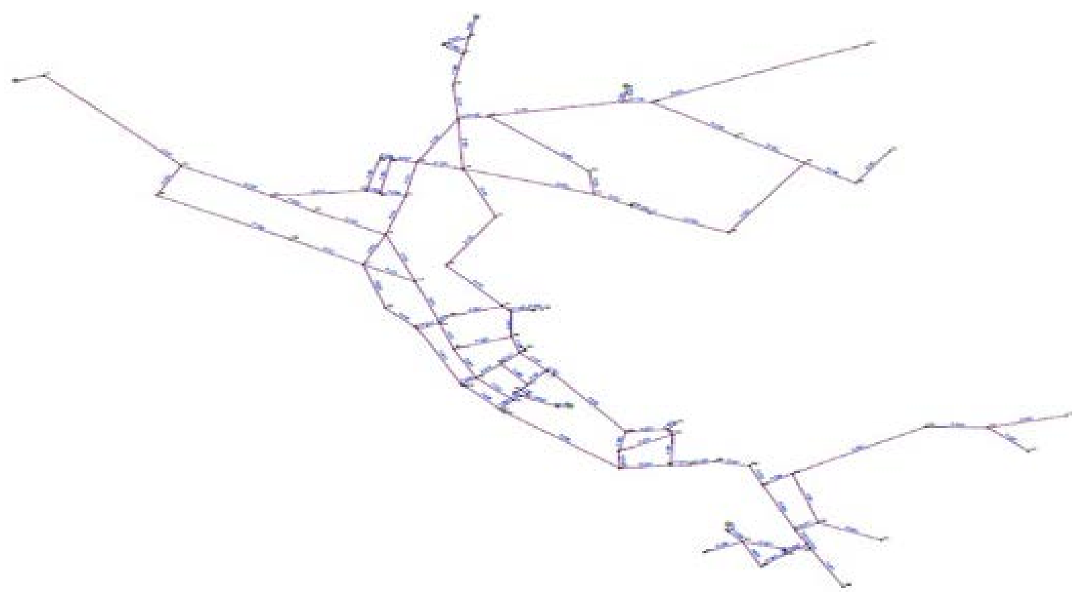


Figure 5 – Network 1

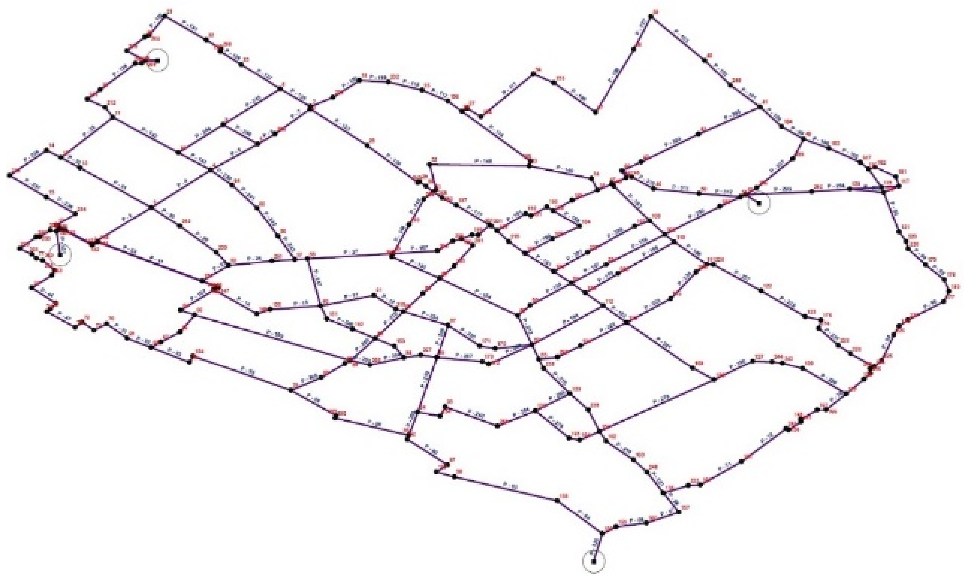


Figure 6 – Network 2

3.1. Accuracy of MILP Recourse Function

First, we evaluate the accuracy of the MILP recourse function. For Network 1, we consider a single scenario case in which the earthquake does not break any pipes. We solve the MILP using Gurobi to determine the flows in Network 1. In this case model does not recommend any rehabilitation policy. Then, we use FMINCON function in MATLAB to find an optimal solution for the nonlinear recourse function. We should mention that the result from FMINCON function may not be global optimal. The results show the linear approximated flow from the MILP recourse function is 210.5 liters/second, while the local optimal solution for the non-linear recourse function is 209.3 liters/second. Consequently, in this case the piecewise linear approximation is 99.4% accurate.

