THE UNIVERSITY OF TEXAS AT ARLINGTON
DOCTORAL THESIS

A Study of Behaviors in Procurement , Supply Chain and Artificial Intelligence

Author:
Xianghua (Jason) Wu

Supervisor:
Kay Yut Chen

A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy

in the

Department of Information Systems and Operations Management
The University of Texas at Arlington

August 8, 2021

Acknowledgements

First I want to thank my advisor, Dr. Kay Yut Chen. I learned from you not only how to do research, but more importantly how to be a researcher. I also want to thank other professors who helped me to complete my dissertation, Dr. Shan Li, Dr. Diana Wu, Dr. Alper Nakkas and Dr. Jennifer Zhang.

Without the support from my family and friends, I cannot imagine how far I have grown in terms of both academic and mental capability in the past few years. I love you!

A special thank to my grandmother, Wang Junxiu. Inspired by your optimism and love since childhood, I can face all the difficulties with great resilience, have all the courage to be myself and write my own life story.

Contents

Chapter 1

Designing Reimbursement Policy for Multidimensional Auction with Loss-Averse Workers in Online Labor Markets

1.1 Introduction

Online labor markets, bridging service buyers with workers, have emerged as a viable means for service procurement. Freelancer (https://www.freelancer.com/) , Upwork (https://www.upwork.com/), and 99 Designs (https://99designs.com/), are examples of the freelancing platforms that mediate between buyers and workers of remotely deliverable cognitive work (Horton and Chilton, 2010).

Reverse auction is the principle behind service procurement on a freelance platform. A buyer will post a project on the platform, then relevant freelancers will bid for the project, write a proposal to persuade the buyer into choosing him/her over the others. Service procurement involving multidimensional bids are ubiquitous. Typically, workers are required to bid on both quality and price, which jointly form single-dimensional scores that are used by the client to determine the winner, whose offer turns out to be the most economically advantageous. For instance, for most projects that have a design component (i.e. app design, logo design, etc), besides submitting the bid on price, the workers are also required to submit a proposal including factors such as idea, design prototype, for the buyer to view. After receiving the bids, the buyer selects the winner bidder, based on both the price and the quality of the proposal. One interesting feature of this market is that it is one of the few real-life examples of non-political all-pay auctions, because the worker must invest effort up-front preparing a quality proposal, regardless of the outcome of the auction.

According to prospect theory, for example, workers are "loss averse" and so react more strongly to losses than to gains of the same magnitude (Kahneman and Tversky, 1979; Ho and Zhang, 2008). Therefore, in these online all-pay auctions, loss aversion can be an important behavioral factor that impacts individuals bidding decisions on both price and the quality of the proposal. The first goal of this research is to examine and measure the impact of workers' loss aversion in the context of service procurement with up-front effort spending in online labor platforms.

On the other hand, in practice, there are reimbursement policies to compensate upfront costs incurred before contract, such as prototyping competition and research contests. Among these cases, there are two types of reimbursement policy used, percentage reimbursement policy-in which the buyer reimburses the bidders' upfront cost with a percentage, and the flat reimbursement policy-in which the buyer reimburses the bidder a fixed amount. In addition, they consider about reimbursing the bidder, contingent on their winning or losing scenarios. Therefore, the second goal of this research is to examine what is the optimal (flat and percentage) reimbursement policy for the buyer and which reimbursement policy the buyer should use.

1.2 Literature Review

Procurement auction has received considerable attention in operations management (OM) research communities. Examples of analytical and laboratory OM studies on procurement auctions include, Chen (2007), Chen, Seshadri, and Zemel (2008), Chu (2009), Chaturvedi and Albéniz (2011), Davis, Katok, and Kwasnica (2014), Shachat and Tan (2015), Chaturvedi, Katok, and Beil (2019), and Fan, Chen, and Tang (2020), to name a few.

Che, 1993 is a pioneering paper studying the multidimensional reverse auction, where bidders bids on the quality in addition to the price, and characterizes the optimal scoring rules which can bring the second-best outcome for the buyer. Researchers have since explored the multidimensional auction in different directions, such as correlated private types (Branco, 1997), properties of the general scoring auctions (Asker and Cantillon, 2008), comparison of a multidimensional auction and a price-based auction under which the buyer selects the winner bidder based on price bidding only (Bichler, 2000; Chen-Ritzo et al., 2005; Engelbrecht-Wiggans, Haruvy, and Katok, 2007), the effect of information transparency (Haruvy and Katok, 2013), collusion and trust (Fugger, Katok, and Wambach, 2016; Fugger, Katok, and Wambach, 2019).

However, this literature on the multidimensional auction only considers the winner-pay costs of the bidders, i.e., the costs incurred by a bidder only if he wins the auction. What differentiates our paper from this literature is that we consider the all-pay cost invested upfront in quality in addition to the winner-pay cost, and investigate impact of loss aversion, which is closely related to the all-pay cost.

We are aware of only one study of multidimensional auction with all-pay quality cost (Kovenock and Lu, 2020). Our paper is different from theirs in two main aspects. While their paper assume that bidders are rational expected profit maximizer, we focus on investigating the impact of a relevant behavioral factors closely associated with the all-pay costs, i.e., loss aversion. Moreover, we design and compare reimbursement policies that can mitigate the negative impact of loss aversion.

Kahneman and Tversky (1979) is a seminal paper establishing prospect theory (PT), in which a decision maker has a tendency of "loss aversion", i.e., his/her pain in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount. Kőszegi and Rabin (2006) and Kőszegi and Rabin (2007) develop a generally more applicable theory, combining the "outcome-based" utility from orthodox economics and the "gain–loss" utility from PT, and illustrate its applicability by establishing some implications of loss aversion for consumer behavior and labor effort.

Based on these two theoretical frameworks incorporating loss aversion as a relevant behavioral factor, other literature have applied their theories to study its implications in different directions and settings, such as endowment effect and reluctance to trade (Thaler, 1980; Kahneman, Knetsch, and Thaler, 1990; Okada, 2001), contract theory (Herweg, Müller, and Weinschenk, 2010; Koszegi, 2014), pricing problems in products and markets (Heidhues and Kőszegi, 2008; Nasiry and Popescu, 2011; Courty and Nasiry, 2018; Hu and Nasiry, 2018; Chen and Nasiry, 2020), decision making in newsvendor problems (Schweitzer and Cachon, 2000; Ho and Zhang, 2008; Wang and Webster, 2009; Herweg, 2013; Long and Nasiry, 2015; Baron et al., 2015).

Our study is mostly related to the literature, investigating the implications of the presence of loss averse bidders in auction settings. Some researchers (Eisenhuth, 2010; Lange and Ratan, 2010; Banerji and Gupta, 2014) showed that the first price auction brings less expected revenue for the auctioneer than the second price auction, when bidders are loss averse. Rosato and Tymula (2019) suggested that loss-averse bidders in the second price auction may behave differently in real-object auctions from induced-value ones. Balzer and Rosato (2020) analyzed the bidding behavior of loss-averse bidders in a common-value auction.

All these auction literature, however, investigated one-dimensional auctions with loss-averse bidders' winner-pay cost. Our paper complements these researches, by studying the effect of loss aversion and the corresponding reimbursement mechanisms to improve auctioneer's expected utility, in a two-dimensional auction with loss-averse bidders' all-pay cost.

1.3 A Model of Multidimensional Auction with Loss Aversion

1.3.1 Model Setup

We consider a single buyer ("she") who is soliciting bids from N ($N \geq 2$) labor workers ("he") to complete a project. Our model characterizes the following key features of a service procurement auction in online labor markets.

First Score Sealed Bid Mechanism[1] with Multi-dimensional Bids:

Each worker submits a sealed bid which consists of a quality and a price. The buyer determines the winner based on the bids she receives. In practice, most projects requires a proposal, in addition to price, such as a design component (i.e. app design, logo design, etc), as part of the bid. The quality dimension of the bid captures how much the buyer values this proposal component.

We use p_i and q_i to denote the bidding price and the quality of the proposal of worker $i \in \{1, 2, ..., N\}$, respectively. Upon winning, a worker provides his service according to the proposal at the offered price.

In online labor markets, the buyer usually does not have full scoring rule commitment power, that is, she is not able to ex ante commit to a scoring rule, other than her own utility function. Note that committing to a scoring rule other than the buyer's utility function is not sub-game perfect when, as part of the process, the buyer chooses the winner after reviewing all the submitted bids. In the main model, we only consider mechanisms that are sub-game perfect, and hence the buyer will only select the bid the maximizes her utility. However, to be comprehensive, we include an analysis of the optimal scoring rule, under the assumption that the buyer can commit to such a rule, in Appendix.

The buyer utility function, $U(q, p)$, is assumed to be quasi-linear (Che, 1993):

$$U(q, p) = V(q) - p,$$

[1]In online labor markets, buyers usually have the choice of adopting either an open or a sealed bid auction mechanism. In this paper, for the exposition purpose, we focus on the sealed bid auction mechanism, and leave the case of open bid auction to future research.

where q is the quality of a worker's proposal, $V(q)$ is the buyer's quality value function and p is the price the worker bids. We assume the trade always takes place[2]. We also make the following general assumptions on the quality value function $V(q)$.

Assumption 1 $V'(q) > 0$, $V''(q) < 0$, $\lim_{q \to 0} V'(q) = +\infty$, *and* $\lim_{q \to +\infty} V'(q) = 0$.

$V'(q) > 0$ and $V''(q) < 0$ imply the quality value function increases with quality at a decreasing rate, that is, the quality value function has the property of diminishing return. $\lim_{q \to 0} V'(q) = +\infty$ and $\lim_{q \to +\infty} V'(q) = 0$ are Inada conditions, which ensure an interior solution (Inada, 1963). This assumption is widely adopted for the quality value function in the multi-dimensional auction literature (Che, 1993; Asker and Cantillon, 2010).

All-Pay Quality Spending: A worker must spend resources up-front preparing a quality proposal. Without loss of generality, we assume this non-recoverable up-front cost in the bidding to be q_i[3], $i \in \{1, 2, ..., N\}$, incurs to all bidders, regardless of the outcome of the auction.

Winner-Pay Production Cost with Private Type: Worker i, upon winning, incurs a production cost $\theta_i q_i$, in order to produce quality q_i. In contrast to the quality investment cost which incurs to all bidders (i.e., all-pay), this production cost only incurs to the winning worker (i.e., winner-pay). We consider workers who are heterogeneous in their expertise in production, which is reflected in this production cost. In particular, θ_i is worker i's private production cost parameter, also referred to as *private type*. The distribution of θ_i is public knowledge for all parties in the auction, but the cost parameter θ_i is the private information of worker i. The multiplicative structure of the production cost that employed in our model (i.e., $\theta_i q_i$) is consistent with prior research in the auction literature (Chen-Ritzo et al., 2005; Kostamis, Beil, and Duenyas, 2009; Kovenock and Lu, 2020)[4]. We remark that a worker with a high θ is less efficient, and a low θ indicates a more efficient worker.

We note that in reality the private type can be on the production cost and/or the quality spending. We have no empirical evidence of which scenario to be more prevalent in practice. Hence, we arbitrarily pick the heterogeneous production scenario as the main case to streamline and focus the exposition. We find that our main conclusions continue to hold if workers are heterogeneous in producing quality proposal instead. Please refer to Section 1.5 for details.

Loss Aversion: As the worker must spend resources up-front preparing a quality proposal regardless of the outcome of the auction, workers who lose in the auction would experience a loss of q_i. We assume all the workers are risk neural but averse to profit losses (Kahneman and Tversky, 1979; Ho and Zhang, 2008). We use λ to denote the degree of loss aversion of the worker, and assume it is exogenous, homogeneous among all workers, and common knowledge for all players (Eisenhuth, 2010; Rosato and Tymula, 2019; Balzer and Rosato, 2020).

Therefore, a worker i of private type θ_i, bidding price p_i and quality spending q_i, earns the interim expected utility:

$$\pi(q_i, p_i) = \big(p_i - \theta_i q_i - q_i\big) P\big(\text{win}|q_i, p_i\big) - \lambda q_i \big(1 - P\big(\text{win}|q_i, p_i\big)\big) \tag{1.1}$$

[2]This is equivalent to the buyer having a reserved utility of $V(0)$.

[3]All of our results continue to hold qualitatively if this upfront quality spending is modelled as any strictly increasing function of quality q_i.

[4]All of our results continue to hold qualitatively if the production cost is modelled as $C + \theta_i q_i$, where C is an non-negative constant.

where q_i is the quality of the proposal, $\theta_i q_i$ is the production cost incurred for worker i who submits a quality bid q_i and wins the auction, $P(\text{win}|q_i, p_i)$ is the probability of winning the auction when the worker bids price p_i and quality q_i, and $\lambda \geq 1$ denotes the degree of loss aversion. It is worth noting that the model is reduced to a special case of no loss aversion when $\lambda = 1$, and it hypothetically represents a special case of a two-dimensional winner-pay auction (Che, 1993) when $\lambda = 0$.

We assume that the private type θ_i ($i \in \{1, 2, ..., N\}$) is independently and identically distributed over a bounded interval $[\underline{\theta}, \overline{\theta}]$ ($0 < \underline{\theta} < \overline{\theta} < +\infty$), according to a distribution function $F(\cdot)$ for which there exists a positive, continuously differentiable density $f(\cdot)$. This assumption is standard for independent private value (IPV) auctions (see Kagel and Levin (2008), Klemperer (2014), and Kagel (2020) for reviews).

Assumption 2 $\frac{F(\cdot)}{f(\cdot)}$ *is non-decreasing.*

This assumption of regularity condition is widely adopted in the literature; see, for example, Myerson, 1981 and Laffont and Tirole, 1987. Many commonly used distributions, such as uniform and normal distribution truncated to a finite interval, satisfy this assumption (Bagnoli and Bergstrom, 2005).

We use $(q(\cdot), p(\cdot))$ to denote the symmetric equilibrium of this auction. We remark that, in the equilibrium, the worker of the lowest private type always wins the auction. Therefore, the buyer's expected utility can be written as:

$$EU_b = \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} \tag{1.2}$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$.

1.3.2 Equilibrium

In this section, we derive the loss-averse workers' equilibrium bidding behavior in this service procurement auction. Using the characterized bidding behavior, we then investigate the impact of loss aversion on the expected utility of the worker as well as the buyer in equilibrium.

How loss aversion impacts workers' equilibrium bidding behavior

The auction can be considered as a Bayesian game where each worker picks a quality-price combination as a function of its private type. In particular, given other workers' bidding strategy, worker i ($i \in \{1, 2, ..., N\}$) with type θ_i chooses a quality-price pair (q_i, p_i) to maximize his interim expected utility in (1.1). The resulting quality-price pairs $(q_i(\theta_i), p_i(\theta_i))(i \in \{1, 2, ..., N\})$ form a Bayesian Nash Equilibrium. Due to the complete symmetry of the workers, we only consider symmetric equilibrium of the game and drop the subscript i in the rest of the paper, when discussing about the equilibrium.

Definition 1 *(Quality Elasticity of Marginal Value)* $M(q)\frac{-qV''(q)}{V'(q)} = -\frac{dV'(q)}{V'(q)} / \frac{dq}{q}$, *where q is a quality bid.*

$M(q)$ measures the percentage decrease (or increase) of the marginal value $V'(q)$ when quality bid q increases (or decreases) by one percent[5]. One can infer that a

[5] This definition is similar in spirit to the definition of the coefficient of relative risk aversion(Simon and Blume, 1995).

low quality-elasticity of marginal value $M(q)$ implies a low degree of diminishing return of the value function (i.e., $-V''(q)$). From assumption 1, we have $M(q) \geq 0$ throughout the paper.

Proposition 1 *(1) In the procurement auction where workers are loss-averse, there exists a unique symmetric equilibrium in which a loss-averse worker of private type θ makes the following bids for quality and price:*

$$q(\theta) = \quad V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big),$$
$$p(\theta) = \quad \theta q(\theta) + q(\theta) + \frac{\int_{\theta}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{N-1}d\tilde{\theta}}{(1-F(\theta))^{N-1}} + \lambda q(\theta)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}},$$

where $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$.

(2) Worker's quality bid in equilibrium decreases with the degree of loss aversion (i.e., $\frac{\partial q(\theta)}{\partial \lambda} = \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}V''(q(\theta))} \leq 0$).

(3) If $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta}+1)]$, worker's price bid in equilibrium decreases with the degree of loss aversion.

Part (1) of Proposition 1 characterizes a unique symmetric equilibrium of this service procurement auction. To provide some intuition of this result, we start with the quality bid in equilibrium. On one hand, a worker of private type θ increasing one unit of quality in his bid incurs θ units of production cost and one unit of quality cost upon winning, and λ units of quality cost upon losing, so the expected marginal cost is $(\theta+1)(1-F(\theta))^{N-1} + \lambda(1-(1-F(\theta))^{N-1})$, where $(1-F(\theta))^{N-1}$ is the probability of winning the auction. On the other hand, increasing one unit of quality bid can make the price bid go up by $V'(q)$ units while keeping the score and the probability of winning unchanged, and this leads to $V'(q)$ units of marginal benefit for the winner, so the expected marginal benefit can be written as $V'(q)(1-F(\theta))^{N-1}$. At the equilibrium, the marginal cost of the quality bid equals to its marginal benefit. Regarding the equilibrium price bid, it is worth noting that if $\lambda = 0$, our solution has the same characterization of the equilibrium price bid in the winner-pay auction as Che, 1993 with production cost $(\theta+1)q$. The last term $\lambda q(\theta)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}$ compensates for the potential loss from all-pay quality spending for loss-averse workers ($\lambda \geq 1$).

Part (2) of Proposition 1 shows the impact of loss aversion on the quality bid in equilibrium. As a higher degree of loss aversion increases the marginal cost of the quality bid and it has no impact on the marginal benefit, it follows directly that the equilibrium quality bid is lower if workers are more averse to loss. In addition, the expression of $\frac{\partial q(\theta)}{\partial \lambda}$ implies a bigger impact of loss aversion on the quality bid when value function has a small diminishing return.

The impact of loss aversion on equilibrium price bid is two-fold. On one hand, there is a *direct effect* - worker needs to bid a higher price to compensate for his potential loss if he is more loss averse; On the other hand, based on part (2) of Proposition 1, high loss aversion lowers quality bid, which in turn reduces the potential loss to be compensated for, and we refer to it as an *indirect effect*. As characterized in part (3) of Proposition 1, when the quality-elasticity of marginal value is low, which also implies a low degree of diminishing return of the value function, the indirect effect is large and outweighs the direct effect, and thus price bid is lower with a high degree of loss aversion. The condition of low quality-elasticity of marginal value in part (3)

of Proposition 1 can be satisfied by multiple classes of functions (which also satisfy Assumption 1), and below we present a couple of examples.
Example 1 (Battermann, Broll, and Wahl, 1997): The quality value function $V(q) = b + a\frac{q^{1-\gamma}}{1-\gamma}$, where $a > 0$ and $0 < \gamma < 1$.
Example 2 (Saha, 1993): The quality value function $V(q) = b - e^{-\beta q^{\alpha}}$, where $0 < \alpha < 1$, $\beta > 0$, and $1 - \alpha + \alpha\beta\big(V'^{-1}(\underline{\theta} + 1)\big)^{\alpha} \leq 1$.

How loss aversion impacts expected utility in equilibrium

So far, we have characterized loss-averse workers' bidding behavior in equilibrium in this service procurement auction. Using the characterized bidding behavior, the following proposition characterizes the impact of loss aversion on the expected utility of the worker and the buyer.

Proposition 2 *In the equilibrium of the procurement auction where workers are loss-averse,*
(1) worker's expected utility decreases with the degree of loss aversion;
(2) if $\lim_{q\to 0+} M(q) \neq 0$[6]*, buyer's expected utility decreases with loss aversion when loss aversion is sufficiently high (i.e.* $\lambda > \lambda_0$*, where* $\lambda_0 \equiv \sup_{\substack{\lambda \geq 1 \\ \theta \in [\underline{\theta}, \overline{\theta}]}} \frac{(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1)(1-F(\theta))^{N-1}}{1-(1-F(\theta))^{N-1}}$ *and* $q(\theta) = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$*).*

As loss aversion incurs additional cost for the worker, in equilibrium, it is intuitive that worker's expected utility is lower if he is more loss averse. Two forces, however, determine the impact of loss aversion on buyer's expected utility. On one hand, loss aversion reduces the equilibrium quality bid as shown in part (2) of Proposition 1, which subsequently lowers buyer's quality value $V(q)$ and therefore decreases buyer's expected utility. We refer to this effect as *quality value effect*. It is important to point out that the quality value effect is always negative (i.e., it always reduces buyer's expected utility), and it becomes stronger as loss aversion increases as a result of the diminishing return of the value function stated in Assumption 1.

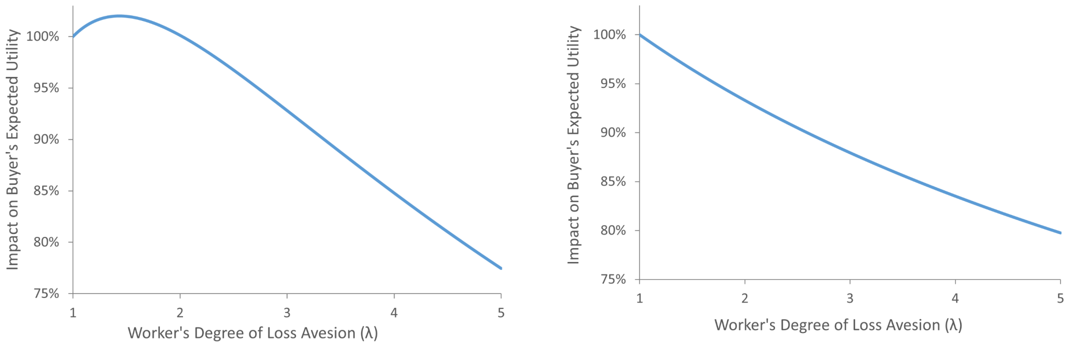
On the other hand, loss aversion can impact the equilibrium price bid and subsequently influence buyer's expected utility, which we refer to as *price effect*. As we show in part (3) of Proposition 1, the price effect can be positive (i.e., higher degree of loss version decreases the equilibrium price and thus increases buyer's expected utility) under some conditions. However, when loss aversion is sufficiently high, the price effect is either negative or dominated by the quality value effect, so the buyer's expected utility always decreases with loss aversion in this case.

[6]Multiple classes of quality value functions (which also satisfy Assumption 1), including Examples 1 and 2, can satisfy this condition.

FIGURE 1.1: Impact of Loss Aversion on Buyer's Expected Utility

(A) Value Function with a Low Diminishing Return $V(q) = 100 + 600q^{0.9}$

(B) Value Function with a High Diminishing Return $V(q) = 100 + 600q^{0.4}$



As an illustration, Figure 1.1 plots how loss aversion impacts buyer's expected utility under two value functions, with low and high diminishing return, respectively. We use $N = 4$ and $\theta \sim U[1,7]$ in both figures. We also use the expected utility of the buyer when workers are not loss-averse (i.e. $\lambda = 1$) as the benchmark, and plot the percentage impact of worker's degree of loss aversion on buyer's expected utility. Prior literature has shown that the reported loss aversion coefficient typically ranges between 1 and 5[7], we therefore use it as the range for the loss aversion coefficient of all plots throughout the paper. Interestingly, as shown in Figure 1.1(a), when the value function has a low diminishing return, buyer can benefit from workers' loss aversion behavior when their degree of loss aversion is low. This is because when the value function has a low diminishing return, the strength of the positive price effect is stronger than the strength of the negative quality value effect, for low degree of loss aversion. As loss aversion increases, the quality value effect is strengthened and eventually outweighs the price effect, which makes buyer's expected utility decrease with loss aversion. When the value function has a high diminishing return as in Figure 1.1(b), the price effect is either negative or dominated by the negative quality value effect, as a result, buyer's expected utility always decreases with the degree of the loss aversion in this case.

1.4 Reimbursement Policy

Thus far, we have shown that the workers' loss aversion behavior not only reduces the expected utility of the worker, it can also decrease the expected utility of the buyer, especially when loss aversion is high. To remedy the impact of loss aversion, a buyer may (partially) reimburse the quality spending incurred by the worker and therefore reduce the negative effect of loss aversion. In this section, we explore several variants of reimbursement policies.

To the best of our knowledge, there is no prior literature to investigate and compare reimbursement policies in multi-dimensional auction with all-pay spending. However, reimbursement policies have been intensively studied in a mechanism similar to our procurement auction, i.e., contest, where the participants determine levels of effort, which is also modeled as an up-front costs and usually the prize they

[7] According to Brown et al., 2021, which examines 607 empirical estimates of loss aversion from 150 articles in economics, psychology, neuroscience, and several other disciplines, the reported loss aversion coefficient typically ranges between 1 and 5, with a mean of 1.97.

can win is exogenously given. Particularly, two types of reimbursement policies, i.e., percentage reimbursement policy (Cohen and Sela, 2005; Matros and Armanios, 2009), and flat reimbursement policy (Fu, Lu, and Lu, 2012; Lichtenberg, 1988) are popular in such studies and found to be relevant to contest designer's revenue in the contest literature.

Intuitively, reimbursing workers' quality spending can be relevant to buyer's expected utility in our procurement auction, since the quality bidding incurs upfront costs similar to how effort is modeled in contest settings. Therefore, we study these two types of reimbursement policies in our procurement auction. In particular, under the percentage reimbursement policy, the buyer can reimburse the worker a certain percentage of his quality spending. Under the flat reimbursement policy, the buyer can decide a reimbursement threshold and amount. If a worker's quality bidding is higher than this threshold, the buyer would reimburse the worker a capped amount no more than this threshold.

Moreover, since the quality investment cost is deemed only as a loss only if the worker loses the auction, the buyer need to consider reimbursement, contingent on whether a worker wins or lose its bid. Not surprisingly, prior contest literature have studied reimbursing upfront (all-pay) cost conditioned on winning and losing (Matros and Armanios, 2009; Minchuk, 2018; Liu and Liu, 2019; Kovenock and Lu, 2020). Therefore, for both types of policies, we consider the buyer can choose different reimbursement percentage or amount for the worker's quality spending, contingent on whether he wins or loses the auction.

1.4.1 Percentage Reimbursement Policy

Under the percentage reimbursement policy, prior to the bidding, the buyer commits to reimburse the worker $\rho_l \in [0\%, 100\%]$ percent of his quality spending if he loses the auction, and $\rho_w \in [0\%, 100\%]$ percent if he wins. Note that our model includes the case of $\rho_l = 0$ ($\rho_w = 0$) where the losing (winning) worker is not reimbursed. Therefore, under a given percentage reimbursement policy (ρ_w, ρ_l), the interim respected utility of a worker i of private type θ_i with price bid p_i and quality spending q_i can be revised from (1.1):

$$\pi^{pr}(q_i, p_i) = \big(p_i - \theta_i q_i - (1-\rho_w)q_i\big)P\big(\text{win}|q_i, p_i\big) - \lambda(1-\rho_l)q_i\big(1 - P\big(\text{win}|q_i, p_i\big)\big), \tag{1.3}$$

where we use superscript pr to refer to the scenario of the percentage reimbursement policy. We denote the symmetric equilibrium of this auction as $(q(\cdot), p(\cdot))$. Given the workers' equilibrium strategy, the buyer decides on a reimbursement percentage (ρ_w, ρ_l), to maximize her interim expected utility:

$$\begin{aligned} EU_b^{pr}(\rho_w, \rho_l) = \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} &- N\int_{\underline{\theta}}^{\bar{\theta}} \rho_l q(\theta)(1-(1-F(\theta))^{N-1})dF(\theta) \\ &- N\int_{\underline{\theta}}^{\bar{\theta}} \rho_w q(\theta)(1-F(\theta))^{N-1}dF(\theta), \end{aligned} \tag{1.4}$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$. In (1.4), the first term captures the expected utility of the buyer excluding the cost for the reimbursement. The second term reflects the buyer's expected cost of reimbursing the losing worker(s), and the last term denotes the expected reimbursement cost for the winning worker. Proposition 3 shows the impact of the percentage reimbursement policy on workers' equilibrium quality bid, price bids and expected utility.

Proposition 3 *Under a percentage reimbursement policy* (ρ_w, ρ_l)*,*

(1) a worker's quality bid in equilibrium increases with the reimbursement percentage ρ_w *and* ρ_l*.*

(2) if $M(q) \leq 1$ *for all* $q \in [0, V'^{-1}(\underline{\theta})]$[8]*, a worker's price bid in equilibrium increases with the reimbursement percentage* ρ_w *and* ρ_l*.*

(3) a worker's expected utility in equilibrium increases with the reimbursement percentage ρ_w *and* ρ_l*.*

The results of Proposition 3 are in the same vein as parts (2) and (3) of Proposition 1. Reimbursing any worker by a certain percentage, regardless of him winning or losing, always decreases his marginal cost of bidding one unit of quality[9], and thus subsequently increases his quality bid in equilibrium. The effect of the reimbursement percentage on the price bid, however, has the same conflicting direct and indirect effect as described in Proposition 1.3. We can show that the indirect effect always dominates the direct effect under the same condition as in Proposition 1.3, and therefore worker's price bid increases with the reimbursement percentage in this situation. Finally, since raising the reimbursement percentage increases the reimbursement of the worker's quality spending, it is not surprising that a worker's expected utility in equilibrium increases with reimbursement percentage.

The following proposition characterizes the optimal percentage reimbursement policy for the buyer.

Proposition 4 *(1) The buyer should never reimburse the winning worker, i.e.,* $\rho_w^* = 0$*.*

(2) Let

$$B \equiv \quad -\frac{1-\rho_l}{\lambda}\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta < 0$$

where $q(\theta) = V'^{-1}\big(\lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$ *and* $V'^{-1}(\cdot)$ *is the inverse of* $V'(\cdot)$*.*

(2.1) If $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda} > B$ *for all* $\rho_l \in [0,1]$*, then the buyer should never reimburse the losing worker, i.e.,* $\rho_l^* = 0$*.*

(2.2) If $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda} < B$ *for all* $\rho_l \in [0,1]$*, then the buyer should fully reimburse the losing worker, i.e.,* $\rho_l^* = 1$*.*

(2.3) If $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda}\Big|_{\rho_l=1} > B|_{\rho_l=1} = 0$ *and* $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda}\Big|_{\rho_l=0} < B|_{\rho_l=0}$*, then the buyer should reimburse the losing worker with the optimal reimbursement percentage* $\rho_l^* \in (0,1)$*, where* ρ_l^* *satisfies* $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda}\Big|_{\rho_l=\rho_l^*} = B|_{\rho_l=\rho_l^*}$*.*

A percentage reimbursement policy impacts the buyer's expected utility from two aspects. On one side, reimbursing a worker by a certain percentage always increases workers' quality bid in equilibrium, as shown in Proposition 3.1. A higher quality bid subsequently increases the quality value of the buyer as well as her expected utility. We refer to this positive effect of the percentage reimbursement policy

[8]Under all possible percentage reimbursement policy ($\rho_w \in [0,1], \rho_l \in [0,1]$), the highest possible equilibrium quality bidding is $V'^{-1}(\underline{\theta})$.

[9]The expected marginal cost of bidding one unit of quality for a worker of type θ under the percentage reimbursement policy is $(\theta + 1 - \rho_w)(1-F(\theta))^{N-1} + \lambda(1-\rho_l)(1-(1-F(\theta))^{N-1})$.

as *value effect*. On the flip side, increasing the reimbursement percentage leads to a higher reimbursement cost, in addition, it also induces a higher quality bid which in turn raises the price bid in equilibrium under some condition as shown in part (2) of Proposition 3, therefore both effects add more cost to the buyer and thus lowers her expected utility. This negative effect of the percentage reimbursement policy is referred to as *payment effect*.

For the case of reimbursing the winner worker, we show that the strength of the payment effect always outweighs the strength of the value effect, for any positive reimbursement percentage of the quality spending of the winning worker. Therefore, as stated in part (1) of Proposition 4, the buyer should never reimburse the winner. The analysis of reimbursing the losing worker is more complex as a result of their loss aversion behavior. In particular, the comparison of the strength of the value effect and that of the payment effect can change depending on the effect of losing workers' loss aversion on the buyer's expected utility, i.e., $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}$, where $EU_b^{pr}(0,\rho_l)$ is the buyer's expected utility reimbursing 0 to the winning workers and ρ_l percentage to the losing worker(s), given the equilibrium biding strategy of the workers in the auction. Part (2) of Proposition 4 discusses the optimal reimbursement percentage for the losing worker in three scenarios where losing worker's loss aversion has a different effect on buyer's expected utility under the percentage reimbursement policy.

It is worth nothing that the percentage reimbursement policy is only useful when loss aversion is harmful for the buyer. When loss aversion is beneficial for the buyer as illustrated for low λ values in Figure 1.1(a), the reimbursement percentages should always be 0. In addition, it is important to see that, when the optimal reimbursement percentage is positive, the percentage reimbursement policy creates a win-win situation for both the buyer and the worker, that is, the expected utility of both parties are improved by using the percentage reimbursement.

The following corollary characterizes a special case where the condition stated in part (2.1) of Proposition 4 is implicitly satisfied.

If the degree of loss aversion is low enough, i.e., $1 \leq \lambda \leq \lambda_l$, then the buyer should never reimburse the losing worker, i.e., $\rho_l^* = 0$, where

$$\lambda_l \equiv \quad \min\Big\{3, \min_{\substack{\lambda\in[1,3]\\ \rho_l\in[0,1]}} \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{q(\theta)NF(\theta)(1-(1-F(\theta))^{N-1})}{M(q(\theta))\left(\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta+1\right)}d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\left(1-(1-F(\theta))^{N-1}\right)d\theta} + 1\Big\} > 1,$$

$q(\theta) = V'^{-1}\left(\lambda(1-\rho_l)\frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} + \theta + 1\right)$ and $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$.

In addition, Corollary 1.4.1 presents a special case where the condition stated in part (2.2) of Proposition 4 is implicitly satisfied.

If $\lim_{q\to 0+} M(q) \neq 0$, there exists $\lambda_m \in (1, +\infty)$, such that when $\lambda \geq \lambda_m$, the buyer should fully reimburse the losing worker, i.e., $\rho_l^* = 1$.

FIGURE 1.2: Optimal Reimbursement Percentage for the Losing Workers
$V(q) = 100 + 600q^{0.4}$

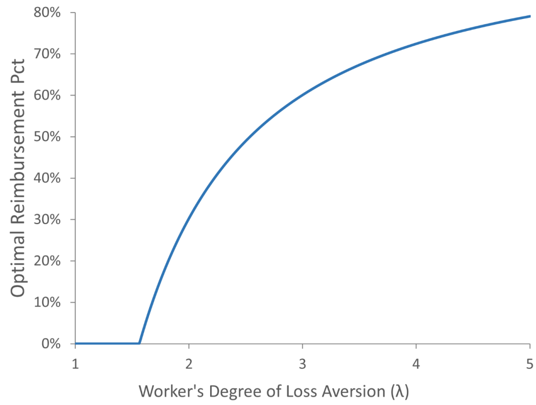


Figure 1.2 plots the optimal reimbursement percentage for the losing workers. For ease of comparison, in this illustration we use the same parameters as in Figure 1.1, where $N = 4$ and $\theta \sim U[1,7]$. One can make two observations. First, the buyer should never reimburse the losing workers when the degree of loss aversion is low enough, consistent with the results in Corollary 1.4.1. Second, as the degree of loss aversion increases, the optimal reimbursement percentage to the losing workers is higher. This is because a higher loss aversion does more harm to the buyer due to the exacerbated quality value effect as discussed in Proposition 2, so there is more to be compensated for.

1.4.2 Flat Reimbursement Policy

Under the flat reimbursement policy, prior to the bidding, the buyer commits to reimburse a fixed amount if the worker's quality spending exceeds a pre-determined threshold x ($x \geq 0$). The buyer can potentially reimburse different amount depending on the winning/losing status of the worker. We use y_w and y_l to denote the reimbursement amount for the winning worker and losing worker, respectively. Specifically, if a type θ worker with quality bidding $q(\theta) \geq x$ loses the auction, the buyer offers him a fixed amount of reimbursement y_l ($y_l \leq x$). If a type θ worker with quality bidding $q(\theta) \geq x$ wins the auction, the buyer reimburses him y_w ($y_w \leq x$).

Therefore, under a given flat reimbursement policy (x, y_w, y_l), the interim respected utility of a worker i of private type θ_i with price bid p_i and quality spending q_i can be revised from (1.1):

$$\pi^{fr}(q_i, p_i) = \big(p_i - \theta_i q_i - (q_i - y_w \cdot 1_{q_i \geq x})\big) P\big(\text{win}|q_i, p_i\big) - \lambda(q_i - y_l \cdot 1_{q_i \geq x})\big(1 - P\big(\text{win}|q_i, p_i\big)\big), \tag{1.5}$$

where we use superscript fr to refer to the scenario of the flat reimbursement. policy. We denote the symmetric equilibrium of this auction as $(q(\theta), p(\theta))$. Given the workers' best response, the buyer makes decisions for the parameters of a flat reimbursement policy (x, y_w, y_l), to maximize her interim expected utility:

$$EU_b^{fr}(x, y_w, y_l) = \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} - N\int_{\underline{\theta}}^{\overline{\theta}} 1_{q(\theta)\geq x}\cdot y_l(1-(1-F(\theta))^{N-1})dF(\theta)$$
$$- N\int_{\underline{\theta}}^{\overline{\theta}} 1_{q(\theta)\geq x}\cdot y_w(1-F(\theta))^{N-1}dF(\theta), \tag{1.6}$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$. In (1.6), the first term comes from (1.2) reflecting the net value the buyer obtains from the winning worker in the auction without considering the cost of the reimbursement. The second term captures the buyer's expected reimbursement amount to the losing worker(s), and the last term denotes her expected reimbursement amount to the winning worker.

We first show the impact of the flat reimbursement policy on worker's equilibrium quality bid, price bid and expected utility in Proposition 5.

Proposition 5 *Under a flat reimbursement policy (x, y_w, y_l),*

(1) the worker's quality bid in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

(2) the worker's price bid in equilibrium decreases with reimbursement amount y_w and y_l if his quality bid exceeds the reimbursement threshold x.

(3) the worker's expected utility in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

Unlike the percentage reimbursement policy which increases loss-averse worker's equilibrium quality bids, Proposition 5 shows that the worker's equilibrium quality bid stays the same under the flat reimbursement policy regardless of its parameters, because the flat reimbursement policy does not change the worker's marginal benefit and marginal cost of bidding one unit of quality. On the other hand, when a worker's quality bid exceeds the reimbursement threshold, the more the reimbursement amount (y_l or y_w) that he receives, the lower his price bid is in equilibrium due to competition. Moreover, the worker's expected utility in equilibrium does not change with respect to the parameters of the flat reimbursement policy. This is because the increase of the worker's expected utility from the flat reimbursement policy completely offsets with the decrease in his price bid.

Now, we characterize the optimal flat reimbursement policy for the buyer.

Proposition 6 *(1) The buyer's expected utility stays the same with respect to any reimbursement amount to the winning worker y_w.*

(2) The buyer should reimburse the maximum amount to the losing worker(s) (i.e., $y_l^* = x^*$).
The optimal threshold for quality spending reimbursement $x^* = argmax_{x\in[0,q(\underline{\theta})]} \int_{\underline{\theta}}^{\overline{\theta}}(\lambda - 1)1_{q(\theta)\geq x}\cdot xNf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta$, where $q(\theta) = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$ is the equilibrium quality bidding function.

Under the flat reimbursement policy, any quality spending reimbursement threshold and any reimbursement amount result in the same equilibrium quality bid, as shown in Lemma 5. Therefore, from the buyer's perspective, in order to maximize her expected utility, it all depends on the price bids as well as the reimbursement amount that she pays to the workers.

Part (1) of Proposition 6 shows that the reimbursement amount to the winning worker does not impact the buyer's expected utility. This is because, comparing with the basic scenario without any reimbursement policy, under the flat reimbursement policy, all workers' price bids in equilibrium will decrease by the same magnitude, which is in fact identical to the reimbursement amount to the winning worker. In other words, when the buyer commits to reimburse the winning worker based on the flat reimbursement policy, the decrease in her price payment completely offsets the increase in her reimbursement payment, and thus her expected utility stays the same.

The effect of the reimbursement amount to the losing worker on the buyer's expected utility, however, depends on the degree of the loss aversion, as shown in part (2) of Proposition 6. For an losing worker, one unit of reimbursement can increase his utility by λ units, which subsequently decreases his price bid in equilibrium by λ units. If $\lambda > 1$ (i.e. workers are loss-averse), the resultant decrease in price bid exceeds the reimbursement amount offered to the losing worker, and thus the buyer is better off reimbursing the losing worker as much as possible by setting the reimbursement amount at the reimbursement threshold (i.e. $y_l^* = x^*$). Below we remark two results which can be derived from part (2) of Proposition 6.

Remark 1 *If workers are not loss-averse ($\lambda = 1$), buyer's expected utility stays the same with respect to any flat reimbursement policy $(x \in [0, V'^{-1}(\underline{\theta}+1)], y_w \le x, y_l \le x)$[10].*

Remark 2 *If the workers are loss-averse ($\lambda > 1$), the optimal threshold for quality spending reimbursement x^* always lies between the highest and lowest equilibrium quality bid (i.e., $q(\overline{\theta}) < x^* < q(\underline{\theta})$, where the equilibrium quality bid $q(\theta) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$).*

If workers are not loss-averse (i.e., $\lambda = 1$), the resultant decrease in price bid is identical to the reimbursement amount offered to the losing worker, and therefore the buyer's expected utility always stays the same, regardless of the reimbursement amount to the losing worker.

Regarding the reimbursement threshold in the case of $\lambda > 1$, we note that a very high threshold decreases the chance of a losing worker receiving the reimbursement, on the other hand, a very low threshold limits the highest possible reimbursement amount. So the optimal reimbursement threshold x^* lies in between of the highest and lowest quality bids. At the optimal reimbursement threshold x^*, the quality bids below x^* are not reimbursed, whereas the quality bids exceeding x^* are partially reimbursed with a reimbursement amount of x^*. Recall that Proposition 6 says $y_l^* = x^*$. Together with Remark 2, one can imply that the optimal reimbursement amount to the losing worker is always positive as long as $\lambda > 1$. Since the buyer's expected utility is higher at the optimal reimbursement amount than at any other feasible reimbursement amount, including a zero reimbursement, we conclude that the buyer is always better off by offering a flat reimbursement policy when $\lambda > 1$.

Finally, as pointed out in Proposition 5.3 and Remark 2, we can see that in the presence of worker's loss aversion behavior, the flat reimbursement policy offers a win-even solution for the buyer and the worker. Specifically, by using the flat reimbursement, the buyer's expected utility is increased, whereas the worker's expected utility is unchanged.

[10] $V'^{-1}(\underline{\theta}+1)$ is the highest equilibrium quality bidding, i.e., $V'^{-1}(\underline{\theta}+1) = q(\underline{\theta})$, where $q(\theta) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$ is the equilibrium quality bidding under the flat reimbursement policy.

1.4.3 Comparison

We have shown that the percentage reimbursement policy in Section 1.4.1 and flat reimbursement policy in Section 1.4.2 can both increase the buyer's expected utility in the interaction with loss-averse workers. More importantly, the buyer and the workers are win-win with the percentage reimbursement, and win-even with the flat reimbursement. In other others, under both types of reimbursement, the buyer is better off without the expense of the worker. In this section, we compare the two reimbursement policies from the buyer's perspective. In the presence of workers' loss aversion behavior, we aim to answer the question of which reimbursement policy would yield the highest benefit for the buyer.

Compared with the percentage reimbursement, the impact of the flat reimbursement has two major differences. First, the worker's quality bid in equilibrium is increased under the percentage reimbursement, whereas it stays unchanged under the flat reimbursement. Second, the percentage reimbursement is only useful when the worker's loss aversion behavior has a negative impact on the buyer (otherwise the percentages should be set as 0). However, as long as the workers are loss averse (i.e. $\lambda > 1$), the flat reimbursement policy always increases buyer's utility, regardless of the negative or positive impact of worker's loss aversion on her (see the case of low λ values in Figure 1.1(a)), in view of Remark 2. The following proposition formally compares the two reimbursement policies for the buyer.

Proposition 7 *(1) If $1 < \lambda \leq \lambda_l$*[11]*, then the buyer should use the flat reimbursement policy.*

(2) If $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$[12]*, then there exists a $\lambda_s \in (1, +\infty)$, such that when $\lambda \geq \lambda_s$, the buyer should use the percentage reimbursement policy.*

If the worker's degree of loss aversion is low enough (i.e., $1 < \lambda < \lambda_l$), according to Corollary 1.4.1, the buyer should offer zero under the percentage reimbursement policy. Under the flat reimbursement policy, however, Proposition 6 states the buyer should set a positive reimbursement amount to the losing worker to maximize her expected utility, the resultant buyer's expected utility is higher than not offering any reimbursement. So in this scenario the buyer's expected utility is always higher under the flat reimbursement policy, compared with the percentage reimbursement policy.

Under the percentage reimbursement policy, Corollary 1.4.1 implies that when loss aversion is strong enough, the buyer should fully reimburse the quality spending of all types of workers. In that situation, the negative effect of loss aversion on buyer's expected utility can be fully eliminated. Under the flat reimbursement policy, however, Remark 2 states that the buyer sets an optimal reimbursement threshold that lies in between of the highest and lowest quality bids, for which low-bids losing workers do not receive any reimbursement and high-bid losing workers only receive partial reimbursement, as a result, the negative effect of loss aversion on buyer's expected utility can only be partially mitigated. Therefore, when loss aversion is strong enough, the percentage reimbursement policy provides more advantages to the buyer.

Figure 1.3(a) depicts the optimal choice between the percentage and the flat reimbursement for the buyer, and the impact of such an optimal choice on buyer's

[11] λ_l is defined in Corollary 1.4.1.

[12] This condition can be satisfied by multiple classes of functions (which also satisfy Assumption 1), including Example 1 and Example 2.

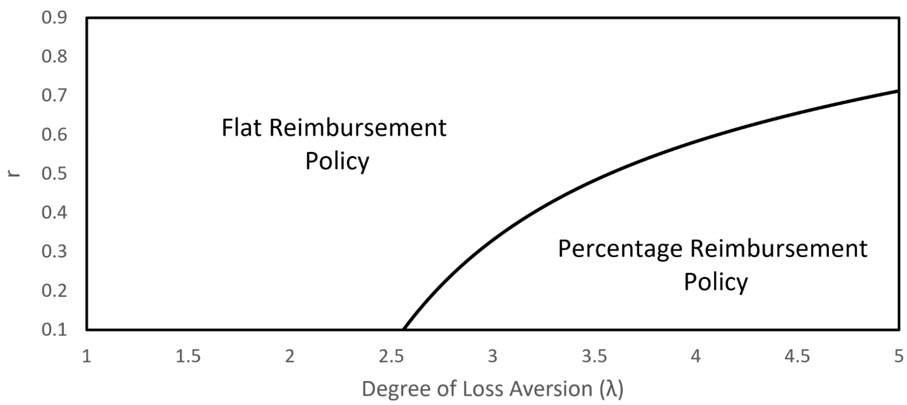
expected utility (in terms of the percentage increase in buyer's expected utility compared with the situation where neither reimbursement is used) is illustrated in Figure 1.3(b). We use this class of value function $V(q) = 100 + 600q^r$, where a high r implies a low diminishing return. We aim to examine the impact of the diminishing return of the value function, as well as the strength of the loss aversion, on the buyer's choice between the two types of reimbursement. We plot the curve where the two reimbursement policies are indifferent for $N = 4$ and $\theta \sim U[1, 7]$.

The indifference curve in Figure 1.3(a) divides the plot into two regions. The buyer should adopt the flat reimbursement if and only if the parameter value of r in the value function and the degree of loss aversion lie in the left region. One can observe that, consistent with Proposition 7, the buyer is better off with the flat reimbursement when worker's loss aversion is low enough, otherwise the percentage reimbursement offers more benefit to the buyer. It is also worth pointing out that as r increases, the region of the percentage reimbursement is smaller. This is because a high r implies a low diminishing return of the value function, in which case the negative effect of loss aversion to the buyer is smaller. Since the percentage reimbursement increases the equilibrium quality bid (whereas the flat reimbursement does not) and it is more effective in mitigating the negative effect of loss aversion when loss aversion is strong, the lessened need of reducing the negative effect under a high r makes the percentage reimbursement less attractive.

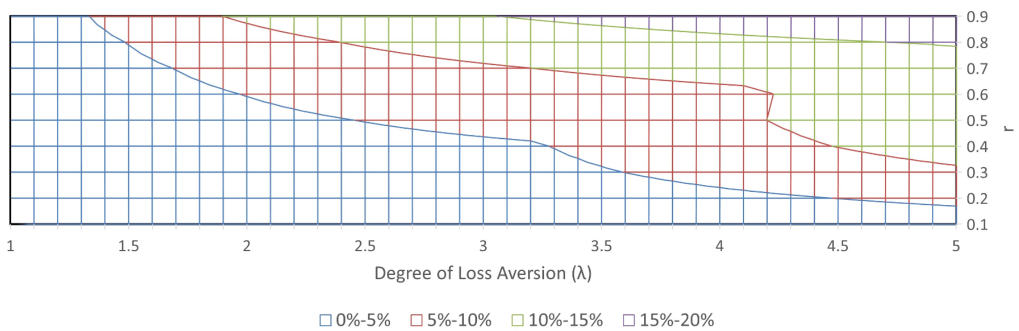
Figure 1.3(b) reveals a significant increase (up to 18.5%) in buyer's expected utility when the buyer adopts the optimal reimbursement form between the flat and the percentage reimbursement. The improvement seems stronger when workers are more loss averse or when r is higher. It is also worth mentioning that the improvement for buyer from using reimbursement is *not* at the expense of the worker.

FIGURE 1.3

(A) Percentage Reimbursement or Flat Reimbursement?



(B) Impact of Reimbursement on Buyer's Expected Utility



1.5 Extension: procurement auction when private type is on quality spending

Our results so far are obtained in the main case where workers have homogeneous expertise in producing quality proposal but heterogeneous one in the production process. In this subsection, we compare the results in the main case with those in another case where the quality producing expertise is heterogeneous (i.e, the private type is on quality spending), keeping all other assumptions in the main case.

In procurement auction when private type is on quality spending, a worker i of private type θ_i bidding price p_i and quality spending q_i earns the interim expected utility[13]:

$$\pi(q_i, p_i) = \big(p_i - \theta_i q_i\big) P\big(\text{win}|q_i, p_i\big) - \lambda \theta_i q_i \big(1 - P\big(\text{win}|q_i, p_i\big)\big) \tag{1.7}$$

where $\theta_i q_i$ is the quality spending of the proposal, θ_i is the private type, denoting workers' heterogeneous expertise in preparing the proposal. $P\big(\text{win}|q_i, p_i\big)$ is the probability of winning the auction when the worker bids price p_i and quality q_i, and $\lambda \geq 1$ denotes the degree of loss aversion.