3.2. Sequential Heuristic Revised Two-Stage Stochastic Programming

We apply algorithm 1 in network 2 to evaluate our sequential algorithm. Since network 2 is huge, we divide the 1505 unique scenarios into groups with five scenarios each. We assume rehabilitation cost is proportional to the pipe length, and the budget limit is 1500 meters. First, we solve the MILP over the most probable five scenarios. Then, we reduce budget limit by the length of the pipes in the solution. We repeat this process until we have either no remaining scenario groups or budget limit. Table 1 shows the results. The final policy rehabilitates 4 pipes with a total length of 1498.97 meters and a serviceability of 373.25 liters/second (figure 7).

We use the method in (Mak, Morton, and Wood 1999) and determine the following optimality bounds for the solution as discussed in previous chapter:

$$373.25\left(\frac{\text{liters}}{\text{second}}\right) \leq z^{*} \leq 386.64\left(\frac{\text{liters}}{\text{second}}\right) \quad (20)$$

Consequently, the solution we find using the sequential algorithm is within 2% of optimality.

Table 1- Sequential MILP

Iteration	Sum Rehab	Remaining Rehab Length	# of Rehabilitated Pipes	Pipes get Rehabilitated
1 to 27	1411.84	88.16	2	22, 154
28 to 60	1492.94	7.06	3	22, 154, 26
61	1498.97	1.03	4	22, 154, 26, 13

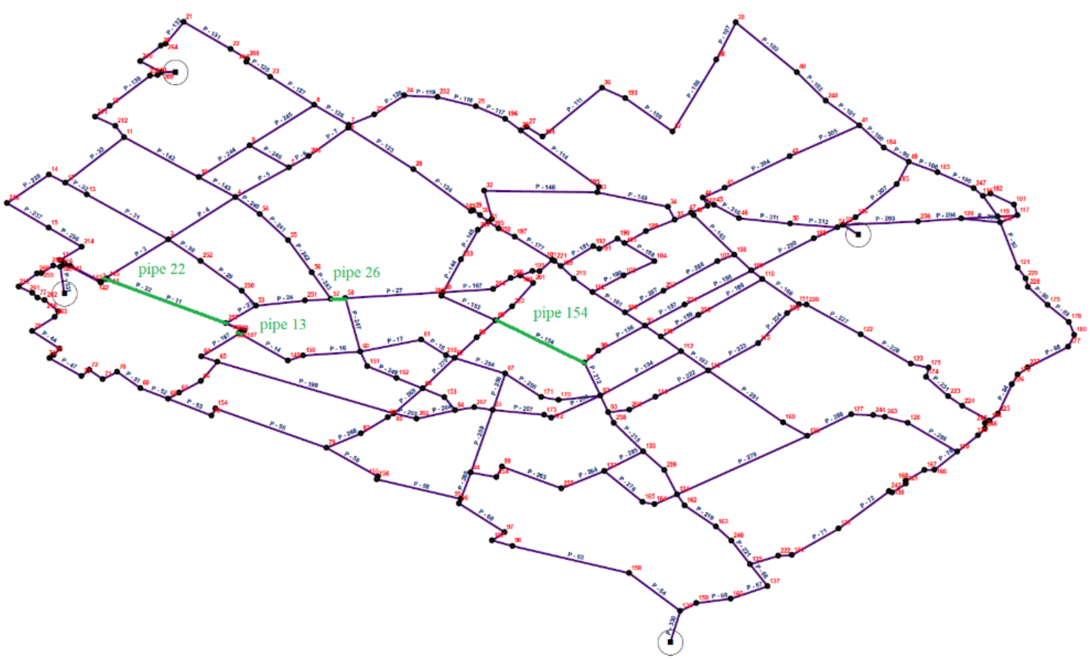


Figure 7 – Rehabilitation policy *X*

3.3. Evaluation with Previous Results

As we mentioned before, Network 2 has been considered before by Shahandashti and Pudasaini (2019). They employed Simulated Annealing (SA) to solve the same problem. For evaluating sequential algorithm solution from algorithm 1, we compare its second-stage recourse function results for its rehabilitation policy with the ones for rehabilitation policy concluded by SA. Table 2 shows the solutions of the two approaches:

Table 2 – comparisons of results

Method	Rehab Length (m)	Remaining Rehab Length (m)	Rehabilitated Pipes	Expected Flow (l/s)
Sequential Algorithm solution	1,498.97	1.03	13, 22, 26, 154	373.3
Simulated Annealing (SA) solution	1,489.06	10.94	3, 48, 79, 137, 160, 273	372.5

As we can see in table 2 sequential algorithm solution provides 0.8 (liter/second) outflow more than SA solution for the same network and earthquake.

Note that, SA solution did not use up the entire budget. There are multiple pipes that are under 10 m in length, like pipe 13, that breaks in at least one scenario out of 3000 scenarios. From a knapsack perspective, we know there is better results than SA solution, since any pipe under 10 m could be added to the rehabilitation policy. On the other side, the sequential algorithm solution has a remaining rehabilitation length that is smaller than every pipe that breaks in the 3000 scenarios in the network.

3.4. Problem Instances

In this study, we created 30 new networks by deleting certain pipes from network 2 to evaluate the performance of the sequential algorithm. Since network 2 is a water pipe network, we were very careful about the pipes we deleted to create new networks. Pipes were deleted according to their position in a loop. Therefore, created new loops were perfect closed-loop, and the new networks had rational structure. In other words, to avoid extraneous pipes, if a sequence of pipes is only connected by couplings, and one of the pipes is deleted, then the remaining pipes in the sequence are deleted as well. Then we reduced the network length by the total deleted pipe length. For determining new sets of unique scenarios in each new network, we removed the deleted pipes in the 3000 original scenarios, and then we found unique scenarios in each new network.

Figure 8 and 9 are summary statistics of the 30 new networks. Figure 8 shows the number of nodes and pipes in the 30 new networks, while figure 9 summarizes the network lengths. Figure 10 shows drawing of 30 new networks.

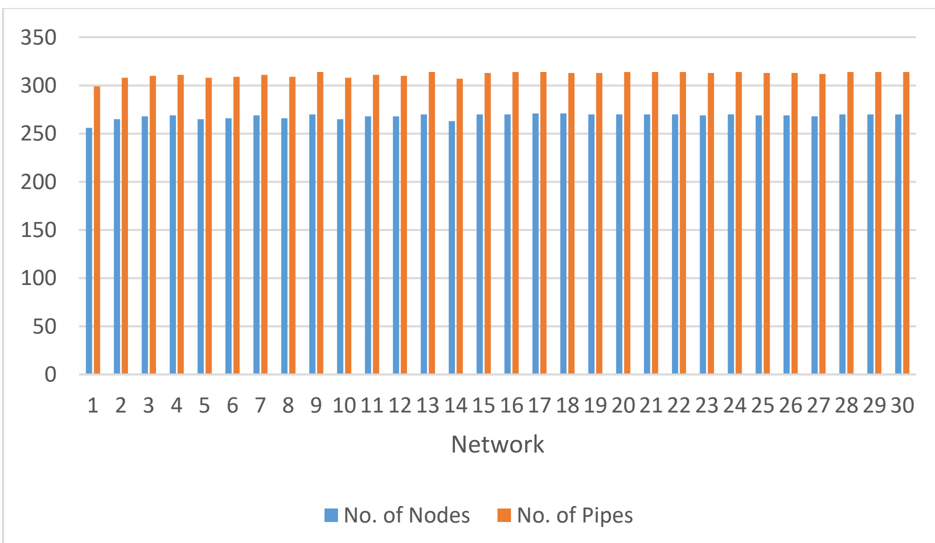


Figure 8 – Number of nodes and pipes in 30 networks

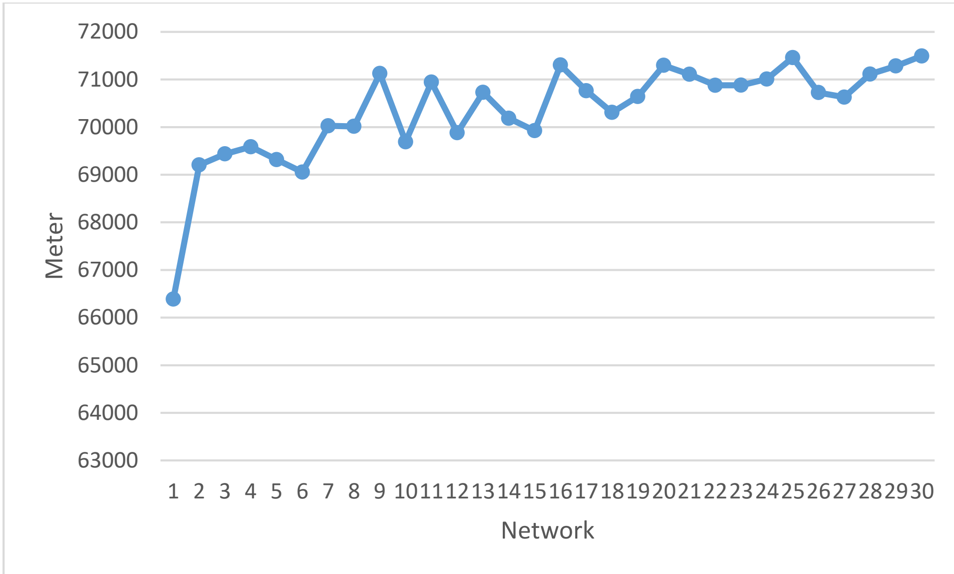
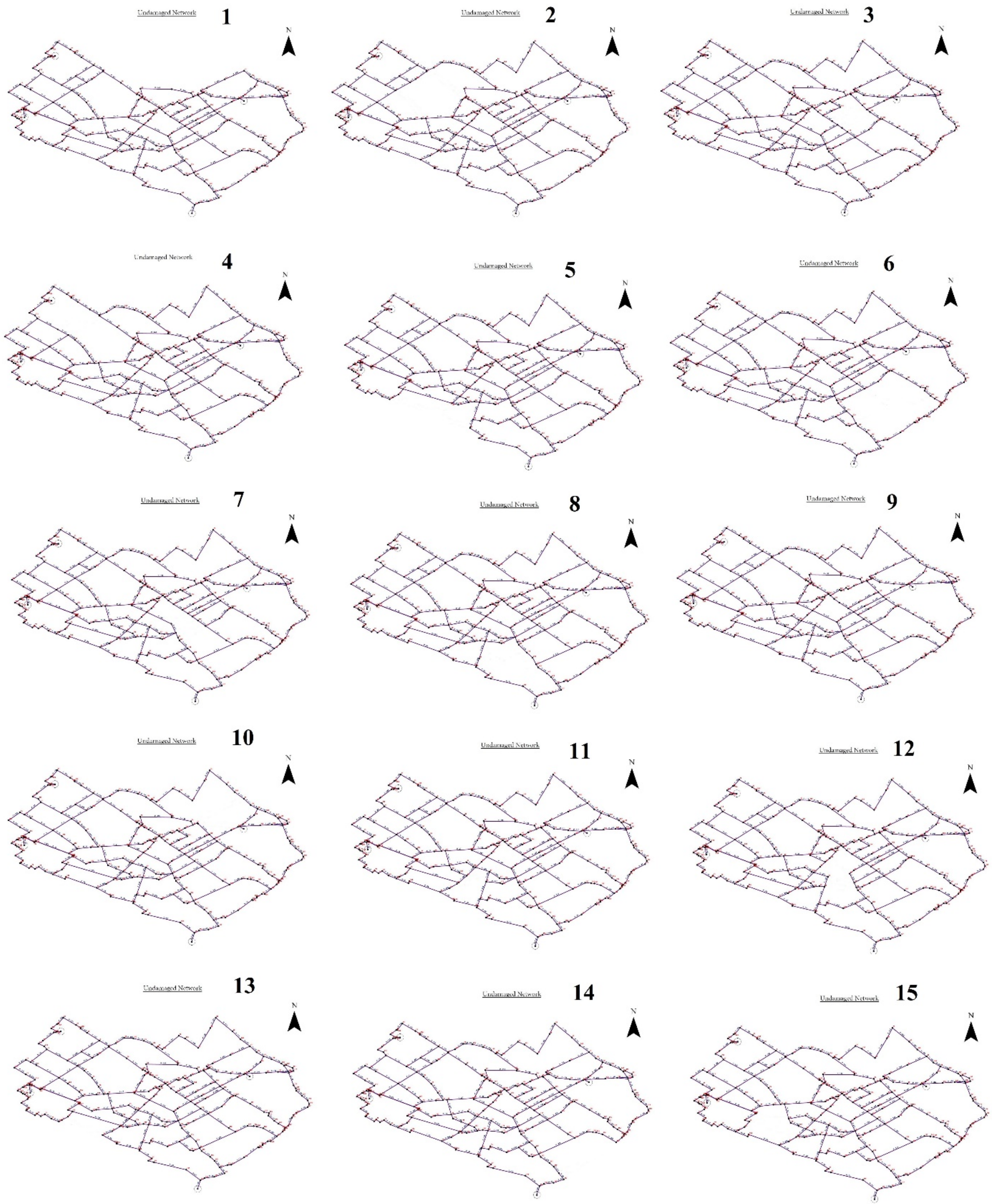