Proposition 8 *(1) In the service procurement auction, there exists a unique symmetric equilibrium in which a loss-averse worker of private type θ makes the following bids for quality and price:*

$$\begin{aligned} q(\theta) = & \quad V'^{-1}\big(\lambda\theta \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big) \\ p(\theta) = & \quad -(\lambda-1)\theta q(\theta) - (\lambda-1)\frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} + \lambda\frac{\theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}}, \end{aligned}$$

where $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$.

(2) Worker's quality bid in equilibrium always decreases with the degree of loss aversion (i.e., $\frac{\partial q(\theta)}{\partial \lambda} = \theta \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1} V''(q(\theta))} \leq 0$).

(3) If $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, worker's price bid in equilibrium decreases with the degree of loss aversion.

Proposition 9 *In the equilibrium of the service procurement auction under which the buyer has no scoring rule commitment power,*

(1) worker's expected utility decreases with the degree of loss aversion;

(2) if $M(q) > m_0$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, buyer's expected utility decreases with loss aversion, where

$$m_0 \equiv \sup_{\lambda \geq 1} \frac{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) N F(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) N F(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) N \theta f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta} \in (0,1),$$

and $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is the equilibrium quality bidding.

Proposition 10 *Under a percentage reimbursement policy (ρ_w, ρ_l),*

[13] We assume the production cost is zero here. However, in general we can consider a constant production cost $c \geq 0$ for this case and all our results would keep unchanged except the equilibrium price bidding increases by c.

(1) a worker's quality bid in equilibrium increases with the reimbursement percentage ρ_w and ρ_l.

(2) if $M(q) \leq 1$ for all $q \geq 0$[14]*, a worker's price bid in equilibrium increases with the reimbursement percentage ρ_w and ρ_l.*

(3) if $M(q) \leq 1$ for all $q \geq 0$, a worker's expected utility in equilibrium increases with the reimbursement percentage ρ_w and ρ_l.

Proposition 11 *If $M(q) \leq 1$ for all $q \geq 0$,*

(1) *the buyer should never reimburse the quality spending of the winning worker, i.e., $\rho_w^* = 0$.*

(2) *Let*

$$B \equiv \quad -\frac{1-\rho_l}{\lambda}\int_{\underline{\theta}}^{\overline{\theta}} \theta q(\theta) N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta$$

where $q(\theta) = V'^{-1}\big(\lambda(1-\rho_l)\theta\frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} + \theta\big)$ and $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$.

(2.1) *If $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} > B$ for all $\rho_l \in [0,1]$, then the buyer should never reimburse losing worker(s), i.e., $\rho_l^* = 0$.*

(2.2) *If $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} < B$ for all $\rho_l \in [0,1]$, then the buyer should fully reimburse the losing worker(s), i.e., $\rho_l^* = 1$.*

(2.3) *If $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}\Big|_{\rho_l=1} > B|_{\rho_l=1} = 0$ and $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}\Big|_{\rho_l=0} < B|_{\rho_l=0}$, then the buyer should use the optimal reimbursement percentage $\rho_l^* \in (0,1)$, where ρ_l^* satisfies $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}\Big|_{\rho_l=\rho_l^*} = B|_{\rho_l=\rho_l^*}$.*

Proposition 12 *If $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, under a flat reimbursement policy (x, y_w, y_l),*

(1) a worker's quality bid in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

(2) a worker's price bid in equilibrium decreases with reimbursement amount y_w and y_l if his quality bid exceeds the reimbursement threshold x.

(3) a worker's expected utility in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

Proposition 13 *If $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$,*

(1) *the buyer's expected utility stays the same with respect to any reimbursement amount to the winning worker y_w $(x \in [0, V'^{-1}(\underline{\theta})], y_w \leq xq^{-1}(x), y_l \leq xq^{-1}(x))$*[15]*, where $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is the equilibrium quality bid and $q^{-1}(\cdot)$ is the inverse of $q(\cdot)$.*

[14] In the extreme case where the buyer chooses to fully reimburse the quality spending whenever a worker wins or loses the auction, a buyer should bid an infinite large quality.

[15] $V'^{-1}(\underline{\theta})$ is the highest equilibrium quality bidding, i.e., $V'^{-1}(\underline{\theta}) = q(\underline{\theta})$, where $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is the equilibrium quality bidding under the flat reimbursement policy.

(2) *the buyer should reimburse the maximum amount to the losing worker(s) (i.e., $y_l^* = x^* q^{-1}(x^*)$). The optimal threshold for quality spending reimbursement:* $x^* = argmax_{x \in [0, q(\underline{\theta})]} \int_{\underline{\theta}}^{\overline{\theta}} (\lambda - 1) 1_{q(\theta) \geq x} \cdot x q^{-1}(x) N f(\theta) \big(1 - (1 - F(\theta))^{N-1}\big) d\theta$, *where* $q(\theta) = V'^{-1}\big(\lambda \theta \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ *is the equilibrium quality bid and* $q^{-1}(\cdot)$ *is the inverse of* $q(\cdot)$.

Proposition 14 *If* $M(q) \leq 1$ *for all* $q \geq 0$, *when* $1 < \lambda \leq \lambda_l$, *the buyer should use the flat reimbursement policy, where*

$$\lambda_l \equiv \min\Big\{3, \min_{\substack{\lambda \in [1,3] \\ \rho_l \in [0,1]}} \frac{\int_{\underline{\theta}}^{\overline{\theta}} \big(\frac{1}{M(q(\theta))} - 1\big) \frac{F(\theta)}{f(\theta)} q(\theta) N f(\theta) \big(1 - (1 - F(\theta))^{N-1}\big) d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \theta q(\theta) N f(\theta) \big(1 - (1 - F(\theta))^{N-1}\big) d\theta} + 1\Big\} > 1,$$

$q(\theta) = V'^{-1}\big(\lambda(1-\rho_l)\frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} + \theta\big)$ *and* $V'^{-1}(\cdot)$ *is the inverse of* $V'(\cdot)$.

TABLE 1.1: Results Summary

Research questions	**Main Case: Private type on production cost**	**Extension Case: Private type on quality spending**	**Difference Summary**
How loss aversion affects workers' equilibrium bidding?	Proposition 1	Proposition 8	Part (3): the range of q where condition "$M(q) \leq 1$" satisfied
How loss aversion affects players' expected utility in equilibrium?	Proposition 2	Proposition 9	Part (2): the condition under which loss aversion is harmful
How percentage reimbursement policy impacts workers' equilibrium?	Proposition 3	Proposition 10	Part (2) and (3): the conditions about "$M(q) \leq 1$"
What is the optimal reimbursement percentage policy for the buyer?	Proposition 4	Proposition 11	All parts: the condition about "$M(q) \leq 1$"
How flat reimbursement policy impacts workers' equilibrium?	Proposition 5	Proposition 12	All parts: the condition about "$M(q) \leq 1$"
What is the optimal flat percentage policy for the buyer?	Proposition 6	Proposition 13	All parts: the condition about "$M(q) \leq 1$"
Which reimbursement policy should the buyer use?	Proposition 7	Proposition 14	In extension case: no part (2) of the main case

1.6 Managerial Insights and Concluding Remarks

This paper models the service procurement in online labor markets as a multidimensional auction combining the winner-pay part for the price bidding and all-pay part for the upfront quality spending which is associated with the loss aversion behavior of workers. Our analysis shows that while loss aversion always decreases workers' equilibrium expected utility, it can increase or decrease the buyer's equilibrium expected utility, depending on the degree of loss aversion and the diminishing return of the quality value function, as it can decrease the equilibrium quality and price bidding.

We further study two common reimbursement policies, i.e, the percentage reimbursement policy and flat reimbursement policy, for the service procurement auction with the loss aversion behavior. We find that in both policies, the buyer should only reimburse the losing worker(s) without any reimbursement to the winning worker. In addition, both policies can improve the buyer's expected utility without harming the workers'. Nevertheless, the two policies work differently. The percentage reimbursement policy can improve the buyer's expected utility only if the effect of loss aversion is harmful, as it counters this harmful effect by increasing her equilibrium quality value. The flat reimbursement policy, on the other hand, is useful for the buyer as long as workers are loss averse, because it can make use of loss aversion's multiplier effect (i.e., one unit of reimbursement amount can decrease $\lambda > 1$ units workers' price bidding) to reduce her equilibrium price payment. Therefore, when the effect of loss aversion is harmful enough (the degree of loss aversion and diminishing return of quality value function is high), it is always more profitable for the buyer to choose the percentage reimbursement policy, otherwise the the buyer should use the flat reimbursement policy.

The managerial implications are straightforward from our research. First, considering workers' loss aversion behavior, the online labor platforms should always encourage the use of reimbursement policies since they can always increase the total social welfare without harming any side. Second, it is crucial for the buyer to choose between the percentage reimbursement policy and the flat reimbursement policy, depending on the degree of loss aversion and the diminishing return of her quality value function.

Several directions of future research are possible. In addition to the first score sealed bid auction we considered in this paper, contest mechanism in which workers bid only the quality for a fixed price payment is also used in online labor markets. It will be interesting to investigate the contest mechanism and compare it with the multidimensional auction mechanism we studied. From a behavioral perspective, it will be interesting to empirically test the reimbursement policies for the procurement auction using human experiments, and design reimbursement policies based on the calibration of loss aversion behavior. Moreover, we have only introduced loss aversion into the model. A large body of literature show that a multitude of behavioral factors, such as risk aversion, utility of winning and regret, can be important in an auction context, and thus there is ample room to incorporate additional behavioral thinking into this line of research.

1.7 Proofs and Extension Cases

1.7.1 Proof of Proposition 1

Proof of Proposition 1 part (1)

We prove the unique symmetric equilibrium in part (1) of Proposition 1 by the following three steps. First, we identify the possible symmetric equilibrium from the first order conditions. Second, we prove the bidding strategy identified in the first step is an equilibrium (proof of sufficiency). Last, we prove the uniqueness of the symmetric equilibrium.

Step 1: possible symmetric equilibrium from first order conditions. We assume the symmetric bidding strategy in equilibrium is $(q(\cdot), p(\cdot))$ and a corresponding strictly decreasing scoring bidding function $S(\cdot) = s(q(\cdot)) - p(\cdot)$ (Note that in this case, $s(q(\cdot)) = V(q(\cdot))$, when buyer has no scoring rule commitment power). Given other workers' bidding strategy $(q(\cdot), p(\cdot))$, if a worker i of type θ_i bids quality q_i and price p_i (the corresponding scoring bid $S_i = V(q_i) - p_i$), based on (1.1), he can earn interim expected utility $\pi(q_i, p_i)$:

$$\pi(q_i, p_i) = (p_i - \theta_i q_i - q_i)\prod_{j\neq i} prob(S_i > S(\theta_j)) - \lambda q_i(1 - \prod_{j\neq i} prob(S_i > S(\theta_j)))$$
$$= (p_i - \theta_i q_i - q_i)\left[1 - F(S^{-1}(S_i))\right]^{N-1} - \lambda q_i(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1})$$

Given θ_i and other workers' bidding strategy, the worker i maximizes the expected utility $\pi_i(q_i, p_i)$ by choosing p_i and q_i. We calculate the derivative of $\pi_i(q_i, p_i)$ with respect to p_i and q_i:

$$\frac{\partial \pi(q_i, p_i)}{\partial p_i} = (p_i - \theta_i q_i - q_i)(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))} + \left[1 - F(S^{-1}(S_i))\right]^{N-1}$$
$$+ \lambda q_i(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}$$

$$\frac{\partial \pi(q_i, p_i)}{\partial q_i} = (p_i - \theta_i q_i - q_i)(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)$$
$$- (\theta_i + 1)\left[1 - F(S^{-1}(S_i))\right]^{N-1}$$
$$- \lambda(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1})$$
$$+ \lambda q_i(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)$$

From the first order conditions $\frac{\partial \pi(q_i,p_i)}{\partial p_i} = 0$ and $\frac{\partial \pi(q_i,p_i)}{\partial q_i} = 0$, we have:

$$\left[1 - F(S^{-1}(S_i))\right]^{N-1} V'(q_i) - (\theta_i + 1)\left[1 - F(S^{-1}(S_i))\right]^{N-1} - \lambda(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1}) = 0$$

Because of the symmetry of the equilibrium, we have $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$ and

$$\left[1 - F(\theta_i)\right]^{N-1} V'(q_i) - (\theta_i + 1)\left[1 - F(\theta_i)\right]^{N-1} - \lambda(1 - \left[1 - F(\theta_i)\right]^{N-1}) = 0$$

Let $G(q_i) \equiv \left[1 - F(\theta_i)\right]^{N-1} V'(q_i) - (\theta_i + 1)\left[1 - F(\theta_i)\right]^{N-1} - \lambda(1 - \left[1 - F(\theta_i)\right]^{N-1})$. Since $V''(q_i) < 0$, we can get: $\frac{\partial G(q_i)}{\partial q_i} = V''(q_i)\left[1 - F(\theta_i)\right]^{N-1} < 0$. In addition, since $\lim_{q\to 0} V'(q) = +\infty$ and $\lim_{q\to +\infty} V'(q) = 0$ from Assumption 1. We have $\lim_{q_i\to +\infty} G(q_i) < 0$, and $\lim_{q_i\to 0} G(q_i) > 0$. According to the intermediate value theorem, there is a unique $q_i^* \in (0, +\infty)$, such that $G(q_i^*) = 0$. Specifically, we can denote worker i's price bidding function as

$$q(\theta_i) = q_i^* = V'^{-1}(\lambda \frac{1 - (1 - F(\theta_i))^{N-1}}{(1 - F(\theta_i))^{N-1}} + \theta_i + 1)$$

where $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$. Note we can easily check that $q(\bar{\theta}) = V'^{-1}(+\infty) = 0$ and $q'(\theta_i) < 0$ from assumption 1 in our paper.

Moreover, from $\frac{\partial \pi(q_i,p_i)}{\partial p_i} = 0$, we can get the following differential equation:

$$\big(p_i - \theta_i q(\theta_i) - q(\theta_i) + \lambda q(\theta_i)\big)(N-1) f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))} + \big[1 - F(S^{-1}(S_i))\big] = 0$$

Because of the symmetry of the equilibrium, we have $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$ and

$$\big(p_i - \theta_i q(\theta_i) - q(\theta_i) + \lambda q(\theta_i)\big)(N-1) f(\theta_i) \frac{1}{S'(\theta_i)} + \big(1 - F(\theta_i)\big) = 0$$

In addition, because $S'(\theta_i) = V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)$, we have:

$$\big(p_i - \theta_i q(\theta_i) - q(\theta_i) + \lambda q(\theta_i)\big)(N-1) f(\theta_i) \frac{1}{V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)} + \big(1 - F(\theta_i)\big) = 0$$

With the boundary condition we solve the above differential equation:

$$p(\theta_i) = \theta_i q(\theta_i) + q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \lambda q(\theta_i) \frac{1-(1-F(\theta_i))^{(N-1)}}{(1-F(\theta_i))^{(N-1)}}$$

We can confirm that $S(\theta_i) = V(q(\theta_i)) - p(\theta_i)$ is strictly decreasing with the private type θ_i, because

$$\frac{dS(\theta_i)}{d\theta_i} = -(N-1)(1-F(\theta_i))^{-N} f(\theta_i) \bigg(\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) d\tilde{\theta} + \lambda q(\theta_i) \bigg) < 0$$

Step 2: $(q(\cdot), p(\cdot))$ **is a symmetric equilibrium.**

Following the approach in Hanazono, Nakabayashi, and Tsuruoka, 2013, **we can check the sufficiency based on two sub-steps. In the first sub-step,** given the score $S(\theta_i) = V(q(\theta_i)) - p(\theta_i)$ we identified in the first step, we need to check:

$$\begin{aligned}(q(\theta_i), p(\theta_i)) = \arg\max_{q_i,p_i} \pi(q_i,p_i) &= \big(p_i - \theta_i q_i - q_i\big)\big(1-F(\theta_i)\big)^{N-1} - \lambda q_i\big(1 - \big(1-F(\theta_i)\big)^{N-1}\big) \\ s.t. \quad & V(q_i) - p_i = S(\theta_i)\end{aligned}$$

Based on the definition of $S(\theta_i)$, the above maximization problem can be transformed into the following equivalent maximization problem:

$$q(\theta_i) = \arg\max_{q_i} \big(V(q_i) - S(\theta_i) - \theta_i q_i - q_i\big)\big(1-F(\theta_i)\big)^{N-1} - \lambda q_i\big(1 - \big(1-F(\theta_i)\big)^{N-1}\big)$$

Let $\pi(q_i, V(q_i) - S(\theta_i)) \equiv \big(V(q_i) - S(\theta_i) - \theta_i q_i - q_i\big)\big(1-F(\theta_i)\big)^{N-1} - \lambda q_i\big(1 - \big(1-F(\theta_i)\big)^{N-1}\big)$.

The first order condition:

$$\frac{\partial \pi(q_i, V(q_i) - S(\theta_i))}{\partial q_i} = \big(1-F(\theta_i)\big)^{N-1}\big(V'(q_i) - 1 - \theta_i\big) - \lambda\big(1 - \big(1-F(\theta_i)\big)^{N-1}\big) = 0,$$

and second order condition:

$$\frac{\partial^2 \pi(q_i, V(q_i) - S(\theta_i))}{\partial q_i^2} = \big(1-F(\theta_i)\big)^{N-1} V''(q_i) < 0.$$

Therefore, we have $q(\theta_i) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i + 1\big) = \arg\max_{q_i} \pi(q_i, V(q_i) -$

$S(\theta_i), \theta_i)$. In the first sub-step, we optimized a worker's quality and price bidding give a score $S(\theta_i)$. The resultant $q(\theta_i)$ and price bidding $p(\theta_i)$ can be both written as an implicit function of $S(\theta_i)$ and θ_i. Therefore, the original problem with multi-dimensional bids can be transformed into one-dimensional problem with a score bid (Hanazono, Nakabayashi, and Tsuruoka, 2013; Kovenock and Lu, 2020).

In the second sub-step, with the fact that $S(\theta_i)$ is strictly decreasing from step 1, we need to check that worker i will report the truthful information in a direct mechanism (Myerson, 1981) under which the worker i can make announcement $\tilde{\theta}_i$ to maximize his expected utility when the other workers are truthful:

$$\theta_i \in \arg\max_{\tilde{\theta}_i} \ \pi(\theta_i, \tilde{\theta}_i) = \left(p(\tilde{\theta}_i) - \theta_i q(\tilde{\theta}_i) - q(\tilde{\theta}_i)\right)\left[1 - F(\tilde{\theta}_i)\right]^{N-1} - \lambda q(\tilde{\theta}_i)\left(1 - \left[1 - F(\tilde{\theta}_i)\right]^{N-1}\right)$$

Let $\pi(\theta_i, \tilde{\theta}_i) \equiv \left(p(\tilde{\theta}_i) - \theta_i q(\tilde{\theta}_i) - q(\tilde{\theta}_i)\right)\left[1 - F(\tilde{\theta}_i)\right]^{N-1} - \lambda q(\tilde{\theta}_i)\left(1 - \left[1 - F(\tilde{\theta}_i)\right]^{N-1}\right)$. Using the form of $(q(\cdot), p(\cdot))$ we derived in Step 1, we have:

$$\pi(\theta_i, \tilde{\theta}_i) = \left(\tilde{\theta}_i q(\tilde{\theta}_i) - \theta_i q(\tilde{\theta}_i)\right)\left[1 - F(\tilde{\theta}_i)\right]^{N-1} + \int_{\tilde{\theta}_i}^{\overline{\theta}} q(\tilde{\theta})(1 - F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}$$

We can take derivative with respect to $\tilde{\theta}_i$ for the two sides of the above equality and get:

$$\frac{\partial \pi(\theta_i, \tilde{\theta}_i)}{\partial \tilde{\theta}_i} = (\tilde{\theta}_i - \theta_i)(1 - F(\tilde{\theta}))^{N-2}\left(q'(\tilde{\theta}_i)(1 - F(\tilde{\theta}_i)) - (N-1)q(\tilde{\theta}_i)f(\tilde{\theta}_i)\right)$$

Because $q'(\tilde{\theta}_i) < 0$, we have when $\tilde{\theta}_i < \theta_i$, $\frac{\pi(\theta_i, \tilde{\theta}_i)}{\partial \tilde{\theta}_i} > 0$; when $\tilde{\theta}_i > \theta_i$, $\frac{\pi(\theta_i, \tilde{\theta}_i)}{\partial \tilde{\theta}_i} < 0$. Therefore, $\theta_i = \arg\max_{\tilde{\theta}_i} \pi(\theta_i, \tilde{\theta}_i)$.

Step 3: uniqueness of the symmetric equilibrium $(q(\cdot), p(\cdot))$**.**

First, we can prove that a type θ_i worker i's quality bid in a symmetric equilibrium can only be $q(\theta_i)$ (Note that $\left[1 - F(\theta_i)\right]^{N-1} V'(q(\theta_i)) = (\theta_i + 1)\left[1 - F(\theta_i)\right]^{N-1} + \lambda\left(1 - \left[1 - F(\theta_i)\right]^{N-1}\right)$). We can prove this claim by contradiction. Suppose in a symmetric equilibrium, the type θ_i worker i's bidding is (q_i, p_i), and $\left[1 - F(\theta_i)\right]^{N-1} V'(q_i) \neq (\theta_i + 1)\left[1 - F(\theta_i)\right]^{N-1} + \lambda\left(1 - \left[1 - F(\theta_i)\right]^{N-1}\right)$

Case 1: If $\left[1 - F(\theta_i)\right]^{N-1} V'(q_i) - (\theta_i + 1)\left[1 - F(\theta_i)\right]^{N-1} - \lambda\left(1 - \left[1 - F(\theta_i)\right]^{N-1}\right) > 0$**,** then the worker i can increase a small amount of quality investment by $\Delta q_i \to 0^+$ and a small amount of price by $V'(q_i)\Delta q_i$, keeping the score unchanged. Specifically, we denote $q_i' = q_i + \Delta q_i$, and $p_i' = p_i + V'(q_i)\Delta q_i$. We have $S(q_i', p_i') = V(q_i') - p_i' = V(q_i + \Delta q_i) - (p_i + V'(q_i)\Delta q_i) = (V(q_i) + V'(q_i)\Delta q_i) - (p_i + V'(q_i)\Delta q_i) = V(q_i) - p_i = S(q_i, p_i)$. Now,

$$\begin{aligned}
\pi(q_i', p_i'|\theta_i) &= (p_i' - \theta_i q_i' - q_i')Prob\{win|S(q_i', p_i')\} - \lambda q_i'(1 - Prob\{win|S(q_i', p_i')\}) \\
&= \left(p_i + V'(q_i)\Delta q_i - \theta_i(q_i + \Delta q_i) - (q_i + \Delta q_i)\right)Prob\{win|S(q_i, p_i)\} \\
&- \lambda(q_i + \Delta q_i)(1 - Prob\{win|S(q_i, p_i)\}) \\
&= (p_i - \theta_i q_i - q_i)Prob\{win|S(q_i, p_i)\} - \lambda q_i(1 - Prob\{win|S(q_i, p_i)\}) \\
&+ \left(\left[1 - F(\theta_i)\right]^{N-1} V'(q_i) - (\theta_i + 1)\left[1 - F(\theta_i)\right]^{N-1} - \lambda\left(1 - \left[1 - F(\theta_i)\right]^{N-1}\right)\right)\Delta q_i \\
&> \pi(q_i, p_i|\theta_i),
\end{aligned}$$

where $\pi(q_i', p_i'|\theta_i)$ is the type θ_i worker i's expected utility when bidding (q_i', p_i'), $\pi(q_i, p_i|\theta_i)$ is the type θ_i worker i's expected utility when bidding (q_i, p_i), the third equality uses the fact that in a symmetric equilibrium in which the type θ_i worker i's bidding is (q_i, p_i) and therefore $Prob\{win|S(q_i, p_i)\} = \big[1 - F(\theta_i)\big]^{N-1}$. The last inequality uses the fact that $\big[1 - F(\theta_i)\big]^{N-1} V'(q_i) - (\theta_i + 1)\big[1 - F(\theta_i)\big]^{N-1} - \lambda\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big) > 0$ and $\Delta q_i \to 0^+$.

Case 2: If $\big[1 - F(\theta_i)\big]^{N-1} V'(q_i) - (\theta_i + 1)\big[1 - F(\theta_i)\big]^{N-1} - \lambda\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big) <$ 0**,** then the worker i can decrease a small amount of quality investment by $\Delta q_i \to 0^-$ and a small amount of price by $V'(q_i)\Delta q_i$, keeping the score unchanged. Specifically, we denote $q_i' = q_i + \Delta q_i$, and $p_i' = p_i + V'(q_i)\Delta q_i$. We have $S(q_i', p_i') = V(q_i') - p_i' = V(q_i + \Delta q_i) - (p_i + V'(q_i)\Delta q_i) = (V(q_i) + V'(q_i)\Delta q_i) - (p_i + V'(q_i)\Delta q_i) = V(q_i) - p_i = S(q_i, p_i)$. Now,

$$
\begin{aligned}
\pi(q_i', p_i'|\theta_i) &= \big(p_i' - \theta_i q_i' - q_i'\big) Prob\{win|S(q_i', p_i')\} - \lambda q_i'\big(1 - Prob\{win|S(q_i', p_i')\}\big) \\
&= \big(p_i + V'(q_i)\Delta q_i - \theta_i(q_i + \Delta q_i) - (q_i + \Delta q_i)\big) Prob\{win|S(q_i, p_i)\} \\
&- \lambda(q_i + \Delta q_i)\big(1 - Prob\{win|S(q_i, p_i)\}\big) \\
&= \big(p_i - \theta_i q_i - q_i\big) Prob\{win|S(q_i, p_i)\} - \lambda q_i\big(1 - Prob\{win|S(q_i, p_i)\}\big) \\
&+ \Big(\big[1 - F(\theta_i)\big]^{N-1} V'(q_i) - (\theta_i + 1)\big[1 - F(\theta_i)\big]^{N-1} - \lambda\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big)\Big)\Delta q_i \\
&> \pi(q_i, p_i|\theta_i),
\end{aligned}
$$

where $\pi(q_i', p_i'|\theta_i)$ is the type θ_i worker i's expected utility when bidding (q_i', p_i'), $\pi(q_i, p_i|\theta_i)$ is the type θ_i worker i's expected utility when bidding (q_i, p_i), the third equality uses the fact that in a symmetric equilibrium in which the type θ_i worker i's bidding is (q_i, p_i) and therefore $Prob\{win|S(q_i, p_i)\} = \big[1 - F(\theta_i)\big]^{N-1}$. The last inequality uses the fact that $\big[1 - F(\theta_i)\big]^{N-1} V'(q_i) - (\theta_i + 1)\big[1 - F(\theta_i)\big]^{N-1} - \lambda\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big) < 0$ and $\Delta q_i \to 0^-$.

Therefore, the bid (q_i, p_i) is strictly dominated by an alternative bid (q_i', p_i') in both cases. Thus, the type θ_i worker i's bidding (q_i, p_i) where $\big[1 - F(\theta_i)\big]^{N-1} V'(q_i) \neq (\theta_i + 1)\big[1 - F(\theta_i)\big]^{N-1} + \lambda\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big)$ cannot be in a symmetric equilibrium.

Therefore, $q(\cdot)$ we derived from Step 1 is the only possible symmetric pure equilibrium quality bidding. Moreover, given $q(\cdot)$ is used as the unique quality bidding strategy for all the workers in equilibrium, the uniqueness of the symmetric equilibrium price bidding $p(\cdot)$ follows from the usual equilibrium result in first-price sealed bid auctions.

Proof of Proposition 1 part (2)

Since the equilibrium quality bidding
$q(\theta) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$. It should satisfy the equation:

$$
\big[1 - F(\theta)\big]^{N-1} V'(q(\theta)) = (\theta + 1)\big[1 - F(\theta)\big]^{N-1} + \lambda\big(1 - \big[1 - F(\theta)\big]^{N-1}\big).
$$

We can take derivative with respect to λ for the two sides of the above equality and get: $\frac{\partial q(\theta)}{\partial \lambda} = \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1} V''(q(\theta))} \leq 0$ (Note $V''(\cdot) < 0$ from Assumption 1), the equality holds if and only if $\theta = \overline{\theta}$ or $\theta = \underline{\theta}$.

Proof of Proposition 1 part (3)

Since the equilibrium price bidding

$$p(\theta) = \theta q(\theta) + q(\theta) + \frac{\int_{\theta}^{\bar{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} + \lambda q(\theta) \frac{1-(1-F(\theta))^{(N-1)}}{(1-F(\theta))^{(N-1)}}.$$

We can take derivative with respect to λ for the two sides of the above equality and get:

$$\begin{aligned}
\frac{\partial p(\theta)}{\partial \lambda} &= \left(\theta - (\lambda - 1) + \frac{\lambda}{(1-F(\theta))^{(N-1)}}\right) \frac{\left(1-\left[1-F(\theta)\right]^{N-1}\right)}{\left[1-F(\theta)\right]^{N-1} V''(q(\theta))} \\
&+ \frac{\left(1-\left[1-F(\theta)\right]^{N-1}\right)}{\left[1-F(\theta)\right]^{N-1}} q(\theta) + \frac{\int_{\theta}^{\bar{\theta}} \frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \\
&= V'(q(\theta)) \frac{\left(1-\left[1-F(\theta)\right]^{N-1}\right)}{\left[1-F(\theta)\right]^{N-1} V''(q(\theta))} + \frac{\left(1-\left[1-F(\theta)\right]^{N-1}\right)}{\left[1-F(\theta)\right]^{N-1}} q(\theta) + \frac{\int_{\theta}^{\bar{\theta}} \frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \\
&= \frac{\left(1-\left[1-F(\theta)\right]^{N-1}\right)}{\left[1-F(\theta)\right]^{N-1}} q(\theta) \left(1 + \frac{V'(q(\theta))}{q(\theta) V''(q(\theta))}\right) + \frac{\int_{\theta}^{\bar{\theta}} \frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \\
&= \frac{\left(1-\left[1-F(\theta)\right]^{N-1}\right)}{\left[1-F(\theta)\right]^{N-1}} q(\theta) \left(1 - \frac{1}{M(q(\theta))}\right) + \frac{\int_{\theta}^{\bar{\theta}} \frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta))^{(N-1)}},
\end{aligned}$$

where the first equality uses the fact that $\frac{\partial q(\theta)}{\partial \lambda} = \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1} V''(q(\theta))}$, the second equality uses the fact that $V'(q(\theta)) = \theta + 1 + \lambda \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}$, the last equality uses the definition that $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$.

Note that $\frac{\int_{\theta_i}^{\bar{\theta}} \frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \leq 0$ because $V''(\cdot) < 0$. Moreover, it is easy to check that the equilibrium quality bidding function $q(\theta) \in [0, V'^{-1}(\underline{\theta}+1)]$, for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta}+1)]$, we can know that $\frac{\partial p(\theta)}{\partial \lambda} \leq 0$, the equality holds if and only if $\theta = \bar{\theta}$.

1.7.2 Proof of Proposition 2

Proof of Proposition 2 part (1)

From the equilibrium quality and price bidding function $q(\theta_i)$ and $p(\theta_i)$ we derived in 1.7.1, we can get worker i's expected utility in equilibrium $\pi(q(\theta_i), p(\theta_i)) = \int_{\theta_i}^{\bar{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}$. Therefore, we have:

$$\frac{d\pi(q(\theta_i), p(\theta_i))}{d\lambda} = \int_{\theta_i}^{\bar{\theta}} \frac{\left(1-(1-F(\tilde{\theta}))^{N-1}\right)}{V''(q(\tilde{\theta}))} d\tilde{\theta} \leq 0,$$

where the equality holds if and only if $\theta_i = \bar{\theta}$, the equality uses the fact that $\frac{\partial q(\theta_i)}{\partial \lambda} = \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1} V''(q(\theta_i))}$ and the inequality follows from the fact that $V''(\cdot) < 0$. Therefore, a worker's expected utility decreases with the degree of loss aversion.

Proof of Proposition 2 part (2)

Based on the price and quality bidding in part (1) of Proposition 1, the buyer's expected utility in equilibrium EU_b should be:

$$\begin{aligned}
EU_b &= \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{V(q(\theta)) - \theta q(\theta) + (\lambda-1)q(\theta) \\
&\quad - \frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \frac{\lambda q(\theta)}{(1-F(\theta))^{(N-1)}}\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{V(q(\theta)) - \theta q(\theta) + (\lambda-1)q(\theta) - \frac{F(\theta)}{f(\theta)}q(\theta) - \frac{\lambda q(\theta)}{(1-F(\theta))^{(N-1)}}\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta,
\end{aligned}$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$ and the last equality follows from integration by parts with the fact that $q(\overline{\theta}) = 0$. Therefore, we have:

$$\begin{aligned}
\frac{\partial EU_b}{\partial \lambda} &= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{\Big(V'(q(\theta)) - \theta + (\lambda-1) - \frac{\lambda}{\big(1-F(\theta)\big)^{N-1}} - \frac{F(\theta)}{f(\theta)}\Big)\frac{dq(\theta)}{d\lambda} \\
&\quad - \frac{\big(1-\big[1-F(\theta)\big]^{N-1}\big)}{\big[1-F(\theta)\big]^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{-\frac{F(\theta)}{f(\theta)}\frac{dq(\theta)}{d\lambda} - \frac{\big(1-\big[1-F(\theta)\big]^{N-1}\big)}{\big[1-F(\theta)\big]^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{-\frac{F(\theta)}{f(\theta)}\frac{\big(1-\big[1-F(\theta)\big]^{N-1}\big)}{\big[1-F(\theta)\big]^{N-1}V''(q(\theta))} - \frac{\big(1-\big[1-F(\theta)\big]^{N-1}\big)}{\big[1-F(\theta)\big]^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{-\frac{F(\theta)}{f(\theta)}\frac{1}{V''(q(\theta))q(\theta)} - 1\Big\} q(\theta)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{\frac{F(\theta)}{f(\theta)}\frac{1}{M(q(\theta))} - (\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1)\Big\}\frac{q(\theta)}{V'(q(\theta))}Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta,
\end{aligned}$$

where the second equality uses the fact that $V'(q(\theta)) = \theta + 1 + \lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}$, the third equality uses the fact that $\frac{\partial q(\theta)}{\partial \lambda} = \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}V''(q(\theta))}$, the last equality uses the facts that $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$ and $V'(q(\theta)) = \lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1$.

First note if $\lim_{q\to 0+} M(q) \neq 0$, then we have $\lim_{q\to 0+} M(q) = \lim_{\lambda\to+\infty} M(q(\theta))(\forall\theta \in (\underline{\theta},\overline{\theta})) = \lim_{\theta\to\overline{\theta}} M(q(\theta)) > 0$ (Note we have $M(q) \geq 0$ throughout the paper from Assumption 1).

Therefore, for any $\theta \in [\underline{\theta},\overline{\theta}]$ and any $\lambda \geq 1$, we have: $\frac{\left(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1\right)\left(1-F(\theta)\right)^{N-1}}{1-\left(1-F(\theta)\right)^{N-1}} < +\infty$ and thus $\sup_{\substack{\lambda\geq 1\\ \theta\in[\underline{\theta},\overline{\theta}]}} \frac{(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1)(1-F(\theta))^{N-1}}{1-(1-F(\theta))^{N-1}}$ exists.

Moreover, when $\lambda > \sup_{\substack{\lambda \geq 1 \\ \theta \in [\underline{\theta}, \overline{\theta}]}} \frac{(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1)(1-F(\theta))^{N-1}}{1-(1-F(\theta))^{N-1}}$, we have:

$$\begin{aligned}
\frac{\partial EU_b}{\partial \lambda} &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{F(\theta)}{f(\theta)} \frac{1}{M(q(\theta))} - \theta - 1 - \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} \lambda \right\} \frac{q(\theta)}{V'(q(\theta))} N f(\theta) (1 - (1-F(\theta))^{N-1}) d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1)(1-F(\theta))^{N-1}}{1-(1-F(\theta))^{N-1}} - \lambda \right\} \frac{q(\theta)}{V'(q(\theta))} N f(\theta) \frac{(1-(1-F(\theta))^{N-1})^2}{(1-F(\theta))^{N-1}} d\theta \\
&< 0,
\end{aligned}$$

where the inequality comes from the facts $\lambda > \sup_{\substack{\lambda \geq 1 \\ \theta \in [\underline{\theta}, \overline{\theta}]}} \frac{(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1)(1-F(\theta))^{N-1}}{1-(1-F(\theta))^{N-1}}$, $q(\theta) > 0$ and $V'(q(\theta)) > 0$.

Therefore, if $\lim_{q \to 0+} M(q) \neq 0$, then when $\lambda > \sup_{\substack{\lambda \geq 1 \\ \theta \in [\underline{\theta}, \overline{\theta}]}} \frac{(\frac{1}{M(q(\theta))}\frac{F(\theta)}{f(\theta)} - \theta - 1)(1-F(\theta))^{N-1}}{1-(1-F(\theta))^{N-1}}$, we always have $\frac{dEU_b}{d\lambda} < 0$, i.e., buyer's expected utility decreases with loss aversion when loss aversion is sufficiently high.

1.7.3 Proof of Proposition 3

Consider the buyer can reimburse the losing workers with $\rho_l \in [0\%, 100\%]$ percent of their quality spending, and the winning workers with $\rho_w \in [0\%, 100\%]$ percent of their quality spending. We will prove this part by two steps. Given any reimbursement percentage ρ_w and ρ_l, we solve the workers' quality and price bidding in equilibrium (Here we identify the symmetric equilibrium from the first order conditions. Its sufficiency and uniqueness can be checked by the same methods in 1.7.1).

We assume the symmetric bidding strategy in equilibrium is $(q(\cdot), p(\cdot))$ and a corresponding strictly decreasing scoring bidding function $S(\cdot) = s(q(\cdot)) - p(\cdot)$ (Note that in this case, $s(q(\cdot)) = V(q(\cdot))$).Given other workers' bidding strategy $(q(\cdot), p(\cdot))$, if a worker i of type θ_i bids quality q_i and price p_i (the corresponding scoring bid $S_i = s(q_i) - p_i$), under the percentage reimbursement policy (ρ_w, ρ_l) he can earn interim expected utility $\pi^{pr}(q_i, p_i)$:

$$\begin{aligned}
\pi^{pr}(q_i, p_i) &= (p_i - \theta_i q_i - (1-\rho_w) q_i) \prod_{j \neq i} prob(S_i > S(\theta_j)) - \lambda (1-\rho_l) q_i (1 - \prod_{j \neq i} prob(S_i > S(\theta_j))) \\
&= (p_i - \theta_i q_i - (1-\rho_w) q_i) \left[1 - F(S^{-1}(S_i))\right]^{N-1} - \lambda (1-\rho_l) q_i (1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1})
\end{aligned}$$

Given θ_i and other workers' bidding strategy, the worker i maximizes his expected profit by choosing p_i and q_i. We calculate the derivative of $\pi_i(q_i, p_i)$ with respect to p_i and q_i:

$$\begin{aligned}
\frac{\partial \pi^{pr}(q_i, p_i)}{\partial p_i} =& (p_i - \theta_i q_i - (1-\rho_w) q_i)(N-1) \left[1 - F(S^{-1}(S_i))\right]^{N-2} f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))} \\
&+ \left[1 - F(S^{-1}(S_i))\right]^{N-1} + \lambda (1-\rho_l) q_i (N-1) \left[1 - F(S^{-1}(S_i))\right]^{N-2} f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))}
\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi^{pr}(q_i,p_i)}{\partial q_i} &= \big(p_i-\theta_i q_i-(1-\rho_w)q_i\big)(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2}(-1)f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)\\ &-(\theta_i+(1-\rho_w))\big[1-F(S^{-1}(S_i))\big]^{N-1}\\ &-\lambda(1-\rho_l)\big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\big)\\ &+\lambda(1-\rho_l)q_i(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2}(-1)f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)\end{aligned}$$

From the first order condition $\frac{\partial \pi^{pr}(q_i,p_i)}{\partial p_i}=0$ and $\frac{\partial \pi^{pr}(q_i,p_i)}{\partial q_i}=0$, we have

$$\begin{aligned}\big[1-F(S^{-1}(S_i))\big]^{N-1}V'(q_i)-(\theta_i+(1-\rho_w))\big[1-F(S^{-1}(S_i))\big]^{N-1}\\ -\lambda(1-\rho_l)\big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\big)=0.\end{aligned}$$

Because of the symmetry of the equilibrium, we have $S^{-1}(S_i)=S^{-1}(S(\theta_i))=\theta_i$. Therefore:

$$\big[1-F(\theta_i)\big]^{N-1}V'(q_i)-(\theta_i+(1-\rho_w))\big[1-F(\theta_i)\big]^{N-1}-\lambda(1-\rho_l)\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)=0$$

Let $G(q_i)\equiv\big[1-F(\theta_i)\big]^{N-1}V'(q_i)-(\theta_i+(1-\rho_w))\big[1-F(\theta_i)\big]^{N-1}-\lambda(1-\rho_l)\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)$. Since $V''(\cdot)<0$, we can get: $\frac{dG(q_i)}{dq_i}=V''(q_i)\big[1-F(\theta_i)\big]^{N-1}<0$.

In addition, since $\lim_{q\to 0}V'(q)=+\infty$ and $\lim_{q\to+\infty}V'(q)=0$. We have $\lim_{q_i\to+\infty}G(q_i)<0$, and $\lim_{q_i\to 0}G(q_i)>0$. According to the intermediate value theorem, there is an unique q_i^*, such that $G(q_i^*)=0$. Specifically, we can denote the equilibrium price bidding :

$$q(\theta_i)=q_i^*=V'^{-1}\Big(\lambda(1-\rho_l)\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}}+\theta_i+(1-\rho_w)\Big),$$

where $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$. Taking derivative with respect to ρ_w and ρ_l for the equilibrium price bidding function $q(\theta_i)$, we have:

$$\begin{aligned}\frac{dq(\theta_i)}{d\rho_w}&=\frac{-1}{V''(q(\theta_i))}>0,\\ \frac{dq(\theta_i)}{d\rho_l}&=\frac{-\lambda\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}V''(q(\theta_i))}\geq 0,\end{aligned}$$

where the equality (of the last inequality) holds if and only if $\theta_i=\underline{\theta}$ or $\theta_i=\overline{\theta}$, both inequality follows from the fact that $V''(\cdot)<0$. Therefore, a worker's quality bidding in equilibrium increases with reimbursement percentage ρ_w and ρ_l.

In addition, from $\frac{\partial \pi^{pr}(q_i,p_i)}{\partial p_i}=0$, we can get the derivative equation

$$\big(p_i-\theta_i q(\theta_i)-(1-\rho_w)q(\theta_i)+\lambda(1-\rho_l)q(\theta_i)\big)(N-1)f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}+\big[1-F(S^{-1}(S_i))\big]=0$$

Because of the symmetry of the equilibrium, we have $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore,

$$\big(p_i - \theta_i q(\theta_i) - (1-\rho_w)q(\theta_i) + \lambda(1-\rho_l)q(\theta_i)\big)(N-1)f(\theta_i)\frac{1}{S'(\theta_i)} + \big[1 - F(\theta_i)\big] = 0$$

Moreover, because $S'(\theta_i) = V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)$. We have:

$$\big(p_i - \theta_i q(\theta_i) - (1-\rho_w)q(\theta_i) + \lambda(1-\rho_l)q(\theta_i)\big)(N-1)f(\theta_i)\frac{1}{V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)} + \big(1 - F(\theta_i)\big) = 0$$

With the boundary condition we can solve this differential equation and get:

$$p(\theta_i) = \theta_i q(\theta_i) + (1-\rho_w)q(\theta_i) - \lambda(1-\rho_l)q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \frac{\lambda(1-\rho_l)q(\theta_i)}{(1-F(\theta_i))^{(N-1)}}$$

We can confirm $S(\theta_i) = V(q(\theta_i)) - p(\theta_i)$ strictly decreases with private type θ_i, because

$$\frac{dS(\theta_i)}{d\theta_i} = -(N-1)(1-F(\theta_i))^{-N}f(\theta_i)\bigg(\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta} + \lambda(1-\rho_l)q(\theta_i)\bigg) < 0$$

Taking derivative with respect to ρ_w and ρ_l for the equilibrium price bidding function $p(\theta_i)$, we have:

$$\begin{aligned}
\frac{\partial p(\theta_i)}{\partial \rho_w} &= \big(\theta_i - \lambda(1-\rho_l) + (1-\rho_w) + \frac{\lambda(1-\rho_l)}{(1-F(\theta_i))^{(N-1)}}\big)\frac{-1}{V''(q(\theta_i))} \\
&- q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}} \frac{-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= V'(q(\theta_i))\frac{-1}{V''(q(\theta_i))} - q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}} \frac{(1-F(\tilde{\theta}))^{(N-1)}}{-V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= q(\theta_i)\big(\frac{-V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))} - 1\big) + \frac{\int_{\theta_i}^{\overline{\theta}} \frac{(1-F(\tilde{\theta}))^{(N-1)}}{-V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= q(\theta_i)\big(\frac{1}{M(q(\theta_i))} - 1\big) + \frac{\int_{\theta_i}^{\overline{\theta}} \frac{(1-F(\tilde{\theta}))^{(N-1)}}{-V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}},
\end{aligned}$$

where the first equality uses the fact that $\frac{dq(\theta_i)}{d\rho_w} = \frac{-1}{V''(q(\theta_i))}$, the second equality uses the fact $V'(q(\theta_i)) = \lambda(1-\rho_l)\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i + (1-\rho_w)$, the last equality uses the definition that $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$. Note that $\frac{\int_{\theta_i}^{\overline{\theta}} \frac{(1-F(\tilde{\theta}))^{(N-1)}}{-V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} > 0$ because $V''(\cdot) < 0$. Moreover, it is easy to check that the equilibrium quality bidding function $q(\theta_i) \in [0, V'^{-1}(\underline{\theta})], \forall \theta_i \in [\underline{\theta}, \overline{\theta}], \forall \rho_w \in [0,1], \forall \rho_l \in [0,1]$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, we can know that $\frac{\partial p(\theta_i)}{\partial \rho_w} \geq 0$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, a worker's price bid in equilibrium increases with reimbursement percentage ρ_w.

$$\begin{aligned}
\frac{\partial p(\theta_i)}{\partial \rho_l} &= \Big(\theta_i - \lambda(1-\rho_l) + (1-\rho_w) + \frac{\lambda(1-\rho_l)}{(1-F(\theta_i))^{(N-1)}}\Big)\frac{-\lambda(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}V''(q(\theta_i))} \\
&+ \frac{-\lambda(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}}q(\theta_i) + \frac{\int_{\theta_i}^{\bar\theta} -\lambda\frac{1-(1-F(\tilde\theta))^{(N-1)}}{V''(q(\tilde\theta))}d\tilde\theta}{(1-F(\theta_i))^{(N-1)}} \\
&= \frac{\lambda(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}}\Big(V'(q(\theta_i))\frac{-1}{V''(q(\theta_i))} - q(\theta_i)\Big) + \frac{\int_{\theta_i}^{\bar\theta} -\lambda\frac{1-(1-F(\tilde\theta))^{(N-1)}}{V''(q(\tilde\theta))}d\tilde\theta}{(1-F(\theta_i))^{(N-1)}} \\
&= \frac{\lambda(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}}q(\theta_i)\Big(\frac{-V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))} - 1\Big) + \frac{\int_{\theta_i}^{\bar\theta} -\lambda\frac{1-(1-F(\tilde\theta))^{(N-1)}}{V''(q(\tilde\theta))}d\tilde\theta}{(1-F(\theta_i))^{(N-1)}} \\
&= \frac{\lambda(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}}q(\theta_i)\Big(\frac{1}{M(q(\theta_i))} - 1\Big) + \frac{\int_{\theta_i}^{\bar\theta} -\lambda\frac{1-(1-F(\tilde\theta))^{(N-1)}}{V''(q(\tilde\theta))}d\tilde\theta}{(1-F(\theta_i))^{(N-1)}},
\end{aligned}$$

where the first equality uses the fact that $\frac{dq(\theta_i)}{d\rho_l} = \frac{-\lambda(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}V''(q(\theta_i))}$, the second equality uses the fact $V'(q(\theta_i)) = \lambda(1-\rho_l)\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i + (1-\rho_w)$, the last equality uses the definition that $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$. Note that $\frac{\int_{\theta_i}^{\bar\theta} -\lambda\frac{1-(1-F(\tilde\theta))^{(N-1)}}{V''(q(\tilde\theta))}d\tilde\theta}{(1-F(\theta_i))^{(N-1)}} > 0$ because $V''(\cdot) < 0$. Moreover, it is easy to check that the equilibrium quality bidding function $q(\theta_i) \in [0, V'^{-1}(\underline{\theta})], \forall\theta_i \in [\underline{\theta}, \bar\theta], \forall\rho_w \in [0,1], \forall\rho_l \in [0,1]$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, we can know that $\frac{\partial p(\theta_i)}{\partial\rho_l} \geq 0$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, a worker's price bidding in equilibrium increases with reimbursement percentage ρ_l.

Moreover, from the equilibrium quality and price bidding function $q(\theta_i)$ and $p(\theta_i)$ we derived, we can get worker i's expected utility in equilibrium $\pi^{pr}(q(\theta_i), p(\theta_i)) = \int_{\theta_i}^{\bar\theta} q(\tilde\theta)(1-F(\tilde\theta))^{(N-1)}d\tilde\theta$. Therefore, we have:

$$\begin{aligned}
\frac{d\pi^{pr}(q(\theta_i), p(\theta_i))}{d\rho_w} &= \int_{\theta_i}^{\bar\theta} \frac{-1}{V''(q(\tilde\theta))}(1-F(\tilde\theta))^{(N-1)}d\tilde\theta \geq 0, \\
\frac{d\pi^{pr}(q(\theta_i), p(\theta_i))}{d\rho_l} &= \int_{\theta_i}^{\bar\theta} \frac{-\lambda\big(1-[1-F(\tilde\theta)]^{N-1}\big)}{V''(q(\tilde\theta))}d\tilde\theta \geq 0,
\end{aligned}$$

where the equality holds if and only if $\theta_i = \bar\theta$, both inequality follows from the fact that $V''(\cdot) < 0$. Therefore, a worker's expected utility in equilibrium increases with reimbursement percentage ρ_w and ρ_l.

1.7.4 Proof of Proposition 4

Proof of Proposition 4 part (1)

Given workers' best response (equilibrium price and quality bidding we characterized in 1.7.3), the buyer's expected utility when she reimburses the winning worker for ρ_w percentage of quality cost and the losing workers for ρ_l percentage of quality cost:

$$
\begin{aligned}
EU_b^{pr}(\rho_w,\rho_l) &= \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} - N\int_{\underline{\theta}}^{\overline{\theta}} \rho_l q(\theta)(1-(1-F(\theta))^{N-1})dF(\theta) \\
&\quad - N\int_{\underline{\theta}}^{\overline{\theta}} \rho_w q(\theta)(1-F(\theta))^{N-1}dF(\theta) \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{V(q(\theta)) - \theta q(\theta) - (1-\rho_w)q(\theta) + \lambda(1-\rho_l)q(\theta) \\
&\quad - \frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \\
&\quad - \frac{\lambda(1-\rho_l)q(\theta)}{(1-F(\theta))^{(N-1)}} - \rho_w q(\theta) - \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}\rho_l q(\theta)\Bigg\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{V(q(\theta)) - \theta q(\theta) - q(\theta) + \lambda(1-\rho_l)q(\theta) \\
&\quad - \frac{F(\theta)}{f(\theta)}q(\theta) - \frac{\lambda(1-\rho_l)q(\theta)}{(1-F(\theta))^{(N-1)}} - \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}\rho_l q(\theta)\Bigg\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta
\end{aligned}
$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$ and the last equality follows from integration by parts.