Figure 9 – Length of new networks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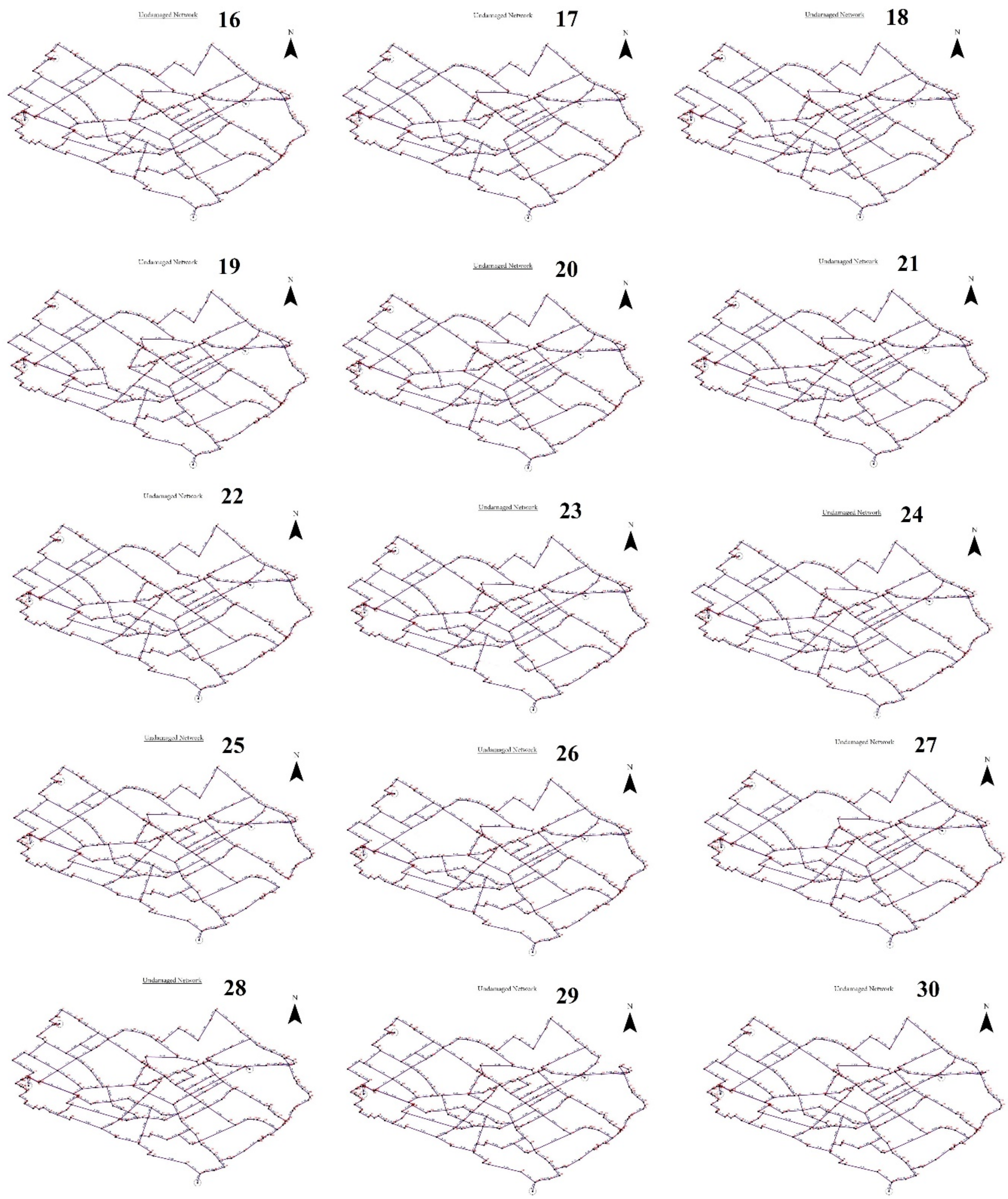

Figure 10 – Drawings of 30 new networks

For each network, the sequential algorithm was used to achieve bounds according to the method described in (Mak, Morton, and Wood 1999). Table 3 shows results.

Table 3- Rehabilitation policies of 30 new networks

Network 1:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 5	2	22, 142	1269.39	230.61
6 to 30	3	22, 142, 254	1457.08	42.92
31 to 275	4	22, 142, 254, 8	1496.99	3.01

Network 2:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 113	1456.99	43.01
30 to 290	3	22, 113, 8	1496.9	3.1

Network 3:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 292	3	22, 114, 8	1496.9	3.1

Network 4:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 21	2	19, 111	1456.99	43.01
22 to 291	3	19, 111, 5	1496.9	3.1

Network 5:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 24	2	22, 148	1411.84	88.16
25 to 30	3	22, 148, 220	1455.09	44.91
31 to 290	4	22, 148, 220, 8	1495	5

Network 6:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 289	3	22, 114, 8	1496.9	3.1

Network 7:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 290	3	22, 114, 8	1496.9	3.1

Network 8:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 290	3	22, 114, 8	1496.9	3.1

Network 9:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 20	2	22, 154	1411.84	88.16
21 to 49	3	22, 154, 81	1463.88	36.12
50 to 275	4	22, 154, 81, 310	1492.33	7.67
276 to 298	5	22, 154, 81, 310, 13	1498.36	1.64

Network 10:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 293	3	22, 114, 8	1496.9	3.1

Network 11:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 295	3	22, 114, 8	1496.9	3.1

Network 12:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 30	2	22, 114	1456.99	43.01
31 to 290	3	22, 114, 8	1496.9	3.1

Network 13:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 111	1456.99	43.01
30 to 297	3	22, 111, 8	1496.9	3.1

Network 14:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 27	2	22, 144	1411.84	88.16
28 to 271	3	22, 144, 73	1491.31	8.69
272 to 293	4	22, 144, 73, 13	1497.34	2.66

Network 15:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	19, 151	1411.84	88.16
29 to 270	3	19, 151, 80	1491.31	8.69
271 to 292	4	19, 151, 80, 13	1497.34	2.66

Network 16:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 299	3	22, 114, 8	1496.9	3.1

Network 17:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 295	3	22, 114, 8	1496.9	3.1

Network 18:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	21, 110	1456.99	43.01
30 to 294	3	21, 110, 7	1496.9	3.1

Network 19:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	20, 111	1456.99	43.01
30 to 295	3	20, 111, 8	1496.9	3.1

Network 20:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 154	1411.84	88.16
29 to 49	3	22, 154, 81	1463.88	36.12
50 to 274	4	22, 154, 81, 310	1492.33	7.67
275 to 297	5	22, 154, 81, 310, 13	1498.36	1.64

Network 21:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 151	1411.84	88.16
29 to 49	3	22, 151, 81	1463.88	36.12
50 to 274	4	22, 151, 81, 310	1492.33	7.67
275 to 297	5	22, 151, 81, 310, 13	1498.36	1.64

Network 22:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 151	1411.84	88.16
29 to 49	3	22, 151, 115	1462.6	37.4
50 to 273	4	22, 151, 115, 310	1491.05	8.95
274 to 296	5	22, 151, 115, 310, 13	1497.08	2.92

Network 23:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 154	1411.84	88.16
29 to 271	3	22, 154, 83	1491.31	8.69
272 to 294	4	22, 154, 83, 13	1497.34	2.66

Network 24:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 154	1411.84	88.16
29 to 295	3	22, 154, 98	1499.38	0.62

Network 25:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 110	1456.99	43.01
30 to 299	3	22, 110, 8	1496.9	3.1

Network 26:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 296	3	22, 114, 8	1496.9	3.1

Network 27:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 29	2	22, 114	1456.99	43.01
30 to 295	3	22, 114, 8	1496.9	3.1

Network 28:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 154	1411.84	88.16
29 to 48	3	22, 154, 81	1463.88	36.12
49 to 275	4	22, 154, 81, 310	1492.33	7.67
276 to 298	5	22, 154, 81, 310, 13	1498.36	1.64

Network 29:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 151	1411.84	88.16
29 to 48	3	22, 151, 81	1463.88	36.12
49 to 275	4	22, 151, 81, 310	1492.33	7.67
276 to 298	5	22, 151, 81, 310, 13	1498.36	1.64

Network 30:

Iteration	No. of Rehab. Pipes	Rehab. Pipes	Sum of their Length	Remaining Budget Limit
1 to 28	2	22, 151	1411.84	88.16
29 to 49	3	22, 151, 81	1463.88	36.12
50 to 276	4	22, 151, 81, 310	1492.33	7.67
277 to 299	5	22, 151, 81, 310, 13	1498.36	1.64

Table 4 summarizes features and results of the new networks. The rehabilitation length was fixed to be 1500 m. We evaluated CPU time for LB and UB.

Table 4- Features and results of 30 created networks

Network	No. of Nodes	No. of Pipes	Network Length	Lower Bound	CPU Time LB	Upper Bound	CPU Time UB	No. of Unique Scenarios	% of Optimality
1	256	299	66387.56	353.86	6205.3	364.24	3660.2	1378	0.014
2	265	308	69204.70	353.75	7565.8	366.12	4616.8	1450	0.017
3	268	310	69435.74	304.52	8546.8	315.52	5467.9	1462	0.017
4	269	311	69585.41	370.05	8836.2	383.27	5795.2	1457	0.017
5	265	308	69316.98	362.58	7879.3	374.94	4865.4	1452	0.016
6	266	309	69053.98	345.14	6929.1	357.57	4674.3	1449	0.017
7	269	311	70026.56	347.52	8114.9	359.66	5234	1453	0.017
8	266	309	70014.29	372.36	8116.9	385.28	5239.6	1450	0.017
9	270	314	71123.78	375.17	9159.8	388.36	5876.6	1490	0.017
10	265	308	69689.81	368.57	8426	381.23	5440.1	1466	0.017
11	268	311	70945.06	372.77	8957.5	385.69	5791.3	1478	0.017
12	268	310	69880.05	334.16	8905.9	346.97	5866.3	1450	0.018
13	270	314	70729.64	366.54	8930.3	379.86	5723.3	1487	0.018
14	263	307	70183.87	372.02	8046.2	384.40	4917.7	1468	0.016
15	270	313	69923.61	365.74	9379.8	378.71	6310.3	1464	0.017
16	270	314	71303.06	371.84	10071	385.12	6731.1	1496	0.017
17	271	314	70763.63	367.41	10241	380.88	6791.3	1476	0.018
18	271	313	70306.57	367.62	9856.3	380.92	6555.8	1471	0.017
19	270	313	70642.50	373.20	9912.1	386.53	6620.4	1478	0.017
20	270	314	71295.31	374.04	7660.8	387.18	5288.9	1485	0.017
21	270	314	71107.37	371.26	8767	384.40	5640.8	1487	0.017

22	270	314	70877.79	374.07	7559	387.12	5198.8	1482	0.017
23	269	313	70878.43	374.89	8403.3	387.96	5354.9	1470	0.017
24	270	314	71008.02	366.41	8852.8	379.23	5614.2	1478	0.017
25	269	313	71458.63	376.08	7090.4	389.17	4672.9	1497	0.017
26	269	313	70724.95	366.53	8642.3	379.18	3646.2	1482	0.017
27	268	312	70628.24	367.96	7397	380.93	5467.6	1477	0.017
28	270	314	71111.92	343.80	8978.1	356.12	5772.3	1490	0.017
29	270	314	71282.24	370.77	8612	383.99	5286.6	1490	0.017
30	270	314	71493.17	376.40	8762.6	389.70	5422.1	1495	0.017
Average	268.16	311.5	70346.10	363.57	8493.52	376.34	5451.43	1470.26	0.017

In addition, figures 11 and 12 are boxplots reporting LB, UB and their CPU time. Figure 13 shows box and whisker plot of number of unique scenarios in 30 new networks.