Taking derivative with respect to ρ_w for the buyer's expected utility $EU_b^{pr}(\rho_w,\rho_l)$, we have:

$$
\begin{aligned}
\frac{\partial EU_b^{pr}(\rho_w,\rho_l)}{\partial\rho_w} &= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{\Bigg(V'(q(\theta)) - \theta + \lambda(1-\rho_l) - (1-\rho_l) \\
&\quad - \frac{F(\theta)}{f(\theta)} - \frac{\lambda(1-\rho_l)}{\big(1-F(\theta)\big)^{N-1}} - \frac{\rho_l}{(1-F(\theta))^{N-1}}\Bigg)\frac{dq(\theta)}{d\rho_w}\Bigg\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{\Bigg(-\rho_w - \frac{F(\theta)}{f(\theta)} - \rho_l\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}\Bigg)\frac{dq(\theta)}{d\rho_w}\Bigg\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{\Bigg(\rho_w + \frac{F(\theta)}{f(\theta)} + \rho_l\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}\Bigg)\frac{1}{V''(q(\theta))}\Bigg\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta < 0
\end{aligned}
$$

where the second equality uses the fact that $V'(q(\theta)) = \lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + (1-\rho_w)$, the third equality uses the fact that $\frac{\partial q(\theta)}{\partial\rho_w} = \frac{-1}{V''(q(\theta))}$. The last inequality follows from $V''(\cdot) < 0$.

Since $\frac{\partial EU_b^{pr}(\rho_w,\rho_l)}{\partial\rho_w} < 0$, the optimal reimbursement percentage of reimbursing the winner worker for the buyer should be $\rho_w^* = 0$.

Proof of Proposition 4 part (2)

Based on 1.7.4, when the buyer can choose to reimburse the losing workers with $\rho_l \in [0\%, 100\%]$ percent of their quality cost, and the winning worker with $\rho_w \in [0\%, 100\%]$ percent of their quality cost, then the optimal reimbursement percentage for the winning worker should be $\rho_w^* = 0$.

Therefore, the buyer should only reimburse the losing workers. In this subsection we consider the optimal reimbursement percentage to the losing workers under the condition that the optimal reimbursement percentage to the winning worker is

$\rho_w^* = 0$. Based on the results in 1.7.4, the buyer's expected utility when he commits to reimburses only the losing workers' quality spending with $\rho_l \in [0,1]$ percent should be:

$$EU_b^{pr}(0,\rho_l) = \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ V(q(\theta)) - q(\theta) - \theta q(\theta) - \frac{F(\theta)}{f(\theta)} q(\theta) - \lambda(1-\rho_l)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) - \rho_l \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta$$

Taking derivative with respect to λ for the buyer's expected utility $EU_b^{pr}(0,\rho_l)$, we have:

$$\begin{aligned}
\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} &= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \Big(V'(q(\theta)) - 1 - \theta - \frac{F(\theta)}{f(\theta)} - (\lambda(1-\rho_l)+\rho_l)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} \Big) \frac{dq(\theta)}{d\lambda} \\
&\quad - (1-\rho_l)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \big(-\rho_l \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} - \frac{F(\theta)}{f(\theta)} \big) \frac{dq(\theta)}{d\lambda} \\
&\quad - (1-\rho_l)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \big(-\rho_l \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} - \frac{F(\theta)}{f(\theta)} \big) \frac{(1-\rho_l)\big(1-\big(1-F(\theta)\big)^{N-1}\big)}{\big(1-F(\theta)\big)^{N-1} V''(q(\theta))} \\
&\quad - (1-\rho_l)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta
\end{aligned}$$

where the second equality uses the fact that $V'(q(\theta)) = \lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1$, the last equality uses the fact that $\frac{dq(\theta)}{d\lambda} = \frac{(1-\rho_l)(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1}V''(q(\theta))}$.

Moreover, taking derivative with respect to ρ_l for the buyer's expected utility $EU_b^{pr}(0,\rho_l)$, we have:

$$
\begin{aligned}
\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l} &= \int_{\underline{\theta}}^{\bar{\theta}} \Big\{ \Big(V'(q(\theta)) - 1 - \theta - \frac{F(\theta)}{f(\theta)} - (\lambda(1-\rho_l)+\rho_l) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} \Big) \frac{dq(\theta)}{d\rho_l} \\
&+ (\lambda-1) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \Big\{ \big(-\rho_l \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} - \frac{F(\theta)}{f(\theta)} \big) \frac{dq(\theta)}{d\rho_l} + (\lambda-1) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \Big\{ \big(-\rho_l \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} - \frac{F(\theta)}{f(\theta)} \big) \frac{-\lambda\big(1-\big(1-F(\theta)\big)^{N-1}\big)}{\big(1-F(\theta)\big)^{N-1} V''(q(\theta))} \\
&+ (\lambda-1) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
&= \frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} \frac{-\lambda}{1-\rho_l} \\
&+ \int_{\underline{\theta}}^{\bar{\theta}} \Big\{ -\lambda \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) + (\lambda-1) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
&= \frac{-\lambda}{1-\rho_l} \Big(\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} + \frac{1-\rho_l}{\lambda} \int_{\underline{\theta}}^{\bar{\theta}} q(\theta) Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big) d\theta \Big)
\end{aligned}
$$

where the second equality uses the fact that $V'(q(\theta)) = \lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1$, the third equality uses the fact that $\frac{dq(\theta)}{d\rho_l} = \frac{-\lambda(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1}V''(q(\theta))}$, the fourth equality uses the $\frac{dEU_b^{pr}(0,\rho_l)}{d\lambda}$ result we derived above. Therefore, we have:

When $\frac{dEU_b^{pr}(0,\rho_l)}{d\lambda} > -\frac{1-\rho_l}{\lambda}\int_{\underline{\theta}}^{\bar{\theta}} q(\theta)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \equiv B$ for all $\rho_l \in [0,1]$, we have $\frac{dEU_b^{pr}(0,\rho_l)}{d\rho_l} < 0$ for all $\rho_l \in [0,1]$. Therefore, the optimal reimbursement percentage $\rho_l^* = 0$.

When $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda} < -\frac{1-\rho_l}{\lambda}\int_{\underline{\theta}}^{\bar{\theta}} q(\theta)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \equiv B$, for all $\rho_l \in [0,1]$, we have $\frac{dEU_b^{pr}(0,\rho_l)}{d\rho_l} > 0$ for all $\rho_l \in [0,1]$. Therefore, the optimal reimbursement percentage $\rho_l^* = 1$.

When $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda}\Big|_{\rho_l=1} > B|_{\rho_l=1}$, we have $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\rho_l}\Big|_{\rho_l=1} < 0$, the buyer should decrease the reimbursement percentage from 100%, which can increase his expected utility. When $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda}\Big|_{\rho_l=0} < B|_{\rho_l=0}$, we have $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\rho_l}\Big|_{\rho_l=0} < 0$, the buyer should increase the reimbursement percentage from zero percent, which can increase his expected utility. Therefore, the buyer should choose the optimal reimbursement percentage $\rho_l^* \in (0,1)$, where ρ_l^* should satisfy $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\rho_l}\Big|_{\rho_l=\rho_l^*} = 0$, or equivalently $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial\lambda}\Big|_{\rho_l=\rho_l^*} = B|_{\rho_l=\rho_l^*}$.

1.7.5 Proof of Corollary 1.4.1

From 1.7.4, we have:

$$\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l}$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left(-\rho_l \frac{1-\left(1-F(\theta)\right)^{N-1}}{\left(1-F(\theta)\right)^{N-1}} - \frac{F(\theta)}{f(\theta)} \right) \frac{-\lambda}{V''(q(\theta))} + (\lambda-1)q(\theta) \right\} Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right) d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{-\lambda\left(-\rho_l \frac{1-\left(1-F(\theta)\right)^{N-1}}{\left(1-F(\theta)\right)^{N-1}} - \frac{F(\theta)}{f(\theta)} \right) V'(q(\theta))}{q(\theta)V''(q(\theta))\left(\lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\right)} + (\lambda-1) \right\} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right) d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{-\left(\rho_l \frac{1-\left(1-F(\theta)\right)^{N-1}}{\left(1-F(\theta)\right)^{N-1}} + \frac{F(\theta)}{f(\theta)} \right)}{M(q(\theta))\left((1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{\theta+1}{\lambda}\right)} + (\lambda-1) \right\} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right) d\theta,$$

where the second equality uses the fact that $V'(q(\theta)) = \lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1$, the last equality uses the definition that $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$.

Let's define:

$$c_l \equiv \min_{\substack{\lambda\in[1,3] \\ \rho_l\in[0,1]}} \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{q(\theta)NF(\theta)(1-(1-F(\theta))^{N-1})}{M(q(\theta))\left(\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta+1\right)} d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta} > 0,$$

From the Weierstrass extreme value theorem, c_l exists. Moreover, we can define:

$$\lambda_l \equiv min\{3, c_l+1\}$$

Obviously, we have $1 < \lambda_l \leq 3$. Therefore, when $1 \leq \lambda \leq \lambda_l$, we have:

$$\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l} = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{-\left(\rho_l \frac{1-\left(1-F(\theta)\right)^{N-1}}{\left(1-F(\theta)\right)^{N-1}} + \frac{F(\theta)}{f(\theta)} \right)}{M(q(\theta))\left((1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{\theta+1}{\lambda}\right)} + (\lambda-1) \right\} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)d\theta \left\{ \lambda - 1 - \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\left(\rho_l \frac{1-\left(1-F(\theta)\right)^{N-1}}{\left(1-F(\theta)\right)^{N-1}} + \frac{F(\theta)}{f(\theta)} \right) q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)}{M(q(\theta))\left((1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{\theta+1}{\lambda}\right)} d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)d\theta} \right\}$$
$$\leq \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)d\theta \left\{ \lambda - 1 - \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{q(\theta)NF(\theta)\left(1-(1-F(\theta))^{N-1}\right)}{M(q(\theta))\left(\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta+1\right)} d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)d\theta} \right\}$$
$$\leq (\lambda - 1 - c_l) \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)Nf(\theta)\left(1-\left(1-F(\theta)\right)^{N-1}\right)d\theta$$
$$\leq 0,$$

where the first inequality uses the conditions $1 \leq \lambda \leq \lambda_l \leq 3$ and $0 \leq \rho_l \leq 1$, the second inequality uses the definition of c_l and the last inequality uses the condition that $\lambda \leq \lambda_l \leq (c_l + 1)$.

Therefore, when $1 \leq \lambda < \lambda_l$, we have $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l} < 0$ for any $\rho_l \in [0,1]$, and the optimal reimbursement percentage for the loser worker $\rho_l^* = 0$.

1.7.6 Proof of Corollary 1.4.1

From 1.7.4, we have:

$$\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l}$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left(-\rho_l \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} - \frac{F(\theta)}{f(\theta)} \right) \frac{-\lambda}{V''(q(\theta))} + (\lambda-1)q(\theta) \right\} Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{-\lambda\left(-\rho_l \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} - \frac{F(\theta)}{f(\theta)} \right) V'(q(\theta))}{q(\theta)V''(q(\theta))\left(\lambda(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\right)} + (\lambda-1) \right\} q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ -\frac{\rho_l \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{F(\theta)}{f(\theta)}}{M(q(\theta))\left((1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{\theta+1}{\lambda}\right)} + (\lambda-1) \right\} q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta.$$

First note if $\lim_{q\to 0+} M(q) \neq 0$, then for any $\rho_l < 1$, we have $\lim_{q\to 0+} M(q) = \lim_{\lambda\to+\infty} M(q(\theta))(\forall\theta \in (\underline{\theta}, \overline{\theta})) = \lim_{\theta\to\overline{\theta}} M(q(\theta)) > 0$ (we have $M(q) \geq 0$ throughout the paper from Assumption 1).

Therefore, we have $\frac{\rho_l \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{F(\theta)}{f(\theta)}}{M(q(\theta))\left((1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \frac{\theta+1}{\lambda}\right)} < +\infty$, for any $\theta \in [\underline{\theta}, \overline{\theta}]$, any $\lambda \geq 1$ and any $\rho_l < 1$.

Therefore, there exists a $\lambda_m \in (1, +\infty)$, such that when $\lambda \geq \lambda_m$, we have $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l} > 0$, which implies the buyer should fully reimburse the loser worker (i.e., $\rho_l^* = 1$).

1.7.7 Proof of Proposition 5

Under the flat reimbursement policy, the buyer will reimburse a fixed amount $y_w \leq x$ to the winner worker with type θ if his quality bidding $q(\theta) \geq x$, and a fixed amount $y_l \leq x$ to the loser worker with type θ if his quality bidding $q(\theta) \geq x$. Here, given a certain flat reimbursement policy, we characterize the workers' quality and price bidding in equilibrium (Again we identify the symmetric equilibrium from the first order conditions. Its sufficiency and uniqueness can be checked by the same methods in 1.7.1).

We assume the symmetric bidding strategy in equilibrium is $(q(\cdot), p(\cdot))$ and a corresponding strictly decreasing scoring bidding function $S(\cdot) = s(q(\cdot)) - p(\cdot)$ (Note that in this case, $s(q(\cdot)) = V(q(\cdot))$, when buyer has no scoring rule commitment power).

Therefore, under a given flat reimbursement policy (x, y_w, y_l), given other workers' bidding strategy $(q(\cdot), p(\cdot))$, if a worker i of type θ_i bids quality q_i and price p_i

(the corresponding scoring bid $S_i = s(q_i) - p_i$), he can earn interim expected utility:

$$\pi^{fr}(q_i, p_i) = \big(p_i - \theta_i q_i - (q_i - y_w \cdot 1_{q_i \geq x})\big) P\big(\text{win}|q_i, p_i\big) - \lambda(q_i - y_l \cdot 1_{q_i \geq x})\big(1 - P\big(\text{win}|q_i, p_i\big)\big)$$
$$= \big(p_i - \theta_i q_i - (q_i - y_w \cdot 1_{q_i \geq x})\big) \prod_{j \neq i} prob\big(S_i > S(\theta_j)\big) - \lambda(q_i - y_l \cdot 1_{q_i \geq x})\big(1 - \prod_{j \neq i} prob\big(S_i > S(\theta_j)\big)\big)$$
$$= \big(p_i - \theta_i q_i - (q_i - y_w \cdot 1_{q_i \geq x})\big) \big[1 - F(S^{-1}(S_i))\big]^{N-1} - \lambda(q_i - y_l \cdot 1_{q_i \geq x})\big(1 - \big[1 - F(S^{-1}(S_i))\big]^{N-1}\big)$$

Therefore,

$$\frac{\partial \pi^{fr}(q_i, p_i)}{\partial p_i} = \big(p_i - \theta_i q_i - (q_i - y_w \cdot 1_{q_i \geq x})\big)(N-1)\big[1 - F(S^{-1}(S_i))\big]^{N-2} f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))}$$
$$+ \big[1 - F(S^{-1}(S_i))\big]^{N-1}$$
$$+ \lambda(q_i - y_l \cdot 1_{q_i \geq x})(N-1)\big[1 - F(S^{-1}(S_i))\big]^{N-2} f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))}$$

$$\frac{\partial \pi^{fr}(q_i, p_i)}{\partial q_i} = \big(p_i - \theta_i q_i - (q_i - y_w \cdot 1_{q_i \geq x})\big)(N-1)\big[1 - F(S^{-1}(S_i))\big]^{N-2}(-1) f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))} V'$$
$$- (\theta_i + 1)\big[1 - F(S^{-1}(S_i))\big]^{N-1} - \lambda\big(1 - \big[1 - F(S^{-1}(S_i))\big]^{N-1}\big)$$
$$+ \lambda(q_i - y_l \cdot 1_{q_i \geq x})(N-1)\big[1 - F(S^{-1}(S_i))\big]^{N-2}(-1) f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))} V'(q_i)$$

From the first order conditions $\frac{\partial \pi^{fr}(q_i, p_i)}{\partial p_i} = 0$ and $\frac{\partial \pi^{fr}(q_i, p_i)}{\partial q_i} = 0$, we have:

$$\big[1 - F(S^{-1}(S_i))\big]^{N-1} V'(q_i) - (\theta_i + 1)\big[1 - F(S^{-1}(S_i))\big]^{N-1} - \lambda\big(1 - \big[1 - F(S^{-1}(S_i))\big]^{N-1}\big) = 0$$

Because of the symmetry of the equilibrium, $S^{-1}((S_i)) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore,

$$\big[1 - F(\theta_i)\big]^{N-1} V'(q_i) - \theta_i \big[1 - F(\theta_i)\big]^{N-1} - \big[1 - F(\theta_i)\big]^{N-1} - \lambda\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big) = 0$$

Therefore,

$$q(\theta_i) = V'^{-1}\big(\lambda \frac{1 - (1 - F(\theta_i))^{N-1}}{(1 - F(\theta_i))^{N-1}} + \theta_i + 1\big)$$

Therefore, a worker's quality bid in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

Moreover, from $\frac{\partial \pi^{fr}(q_i, p_i)}{\partial p_i} = 0$, we can get the differential equation:

$$\big(p_i - \theta_i q(\theta_i) - (q(\theta_i) - y_w \cdot 1_{q(\theta_i) \geq x}) + \lambda(q(\theta_i) - y_l \cdot 1_{q(\theta_i) \geq x})\big)(N-1) f(S^{-1}(S_i)) \frac{1}{S'(S^{-1}(S_i))}$$
$$+ \big[1 - F(S^{-1}(S_i))\big] = 0$$

Because $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$ due to symmetry and $S'(\theta_i) = V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)$, we have:

$$\big(p_i - \theta_i q(\theta_i) - (q(\theta_i) - y_w \cdot 1_{q(\theta_i) \geq x}) + \lambda(q(\theta_i) - y_l \cdot 1_{q(\theta_i) \geq x})\big)(N-1) f(\theta_i) \frac{1}{V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)}$$
$$+ \big[1 - F(\theta_i)\big] = 0$$

Solving the above differential equation with the boundary condition, we have:

$$\begin{aligned}
p(\theta_i) &= \theta_i q(\theta_i) + (q(\theta_i) - y_w \cdot 1_{q(\theta_i)\geq x}) + \frac{\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&\quad + \lambda(q(\theta_i) - y_l \cdot 1_{q(\theta_i)\geq x}) \frac{1-(1-F(\theta_i))^{(N-1)}}{(1-F(\theta_i))^{(N-1)}} \\
&= \theta_i q(\theta_i) - (\lambda-1)q(\theta_i) - y_w \cdot 1_{q(\theta_i)\geq x} + \lambda y_l \cdot 1_{q(\theta_i)\geq x} \\
&\quad + \frac{\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \frac{\lambda(q(\theta_i) - y_l \cdot 1_{q(\theta_i)\geq x})}{(1-F(\theta_i))^{(N-1)}}
\end{aligned}$$

We can confirm that $S(\theta_i) = V(q(\theta_i)) - p(\theta_i)$ is strictly decreasing with the private type θ_i, because

$$\frac{dS(\theta_i)}{d\theta_i} = -(N-1)(1-F(\theta_i))^{-N} f(\theta_i) \left(\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) d\tilde{\theta} + \lambda(q(\theta_i) - y_l \cdot 1_{q(\theta_i)\geq x}) \right) < 0$$

where the inequality uses the assumption that $y_l \leq x$ and thus $(q(\theta_i) - y_l \cdot 1_{q(\theta_i)\geq x}) \geq 0$.

Since the equilibrium quality bidding $q(\theta_i) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i + 1\big)$ under the flat reimbursement policy, it is easy to know that a worker's quality bid in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_l and y_w. When a worker of type θ_i bids quality higher than the reimbursement threshold, i.e., $q(\theta_i) \geq x$, his equilibrium price bidding:

$$p(\theta_i) = \theta_i q(\theta_i) - (\lambda-1)q(\theta_i) - y_w + \lambda y_l + \frac{\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \frac{\lambda(q(\theta_i) - y_l)}{(1-F(\theta_i))^{(N-1)}}$$

Therefore, when $q(\theta_i) \geq x$, we have $\frac{\partial p(\theta_i)}{\partial y_w} = -1 < 0$ and $\frac{\partial p(\theta_i)}{\partial y_l} = -\lambda \frac{1-(1-F(\theta_i))^{(N-1)}}{(1-F(\theta_i))^{(N-1)}} \leq 0$ (the equality holds if and only if $\theta_i = \underline{\theta}$ or $\theta_i = \overline{\theta}$). Therefore, a worker's price bid in equilibrium decreases with reimbursement amount y_l and y_w if his quality bidding exceeds the reimbursement threshold x.

Moreover, from the equilibrium quality and price bidding function we derived, we can get worker i's expected utility in equilibrium $\pi^{fr}(q(\theta_i), p(\theta_i)) = \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}$. Obviously, $\pi^{fr}(q(\theta_i), p(\theta_i))$ stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l, because $q(\tilde{\theta})$ stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

1.7.8 Proof of Proposition 6

In 1.7.7, we have characterized workers' equilibrium price and quality bidding, given a flat reimbursement policy. In this subsection, we aim to solve the optimal flat reimbursement policy for the buyer, given workers' best response we characterized in 1.7.7.

Since the equilibrium quality bidding under the flat reimbursement policy $q(\theta) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$ is strictly decreasing with θ, the buyer should not choose the reimbursement threshold $x > q(\underline{\theta})$, otherwise no workers will get the reimbursement and there will be no benefit for the buyer to use the reimbursement policy.

Therefore, the buyer should choose $x \in [q(\overline{\theta}), q(\underline{\theta})]$, in order to maximize her expected utility.

Because the equilibrium quality bidding $q(\theta) = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$ is strictly decreasing with θ, we can find an unique $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ such that $x = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0 + 1\big)$, $\forall x \in [q(\overline{\theta}), q(\underline{\theta})]$.

Using a flat reimbursement policy $(x \in [q(\overline{\theta}), q(\underline{\theta})], y_w, y_l)$, given workers' best response, the buyer's expected utility should be:

$$
\begin{aligned}
& EU_b^{fr}(x, y_w, y_l) \\
=& \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} - N\int_{\underline{\theta}}^{\overline{\theta}} 1_{q(\theta)\geq x}\cdot y_l(1-(1-F(\theta))^{N-1})dF(\theta) - N\int_{\underline{\theta}}^{\overline{\theta}} 1_{q(\theta)\geq x}\cdot y_w(1-F(\theta))^{N-1} \\
=& \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} - N\int_{\underline{\theta}}^{\theta_0} y_w(1-F(\theta))^{N-1}dF(\theta) - N\int_{\underline{\theta}}^{\theta_0} y_l(1-(1-F(\theta))^{N-1})dF(\theta) \\
=& \int_{\theta_0}^{\overline{\theta}}\Big\{V(q(\theta)) - \theta q(\theta) + (\lambda-1)q(\theta) - \frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \frac{\lambda q(\theta)}{(1-F(\theta))^{(N-1)}}\Big\}Nf(\theta)\big(1-F(\theta) \\
&+ \int_{\underline{\theta}}^{\theta_0}\Big\{V(q(\theta)) - \theta q(\theta) + (\lambda-1)q(\theta) + y_w - \lambda y_l - \frac{\int_{\theta}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \\
&- \frac{\lambda(q(\theta)-y_l)}{(1-F(\theta))^{(N-1)}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&- \int_{\underline{\theta}}^{\theta_0}\Big\{y_l\frac{(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta - \int_{\underline{\theta}}^{\theta_0} y_w Nf(\theta)(1-F(\theta))^{N-1}d\theta \\
=& \int_{\underline{\theta}}^{\overline{\theta}}\Big\{V(q(\theta)) - \theta q(\theta) + (\lambda-1)q(\theta) - \frac{\int_{\theta}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \frac{\lambda q(\theta)}{(1-F(\theta))^{(N-1)}}\Big\}Nf(\theta)\big(1-F(\theta \\
&+ \int_{\underline{\theta}}^{\theta_0}(\lambda-1)y_l Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta
\end{aligned}
$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$.

We have $\frac{\partial EU_b^{fr}(x,y_w,y_l)}{\partial y_w} = 0$, $\frac{\partial EU_b^{fr}(x,y_w,y_l)}{\partial y_l} = \int_{\underline{\theta}}^{\theta_0}(\lambda-1)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \geq 0$. Therefore, The buyer's expected utility stays the same with respect to any reimbursement amount to the winning worker y_w. Moreover, the buyer should always choose the highest possible y_l, i.e., $y_l = x = V'^{-1}\big(\lambda\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0 + 1\big)$ to maximize his expected utility. Considering this, the buyer can choose θ_0 to maximizes her expected utility, based on the following maximization problem:

$$
\max_{\theta_0\in[\underline{\theta},\overline{\theta}]} \int_{\underline{\theta}}^{\theta_0}(\lambda-1)V'^{-1}\big(\lambda\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0 + 1\big)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta
$$

Since $x = V'^{-1}\big(\lambda\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0 + 1\big)$ is a strictly decreasing function of θ_0, the above maximization problem is equivalent to:

$$
\max_{x\in[0,q(\underline{\theta})]} \int_{\underline{\theta}}^{\overline{\theta}}(\lambda-1)1_{q(\theta)\geq x}\cdot xNf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta
$$

where $q(\theta) = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$.

Therefore, the buyer should choose the optimal reimbursement amount to the loser workers $y_l^* = x^*$, and the optimal quality reimbursement threshold $x^* = \text{argmax}_{x\in[0,q(\underline{\theta})]} \int_{\underline{\theta}}^{\overline{\theta}} (\lambda-1)1_{q(\theta)\geq x}\cdot xNf(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta$, where $q(\theta) = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta+1\big)$ is the equilibrium quality bidding.

1.7.9 Proof of Proposition 7

Proof of Proposition 7 part (1)

According to Corollary 1.4.1, if the degree of loss aversion is low enough, i.e., $1 \leq \lambda < \lambda_l$, then the buyer should never reimburse loser worker, i.e., $\rho_l^* = 0$, when using the percentage reimbursement policy. On the other hand, from 1.7.8 , when $\lambda > 1$, the buyer should always choose a positive reimbursement amount under the optimized threshold, and thus get the expected utility higher than that when there is zero reimbursement amount.

Therefore, when $1 < \lambda < \lambda_l$, the flat reimbursement policy can always bring higher expected utility to the buyer, since buyer's expected utility are the same when reimbursing zero percentage under the percentage reimbursement policy or reimbursing zero amount under the flat reimbursement policy.

Proof of Proposition 7 part (2)

According to Corollary 1.4.1, if $\lim_{q\to 0+} M(q) \neq 0$, there exists a $\lambda_m \in (1,+\infty)$, such that when $\lambda \geq \lambda_m$, the buyer should fully reimburse the loser worker (i.e., $\rho_l^* = 100\%$), using the percentage reimbursement policy. From 1.7.4, we can know when $\lambda \geq \lambda_m$, we have $EU_b^{pr}(\rho_w = 0, \rho_l = 1) > EU_b^{pr}(\rho_w = 0, \rho_l = 0)$, since $\rho_l^* = 100\%$ and it is strictly better for the buyer to increase the reimbursement percentage to the losing workers when $\lambda \geq \lambda_m$. In addition, from 1.7.4, we have:

$$EU_b^{pr}(\rho_w = 0, \rho_l = 1) = \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ V(q(\theta)) - q(\theta) - \theta q(\theta) - \frac{F(\theta)}{f(\theta)} q(\theta) - \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} q(\theta) \Big\} Nf(\theta)(1-F(\theta))^{N-1} d\theta$$

where the equilibrium quality bidding when $\rho_l^* = 100\%$ is $q(\theta) = V'^{-1}(\theta+1)$, which is independent of loss aversion.

Moreover, according to Proposition 6, if the buyer uses the optimal flat reimbursement policy, her expected utility:

$$EU_b^{fr}(x = x^*, y_w = 0, y_l = x^*) = \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ V(q(\theta)) - \theta q(\theta) - q(\theta) - \frac{F(\theta)}{f(\theta)} q(\theta) - \lambda q(\theta) \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} \Big\} Nf(\theta)(1-F(\theta))^{N-1} d\theta + \int_{\underline{\theta}}^{\theta_0^*} (\lambda-1) q(\theta_0^*) Nf(\theta)\big(1-(1-F(\theta))^{N-1}\big) d\theta$$

where $q(\theta) = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1\big)$, $x^* = q(\theta_0^*)$ and $\theta_0^* = \text{argmax}_{\theta_0\in[\underline{\theta},\overline{\theta}]} \int_{\underline{\theta}}^{\theta_0} (\lambda - 1)V'^{-1}\big(\lambda\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0 + 1\big)Nf(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta$.

First note if $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$, we have $0 < \lim_{q\to 0+} M(q) = \lim_{\lambda\to+\infty} M(q(\theta))(\forall\theta \in (\underline{\theta},\overline{\theta})) = \lim_{\theta\to\overline{\theta}} M(q(\theta)) < 1$ (we have $M(q) \geq 0$ throughout the paper from Assumption 1).

Therefore, if $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$, for equilibrium quality bidding $q(\theta)$ $(\forall\theta \in (\underline{\theta},\overline{\theta}))$ under the flat reimbursement policy, we have:

$$\begin{aligned}
\lim_{\lambda\to+\infty} \lambda q(\theta) &= \lim_{\lambda\to+\infty} \frac{\lambda}{\frac{1}{q(\theta)}} = \lim_{\lambda\to+\infty} \frac{q(\theta)^2 V''(q(\theta))}{\frac{-\left(1-(1-F(\theta))^{N-1}\right)}{(1-F(\theta))^{N-1}}} = \lim_{\lambda\to+\infty} \frac{\lambda q(\theta)(-1)V''(q(\theta))q(\theta)}{\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}} \\
&= \lim_{\lambda\to+\infty} \frac{\lambda q(\theta)(-1)V''(q(\theta))q(\theta)}{\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1} = \lim_{\lambda\to+\infty} \frac{\lambda q(\theta)(-1)V''(q(\theta))q(\theta)}{V'(q(\theta))} \\
&= \lim_{\lambda\to+\infty} \lambda q(\theta) \lim_{\lambda\to+\infty} M(q(\theta)) = \lim_{q\to 0+} M(q) \lim_{\lambda\to+\infty} \lambda q(\theta) \\
&= (\lim_{q\to 0+} M(q))^2 \lim_{\lambda\to+\infty} \lambda q(\theta) = (\lim_{q\to 0+} M(q))^3 \lim_{\lambda\to+\infty} \lambda q(\theta) \\
&= ... = (\lim_{q\to 0+} M(q))^{N_\lambda} \lim_{\lambda\to+\infty} \lambda q(\theta) = \lim_{\lambda\to+\infty} (\lim_{q\to 0+} M(q))^{N_\lambda} \lambda q(\theta) = 0,
\end{aligned}$$

where the second equality follows from the L'Hospital's rule and the fact that $\frac{dq(\theta)}{d\lambda} = \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}V''(q(\theta))}$, the fifth equality follows from the fact that $V'(q(\theta)) = \lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1$, the sixth equality uses the definition that $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$, the seventh uses the fact that $\lim_{\lambda\to+\infty} M(q(\theta)) = \lim_{q\to 0+} M(q)$. For the eighth to the last equality, we iteratively use $\lim_{\lambda\to+\infty} \lambda q(\theta) = \lim_{q\to 0+} M(q) \cdot \lim_{\lambda\to+\infty} \lambda q(\theta)$ for N_λ large number of times (e.g., $N_\lambda = [\lambda] + 1, \forall\lambda > 1$), the last equality uses the condition that $0 < \lim_{q\to 0+} M(q) < 1$.

Therefore, if $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$, we have $\lim_{\lambda\to+\infty} q(\theta) = 0$ and $\lim_{\lambda\to+\infty} \lambda q(\theta) = 0, \forall\theta \in (\underline{\theta},\overline{\theta})$, and thus $\lim_{\lambda\to+\infty} EU_b^{fr}(x = x^*, y_w = 0, y_l = x^*) = V(0)$.

On the one hand, if $\lim_{q\to 0+} M(q) \neq 0$, there exists a $\lambda_m \in (1,+\infty)$, such that when $\lambda \geq \lambda_m$, the buyer's expected utility under the optimal percentage reimbursement policy is $EU_b^{pr}(\rho_w = 0, \rho_l = 1)$. On the other hand, if $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$, we have $\lim_{\lambda\to+\infty} EU_b^{fr}(x = x^*, y_w = 0, y_l = x^*) = V(0)$.

Therefore, if $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$, we have $\lim_{\lambda\to+\infty} \left(EU_b^{pr}(\rho_w = \rho_w^*, \rho_l = \rho_l^*) - EU_b^{fr}(x = x^*, y_w = y_w^*, y_l = x^*)\right) = EU_b^{pr}(\rho_w = 0, \rho_l = 1) - V(0) > EU_b^{pr}(\rho_w = 0, \rho_l = 0) - V(0) > 0$ (the last inequality follows from the fact that the workers' equilibrium score bidding function $S(\theta) = U(q(\theta), p(\theta))$ strictly decreases with type θ, with the lowest score $S(\overline{\theta}) = V(0)$). In other words, if $\lim_{q\to 0+} M(q) \neq 0$ and $\lim_{q\to 0+} M(q) < 1$, there exists a $\lambda_s \in (1,+\infty)$, such that when $\lambda \geq \lambda_s$, the percentage reimbursement policy brings higher expected utility to the buyer than the flat reimbursement policy does.

1.7.10 Procurement auction when buyer has scoring rule commitment power

In this section, we consider the service procurement auction when the buyer has the scoring rule commitment power. In particular, prior to all workers submit their bidding, the buyer can announce and commit to a scoring rule that is different from her own utility function. After all workers completed their bidding, the buyer selects the winning worker whose offer (q, p) achieves the highest score. Each worker, upon winning, provides the service according to the proposal and receives the offered price.

In general, we assume the scoring rule is $S(q, p)$ under which the procurement auction has an unique symmetric equilibrium. Many scoring rules can satisfy this assumption, including the naive scoring rule $U(q, p)$ and the optimal scoring rule we designed.

In the following section, we first discuss the design of the optimal scoring which can bring the highest expected utility for the buyer, among all possible scoring rule $S(q, p)$, then for the auction in which the buyer commits to the optimal scoring rule (we refer to this auction as OSR auction), we study the effect of loss aversion behavior on all parties' expected utility.

Optimal scoring rule (OSR)

The following proposition characterizes the optimal scoring rule (OSR). Among all possible scoring rules belonging to the category of $S(q, p)$, the buyer can obtaining the highest expected utility under the designed optimal scoring rule (OSR).

Proposition 15 *(1) We can design the optimal scoring rule as buyer's utility function with an adjustment. In particular, $S^*(q, p) = U(q, p) - \Delta(q)$ is the optimal scoring rule which can bring the highest expected utility for the buyer among all possible scoring rules. The adjustment*

$$\Delta(q) = \begin{cases} \int_0^q \frac{F(q^{*-1}(s))}{f(q^{*-1}(s))} ds & q \in [q^*(\overline{\theta}), q^*(\underline{\theta})] \\ +\infty & q \notin [q^*(\overline{\theta}), q^*(\underline{\theta})] \end{cases}$$

where $q^{*-1}(\cdot)$ is the inverse of $q^*(\cdot)$ and the equilibrium quality bidding $q^*(\theta) = V'^{-1}(\lambda \frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} + \theta + 1 + \frac{F(\theta)}{f(\theta)})$.

(2) The adjustment $\Delta(q)$ decreases with loss aversion.

See Proof of Proposition 15 in 1.7.11.

From part (1) of Proposition 15, in the OSR auction the buyer should announce and commit to a scoring rule that understates the value of quality, so as to limit the informational rents collected by the low-cost workers. This intuition is consistent with the prior winner-pay multi-dimensional auction design research (Che, 1993; Asker and Cantillon, 2010).

From part (2) of Proposition 15, as the quality bids decrease with loss aversion in both the OSR auction and the auction without buyer's scoring rule commitment power. Therefore, the adjustment $\Delta(q)$ inducing the quality bid difference in these two auctions decreases, as the degree of loss aversion increases.

How loss aversion impacts buyer's expected utility in the OSR auction?

Proposition 16 *In equilibrium of the OSR auction, the expected utility of both the buyer and workers decrease with loss aversion.*

See proof of Proposition 16 in 1.7.12

Similar to (1) in proposition 2, worker expected utility decreases with loss aversion in the OSR auction as loss aversion incurs additional cost for the worker. Moreover, compared with the auction without buyer's scoring rule commitment power, the OSR auction has a lower quality bidding in equilibrium, as the buyer in the OSR auction understate the quality value in the scoring rule[16]. When quality bidding is lower, the "quality value" effect of loss aversion on the buyer expected utility is higher, which means the effect of loss aversion on buyer's expected utility is more harmful. Therefore, loss aversion always decreases with buyer expected utility in the OSR auction.

1.7.11 Proof of Proposition 1.7.10

Proof of Proposition 15 part (1)

We identify the optimal scoring rule based on two steps. In the first step, under any possible scoring rule under which there exists a unique symmetric equilibrium for the procurement auction, we can always pin down the equilibrium price bidding $p(\cdot)$ as a function of the equilibrium quality bidding $q(\cdot)$, using the Envelope theorem. Then we can find the optimal quality bidding function, maximizing the buyer's expected utility which only depends on the quality bidding function. In the second step, we can design the optimal scoring rule under which the equilibrium quality bidding is the optimal quality bidding function we identified in the first step.

Step 1: Identify the optimal quality bidding function

Let $(q(\cdot), p(\cdot))$ be a symmetric equilibrium under a scoring rule which can lead to a unique symmetric equilibrium for the procurement auction, and the corresponding scoring function be $S(\cdot)$. Given other workers' bidding strategy $((q(\cdot), p(\cdot)))$, a type θ_i worker i maximizes his expected profit by choosing q_i and p_i (we denote the corresponding score he get is S_i under the scoring rule):

$$\begin{aligned}\pi(\theta_i) &= \max_{q_i,p_i} (p_i - \theta_i q_i - q_i)\prod_{j\neq i} prob(S_i > S(\theta_j)) - \lambda q_i(1 - \prod_{j\neq i} prob(S_i > S(\theta_j)))\\ &= \max_{q_i,p_i} (p_i - \theta_i q_i - q_i)\left[1 - F(S^{-1}(S_i))\right]^{N-1} - \lambda q_i(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1})\end{aligned}$$

According to the Envelope theorem and the symmetry of the equilibrium:

$$\frac{d\pi(\theta_i)}{d\theta_i} = -q(\theta_i)\big(1 - F(\theta_i)\big)^{N-1}$$

With the boundary condition $\pi(\overline{\theta}) = 0$,

$$\pi(\theta_i) = \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1 - F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}$$

[16] Note $q^*(\theta) = V'^{-1}(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1 + \frac{F(\theta)}{f(\theta)}) < q(\theta) = V'^{-1}(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1)$, $\forall\theta \in (\underline{\theta}, \overline{\theta})$.

On the other hand,

$$\pi(\theta_i) = \big(p(\theta_i) - \theta_i q(\theta_i) - q(\theta_i)\big)\big[1 - F(\theta_i)\big]^{N-1} - \lambda q(\theta_i)\big(1 - \big[1 - F(\theta_i)\big]^{N-1}\big)$$

Therefore,

$$p(\theta_i) = \theta_i q(\theta_i) - (\lambda - 1)q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1 - F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}}{(1 - F(\theta_i))^{(N-1)}} + \frac{\lambda q(\theta_i)}{(1 - F(\theta_i))^{(N-1)}}$$

Note that we have pinned down the equilibrium price bidding $p(\cdot)$ as a function of the equilibrium quality bidding $q(\cdot)$, Therefore, the buyer's expected utility under the scoring rule:

$$\begin{aligned} EU_b &= \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{V(q(\theta)) - \theta q(\theta) + (\lambda - 1)q(\theta) \\ &\quad - \frac{\int_{\theta}^{\overline{\theta}} (1 - F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) d\tilde{\theta}}{(1 - F(\theta))^{(N-1)}} - \frac{\lambda q(\theta)}{(1 - F(\theta))^{(N-1)}}\Big\} N f(\theta)\big(1 - F(\theta)\big)^{N-1} d\theta \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{V(q(\theta)) - \theta q(\theta) + (\lambda - 1)q(\theta) - \frac{F(\theta)}{f(\theta)} q(\theta) - \frac{\lambda q(\theta)}{(1 - F(\theta))^{(N-1)}}\Big\} N f(\theta)\big(1 - F(\theta)\big)^{N-1} d\theta \end{aligned}$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$ and the last equality follows from integration by parts, with the fact that $q(\overline{\theta}) = 0$[17].

Maximizing $\Big\{V(q(\theta)) - \theta q(\theta) + (\lambda - 1)q(\theta) - \frac{F(\theta)}{f(\theta)} q(\theta) - \frac{\lambda q(\theta)}{(1-F(\theta))^{(N-1)}}\Big\}$ point-wise, we can get that the optimal quality bidding function which can maximize the buyer's expected utility EU_b is $q^*(\theta) = V'^{-1}\big(\lambda \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1 + \frac{F(\theta)}{f(\theta)}\big)$. Note that we have $q^*(\overline{\theta}) = 0$ and $q^{*\prime}(\theta) < 0$ from Assumption 1 and Assumption 2 in the paper.

Step 2: Design the optimal scoring rule to induce the optimal quality bidding. We can design the optimal scoring rule as: $S^*(q, p) = V(q) - \Delta q - p = s^*(q) - p$ (let $s^*(q) \equiv V(q) - \Delta q$), where

$$\Delta q = \begin{cases} \int_k^q \frac{F(q^{*-1}(s))}{f(q^{*-1}(s))} ds & q \in [q^*(\overline{\theta}), q^*(\underline{\theta})] \\ +\infty & q \notin [q^*(\overline{\theta}), q^*(\underline{\theta})] \end{cases}$$

k can be any real number (without loss of generality, we set $k = 0$ here), and $q^{*-1}(\cdot)$ is the inverse of $q^*(\cdot)$.

In order to prove that $S^*(q, p)$ is the optimal scoring rule, we only need to prove that the equilibrium quality bidding under the designed scoring rule $S^*(q, p)$ is $q^*(\cdot)$. Below we solve the equilibrium quality bidding under the optimal scoring rule $S^*(q, p)$ from the first order conditions. Its sufficiency and uniqueness can be checked by the same methods in 1.7.1.

Let's assume the symmetric equilibrium bidding strategy under the optimal scoring rule $S^*(q, p)$ is $(q(\cdot), p(\cdot))$ and a corresponding strictly decreasing scoring bidding function $S^*(\cdot) = s^*(q(\cdot)) - p(\cdot)$. Given other workers' bidding strategy $(q(\cdot), p(\cdot))$,

[17] Note a type $\overline{\theta}$ worker always loses the auction in a symmetric equilibrium, he must bid $q(\overline{\theta}) = 0$ to avoid any losses.

if a worker i of type θ_i bids quality q_i and price p_i (the corresponding scoring bid $S_i = s^*(q_i) - p_i$), based on (1.1), he can earn interim expected utility $\pi_i(q_i, p_i)$:

$$\begin{aligned}
\pi_i(q_i, p_i) &= \big(p_i - \theta_i q_i - q_i\big)\prod_{j\neq i} prob\big(S_i > S^*(\theta_j)\big) - \lambda q_i\big(1 - \prod_{j\neq i} prob\big(S_i > S^*(\theta_j)\big)\big) \\
&= \big(p_i - \theta_i q_i - q_i\big)\big[1 - F(S^{*-1}(S_i))\big]^{N-1} - \lambda q_i\Big(1 - \big[1 - F(S^{*-1}(S_i))\big]^{N-1}\Big)
\end{aligned}$$

Given θ_i and other workers' bidding strategy, the worker i maximizes the expected utility $\pi_i(q_i, p_i)$ by choosing p_i and q_i. We calculate the derivative of $\pi_i(q_i, p_i)$ with respect to p_i and q_i:

$$\begin{aligned}
\frac{\partial \pi_i(q_i, p_i)}{\partial p_i} =& \big(p_i - \theta_i q_i - q_i\big)(N-1)\big[1 - F(S^{*-1}(S_i))\big]^{N-2} f(S^{*-1}(S_i))\frac{1}{S^{*\prime}(S^{*-1}(S_i))} + \big[1 - F(S^{*-1}(S_i)) \\
&+ \lambda q_i(N-1)\big[1 - F(S^{*-1}(S_i))\big]^{N-2} f(S^{*-1}(S_i))\frac{1}{S^{*\prime}(S^{*-1}(S_i))}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_i(q_i, p_i)}{\partial q_i} =& \big(p_i - \theta_i q_i - q_i\big)(N-1)\big[1 - F(S^{*-1}(S_i))\big]^{N-2}(-1) f(S^{*-1}(S_i))\frac{1}{S^{*\prime}(S^{*-1}(S_i))} s^{*\prime}(q_i) \\
&- (\theta_i + 1)\big[1 - F(S^{*-1}(S_i))\big]^{N-1} \\
&- \lambda\big(1 - \big[1 - F(S^{*-1}(S_i))\big]^{N-1}\big) \\
&+ \lambda q_i(N-1)\big[1 - F(S^{*-1}(S_i))\big]^{N-2}(-1) f(S^{*-1}(S_i))\frac{1}{S^{*\prime}(S^{*-1}(S_i))} s^{*\prime}(q_i)
\end{aligned}$$

From the first order conditions $\frac{\partial \pi_i(q_i,p_i)}{\partial p_i} = 0$ and $\frac{\partial \pi_i(q_i,p_i)}{\partial q_i} = 0$, we have:

$$\big[1 - F(S^{*-1}(S_i))\big]^{N-1} s^{*\prime}(q_i) - (\theta_i + 1)\big[1 - F(S^{*-1}(S_i))\big]^{N-1} - \lambda\big(1 - \big[1 - F(S^{*-1}(S_i))\big]^{N-1}\big) = 0$$

Therefore, in order to prove the equilibrium quality bidding under the optimal scoring rule $S^*(q, p)$ is $q^*(\cdot)$, we only need to prove the following equation has the unique root $q = q^*(\theta)$, for any $\theta \in [\underline{\theta}, \overline{\theta}]$:

$$(1 - F(\theta))^{N-1} s^{*\prime}(q) - (\theta + 1)\big[1 - F(\theta)\big]^{N-1} - \lambda\big(1 - \big[1 - F(\theta)\big]^{N-1}\big) = 0$$

First, we can check that $q = q^*(\theta)$ is the root of the quation by the definition of $s^*(q)$ and $q^*(\theta)$.

Second, we can check the function $s^{*\prime}(q)$ is strictly decreasing with q for $q \in [q^*(\overline{\theta}), q^*(\underline{\theta})]$, then the root $q = q^*(\theta)$ must be unique, for any $\theta \in [\underline{\theta}, \overline{\theta}]$. In fact, we have:

$$\begin{aligned}
s^{*\prime\prime}(q) = \frac{d^2(V(q) - \Delta q)}{dq^2} &= V''(q) - \frac{d\frac{F}{f}}{d\theta}\frac{d\theta}{dq} \\
&= V''(q) - \frac{d\frac{F}{f}}{d\theta}\frac{V''(q)}{\frac{d(\frac{F}{f} + \lambda\frac{1-(1-F)^{N-1}}{(1-F)^{N-1}} + \theta + 1)}{d\theta}} \\
&= V''(q)\Bigg(1 - \frac{\frac{d\frac{F}{f}}{d\theta}}{\frac{d\frac{F}{f}}{d\theta} + \frac{d(\lambda\frac{1-(1-F)^{N-1}}{(1-F)^{N-1}} + \theta + 1)}{d\theta}}\Bigg) < 0
\end{aligned}$$

where $\theta \equiv q^{*-1}(q)$ for any $q \in [q^*(\overline{\theta}), q^*(\underline{\theta})]$, the third equality follows from the definition of $q^*(\cdot)$ and the last inequality follows from the facts that $V''(\cdot) < 0$, $\frac{d(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta+1)}{d\theta} > 0$ and $\frac{d\frac{F(\theta)}{f(\theta)}}{d\theta} \geq 0$ (Assumption 2 in the paper).

Proof of Proposition 15 part (2)

From 1.7.11, we know that the optimal scoring rule $S^*(q,p) = V(q) - \Delta q - p = s^*(q) - p$ ($s^*(q) \equiv V(q) - \Delta q$), where

$$\Delta q = \begin{cases} \int_0^q \frac{F(q^{*-1}(s))}{f(q^{*-1}(s))}ds & q \in [q^*(\overline{\theta}), q^*(\underline{\theta})] \\ +\infty & q \notin [q^*(\overline{\theta}), q^*(\underline{\theta})] \end{cases}$$

and $q^{*-1}(\cdot)$ is the inverse of $q^*(\cdot)$.

Because

$$\begin{aligned} \frac{\partial \frac{F(q^{*-1}(s))}{f(q^{*-1}(s))}}{\partial \lambda} &= \frac{\partial \frac{F}{f}}{\partial \theta}\frac{\partial \theta}{\partial \lambda} \\ &= \frac{\partial \frac{F}{f}}{\partial \theta}\frac{(-1)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}}{\lambda\frac{d\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}}{d\theta} + \frac{d(\theta+1+\frac{F(\theta)}{f(\theta)})}{d\theta}} < 0 \end{aligned}$$

where $\theta \equiv q^{*-1}(s)$ for any $s \in [q^*(\overline{\theta}), q^*(\underline{\theta})]$, the second equality follows from the definition of $q^*(\cdot)$, the inequality follows from the facts that $\frac{d\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}}{d\theta} > 0$ and $\frac{d\frac{F(\theta)}{f(\theta)}}{d\theta} \geq 0$ (Assumption 2 in the paper).

Therefore, when $q \in [q^*(\overline{\theta}), q^*(\underline{\theta})]$, $\frac{\partial \Delta(q)}{\partial \lambda} = \int_0^q \frac{\partial \frac{F(q^{*-1}(s))}{f(q^{*-1}(s))}}{\partial \lambda}ds \leq 0$ the equality holds if and only if $q = 0$.

1.7.12 Proof of Proposition 16

From the optimal quality bidding function $q^*(\theta_i)$ and the price bidding function we derived in 1.7.11, we can get worker i's expected utility in equilibrium $\pi(q^*(\theta_i), p(\theta_i)) = \int_{\theta_i}^{\overline{\theta}} q^*(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta}$. Therefore, we have:

$$\frac{d\pi(q^*(\theta_i), p(\theta_i))}{d\lambda} = \int_{\theta_i}^{\overline{\theta}} \frac{\left(1-\left(1-F(\tilde{\theta})\right)^{N-1}\right)}{V''(q^*(\tilde{\theta}))}d\tilde{\theta} \leq 0,$$

where the equality holds if and only if $\theta_i = \overline{\theta}$, the equality uses the fact that $\frac{\partial q^*(\theta_i)}{\partial \lambda} = \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}V''(q^*(\theta_i))}$ and the inequality follows from the fact that $V''(\cdot) < 0$. Therefore, a worker's expected utility decreases with the degree of loss aversion.

From 1.7.11, the buyer's expected utility under the optimal scoring rule:

$$EU_b = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ V(q^*(\theta)) - \theta q^*(\theta) + (\lambda-1)q^*(\theta) - \frac{F(\theta)}{f(\theta)}q^*(\theta) - \frac{\lambda q^*(\theta)}{(1-F(\theta))^{(N-1)}} \right\} Nf(\theta)\left(1-F(\theta)\right)^{N-1}d\theta,$$

where $q^*(\theta) = V'^{-1}(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1 + \frac{F(\theta)}{f(\theta)})$.

Using the fact that $V'(q^*(\theta)) = \lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta + 1 + \frac{F(\theta)}{f(\theta)}$, we have:

$$\frac{dEU_b}{d\lambda}\bigg|_{q(\theta)=q^*(\theta)} = \int_{\underline{\theta}}^{\overline{\theta}} \left\{q^*(\theta) - \frac{q^*(\theta)}{(1-F(\theta))^{N-1}}\right\} Nf(\theta)(1-F(\theta))^{N-1} d\theta < 0.$$

Therefore, in equilibrium of the OSR auction, the expected utility of both the buyer and workers decrease with loss aversion.

1.7.13 Procurement auction when private type is on quality spending

Proof of Proposition 8

In the procurement auction when private type is on quality spending, below we solve the workers' quality and price bidding in equilibrium (Here we identify the symmetric equilibrium from the first order conditions. Its sufficiency and uniqueness can be checked by the same methods in 1.7.1).

Given other workers' bidding strategy, if the worker with quality producing efficiency type θ_i bid (p_i, q_i), his interim expected profit is:

$$\begin{aligned}\pi(q_i, p_i) &= (p_i - \theta_i q_i)\prod_{j\neq i} prob(S_i > S(\theta_j)) - \lambda\theta_i q_i\left(1 - \prod_{j\neq i} prob(S_i > S(\theta_j))\right)\\ &= (p_i - \theta_i q_i)\left[1 - F(S^{-1}(S_i))\right]^{N-1} - \lambda\theta_i q_i\left(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1}\right)\end{aligned}$$

Given θ_i and other worker's bidding strategy, the worker i maximizes the expected profit by choosing p_i and q_i. We calculate the derivative of $\pi(q_i, p_i)$ with respect to p_i and q_i:

$$\begin{aligned}\frac{\partial\pi(q_i,p_i)}{\partial p_i} =&(p_i - \theta_i q_i)(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))} + \left[1 - F(S^{-1}(S_i))\right]^{N-1}\\ &+ \lambda\theta_i q_i(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))},\end{aligned}$$

$$\begin{aligned}\frac{\partial\pi(q_i,p_i)}{\partial q_i} =& (p_i - \theta_i q_i)(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)\\ &- \theta_i\left[1 - F(S^{-1}(S_i))\right]^{N-1}\\ &- \lambda\theta_i\left(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1}\right)\\ &+ \lambda\theta_i q_i(N-1)\left[1 - F(S^{-1}(S_i))\right]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i).\end{aligned}$$

From the first order condition $\frac{\partial\pi(q_i,p_i)}{\partial p_i} = 0$ and $\frac{\partial\pi(q_i,p_i)}{\partial q_i} = 0$, we have:

$$(1 - F(S^{-1}(S_i)))^{N-1}V'(q_i) - \theta_i\left[1 - F(S^{-1}(S_i))\right]^{N-1} - \lambda\theta_i\left(1 - \left[1 - F(S^{-1}(S_i))\right]^{N-1}\right) = 0.$$

Because of the symmetry of the equilibrium, $S^{-1}(S_i)) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore:

$$(1 - F(\theta_i))^{N-1}V'(q_i) - \theta_i(1 - F(\theta_i))^{N-1} - \lambda\theta_i\left(1 - \left[1 - F(\theta_i)\right]^{N-1}\right) = 0.$$

Let $G(q_i) \equiv \left[1-F(\theta_i)\right]^{N-1}V'(q_i) - \theta_i\left[1-F(\theta_i)\right]^{N-1} - \lambda\theta_i\left(1-\left[1-F(\theta_i)\right]^{N-1}\right)$. Since $V''(q_i)<0$, we can get: $\frac{dG(q_i)}{dq_i} = V''(q_i)\left[1-F(\theta_i)\right]^{N-1} < 0$.

In addition, since $\lim_{q\to 0} V'(q) = +\infty$ and $\lim_{q\to+\infty} V'(q) = 0$, we have $\lim_{q_i\to+\infty} G(q_i) < 0$, and $\lim_{q_i\to 0} G(q_i) > 0$. According to the intermediate value theorem, there is a unique q_i^*, such that $G(q_i^*) = 0$. Specifically, we can denote:

$$q(\theta_i) = q_i^* = V'^{-1}\left(\lambda\theta_i \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i\right)$$

where $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$. Note we can easily check that $q(\overline{\theta}) = V'^{-1}(+\infty) = 0$ and $q'(\theta_i) < 0$ from assumption 1 in our paper. Taking derivatives for $q(\theta_i)$ with respect to λ , we have:

$$\frac{dq(\theta_i)}{d\lambda} = \frac{\theta_i\left(1-\left(1-F(\theta_i)\right)^{N-1}\right)}{\left(1-F(\theta_i)\right)^{N-1} V''(q(\theta_i))} \leq 0$$

where the equality holds if and only if $\theta_i = \overline{\theta}$ or $\theta_i = \underline{\theta}$. Therefore, a worker's quality bid in equilibrium always decreases with the degree of loss aversion.

Moreover, from $\frac{\partial \pi(q_i,p_i)}{\partial p_i} = 0$, we can get the derivative equation:

$$\left(p_i - \theta_i q_i + \lambda\theta_i q_i\right)(N-1)f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))} + \left[1-F(S^{-1}(S_i))\right] = 0$$

Because of the symmetry of the equilibrium, $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore,

$$\left(p_i - \theta_i q(\theta_i) + \lambda\theta_i q(\theta_i)\right)(N-1)f(\theta_i)\frac{1}{S'(\theta_i)} + \left[1-F(\theta_i)\right] = 0$$

Because $S'(\theta_i) = V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)$. Therefore,

$$\left(p_i - \theta_i q(\theta_i) + \lambda\theta_i q(\theta_i)\right)(N-1)f(\theta_i)\frac{1}{V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)} + \left[1-F(\theta_i)\right] = 0$$

We can solve the above differential equation with boundary condition and get:

$$p(\theta_i) = -(\lambda-1)\theta_i q(\theta_i) - (\lambda-1)\frac{\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \lambda\frac{\theta_i q(\theta_i) + \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}$$

We have:

$$\begin{aligned}\frac{dS(\theta_i)}{d\theta_i} &= -(N-1)(1-F(\theta_i))^{-N}f(\theta_i)\Bigg(-(\lambda-1)\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta} + \lambda\left(\theta_i q(\theta_i) + \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}\right)\Bigg) \\ &= -(N-1)(1-F(\theta_i))^{-N}f(\theta_i)\Bigg(\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta} + \lambda\theta_i q(\theta_i) \\ &\quad + \lambda\Big(\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta} - \int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}\Big)\Bigg) < 0\end{aligned}$$

We can confirm that $S(\theta)$ is strictly decreasing with the private type θ.

From the equilibrium price bidding function $p(\theta_i)$, we have:

$$\frac{dp(\theta_i)}{d\lambda} = \big(-(\lambda-1)\theta_i + \lambda\frac{\theta_i}{(1-F(\theta_i))^{(N-1)}}\big)\frac{\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{\big[1-F(\theta_i)\big]^{N-1}V''(q(\theta_i))}$$
$$+\frac{\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{(1-F(\theta_i))^{(N-1)}}q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}}\tilde{\theta}\frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q^*(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}$$
$$+\frac{\int_{\theta_i}^{\overline{\theta}}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \frac{\int_{\theta_i}^{\overline{\theta}}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\lambda\frac{\tilde{\theta}\big(1-\big[1-F(\tilde{\theta})\big]^{N-1}\big)}{\big[1-F(\tilde{\theta})\big]^{N-1}V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}$$
$$=V'(q(\theta_i))\frac{\theta_i\big(1-(1-F(\theta_i))^{N-1}\big)}{(1-F(\theta_i))^{N-1}V''(q(\theta_i))} + \frac{\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{(1-F(\theta_i))^{(N-1)}}q(\theta_i)$$
$$+\frac{\int_{\theta_i}^{\overline{\theta}}\big(\tilde{\theta}+\lambda\frac{\tilde{\theta}\big(1-\big[1-F(\tilde{\theta})\big]^{N-1}\big)}{\big[1-F(\tilde{\theta})\big]^{N-1}}\big)\frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \frac{\int_{\theta_i}^{\overline{\theta}}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}$$
$$=\frac{\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{(1-F(\theta_i))^{(N-1)}}q(\theta_i)\big(1+\frac{V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))}\big)$$
$$+\frac{\int_{\theta_i}^{\overline{\theta}}V'(q(\tilde{\theta}))\frac{1-(1-F(\tilde{\theta}))^{(N-1)}}{V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \frac{\int_{\theta_i}^{\overline{\theta}}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}$$
$$=\frac{\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{(1-F(\theta_i))^{(N-1)}}q(\theta_i)\big(1-\frac{1}{M(q(\theta_i))}\big) + \frac{\int_{\theta_i}^{\overline{\theta}}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)q(\tilde{\theta})\big(1-\frac{1}{M(q(\tilde{\theta}))}\big)d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}$$

where the second the third equality uses the fact that $\big(1-F(\theta_i)\big)^{N-1}V'(q(\theta_i)) - \theta\big(1-F(\theta_i)\big)^{N-1} - \lambda\theta_i\big(1-\big(1-F(\theta_i)\big)^{N-1}\big) = 0$, and the last equality follows from the definition that $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$. Moreover, it is easy to check that the equilibrium quality bidding function $q(\theta_i) \in [0, V'^{-1}(\underline{\theta})], \forall\theta_i \in [\underline{\theta}, \overline{\theta}]$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, we can know that $\frac{\partial p(\theta_i)}{\partial\lambda} \leq 0$, the equality holds if and only if $\theta_i = \overline{\theta}$ or $M(q) = 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$. Therefore, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, a worker's price bidding in equilibrium decreases with loss aversion.

Proof of Proposition 9

From the equilibrium quality and price bidding function $q(\theta_i)$ and $p(\theta_i)$ we derived in 1.7.13, we can get worker i's expected utility in equilibrium $\pi(q(\theta_i), p(\theta_i)) = \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta}$. Therefore, we have:

$$\frac{d\pi(q(\theta_i), p(\theta_i))}{d\lambda} = \int_{\theta_i}^{\overline{\theta}}\frac{\tilde{\theta}\big(1-\big(1-F(\tilde{\theta})\big)^{N-1}\big)}{V''(q(\tilde{\theta}))}d\tilde{\theta} \leq 0,$$

where the equality holds if and only if $\theta_i = \overline{\theta}$, the equality uses the fact that $\frac{\partial q(\theta_i)}{\partial\lambda} = \frac{\theta_i\big(1-(1-F(\theta_i))^{N-1}\big)}{(1-F(\theta_i))^{N-1}V''(q(\theta_i))}$ and the inequality follows from the fact that $V''(\cdot) < 0$. Therefore, a worker's expected utility decreases with the degree of loss aversion.

On the other hand, from the equilibrium quality and price bidding function $q(\cdot)$ and $p(\cdot)$ we derived in 1.7.13, we have the buyer's expected utility in equilibrium:

$$\begin{aligned}
EU_b &= \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{V(q(\theta)) + (\lambda-1)\theta q(\theta) \\
&+ (\lambda-1)\frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \lambda\frac{\theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}}\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{V(q(\theta)) + (\lambda-1)\theta q(\theta) + (\lambda-1)\frac{F(\theta)}{f(\theta)}q(\theta) - \lambda\frac{\frac{F(\theta)}{f(\theta)}+\theta}{\big(1-F(\theta)\big)^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta
\end{aligned}$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$ and the last equality follows from integration by parts.

Therefore, we have:

$$\begin{aligned}
\frac{dEU_b}{d\lambda} &= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{\big(V'(q(\theta)) + (\lambda-1)\theta + (\lambda-1)\frac{F(\theta)}{f(\theta)} - \lambda\frac{\frac{F(\theta)}{f(\theta)}+\theta}{\big(1-F(\theta)\big)^{N-1}}\big)\frac{dq(\theta)}{d\lambda} \\
&+ \theta q(\theta) + \frac{F(\theta)}{f(\theta)}q(\theta) - \frac{\frac{F(\theta)}{f(\theta)}+\theta}{\big(1-F(\theta)\big)^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ -\big(1+\lambda\frac{\big(1-\big(1-F(\theta)\big)^{N-1}\big)}{\big(1-F(\theta)\big)^{N-1}}\big)\frac{F(\theta)}{f(\theta)}\frac{dq(\theta)}{d\lambda} \\
&- \big(\theta + \frac{F(\theta)}{f(\theta)}\big)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ -\big(1+\lambda\frac{\big(1-\big(1-F(\theta)\big)^{N-1}\big)}{\big(1-F(\theta)\big)^{N-1}}\big)\frac{F(\theta)}{f(\theta)}\frac{\theta\big(1-\big[1-F(\theta)\big]^{N-1}\big)}{\big[1-F(\theta)\big]^{N-1}V''(q(\theta))} \\
&- \big(\theta + \frac{F(\theta)}{f(\theta)}\big)\frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}}q(\theta)\Big\} Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ -\big(1+\lambda\frac{\big(1-\big(1-F(\theta)\big)^{N-1}\big)}{\big(1-F(\theta)\big)^{N-1}}\big)\frac{F(\theta)}{f(\theta)}\frac{\theta}{V''(q(\theta))} \\
&- \big(\theta + \frac{F(\theta)}{f(\theta)}\big)q(\theta)\Big\} Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ -\frac{F(\theta)}{f(\theta)}\frac{V'(q(\theta))}{q(\theta)V''(q(\theta))} - \big(\theta + \frac{F(\theta)}{f(\theta)}\big)\Big\} q(\theta)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \frac{F(\theta)}{f(\theta)}\frac{1}{M(q(\theta))} - \big(\theta + \frac{F(\theta)}{f(\theta)}\big)\Big\} q(\theta)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \frac{1}{M(q(\theta))} - \frac{\theta f(\theta)+F(\theta)}{F(\theta)}\Big\} q(\theta)NF(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta
\end{aligned}$$

where the second equality uses the fact that $V'(q(\theta)) = \lambda\theta\frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} + \theta$, the third equality uses the fact that $\frac{dq(\theta)}{d\lambda} = \frac{\theta(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1}V''(q(\theta))}$, the fifth equality uses the fact that $V'(q(\theta)) = \lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta$, the sixth equality follows from the fact that $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$.

Let's define:

$$m_0 \equiv \sup_{\lambda\geq 1} \frac{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)N\theta f(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta} \in (0,1),$$

where $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is the equilibrium quality bidding. It is also easy to check that the equilibrium quality bidding function $q(\theta_i) \in [0, V'^{-1}(\underline{\theta})], \forall\theta_i \in [\underline{\theta}, \overline{\theta}]$.

If $M(q) > m_0$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, then:

$$\begin{aligned}
\frac{dEU_b}{d\lambda} =& \int_{\underline{\theta}}^{\overline{\theta}} \left\{\frac{1}{M(q(\theta))} - \frac{\theta f(\theta)+F(\theta)}{F(\theta)}\right\} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta \\
<& \int_{\underline{\theta}}^{\overline{\theta}} \left\{\frac{1}{m_0} - \frac{\theta f(\theta)+F(\theta)}{F(\theta)}\right\} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta \\
=& \frac{1}{m_0}\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta \\
&- \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)N\theta f(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta\right) \\
=& \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)N\theta f(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta\right) \\
&\times \left(\frac{1}{m_0}\frac{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)NF(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)N\theta f(\theta)\big(1-(1-F(\theta))^{N-1}\big)d\theta} - 1\right) \\
\leq& 0
\end{aligned}$$

where the first inequality uses the fact that $M(q) > m_0$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, the last inequality follows from the definition of m_0. Therefore, if $M(q) > m_0$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, we have $\frac{dEU_b}{d\lambda} < 0$, i.e., buyer's expected utility decreases with loss aversion.

Proof of Proposition 10

Given any reimbursement percentage ρ_w and ρ_l, we solve the workers' quality and price bidding in equilibrium (Here we identify the symmetric equilibrium from the first order conditions. Its sufficiency and uniqueness can be checked by the same methods in 1.7.1).

We assume the symmetric bidding strategy in equilibrium is $(q(\cdot), p(\cdot))$ and a corresponding strictly decreasing scoring bidding function $S(\cdot) = s(q(\cdot)) - p(\cdot)$ (Note that in this case, $s(q(\cdot)) = V(q(\cdot))$).Given other workers' bidding strategy $(q(\cdot), p(\cdot))$, if a worker i of type θ_i bids quality q_i and price p_i (the corresponding scoring bid $S_i = s(q_i) - p_i$), under the percentage reimbursement policy (ρ_w, ρ_l) he can earn interim expected utility $\pi^{pr}(q_i, p_i)$:

$$\pi^{pr}(q_i,p_i) = \big(p_i - \theta_i(1-\rho_w)q_i\big)\prod_{j\neq i} prob\big(S_i > S(\theta_j)\big) - \lambda\theta_i(1-\rho_l)q_i\big(1-\prod_{j\neq i} prob\big(S_i > S(\theta_j)\big)\big)$$
$$= \big(p_i - \theta_i(1-\rho_w)q_i\big)\big[1-F(S^{-1}(S_i))\big]^{N-1} - \lambda\theta_i(1-\rho_l)q_i\Big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\Big)$$

Given θ_i and other worker's bidding strategy, the worker i maximizes the expected profit by choosing p_i and q_i. We calculate the derivative of $\pi^{pr}(q_i,p_i)$ with respect to p_i and q_i:

$$\frac{\partial\pi^{pr}(q_i,p_i)}{\partial p_i} = \big(p_i - \theta_i(1-\rho_w)q_i\big)(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}$$
$$+ \big[1-F(S^{-1}(S_i))\big]^{N-1} + \lambda\theta_i(1-\rho_l)q_i(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}$$

$$\frac{\partial\pi^{pr}(q_i,p_i)}{\partial q_i} = \big(p_i - \theta_i(1-\rho_w)q_i\big)(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)$$
$$- \theta_i(1-\rho_w)\big[1-F(S^{-1}(S_i))\big]^{N-1}$$
$$- \lambda\theta_i(1-\rho_l)\Big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\Big)$$
$$+ \lambda\theta_i(1-\rho_l)q_i(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)$$

From the first order condition $\frac{\partial\pi^{pr}(q_i,p_i)}{\partial p_i} = 0$ and $\frac{\partial\pi^{pr}(q_i,p_i)}{\partial q_i} = 0$, we have

$$\big[1-F(S^{-1}(S_i))\big]^{N-1}V'(q_i) - \theta_i(1-\rho_w)\big[1-F(S^{-1}(S_i))\big]^{N-1}$$
$$- \lambda\theta_i(1-\rho_l)\big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\big) = 0$$

Because of the symmetry of the equilibrium, $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore:

From the first order condition $\frac{\partial\pi^{pr}(q_i,p_i)}{\partial p_i} = 0$ and $\frac{\partial\pi^{pr}(q_i,p_i)}{\partial q_i} = 0$, we have:

$$\big[1-F(\theta_i)\big]^{N-1}V'(q_i) - \theta_i(1-\rho_w)\big[1-F(\theta_i)\big]^{N-1} - \lambda\theta_i(1-\rho_l)\big(1-\big[1-F(\theta_i)\big]^{N-1}\big) = 0$$

Let $G(q_i) \equiv \big[1-F(\theta_i)\big]^{N-1}V'(q_i) - \theta_i(1-\rho_w)\big[1-F(\theta_i)\big]^{N-1} - \lambda\theta_i(1-\rho_l)\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)$. Since $V''(q_i) < 0$, we can get: $\frac{dG(q_i)}{dq_i} = V''(q_i)\big[1-F(\theta_i)\big]^{N-1} < 0$. In addition, since $\lim_{q\to 0} V'(q) = +\infty$ and $\lim_{q\to+\infty} V'(q) = 0$, we have $\lim_{q\to+\infty} G(q_i) < 0$, and $\lim_{q\to 0} G(q_i) > 0$. According to the intermediate value theorem, there is an unique q_i^*, such that $G(q_i) = 0$. Specifically, we can denote

$$q_i^* = q(\theta_i) = V'^{-1}\big(\lambda\theta_i(1-\rho_l)\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)\big)$$

where $V'^{-1}(\cdot)$ is the inverse of $V'(\cdot)$. Taking derivative with respect to ρ_w and ρ_l for the equilibrium price bidding function $q(\theta_i)$, we have:

$$\frac{dq(\theta_i)}{d\rho_w} = \frac{-\theta_i}{V''(q(\theta_i))} > 0,$$
$$\frac{dq(\theta_i)}{d\rho_l} = \frac{-\lambda\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}V''(q(\theta_i))} \geq 0,$$

where the equality (of the last inequality) holds if and only if $\theta_i = \underline{\theta}$ or $\theta_i = \overline{\theta}$, both inequality follows from the fact that $V''(\cdot) < 0$. Therefore, a worker's quality bidding in equilibrium increases with reimbursement percentage ρ_w and ρ_l.

Moreover, from $\frac{\partial \pi^{pr}(q_i,p_i)}{\partial p_i} = 0$, we can get the derivative equation:

$$\big(p_i - \theta_i(1-\rho_w)q(\theta_i) + \lambda\theta_i(1-\rho_l)q(\theta_i)\big)(N-1)f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))} + \big[1-F(S^{-1}(S_i))\big] = 0$$

Because of the symmetry of the equilibrium, $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore,

$$\big(p_i - \theta_i(1-\rho_w)q(\theta_i) + \lambda\theta_i(1-\rho_l)q(\theta_i)\big)(N-1)f(\theta_i)\frac{1}{S'(\theta_i)} + \big[1-F(\theta_i)\big] = 0$$

Because $S'(\theta_i) = V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)$. Therefore,

$$\big(p_i - \theta_i(1-\rho_w)q(\theta_i) + \lambda\theta_i(1-\rho_l)q(\theta_i)\big)(N-1)f(\theta_i)\frac{1}{V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)} + \big[1-F(\theta_i)\big] = 0$$

Solve this differential equation with the boundary condition, we can get:

$$p(\theta_i) = -\big((1-\rho_l)\lambda - (1-\rho_w)\big)\theta_i q(\theta_i) - \big((1-\rho_l)\lambda - (1-\rho_w)\big)\frac{\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde\theta))^{(N-1)}q(\tilde\theta)d\tilde\theta}{(1-F(\theta_i))^{(N-1)}}$$
$$+\lambda(1-\rho_l)\frac{\theta_i q(\theta_i) + \int_{\theta_i}^{\overline{\theta}} q(\tilde\theta)d\tilde\theta}{(1-F(\theta_i))^{(N-1)}}$$

We have:

$$\begin{aligned}\frac{dS(\theta_i)}{d\theta_i} =& -(N-1)(1-F(\theta_i))^{-N}f(\theta_i)\Bigg((1-\rho_w)\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde\theta))^{(N-1)}q(\tilde\theta)d\tilde\theta \\ &+\lambda(1-\rho_l)\big(\theta_i q(\theta_i) + \int_{\theta_i}^{\overline{\theta}}\big(1-(1-F(\tilde\theta))^{(N-1)}\big)q(\tilde\theta)d\tilde\theta\big)\Bigg) \\ <&0\end{aligned}$$

We can confirm that $S(\theta_i)$ strictly decreases with the private type θ_i.

Taking derivative with respect to ρ_w for the equilibrium price bidding function $p(\theta_i)$, we have:

$$\begin{aligned}
\frac{\partial p(\theta_i)}{\partial \rho_w} &= \big(-\lambda(1-\rho_l)\theta_i + (1-\rho_w)\theta_i + \frac{\lambda(1-\rho_l)\theta_i}{(1-F(\theta_i))^{(N-1)}} \big) \frac{dq(\theta_i)}{d\rho_w} \\
&- \big((1-\rho_l)\lambda - (1-\rho_w)\big) \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} \frac{dq(\tilde{\theta})}{d\rho_w} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} - \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} - \theta_i q(\theta_i) \\
&+ \lambda(1-\rho_l) \frac{\int_{\theta_i}^{\overline{\theta}} \frac{dq(\tilde{\theta})}{d\rho_w} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= \frac{-\theta_i V'(q(\theta_i))}{V''(q(\theta_i))} - \theta_i q(\theta_i) + (1-\rho_w) \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} \frac{-\tilde{\theta}}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&+ \lambda(1-\rho_l) \frac{\int_{\theta_i}^{\overline{\theta}} \frac{-\tilde{\theta}\left(1-(1-F(\tilde{\theta}))^{(N-1)}\right)}{V''(q(\tilde{\theta}))} d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} - \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= \frac{-\theta_i V'(q(\theta_i))}{V''(q(\theta_i))} - \theta_i q(\theta_i) \\
&+ \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) \big(\frac{-1}{V''(q(\tilde{\theta}))q(\tilde{\theta})} (\lambda(1-\rho_l)\tilde{\theta} \frac{1-(1-F(\tilde{\theta}))^{N-1}}{(1-F(\tilde{\theta}))^{N-1}} + \tilde{\theta}(1-\rho_w)) - 1 \big) d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= \frac{-\theta_i V'(q(\theta_i))}{V''(q(\theta_i))} - \theta_i q(\theta_i) + \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) \big(\frac{-V'(q(\tilde{\theta}))}{V''(q(\tilde{\theta}))q(\tilde{\theta})} - 1 \big) d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&= \theta_i q(\theta_i) \big(\frac{1}{M(q(\theta_i))} - 1 \big) + \frac{\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta}) \big(\frac{1}{M(q(\tilde{\theta}))} - 1 \big) d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}
\end{aligned}$$

where the second equality uses the facts that $\frac{dq(\theta_i)}{d\rho_w} = \frac{-\theta_i}{V''(q(\theta_i))}$ and $V'(q(\theta_i)) = \lambda(1-\rho_l)\theta_i \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)$, the fourth equality follows from the fact that $V'(q(\theta_i)) = \lambda(1-\rho_l)\theta_i \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)$, the last equality uses the definition that $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$.

Therefore, if $M(q) \leq 1$ for all $q \geq 0$, we can know that $\frac{\partial p(\theta_i)}{\partial \rho_w} \geq 0$, and thus a worker's price bid in equilibrium increases with reimbursement percentage ρ_w.

Taking derivative with respect to ρ_l for the equilibrium price bidding function $p(\theta_i)$, we have:

$$
\begin{aligned}
\frac{\partial p(\theta_i)}{\partial \rho_l} =& \big(-\lambda(1-\rho_l)\theta_i + (1-\rho_w)\theta_i + \frac{\lambda(1-\rho_l)\theta_i}{(1-F(\theta_i))^{(N-1)}}\big)\frac{dq(\theta_i)}{d\rho_l} + \lambda\rho_l\theta_i q(\theta_i) \\
&- \big((1-\rho_l)\lambda - (1-\rho_w)\big)\frac{\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}\frac{dq(\tilde{\theta})}{d\rho_l}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&+ \lambda\rho_l\frac{\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta} - \theta_i q(\theta_i) - \int_{\theta_i}^{\overline{\theta}}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \lambda(1-\rho_l)\frac{\int_{\theta_i}^{\overline{\theta}}\frac{dq(\tilde{\theta})}{d\rho_l}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
=& \frac{\lambda\theta_i q(\theta_i)\big(1-(1-F(\theta_i))^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}}\big(\frac{-V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))} - \rho_l\big) \\
&- \big((1-\rho_l)\lambda - (1-\rho_w)\big)\frac{\int_{\theta_i}^{\overline{\theta}}\lambda\tilde{\theta}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\frac{-1}{V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
&+ \lambda\rho_l\frac{\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta} - \int_{\theta_i}^{\overline{\theta}}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \lambda(1-\rho_l)\frac{\int_{\theta_i}^{\overline{\theta}}\frac{-\lambda\tilde{\theta}(1-(1-F(\tilde{\theta}))^{N-1})}{(1-F(\tilde{\theta}))^{N-1}V''(q(\tilde{\theta}))}d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
=& \frac{\lambda\theta_i q(\theta_i)\big(1-(1-F(\theta_i))^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}}\big(\frac{-V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))} - \rho_l\big) \\
&+ \frac{\int_{\theta_i}^{\overline{\theta}}\lambda\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{-1}{V''(q(\tilde{\theta}))}\big(\lambda(1-\rho_l)\tilde{\theta}\frac{1-(1-F(\tilde{\theta}))^{N-1}}{(1-F(\tilde{\theta}))^{N-1}} + \tilde{\theta}(1-\rho_w)\big) - \rho_l q(\tilde{\theta})\big)d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
=& \frac{\lambda\theta_i q(\theta_i)\big(1-(1-F(\theta_i))^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}}\big(\frac{-V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))} - \rho_l\big) \\
&+ \frac{\int_{\theta_i}^{\overline{\theta}}\lambda\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{-V'(q(\tilde{\theta}))}{V''(q(\tilde{\theta}))} - \rho_l q(\tilde{\theta})\big)d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
=& \frac{\lambda\theta_i q(\theta_i)\big(1-(1-F(\theta_i))^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}}\big(\frac{-V'(q(\theta_i))}{q(\theta_i)V''(q(\theta_i))} - \rho_l\big) \\
&+ \frac{\int_{\theta_i}^{\overline{\theta}}\lambda q(\tilde{\theta})\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{-V'(q(\tilde{\theta}))}{q(\tilde{\theta})V''(q(\tilde{\theta}))} - \rho_l\big)d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} \\
=& \frac{\lambda\theta_i q(\theta_i)\big(1-(1-F(\theta_i))^{N-1}\big)}{\big(1-F(\theta_i)\big)^{N-1}}\big(\frac{1}{M(q(\theta_i))} - \rho_l\big) + \frac{\int_{\theta_i}^{\overline{\theta}}\lambda q(\tilde{\theta})\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{1}{M(q(\tilde{\theta}))} - \rho_l\big)d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}},
\end{aligned}
$$

where the second equality uses the facts that $\frac{dq(\theta_i)}{d\rho_l} = \frac{-\lambda\theta_i(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}V''(q(\theta_i))}$ and $V'(q(\theta_i)) = \lambda(1-\rho_l)\theta_i\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)$, the fourth equality follows from the fact that $V'(q(\theta_i)) = \lambda(1-\rho_l)\theta_i\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)$, the last equality uses the definition that $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$.

Therefore, if $M(q) \leq 1$ for all $q \geq 0$, we can know that $\frac{\partial p(\theta_i)}{\partial \rho_l} \geq 0$, and thus a worker's price bid in equilibrium increases with reimbursement percentage ρ_l.

Moreover, from the equilibrium quality and price bidding function $q(\theta_i)$ and $p(\theta_i)$ we derived, we can get worker i's expected utility in equilibrium:

$$\pi^{pr}(q(\theta_i), p(\theta_i)) = (1-\rho_w)\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} + \lambda(1-\rho_l)\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-(1-F(\tilde{\theta}))^{(N-1)})d\tilde{\theta}$$

Taking derivative with respect to ρ_w for $\pi^{pr}(q(\theta_i), p(\theta_i))$, we have:

$$\begin{aligned}
&\frac{d\pi^{pr}(q(\theta_i), p(\theta_i))}{d\rho_w} \\
=& -\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} + (1-\rho_w)\int_{\theta_i}^{\overline{\theta}} \frac{-\tilde{\theta}}{V''(q(\tilde{\theta}))}(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} \\
&+ \lambda(1-\rho_l)\int_{\theta_i}^{\overline{\theta}} \frac{-\tilde{\theta}}{V''(q(\tilde{\theta}))}(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} \\
=& \int_{\theta_i}^{\overline{\theta}} \frac{-\tilde{\theta}}{V''(q(\tilde{\theta}))}(1-F(\tilde{\theta}))^{(N-1)}\big(\lambda(1-\rho_l)\tilde{\theta}\frac{1-(1-F(\tilde{\theta}))^{N-1}}{(1-F(\tilde{\theta}))^{N-1}} + \tilde{\theta}(1-\rho_w)\big)d\tilde{\theta} - \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} \\
=& \int_{\theta_i}^{\overline{\theta}} \frac{-\tilde{\theta}V'(q(\tilde{\theta}))}{V''(q(\tilde{\theta}))}(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} - \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} \\
=& \int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})\big(\frac{1}{M(q(\tilde{\theta}))} - 1\big)d\tilde{\theta} \geq 0
\end{aligned}$$

where the first equality uses the fact that $\frac{dq(\theta_i)}{d\rho_w} = \frac{-\theta_i}{V''(q(\theta_i))}$, the third equality uses the fact that $V'(q(\theta_i)) = \lambda(1-\rho_l)\theta_i\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)$, the last equality follows from the definition $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$, the last inequality follows from the assumption that $M(q) \leq 1$ for all $q \geq 0$.

Taking derivative with respect to ρ_l for $\pi^{pr}(q(\theta_i), p(\theta_i))$, we have:

$$\begin{aligned}
&\frac{d\pi^{pr}(q(\theta_i), p(\theta_i))}{d\rho_l} \\
=&(1-\rho_w)\int_{\theta_i}^{\overline{\theta}} \frac{-\lambda\tilde{\theta}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)}{(1-F(\tilde{\theta}))^{(N-1)}V''(q(\tilde{\theta}))}(1-F(\tilde{\theta}))^{(N-1)}d\tilde{\theta} - \lambda\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)d\tilde{\theta} \\
&+ \lambda(1-\rho_l)\int_{\theta_i}^{\overline{\theta}} \frac{-\lambda\tilde{\theta}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)}{(1-F(\tilde{\theta}))^{(N-1)}V''(q(\tilde{\theta}))}\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)d\tilde{\theta} \\
=& \int_{\theta_i}^{\overline{\theta}} \lambda\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{-1}{V''(q(\tilde{\theta}))}\big(\lambda(1-\rho_l)\tilde{\theta}\frac{1-(1-F(\tilde{\theta}))^{N-1}}{(1-F(\tilde{\theta}))^{N-1}} + \tilde{\theta}(1-\rho_w)\big) - q(\tilde{\theta})\big)d\tilde{\theta} \\
=& \int_{\theta_i}^{\overline{\theta}} \lambda q(\tilde{\theta})\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{-V'(q(\tilde{\theta}))}{q(\tilde{\theta})V''(q(\tilde{\theta}))} - 1\big)d\tilde{\theta} \\
=& \int_{\theta_i}^{\overline{\theta}} \lambda q(\tilde{\theta})\big(1-(1-F(\tilde{\theta}))^{(N-1)}\big)\big(\frac{1}{M(q(\tilde{\theta}))} - 1\big)d\tilde{\theta} \geq 0
\end{aligned}$$

where the first equality uses the fact that $\frac{dq(\theta_i)}{d\rho_l} = \frac{-\lambda\theta_i(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1}V''(q(\theta_i))}$, the third equality uses the fact that $V'(q(\theta_i)) = \lambda(1-\rho_l)\theta_i\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i(1-\rho_w)$, the last equality follows from the definition $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$,the last inequality follows from the assumption that $M(q) \leq 1$ for all $q \geq 0$.

Therefore, if $M(q) \leq 1$ for all $q \geq 0$, a worker's expected utility in equilibrium increases with the reimbursement percentage ρ_w and ρ_l.

Proof of Proposition 11 part (1)

Given workers' best response (equilibrium price and quality bidding we characterized in 1.7.13), the buyer's expected utility when she reimburses the winning worker for ρ_w percentage of quality cost and the losing workers for ρ_l percentage of quality cost:

$$
\begin{aligned}
&EU_b^{pr}(\rho_w,\rho_l) = \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} \\
&- N\int_{\underline{\theta}}^{\overline{\theta}} \rho_w \theta q(\theta)\big(1-F(\theta)\big)^{N-1} dF(\theta) - N\int_{\underline{\theta}}^{\overline{\theta}} \rho_l \theta q(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big) dF(\theta) \\
= &\int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{V(q(\theta)) + \big((1-\rho_l)\lambda - (1-\rho_w)\big)\theta q(\theta) \\
&+ \big((1-\rho_l)\lambda - (1-\rho_w)\big)\frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \lambda(1-\rho_l)\frac{\theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} \\
&- \rho_l \theta q(\theta)\frac{1-(1-F(\theta))^{(N-1)}}{(1-F(\theta))^{(N-1)}} - \rho_w \theta q(\theta)\Bigg\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
= &\int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{V(q(\theta)) + \big((1-\rho_l)\lambda - (1-\rho_w)\big)\theta q(\theta) + \big((1-\rho_l)\lambda - (1-\rho_w)\big)\frac{F(\theta)}{f(\theta)} q(\theta) \\
&- \lambda(1-\rho_l)\frac{\frac{F(\theta)}{f(\theta)} + \theta}{\big(1-F(\theta)\big)^{N-1}} q(\theta) - \rho_l \theta q(\theta)\frac{1-(1-F(\theta))^{(N-1)}}{(1-F(\theta))^{(N-1)}} - \rho_w \theta q(\theta)\Bigg\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta
\end{aligned}
$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$ and the last equality follows from integration by parts.

Taking derivative with respect to ρ_w for the buyer's expected utility $EU_b^{pr}(\rho_w, \rho_l)$, we have:

$$\frac{dEU_b^{pr}(\rho_w, \rho_l)}{d\rho_w}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{ \Bigg(V'(q(\theta)) + ((1-\rho_l)\lambda - (1-\rho_w))(\theta + \frac{F(\theta)}{f(\theta)}) - \lambda \frac{\frac{F(\theta)}{f(\theta)} + \theta}{(1-F(\theta))^{N-1}}$$

$$- \rho_w \theta - \rho_l \theta \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} \Bigg) \frac{dq(\theta)}{d\rho_w} + \frac{F(\theta)}{f(\theta)} q(\theta) \Bigg\} N f(\theta) (1-F(\theta))^{N-1} d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{ \Bigg(-\rho_w \theta^2 - \frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} \rho_l \theta^2 - \frac{\lambda \theta^2 \rho_l}{(1-F(\theta)^{N-1}} - \frac{F(\theta)}{f(\theta)} \frac{\lambda \rho_l \theta}{(1-F(\theta))^{N-1}}$$

$$- \frac{F(\theta)}{f(\theta)} \Big(\frac{\lambda(1-\rho_l)\theta}{(1-(1-F(\theta))^{N-1}} + (1-\rho_w)\theta - (1-\rho_l)\lambda\theta \Big) \Bigg) \frac{-1}{V''(q(\theta))q(\theta)}$$

$$+ \frac{F(\theta)}{f(\theta)} \Bigg\} q(\theta) N f(\theta) (1-(1-F(\theta))^{N-1}) d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{ \Bigg(\Big(-\rho_w \theta^2 - \frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} \rho_l \theta^2 - \frac{\lambda \theta^2 \rho_l}{(1-F(\theta)^{N-1}} - \frac{F(\theta)}{f(\theta)} \frac{\lambda \rho_l \theta}{(1-F(\theta))^{N-1}} \Big) \frac{1}{V'(q(\theta))}$$

$$- \frac{F(\theta)}{f(\theta)} \Bigg) \frac{-V'(q(\theta))}{V''(q(\theta))q(\theta)} + \frac{F(\theta)}{f(\theta)} \Bigg\} q(\theta) N f(\theta) (1-(1-F(\theta))^{N-1}) d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Bigg\{ \Bigg(\Big(-\rho_w \theta^2 - \frac{1-(1-F(\theta)^{N-1}}{(1-F(\theta)^{N-1}} \rho_l \theta^2 - \frac{\lambda \theta^2 \rho_l}{(1-F(\theta)^{N-1}} - \frac{F(\theta)}{f(\theta)} \frac{\lambda \rho_l \theta}{(1-F(\theta))^{N-1}} \Big) \frac{1}{V'(q(\theta))}$$

$$- \frac{F(\theta)}{f(\theta)} \Bigg) \frac{1}{M(q(\theta))} + \frac{F(\theta)}{f(\theta)} \Bigg\} q(\theta) N f(\theta) (1-(1-F(\theta))^{N-1}) d\theta$$

where the second and third equality use the facts that $\frac{dq(\theta)}{d\rho_w} = \frac{-\theta}{V''(q(\theta))}$ and $V'(q(\theta)) = \lambda\theta(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta(1-\rho_w)$, the last equality follows from the definition $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$.

Obviously, if $M(q) \leq 1$ for all $q \geq 0$, then $\frac{dEU_b^{pr}(\rho_w,\rho_l)}{d\rho_w} < 0$. the buyer should never reimburse the quality spending of the winning worker, i.e., $\rho_w^* = 0$.

Proof of Proposition 11 part (2)

Based on 1.7.13, when the buyer can choose to reimburse the losing workers with $\rho_l \in [0\%, 100\%]$ percent of their quality cost, and the winning worker with $\rho_w \in [0\%, 100\%]$ percent of their quality cost, then the optimal reimbursement percentage for the winning worker should be $\rho_w^* = 0$ if $M(q) \leq 1$ for all $q \geq 0$.

Therefore, under the condition "$M(q) \leq 1$ for all $q \geq 0$", the buyer should only reimburse the losing workers. In this subsection we consider the optimal reimbursement percentage to the losing workers under the condition that the optimal reimbursement percentage to the winning worker is $\rho_w^* = 0$. Based on the results in 1.7.13, the buyer's expected utility when he commits to reimburses only the losing workers' quality spending with $\rho_l \in [0,1]$ percent should be:

$$EU_b^{pr}(0,\rho_l) = \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ V(q(\theta)) + \big((1-\rho_l)\lambda - 1\big)\theta q(\theta) + \big((1-\rho_l)\lambda - 1\big)\frac{F(\theta)}{f(\theta)} q(\theta) \\ - \lambda(1-\rho_l)\frac{\frac{F(\theta)}{f(\theta)} + \theta}{\big(1-F(\theta)\big)^{N-1}} q(\theta) - \rho_l \theta q(\theta) \frac{1-(1-F(\theta))^{(N-1)}}{(1-F(\theta))^{(N-1)}} \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta$$

Taking derivative with respect to λ for the buyer's expected utility $EU_b^{pr}(0,\rho_l)$, we have:

$$\begin{aligned}
&\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} \\
=& \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \Big(V'(q(\theta)) - (\theta + \frac{F(\theta)}{f(\theta)}) - (\lambda(1-\rho_l)(\theta + \frac{F(\theta)}{f(\theta)}) + \rho_l\theta) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} \Big) \frac{dq(\theta)}{d\lambda} \\
&- (1-\rho_l)(\theta + \frac{F(\theta)}{f(\theta)}) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
=& \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \Big(-\frac{F(\theta)}{f(\theta)} - (\lambda(1-\rho_l)\frac{F(\theta)}{f(\theta)} + \rho_l\theta) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} \Big) \frac{(1-\rho_l)\theta}{V''(q(\theta))} \\
&- (1-\rho_l)(\theta + \frac{F(\theta)}{f(\theta)}) q(\theta) \Big\} N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big) d\theta
\end{aligned}$$

where the second equality uses the facts that $\frac{\partial q(\theta)}{\partial \lambda} = \frac{(1-\rho_l)\theta(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1} V''(q(\theta))}$ and $V'(q(\theta)) = \lambda\theta(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta$.

Moreover, Taking derivative with respect to ρ_l for the buyer's expected utility $EU_b^{pr}(0,\rho_l)$, we have:

$$\begin{aligned}
\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l} =& \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \Big(V'(q(\theta)) - (\theta + \frac{F(\theta)}{f(\theta)}) - (\lambda(1-\rho_l)(\theta + \frac{F(\theta)}{f(\theta)}) + \rho_l\theta) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} \Big) \frac{dq(\theta)}{d\rho_l} \\
&+ \big(\lambda(\theta + \frac{F(\theta)}{f(\theta)}) - \theta\big) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} q(\theta) \Big\} N f(\theta)\big(1-F(\theta)\big)^{N-1} d\theta \\
=& \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ \Big(-\frac{F(\theta)}{f(\theta)} - (\lambda(1-\rho_l)\frac{F(\theta)}{f(\theta)} + \rho_l\theta) \frac{1-\big(1-F(\theta)\big)^{N-1}}{\big(1-F(\theta)\big)^{N-1}} \Big) \frac{-\lambda\theta}{V''(q(\theta))} \\
&+ \big(\lambda(\theta + \frac{F(\theta)}{f(\theta)}) - \theta\big) q(\theta) \Big\} N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big) d\theta \\
=& \frac{\partial EU_{fs}}{\partial \lambda} \frac{-\lambda}{1-\rho_l} - \int_{\underline{\theta}}^{\overline{\theta}} \theta q(\theta) N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big) d\theta \\
=& \frac{-\lambda}{1-\rho_l} \Big(\frac{\partial EU_{fs}}{\partial \lambda} + \frac{1-\rho_l}{\lambda} \int_{\underline{\theta}}^{\overline{\theta}} \theta q(\theta) N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big) d\theta \Big)
\end{aligned}$$

where the second equality follows from the facts that $\frac{dq(\theta_i)}{d\rho_l} = \frac{-\lambda\theta_i(1-(1-F(\theta_i))^{N-1})}{(1-F(\theta_i))^{N-1} V''(q(\theta_i))}$ and $V'(q(\theta)) = \lambda\theta(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta$, the third equality uses the formula about

$\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}$ we derived above.

When $\frac{dEU_b^{pr}(0,\rho_l)}{d\lambda} > -\frac{1-\rho_l}{\lambda}\int_{\underline{\theta}}^{\overline{\theta}} \theta q(\theta) N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \equiv B$ for all $\rho_l \in [0,1]$, we have $\frac{dEU_b^{pr}(0,\rho_l)}{d\rho_l} < 0$ for all $\rho_l \in [0,1]$. Therefore, the optimal reimbursement percentage $\rho_l^* = 0$.

When $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda} < -\frac{1-\rho_l}{\lambda}\int_{\underline{\theta}}^{\overline{\theta}} \theta q(\theta) N f(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \equiv B$, for all $\rho_l \in [0,1]$, we have $\frac{dEU_b^{pr}(0,\rho_l)}{d\rho_l} > 0$ for all $\rho_l \in [0,1]$. Therefore, the optimal reimbursement percentage $\rho_l^* = 1$.

When $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}\Big|_{\rho_l=1} > B|_{\rho_l=1}$, we have $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l}\Big|_{\rho_l=1} < 0$, the buyer should decrease the reimbursement percentage from 100%, which can increase his expected utility. When $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}\Big|_{\rho_l=0} < B|_{\rho_l=0}$, we have $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l}\Big|_{\rho_l=0} < 0$, the buyer should increase the reimbursement percentage from zero percent, which can increase his expected utility. Therefore, the buyer should choose the optimal reimbursement percentage $\rho_l^* \in (0,1)$, where ρ_l^* should satisfy $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l}\Big|_{\rho_l=\rho_l^*} = 0$, or equivalently $\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \lambda}\Big|_{\rho_l=\rho_l^*} = B|_{\rho_l=\rho_l^*}$.

Proof of Proposition 12

.

Let $q^{-1}(\cdot)$ is the inverse of function $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$. Under the following flat reimbursement policy, we would prove the equilibrium quality bidding of a worker with type θ is $q(\theta)$[18].

Under the flat reimbursement policy, the buyer can reimburse a fixed amount $y_w \le q^{-1}(x)x$[19] to the winning worker with type θ if his quality bidding is bigger than or equal to x, and a fixed amount $y_l \le q^{-1}(x)x$ to the losing worker with type θ if his quality bidding is bigger than or equal to x.

Here, given a certain flat reimbursement policy, we characterize the workers' quality and price bidding in equilibrium (Again we identify the symmetric equilibrium from the first order conditions. Its sufficiency and uniqueness can be checked by the same methods in 1.7.1).

We assume the symmetric bidding strategy in equilibrium is $(q(\cdot), p(\cdot))$ and a corresponding strictly decreasing scoring bidding function $S(\cdot) = s(q(\cdot)) - p(\cdot)$ (Note that in this case, $s(q(\cdot)) = V(q(\cdot))$, when buyer has no scoring rule commitment power).

Therefore, under a given flat reimbursement policy (x, y_w, y_l), given other workers' bidding strategy $(q(\cdot), p(\cdot))$, if a worker i of type θ_i bids quality q_i and price p_i (the corresponding scoring bid $S_i = s(q_i) - p_i$), he can earn interim expected utility:

$$\begin{aligned}
\pi^{fr}(q_i,p_i) &= \big(p_i - (\theta_i q_i - y_w\cdot 1_{q_i\ge x})\big)P\big(\text{win}|q_i,p_i\big) - \lambda(\theta_i q_i - y_l\cdot 1_{q_i\ge x})\big(1-P\big(\text{win}|q_i,p_i\big)\big)\\
&= \big(p_i - (\theta_i q_i - y_w\cdot 1_{q_i\ge x})\big)\prod_{j\ne i} prob\big(S_i > S(\theta_j)\big) - \lambda(\theta_i q_i - y_l\cdot 1_{q_i\ge x})\big(1-\prod_{j\ne i} prob\big(S_i > S(\theta_j)\big)\big)\\
&= \big(p_i - (\theta_i q_i - y_w\cdot 1_{q_i\ge x})\big)\big[1-F(S^{-1}(S_i))\big]^{N-1} - \lambda(\theta_i q_i - y_l\cdot 1_{q_i\ge x})\big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\big)
\end{aligned}$$

[18] We can prove later that the equilibrium quality spending $\theta q(\theta)$ strictly decreases with type θ with the assumption $M(q) \le 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$.

[19] As we can prove later in this subsection, we have $q^{-1}(x)x$ strictly increases with x with the assumption that $M(q) \le 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$.