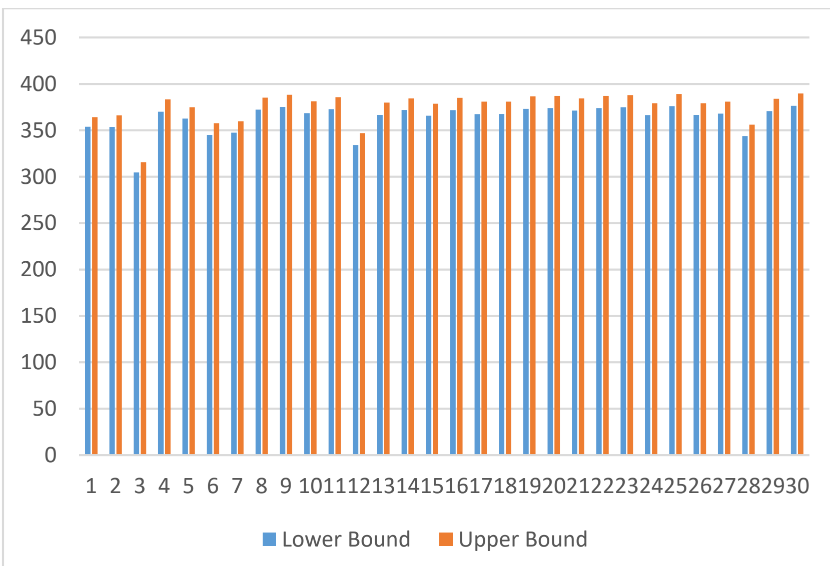

Figure 11 – LB and UB of 30 new networks

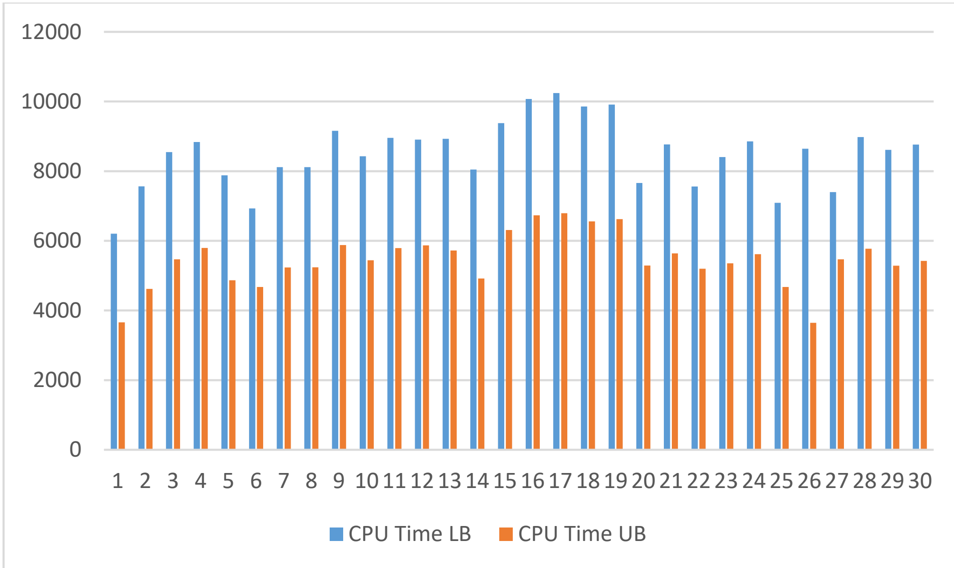

Figure 12 – CPU time LB and CPU time UB of 30 new networks

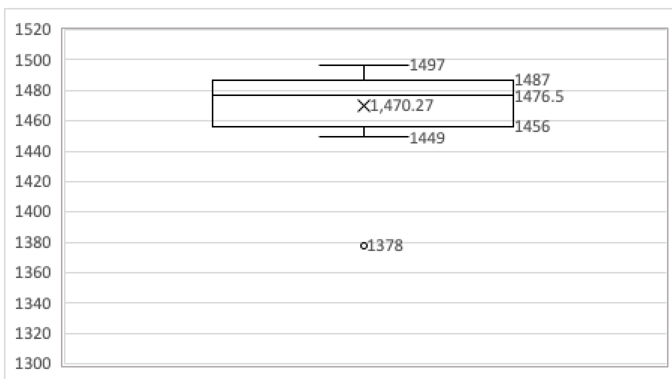

Figure 13 – Number of unique scenarios in the new networks

Figure 14 shows the box plot of percentage of optimality in these 30 networks. Figure 15 compares CPU time LB and CPU time UB in each network. CPU time LB is greater than CPU time UB in all 30 networks.