Therefore,

$$\begin{aligned}\frac{\partial \pi^{fr}(q_i,p_i)}{\partial p_i} =& \big(p_i - (\theta_i q_i - y_w \cdot 1_{q_i \geq x})\big)(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))} \\ &+ \big[1-F(S^{-1}(S_i))\big]^{N-1} \\ &+ \lambda(\theta_i q_i - y_l \cdot 1_{q_i \geq x})(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2} f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi^{fr}(q_i,p_i)}{\partial q_i} =& \big(p_i - (\theta_i q_i - y_w \cdot 1_{q_i \geq x})\big)(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i) \\ &- \theta_i\big[1-F(S^{-1}(S_i))\big]^{N-1} - \lambda\theta_i\big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\big) \\ &+ \lambda(\theta_i q_i - y_l \cdot 1_{q_i \geq x})(N-1)\big[1-F(S^{-1}(S_i))\big]^{N-2}(-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))}V'(q_i)\end{aligned}$$

From the first order conditions $\frac{\partial \pi^{fr}(q_i,p_i)}{\partial p_i} = 0$ and $\frac{\partial \pi^{fr}(q_i,p_i)}{\partial q_i} = 0$, we have:

$$\big[1-F(S^{-1}(S_i))\big]^{N-1}V'(q_i) - \theta_i\big[1-F(S^{-1}(S_i))\big]^{N-1} - \lambda\theta_i\big(1-\big[1-F(S^{-1}(S_i))\big]^{N-1}\big) = 0$$

Because of the symmetry of the equilibrium, $S^{-1}((S_i)) = S^{-1}(S(\theta_i)) = \theta_i$. Therefore,

$$\big[1-F(\theta_i)\big]^{N-1}V'(q_i) - \theta_i\big[1-F(\theta_i)\big]^{N-1} - \lambda\theta_i\big(1-\big[1-F(\theta_i)\big]^{N-1}\big) = 0$$

Therefore,

$$q(\theta_i) = V'^{-1}\big(\lambda\theta_i\frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i\big)$$

Moreover, from $\frac{\partial \pi^{fr}(q_i,p_i)}{\partial p_i} = 0$, we can get the differential equation:

$$\begin{aligned}&\big(p_i - (\theta_i q(\theta_i) - y_w \cdot 1_{q(\theta_i) \geq x}) + \lambda(\theta_i q(\theta_i) - y_l \cdot 1_{q(\theta_i) \geq x})\big)(N-1) f(S^{-1}(S_i))\frac{1}{S'(S^{-1}(S_i))} \\ &+ \big[1-F(S^{-1}(S_i))\big] = 0\end{aligned}$$

Because $S^{-1}(S_i) = S^{-1}(S(\theta_i)) = \theta_i$ due to symmetry and $S'(\theta_i) = V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)$, we have:

$$\begin{aligned}&\big(p_i - \theta_i(q(\theta_i) - y_w \cdot 1_{q(\theta_i) \geq x}) + \lambda\theta_i(q(\theta_i) - y_l \cdot 1_{q(\theta_i) \geq x})\big)(N-1) f(\theta_i)\frac{1}{V'(q(\theta_i))q'(\theta_i) - p'(\theta_i)} \\ &+ \big[1-F(\theta_i)\big] = 0\end{aligned}$$

Solving the above differential equation with the boundary condition, we have:

$$\begin{aligned}p(\theta_i) =& (\theta_i q(\theta_i) - y_w \cdot 1_{q(\theta_i) \geq x}) - \lambda(\theta_i q(\theta_i) - y_l \cdot 1_{q(\theta_i) \geq x}) \\ &- (\lambda-1)\frac{\int_{\theta_i}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}} + \lambda\frac{(\theta_i q(\theta_i) - y_l \cdot 1_{q(\theta_i) \geq x}) + \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta_i))^{(N-1)}}\end{aligned}$$

Therefore, when $q(\theta_i) \geq x$, we have $\frac{\partial p(\theta_i)}{\partial y_w} = -1 < 0$ and $\frac{\partial p(\theta_i)}{\partial y_l} = -\lambda\frac{1-(1-F(\theta_i))^{(N-1)}}{(1-F(\theta_i))^{(N-1)}} \leq$

0 (the equality holds if and only if $\theta_i = \underline{\theta}$ or $\theta_i = \overline{\theta}$). Therefore, a worker's price bid in equilibrium decreases with reimbursement amount y_l and y_w if his quality bidding exceeds the reimbursement threshold x.

First, with the assumption $M(q(\theta_i)) \leq 1$ for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ we can check $\theta_i q(\theta_i)$ strictly decreases with θ_i (or equivalently $q^{-1}(x)x$ strictly increases with x since $q(\theta_i) = V'^{-1}\big(\lambda\theta_i \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i\big)$ is a strictly decreasing function of θ_i). In fact, if $M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$, we have:

$$\begin{aligned}
\frac{\partial \theta_i q(\theta_i)}{\partial \theta_i} &= q(\theta_i) + \theta_i \frac{\partial q(\theta_i)}{\partial \theta_i} \\
&= q(\theta_i) + \theta_i \frac{\lambda \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + 1 + \frac{\lambda\theta_i (N-1) f(\theta_i)}{(1-F(\theta_i))^N}}{V''(q(\theta_i))} \\
&= q(\theta_i) + \frac{V'(q(\theta))}{V''(q(\theta_i))} + \frac{\frac{\lambda\theta_i^2 (N-1) f(\theta_i)}{(1-F(\theta_i))^N}}{V''(q(\theta_i))} \\
&= q(\theta_i)\big(1 - \frac{1}{M(q(\theta_i))}\big) + \frac{\frac{\lambda\theta_i^2 (N-1) f(\theta_i)}{(1-F(\theta_i))^N}}{V''(q(\theta_i))} < 0,
\end{aligned}$$

where the third equality uses the fact that $V'(q(\theta_i)) = \lambda\theta_i \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i$, the fourth equality uses the definition $M(q(\theta_i)) = \frac{-q(\theta_i)V''(q(\theta_i))}{V'(q(\theta_i))}$, the inequality uses the assumption that "$M(q) \leq 1$ for all $q \in [0, V'^{-1}(\underline{\theta})]$" and $V''(\cdot) < 0$.

Moreover, we can confirm that $S(\theta_i) = V(q(\theta_i)) - p(\theta_i)$ is strictly decreasing with the private type θ_i, because

$$\begin{aligned}
\frac{dS(\theta_i)}{d\theta_i} = &-(N-1)(1-F(\theta_i))^{-N} f(\theta_i)\Bigg(\lambda\Big(\int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta} - \int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta})d\tilde{\theta}\Big) + \\
&\int_{\theta_i}^{\overline{\theta}} (1-F(\tilde{\theta}))^{(N-1)} q(\tilde{\theta})d\tilde{\theta} + \lambda(\theta_i q(\theta_i) - y_l \cdot 1_{q(\theta_i)\geq x})\Bigg) < 0,
\end{aligned}$$

where the inequality uses the condition that $y_w \leq q^{-1}(x)x$ and $y_l \leq q^{-1}(x)x$, the fact that $\theta_i q(\theta_i)$ strictly decreases with θ_i (or equivalently $q^{-1}(x)x$ strictly increases with x), and thus $(\theta_i q(\theta_i) - y_l \cdot 1_{q(\theta_i)\geq x}) \geq 0, (\theta_i q(\theta_i) - y_w \cdot 1_{q(\theta_i)\geq x}) \geq 0, \forall \theta_i \in [\underline{\theta}, \overline{\theta}]$.

Since the equilibrium quality bidding $q(\theta_i) = V'^{-1}\big(\lambda\theta_i \frac{1-(1-F(\theta_i))^{N-1}}{(1-F(\theta_i))^{N-1}} + \theta_i\big)$ under the flat reimbursement policy, it is easy to know that a worker's quality bid in equilibrium stays the same with respect to any reimbursement threshold x and any reimbursement amount y_l and y_w.

Moreover, from the equilibrium quality and price bidding function we derived, we can get worker i's expected utility in equilibrium $\pi^{fr}(q(\theta_i), p(\theta_i)) = \lambda \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})\big(1 - (1-F(\tilde{\theta}))^{(N-1)}\big)d\tilde{\theta} + \int_{\theta_i}^{\overline{\theta}} q(\tilde{\theta})(1-F(\tilde{\theta}))^{(N-1)} d\tilde{\theta}$. Obviously, $\pi^{fr}(q(\theta_i), p(\theta_i))$ stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l, because $q(\tilde{\theta})$ stays the same with respect to any reimbursement threshold x and any reimbursement amount y_w and y_l.

Proof of Proposition 13

We have characterized workers' equilibrium price and quality bidding, given a flat reimbursement policy. In this following, we aim to solve the optimal flat reimbursement policy for the buyer, given workers' best response we have characterized in 1.7.13.

Since the equilibrium quality bidding under the flat reimbursement policy $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is strictly decreasing with θ, the buyer should not choose the reimbursement threshold $x > q(\underline{\theta})$, otherwise no workers will get the reimbursement and there will be no benefit for the buyer to use the reimbursement policy. Therefore, the buyer should choose $x \in [q(\overline{\theta}), q(\underline{\theta})]$, in order to maximize her expected utility.

Because the equilibrium quality bidding $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is strictly decreasing with θ, we can find an unique $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ such that $x = V'^{-1}\big(\lambda\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0\big), \forall x \in [q(\overline{\theta}), q(\underline{\theta})]$.

Using a flat reimbursement policy $(x \in [q(\overline{\theta}), q(\underline{\theta})], y_w, y_l)$, given workers' best response, the buyer's expected utility should be:

$$
\begin{aligned}
& EU_b^{fr}(x, y_w, y_l) \\
=& \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} - N\int_{\underline{\theta}}^{\overline{\theta}} 1_{q(\theta)\geq x}\cdot y_l(1-(1-F(\theta))^{N-1})dF(\theta) - N\int_{\underline{\theta}}^{\overline{\theta}} 1_{q(\theta)\geq x}\cdot y_w(1-F(\theta))^{N-1}d \\
=& \mathbb{E}\Big\{V(q(\theta_1)) - p(\theta_1)\Big\} - N\int_{\underline{\theta}}^{\theta_0} y_w(1-F(\theta))^{N-1}dF(\theta) - N\int_{\underline{\theta}}^{\theta_0} y_l(1-(1-F(\theta))^{N-1})dF(\theta) \\
=& \int_{\theta_0}^{\overline{\theta}}\Big\{V(q(\theta)) - \theta q(\theta) + \lambda\theta q(\theta) \\
& + (\lambda-1)\frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \lambda\frac{\theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
& + \int_{\underline{\theta}}^{\theta_0}\Big\{V(q(\theta)) - (\theta q(\theta) - y_w) + \lambda(\theta q(\theta) - y_l) \\
& + (\lambda-1)\frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \lambda\frac{(\theta q(\theta) - y_l) + \int_{\theta}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
& - \int_{\underline{\theta}}^{\theta_0}\Big\{y_l\frac{(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta - \int_{\underline{\theta}}^{\theta_0} y_w Nf(\theta)(1-F(\theta))^{N-1}d\theta \\
=& \int_{\underline{\theta}}^{\overline{\theta}}\Big\{V(q(\theta)) + (\lambda-1)\theta q(\theta) + \\
& (\lambda-1)\frac{\int_{\theta}^{\overline{\theta}}(1-F(\tilde{\theta}))^{(N-1)}q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}} - \lambda\frac{\theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tilde{\theta})d\tilde{\theta}}{(1-F(\theta))^{(N-1)}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta \\
& + \int_{\underline{\theta}}^{\theta_0}\Big\{(\lambda-1)y_l\frac{(1-(1-F(\theta))^{N-1})}{(1-F(\theta))^{N-1}}\Big\}Nf(\theta)\big(1-F(\theta)\big)^{N-1}d\theta
\end{aligned}
$$

where θ_1 is the lowest order statistic, i.e., $\theta_1 = \min\{\theta_i\}_{i=1}^N$.

We have $\frac{\partial EU_b^{fr}(x,y_w,y_l)}{\partial y_w} = 0$, $\frac{\partial EU_b^{fr}(x,y_w,y_l)}{\partial y_l} = \int_{\underline{\theta}}^{\theta_0}(\lambda-1)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta \geq 0$. Therefore, The buyer's expected utility stays the same with respect to any reimbursement amount to the winning worker y_w. Moreover, the buyer should always choose the highest possible y_l, i.e., $y_l = \theta_0 V'^{-1}\big(\lambda\theta_0\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0\big) = q^{-1}(x)x$ to maximize his expected utility. Considering this, the buyer can choose θ_0 to maximizes her expected utility, based on the following maximization problem:

$$\max_{\theta_0\in[\underline{\theta},\overline{\theta}]} \int_{\underline{\theta}}^{\theta_0}(\lambda-1)\theta_0 V'^{-1}\big(\lambda\theta_0\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0\big)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta$$

Since $x = V'^{-1}\big(\lambda\frac{1-(1-F(\theta_0))^{N-1}}{(1-F(\theta_0))^{N-1}} + \theta_0 + 1\big)$ is a strictly decreasing function of θ_0, the above maximization problem is equivalent to:

$$\max_{x\in[0,q(\underline{\theta})]} \int_{\underline{\theta}}^{\overline{\theta}}(\lambda-1)1_{q(\theta)\geq x}\cdot xq^{-1}(x)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta$$

where $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$.

Therefore, the buyer should choose the optimal reimbursement amount to the loser workers $y_l^* = x^*q^{-1}(x^*)$, and the optimal quality reimbursement threshold $x^* = \text{argmax}_{x\in[0,q(\underline{\theta})]} \int_{\underline{\theta}}^{\overline{\theta}}(\lambda-1)1_{q(\theta)\geq x}\cdot xq^{-1}(x)Nf(\theta)\big(1-\big(1-F(\theta)\big)^{N-1}\big)d\theta$, where $q(\theta) = V'^{-1}\big(\lambda\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}} + \theta\big)$ is the equilibrium quality bidding.

Proof of proposition 14

Let's define $\lambda_l \equiv \min\left\{3, \min_{\lambda\in[1,3]\rho_l\in[0,1]} \frac{\int_{\underline{\theta}}^{\overline{\theta}}(\frac{1}{M(q(\theta))}-1)\frac{F(\theta)}{f(\theta)}q(\theta)Nf(\theta)\left(1-(1-F(\theta))^{N-1}\right)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)\left(1-(1-F(\theta))^{N-1}\right)d\theta}+1\right\}$. We can easily know that $1 < \lambda_l \leq 3$. From 1.7.13, under the condition "$M(q) \leq 1$ for all $q \geq 0$", the buyer should only reimburse the losing workers in the percentage reimbursement policy. The buyer's expected utility when he only reimburses the losing worker(s) is $EU_b^{pr}(0,\rho_l)$. When $1 \leq \lambda \leq \lambda_l$, we have:

$$\begin{aligned}
&\frac{\partial EU_b^{pr}(0,\rho_l)}{\partial \rho_l}\\
=&\int_{\underline{\theta}}^{\overline{\theta}}\Big\{\Big(-\frac{F(\theta)}{f(\theta)}-(\lambda(1-\rho_l)\frac{F(\theta)}{f(\theta)}+\rho_l\theta)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}\Big)\frac{-\lambda\theta}{V''(q(\theta))q(\theta)}\\
&+(\lambda(\theta+\frac{F(\theta)}{f(\theta)})-\theta)\Big\}q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\\
=&\int_{\underline{\theta}}^{\overline{\theta}}\Big\{\Big(\frac{F(\theta)}{f(\theta)}(\lambda(1-\rho_l)\theta\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta)+\rho_l\theta^2\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}\Big)\frac{\lambda}{V''(q(\theta))q(\theta)}\\
&+(\lambda(\theta+\frac{F(\theta)}{f(\theta)})-\theta)\Big\}q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\\
=&\int_{\underline{\theta}}^{\overline{\theta}}\Big\{\Big(\frac{F(\theta)}{f(\theta)}+\rho_l\theta^2\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}V'(q(\theta))}\Big)\frac{\lambda V'(q(\theta))}{V''(q(\theta))q(\theta)}\\
&+(\lambda(\theta+\frac{F(\theta)}{f(\theta)})-\theta)\Big\}q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\\
=&\int_{\underline{\theta}}^{\overline{\theta}}\Big\{\Big(\frac{F(\theta)}{f(\theta)}+\rho_l\theta^2\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}V'(q(\theta))}\Big)\frac{-\lambda}{M(q(\theta))}+(\lambda(\theta+\frac{F(\theta)}{f(\theta)})-\theta)\Big\}q(\theta)Nf(\theta)(1-(1-F(\theta
\\
\leq&\lambda\int_{\underline{\theta}}^{\overline{\theta}}\Big\{\frac{F(\theta)}{f(\theta)}\frac{-1}{M(q(\theta))}+(\theta+\frac{F(\theta)}{f(\theta)})\Big\}q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\\
&-\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\\
=&-\lambda\int_{\underline{\theta}}^{\overline{\theta}}(\frac{1}{M(q(\theta))}-1)q(\theta)NF(\theta)(1-(1-F(\theta))^{N-1})d\theta+(\lambda-1)\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-
\\
=&\lambda\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\Big(\frac{\lambda-1}{\lambda}-\frac{\int_{\underline{\theta}}^{\overline{\theta}}(\frac{1}{M(q(\theta))}-1)\frac{F(\theta)}{f(\theta)}q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})}{\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta}\\
\leq&\lambda\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\Big(\lambda-1-\frac{\int_{\underline{\theta}}^{\overline{\theta}}(\frac{1}{M(q(\theta))}-1)\frac{F(\theta)}{f(\theta)}q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})}{\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta}\\
\leq&(\lambda-\lambda_l)\lambda\int_{\underline{\theta}}^{\overline{\theta}}\theta q(\theta)Nf(\theta)(1-(1-F(\theta))^{N-1})d\theta\leq 0
\end{aligned}$$

where $q(\theta) = V'^{-1}\left(\lambda\theta(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta\right)$, the first equality is from 1.7.13, the third equality follows from the fact that $V'(q(\theta)) = \lambda\theta(1-\rho_l)\frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}}+\theta$, the fourth equality uses the definition that $M(q(\theta)) = \frac{-q(\theta)V''(q(\theta))}{V'(q(\theta))}$, the first inequality follows from $\rho_l \theta^2 \frac{1-(1-F(\theta))^{N-1}}{(1-F(\theta))^{N-1}V'(q(\theta))} \geq 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\rho_l \in [0,1]$, the second equality follows from the condition that $1 \leq \lambda$, the third equality uses the definition of λ_l and the last equality follows from the condition that $\lambda \leq \lambda_l$.

Therefore, if the degree of loss aversion is low enough, i.e., $1 \leq \lambda < \lambda_l$, then the buyer should never reimburse loser worker, i.e., $\rho_l^* = 0$, when using the percentage reimbursement policy. On the other hand, from Proposition 1.7.13, when $\lambda > 1$, under the flat reimbursement policy the buyer should always choose a positive reimbursement amount under the optimized threshold, and thus get the expected utility higher than that when there is zero reimbursement amount.

Therefore, when $1 < \lambda < \lambda_l$, the flat reimbursement policy can always bring higher expected utility to the buyer, since buyer's expected utility are the same when reimbursing zero percentage under the percentage reimbursement policy or reimbursing zero amount under the flat reimbursement policy.

Chapter 2

Trust and Trustworthiness: Experiments withArtificial Intelligence (AI) Agents

2.1 Introduction

There has been rapid development of artificial intelligence (AI) research and applications. Some focus on constructing "superhuman" AIs that are capable to defeat human professionals in increasingly complex games such as chess (Campbell, Hoane Jr, and Hsu, 2002), poker (Bowling et al., 2015) and Go (Silver et al., 2016; Silver et al., 2017). Different forms of self-play, where an artificial agent trains against copies and variations of itself, have been applied to efficiently approximate game theoretic solutions in these games. Others incorporate AIs into decision support system (DSS) to facilitate human decision makers in fields of healthcare, transportation, cybersecurity, and different business domains such as finance, marketing and supply chain operations (Gupta et al., 2021). According to a recent survey, more than 20% of businesses are planing or implementing AIs in their DSSs to help simulate human intelligence, optimize and automate decision-making activities (Lynkova, 2021).

We report a series of experiments with artificial agents playing the "trust game" (also known as the investment game) introduced in Berg, Dickhaut, and McCabe, 1995. We are interested in exploring conditions for AIs to mimic social interactions of humans, and more specifically, for them to behave as if they would trust and be trustworthy in the game. Trust and trustworthiness are economic primitives that influence human behaviors (Berg, Dickhaut, and McCabe, 1995), organizational performance (Jeffries and Reed, 2000; Dirks and Ferrin, 2001), social and business relationships (Rempel, Holmes, and Zanna, 1985; Ring and Ven, 1994), and efficiency of markets and channels (Bolton, Katok, and Ockenfels, 2004; Bolton, Greiner, and Ockenfels, 2013; Beer, Ahn, and Leider, 2018). Their characteristics, expressions, and implications have been studied in many disciplines including biology, psychology, sociology, economics and management. Empirical evidence from laboratory experiments, surveys and interviews suggests that determinants for trust can be biological (for example, hormones or genes (Kosfeld et al., 2005; Fehr, Fischbacher, and Kosfeld, 2005; Riedl and Javor, 2012)) and environmental (such as cultures and institutions (Croson and Buchan, 1999; Gächter, Herrmann, and Thöni, 2004; Engle-Warnick and Slonim, 2004)).

We build deep neural network-based artificial agents and have them trained by playing with one another in the trust game without any prior knowledge or assumption regarding trust or trustworthiness. The use of AIs as experimental subjects completely removes any influence from biological and demographic differences (eg., gender or race) inherent in humans. We identify conditions under which artificial agents can discover trust/trustworthiness through interactive learning and produce outcomes better than economic behaviors based on self-interests in the game. Results from this research can help deepen our understanding of trust, and offer insights on how AIs can be built to overcome incentive barriers that hinder long-term cooperation. Hence, this study constitutes an important step for developing AI-integrated decision support systems capable of going beyond self-interested optimization and making use of social behaviors to achieve better outcomes.

2.2 Trust Game

To establish a clear and simple measure of trust and trustworthiness, we follow the standard behavioral economics approach to conduct experiments using the trust game, a sequential-move non-zero-sum game in which two players send money back and forth (Berg, Dickhaut, and McCabe, 1995). Player 1 (i.e., the trustor) is given a sum of money (an endowment) and decides how much to send to the other player, knowing that the amount sent will be tripled. Player 2 (i.e., the trustee) then decides how much to send back, which the trustor has no control of. Trust is measured by the *amount sent* and trustworthiness is measured by the *amount returned*.

From a rational choice perspective that assumes self-interest, the trustee should not return any money. Anticipating that, the trustor should therefore never send any money. In the controlled lab experiments of Berg, Dickhaut, and McCabe, 1995 and numerous follow-up studies (Johnson and Mislin, 2011), however, human subjects are found to send and return significantly positive amounts albeit variations across individuals. These results help demonstrate that trust and trustworthiness can allow for mutual gains to be realized without enforcement in human society (Alós-Ferrer and Farolfi, 2019).

It should be noted that, while trust may involve a “psychological state comprising of the intention to accept vulnerability" (Rousseau et al., 1998), such intentions or perceptions of individuals cannot be observed or measured directly. Past research has attempted to correlate surveyed attitudes toward trust (Glaeser et al., 2000) or brain signals (King-Casas et al., 2005; Riedl et al., 2014) with results from the trust game. In this study, we evaluate actions of AIs exclusively by outcomes from the trust game, i.e., the amount sent and amount returned by artificial agents in the experiments.

2.3 Deep Q-network (DQN) Artificial Agents

Reinforcement learning (RL), one of the basic machine learning paradigms, provides a framework of how an intelligent agent learns to optimize actions through interactions with the environment in order to maximize the expected cumulative reward (Sutton, Barto, et al., 1998). Deep reinforcement learning (DRL, see reviews (Shrestha and Mahmood, 2019; Wang et al., 2020)), combining the power of deep neural networks and the RL paradigm, has obtained striking success in challenges such as AlphaGo (Silver et al., 2016).

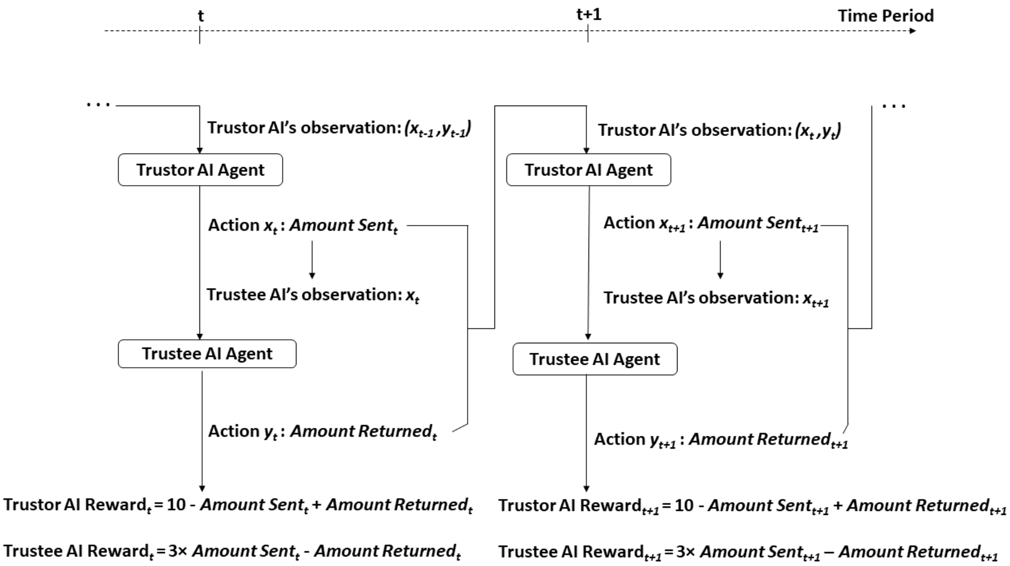
We use the Deep-Q-network (DQN) method, an epoch-making value based DRL algorithm (Mnih et al., 2015), to develop the artificial agents. In particular, two types of agents, the trustor AI and the trustee AI, are created and interact in an environment identical to the trust game introduced by Berg, Dickhaut, and McCabe, 1995 . Fig.2.1a illustrates the timeline for their learning process. Fig.2.1b and Fig.2.1c present schematic of how the neural networks are structured for the trustor and the trustee AIs accordingly (a full description of the DQN algorithm is provided in supplementary materials).

In a time period t, the trustor AI receives \$10 of endowment, observes the amount sent and the amount returned in the previous period (x_{t-1}, y_{t-1}), and then decides the amount to send to the trustee AI in the current period (x_t). The trustee AI observes such an amount being tripled, and then decides the amount to return to the trustor AI (y_t). Reward to the artificial agent in the period t ($reward_t$) is characterized as: $10 - x_t + y_t$ for the trustor AI and $3 \times x_t - y_t$ for the trustee AI, respectively.

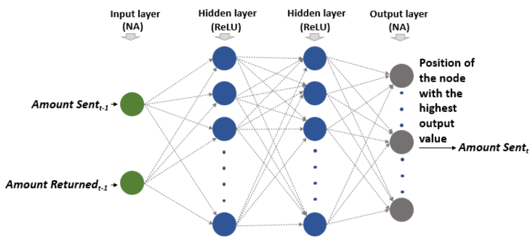
To capture the potential impact of future reward on AI's actions, we use the recursive formulation of the Bellman Equation to define the objective (action-value) functions of the trustor and the trustee AIs. This formulation is theoretically identical to the sum of an infinite stream of discounted per-period rewards (Bertsekas et al., 1995). The discount factor (γ) ranges between 0 and 1, with 0 implying that an agent completely ignores the future and only learn about actions that produce an immediate reward. There have been research attempts to elicit discount rates from people under different settings, which turn out to vary substantially with respect to demographic factors (Coller and Williams, 1999; Harrison, Lau, and Williams, 2002; Warner and Pleeter, 2001). As a baseline, we train the artificial agents with a discount rate similar to the average level estimated for human subjects ($\gamma = 0.75$). This variable is further manipulated in subsequent experiments to examine its impact on AI's actions.

FIGURE 2.1: Reinforcement Learning Process and Neural Network Structure

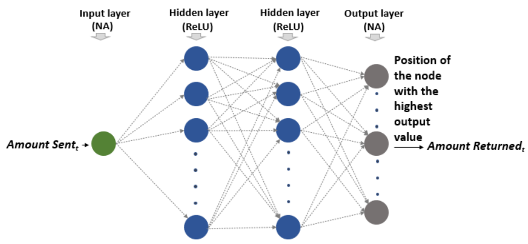
(A) Reinforcement Learning Process of the Trust Game



(B) Schematic of Trustor AI agent's Neural Network



(C) Schematic of Trustee AI agent's Neural Network



(a)Timeline for how artificial agents interact under the trust game through a sequence of observations, actions and rewards. (b) and (c) Neural network architectures for the trustor and trustee AI, respectively. It has been shown that credit assignment path (CAP) of depth two can be an universal approximator to emulate any function (Sugiyama, 2019). Given that the environment examined is relatively simple, we choose a deep neural network with depth three, and use only dense layers to approximate the action-value functions. Input to the neural network is observations of the respective type of AIs. The input layer is followed by two fully connected density layers with ReLU activation function (i.e.,$\max(0, x)$). The action taken by the AI agent corresponds to the position of the output unit which has the highest output value (i.e., the estimated action value). Thus, the number of neurons in the output layer equals to the number of actions the AI agent can take. More architectural details can be found in supplementary materials.

2.4 Training of Artificial Agents and Experimental Design

We build multiple artificial agents independently, 20 as the trustor and 20 as the trustee, to play the game. A series of experiments is conducted using these agents with two research objectives: 1) to determine whether or not human-like behaviors of trust and trustworthiness can emerge from artificial agents, and 2) to explore conditions under which levels of trust and trustworthiness by AIs can be increased or decreased. Each experiment consists of a training stage that lasts for 1,000,000 periods and a playing stage that lasts for 10,000 periods. Similar to human subject experiments, choices of artificial agents are restricted to be integers in all experiments.

In each period of the training stage, one trustor AI interacts with one trustee AI according to the process illustrated by Fig.2.1a. We initialize the training stage with 200 periods where both the trustor and the trustee AIs choose random actions. Thereafter, training of the neural networks starts. Using a back-propagation algorithm, artificial agents adjust parameters of the neural networks to reduce mean-squared errors in the Bellman equation every two time periods (more details of the training process can be found in supplementary materials). We configure the matching between two types of AIs during training in two ways. Under "fixed training", random pairing between the trustor and trustee AIs occur only in the first period, and artificial agents play and train with fixed partners for one million periods. In contrast, under "random training", the 20 trustor and 20 trustee AIs are randomly re-matched in every time period. Recall that the trustor AIs always observe (x_{t-1}, y_{t-1}) as training inputs. This implies that under random training, the amount returned in the previous period observed by the trustor AI may not belong to the trustee AI it currently plays with. The above design is consistent with the partner/stranger matching protocols commonly used in human subject experiments to approximate repeated/one-shot interactions (Bohnet and Huck, 2004).

After one million periods, training of the artificial agents' neural networks stops, and the playing stage starts. Similarly, we vary the partner configuration to create the "fixed playing" versus "random playing" treatments. Under the fixed playing, artificial agents who have trained together in the training stage continue to play the game, with the same partners, for another ten thousands periods; whereas under the random playing, the 20 trustor and 20 trustee agents are randomly re-matched for every playing period. Similar to Berg, Dickhaut, and McCabe, 1995, we are interested in how history affects trust and trustworthiness. In the treatment called "no-history", we make information on the past actions unavailable to the trustor AI in the playing stage by setting x_{t-1} and y_{t-1} to be zeros. Lastly, we also investigate the impact of future rewards on the emergence of trust and trustworthy behaviors by varying the discount factor (γ). In particular, γ is set to be 0.75 at first, which is close to the average discount rate observed in humans (Harrison, Lau, and Williams, 2002), and then is varied between 0 and 1 systematically to test for its influence in follow-up experiments.

We refer to the experiment, in which same pairs of artificial agents are trained together with $\gamma = 0.75$ and played with past actions available to trustor AIs, as the Baseline. We believe this treatment offers the most promising condition for trust and trustworthiness to be discovered by AIs as it mimics conditions where humans exhibit trust and trustworthiness. In the rest of experiments, we change one of the treatment variables (i.e., training partners, playing partners, discount factor, and history) at a time in comparison with the Baseline. Table 2.1 provides a summary of the experimental treatments. All artificial agents are reset before participating in a different experiment. The AIs are not aware of how many periods that the training or the playing stage lasts, nor are they provided with information to identify one another.

TABLE 2.1: Summary of experimental treatments

		Baseline	Fixed training Random playing	Random training Fixed playing	Random training Random playing	No-history	Discount rate
Training Stage	Discount rate γ	$\gamma = 0.75$	$\gamma = 0.75$	$\gamma = 0.75$	$\gamma = 0.75$	$\gamma = 0.75$	$\gamma \in (0, 1)$
	Training partner configuration	Fixed	Fixed	Random	Random	Fixed	Fixed
Playing Stage	Playing partner configuration	Fixed	Random	Fixed	Random	Fixed	Fixed
	Past action information available	Yes	Yes	Yes	Yes	No	Yes

γ = 0.75 for all experiments except for the "Discount rate" treatment where we explicitly manipulate the discount factor. This default value is consistent with the average of individual estimates reported in humans (Harrison, Lau, and Williams, 2002).

2.5 Results

We focus on observations on artificial agents in the playing stage after training of the neural networks stops. Student's t-test and Wilcoxon test are used for comparisons of means and medians, respectively.

Result 1: Artificial agents discover trust and trustworthiness through interactions with each other; and the resulting levels of trust and trustworthiness are similar to what have been observed in human subject experiments.

Table 2.2 reports summary statistics of the Baseline. In the first period of the playing stage, the amount sent and the amount returned by artificial agents are both significantly positive (n= 20 pairs, p-values< 0.01 by two-sided t-test and Wilcoxon signed-rank test). Decisions of the trustor AI are statistically different from random draws of a uniform distribution over possible amounts $\{0, 1, 2,..., 10\}$ (p-values < 0.01 by randomization tests, see details of this test in supplementary materials). Amounts returned by the trustee AI are higher than amounts sent by the trustor AI (p-values < 0.01 by t-test and Wilcoxon test for matched pairs), and they are positively correlated (Spearman's rank correlation coefficient r_s = 0.98 with p-value< 0.01). We also evaluate decisions by artificial agents averaged over 10,000 periods in the playing stage and find them to be insignificant from observations in the first playing period.
.

Next, we compare results from our AI experiments with those from human subject experiments. We plot data from Berg, Dickhaut, and McCabe, 1995 at the aggregate and individual levels in Fig.2.2a and Fig.2.2b, correspondingly. We find that decisions of both the trustor AI and the trustee AI are not statistically different from their respective human counterparts (by either independent t-test or Wilcoxon rank-sum test). It is also important to point out that, with demographic differences being removed completely from artificial agents, their behaviors still exhibit heterogeneity yet with smaller variability than human subjects. A research (Johnson and Mislin, 2011) surveyed 162 replications of the trust game involving more than 23,000 participants. It measures trust by the amount sent divided by endowment and trustworthiness by the amount returned as a proportion of the amount available to return. An average of 50% of trust and 37% of trustworthiness result from this meta-analysis. By the same measures, close levels of 53% of trust and 39% of trustworthiness are found in our AI experiments (although formal statistical tests cannot be performed due to lack of data access).

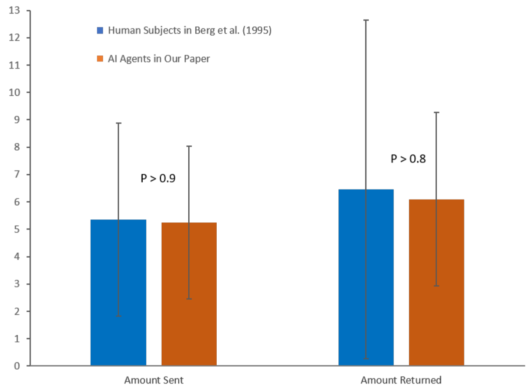
TABLE 2.2: Trust/Trustworthiness Comparison Between Human and AI

Human Experiment Berg et al. (1995)		Baseline Treatment the first period of playing		Baseline Treatment average for the 10000 playing periods	
Amount Sent	Amount Returned	Amount Sent	Amount Returned	Amount Sent	Amount Returned
5.36 (3.53)	6.46 (6.19)	5.25 (2.79)	6.10 (3.16)	5.45 (2.54)	6.20 (2.98)

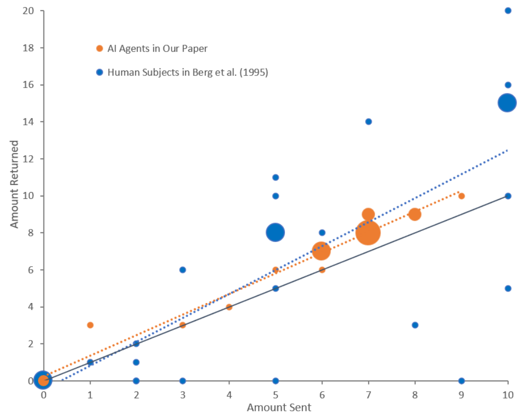
Human experiment: results from the "social history treatment" in Berg, Dickhaut, and McCabe, 1995, where each subject was given a report summarizing decisions of a previous treatment without history information before playing a one-shot trust game (n=28 pairs, standard deviation in parentheses).

FIGURE 2.2: Trust/Trustworthiness comparisons: Human versus AI agents

(A)



(B)



(a) Comparisons between experiments of Berg, Dickhaut, and McCabe, 1995 with human subjects and the first playing period of our Baseline with AI agents show insignificant differences. (b) The scatter plot is weighted by the number of observations to account for duplicates. The solid line represents when the amount of sent equals to the amount returned. Linear trend lines are added to show the estimated relationship between "Amount Sent" and "Amount Returned" by human subjects in Berg, Dickhaut, and McCabe, 1995 versus AI agents in our Baseline.

Result 2: Training with fixed partners is a necessary condition for artificial agents to trust and to be trustworthy.

Table 2.3 summarizes results from our manipulations of the partner configuration (fixed vs. random) in the experiments (training stage vs. playing stage). First, we observe that no trust or trustworthiness is developed when a trustor AI interacts with a random trustee AI in each training period. This result is independent of the partner configuration used in the playing stage. In other words, if an artificial agent has been trained with random strangers, no trust or trustworthiness would arise later even if the AI plays with a fixed partner repeatedly for 10,000 periods. On the other hand, if artificial agents are trained as fixed pairs, independent of the partner configuration in the playing stage, they send and return amounts that are significantly positive (p-values$<$ 0.01 by t-test or Wilcoxon rank-sum test). When playing with random strangers, however, artificial agents exhibit lower levels of trust and trustworthiness than those in the Baseline given fixed pairs (p-values $<$ 0.01 by t-test or Wilcoxon signed-rank test); and the amount returned appears to be lower than the amount sent but not statistically so. Again, we do not find any significant time trend over the 10,000 playing periods.

TABLE 2.3: Effect of matching configurations

	Training with fixed partners		Training with random strangers	
	Amount Sent	Amount Returned	Amount Sent	Amount Returned
Playing with fixed partners	5.45 (2.54)	6.20 (2.98)	0.00 (0.00)	0.00 (0.00)
Playing with random strangers	3.79 (2.09)	3.25 (0.62)	0.00 (0.00)	0.00 (0.00)

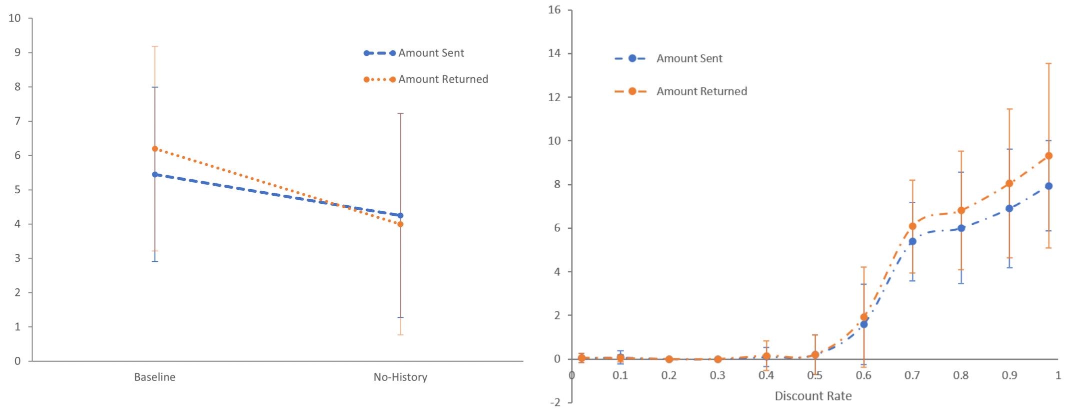
Amounts sent and returned are averaged over the 10,000 playing periods with standard deviation in the parentheses to account for the matching difference across treatments. All numbers are rounded to the second decimal place. In the two "training with random strangers" treatments, numbers are rounding zeroes, while the original observations include very few periods of positive amounts.

Result 3: Information regarding past actions and incentives for future rewards both affect levels of trust and trustworthiness at present.

Artificial agents in experiments reported in Fig.2.2 and Table 2.3. are trained with a discount rate of 0.75 and the amount sent and returned in the previous period observable to the trustor AI. We run additional experiments to explore impact of deviations from these two default settings while keeping the same pairs of artificial agents trained and played together. Fig.2.3a displays results from the treatment of "no history". In this experiment, the trustor AI trains the the neural network with observations on the amount sent and returned in the previous period, yet plays with these past actions replaced by zeroes. We find that both trust and trustworthiness tend to decrease without such information in the playing stage (for amount sent, p-value = 0.093 by t-test, and p-value=0.154 by Wilcoxon signed-rank test; for amount returned, p-value=0.009 by t-test, and p-value=0.031 by Wilcoxon signed-rank test). Fig.2.3b presents results from a series of experiments in which we systematically vary the discount rate between 0 and 1. It is not surprising to see that both trust and trustworthiness increase with more weights on the future rewards. Interestingly, once the discount rate is below 0.5, i.e., when future reward is weighted less than half of the immediate reward, both trust and trustworthiness drop to almost zeros.

FIGURE 2.3: Impact of the Past and the Future

(A) Past Affects Trust/Trustworthiness **(B) Future Affects Trust/Trustworthiness**



(a) The "no-history" treatment differs from the Baseline only in that the trustor AI always observes no amount is sent or returned in the previous period in the Playing stage. The amount sent and the amount returned plotted are averages across fixed pairs of artificial agents (n = 20) over the 10,000 playing periods. We include error bars for the standard deviations. (b) The lowest discount rate used in the experiments is $\gamma = 0.02$, and the highest is $\gamma = 0.98$.

2.6 Conclusion and Discussion

This study establishes that deep neural network-based artificial agents can discover trust and trustworthiness through an interactive trial-and-error learning process without any prior knowledge or assumption regarding the social interactions. We identify two necessary conditions for trust and trustworthiness to arise in the AI experiments. First, artificial agents have to train the neural networks as fixed partners together. Second, they have to "care" about future rewards to at least some degree instead of being entirely myopic. Moreover, levels of trust and trustworthiness can be influenced by the ability of observing past actions.

These findings are eerily similar to our understanding of trust and trustworthiness from existing literatures. Studies in behavioral economics, psychology, sociology have shown that a stable family environment is conducive to develop trusting relationships (Bernath and Feshbach, 1995; Bowlby, 1969; Erikson, 1993); and considerations for the future is a key driver of trust and trustworthiness to foster long-term cooperation (Engle-Warnick and Slonim, 2006; Engle-Warnick and Slonim, 2004; Mahajna et al., 2008). At the same time, reputation, a reflection of someone's past actions, has been known as an important ingredient to build trust (Bohnet and Croson, 2004; Bolton, Katok, and Ockenfels, 2004; Charness, Du, and Yang, 2011; King-Casas et al., 2005).

Our results indicate that artificial agents are capable of arriving at decisions similar to those of human subjects under the influence of social interactions. From a managerial perspective, this suggests that AI incorporated DSSs can be applied to business scenarios where social considerations such as trust and trustworthiness are important to decision making. Examples may include new product or technology development, collaborative forecasting, humanitarian operations and supply chain relationship management. This study is a first step to explore the possibility to integrate different AI systems that can go beyond self-interested optimization and make use of social behaviors to achieve better outcomes collectively.

While artificial agents are different from humans in many substantial aspects, findings of the study may also help shed light on how social behaviors take place. The AI experiments completely removes biological or demographic differences that have been known to account for trust/trustworthiness observed in individuals. Artificial agents are found to produce actions close to human subjects at the aggregate level in the trust game, and with a certain degree of heterogeneity as well. These results seem to suggest a more algorithmic origins of trust. Ultimately, both artificial intelligence and human brains are built upon nonlinear and densely connected networks with learning capabilities. This study is merely scratching the surface of this direction of research on how artificial intelligence and behavioral economics may interact to influence business decision making activities (Camerer, 2018). The natural next step is to investigate whether or not other types of human behaviors such as fairness, reciprocity or different risk preferences may emerge from similarly constructed AI experiments.

APaper

2.7 Supplementary Materials

Objective Function Formulation: Bellman Equation

We created two types of artificial agents, referred to as the trustor AI and trustee AI, to play the respective roles in the trust game. In this section, we describe the corresponding objective functions that each type of AI is trained to optimize, and the associated temporal structures.

Let x_t be the amount sent and y_t be the amount returned in period t. The reward in period t for the trustor AI is $R - x_t + y_t$, where R is the initial endowment of the trustor. The reward in period t for the trustee AI is $\alpha x_t - y_t$, where α is the multiplier for the amount sent. In all experiments of this study, we have $R = 10$ and $\alpha = 3$, which are the same as Berg, Dickhaut, and McCabe, 1995.

To capture the potential impact of future rewards, we use the recursive formulation of the Bellman Equation (Bertsekas et al., 1995) to define the objectives of the artificial agents. This formulation is theoretically identical to the sum of an infinite stream of discounted per-period rewards. Let the objective functions, also known as the action-value functions, of the trustor AI and the trustee AI be $Q^*_{trustor}$ and $Q^*_{trustee}$ respectively. In each period, a pair of artificial agents interact through observing information, taking actions and receiving rewards. The following table summarizes these components.

TABLE S1: Observation, Action and Reward for Artificial Agents

	Trustor AI	**Trustee AI**
Observations (in period t)	(x_{t-1}, y_{t-1}) x_{t-1}: Amount sent by the trustor AI itself in period $t-1$ y_{t-1}: Amount returned by the trustee AI matched in period $t-1$	x_t x_t: Amount sent by the trustor AI matched in period t
Action (in period t)	x_t	y_t
Reward (in period t)	$R - x_t + y_t$	$\alpha \times x_t - y_t$

Hence, the respecitve action-value functions for the trustor AI and the trustee AI agents are defined as:

$$Q^*_{trustor}((x_{t-1}, y_{t-1}), x_t) = \mathbf{E}\{R - x_t + y_t + \gamma \cdot \mathbf{max}_{x_{t+1}} Q^*_{trustor}((x_t, y_t), x_{t+1}) | (x_{t-1}, y_{t-1}), x_t\},$$

and

$$Q^*_{trustee}(x_t, y_t) = \mathbf{E}\{\alpha x_t - y_t + \gamma \cdot \mathbf{max}_{y_{t+1}} Q^*_{trustee}(x_{t+1}, y_{t+1}) | x_t, y_t\},$$

where γ is the discount rate.

Deep Q-network (DQN)

We apply deep-Q network method (Mnih et al., 2015) to build the artificial agents. Specifically, the trustor AI uses a deep neural network $Q_{trustor}((x_{t-1}, y_{t-1}), x_t, \theta_{trustor})$ to approximate the optimal action-value function, i.e., $Q_{trustor}((x_{t-1}, y_{t-1}), x_t, \theta_{trustor}) \approx Q^*_{trustor}((x_{t-1}, y_{t-1}), x_t)$, where $\theta_{trustor}$ is the weights of the trustor AI's neural network.The optimal target value $R - x_t + y_t + \gamma \cdot \mathbf{max}_{x_{t+1}} Q^*_{trustor}((x_t, y_t), x_{t+1})$ is approximated by target values $T_{trustor} = R - x_t + y_t + \gamma \cdot \mathbf{max}_{x_{t+1}} Q_{trustor}((x_t, y_t), x_{t+1}, \theta^{-1}_{trustor})$, where the deep neural network $Q_{trustor}((x_t, y_t), x_{t+1}, \theta^{-1}_{trustor})$ has exactly the same neural network structure as
$Q_{trustor}((x_{t-1}, y_{t-1}), x_t, \theta_{trustor})$ and its parameters $\theta^{-1}_{trustor}$ are copied from $\theta_{trustor}$ every C training iterations (see the hyper-parameter values we used in Table S3). At training iteration k, the parameter $\theta_{trustor}$ of the neural network
$Q_{trustor}((x_{t-1}, y_{t-1}), x_t, \theta_{trustor})$ is adjusted to minimize the mean-squared error in the Bellman equation. Following the method (Mnih et al., 2015), we define the loss function based on a mean-squared error:

$$\begin{aligned} L(\theta_{trustor}) =& \mathbf{E}_{x_{t-1}, y_{t-1}, x_t, y_t} \big(R - x_t + y_t + \gamma \cdot \mathbf{max}_{x_{t+1}} Q_{trustor}((x_t, y_t), x_{t+1}, \theta^{-1}_{trustor}) - \\ & Q_{trustor}((x_{t-1}, y_{t-1}), x_t, \theta_{trustor})\big)^2, \end{aligned}$$

where parameters set $\theta^{-1}_{trustor}$ is updated to equal to $\theta_{trustor}$ only every C training iterations and thus fixed when we calculate the loss $L(\theta_{trustor})$ in training iteration k. Once we have this well-defined loss function, we can use gradient descent method to train the neural network, i.e., to update $\theta_{trustor}$.

Similarly, for the trustee AI agents, we use a deep neural network $Q_{trustee}(x_t, y_t, \theta_{trustee})$ to approximate the objective function, i.e., $Q_{trustee}(x_t, y_t, \theta_{trustee}) \approx Q^*_{trustee}(x_t, y_t)$, where $\theta_{trustee}$ is the weights for the trustee neural network. On the other hand, the optimal target value $\alpha x_t - y_t + \gamma \cdot \mathbf{max}_{y_{t+1}} Q^*_{trustee}(x_{t+1}, y_{t+1})$ is approximated by target values $T_{trustee} = \alpha x_t - y_t + \gamma \cdot \mathbf{max}_{y_{t+1}} Q_{trustee}(x_{t+1}, y_{t+1}, \theta^{-1}_{trustee})$, where the deep neural network $Q_{trustee}(x_{t+1}, y_{t+1}, \theta^{-1}_{trustee})$ has exactly the same neural network structure as $Q_{trustee}(x_t, y_t, \theta_{trustee})$ and its parameters $\theta^{-1}_{trustee}$ are copied from $\theta_{trustee}$ every C training iterations. At training iteration k, we aim to adjust the parameter $\theta_{trustee}$ in neural network $Q_{trustee}(x_t, y_t, \theta_{trustee})$ to minimize the mean-squared error in the Bellman equation. Following the method in (Mnih et al., 2015), we define the loss function based on a mean-squared error:

$$L(\theta_{trustee}) = \mathbf{E}_{x_t, y_t, x_{t+1}} \big(\alpha x_t - y_t + \gamma \cdot \mathbf{max}_{y_{t+1}} Q_{trustee}(x_{t+1}, y_{t+1}, \theta^{-1}_{trustee}) - Q_{trustee}(x_t, y_t, \theta_{trustee})\big)^2,$$

where parameters set $\theta^{-1}_{trustee}$ is updated to equal to $\theta_{trustee}$ only every C training iterations and thus fixed when we calculate the loss $L(\theta_{trustee})$ at training iteration k.

Once we have this well-defined loss function, we can use gradient descent method to train this deep neural network, i.e., to update $\theta_{trustee}$.

The architecture of the trustor AI and trustee AI neural network, illustrated schematically in Fig. 1.(B), is detailed in Table S2.

TABLE S2: Structural Details of the Neural Network

Layer	**Activation Function**	**Number of Nodes in the Neural Network**	
		Trustor AI	Trustee AI
Input layer	NA	2	1
Hidden layer 1	ReLU $\max(0, x)$	800	800
Hidden layer 2	ReLU $\max(0, x)$	1000	1000
Output layer	NA	11	31

[1] Structures of the neural networks of the trustor and the trustee AIs are exactly the same except the number of neural nodes in the input layer and output layer. The trustor AI observe actions of both players in the previous period, and therefore its neural network input layer has two nodes. The trustee AI always observes the amount sent by the trustor AI in the current period, and thus its neural network input layer has only one node.

[2] Their input layers are followed by two fully-connected layers with the "ReLU" activation function (the number of neuron nodes are 800 and 1000 respectively). The output layer is a fully-connected linear layer with no activation function and the position of each single output unit corresponds to a valid action. Since the trustor AI agent can take any integer decision from $\{0, 1, 2, ..., 10\}$, its neuron network has 11 output layer nodes. Since the trustee AI agent may take any integer decision from $\{0, 1, 2, ..., 29, 30\}$, its neuron network has 31 output layer nodes.

Training details

We created twenty trustor and twenty trustee AIs, each with a unique and independent neural network. Pairs of artificial agents are configured to be either " fixed" or "random" according to the experimental treatment. There is an *initialization phase* of 200 periods in the training stage. In the first period of the initialization phase, both types of artificial agents take random actions. From the second time period and onward, the trustor AI can observe actions of its own and the matched trustee AI from the previous period. The trustor AI takes an action based on an ε-greedy policy: with a probability of $1 - \varepsilon$, the trustor AI selects the action which corresponds to the position of the neural node with the highest output of its updated neural network (i.e., the estimated action value); and with a probability of ε, the trustor AI selects a random action from a discrete uniform distribution over $\{0, 1, 2, ..., 10\}$. ε decreases with training iterations at a diminishing rate, i.e., $\varepsilon_t = e^{-\phi \Delta t}$, where ε_t is probability at which the AI agent chooses random action in period t, ϕ is the decayed rate and Δt is the number of training iterations accumulated up to period t. In addition, we use a small probability as the lower bound of ε (see all hyper-parameter values in Table S3).

After the trustor AI takes an action in time period t, the trustee AI can observe this action, and then take an action based on the ε-greedy policy as the trustor AI does. Rewards to the artificial agents in period t follow the standard trust game

(Berg, Dickhaut, and McCabe, 1995). For the trustor AI's reward in period t, $reward_t = R - x_t + y_t$. For the trustee AI's reward in period t, $reward_t = \alpha \times x_t - y_t$, where x_t is amount sent by the trustor AI in period t and y_t is amount returned by the trustee AI in period t. Training of the artificial agents starts after the initialization phase. The neural networks are updated every two periods with the deep-Q learning algorithm (Mnih et al., 2015) (its full structure is shown in Algorithm 1 below). Specifically, we use two key techniques in this algorithm:

1) Experience replay

For each time period $t \geq 1$ we can store trustor AI agents' action experience tuple $e^t_{trustor} = ((x_{t-1}, y_{t-1}), x_t, R - x_t + y_t, (x_t, y_t))$ in a data set $D^t_{trustor} = \{e^1_{trustor}, e^2_{trustor}, ..., e^t_{trustor}\}$, where x_{t-1} is the amount sent by trustor AI agent itself in the previous time period $t-1$, y_{t-1} is the amount returned by trustee AI agent with who the trustor AI agent matched in the previous time period $t-1$, (x_{t-1}, y_{t-1}) is the trustor AI agent's observation in time period t, x_t is trustor AI agent's action taken in time period t, $R - x_t + y_t$ is the trustor AI agent's reward in time period t, (x_t, y_t) is the trustor AI agent's observation in the next time period $t+1$.

Similarly, for the trustee AI agents, we store their action experience tuple $e^t_{trustee} = (x_t, y_t, \alpha x_t - y_t, x_{t+1})$ in a data set $D^t_{trustee} = \{e^1_{trustee}, e^2_{trustee}, ..., e^t_{trustee}\}$, where x_t is the amount sent by trustor AI agent with who the trustee AI agent matched in the current time period t (x_t is also the trustee AI agent's observation in the current time period t), y_t is the amount returned by trustee AI agent itself in the current time period t (y_t is also the action taken by the trustee AI agent in the current time period t), x_{t+1} is the amount sent by trustor AI agent with who the trustee AI agent matched in the next time period $t+1$ (x_{t+1} is also the trustee AI agent's observation in the next time period $t+1$).

The data set $D^t_{trustor}$ and $D^t_{trustee}$ are pooled into a replay memory $D_{trustor}$ and $D_{trustee}$ with capacity K (new data samples will gradually replace old samples into the replay memory when its capacity is full). At each training iteration, we randomly draw a mini batch of samples stored in the replay memory. Note that all experiences are uniformly distributed in the data set, i.e., $e_{trustor} \sim U(D_{trustor})$ and $e_{trustee} \sim U(D_{trustee})$. This technique can increase data efficiency, reduce updating variance and smooth out the learning process (Mnih et al., 2015).

2) Fixed Q-targets network

We denote the neural network $Q_{trustor}((x_{t-1}, y_{t-1}), x_t, \theta_{trustor})$ and $Q_{trustee}(x_t, y_t, \theta_{trustee})$ in the loss functions as $Q_{trustor}$ and $Q_{trustee}$, which is updated by performing gradient descent in each training iteration. For the target neural network $Q_{trustor}((x_t, y_t), x_{t+1}, \theta^{-1}_{trustor})$ and $Q_{trustee}(x_{t+1}, y_{t+1}, \theta^{-1}_{trustee})$ in the loss functions, we denote them by $\hat{Q}_{trustor}$ and $\hat{Q}_{trustee}$ respectively. They have exactly the same neural network structure as $Q_{trustor}$ and $Q_{trustee}$. We copy all the weight of $Q_{trustor}$ and $Q_{trustee}$ to $\hat{Q}_{trustor}$ and $\hat{Q}_{trustee}$ every C training iterations, and then use $\hat{Q}_{trustor}$ and $\hat{Q}_{trustee}$ to generate the Q-learning targets $T_{trustor}$ and $T_{trustee}$ for the next C training iterations. This technique can further improve the training stability of the agents' neural networks (Mnih et al., 2015).

Algorithm 1

For $t = 1$,T **Do**

{

For each pair of the trustor and trustee AI agents (20 pairs in total) **Do**

{

Initialize neural networks $Q_{trustor}$, $\hat{Q}_{trustor}$,$Q_{trustee}$, $\hat{Q}_{trustee}$;

Initialize replay memory $D_{trustor}$ and $D_{trustee}$ to capacity K;

If time period t **equals to one**

{ Assign x^t and y^t as random actions; }
Else {
Trustor AI agent i selects a random action x^t with probability ε_t ;
otherwise choose action $x^t = \mathbf{argmax_x} Q_{trustor_i}((x^{t-1}, y^{t-1}), x, \theta_{trustor})$;

Trustee AI agent j selects a random action y^t with probability ε_t ;
otherwise choose action $y^t = \mathbf{argmax_y} Q_{trustee_j}(x^t, y, \theta_{trustee})$;

If time period $t > 200$ **and** t **is even**
{ Sample mini-batch of transitions $e^k_{trustor} = ((x^{k-1}, y^{k-1}), x^k, R - x^k + y^k, (x^k, y^k))$ from $D_{trustor}$;
Calculate target value $T^k_{trustor} = R - x^k + y^k + \gamma \cdot \mathbf{max}_x \hat{Q}_{trustor}((x^k, y^k), x, \theta^{-1}_{trustor})$;
Perform a gradient descent on loss $\left(T^k_{trustor} - Q_{trustor}((x^{k-1}, y^{k-1}), x^k, \theta_{trustor})\right)^2$ with respect to weight $\theta_{trustor}$;
Every C training iterations reset $\hat{Q}_{trustor} = Q_{trustor}$;
Sample mini-batch of transitions $e^k_{trustee} = \left(x^k, y^k, \alpha x^k - y^k, x^{k+1}\right)$ from $D_{trustee}$;
Calculate target value $T^k_{trustee_j} = \alpha x^k - y^k + \gamma \cdot \mathbf{max}_y \hat{Q}_{trustee}(x^{k+1}, y, \theta^{-1}_{trustee})$;
Perform a gradient descent on loss $\left(T^k_{trustee} - Q_{trustee}(x^k, y^k, \theta_{trustee})\right)^2$ with respect to weight $\theta_{trustee}$;
Every C training iterations reset $\hat{Q}_{trustee} = Q_{trustee}$;
}
Store transition $e^t_{trustor}$ in $D_{trustor}$ (Store transition $e^{t-1}_{trustor}$ in $D_{trustor}$ in "random training" treatments);
Store transition $e^t_{trustee}$ in $D_{trustee}$ (Store transition $e^{t-1}_{trustee}$ in $D_{trustee}$ in "random training" treatments);
}
}
End For
}
End For

Values of some parameters, known as hyperparameters, are tuned to control the neural network training process. The descriptions of these hyperparameters and their values used in the study are listed in Table S3 below.

TABLE S3: Hyperparameters

Hyperparameter	**Value used**	**Description**
Time periods in initialization phase	200	In the first 200 periods of the training stage, the neural networks are not trained.
Time periods in the training stage	1000000	Total number of periods in the training stage
Time periods in the playing stage	10000	Total number of periods in the playing stage
Training frequency	2	The neural networks are updated every two periods after the initialization phase in the training stage.
Learning rate	0.0016	The learning rate used by the RMSprop optimizer
Initial exploration	1	Initial value of ε in ε-greedy policy.
Final exploration	0.00001	Final value of ε in ε-greedy policy.
Decayed rate	0.0001	ε in ε-greedy policy is exponentially decayed by this rate with training iterations.
Target network updating frequency (C)	3000	The target network weights are updated every 3000 training iterations.
Replay memory size	300000	Stochastic gradient descent (SGD) samples update from this number of most recent combinations of game information.
Mini-batch size	200	The number of training cases over which the SGD update is computed.

Randomization Test

Similar to Berg, Dickhaut, and McCabe, 1995, we performed a randomization test for the null hypothesis that actions of the artificial agent are randomly drawn from some uniform distribution. For a trustor AI agent i ($i = 1, 2, 3, ..., 20$), we randomly draw a sample with 10,000 observations (which equal to the number of playing periods) from the discrete uniform distribution over the amounts $\{0, 1, 2, ..., 10\}$. We denote the sample as s_i and the frequency of each amount $m \in \{0, 1, 2, ..., 10\}$ in this sample as f^i_m. We measure the variance of the sample s_i as: $v(s_i) = \sum_{m=1}^{11}(f^i_m - \frac{N}{11})^2$, where $N = 10,000$. Given a trustor AI agent's actual decisions in the playing stage denoted d_i (which also include 10,000 observations), we have $v(d_i)$. We then can calculate the probability of $v(s_i) \geq v(d_i)$, which is the p-value of the randomization test, based upon 100,000 times of the random sampling. For each trustee AI, the same test procedure is repeated with a discrete uniform distribution over the amounts $\{0, 1, 2, ..., 30\}$. All p-values from the above randomization tests are smaller than 0.01. We thus reject the null hypothesis that the artificial agent takes random actions.

Chapter 3

Trust in Supply Chain with Double Marginalization

3.1 Introduction

Demand information sharing is one of the most active and important areas of research because it has profound effects on the performances of supply chains. In the capacity management related research, information sharing is always assumed to be desirable and the issue is to find ways to overcome the associating incentive problems. In practice, this view may not be always true. One such example is the Vendor Managed Inventory (VMI) system, popularized by Walmart, where the retailer provides dynamical inventory and demand (often in forms of sales) information of products to supplier who takes full responsibility for maintaining inventory. Murray (2018 (accessed December 5, 2018)) suggests that such a system may result in a scenario where "a supply chain manager becomes too reliant on a supplier to manage its inventory, the supply chain manager may live with higher prices, reduced quality or other supplier-related issues". Clearly, a dominating retailer like Walmart will not "live with higher prices", but what about smaller retailers? Indeed, in the information sharing literature, there are papers pointing out the downside. That is, information sharing can exacerbate double marginalization. Shang, Ha, and Tong (2015) shows that demand information helps the supplier to pin down the retailer's willingness to pay. The higher the potential demand, the more willing is the retailer to stock up. As a result, the supplier can squeeze the retailer, with the use of the wholesale price, more effectively. The first goal of this paper is to analyze the tradeoffs between these two views: information sharing helps the capacity misalignment (*CM*) problem but exacerbates the double marginalization (*DM*) problem.

The classical incentive issue of demand information sharing can be summarized in a deceptively simple two-tier supply chain setting where a supplier is deciding the amount of capacity prior to a binding order from a retailer who has private demand information. In this setting, forecast communications that are costless, nonbinding and non-verifiable, referred to as "cheap talk", while popular in practice, should result in no information sharing because of incentive misalignment (Crawford and Sobel, 1982). The retailer prefers the supplier to build more capacity to ensure enough supply, and hence always has the incentive to inflate its forecasts. Anticipating this, the supplier takes the retailer's report as incredible and decides the capacity according to the prior belief, which can lead to too much or too little capacity and harm the supply chain efficiency. We call this incentive problem as the "capacity misalignment". This forecast information sharing problem not only has been reported in industries (The Economist, 2012), but has also spawned a large body of research. Some, by the use of game theoretic analysis, aim to find mechanisms, such as contracts, that can enable credible information sharing (Cachon and

Lariviere, 2001; Özer and Wei, 2006; Oh and Özer, 2013). Others, notably Özer, Zheng, and Chen (2011), reconciles the puzzling popularity of "cheap talk" forecast communications with the seemingly insurmountable incentive conflict, and show that "cheap talk" with a simple wholesale price contract can result in effective information sharing because individuals are not only driven by pecuniary motivations. In particular, Özer, Zheng, and Chen (2011) demonstrated with behavioral experiments that innate human qualities, trust (the ability to "believe" cheap talk forecasts in this context) and trustworthiness (the restraint from lying), play important roles to negate the misalignment of pecuniary incentives. Their results show that trust and trustworthiness exist in shades of grey, rather than all-or-nothing binary states (i.e. a person can either be trusted 100% or not at all). They developed a behavioral model, referred to as the trust-embedded model, to capture and measure trust and trustworthiness.

The second goal of this paper is to investigate how trust and trustworthiness impacts the upside and the downside of information sharing. We operationalize a setting common to the *CM* and the *DM* problems by two additions to the aforementioned capacity game setting used in Özer, Zheng, and Chen (2011). First, the order of the retailer is placed *before* demand realization. Hence, the retailer also acts as a newsvendor, similar to the supplier, but with better demand information. Secondly, the wholesale price is not exogenous and, instead, is set by the supplier. Note that these additional assumptions also make the scenario more aligned with practice. After all, it is not common that wholesale prices are completely exogenous and retailers only order after the customer shows up.

We confirm that information sharing can exacerbate *DM* problem and can be harmful to a supply chain. We develop a measure, called the double marginalization index (DMI) to decide how information sharing change the severity of the problem, and provide associating conditions of when the exacerbation of double marginalization overwhelms the benefit to aligning capacity.

Trust and trustworthiness introduces additional considerations. We endogenize information sharing by applying the trust-embedded model (Özer, Zheng, and Chen 2011) in a "cheap talk" environment, to determine trust's role. We are able to show that even when the retailer is highly trustworthy, trust, on the part of the supplier, does not always benefit the supply chain. This mirrors the result that information sharing is not always helpful. More interestingly, we find that too little or too much trust can reduce supply chain efficiency, and we are able to characterize the conditions of balancing the two, and find the optimal trust level for the supply chain [1]. The most counter-intuitive result is that, under some conditions, an untrustworthy retailer, coupled with a trusting supplier, can be beneficial to the supply chain because the retailer has the opportunity to manipulate the double marginalization problem away!

The rest of this paper is organized as follows. In §2, related literature is reviewed. In §3, we describe the game setting. In §4, we investigate the role of information sharing in supply chain efficiency. The trust-embedded models in §5 are demonstrated to study the role of trust in supply chain efficiency. We conclude the paper in §6.

[1] It is an open issue, beyond the scope of this paper, of whether trust is a decision variable that the supplier can control or if it is a behavioral response driven by social factors. In the following discussion of this paper, an optimal trust level maximizing the total supply chain efficiency highlights trust's balancing in the two incentive problems and the fact that a higher level of trust is not necessarily beneficial, counter to the traditional wisdom that trust is always beneficial.

3.2 Literature Review

In this part, we first review the literature about demand information sharing and the capacity misalignment problem. Then we point out the literature regarding the double marginalization problem under the prevalent wholesale price contract. Lastly, we review the literature about trust.

Since capacity decision is made in anticipation of the end customer demand, it critically depends on accurate demand forecasts (see Van Mieghem 2003 for a review). Although more accurate demand forecast leads to better capacity decision, forecast information sharing is challenging in a decentralized supply chain with incomplete demand information. This is because the downstream player has incentive to offer a rosy demand forecast in order to ensure enough capacity from the supplier. Anticipating this, the supplier considers retailer's demand forecast as incredible and sets the capacity according to his prior belief. Therefore, the supplier can set too much or too little capacity without accurate forecast information, which harms the supply chain efficiency. We refer to this as "capacity misalignment problem". To address this problem, two seminal research paper focused on designing theoretical contracts in a signaling or screening game that can help elicit credible information sharing and remedy the aforementioned incentive problem (Cachon and Lariviere 2001, Özer and Wei 2006). Literature has since extended the research about strategic issues in information sharing towards different theoretical directions, such as supply chain competition (Ha and Tong 2008), information confidentiality (Li and Zhang 2008), forecasting investment (Shin and Tunca 2010), forecasting accuracy (Taylor and Xiao 2010), and dynamic environment (Oh and Özer 2013).

Wholesale price contract has been widely used in industry due to its simplicity. However, it causes a double marginalization problem. Spengler (1950) is a well-know illustration of the double marginalization problem. To maximize the profit, the upstream firm charges a higher price than its marginal cost. The downstream firm thus faces a higher cost than the vertical structure's cost. Then the downstream firm as a monopolist charges a price above its cost. These two successive markups have named the "double marginalization" problem. Taking no consideration of the upstream firm's profit, the downstream firm charges a higher price and lower order quantity than the channel optimal one. This vertical externality implies all firms forgo potential profits, incurring inefficiency for the supply chain. Various contractual arrangements have been investigated to fix the double marginalization problem and to coordinate the supply chain (see Cachon 2003 for a review).

Research papers (Cachon and Lariviere 2001, Özer and Wei 2006, Oh and Özer 2013) focused on designing contracts to remedy the information coordination problem regarding capacity decision under the wholesale price contract. However, they did not consider the aforementioned double marginalization problem under the wholesale price contract. In particular, they assume that the downstream firm's order quantity always equals to the minimum of the capacity and realized demand, since it is submitted after the realization of the end market demand. Therefore, the order quantity will always be optimal both for the downstream firm and the total supply chain.

In additional to the capacity misalignment problem they considered, our paper studied the double marginalization problem. It assumes that the wholesale price is endogenously set by the supplier, and that retailer's order is submitted before the realization of the end market demand. We found that information sharing can benefit the capacity misalignment problem, consistent with the previous literature, but can harm the double marginalization problem. Shang, Ha, and Tong (2015) refers this harmful effect of information sharing as "the double marginalization effect of information sharing". When a upstream player (i.e., the manufacturer in their case) gets access to the demand information from the retailer, the player can extract more profit from the retailer by adjusting the wholesale price corresponding to the demand information (Li and Zhang 2008, Li 2002). This can worsen the double marginalization and harm the supply chain efficiency. Our paper complements this insight from their papers by confirming information can be harmful to double marginalization problem and investigating the specific condition of when and how information can be harmful to the supply chain efficiency in our setting coupled with the capacity misalignment problem.

Moreover, our paper investigates the role of trust and trustworthiness in the capacity misalignment and double marginalization problem. The closest paper to ours regarding trust and trustworthiness is Özer, Zheng, and Chen (2011). Particularly, in their paper, the downstream firm reports the forecast by costless, non-binding, and non-verifiable communications (also known as "cheap talk" Crawford and Sobel (1982)), then the upstream supplier makes the capacity decision. Although the cheap talk is uninformative theoretically, experimental results show that the upstream supplier has a willingness to rely on the downstream firm's cheap talk to determine capacity. In other words, the upstream firm will trust the cheap-talk communication to some extent and update his belief of the private demand forecast information according to it. If the downstream firm is trustworthy (has high dis-utility of deception) true private forecast information will be transmitted by a trusting supplier. This trust-induced information sharing can share true information and thus enhance the supply chain efficiency. We employed the trust-embedded model in this paper to capture the effect of trust and trustworthiness on supply chain efficiency. However, our paper complements their research by studying their effects both on the capacity misalignment problem and the double marginalization problem.

Trust has been widely studied across disciplines. A definition commonly agreed upon different disciplines is "trust is a psychological state comprising the intention to accept vulnerability based upon positive expectations of the intentions or behavior of another"(Rousseau et al. 1998). One group of studies investigates the trust regarding property rights, i.e., the trustor voluntarily transfers the property rights to the trustee in hope of reciprocal returns from the trustee. This group of research is mainly based on the trust game (Kreps 1996) or the investment game (Berg, Dickhaut, and McCabe 1995), and uses experiments to investigate the determinants of trust, such as willingness to take risk (Ben-Ner and Putterman 2001), expectation of return (Ashraf, Bohnet, and Piankov 2006; Eckel and Wilson 2004), betrayal aversion (Bohnet and Croson 2004), gender (Ben-Ner and Halldorsson 2010), social status (Hong and Bohnet 2007), and culture (Croson and Buchan 1999).

A more recent group of studies define trust as the trustor's willingness to rely on the trustee's information claims in strategic information sharing. Özer, Zheng, and Chen (2011) is a seminal paper that investigates the role of this kind of trust in information sharing. Based on this paper's theoretical framework, Özer, Zheng, and Ren (2014) use a real-time, cross-country interactive experiment to investigate how culture affects trust with respect to strategic information sharing. Our paper contributes to this group of research, by analytically and numerically examining the role of trust and trustworthiness in the double marginalization problem, in addition to the well-studied capacity misalignment problem.

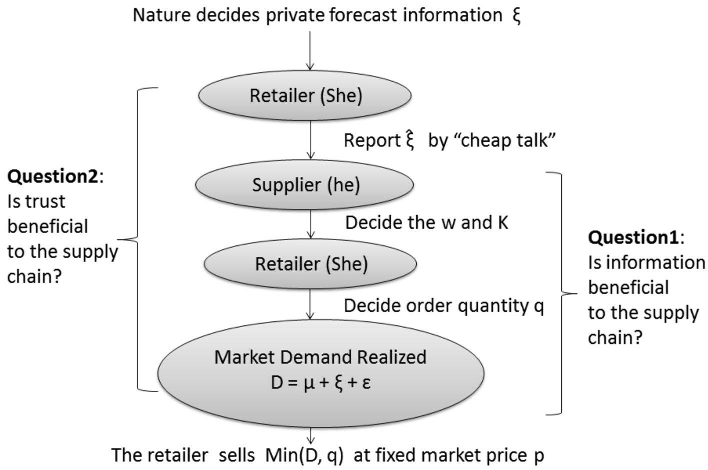


FIGURE S1: Model setting

3.3 Model Setting

We use a formulation similar to other papers, such as Özer and Wei (2006) and Özer, Zheng, and Chen (2011), in this literature. Consider a dyad supply chain with a supplier (he) and a retailer (she). Because of the proximity to the end market, the retailer has a private forecast information ξ, which is decided by nature and privately known to her at the beginning of the game. We assume ξ is a zero mean random variable with cumulative distribution function $F(\cdot)$, and probability density function $f(\cdot)$ supported on $[\underline{\xi}, \overline{\xi}]$, which is supplier's prior belief about ξ. End customer demand $D = \mu + \xi + \epsilon$, where μ is a constant denoting the average demand, ϵ is a zero mean random variable denoting market uncertainty with cumulative distribution function $G(\cdot)$, and probability density function $g(\cdot)$ supported on $[\underline{\epsilon}, \overline{\epsilon}]$. We assume ϵ follows an uniform distribution [2]. The supplier's production cost per unit is denoted as $c \geq 0$, one unit of capacity costs the supplier $c_k \geq 0$. The retailer can sell the product at a fixed price p. In addition, we assume the lowest possible demand $D_{min} = \mu + \underline{\xi} + \underline{\epsilon} \geq 0$, i.e., the demand is always non-negative, and the profit margin for the supply chain is positive, i.e., $p > c + c_k$. Moreover, the supplier and retailer interact under a wholesale price contract, which is widely used in industry due to its simplicity. All these mentioned information is common knowledge to the players.

In section 3.4, we aim to investigate the first research question-"Is information beneficial to the supply chain?" The analysis strategy is to first determine if exogenous coerced information sharing (i.e. the supplier learns the demand signal in some credible exogenous process) improve or decrease supply chain efficiency. Note that, in this scenario, cheap talk is no longer relevant and is not in the setting. The game sequence for this part is as following: (1) Nature decides the private forecast information ξ, privately known to the retailer. (2) The supplier decides the wholesale price w and capacity K. (3) The retailer decides her order quantity q with the constraint $q \leq K$. Then the end market demand $D = \mu + \xi + \epsilon$ realizes, and the retailer sells $\min(D, q)$ to the customers.

[2]In some of the analysis cases, we also need to assume ξ follows an uniform distribution to make the model tractable. If we need this assumption, we will clarify it in the specific analysis.

In section 3.5, we endogenize information sharing by adding the “cheap talk” stage, where the retailer report the forecast information before the supplier deciding the wholesale price and capacity, by the form of “cheap-talk” (cost-less, non-binding, and non-verifiable communication) which is common in industry (Aviv 2003; Holmström et al. 2002). We apply the trust-embedded model, as a more behaviorally realistic model formulated in Özer, Zheng, and Chen (2011), to investigate the second research question “Is trust beneficial to the supply chain?” The game sequence and assumptions of the trust-embedded model are presented in subsection 3.5.1.

3.4 Is Information Beneficial To The Supply Chain?

In this section, we investigate the role of information sharing in the supply chain, to answer the first research question, “’Is information beneficial to the supply chain?”. Information can help the supplier to set a “right” capacity decision and remedy the capacity misalignment problem (*CM* problem), well-studied in the literature. The new insight, a focus of the analysis in this section, is that information can potentially exacerbate the double marginalization problem (*DM* problem). In particular, we first analyze a *DM-only* scenario, where only the *DM* problem exists in the supply chain, by setting the capacity cost to zero. This eliminates the *CM* problem, since the supplier can always provide enough capacity without any capacity cost and the retailer has no incentive to exaggerate her report any more.

Then we analyze the *CM-only* scenario, where only the *CM* problem exists in the supply chain, by setting an exogenous wholesale price. In this case, we expect the wholesale price and thus the *DM* problem no longer interacts with the information, because the wholesale price is exogenously fixed. Since the *CM-only* problem is already well studied in the literature and our conclusion is consistent with the past literature (i.e., information is always beneficial to the *CM* problem), we are omitting a full analysis in this section and provide that in the Appendix 3.7.5 for reference.

The most important part of this section is an analysis of information’s trade-off between the *DM* and the *CM* problem, when both are in play. The model, given the complexity of the interactions of the two problems, is not fully tractable [3]. We are able to provide closed-form solutions for a special case of when the "amount" of private information is "small". We augment this analytical analysis with numerical studies and show the robustness of the conclusion. Please see subsection 3.4.5, for a summary of all the analysis cases in this section.

3.4.1 Complete vs. Incomplete Information

We aim to study the role of information sharing in the double marginalization and capacity misalignment problem, by artificially comparing the theoretical expected supply chain profits under the condition where the supplier has the access to the private forecast information ξ (complete information condition) and the supply chain efficiency under the condition where the supplier does not have the access to the private forecast information ξ (incomplete information condition). We define information as “beneficial" if the complete information condition results in a higher supply chain expected profit.

In the last stage of the game, given the wholesale price w and capacity K the supplier decides, the retailer chooses an optimal order quantity q to maximize his

[3] please see Appendix 3.7.4 for the reason why the model is not fully tractable.

expected profit under the capacity constraint. The retailer faces the following maximization problem.

$$\max_{q} \quad \Pi^r(q,\xi) = p\mathbb{E}_\epsilon \min(\mu+\xi+\epsilon,q) - wq$$
$$s.t. \quad q \le K \tag{3.1}$$

where $\Pi^r(q,\xi)$ is retailer's expected profit. Given w and K, retailer's best response is:

$$q^*(w,K,\xi) = \begin{cases} \xi+\mu+G^{-1}(\frac{p-w}{p}) & w \le p, K > \xi+\mu+G^{-1}(\frac{p-w}{p}) \\ K & w \le p, K \le \xi+\mu+G^{-1}(\frac{p-w}{p}) \\ 0 & w > p \end{cases} \tag{3.2}$$

1. Complete Information Condition

If the supplier has access to the retailer's private forecast information ξ, i.e., when information is complete, the supplier chooses the wholesale price w and capacity K to maximize his profit. By backward induction, the maximization problem for the supplier is:

$$\max_{w,K} \quad \Pi^s_{ci}(w,K,\xi) = (w-c)q^*(w,K,\xi) - c_k K \tag{3.3}$$

where $\Pi^s_{ci}(w,K,\xi)$ is supplier's profit under complete information condition. Note that we index the complete information scenario by "ci".

The equilibrium under this complete information condition is summarized in lemma 3.4.1 (see proof in Appendix 3.7.2).

In equilibrium of a decentralized supply chain with complete information:

(1) the supplier chooses the wholesale price and capacity decision as: $w^*_{ci} = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) > c+c_k$, $K^*_{ci} = \xi+\mu+G^{-1}(\frac{p-w^*_{ci}}{p})$

(2) the retailer chooses the order quantity: $q^*_{ci} = \xi+\mu+G^{-1}(\frac{p-w^*_{ci}}{p})$.

where $w^*_{ci}, K^*_{ci}, q^*_{ci}$ are the wholesale price, capacity decision, and the order quantity in equilibrium when information is complete.

2. Incomplete Information Condition

If the supplier cannot get access to the private forecast information ξ, i.e., when information is incomplete, the supplier chooses the wholesale price w and capacity decision K to maximize his expected profit:

$$\max_{w,K} \quad \Pi^s_{ii}(w,K) = (w-c)\mathbb{E}_\xi q^*(w,K,\xi) - c_k K \tag{3.4}$$

where $\Pi^s_{ii}(w,K,\xi)$ is supplier's expected profit (with respect to ξ) under incomplete information condition. The equilibrium under this incomplete information is summarized in Proposition 17 (see proof in Appendix 3.7.3).

Proposition 17 *If ξ follows a uniform distribution, i.e., $\xi \sim U[\underline{\xi},\bar{\xi}]$, in equilibrium of a decentralized supply chain with incomplete information:*

(1) the supplier chooses the wholesale price and capacity: $w^*_{ii} = \min(w^*_r, p) > c + c_k$, $K^*_{ii} = \xi'^* + \mu + G^{-1}(\frac{p-w^*_{ii}}{p})$, where $\xi'^* = F^{-1}(\frac{w^*_{ii}-c-c_k}{w^*_{ii}-c}) \in [\underline{\xi},\bar{\xi}]$, and w^*_r is the unique real root, satisfying $c+c_k < w^*_r < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$, of equation $-2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}(w^*_r-c)^3 + \frac{p(\mu+\bar{\epsilon})-(\bar{\epsilon}-\underline{\epsilon})(c-c_k)}{p}(w^*_r-c)^2 + c_k^2\underline{\xi} = 0$.

(2) *the retailer with type* ξ *chooses order quantity* $q_{ii}^* = \xi + \mu + G^{-1}\big(\frac{p-w_{ii}^*}{p}\big)$*, if* $\xi < \xi'^*$*;* $q_{ii}^* = K_{ii}^*$*, if* $\xi \geq \xi'^*$*.*

(3) *when* $-\mu + \big(\frac{p-c-c_k}{p} + 0.5\big)\big(\bar{\epsilon} - \underline{\epsilon}\big) - \overline{\xi} \geq 0$*, the wholesale price under incomplete information condition is lower than the expected value (with respect to* ξ*) of the wholesale price under complete information,i.e.,* $w_{ii}^* < \mathbb{E}_{\xi}(w_{ci}^*)$*.*

where $w_{ii}^*, K_{ii}^*, q_{ii}^*$ *are the wholesale price, capacity decision, and the order quantity in equilibrium when information is incomplete.*

3.4.2 The Capacity Misalignment and Double Marginalization Problem

Lemma 3.4.1 and Proposition 17 point to two resources of inefficiency in a decentralized supply chain. **The first type of inefficiency comes from the "wrong" decision of capacity. Information can remedy this capacity misalignment problem (*CM* Problem).** When the supplier knows the private forecast information ξ, he can decide the capacity exactly according to ξ, since K_{ci}^* is a function of ξ and always equal to the order quantity. When information is incomplete, however, capacity decision K_{ii}^* is independent of the true private information type ξ. Without private information, the supplier can build too much or too less capacity, and harm the supply chain efficiency. Specifically, when $\xi'^* > \xi$, the supplier build too much capacity (the capacity is more than what the retailer with type ξ needs), which harms the supply chain efficiency since some capacity are "wasted". When $\xi'^* < \xi$, the supplier build too little capacity (the capacity is less than what the retailer with type ξ needs), resulting in forgone order quantity from the retailer, which also harms the supply chain efficiency. Therefore, information can always help to eliminate the capacity misalignment problem.

The second type of inefficiency comes from the double marginalization problem (*DM* Problem). Information can potentially exacerbate this problem. Note that the wholesale price under complete and incomplete condition (w_{ii}^* and w_{ci}^*) are both larger than $c + c_k$, thus the *DM* Problem exists in a decentralized supply chain under both conditions.[4] However, from Proposition 17, when the condition "$-\mu + \big(\frac{p-c-c_k}{p} + 0.5\big)\big(\bar{\epsilon} - \underline{\epsilon}\big) - \overline{\xi} \geq 0$" is satisfied, the expected wholesale price under incomplete information condition is lower than that under complete information condition, which implies information can increase the wholesale price "averagely" and thus potentially worsen the double marginalization problem. Since this condition is crucial for the role of information in *DM* problem, it motivates us to define and interpret it in subsection 3.4.3.

3.4.3 The role of information in the *DM* problem: Double Marginalization Index (*DMI*)

In order to characterize the condition of when and how information is harmful or beneficial to the supply chain regarding the *DM* problem, we define the ***Double Marginalization Index (DMI)***:

$$DMI \equiv -\mu + \big(\frac{p-c-c_k}{p} + 0.5\big)\big(\bar{\epsilon} - \underline{\epsilon}\big) - \overline{\xi}$$

[4] We analyze a centralized supply chain where the *DM* problem doesn't exist, and compare this "first best" order quantity with that in a decentralized supply, to further illustrate the existence of the *DM* problem in a decentralized supply chain. But we put all these analysis in Appendix 3.7.1, since the emphasis of this section is the role of information in the *DM* problem instead of the *DM* problem itself.

where μ is a constant denoting the mean demand, $\frac{p-c-c_k}{p}$ is the profit margin, $(\bar{\epsilon}-\underline{\epsilon})$ is the market uncertainty, and $\bar{\xi}$ is the upper bound of the private information. The *DMI* is directly motivated by proposition 17 part (3) which implies information can potentially exacerbate the *DM* problem under the condition $-\mu+\left(\frac{p-c-c_k}{p}+0.5\right)(\bar{\epsilon}-\underline{\epsilon})-\bar{\xi}\geq 0$.

In order to formally capture when and how *DMI* can affect the role of information in the *DM* problem, we consider a special case where the capacity cost is zero. When capacity cost is zero, there is no *CM* problem in supply chain because the supplier can simply set capacity to maximum demand without any capacity costs. We are able to show when and how information can worsen the *DM* problem in Proposition 18 (See proof in Appendix 3.7.6).

Proposition 18 *In equilibrium of a decentralized supply chain with no capacity costs:*

(1) *If* $DMI \geq 0$*, information is harmful to the total supply chain efficiency (expected profit with respect to* ξ*), i.e.,* $\mathbb{E}_{\xi}\pi^{sc}_{ci}(\xi) < \mathbb{E}_{\xi}\pi^{sc}_{ii}(\xi)$.

(2) *If* $2\underline{\xi} < DMI \leq \underline{\xi}$*, information is beneficial to the total supply chain efficiency, i.e.,*$\mathbb{E}_{\xi}\pi^{sc}_{ci}(\xi) > \mathbb{E}_{\xi}\pi^{sc}_{ii}(\xi)$;

(3) *If* $DMI \leq 2\underline{\xi}$*, information cannot affect the supply chain efficiency, i.e.,* $\mathbb{E}_{\xi}\pi^{sc}_{ci}(\xi) = \mathbb{E}_{\xi}\pi^{sc}_{ii}(\xi)$;

where $\mathbb{E}_{\xi}\pi^{sc}_{ci}(\xi)$, $\mathbb{E}_{\xi}\pi^{sc}_{ii}(\xi)$ *is the expected profit (with respect to* ξ*) of the total supply chain under compete and incomplete information condition respectively.* [5]

The results of Proposition 18 are illustrated in Figure S2, where *DMI* can decide whether information is beneficial or harmful to the *DM* problem. **First, if *DMI* is high (i.e.,** $DMI \geq 0$**), information is harmful to *DM* problem.** Since *DMI* is increasing with market uncertainty $(\bar{\epsilon}-\underline{\epsilon})$ and profit margin $\frac{p-c-c_k}{p}$, $DMI \geq 0$ implies both market uncertainty and profit margin are high compared with the private forecast information ξ. The high profit margin allows enough room, for the supplier, to adjust wholesale price. The high market uncertainty allows enough room for the retailer to react to the wholesale price. Thus, when $DMI \geq 0$, the supplier has enough "room" to take advantage of the forecast information, if shared, to double marginalize more. Therefore, information can worsen the *DM* problem and harm the supply chain efficiency.

Second, if *DMI* is low (i.e., $DMI \leq -2\bar{\xi}$**), making the private forecast information available to the supplier cannot affect the *DM* problem (i.e., information has no effect on the *DM* problem)**. $DMI \leq 2\underline{\xi}$ implies that market uncertainty or profit margin is low, compared with private information. In this case, the supplier will always set the wholesale price to the fixed market price p, no matter if he can get access to the private forecast information or not (under complete information condition or incomplete information condition), because either the supplier doesn't have enough "room" to adjust wholesale price due to low profit margin, or the retailer doesn't react to wholesale price due to low market uncertainty. Therefore, the double marginalization problem will keep constant no matter whether information is shared or not, since the wholesale price is always the fixed market price p. In other words, information has no effect on supply chain efficiency regarding the *DM* problem.

[5] We also investigated the role of information in supplier and retailer's expected profit (with respect to ξ) in equilibrium, and put the results in Appendix 3.7.6.

Third, if *DMI* is middle (i.e., $-2\overline{\xi} < DMI \leq -\overline{\xi}$), information is beneficial to the DM problem. According to the definition of DMI, we have $\mu + \underline{\xi} < \left(\frac{p-c-c_k}{p} + 0.5\right)\left(\overline{\epsilon} - \underline{\epsilon}\right) \leq \mu + \frac{\overline{\xi}+\underline{\xi}}{2}$, where $\overline{\xi} + \underline{\xi} = 0$ (ξ is a zero-mean random variable). On the one hand, $\left(\frac{p-c-c_k}{p} + 0.5\right)\left(\overline{\epsilon} - \underline{\epsilon}\right) \leq \mu + \frac{\overline{\xi}+\underline{\xi}}{2}$ means that the market uncertainty and profit margin are "averagely" low (the average of the private information). Then when the supplier does not have access to the forecast information (i.e., when information is incomplete), he "averagely" sets the wholesale price $w_{ii}^* = p$ for all types of retailer. On the other hand, $\mu + \underline{\xi} < \left(\frac{p-c-c_k}{p} + 0.5\right)\left(\overline{\epsilon} - \underline{\epsilon}\right)$ means that with private information supplier has enough "room" to set different wholesale price for the retailer with low types of ξ. Therefore, the retailer with low type of private forecast information gets a lower wholesale price under complete information condition than that under incomplete information condition (note $w_{ii}^* = p$). Therefore, information can reduce the wholesale price and benefit the DM problem.

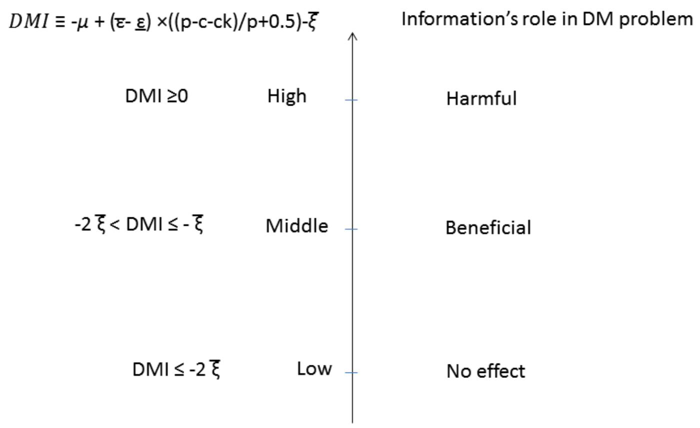


FIGURE S2: The role of information in the *DM* problem

3.4.4 Information's trade-off in the *DM* and *CM* problem

In this subsection, we investigate information's role in the supply chain integrating the DM and CM problem, to address the first research question, "Is information beneficial to the supply chain ?" The main conclusion of this part is summarized as following.

Main Finding: When DMI is high and capacity cost c_k is small [6]***, information is harmful to the total supply chain efficiency, trading off its harmful effect on the DM problem and its small beneficial effect on the CM problem. In all other cases, information is beneficial, consistent with the traditional wisdom.***

[6] According to the definition of *DMI*, high *DMI* implies low capacity cost. However, in order to illustrate information's role in the two problem respectively, we write the condition that "DMI is high" as "DMI is high and capacity cost c_k is small"

On the one hand, based on Proposition 18, when DMI is high, information can be harmful to the DM problem. On the other hand, information is always beneficial to the CM problem, but its beneficial effect is small when capacity cost c_k is small (Please refer to Appendix 3.7.5 for the analysis and proof.). For instance, when capacity cost is zero, there is no CM problem and thus information's role in this problem is also zero. Therefore, when DMI is high and capacity cost c_k is small, information needs to balance between its harmful effect on the DM problem and its small beneficial effect on the CM problem. Thus the total effect of information, trading off these two problems, is harmful.

In all other cases, either information has no effect or benefits the DM problem (when DMI is not high), or the beneficial effect of information on the CM problem out-weights its effect on the DM problem (when capacity cost is high). Therefore, information is beneficial to the supply chain efficiency.

(1) Analytical support

Ideally, we want to capture the effect of information on supply chain efficiency in terms of the CM problem and DM problem simultaneously. Since the general formulation, as far as we know, is not fully analytically tractable, studying a case with the "amount of" private information approaching zero allows us to linearize part of the formulation (specifically the wholesale price) and arrives at an analytic solution.[7]. Proposition 19 formally summarizes the role of information in supply chain efficiency in this setting (See proof in Appendix 3.7.7).

Proposition 19 *When the supply chain has "a small amount of" private information, i.e., $\overline{\xi} \to 0^+$, in equilibrium of a decentralized supply chain: if $DMI \geq 0$, and $c_k = o(\overline{\xi}^N)$ is the N-order infinitesimal of $\overline{\xi}$ (where N is a positive integer and $N \geq 3$), then information is harmful to the supply chain efficiency; ie., when $\overline{\xi} \to 0^+$, $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) < \mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$ [8]; $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi)$ and $\mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$ are the total supply chain's expected profit (with respect to ξ) under complete information condition and incomplete information condition respectively.*

On the one hand, when $DMI \geq 0$, information is harmful to the DM problem, based on proposition 18. On the other hand, since capacity cost is a high order infinitesimal of private information (i.e., $c_k = o(\overline{\xi}^N)$ and $N \geq 3$), capacity cost's speed of approaching to zero is much faster than that of private information. This implies that capacity cost and the role of information in the CM problem is much "smaller" [9]. Therefore, information is harmful to the total supply chain efficiency, trading off these two effects.

(2) Numerical support 1

Numerical results in table S1 further support the main finding (See specific graphs in Appendix 3.8). From subsection 3.4.3, information's role in the DM problem is decided by DMI, which is further decided by the comparison of the market uncertainty ($\overline{\epsilon} - \underline{\epsilon}$), profit margin ($\frac{p-c-c_k}{p}$), and incomplete information range ($\overline{\xi} - \underline{\xi}$). On the other hand, from Appendix 3.7.5, information's role in the DM problem is

[7] In this setting, we can use the Taylor formula to get the analytical solution, by linearizing the wholesale price and omitting the higher order infinitesimal.

[8] Formally, $\exists\, \sigma > 0$, s.t. $\forall \overline{\xi} \in (0, \sigma)$, we have $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) < \mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$. We also investigated the role of information in supplier and retailer's efficiency when the amount of information is "small" and put the results in Appendix 3.7.7.

[9] In this case, when the "amount" of private information is approaching zero (i.e., $\overline{\xi} \to 0$), the role of information in both the DM and CM problem are approaching to zero (note that when $\overline{\xi} = 0$, incomplete information condition is exactly the same as the complete information). Since the capacity cost is very small (its speed of approaching zero is much faster than private information), information's role in the the CM problem is "smaller" than that in the DM problem.

mainly decided by c_k. Therefore, we can manipulate these measures to investigate how they affect information's trade-off between these two problems. Since we fix the market price p and production cost c to make their difference $(p - c)$ high, we can manipulate the profit margin $(\frac{p-c-c_k}{p})$ by changing capacity cost c_k. Therefore, in Table S1 we set the magnitude of market uncertainty $(\bar{\epsilon} - \underline{\epsilon})$, incomplete information $(\bar{\xi} - \underline{\xi})$ to be high and low (eight different cases in total), in order to investigate how they affect information's trade-off in these two problems.

When capacity cost c_k is low (profit margin is high) and market uncertainty is high, DMI is high, and thus information can worsen the DM problem, based on Proposition 18. On the other hand, information can always help the CM problem, but its beneficial effect is small when capacity cost is low, based on Appendix 3.7.5. Trading off these two effects, information is harmful to total supply chain efficiency when capacity cost is low and market uncertainty is high, as we can see in Figure S9[10] and Figure S12.

In all other cases, because either profit margin is low (capacity cost is high) or market uncertainty is low, the supplier has no "room" to double marginalize and always set the wholesale price w to the market price p under both incomplete information and complete information condition [11]. Therefore, information has no effect in the DM problem. Moreover, information is always beneficial to the CM problem. Therefore, information is beneficial to the supply chain integrating these two problems, as we can see in Figures S6, S7, S8, S10, S11, and S13.

[10]In Figure S9, because information is always beneficial to the CM problem, and its total effect on supply chain integrating the CM and DM problem is harmful, based on the numerical analysis, information's effect on the DM problem has to be harmful,otherwise the total effect cannot be harmful (this is also the reason why information is harmful to the DM problem in Figure S12). Note that $DMI \geq 0$ is a sufficient condition for information being harmful to the DM problem. The DMI in Figure S9 is $-21.4 < 0$, but the role of information in the DM problem is still harmful . This is because profit margin and market uncertainty are high enough so that the supplier has "room" to double marginalize more under complete information condition.

[11]In the numerical analysis, the wholesale prices in equilibrium under the two conditions are always market price p, independent of private information ξ.

TABLE S1: Information's trade-off in the two problems

Market Uncertainty: $(\bar{\epsilon}-\underline{\epsilon})$	High	High	Low	High	Low	Low	High	Low
Private Information: $(\bar{\xi}-\underline{\xi})$	High	Low	High	High	Low	High	Low	Low
Capacity Cost: c_k	High	High	High	Low	High	Low	Low	Low
$DMI =$	-380	-220	-556	**-21.4**	-396	-484.28	**138.6**	-324.28
The role of information in *DM* problem	No effect	No effect	No effect	**Harmful**	No effect	No effect	**Harmful**	No effect
The role of information in *CM* problem	Beneficial	Beneficial	Beneficial	Beneficial	Beneficial	Beneficial	Beneficial	Beneficial
The role of information in supply chain efficiency	Beneficial Figure S6	Beneficial Figure S7	Beneficial Figure S8	**Harmful** Figure S9	Beneficial Figure S10	Beneficial Figure S11	**Harmful** Figure S12	Beneficial Figure S13

[1] By setting the range of market uncertainty $(\bar{\epsilon}-\underline{\epsilon})$, incomplete information $(\bar{\xi}-\underline{\xi})$, and capacity cost c_k to be high or low, we studied eight different cases. In all the cases, we fix market price $p = 200$, production cost $c = 10$, and mean demand $\mu = 400$. For the "Low" capacity cost, $c_k = 0.7$. For the "High" capacity cost, $c_k = 180$. For the "High" market uncertainty, $(\bar{\epsilon}-\underline{\epsilon}) = 400$. For the "Low" market uncertainty, $(\bar{\epsilon}-\underline{\epsilon}) = 80$. For the "High" incomplete information, $(\bar{\xi}-\underline{\xi}) = 400$. For the "Low" incomplete information, $(\bar{\xi}-\underline{\xi}) = 80$. We can choose the market price p to be an another number (for instance $p = 2000$), but it doesn't affect the results if we scale the production cost c and capacity cost c_k accordingly. Based on the market price p, we choose the production cost c to keep $p-c$ high, and capacity cost c_k to be high or low. Moreover, we choose the mean demand $\mu = 400$. Then based on this, we choose the market uncertainty $(\bar{\epsilon}-\underline{\epsilon})$ and private information $(\bar{\xi}-\underline{\xi})$ to be high or low.

Numerical support 2

In Table S1, we discretely change the variables that can affect information's role in the *CM* and *DM* problem. In Figure S3, however, we continuously change these variables to characterize the condition when information, trade-offing these two problems, is harmful to the total supply chain efficiency. Specifically, we fix the private information range $(\bar{\xi}-\underline{\xi})$, mean demand μ. Moreover, we fix the market price p and production cost c to keep their difference $(p-c)$ high. Therefore, we can manipulate the capacity cost c_k in the y-axis to change the profit margin $\frac{p-c-c_k}{p}$. On the other hand, market uncertainty $(\bar{\epsilon}-\underline{\epsilon})$ is manipulated in the x-axis.

When market uncertainty is high and capacity cost is low (this implies *DMI* is high [12]), information is harmful to the total supply chain efficiency. When market uncertainty is high and capacity cost is low, the supplier has more "room" to take advantage of the forecast information, if shared, to double marginalize more. Therefore, information is harmful to the *DM* problem. On the other hand, information is beneficial to the *CM* problem, but its effect is inconsequential since the capacity cost c_k is low. Therefore, information is harmful to the total supply chain, trading off these two effects.

[12] "capacity cost is low" implies that profit margin is high, since we fix $(p-c)$ to be high. Because private information range is fixed and both profit margin and market uncertainty are high, *DMI* is high according to its definition.