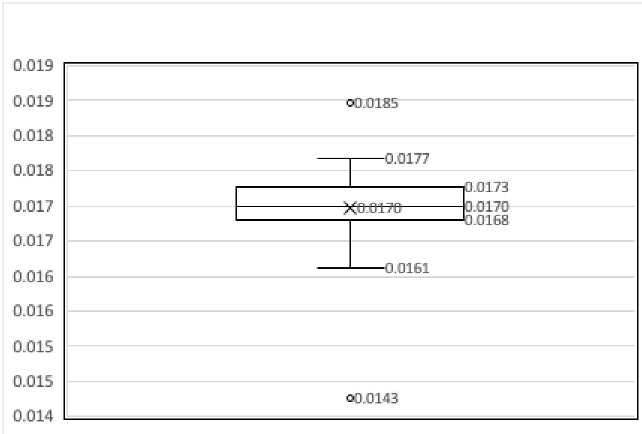

Figure 14 – Percentage of optimality in 30 networks

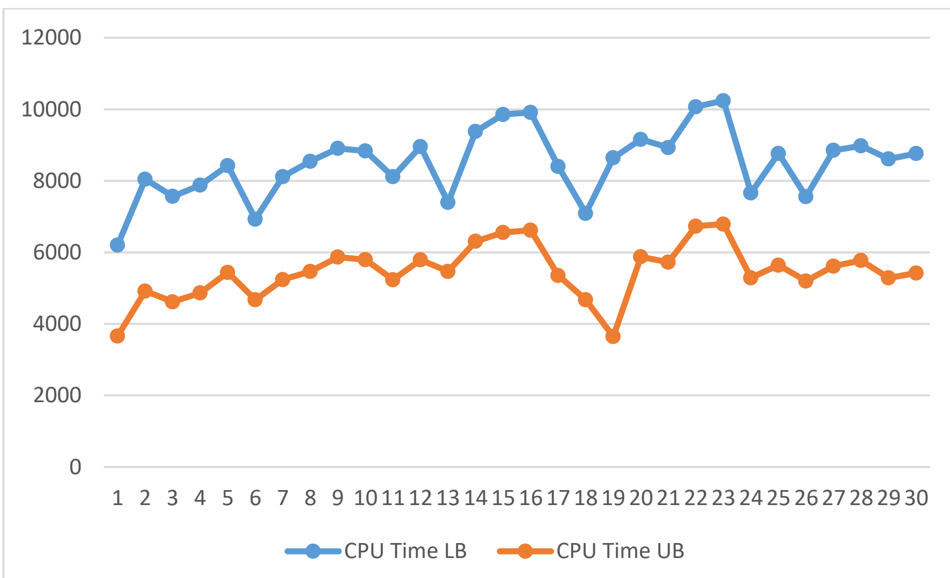

Figure 15 – CPU time for UB and LB in 30 networks

CHAPTER 4: CONCLUSIONS AND FUTURE RESEARCH

This study proposed a two-stage stochastic programming model for the water pipe rehabilitation problem with a recourse flow function to maximize the output flow right after an earthquake. The MINLP model cannot be solved by commercial optimization software, like BARON even for problems with a very small number of scenarios. Consequently, we proposed piecewise linear functions (PLF) to approximate the nonlinearity in the MINLP. Therefore, we formulated a mixed integer linear program (MILP) to approximate the MINLP. The optimization of the MILP is still challenging to solve, so we introduced a sequential algorithm to mitigate this computational issue and find bounds for the approximated optimal solution. The solution we found using the sequential algorithm is within 2% of optimality. We created 30 more networks and tested our sequential algorithm on those. The solution is still within 2% of optimality

In future research, the model can be used for rehabilitation plans of corroded pipes. In addition, we can consider leakage in the model, another objective function, using Benders Decomposition for solving the model, and developing software for helping municipalities to rehabilitate their water pipes, especially in cities where an earthquake may occur with higher probability.

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Appendix

Piecewise Linear Function (Knot selection)

For linearization the first step was finding the maximum possible flow in the network. Consequently, we used our mathematical model, the objective function (11), constraints (4) and (7a) for each pipe before earthquake to achieve maximum flow in each pipe. We described it in section 2.3.1. We found the maximum pipe flow in Modena network is 223 liter/second before earthquake. Then, we visualized drawing of function $x^{1.852}$ for x in [0, 223]. We selected the knots to be in x= 0, 20, 60, and 223. The reason we considered 3 pieces in our linearization is that the model gets huge as the number of PLF gets higher, therefore, we started with the minimum number of pieces that is 3. We described the PLF general formulation in 2.3.3. Studying more pieces and different methods for finding knots are our future research that will affect the result and create better solution. In other words, sensitivity analysis in finding knots is our future research.

Then we considered x's be a number between 0 and 223 with 0.1 step length and we calculated $x^{1.852}$ and we considered 3 pieces [0, 20], [20, 60], [60, 223] as we described before. The PLF must be continuous in the knots so we considered it in our calculation. By using linear regression tool in excel we found the formulation of linear functions fitted to each piece of the curve and finally we concluded slope and intercept of each PLFs. Then we added equations (15- 19) to our model with concluded slopes and intercepts.