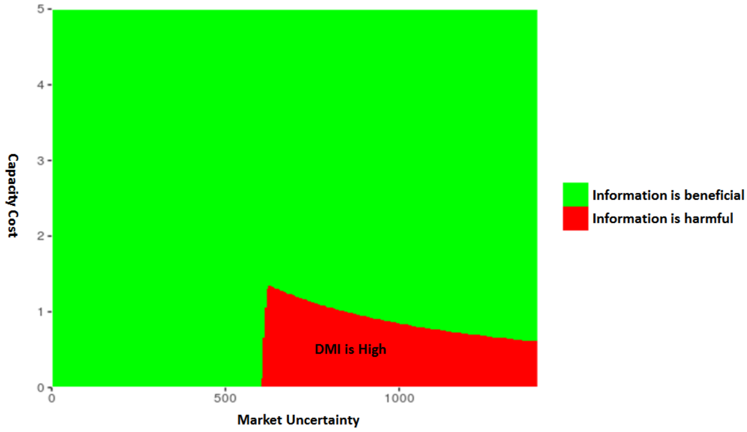


FIGURE S3: Information's role in supply chain trading off the *CM* and *DM* problem.

The red (green) part of the heat-map suggests information is harmful (beneficial) to the supply chain. We fix $p = 200, c = 10, \mu = 800, \underline{\xi} = -100, \overline{\xi} = 100$, and assume ξ and ϵ follow uniform distribution with zero mean. In the horizontal axis, we change the market uncertainty $(\overline{\epsilon} - \underline{\epsilon}) \in [0, 1400]$. The vertical axis is the capacity cost c_k. We can choose the market price p to be an another number (for instance $p = 2000$), but it doesn't affect the results if we scale the production cost c and capacity cost c_k accordingly. Based on the market price p, we choose the production cost c to keep $p - c$ high. Moreover, we choose the mean demand $\mu = 800$ to make sure $D_{min} = \mu + \underline{\xi} + \underline{\epsilon} \geq 0$ for all the manipulated market uncertainty.

In all other cases, either market uncertainty is low or capacity cost is high. On the one hand, due to the low market uncertainty or low profit margin (high capacity cost), supplier with the shared information doesn't has enough "room" to double marginalize more. Therefore, information cannot harm the *DM* problem. On the other hand, information can always help the *CM* problem. Therefore, information is beneficial to the supply chain, integrating these two problems.

3.4.5 Analysis Cases

In this subsection, we summarize in Table S2 all the analysis cases regarding the role of information in the supply chain efficiency (the first research question).

TABLE S2: The role of information sharing –analysis cases

Model Environment	Incentive Problem(s) studied	Method	The role of information Sharing
Decentralized supply chain	*CM* problem and *DM* problem	Analytical Partially solved	Beneficial/harmful See Proposition 17
Decentralized supply chain: Wholesale price is exogenous	*CM* problem only	Analytical Fully solved	Always beneficial See Appendix 3.7.5
Decentralized supply chain: Capacity cost is zero	*DM* problem only	Analytical Fully solved	Beneficial/harmful See Proposition 18
Decentralized supply chain: "A small amount of" private information	*CM* problem and *DM* problem	Analytical Fully solved	Can be harmful See Proposition 19
Decentralized supply chain	*CM* problem and *DM* problem	Numerical	Beneficial/harmful See Table S1 and Figure S3

3.5 Is Trust Beneficial To the Supply Chain?

In this section, we try to answer the second research question "Is trust beneficial to the supply chain?" by applying the trust-embedded model (Özer, Zheng, and Chen 2011). Firstly, we specify the assumption and setting of the trust-embedded model. Similar to section 3.4, we analyze trust's role in the *DM* and *CM* problem respectively, by providing an analytical solution for these two problems in isolation. Again, we analyze the *DM-only* scenario by setting the capacity cost to zero, and the *CM-only* scenario by using an exogenous wholesale price. Since the general trust-embedded model incorporating the *DM* and *CM* problem is not tractable [13], we provide a special case where the capacity cost is "small" augmented with numerical analysis, to address trust's trade-off between these two problems. Please see subsection 3.5.5 for a summary of all cases of analysis.

3.5.1 Trust-embedded Model Setting

In this section, we expand the analysis to investigate the role of trust in the supply chain efficiency, applying the trust-embedded model (Özer, Zheng, and Chen (2011)). Specifically, we make the following assumptions for the trust-embedded model: (1) The private demand forecast information ξ is uniformly distributed on $[\underline{\xi}, \overline{\xi}]$, which is supplier's prior belief about ξ. (2) After receiving retailer's report $\hat{\xi}$ in the first stage of the game, the supplier trusts the report to some extent. Specifically, supplier's posterior belief of ξ has the same distribution as $\alpha^s\hat{\xi} + (1-\alpha^s)\xi$ [14], where $0 \leq \alpha^s \leq 1$ denotes the supplier's degree of trust. We denote the distribution function of this posterior belief as F_t, the corresponding density function as f_t. (3) When the retailer with private type ξ reports $\hat{\xi}$, she gets some dis-utility $-\beta|\hat{\xi} - \xi|$, where $\beta \geq 0$ denotes the degree of trustworthiness of the retailer. The higher β is, the more trustworthy the retailer is.

In the trust model, the sequence of events is as follow: (1) Nature decides the private forecast information ξ, privately known to the retailer. (2) The retailer observes her private forecast information ξ, and reports it as $\hat{\xi}$. (3) The supplier updates his belief of ξ based on the above mentioned assumptions, and then decides the wholesale price w and the capacity K. (4) The retailer with private information ξ decides the order quantity q with the constraint $q \leq K$, then market demand $D = \mu + \xi + \epsilon$ is realized, and sales of $\min(D, q)$ are sold to the end customers at fixed market price p.

In the last stage of the game, given the wholesale price w and capacity K the supplier decides, the retailer chooses an optimal q to maximize his expected profit under the capacity constraint. The retailer faces the same maximization problem as (3.1) and the retailer's best response will be the same as (3.2). Similar to the incomplete information scenario in the general model in section 3.4, the supplier chooses

[13] Please see Appendix 3.7.4 for details about the reason why the model is not tractable.

[14] In our setting, because the retailer has the incentive to under-report to lower the wholesale price, the supplier can no longer eliminate all the possibilities that retailer's true state is higher than the reporting (i.e., under-reporting). Therefore, we assume the supplier's posterior belief of ξ is a combination of the reporting (the supplier trusts retailer's reporting to some extent) and the prior belief of the type (the true type can be higher or lower than the reporting, since the retailer has incentive of both under-reporting and over-reporting). However, from the trust-embedded model in Özer, Zheng, and Chen (2011), the supplier's posterior belief of ξ has the same distribution as $\alpha^s\hat{\xi} + (1-\alpha^s)\xi^T$, where ξ^T has the distribution of ξ truncated on $[\underline{\xi}, \hat{\xi}]$. In other words, they assume that the supplier's posterior belief of the type of retailer is a combination of the reporting (the supplier trusts the reporting to some extent) and ξ^T (the supplier believes that the true type of retailer can only be lower than the reporting, since the retailer only has incentive of over-reporting to ensure more capacity).

the wholesale price w and capacity K to maximize his expected profit according to the maximization problem (3.4) with the posterior belief of ξ after receiving retailer's report $\hat{\xi}$.

The retailer chooses her reporting strategy to maximize her expected profit, by backward induction.

$$\begin{aligned} \max_{\hat{\xi}} \quad & \Pi_t^r(\hat{\xi}) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q^*(w(\hat{\xi}), K(\hat{\xi}), \xi)) - w(\hat{\xi})q^*(w(\hat{\xi}), K(\hat{\xi}), \xi) - \beta|\hat{\xi} - \xi| \\ s.t. \quad & \hat{\xi} \in [\underline{\xi}, \overline{\xi}] \end{aligned} \tag{3.5}$$

where $\Pi_t^r(\hat{\xi})$ is retailer's expected profit if she reports $\hat{\xi}$; $w(\hat{\xi})$ and $K(\hat{\xi})$ are supplier's wholesale price and capacity response respectively, given retailer's report $\hat{\xi}$; $q^*(w(\hat{\xi}), K(\hat{\xi}), \xi))$ is retailer's best response in the last stage of the game, given her private type is ξ; $-\beta|\hat{\xi} - \xi|$ is the dis-utility the retailer with type ξ gets if she reports $\hat{\xi}$. Since retailer's optimal decision of reporting depends on her private type ξ, we denote the solution of this optimization problem as $\hat{\xi}^*(\xi)$.

3.5.2 The role of trust in the DM problem

In this case, we set capacity cost $c_k = 0$. The *CM* problem doesn't exist, because the supplier can always set infinite capacity without any costs. Therefore, we can investigate the effect of trust on the *DM* problem in this setting, without considering the *CM* problem. The results for the role of trust in the *DM* problem is summarized in Proposition 20 (See proof in Appendix 3.7.9).

Proposition 20 *In equilibrium of the trust-embedded model with no capacity cost:*

(a) When $DMI \geq 0$, the effect of trust and trustworthiness is summarized in the following table:

TABLE S3: The effect of trust and trustworthiness on the *DM* problem: $DMI \geq 0$

	$0 < \alpha^s < 1$	$\alpha^s = 0$	$\alpha^s = 1$
When $\beta \geq \frac{(\mu+\overline{\epsilon}+2\overline{\xi})p-(\overline{\epsilon}-\underline{\epsilon})c}{4(\overline{\epsilon}-\underline{\epsilon})}$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = \frac{-2p\alpha^s}{8(\overline{\epsilon}-\underline{\epsilon})} var(\xi) < 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ci}^{sc}(\xi$
When $\beta = 0$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = -\underline{\xi} \cdot \frac{(\mu+\overline{\epsilon}+\alpha^s\underline{\xi})p-(\overline{\epsilon}-\underline{\epsilon})c}{4(\overline{\epsilon}-\underline{\epsilon})} > 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) > \mathbb{E}_\xi \pi_{ci}^{sc}(\xi$

$\mathbb{E}_\xi \pi_t^{sc}(\xi)$ is the total supply chain's expected profit with respect to ξ in equilibrium of the trust-embedded mo
$\mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$ is the expected profit with respect to ξ when information is complete. $\mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$ is the expected profit w
respect to ξ when information is incomplete.

(b) When $2\underline{\xi} < DMI \leq \underline{\xi}$, the effect of trust and trustworthiness is summarized in the following table:

TABLE S4: The effect of trust and trustworthiness on the *DM* problem: $2\underline{\xi} < DMI \leq \underline{\xi}$

	$\frac{\xi_t}{\underline{\xi}} < \alpha^s < 1$	$0 \leq \alpha^s \leq \frac{\xi_t}{\underline{\xi}}$	$\alpha^s = 1$
When $\beta \geq \frac{(\mu+\bar{\epsilon}+3\bar{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}} -\frac{\xi}{2} \cdot \frac{(\mu+\bar{\epsilon}+\alpha^s\xi)p-(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} dF(\xi) > 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$
When $\beta = 0$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = -\underline{\xi} \cdot \frac{(\mu+\bar{\epsilon}+\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) > \mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$

We define $\xi_t \equiv DMI + \bar{\xi} \in (\underline{\xi}, 0]$; $\mathbb{E}_\xi \pi_t^{sc}(\xi)$, is the total supply chain's expected profit with respect to ξ in equilibrium of the trust-embedded model. $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$ is the expected profit with respect to ξ when information is complete. $\mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$ is the expected profit with respect to ξ when information is incomplete.

(c) *When $DMI \leq 2\underline{\xi}$, trust has no effect on the supply chain efficiency, i.e., $\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = 0$, where $\mathbb{E}_\xi \pi_t^{sc}(\xi)$, is the total supply chain's expected profit with respect to ξ in equilibrium of the trust-embedded model.* [15]

When information is harmful to the *DM* problem, trusting a trustworthy retailer can worsen the *DM* problem, and decrease the supply chain efficiency. As we can see from Proposition 20 part (a), when β is high (i.e., $\beta \geq \frac{(\mu+\bar{\epsilon}+2\bar{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$), the trustworthy retailer always reports the true private information, i.e., $\hat{\xi}^*(\xi) = \xi$. On the other hand, when $DMI \geq 0$, based on Proposition 18, information is harmful to *DM* problem. Hence, higher trust level can bring more harmful information, and thus harm the *DM* problem and the supply chain efficiency.

When information is beneficial to the *DM* problem, trusting a trustworthy retailer can reduce the *DM* problem, and increase the supply chain efficiency. As we can see from Proposition 20 part (b), when β is high (i.e., $\beta \geq \frac{(\mu+\bar{\epsilon}+3\bar{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$), the trustworthy retailer always reports the true private information, i.e., $\hat{\xi}^*(\xi) = \xi$. On the other hand, when $2\underline{\xi} < DMI \leq \underline{\xi}$, based on Proposition 18, information is beneficial to *DM* problem. Hence, the higher trust level can make the supplier trust this beneficial information more, leading to more beneficial information shared. This can benefit the *DM* problem and the supply chain efficiency.

Trusting an untrustworthy retailer reduces the *DM* problem, and increases the supply chain efficiency. As we can see from Proposition 20 part (a) and (b), when $\beta = 0$, the untrustworthy retailer can manipulate the wholesale price down without any moral costs, therefore, the retailer will always report the lowest type to reduce the wholesale price, i.e., $\hat{\xi}^* = \underline{\xi}$. When DMI is not too small (i.e, $DMI > -2\bar{\xi}$), there is always some room (profit margin and market uncertainty room) for the retailer to manipulate the wholesale price down. This manipulated information can reduce the wholesale price and the DM problem, which is beneficial to the supply chain efficiency. With higher trust level, more beneficial information (although it is manipulated information) will be shared, which leads to higher supply chain efficiency.

When information has no effect on the *DM* problem, trust has no effect on the *DM* problem. As we can see from Proposition 20 part (c), when $DMI \leq -2\bar{\xi}$, the retailer cannot manipulate the wholesale price down, because the supplier always sets the wholesale price to fixed market price p, no matter the trust level α^s is. Therefore, trust cannot affect the *DM* problem in this case.

[15] For trust and trustworthiness's effect on supplier and retailer's expected profit (with respect to ξ), we put it in Appendix 3.7.9.

3.5.3 The role of trust in the CM problem

Similar to section 3.7.5, we consider the trust-embedded model with exogenous wholesale price, to investigate the role of trust in the *CM* problem. Solving the equilibrium of the game, we can get the following proposition 21, summarizing the role of trust in the *CM* problem (see proof in Appendix 3.7.8):

Proposition 21 *If the wholesale price $w \in [c + c_k, p]$ is exogenous and $\beta > 0$, in the equilibrium of the trust-embedded model:*

(1) For every realization of the private information ξ satisfying $\xi \le \underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})$, the supply chain's expected profit (with respect to ϵ) is independent of α^s.

(2) For every realization of the private information ξ satisfying $\xi > \underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})$, the supply chain's expected profit (with respect to ϵ) strictly increases with trust level α^s i.e., for any α_1^s, α_2^s satisfying $0 \le \alpha_1^s < \alpha_2^s \le 1$, we have $\pi_t^{sc}(\alpha_1^s) < \pi_t^{sc}(\alpha_2^s)$.
where $\pi_t^{sc}(\alpha^s)$ is the expected supply chain profit for a certain realization of ξ when the trust level is α^s under the trust embedded model [16].

Higher trust level makes the retailer with high type of private forecast information easier (take less moral costs) to get enough capacity, and thus increases the supply chain efficiency. [17] When the supplier trusts 100% of retailer's report, the retailer can simply tell the truth and supplier will set the right capacity. In this case, the *CM* problem is totally reduced, since capacity is sufficient for all types of retailers and not wasted. However, the less the supplier trusts, the retailer with high type private forecast information needs to manipulate the supplier more to achieve enough capacity. Since over-reporting brings moral costs, it will be harder for the retailer to achieve enough capacity. Therefore, higher trust level can reduce the *CM* problem and increase the supply chain efficiency.

3.5.4 Trust's trade-off in the *DM* and *CM* problem

The second research question of this paper is "Is trust beneficial to the supply chain ?". Again, this question crucially depends on trust's balancing in the *CM* and *DM* problem. We provide both analytical and numerical results to illustrate this trade-off. As we can see from Figure S4, we have two scenarios [18] in which trust level needs to trade-off between the *CM* and *DM* problem, and thus there exists an optimal trust level $\alpha^{s*} \in (0, 1)$, which can maximize the total supply chain efficiency. We analyze these two scenarios in subsection 3.5.4 and subsection 3.5.4 respectively.

[16] For trust's effect on supplier and retailer's expected profit, we put it in Appendix 3.7.8.

[17] This result is different from the trust-embedded model results in Özer, Zheng, and Chen 2011. In their setting, because of the incentive to over-report, an untrustworthy retailer can over-report a very large forecast information. If the supplier highly trusts retailer's report (e.g. 100% trust level), the capacity can be too much for supplier, and thus be harmful to the supply chain efficiency. In our setting, however, the retailer doesn't have the incentive to manipulate the capacity more than what she needs. This can prevent the retailer from reporting too much, and thus avoid the case where trust can be harmful to the supply chain efficiency regarding the *CM* problem.

[18] In the scenario where the retailer is trustworthy, we set the β to be high enough so that all types of retailer report the true type of information. In the scenario where the retailer is untrustworthy, we set the β to be low so that all types of retailer can report the lowest type under the right condition. In other words, we use extreme cases to illustrate trust's trade-off between the two problems.

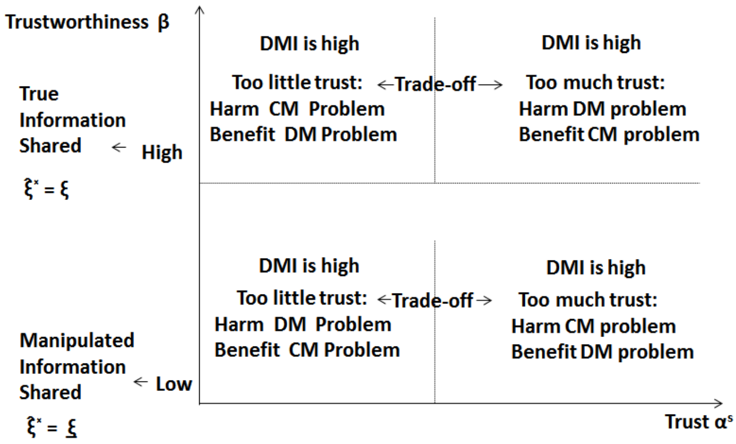


FIGURE S4: Trust's trade-off in the CM and DM problem

TABLE S5: Trust's trade-off in the CM and DM problem

Market Uncertainty: $(\overline{\epsilon}-\underline{\epsilon})$	High	High	Low	High	Low	Low	High	Low
Incomplete Information: $(\overline{\xi}-\underline{\xi})$	High	Low	High	High	Low	High	Low	Low
Capacity Cost: c_k	High	High	High	Low	High	Low	Low	Low
$DMI =$	-380	-220	-556	**-21.4**	-396	-484.28	**138.6**	-324.28
The role of information sharing in supply chain efficiency	Beneficial Figure S6	Beneficial Figure S7	Beneficial Figure S8	Harmful Figure S9	Beneficial Figure S10	Beneficial Figure S11	Harmful Figure S12	Beneficial Figure S13
When β is high Report in equilibrium $\hat{\xi}^*$	ξ	ξ	ξ	ξ	ξ	ξ	ξ	ξ
When β is high The role of trust α^s in supply chain efficiency	$\alpha^s \uparrow$ Figure S14	$\alpha^s \uparrow$ Figure S15	$\alpha^s \uparrow$ Figure S16	An optimal $\alpha^{s*} \in (0,1)$ Figure S17	$\alpha^s \uparrow$ Figure S18	$\alpha^s \uparrow$ Figure S19	An optimal $\alpha^{s*} \in (0,1)$ Figure S20	$\alpha^s \uparrow$ Figure S21
When β is low Report in equilibrium $\hat{\xi}^*$	ξ	ξ	ξ	$\underline{\xi}$	ξ	ξ	$\underline{\xi}$	ξ
When β is low The role of trust α^s in total supply chain efficiency	$\alpha^s \uparrow$ Figure S22	$\alpha^s \uparrow$ Figure S23	$\alpha^s \uparrow$ Figure S24	An optimal $\alpha^{s*} \in (0,1)$ Figure S25	$\alpha^s \uparrow$ Figure S26	$\alpha^s \uparrow$ Figure S27	An optimal $\alpha^{s*} \in (0,1)$ Figure S28	$\alpha^s \uparrow$ Figure S29

By setting the range of market uncertainty $(\overline{\epsilon}-\underline{\epsilon})$, incomplete information $(\overline{\xi}-\underline{\xi})$, and capacity cost c_k to be high or low, we studied eight different cases. In high β case, $\beta = 200$. In the low β case, $\beta = 0.2$ (we use these extreme cases to illustrate trust's trade-off in the *CM* and *DM* problem). All other parameters and the reason why we set them are the same with those in Table S1.

When the retailer is highly trustworthy and DMI is high, there exists an optimal trust level $\alpha^{s*} \in (0,1)$ for the supply chain: Too much trust can harm the DM problem, too little trust can harm the CM problem.

Numerical support

From the numerical analysis in Figure S9 and S12 from Table S5, when DMI is high and the retailer is highly trustworthy (i.e., β is high), there exists an certain optimal trust level $\alpha^{s*} \in (0,1)$, which can maximize the total supply chain efficiency: Too much trust can harm the DM problem, too little trust can harm the CM problem.

When the retailer is highly trustworthy (i.e., β is high), she should always report the true information to avoid high moral costs from lying. Therefore, true information (the true type of the retailer's private information) will be shared. On the one hand, when DMI is high, the true information is harmful to DM problem, based on Proposition 18. Therefore, more trust can bring more true but harmful information, and thus harm the supply chain efficiency. On the other hand, the lack of information can lead to too much or too little capacity, and thus harm the CM problem. Therefore, too little trust brings too little information, and harms the CM problem.

When the retailer is highly untrustworthy and DMI is high, there exists an optimal trust level $\alpha^{s*} \in (0,1)$ for the supply chain: Too much trust can harm the CM problem, too little trust can harm the DM problem.

Numerical support

In the numerical analysis in Figure S28 and S25 from Table S5, when DMI is high and retailer is highly untrustworthy (i.e., β is low), there exists an certain optimal trust level $\alpha^{s*} \in (0,1)$, which can maximize the total supply chain efficiency: Too much trust can harm the CM problem, too little trust can harm the DM problem.

The retailer can report a low type in order to manipulate the wholesale price down and get more profit, since $DMI \geq 0$ and thus there is enough room (market uncertainty and profit margin room) for the retailer to manipulate supplier's wholesale price down. In addition, the retailer has an incentive to over-report the forecast information in order to get enough capacity. Because in the cases of Figure S28 and S25, the incentive to under-report is stronger than the incentive to over-report, since the capacity cost is low (the incentive to manipulate enough capacity is weak, because the supplier should build more capacity with lower capacity cost), and profit margin is high (it will be more profitable to manipulate the profit down). Therefore, the retailer report the lowest type (i.e., $\hat{\xi}^*(\xi) = \underline{\xi}$) because of the incentive to under-report.

For the *DM* problem, the manipulated information is beneficial, since it reduces the wholesale price. For the *CM* problem, the manipulated information is harmful, since it renders a wrong capacity decision. Therefore, too much trust can bring more "wrong" information and harm the *CM* problem, too little trust can forgo a low wholesale price and harm the *DM* problem. Trading off these two effects, there exists an optimal trust level $\alpha^{s*} \in (0,1)$ which maximizes the total supply chain efficiency.

Analytical Support

Because the general trust-embedded model incorporating the *DM* and *CM* problem is not tractable, we give a special case where capacity cost is small (i.e., $c_k \to 0$) [19], to analytically capture the role of trust in the two problem simultaneously [20].

[19]"Capacity cost is zero" is also included as a continuous case. When capacity cost is zero, the trust-embedded model can have multiple equilibriums, since the supplier can set any capacity level which he believes enough for the retailer (for example, infinite many capacity or just what the retailer with a upper bound of private information needs). However, when capacity cost is zero, we assume the supplier set the capacity according to the upper bound of private forecast information to make the two cases ("capacity cost is small" and "capacity cost is zero") continuous.

[20]When $c_k \to 0$, the decisions in equilibrium are approaching results. To keep the notation simple, we omit the approaching, and use equality instead. When $c_k = 0$, all the approaching results become exact value.

Even the capacity cost is small, the supplier doesn't want to waste any extra capacity, and sets the capacity which he believes is just enough for the retailer and according to the upper bound of the belief of private information ξ . In this case, even the capacity cost is zero, the *CM* problem can exist, since the belief of the private information can be manipulated.

Specifically, when capacity cost is zero, the supplier sets the capacity according to retailer's report and the highest type of private information and believes that this capacity is enough for all types of retailer[21]. However this capacity may not be enough for the retailer, since the belief of the private forecast information can be manipulated by the retailer. We summarize the analytical results for this case in Proposition 22 (See the details of this analysis case in Appendix 3.7.10 and proof of Proposition 22 in Appendix 3.7.12).

Proposition 22 *If $c_k \to 0$, $DMI \geq 0$, $\beta = 0$,and $\frac{p-2c}{p}\frac{(\bar{\epsilon}-\epsilon)}{2} + \mu > 5\bar{\xi}$, in equilibrium of the trust model, the following properties regarding the expected profit (with respect to ξ) of the total supply chain, i.e., $\mathbb{E}_\xi \pi_t^{sc}(\alpha^s)$, is satisfied:*

(1) The report strategy of any type of retailer is $\hat{\xi}^*(\xi) = \underline{\xi}$.

(2)

$$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\xi'^*} \underbrace{\frac{\overline{\xi}}{2}\frac{p}{\overline{\epsilon}-\underline{\epsilon}}\Big(\frac{(u+\xi+\overline{\epsilon})p-(\overline{\epsilon}-\underline{\epsilon})c}{p} - q_{1t}^*\Big)}_{>0,\text{trust increases efficiency by lowering wholesale price}} dF(\xi)$$
$$+ \int_{\xi'^*}^{\overline{\xi}} \underbrace{\frac{-3\overline{\xi}}{2}\frac{p}{\overline{\epsilon}-\underline{\epsilon}}\Big(\frac{(u+\xi+\overline{\epsilon})p-(\overline{\epsilon}-\underline{\epsilon})c}{p} - q_{2t}^*\Big)}_{<0,\text{trust reduces efficiency by lowering capacity}} dF(\xi)$$

(3) when $\alpha^s \to 0, \frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} > 0$; when $\alpha^s \to 1, \frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} < 0$;

(4) $\frac{\partial^2 \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial {\alpha^s}^2} < 0$, $\mathbb{E}_\xi \pi_t^{sc}(\alpha^s)$ is concave in α^s.

(5) There exists an unique trust level $\alpha^{s*} \in (0,1)$ s.t. $\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s}|_{\alpha^s=\alpha^{s*}} = 0$ and maximizes $\mathbb{E}_\xi \pi_t^{sc}(\alpha^s)$.

On the one hand, the condition of both $DMI \geq 0$ and $\frac{p-2c}{p}\frac{(\bar{\epsilon}-\epsilon)}{2} + \mu > 5\bar{\xi}$ imply that profit margin and market uncertainty are high. Manipulating the whole-sale price down is profitable for the retailer (e.g.,if the profit margin is low, it is not profitable for the retailer to report the lowest type). On the other hand, when the retailer is untrustworthy (i.e., $\beta = 0$), manipulation brings no moral cost. Therefore, the retailer reports the lowest type to lower the wholesale price.

For the *DM* problem, trust can benefit it: higher trust level can lead to more manipulated information shared, which can lower the wholesale price and increase the supply chain efficiency. For the *CM* problem, trust can harm it: higher trust level can lead to more manipulated information shared, which can lead to a worse capacity decision and decrease the supply chain efficiency. Trading off theses two effects, there exists an optimal trust level maximizing the total supply chain efficiency.

[21] After receiving retailer's report, the supplier believes that retailer's highest possible type is $\alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi}$.

3.5.5 Analysis road map

In this subsection, we presented in Table S6 all the analysis cases regarding the role of trust in the supply chain efficiency (the second research question).

TABLE S6: The role of trust in supply chain–analysis cases

Model Environment	Incentive Problem(s)	Method	The role of trust
Trust-embedded model: Wholesale price is exogenous	*CM* problem only	Analytical	Always beneficial; See Proposition 21
Trust-embedded model: Capacity cost is zero	*DM* problem only	Analytical	Beneficial/harmful; See Proposition 20
Trust-embedded model: Capacity cost is small	*CM* and *DM* problem	Analytical	An optimal trust level $\alpha^{s*} \in (0,1)$ See Proposition 22
Trust-embedded model	*CM* and *DM* problem	Numerical	An optimal trust level $\alpha^{s*} \in (0,1)$ See Table S5

3.6 Conclusion

We investigate the role of information, trust and trustworthiness, in a two-tier supply chain with a supplier and a retailer. The supplier sets the capacity and the linear wholesale price. The retailer faces demand uncertainty before ordering and it has better demand information than the supplier. These elements capture the capacity misalignment problem (supplier building capacity with less demand information than the retailer), and the double marginalization problem (supplier setting wholesale price and "squeeze" the retailer). We find that information is not always beneficial to the supply chain because it can exacerbate the double marginalization problem, even when there is no incentive problem preventing sharing. Intuitively, when the market uncertainty and profit margin is high, the supplier with complete demand information has enough room to set different wholesale price for different types of retailers. Therefore, the supplier will take advantage of this information to squeeze more profits from the retailer, which worsens the double marginalization problem from the perspective of the supply chain.

As in previous literature, information sharing is not automatic, because of incentive barriers. We incorporate the notions of trust and trustworthiness, adapting from the trust-embedded model (Özer, Zheng, and Chen, 2011), into our setting to study how cheap talk forecast communication affect the aforementioned capacity misalignment and double marginalization problems. We find that trust is not always beneficial for the supply chain. When the retailer is trustworthy, the supplier uses the reported information to squeeze the retailer. This worsens the double marginalization problem. At the same time, trust does mitigate the capacity misalignment problem. Because of this trade-off, there is an "optimal" level of trust. Too much trust leads to too much double marginalization. Too little trust leads to too much capacity misalignment. The most striking and counter-intuitive result is that a totally untrustworthy retailer can be "good" from the supply chain efficiency perspective. Under the right conditions, an untrustworthy retailer can manipulate a trusting supplier to set a lower wholesale price, and reduces the double marginalization problem.

This study provides three insights from a practice perspective. First, information sharing is not always beneficial to the supply chain, or to the retailer. This is particular true when the supplier has the market power to "squeeze" the retailers with that information, and when the gain of setting the right capacity is low (e.g. capacity cost is low). Secondly, trust and trustworthiness no longer provide monotonic improvements to the supply chain. Retailers need to pay attention to the business conditions, and sometimes a little manipulation is better for the supply chain and increase the total pie. Finally, the supplier is always the beneficiary of information sharing, and from its point of view, it should always try to exploit a trustworthy retailer.

There are many natural directions for future research. From an engineering point of view, there is a need to develop incentive mechanisms (i.e., contracts) to solve this dual, capacity misalignment and double marginalization problem. There are a large body of literature of using contracts to solve either problem, and it would be interesting to see if both problems, in the same supply chain, can be addressed at the same time. From a behavioral perspective, we have only introduced trust and trustworthiness into the model. A large body of literature show that a multitude of behavioral factors such as bounded rationality, fairness and learning can be important in a supply chain contracting context, and there is ample room to incorporate additional behavioral thinking into this line of research.

3.7 Proofs and Omitted Analysis Cases

3.7.1 Centralized supply chain

We analyze the centralized supply chain as a benchmark in which we assume that the supplier “own” the whole supply chain, and he faces the following maximization problem.

$$\begin{aligned} \max_{q,K} \quad & \Pi^{cs}(K,q,\xi) = p\mathbb{E}_\epsilon \min(\mu+\xi+\epsilon,q) - cq - c_k K \\ s.t. \quad & q \le K \end{aligned}$$

where $\Pi^{cs}(K,q,\xi)$ is the expected profit of the centralized supply chain.

The “First Best” solution, where we let the supplier artificially know the private forecast information ξ, is given:

$$q^{*cs}_{ci} = \xi + \mu + G^{-1}\left(\frac{p-(c+c_k)}{p}\right)$$

$$K^{*cs}_{ci} = \xi + \mu + G^{-1}\left(\frac{p-(c+c_k)}{p}\right)$$

Where q^{*cs}_{ci} and K^{*cs}_{ci} denote the optimal order quantity and capacity for the centralized supply chain when information is complete, ie., supplier knows the private forecast information ξ. Note that we index the complete information scenario by “ci”.

The Capacity Misalignment Problem

When supplier cannot get access to the private information ξ, ie., information is incomplete, the optimal order quantity and capacity for the centralized supply chain are:

$$q_{ii}^{*cs} = \mu + (F \circ G)^{-1}\left(\frac{p-(c+c_k)}{p}\right) \tag{3.6}$$

$$K_{ii}^{*cs} = \mu + (F \circ G)^{-1}\left(\frac{p-(c+c_k)}{p}\right) \tag{3.7}$$

where $F \circ G$ is the distribution function of $\epsilon + \xi$. q_{ii}^{*cs} and K_{ii}^{*cs} denote the optimal order quantity and capacity for the centralized supply chain when information is incomplete. Similarly, we index the incomplete information scenario with"ii".

In this case, the supplier does not use all the information in the system to make the capacity decision, and results in a worse decision, compared to the first best solution. This is consistent with the results in past literature (Özer, Zheng, and Chen (2011)). This result is formally captured in the following proposition.

Proposition 23 *Information is always beneficial to the centralized supply chain efficiency, ie.,* $\pi_{ci}^{cs} \geq \pi_{ii}^{cs}$*, the equality holds if and only if* $q_{ii}^{*cs} = q_{ci}^{*cs}$*, where* $\pi_{ci}^{cs} = \Pi^{cs}(K_{ci}^{*cs}, q_{ci}^{*cs}, \xi)$ *and* $\pi_{ii}^{cs} = \Pi^{cs}(K_{ii}^{*cs}, q_{ii}^{*cs}, \xi)$*.*

The proof of this Proposition is as following. According to the definition of q_{ci}^{*cs}, we can know: $q_{ci}^{*cs} \in \arg\max_q \Pi^{cs}(K = q, q = q, \xi) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q) - cq - c_k q$. Therefore, we can know, $\pi_{ci}^{cs} = \Pi^{cs}(K_{ci}^{*cs}, q_{ci}^{*cs}, \xi) = \Pi^{cs}(q_{ci}^{*cs}, q_{ci}^{*cs}, \xi) \geq \Pi^{cs}(q_{ii}^{*cs}, q_{ii}^{*cs}, \xi) = \Pi^{cs}(K_{ii}^{*cs}, q_{ii}^{*cs}, \xi) = \pi_{ii}^{cs}$. If $q_{ii}^{*cs} = q_{ci}^{*cs}$, we can easily know $\pi_{ci}^{cs} = \pi_{ii}^{cs}$. In addition, because $\Pi^{cs}(K, q, \xi)$ is concave in q, and q_{ci}^{*cs} is the unique point maximizing $\Pi^{cs}(K = q, q = q, \xi)$, if $\pi_{ci}^{cs} = \pi_{ii}^{cs}$, we can get $q_{ii}^{*cs} = q_{ci}^{*cs}$.

3.7.2 Proof of Lemma 3.4.1

Given retailer's best response, the supplier will choose an optimal w and K to maximize his expected profit:

$$\Pi^s(w, K) = (w - c)q^*(K, w) - c_k K$$

First, any $w < c + c_k$ will be dominated by $w = c + c_k$, since any $w < c + c_k$ will lead to negative profit for the supplier. Further, any $w > p$ is weakly dominated by $w = p$, since any $w > p$ will bring zero profit to the supplier. Second, any $K > \xi + \mu + G^{-1}(\frac{p-w}{p})$ will be dominated by $K = \xi + \mu + G^{-1}(\frac{p-w}{p})$, because the possible maximum quantity order of the retailer is $\xi + \mu + G^{-1}(\frac{p-w}{p})$. Third, any $K < \xi + \mu + G^{-1}(\frac{p-w}{p})$ will be dominated by $K = \xi + \mu + G^{-1}(\frac{p-w}{p})$, because the supplier can always increase his profit by increasing his capacity to $K = \xi + \mu + G^{-1}(\frac{p-w}{p})$, when $K < \xi + \mu + G^{-1}(\frac{p-w}{p})$ and $w \in [c + c_k, p]$. Therefore, after the deletion of weakly dominated strategies, the supplier will choose $w \in [c + c_k, p]$, and $K = \xi + \mu + G^{-1}(\frac{p-w}{p})$ We can change the maximization problem of the supplier as following:

$$\begin{aligned} \underset{w}{\text{Maximize}} \quad & \Pi^s(w) = (w - c - c_k)\left(\xi + \mu + G^{-1}(\frac{p-w}{p})\right) \\ \text{s.t.} \quad & w \in [c + c_k, p] \end{aligned}$$

With the assumption that ϵ follows an uniform distribution, we can get: $\frac{\partial \Pi^s(w)}{\partial w} = \mu + \xi + \bar{\epsilon} - 2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}w + \frac{\bar{\epsilon}-\underline{\epsilon}}{p}(c+c_k)$,and $\frac{\partial^2 \Pi^s(w)}{\partial w^2} = -2\frac{\bar{\epsilon}-\underline{\epsilon}}{p} < 0$.Therefore, $\Pi^s(w)$ is concave in w. Let $\frac{\partial \Pi^s(w)}{\partial w} = 0$, we can get $w = \frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$. Considering the constraints, we can get the optimal wholesale price and capacity as,$w^*_{ci} = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right)$, and $K^*_{ci} = \xi + \mu + G^{-1}(\frac{p-w^*}{p})$.

3.7.3 Proof of Proposition 17

In this case, we need a new assumption that ξ follows an uniform distribution on $[\underline{\xi}, \bar{\xi}]$. Note that any $w < c + c_k$ will be dominated by $w = c + c_k$, since any $w < c + c_k$ will lead to negative profit for the supplier. Further, any $w > p$ is weakly dominated by $w = p$, since any $w > p$ will always bring zero profit to the supplier. Next we will consider about the choice of K. First, any $K > \bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})$ will be dominated by $K = \bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})$, because the possible maximum quantity order of the retailer is $\bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})$. For any $K > \bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})$, supplier can always save costs by reducing the capacity to $\bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})$, without affecting the profit from selling the product to the retailer. Second, any $K < \underline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$ will be weakly dominated by $K = \bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})$, because the supplier can always increase his profit by increasing his capacity to $K = \underline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$,when $K < \underline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$ and $w \in [c + c_k, p]$. Therefore, supplier will choose a $K \in [\underline{\xi} + \mu + G^{-1}(\frac{p-w}{p}), \bar{\xi} + \mu + G^{-1}(\frac{p-w}{p})]$ to maximize his profit.

Let $K \equiv \xi' + \mu + G^{-1}(\frac{p-w}{p})$, where $\xi' \in [\underline{\xi}, \bar{\xi}]$. Thus, $\mathbb{E}_\xi q^*(\xi', w, \xi) = \int_{\underline{\xi}}^{\xi'} \left[\xi + \mu + G^{-1}(\frac{p-w}{p})\right] dF(\xi) + \int_{\xi'}^{\bar{\xi}} \left[\xi' + \mu + G^{-1}(\frac{p-w}{p})\right] dF(\xi)$. We can change the maximization problem of the supplier into the following maximization problem:

$$\begin{aligned}
\underset{w,\xi'}{\text{Maximize}} \quad & \Pi^s(\xi', w) = \left\{(w-c)\left\{\int_{\underline{\xi}}^{\xi'} \left[\xi + \mu + G^{-1}(\frac{p-w}{p})\right] dF(\xi) + \int_{\xi'}^{\bar{\xi}} \left[\xi' + \mu + G^{-1}(\frac{p-w}{p})\right] dF(\xi)\right\}\right. \\
& \left. - c_k\left[\xi' + \mu + G^{-1}(\frac{p-w}{p})\right]\right\} \\
\text{s.t.} \quad & \xi' \in [\underline{\xi}, \bar{\xi}], w \in [c + c_k, p]
\end{aligned}$$

Taking no consideration about the constraint first, we can get: $\frac{\partial \Pi^s(\xi', w)}{\partial \xi'} = (w-c)[1 - F(\xi')] - c_k$ and $\frac{\partial^2 \Pi^s(\xi', w)}{\partial \xi'^2} = -(w-c)f(\xi') < 0$. Therefore, given a w, $\Pi^s(\xi', w)$ is concave in ξ'. Therefore, given the optimal wholesale price is w^*_{ii}, $\xi'^* = F^{-1}(\frac{w^*_{ii}-c-c_k}{w^*_{ii}-c}) \in [\underline{\xi}, \bar{\xi}]$, the optimal capacity will be $K^* = F^{-1}(\frac{w^*_{ii}-c-c_k}{w^*_{ii}-c}) + \mu + G^{-1}(\frac{p-w^*_{ii}}{p})$. With the assumption that ξ follows an uniform distribution, we can get the first order condition of w. Without considering the constraints: $\frac{\partial \Pi^s(\xi', w)}{\partial w} = -(2w - c - c_k)\frac{\bar{\epsilon}-\bar{\epsilon}}{p} + \mu + \bar{\epsilon} + \frac{\xi'^2-\underline{\xi}^2}{2(\bar{\xi}-\underline{\xi})} + \xi'\frac{\bar{\xi}-\xi'}{\bar{\xi}-\underline{\xi}}$. $\frac{\partial^2 \Pi^s(\xi', w)}{\partial w^2} - 2\frac{\bar{\epsilon}-\underline{\epsilon}}{p} < 0$. Combine $\frac{\partial \Pi^s(\xi', w)}{\partial w} = 0$, and $\frac{\partial \Pi^s(\xi', w)}{\partial \xi'} = 0$, with the assumption that $\bar{\xi} + \underline{\xi} = 0$, we can get a cubic equation about w:

$$-2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}(w-c)^3 + \frac{p(\mu+\bar{\epsilon}) - (\bar{\epsilon}-\underline{\epsilon})(c-c_k)}{p}(w-c)^2 + c_k^2\underline{\xi} = 0$$

Denote the real root of the above equation is w_r^*. We can get: $w_r^* = \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} + \frac{p\underline{\xi}c_k^2}{2(\bar{\epsilon}-\underline{\epsilon})(w_r^*-c)^2} < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$. **Then, we will prove there exists an unique $w_r^* \in (c+c_k, +\infty)$.**

We define $f(w) = 2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}(w-c)^2\left(w - \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}\right) - c_k^2\underline{\xi}$. Therefore,

$$
\begin{aligned}
f(c+c_k) =& 2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}c_k^2\left(c+c_k - \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}\right) - c_k^2\underline{\xi} \\
=& -2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}c_k^2\left(\frac{p(\mu+\bar{\epsilon}+\underline{\xi})-(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}\right) - c_k^2\underline{\xi} \\
=& -c_k^2\frac{p(\mu+\bar{\epsilon}+\underline{\xi})-(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{p} < -c_k^2\frac{(\bar{\epsilon}-\underline{\epsilon})(p-c-c_k)}{p} < 0
\end{aligned}
$$

Also, we can easily know that $f(+\infty) > 0$, We can get:$f'(w) = 6\frac{\bar{\epsilon}-\underline{\epsilon}}{p}(w-c)\left(w - \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(2c+c_k)}{3(\bar{\epsilon}-\underline{\epsilon})}\right)$. Note that $\frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(2c+c_k)}{3(\bar{\epsilon}-\underline{\epsilon})} > \frac{c(\bar{\epsilon}-\underline{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})2c}{3(\bar{\epsilon}-\underline{\epsilon})} = c$. We denote $\frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(2c+c_k)}{3(\bar{\epsilon}-\underline{\epsilon})} \equiv w_\phi$. Then $f(w)$ is increasing on $[w_\phi, +\infty)$, and decreasing on $[c, w_\phi]$

If $w_\phi > c + c_k$, then according to the intermediate value theorem and the monotonicity of $f(w)$, we can know there exists an unique $w_r^* \in (w_\phi, +\infty)$, s.t. $f(w_r^*) = 0$

If $w_\phi < c + c_k$, then according to the intermediate value theorem and the monotonicity of $f(w)$, we can know there exists an unique $w_r^* \in (c+c_k, +\infty)$, s.t. $f(w_r^*) = 0$

If $w_\phi = c + c_k$, then according to the intermediate value theorem and the monotonicity of $f(w)$, we can know there exists an unique $w_r^* \in (c+c_k, +\infty)$, s.t. $f(w_r^*) = 0$

Therefore, there exists an unique $w_r^* \in (c+c_k, +\infty)$, s.t. $f(w_r^*) = 0$. Now, we consider the constraint of $w \in [c+c_k, p]$, and the fact that $\Pi^s(\xi', w)$ is concave in w, we can get the optimal wholesale price $w_{ii}^* = \min\left(w_r^*, p\right)$. Given the optimal wholesale price w_{ii}^*, the supplier will set the optimal ξ' to be $\xi'^* = F^{-1}\left(\frac{w_{ii}^*-c-c_k}{w_{ii}^*-c}\right) \in [\underline{\xi}, \bar{\xi}]$, and the optimal capacity to be $K_{ii}^* = \xi'^* + \mu + G^{-1}\left(\frac{p-w_{ii}^*}{p}\right)$.

Given the optimal wholesale price and the capacity, we can easily get the second part of the proposition 17, according to retailer's best response. Now we prove the third part of Lemma 17. Because $\frac{p(\mu+\bar{\xi}+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} \leq p$ (i.e., $DMI \geq 0$), then $w_r^* < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} < \frac{p(\mu+\bar{\xi}+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} \leq p$, then $w_{ii}^* = \min\left(w_r^*, p\right) = w_r^*$. Also because $\frac{p(\mu+\bar{\xi}+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} < p$, we can get $\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} \leq \frac{p(\mu+\bar{\xi}+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} \leq p$, then $w_{ci}^* = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) = \frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$ Therefore, we can get $w_{ii}^* = w_r^* < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} = \mathbb{E}_\xi(w_{ci}^*)$.

3.7.4 Intractability of the Model

Based on Proposition 17, in equilibrium of a decentralized supply under the incomplete information case, the supplier chooses the wholesale price $w_{ii}^* = \min\left(w_r^*, p\right) > c + c_k$ where w_r^* is the unique real root, satisfying $c + c_k < w_r^* < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$, of equation:

$$-2\frac{\overline{\epsilon}-\underline{\epsilon}}{p}(w_r^*-c)^3+\frac{p(\mu+\overline{\epsilon})-(\overline{\epsilon}-\underline{\epsilon})(c-c_k)}{p}(w_r^*-c)^2+c_k^2\underline{\xi}=0 \tag{3.8}$$

We solve this cubic equation, using Mathematica (We keep the only real root and omit the two root with imaginary). The only real root of this equation is presented in Figure S5. We can see that w_r^* is pretty long and hard to tackle. What's more, we need to use the wholesale price $w_{ii}^* = \min\left(w_r^*, p\right)$, to further calculate the order quantity and compare the expected profit of the supply chain with respect to ξ with that under complete information condition. To the best of our knowledge, we cannot analytically calculate the total supply chain efficiency with w_r^*.

For the general trust-embedded model, we also need to calculate a similar equation about the wholesale price when supplier has the posterior belief of ξ and needs to decide the wholesale price and capacity. For instance, when the trust level is zero, the trust-embedded model will be the same with the incomplete information model. Since the incomplete information model is not fully tractable because the above mentioned reason, the trust embedded model is not tractable.

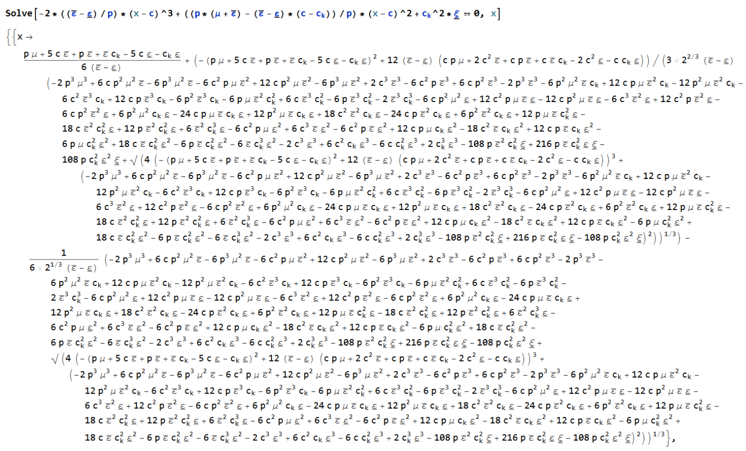


FIGURE S5: The real root of equation 3.8

3.7.5 The role of information in the CM problem

In order to investigate the role of information in the *DM* problem, we consider a setting with an exogenous wholesale price w. In this case, we expect both the wholesale price and the *DM* problem no longer interact with information, since the wholesale price is exogenously fixed. Hence, information is only relevant to the *CM* problem. The following Lemma formally summarizes the role of information in the *CM* problem in this setting.

In equilibrium of a decentralized supply chain with exogenous wholesale price $w \in [c+c_k, p]$:

(1) Information is beneficial to the *CM* problem, i.e., $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi) - \mathbb{E}_\xi \pi_{ii}^{sc}(\xi) \geq 0$; $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$, iff $c_k = 0$, where $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$, $\mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$ is the expected profit (with respect to ξ) of the total supply chain in equilibrium under compete and incomplete information condition respectively.[22]

[22]For information's role in supplier and retailer's expected profit (with respect to ξ), we put it in Proposition 3.7.5

(2) If ξ follows an uniform distribution, i.e., $\xi \sim U[\underline{\xi}, \overline{\xi}]$ $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) - \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) = \frac{\overline{\xi}-\underline{\xi}}{(w-c)^2}\left(\frac{(w-c-c_k)(w-c)}{2} + \frac{p}{6(\overline{\epsilon}-\underline{\epsilon})} \cdot \frac{\overline{\xi}-\underline{\xi}}{w-c} \cdot c_k^2\right)c_k$.

Information is always beneficial to the *CM* problem. This is because information can always help the supplier set the "right" capacity decision according to private forecast information ξ, which prevents the supplier from building too much or too little capacity. This result is consistent with the previous literature regarding information sharing in the capacity misalignment problem (Cachon and Lariviere (2001), Özer and Wei (2006), and Oh and Özer (2013)).

When capacity cost is small, information's role in the *CM* problem is inconsequential. When c_k is zero, the *CM* problem does not exist, since the supplier can always ensure enough capacity by building infinite capacity. In this case, information cannot affect the *CM* problem. From Proposition 3.7.5, information's role in the CM problem is $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) - \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) = \frac{\overline{\xi}-\underline{\xi}}{(w-c)^2}\left(\frac{(w-c-c_k)(w-c)}{2} + \frac{p}{6(\overline{\epsilon}-\underline{\epsilon})} \cdot \frac{\overline{\xi}-\underline{\xi}}{w-c} \cdot c_k^2\right)c_k$, which is continuous in c_k. Therefore, when capacity cost c_k is small, the beneficial role of information in the *CM problem* is inconsequential.

The Proof of the above Lemma is as following:

To capture the role of information sharing in the capacity misalignment problem alone, we examine the special case when the wholesale price is exogenous. In this case, intuitively, information sharing cannot affect the double marginalization problem because the margin is exogenous.

First, we consider the incomplete information scenario. Let the exogenous wholesale price be $w \in [c + c_k, p]$. Given the wholesale price w and capacity K the supplier decides, the retailer chooses an optimal q to maximize his expected profit under the capacity constraint. The retailer faces the following maximization problem.

$$\begin{aligned} \max_q \quad & \Pi^r(q,\xi) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q) - wq \\ s.t. \quad & q \leq K \end{aligned}$$

where $\Pi^r(q,\xi)$ is retailer's expected profit. Given w and K, retailer's best response is:

$$q^*(w,K,\xi) = \begin{cases} \xi + \mu + G^{-1}(\frac{p-w}{p}) & w \leq p, K > \xi + \mu + G^{-1}(\frac{p-w}{p}) \\ K & w \leq p, K \leq \xi + \mu + G^{-1}(\frac{p-w}{p}) \\ 0 & w > p \end{cases}$$

Given retailer's best response, the supplier chooses a capacity K to maximize his expected profit without knowing the private forecast information ξ,

$$\max_K \quad \Pi^s_{ii}(K) = (w-c)\mathbb{E}_\xi q^*(w,K,\xi) - c_k K$$

where $\Pi^s_{ii}(w,K)$ is supplier's expected profit (with respect to ξ), when information is incomplete, i.e., supplier cannot get access to the private information ξ.

For any $K > \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$ will be dominated by $K = \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$, because the possible maximum quantity order of the manufacturer is $\overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$. Second, any $K < \underline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$ will be dominated by $K = \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$, because the supplier can always increase his profit by increasing his capacity to $K = \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$,when $K < \underline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$ and $w \in (c + c_k, p)$. Therefore, the supplier will choose a $K \in [\underline{\xi} + \mu + G^{-1}(\frac{p-w}{p}), \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})]$. We denote that $K \equiv \xi' + \mu + G^{-1}(\frac{p-w}{p})$, where $\xi' \in [\underline{\xi}, \overline{\xi}]$. Thus $\mathbb{E}_{\xi} q^*(K, \xi) = \int_{\underline{\xi}}^{\xi'} \Big[\xi + \mu + G^{-1}(\frac{p-w}{p})\Big] dF(\xi) + \int_{\xi'}^{\overline{\xi}} \Big[\xi' + \mu + G^{-1}(\frac{p-w}{p})\Big] dF(\xi)$. We can change the maximization problem of the supplier into:

$$\underset{K}{\text{Maximize}} \quad \Big\{(w-c)\Big\{\int_{\underline{\xi}}^{\xi'} \Big[\xi + \mu + G^{-1}(\frac{p-w}{p})\Big] dF(\xi) + \int_{\xi'}^{\overline{\xi}} \Big[\xi' + \mu + G^{-1}(\frac{p-w}{p})\Big] dF(\xi)\Big\} - c_k K\Big\}$$

$$\text{subject to} \quad \xi' \in [\underline{\xi}, \overline{\xi}]$$

Taking no consideration about the constraint, we get the first order condition about ξ', $\frac{\partial \Pi^s(\xi')}{\partial \xi'} = (w-c)[1 - F(\xi')] - c_k, \frac{\partial^2 \Pi^s(\xi')}{\partial \xi'^2} = -(w-c)f(\xi') < 0$. Therefore, $\Pi^s(\xi')$ is concave in ξ'. We can get a global optimal $\xi'^* = F^{-1}(\frac{w-c-c_k}{w-c}) \in [\underline{\xi}, \overline{\xi}]$. Therefore, the unique optimal capacity will be $K^*_{ii} = F^{-1}(\frac{w-c-c_k}{w-c}) + \mu + G^{-1}(\frac{p-w}{p})$.

Second, we analyze the scenario where information is complete, i.e., the supplier can get access to the private information. Given retailer's best response, the supplier chooses a capacity K to maximize his expected profit when he can get access to the forecast information ξ,

$$\max_K \quad \Pi^s_{ci}(K, \xi) = (w-c)q^*(w, K, \xi) - c_k K$$

where $\Pi^s_{ci}(w, K)$ is supplier's expected profit (with respect to ξ), when information is complete, i.e., supplier can get access to the private information ξ. Similar to the analysis to incomplete information case, the supplier will choose the optimal capacity: $K^*_{ci} = \xi + \mu + G^{-1}(\frac{p-w}{p})$.

Therefore, if the wholesale price $w \in [c + c_k, p]$ is exogenous:

(1) when information is incomplete, the supplier chooses the optimal capacity: $K^*_{ii} = \xi'^* + \mu + G^{-1}(\frac{p-w}{p})$, where $\xi'^* = F^{-1}(\frac{w-c-c_k}{w-c}) \in [\underline{\xi}, \overline{\xi}]$.

(2) when information is complete, the supplier chooses the optimal capacity: $K^*_{ci} = \xi + \mu + G^{-1}(\frac{p-w}{p})$.

In equilibrium of the game, we define retailer's expected profit (for every possible realization of private information ξ) under complete and incomplete information scenario $\pi^r_{ci}(\xi) = \Pi^r(q^*(w, K^*_{ci}, \xi), \xi)$, $\pi^r_{ii}(\xi) = \Pi^r(q^*(w, K^*_{ii}, \xi), \xi)$; supplier's expected profit (for every possible realization of private information ξ) under complete and incomplete information scenario $\pi^s_{ci}(\xi) = \Pi^s_{ci}(K^*_{ci}, \xi)$, $\pi^s_{ii}(\xi) = \Pi^s_{ii}(K^*_{ii})$; the expected profit of total supply chain under complete and incomplete information scenario $\pi^{sc}_{ci}(\xi) = \pi^s_{ci}(\xi) + \pi^r_{ci}(\xi)$, $\pi^{sc}_{ii}(\xi) = \pi^s_{ii}(\xi) + \pi^r_{ii}(\xi)$.

First, if $w = p$, the retailer's profit will always be zero, and in equilibrium $\pi^r_{ci}(w, \xi) = \pi^r_{ii}(w, \xi) = 0$. If $w = c + c_k$, the supplier's profit will always be zero, and in equilibrium $\pi^s_{ci}(w, \xi) = \pi^s_{ii}(w, \xi) = 0$.

Second, if $c_k = 0$, $\xi'^* = \overline{\xi}$. Therefore, capacity is always enough for the retailer under incomplete information condition, the order quantities of the retailer under both complete information condition and incomplete information condition are the same. Therefore, the retailer's profit will always be the same. In addition, since capacity costs the supplier nothing, the supplier's profit will be the same under both complete information case and incomplete information case, since extra capacity bring no costs to the supplier in the incomplete information condition.

Third, we consider the case where $c_k \neq 0$, $c + c_k < w < p$ and $\xi < \xi'^*$. When information is incomplete, since we can know $K_{ii}^* = F^{-1}(\frac{w-c-c_k}{w-c}) + \mu + G^{-1}(\frac{p-w}{p}) > \xi + \mu + G^{-1}(\frac{p-w}{p})$, therefore $q_{ii}^*(w, K, \xi) = \xi + \mu + G^{-1}(\frac{p-w}{p})$. When information is complete, we can easily get $q_{ci}^*(w, K, \xi) = \xi + \mu + G^{-1}(\frac{p-w}{p})$. Therefore, the optimized profit of the retailer under incomplete information and complete information condition will be the same, ie., $\pi_{ci}^r(w, \xi) = \pi_{ii}^r(w, \xi)$. For the supplier, when information is incomplete, the extra capacity $(\xi'^* - \xi)$ will be wasted. Therefore, $\pi_{ci}^s(w, \xi) > \pi_{ii}^s(w, \xi)$.

Fourth, we consider about the case where $c_k \neq 0$, $c + c_k < w < p$ and $\xi > \xi'^*$. When information is incomplete, since we can know $K_{ii}^* = \xi'^* + \mu + G^{-1}(\frac{p-w}{p}) < \xi + \mu + G^{-1}(\frac{p-w}{p})$, therefore $q_{ii}^*(w, K, \xi) = \xi'^* + \mu + G^{-1}(\frac{p-w}{p})$. When information is complete, we can know that the optimal order quantity will be $q_{ci}^*(w, K, \xi) = \xi + \mu + G^{-1}(\frac{p-w}{p})$. Since the expected profit function $\Pi^r(q, \xi) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q) - wq$ is strictly increasing on $[0, \xi + \mu + G^{-1}(\frac{p-w}{p})]$(with respect to q), we can get $\pi_{ci}^r(w, \xi) > \pi_{ii}^r(w, \xi)$. On the other hand, when $w \in [c + c_k, p]$, the expected profit function of the supplier is strictly increasing with retailer's order quantity if the supplier's capacity is equal to the order quantity. Thus, $\pi_{ci}^s(w, \xi) > \pi_{ii}^s(w, \xi)$.

Fifth, we consider about the case where $c_k \neq 0$, $c + c_k < w < p$ and $\xi = \xi'^*$, $\pi_{ci}^s(w, \xi) = \pi_{ii}^s(w, \xi)$, since both the capacity decision and the order quantity in the equilibrium are the same under both incomplete information condition and complete information condition.

Combining all the cases and taking expectation about the profit with respect to ξ, we can get the following Lemma 3.7.5.

If the wholesale price $w \in [c + c_k, p]$ is exogenous,

(1) Information sharing is beneficial to the total supply chain efficiency: $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi) \geq \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$, the equality establishes if and only if $c_k = 0$.

(2) Information sharing is beneficial to the retailer: $\mathbb{E}_\xi \pi_{ci}^r(\xi) \geq \mathbb{E}_\xi \pi_{ii}^r(\xi)$, the equality establishes if and only if $w = p$ or $c_k = 0$.

(3) Information sharing is beneficial to the supplier: $\mathbb{E}_\xi \pi_{ci}^s(\xi) \geq \mathbb{E}_\xi \pi_{ii}^s(\xi)$, the equality establishes if and only if $w = c + c_k$ or $c_k = 0$.

where $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$, $\mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$ is the expected profit (with respect to ξ) of the total supply chain in equilibrium under compete and incomplete information condition respectively. $\mathbb{E}_\xi \pi_{ci}^r(\xi)$, $\mathbb{E}_\xi \pi_{ii}^r(\xi)$ is the expected profit (with respect to ξ) of the retailer in equilibrium under compete and incomplete information condition respectively. $\mathbb{E}_\xi \pi_{ci}^s(\xi)$, $\mathbb{E}_\xi \pi_{ii}^s(\xi)$ is the expected profit (with respect to ξ) of the retailer in equilibrium under compete and incomplete information case respectively.

In addition, for the total supply chain efficiency (the expect profit with respect to ϵ, for every realization of ξ) under complete information and incomplete information condition: $\pi_{ci}^{sc}(\xi) = p(\mu + \xi) - cq_{ci}^* + \frac{p}{2(\bar{\epsilon} - \epsilon)}\left[-\underline{\epsilon}^2 + 2(q_{ci}^* - \mu - \xi)\bar{\epsilon} - (q_{ci}^* - \mu - \xi)^2 \right] - c_k q_{ci}^*$, and $\pi_{ii}^{sc}(\xi) = p(\mu + \xi) - cq_{ii}^* + \frac{p}{2(\bar{\epsilon} - \epsilon)}\left[-\underline{\epsilon}^2 + 2(q_{ci}^* - \mu - \xi)\bar{\epsilon} - (q_{ii}^* - \mu - \xi)^2 \right] - c_k K_{ii}^*$,

When information is incomplete and $\xi'^* > \xi$, the retailer with private type ξ always gets enough capacity, and the supplier wasted some capacity. Specifically,

$(\xi'^* - \xi)c_k$ will be wasted, where $(\xi'^* - \xi)$ are the wasted quantity and c_k is the cost for each capacity unit.

When information is incomplete and $\xi'^* < \xi$, the retailer with private type ξ cannot get enough capacity. This can reduce the order quantity of the supply chain and lower the supply chain efficiency.

We can get the information's role in the *CM* problem is: $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) - \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) = \int_{\underline{\xi}}^{\xi'^*} (\xi'^* - \xi) c_k dF(\xi) + \int_{\xi'^*}^{\overline{\xi}} (\xi - \xi'^*)\left[(w - c - c_k) + \frac{p}{2(\overline{\epsilon} - \underline{\epsilon})}(\xi - \xi'^*)\right] dF(\xi)$, with the assumption that $\xi \sim U[\underline{\xi}, \overline{\xi}]$:

$$\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) - \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) = \frac{\overline{\xi}-\underline{\xi}}{(w-c)^2}\left(\frac{c_k(w-c-c_k)(w-c)}{2} + \frac{p}{6(\overline{\epsilon}-\underline{\epsilon})}\cdot\frac{\overline{\xi}-\underline{\xi}}{w-c}\cdot c_k^3\right) = \frac{\overline{\xi}-\underline{\xi}}{(w-c)^2}\left(\frac{(w-c-c_k)(w-c)}{2} + \frac{p}{6(\overline{\epsilon}-\underline{\epsilon})}\cdot\frac{\overline{\xi}-\underline{\xi}}{w-c}\cdot c_k^2\right)c_k.$$

When $c_k \to 0$, information's role in the capacity misalignment problem is $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) - \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) \to 0$.

From Lemma 3.7.5, information is always beneficial for both the supplier and retailer in the *CM* problem.

3.7.6 Proof of Proposition 18

We examine the special case where the capacity cost $c_k = 0$ in this case. Intuitively, as the cost of capacity goes to zero, the capacity misalignment problem disappears since the supplier can always, trivially, provide enough capacity for maximum possible demand. Formally, when $c_k = 0$, the supplier can choose any component capacity $K \geq \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$. By doing this, he can always ensure enough capacity for the retailer and maximize his profit. Thus, we don't need to consider the capacity constraint in this model. This will allow us to capture the role of information sharing in double marginalization problem alone, without considering the capacity constraint in the model.

The following proposition summarizes the role of information sharing when we only consider the double marginalization problem.

[Proposition 2.] In equilibrium of a decentralized supply chain with no capacity costs.

(1) If $DMI \geq 0$, information sharing is harmful to the total supply chain efficiency (expected profit with respect to ξ), ie., $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) < \mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$; information sharing is beneficial to the supplier, ie., $\mathbb{E}_\xi \pi^{s}_{ci}(\xi) > \mathbb{E}_\xi \pi^{s}_{ii}(\xi)$; information sharing is harmful to the retailer, ie., $\mathbb{E}_\xi \pi^{r}_{ci}(\xi) < \mathbb{E}_\xi \pi^{r}_{ii}(\xi)$.

(2) If $2\underline{\xi} < DMI \leq \underline{\xi}$, information sharing is beneficial to the total supply chain efficiency, ie.,$\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) > \mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$; information sharing is beneficial to the supplier, ie., $\mathbb{E}_\xi \pi^{s}_{ci}(\xi) > \mathbb{E}_\xi \pi^{s}_{ii}(\xi)$; information sharing is beneficial to the retailer, ie., $\mathbb{E}_\xi \pi^{r}_{ci}(\xi) > \mathbb{E}_\xi \pi^{r}_{ii}(\xi)$

(3) If $DMI \leq 2\underline{\xi}$, information sharing cannot affect the supply chain efficiency, ie., $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) = \mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$; $\mathbb{E}_\xi \pi^{s}_{ci}(\xi) = \mathbb{E}_\xi \pi^{s}_{ii}(\xi)$; $\mathbb{E}_\xi \pi^{r}_{ci}(\xi) = \mathbb{E}_\xi \pi^{r}_{ii}(\xi)$.

where $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi)$, $\mathbb{E}_\xi \pi^{sc}_{ii}(\xi)$ is the expected profit (with respect to ξ) of the total supply chain under compete and incomplete information condition respectively. $\mathbb{E}_\xi \pi^{r}_{ci}(\xi)$, $\mathbb{E}_\xi \pi^{r}_{ii}(\xi)$ is the expected profit (with respect to ξ) of the retailer under compete and incomplete information condition respectively. $\mathbb{E}_\xi \pi^{s}_{ci}(\xi)$, $\mathbb{E}_\xi \pi^{s}_{ii}(\xi)$ is the expected profit (with respect to ξ) of the retailer under compete and incomplete information condition respectively.

The Proof of Proposition 18 is as following. Given the wholesale price w, retailer's best response is:

$$q^*(w,\xi) = \begin{cases} \xi + \mu + G^{-1}(\frac{p-w}{p}) & if\ w \leq p \\ 0 & if\ w > p \end{cases} \tag{3.9}$$

When the supplier does not have access to the private forecast information, ie., information is incomplete, he optimizes his expected profit:

$$\max_{w} \quad \Pi^s_{ii}(w) = (w-c)\mathbb{E}_\xi q^*(w,\xi) \tag{3.10}$$

where $\Pi^s_{ii}(w)$ is supplier's expected profit (with respect to ξ) when information is incomplete. In equilibrium, supplier decides the wholesale price:

$$w^*_{ii} = \min\left(\frac{(\mu+\overline{\epsilon})p + (\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}, p\right) \tag{3.11}$$

where w^*_{ii} is the wholesale price in equilibrium when information is incomplete.

Then we consider the scenario where the supplier has access to the private forecast information, ie., information is complete. The supplier optimizes his profit:

$$\max_{w} \quad \Pi^s_{ci}(w) = (w-c)q^*(w,\xi) \tag{3.12}$$

where $\Pi^s_{ci}(w)$ is supplier's profit when information is complete. The supplier chooses the wholesale price in equilibrium:

$$w^*_{ci} = \min\left(\frac{(\mu+\xi+\overline{\epsilon})p + (\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}, p\right) \tag{3.13}$$

where w^*_{ci} is the wholesale price in equilibrium when information is complete.

In the following analysis, we define $\xi_t \equiv DMI + \overline{\xi}$. Therefore, considering different cases about ξ_t is equivalent to considering cases about DMI.

First, we consider the case where $\xi_t \geq \overline{\xi}$ (i.e., $DMI \geq 0$), we can get $\frac{(\mu+\overline{\xi}+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})} \leq p$. In this case, we maximize the supplier's expected profit and get the optimal wholesale price under complete information and incomplete information condition:$w^*_{ci} = \min\left(\frac{(\mu+\xi+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\overline{\epsilon}-\underline{\epsilon})}, p\right) = \frac{(\mu+\xi+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}$, and $w^*_{ii} = \min\left(\frac{(\mu+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}, p\right) = \frac{(\mu+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}$. According to the best response of the retailer, we can get the order quantity in equilibrium under complete information and incomplete information condition: $q^*_{ii} = \xi + \frac{(\mu+\overline{\epsilon})p-(\overline{\epsilon}-\underline{\epsilon})c}{2p}$,and $q^*_{ci} = \frac{\xi}{2} + \frac{(\mu+\overline{\epsilon})p-(\overline{\epsilon}-\underline{\epsilon})c}{2p}$. Insert these results into the profit function of the retailer and supplier, after taking expectation, we can get: $\mathbb{E}_\xi \pi^{sc}_{ci}(\xi) - \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) = -\frac{p}{8(\overline{\epsilon}-\underline{\epsilon})}\mathbb{E}_\xi \xi^2 = -\frac{p}{8(\overline{\epsilon}-\underline{\epsilon})}Var(\xi) < 0$, $\mathbb{E}_\xi \Pi^s_{ci}(\xi) - \mathbb{E}_\xi \Pi^s_{ii}(\xi) = \frac{p}{4(\overline{\epsilon}-\underline{\epsilon})}\mathbb{E}_\xi \xi^2 = \frac{p}{4(\overline{\epsilon}-\underline{\epsilon})}Var(\xi) > 0$, and $\mathbb{E}_\xi \Pi^r_{ci}(\xi) - \mathbb{E}_\xi \Pi^r_{ii}(\xi) = -\frac{3p}{8(\overline{\epsilon}-\underline{\epsilon})}Var(\xi) < 0$.

Second, we consider the case where $\xi_t \in (\underline{\xi}, 0]$ (i.e., $2\underline{\xi} < DMI \leq \underline{\xi}$), we can get $\frac{(\mu+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})} \leq p$. Therefore when information is incomplete, we always have $w^*_{ii} = \min\left(\frac{(\mu+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}, p\right) = p$, and $q^*_{ii} = \mu + \xi + \underline{\epsilon}$. For complete information case, when $\xi > \xi_t$, we can get the wholesale price in equilibrium $w^*_{ci} = \min\left(\frac{(\mu+\xi+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\overline{\epsilon}-\underline{\epsilon})}, p\right) = p$, and the order quantity in equilibrium $q^*_{ci} = \mu + \xi + \underline{\epsilon}$. Therefore, for $\xi > \xi_t$, the efficiency of both the supplier and the retailer in equilibrium will be the same, ie.,

$\pi^s_{ci}(\xi) = \pi^s_{ii}(\xi)$, $\pi^r_{ci}(\xi) = \pi^r_{ii}(\xi)$, $\pi^{sc}_{ci}(\xi) = \pi^{sc}_{ii}(\xi)$. When $\xi < \xi_t$, we can get the wholesale price in equilibrium $w^*_{ci} = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) = \frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$, and the order quantity in equilibrium $q^*_{ci} = \frac{\xi}{2} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. Because $\xi < \xi_t = \frac{2(\bar{\epsilon}-\underline{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - (\mu+\bar{\epsilon})$, we can know $q^*_{ci} - q^*_{ii} = -\frac{\xi}{2} - \frac{(\mu+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2p} + (\bar{\epsilon}-\underline{\epsilon}) > -\frac{(\mu+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2p} + (\bar{\epsilon}-\underline{\epsilon}) - \frac{1}{2}\left(\frac{2(\bar{\epsilon}-\underline{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - (\mu+\bar{\epsilon})\right) = \frac{\mu+\bar{\epsilon}}{2} > 0$. In the equilibrium, we can get the expected profit for the supply chain under complete information and incomplete information as: $\pi^{sc}_{ii}(\xi) = p(\mu+\xi) - cq^*_{ii} + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q^*_{ii} - \mu - \xi)\bar{\epsilon} - (q^*_{ii}-\mu-\xi)^2\Big]$, and $\pi^{sc}_{ci}(\xi) = p(\mu+\xi) - cq^*_{ci} + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q^*_{ci} - \mu - \xi)\bar{\epsilon} - (q^*_{ci}-\mu-\xi)^2\Big]$. After some algebra, we can get: $\pi^{sc}_{ci}(\xi) - \pi^{sc}_{ii}(\xi) = (q^*_{ci} - q^*_{ii})\frac{(3\bar{\epsilon}-2\underline{\epsilon}+\mu+\xi)p-3(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > (q^*_{ci} - q^*_{ii})\frac{(3\bar{\epsilon}-2\underline{\epsilon}-\epsilon)p-3(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} = (q^*_{ci} - q^*_{ii})\frac{3(p-c)}{4} > 0$. For the supplier, we can get the expected profit under complete information and incomplete information condition in equilibrium: $\pi^s_{ii}(\xi) = (w^*_{ii} - c) * q^*_{ii} = (p-c)(\mu+\xi+\underline{\epsilon})$, $\pi^s_{ci}(\xi) = (w^*_{ci} - c) * q^*_{ci} = \left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})} - c\right)\left(\frac{\xi}{2} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}\right)$ After some algebra, we can get: $\pi^s_{ci}(\xi) - \pi^s_{ii}(\xi) = \frac{(\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{-4(\bar{\epsilon}-\underline{\epsilon})p}\Big(2(\bar{\epsilon}-\underline{\epsilon})(p-c) - 2(\mu+\xi+\underline{\epsilon})p + (\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})c\Big)$. Because $\xi < \xi_t = \frac{2(\bar{\epsilon}-\underline{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - (\mu+\bar{\epsilon})$, we can know $(\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})c < \Big(\mu + \frac{2(\bar{\epsilon}-\underline{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - (\mu+\bar{\epsilon}) - \bar{\epsilon} + 2\underline{\epsilon}\Big)p + (\bar{\epsilon}-\underline{\epsilon}) = 0$ Thus, $\frac{(\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{-4(\bar{\epsilon}-\underline{\epsilon})p} > 0$. Also, because $\xi < \xi_t = \frac{2(\bar{\epsilon}-\underline{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - (\mu+\bar{\epsilon})$, we can get: $2(\bar{\epsilon}-\underline{\epsilon})(p-c) - 2(\mu+\xi+\underline{\epsilon})p + (\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})c > 2(\bar{\epsilon}-\underline{\epsilon})(p-c) - 2(\mu + \frac{2(\bar{\epsilon}-\underline{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - (\mu+\bar{\epsilon}) + \underline{\epsilon})p + (\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})c = 0$. Therefore, $2(\bar{\epsilon}-\underline{\epsilon})(p-c) - 2(\mu+\xi+\underline{\epsilon})p(\mu+\xi-\bar{\epsilon}+2\underline{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})c > 0$.Therefore, we can get: $\pi^s_{ci}(\xi) - \pi^s_{ii}(\xi) > 0$. For the retailer, because $q^*_{ci} > q^*_{ii}$, and because the expected profit function of the retailer is increasing in $[0, q^*_{ci}]$, we can get $\pi^r_{ci}(\xi) - \pi^r_{ii}(\xi) > 0$. Take all the results' expectation on ξ, we have $\mathbb{E}_\xi\pi^{sc}_{ci}(\xi) > \mathbb{E}_\xi\pi^{sc}_{ii}(\xi)$; $\mathbb{E}_\xi\pi^s_{ci}(\xi) > \mathbb{E}_\xi\pi^s_{ii}(\xi)$; $\mathbb{E}_\xi\pi^r_{ci}(\xi) > \mathbb{E}_\xi\pi^r_{ii}(\xi)$.

Third, we consider the case where $\xi_t \le \underline{\xi}$ (i.e., $DMI \le 2\underline{\xi}$). We can get $\frac{(\mu+\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \le p$, thus $\frac{(\mu+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} < p$. Therefore when information is incomplete, we always have $w^*_{ii} = \min\left(\frac{(\mu+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) = p$, and $q^*_{ii} = \mu+\xi+\underline{\epsilon}$. In addition, for complete information case, we can get the wholesale price in equilibrium $w^*_{ci} = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) = p$, and the order quantity in equilibrium $q^*_{ci} = \mu+\xi+\underline{\epsilon}$. Therefore, the efficiency of both the supplier and the retailer in equilibrium will be the same, ie., $\pi^s_{ci}(\xi) = \pi^s_{ii}(\xi)$, $\pi^r_{ci}(\xi) = \pi^r_{ii}(\xi)$, $\pi^{sc}_{ci}(\xi) = \pi^{sc}_{ii}(\xi)$. Take expectation of these results' on ξ, we have $\mathbb{E}_\xi\pi^{sc}_{ci}(\xi) = \mathbb{E}_\xi\pi^{sc}_{ii}(\xi)$; $\mathbb{E}_\xi\pi^s_{ci}(\xi) = \mathbb{E}_\xi\pi^s_{ii}(\xi)$; $\mathbb{E}_\xi\pi^r_{ci}(\xi) = \mathbb{E}_\xi\pi^r_{ii}(\xi)$.

3.7.7 Proof of Proposition 19

In this case, we investigate the effect of information sharing on supply chain efficiency when we add "a small amount of" incomplete information. Ideally, we want to capture the effect of information sharing on supply chain efficiency in terms of capacity misalignment problem and double marginalization problem simultaneously. Since the general formulation, as far as we know, is not fully analytically tractable, studying a case with "amount of" private information approaching zero allows us to linearize part of the formulation (specifically, the wholesale price) and arrives at an analytic solution.

We add "a small amount of" private information by assuming supplier's prior belief about ξ is uniformly distributed on $[-a, a]$ (i.e., $a = \overline{\xi}$), where a is a positive small number, i.e., $a \to 0^+$. We denote the distribution function of the private information as $F_a(\cdot)$. The following Lemma (we can prove this Lemma by directly applying Proposition 17.) describes the decisions of supplier in equilibrium.

When $DMI \geq 0$, and we add "a small amount of" private information, i.e., $a \to 0^+$,

(1) If information is complete, in equilibrium, the supplier sets the wholesale price and capacity: $w^*_{ci} = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) = \frac{(\mu+\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$, $K^*_{ci} = 0 + u + G^{-1}\left(\frac{p-w(0)}{p}\right)$, where w^*_{ci} and K^*_{ci} are the wholesale price and capacity decision in equilibrium when information is complete.

(2) With "a small amount of" incomplete information, the supplier sets the wholesale price w^*_{ii} and capacity K^*_{ii}, where $w^*_{ii} = \min\left(w^*_r, p\right) = w^*_r$, where w^*_r is the real root, satisfying $c + c_k < w^*_r < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$), of equation $2\frac{\bar{\epsilon}-\underline{\epsilon}}{p}(w^*_r - c)^2\left(w^*_r - \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}\right) + c_k^2 a = 0$, $K^*_{ii} = \xi'^* + u + G^{-1}\left(\frac{p-w(a)}{p}\right)$, where $\xi'^* = F_a^{-1}\left(\frac{w^*_{ii}-c-c_k}{w^*_{ii})-c}\right) \in [-a, a]$, where w^*_{ii} and K^*_{ii} are the wholesale price and capacity decision in equilibrium when information is incomplete.

(3) With "a small amount of" incomplete information, information sharing can lower the wholesale price, and thus reduce the double marginalization problem, i.e., when $a \to 0^+$, $w^*_{ii} < \mathbb{E}_\xi w^*_{ii}$[23].

When $DMI \geq 0$, information sharing can exacerbate the double marginalization problem, when the supply chain has "a small amount of" private information. Specifically, when $a \to 0^+$, we have $w(a) < w(0)$, i.e., the wholesale price under complete information condition is higher than that under incomplete information condition, which means information sharing can worsen the double marginalization problem. This is because when $\xi_t \geq a$ (this means the market uncertainty and profit margin are high), the supplier has enough room to set different wholesale price for different types of retailer. This allows the supplier to double marginalize more and makes the double marginalization problem worsen. However, information sharing can help the capacity misalignment problem. Taking these two problems into consideration, we summarize the role of information sharing in supply chain efficiency in the following Proposition.

[Proposition 3.] When the supply chain has "a small amount of" private information, i.e., $a \to 0$, if $c_k = o(a^N)$, where N is a positive integer with $N \geq 3$ and $o(a^N)$ is the N-order infinitesimal of a, and $DMI \geq 0$:

(1) information sharing is harmful to the supply chain efficiency, ie., when $a \to 0^+$, $\pi^{sc}_{ii} > \pi^{sc}_{ci}$ [24], where π^{sc}_{ii} and π^{sc}_{ci} are the total supply chain's expected profit

[23] More formally, $\exists\, \sigma > 0$, s.t. $\forall a \in (0, \sigma)$, $w^*_{ii} < \mathbb{E}_\xi(w^*_{ci})$. Other contents regarding $a \to 0^+$ in this case are similar to this definition.

[24] For notation's conveniences, we have $\mathrm{E}_\xi \pi^{sc}_{ci}(\xi) \equiv \pi^{sc}_{ci}; \mathbb{E}_\xi \pi^{sc}_{ii}(\xi) \equiv \pi^{sc}_{ii}$

(with respect to ξ) under incomplete information condition and under complete information condition respectively.

(2) Information sharing is harmful to the retailer, ie., when $a \to 0^+$, $\pi^r_{ii} > \pi^r_{ci}$, where π^r_{ii} and π^r_{ci} are retailer's expected profit (with respect to ξ) under incomplete information condition and under complete information condition respectively.

(3) Information sharing is beneficial to the supplier, ie., when $a \to 0^+$, $\pi^s_{ii} < \pi^s_{ci}$, where π^s_{ii} and π^s_{ci} are supplier's expected profit (with respect to ξ) under incomplete information condition and under complete information condition respectively.

As we mentioned in section 3.4.3, when $DMI \geq 0$, information sharing will benefit the supply chain efficiency, regarding the double marginalization problem. In addition, when $c_k \to 0$ and its speed to zero is very fast (i.e., $c_k = o(a^N)$, where N is a positive integer with $N \geq 3$ and $o(a^N)$ is the N-order infinitesimal of a), although information sharing is beneficial to the capacity misalignment problem, but its effect is small, since c_k is very small in this case. Integrating these two effects, information sharing is harmful to the total supply chain efficiency.

Moreover, for the supplier, he can always make use of the private information, and make a better wholesale price and capacity decision, and thus get more profits under complete information condition than he does under incomplete information condition. Since information sharing allows the supplier to squeeze more profit from retailer, information sharing can reduce retailer's profit.

The Proof of Proposition 19 is as following.

According to Lemma 3.4.1, when information is complete, the supplier sets the wholesale price and capacity in equilibrium:

$$w^*_{ci} = \min\left(\frac{(\mu+\xi+\bar{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}, p\right) = \frac{(\mu+\xi+\bar{\epsilon})p + (\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$$

$$K^*_{ci} = 0 + u + G^{-1}\left(\frac{p-w^*_{ci}}{p}\right)$$

where w^*_{ci} and K^*_{ci} are the wholesale price and capacity in equilibrium when information is complete. We denote $w(0) \equiv \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$

From Proposition 17, with "a small amount of" incomplete information, the supplier sets the following wholesale price and capacity to maximize expected his profit: $w^*_{ii} = \min(w^*_r, p)$, $K^*_{ii} = \xi'^* + \mu + G^{-1}(\frac{p-w^*_{ii}}{p})$, where $\xi'^* = F_a^{-1}(\frac{w^*_{ii}-c-c_k}{w^*_{ii}-c}) \in [-a, a]$, and w^*_r is the unique solution of the following equation satisfying $w^*_r > c + c_k$:

$$2\frac{\bar{\epsilon}-\epsilon}{p}(w^*_r - c)^2\left(w^*_r - \frac{p(\mu+\bar{\epsilon}) + (\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}\right) + c_k^2 a = 0$$

According to Proposition 17, we can easily know $w^*_r < w(0) < p$ with assumption $DMI \geq 0$. Therefore, with "a small amount of" incomplete information, in equilibrium the supplier sets the wholesale price w^*_{ii} and capacity K^*_{ii} :

$$w^*_{ii} = \min(w^*_r, p) = w^*_r$$

$$K^*_{ii} = \xi'^* + u + G^{-1}\left(\frac{p-w^*_{ii}}{p}\right), \xi'^* = F_a^{-1}\left(\frac{w^*_{ii}-c-c_k}{w^*_{ii}-c}\right) \in [-a, a]$$

Taking derivatives with respect to a on the cubic equation about w_r^*, with $w(0) = \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$, we can get:

$$\frac{\partial w_{ii}^*}{\partial a}|a = 0 = \frac{-c_k^2 p}{2(\bar{\epsilon}-\underline{\epsilon})(w(0)-c)^2}$$

According to the Taylor formula:

$$w_{ii}^* = w(0) + \frac{-c_k^2 p}{2(\bar{\epsilon}-\underline{\epsilon})(w(0)-c)^2} a + o(a^2)$$

When information is complete, the order quantity $q_{ci}^* = \xi + \mu + G^{-1}(\frac{p-w_{ci}^*}{p})$. When information is incomplete, $q_{ii}^* = \xi + \mu + G^{-1}(\frac{p-w_{ii}^*}{p})$, if $\xi < \xi'^*$; $q_{ii}^* = K_{ii}^*$, if $\xi \geq \xi'^*$.

We have $\pi_{ci}^{sc}(\xi) = p(\mu+\xi) - cq_{ci}^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_{ci}^* - \mu - \xi)\bar{\epsilon} - (q_{ci}^* - \mu - \xi)^2 \Big] - c_k q_{ci}^*$,

and $\pi_{ii}^{sc}(\xi) = p(\mu+\xi) - cq_{ii}^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_{ci}^* - \mu - \xi)\bar{\epsilon} - (q_{ii}^* - \mu - \xi)^2 \Big] - c_k K_{ii}^*$,

Therefore, we can get:

$$\pi_{ci}^{sc} - \pi_{ii}^{sc} = \int_{-a}^{\xi'^*} (\xi' - \xi)c_k - \frac{\bar{\epsilon}-\epsilon}{p}(w_{ii}^* - w_{ci}^*)\big((c+c_k) - \frac{1}{2}(w_{ii}^* + w_{ci}^*)\big)dF_a(\xi)$$
$$+ \int_{\xi'^*}^{a} \big((\xi' - \xi) - \frac{\bar{\epsilon}-\epsilon}{p}(w_{ii}^* - w_{ci}^*)\big)\big((c+c_k) - \frac{1}{2}(w_{ii}^* + w_{ci}^*) + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}(\xi' - \xi)\big)\big)dF_a(\xi)$$

$$\pi_{ci}^{sc} - \pi_{ii}^{sc} = \int_{-a}^{\xi'^*} (\xi' - \xi)c_k - \frac{\bar{\epsilon}-\epsilon}{p}(w_{ii}^* - w_{ci}^*)\big((c+c_k) - \frac{1}{2}(w_{ii}^* + w_{ci}^*)\big)dF_a(\xi)$$
$$+ \int_{\xi'^*}^{a} \big((\xi' - \xi) - \frac{\bar{\epsilon}-\epsilon}{p}(w_{ii}^* - w_{ci}^*)\big)\big((c+c_k) - \frac{1}{2}(w_{ii}^* + w_{ci}^*) + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}(\xi' - \xi)\big)\big)dF_a(\xi)$$

where π_{ii}^{sc} and π_{ci}^{sc} are the total supply chain's expected profit (with respect to ξ) under incomplete information condition and under complete information condition respectively..

After some algebra, we can get:

$$\begin{aligned}
\pi_{ci}^{sc} - \pi_{ii}^{sc} =& \frac{1}{2a}\int_{-a}^{a} -\frac{\bar{\epsilon}-\epsilon}{p}(w_{ii}^* - w_{ci}^*)\big((c+c_k) - \frac{1}{2}(w_{ii}^* + w_{ci}^*)\big)d\xi \\
&+ \frac{1}{2a}\int_{-a}^{\xi'^*} (\xi'^* - \xi)c_k d\xi \\
&+ \frac{1}{2a}\int_{\xi'^*}^{a} (\xi'^* - \xi)\big((c+c_k) - \frac{1}{2a}(w_{ii}^* + w_{ci}^*)\big)d\xi \\
&+ \frac{1}{2a}\int_{\xi'^*}^{a} \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}(\xi'^* - \xi)^2 d\xi \\
&+ \frac{1}{2a}\int_{\xi'^*}^{a} \frac{-1}{2}(w_{ii}^* - w_{ci}^*)d\xi
\end{aligned}$$

If $c_k = o(a^3)$, we can get $a - \xi'^* = \frac{2ac_k}{w_{ii}^* - c} = o(a^4)$, Since $(\xi'^* - \xi)\big((c+c_k) - \frac{1}{2}(w^*_{ii} + w^*_{ci})\big)$, $\frac{p}{2(\overline{\epsilon}-\underline{\epsilon})}(\xi'^* - \xi)^2$, $\frac{-1}{2}(w^*_{ii} - w^*_{ci})$, are all bounded when $\xi \in [-a, a]$.

$$\frac{1}{2a}\int_{\xi'^*}^{a}(\xi'^* - \xi)\big((c+c_k) - \frac{1}{2}(w^*_{ii} + w^*_{ci})\big)d\xi = o(a^3)$$

$$\frac{1}{2a}\int_{\xi'^*}^{a}\frac{p}{2(\overline{\epsilon}-\underline{\epsilon})}(\xi'^* - \xi)^2 d\xi = o(a^3)$$

$$\frac{1}{2a}\int_{\xi'^*}^{a}\big((\xi' - \xi) - \frac{\overline{\epsilon}-\epsilon}{p}(w^*_{ii} - w^*_{ci})\big)\big((c+c_k) - \frac{1}{2}(w^*_{ii} + w^*_{ci}) + \frac{p}{2(\overline{\epsilon}-\underline{\epsilon})}(\xi' - \xi)\big)\big)dF_a(\xi) = o(a^3)$$

$$\frac{1}{2a}\int_{-a}^{\xi'^*}(\xi'^* - \xi)c_k d\xi = \frac{(\xi'^* + a)^2}{4a}c_k = o(a^4)$$

$$\frac{1}{2a}\int_{-a}^{a} -\frac{\overline{\epsilon}-\epsilon}{p}(w^*_{ii} - w^*_{ci})\big((c+c_k) - \frac{1}{2}(w^*_{ii} + w^*_{ci})\big)d\xi =$$
$$\frac{1}{2a}\int_{-a}^{a}\frac{1}{2}\big(\xi + \frac{c_k^2}{(w(0)-c)^2}a + 0(a^2)\big)\big(\frac{c+c_k}{2} - \frac{p(\mu+\epsilon)}{2(\overline{\epsilon}-\underline{\epsilon})} - \frac{p\xi}{4(\overline{\epsilon}-\underline{\epsilon})} + \frac{c_k^2 p}{4(\overline{\epsilon}-\underline{\epsilon})(w(0)-c)^2}a + o(a^2)\big)d\xi$$
$$= o(a^2)$$

when $a \to 0^+$, omitting the high order infinitesimal, with $c_k = o(a^3)$ we can get:

$$\begin{aligned}
\pi^{sc}_{ci} - \pi^{sc}_{ii} =& \frac{1}{2a}\int_{-a}^{a} -\frac{\overline{\epsilon}-\epsilon}{p}(w^*_{ii} - w^*_{ci})\big((c+c_k) - \frac{1}{2}(w^*_{ii} + w^*_{ci})\big)d\xi \\
&+ \frac{1}{2a}\int_{-a}^{\xi'^*}(\xi'^* - \xi)c_k d\xi \\
&+ \frac{1}{2a}\int_{\xi'^*}^{a}(\xi'^* - \xi)\big((c+c_k) - \frac{1}{2a}(w^*_{ii} + w^*_{ci})\big)d\xi \\
&+ \frac{1}{2a}\int_{\xi'^*}^{a}\frac{p}{2(\overline{\epsilon}-\underline{\epsilon})}(\xi'^* - \xi)^2 d\xi \\
&+ \frac{1}{2a}\int_{\xi'^*}^{a}\frac{-1}{2}(w^*_{ii} - w^*_{ci})d\xi \\
&\to \frac{1}{2a}\int_{-a}^{a}\frac{1}{2}\big(\xi + \frac{c_k^2}{(w(0)-c)^2}a + 0(a^2)\big)\big(\frac{c+c_k}{2} - \frac{p(\mu+\epsilon)}{2(\overline{\epsilon}-\underline{\epsilon})} - \frac{p\xi}{4(\overline{\epsilon}-\underline{\epsilon})} \\
&+ \frac{c_k^2 p}{4(\overline{\epsilon}-\underline{\epsilon})(w(0)-c)^2}a + o(a^2)\big)d\xi \\
&\to \frac{1}{2a}\int_{-a}^{a}\frac{1}{2}\xi\big(\frac{c}{2} - \frac{p(\mu+\epsilon)}{2(\overline{\epsilon}-\underline{\epsilon})} - \frac{p\xi}{4(\overline{\epsilon}-\underline{\epsilon})}\big)d\xi \\
&= \frac{1}{2a}\int_{-a}^{a} -\frac{p\xi^2}{8(\overline{\epsilon}-\underline{\epsilon})}d\xi < 0
\end{aligned}$$

For the supplier, since the retailer's response functions are always the same under both complete information and incomplete information condition, the supplier can always take advantage of the private information and get more profit when information is complete, i.e., $\pi^s_{ci} > \pi^s_{ii}$, when $a \to 0^+$. Since $\pi^{sc}_{ci} < \pi^{sc}_{ii}$,when $a \to 0^+$. We have $\pi^r_{ci} < \pi^r_{ii}$, when $a \to 0^+$, because $\pi^r_{ci} = \pi^{sc}_{ci} - \pi^s_{ci}$, and $\pi^r_{ii} = \pi^{sc}_{ii} - \pi^s_{ii}$.

When $c_k = o(a^N)$, and integer $N > 3$, obviously we can get the same results.

3.7.8 Proof of Proposition 21

We assume the wholesale price w is exogenous in this case. Based on the assumptions in the trust-embedded model, if retailer report $\hat{\xi}$ in the first stage of the game, the supplier will believe ξ is uniformly distributed on $[\alpha^s\hat{\xi} + (1-\alpha^s)\underline{\xi}, \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi}]$, we denote the distribution function of this belief of ξ as F_t. According to Appendix 3.7.5, the supplier sets the capacity as:

$K(\hat{\xi}) = \xi'(\hat{\xi}) + \mu + G^{-1}(\frac{p-w}{p})$, where $\xi'(\hat{\xi}) = F_t^{-1}(\frac{w-c-c_k}{w-c}) = \alpha^s\hat{\xi} + (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right]$.

Therefore in the third stage of the game, the retailer sets the order quantity as:

$$q(\hat{\xi}) = \begin{cases} \xi + \mu + G^{-1}(\frac{p-w}{p}) & if\ \xi \leq \xi'(\hat{\xi}) \\ \xi'(\hat{\xi}) + \mu + G^{-1}(\frac{p-w}{p}) & if\ \xi > \xi'(\hat{\xi}) \end{cases} \tag{3.14}$$

By backward induction, the retailer will choose her reporting strategy in the first stage of the game to maximize her expected profit:

$$\begin{aligned} \max_{\hat{\xi}} \quad & \Pi_t^r(\hat{\xi}) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q(\hat{\xi})) - wq(\hat{\xi}) - \beta|\hat{\xi} - \xi| \\ s.t. \quad & \hat{\xi} \in [\underline{\xi}, \overline{\xi}] \end{aligned} \tag{3.15}$$

For the retailer's reporting strategy in the first state of the game, any $\hat{\xi} < \xi$ will be dominated by $\hat{\xi} = \xi$. This is because reporting the true type can avoid the moral cost and at the same time increasing ξ', which can possibly increase the capacity decision and thus improve the retailer's profit. Then, we consider different cases to get the best reporting strategy.

Case 1: For $\xi \leq \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi}) + \underline{\xi}$. When $\xi \leq \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi}) + \underline{\xi}$, we can easily get that $\xi \leq \alpha^s\xi + (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right]$, which means reporting the true type will ensure sufficient capacity for retailer. Therefore, the best reporting strategy will be:

(1) $\hat{\xi}^* = \xi$ if $\beta > 0$;

(2) $\hat{\xi}^*$ can be any $\hat{\xi} \in [\xi, \overline{\xi}]$, which satisfies $\alpha^s\hat{\xi} + (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right] > \xi$ if $\beta = 0$.

Case 2: For $\xi > \alpha^s\overline{\xi} + (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right]$. We can know that even the retailer report the highest type, she will still suffer from capacity insufficiency, since $\xi'(\hat{\xi}) = \alpha^s\hat{\xi} + (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right] \leq \alpha^s\overline{\xi} + (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right] < \xi$. Therefore, the retailer will order $q(\hat{\xi}) = \xi'(\hat{\xi}) + \mu + G^{-1}(\frac{p-w}{p})$. Retailer's maximization problem in the first stage will be:

$$\begin{aligned} \max_{\hat{\xi}} \quad & \Pi_t^r(\hat{\xi}) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q(\hat{\xi})) - wq(\hat{\xi}) - \beta|\hat{\xi} - \xi| \\ s.t. \quad & \hat{\xi} \in [\xi, \overline{\xi}] \end{aligned}$$

Without considering constraint, we can get: $\frac{\partial \Pi_t^r(\hat{\xi})}{\partial \hat{\xi}} = \frac{(\xi - \xi')p}{\overline{\epsilon} - \underline{\epsilon}}\alpha^s - \beta$, $\frac{\partial^2 \Pi_t^r(\hat{\xi})}{\partial \hat{\xi}^2} = -\frac{\alpha^{s2} p}{\overline{\epsilon} - \underline{\epsilon}} < 0$. Considering the constraint, the best reporting strategy will be:

(1) $\hat{\xi}^* = \xi$, if $\beta \geq \frac{(1-\alpha^s)\xi - (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right]}{\overline{\epsilon} - \underline{\epsilon}}\alpha^s p$

(2) $\hat{\xi}^* = \overline{\xi}$, if $\beta \leq \frac{\xi - \alpha^s\overline{\xi} - (1-\alpha^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi} - \underline{\xi})\right]}{\overline{\epsilon} - \underline{\epsilon}}\alpha^s p$

(3) $\hat{\xi}^* = \frac{\xi-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]-\frac{(\overline{\epsilon}-\underline{\epsilon})\beta}{p\alpha^s}}{\alpha^s}$,
if $\frac{\xi-\alpha^s\overline{\xi}-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]}{\overline{\epsilon}-\underline{\epsilon}}\alpha^s p \le \beta \le \frac{(1-\alpha^s)\xi-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]}{\overline{\epsilon}-\underline{\epsilon}}\alpha^s p.$

Case 3: For $\frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})+\underline{\xi} < \xi \le \alpha^s\overline{\xi}+(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})\right]$. In this case, the retailer can manipulate her reporting to achieve sufficient capacity. We can derive the minimum reporting the retailer should report in order to achieve sufficient capacity, we denote it as ξ_0:

$$\xi_0 \equiv \frac{\xi-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]}{\alpha^s} \in \left(\frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})+\underline{\xi},\overline{\xi}\right)$$

Obviously, $\hat{\xi}=\xi_0$ will dominate any $\hat{\xi}>\xi_0$ when $\beta>0$, because $\hat{\xi}=\xi_0$ already ensure enough capacity for the retailer and larger reporting will bring more moral costs when $\hat{\xi}>\xi$. We only need to consider $\hat{\xi}\le\xi_0$. When $\hat{\xi}\le\xi_0$, we only need to consider $\xi'(\hat{\xi})=\alpha^s\hat{\xi}+(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})\right]\le\xi$. Thus, retailer's order quantity will be $q(\hat{\xi})=\xi'(\hat{\xi})+\mu+G^{-1}\left(\frac{p-w}{p}\right)$. Similar to case 2, in the first state of the game, the retailer will optimize her expected profit by choosing the best reporting strategy.

$$\max_{\hat{\xi}} \quad \Pi_t^r(\hat{\xi}) = p\mathbb{E}_\epsilon \min(\mu+\xi+\epsilon, q(\hat{\xi})) - wq(\hat{\xi}) - \beta|\hat{\xi}-\xi|$$
$$s.t. \quad \hat{\xi}\in[\xi,\xi_0]$$

Without considering the constraint, we can get $\frac{\partial\Pi_t^r(\hat{\xi})}{\partial\hat{\xi}} = \frac{(\xi-\xi')p}{\overline{\epsilon}-\underline{\epsilon}}\alpha^s-\beta$,and $\frac{\partial^2\Pi_t^r(\hat{\xi})}{\partial\hat{\xi}^2} = -\frac{\alpha^{s2}p}{\overline{\epsilon}-\underline{\epsilon}}<0$. Considering the constraint, we can get the optimal reporting strategy for the retailer:

(1) $\hat{\xi}^*=\xi$, if $\beta \ge \frac{(1-\alpha^s)\xi-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]}{\overline{\epsilon}-\underline{\epsilon}}\alpha^s p$

(2) $\hat{\xi}^* = \frac{\xi-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]-\frac{(\overline{\epsilon}-\underline{\epsilon})\beta}{p\alpha^s}}{\alpha^s}$, if $0<\beta\le\frac{(1-\alpha^s)\xi-(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]}{\overline{\epsilon}-\underline{\epsilon}}\alpha^s p.$

(3) $\hat{\xi}^*$ can be any $\hat{\xi}\in[\underline{\xi},\overline{\xi}]$, which satisfies $\alpha^s\hat{\xi}+(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]\ge\xi$, if $\beta=0$.

From the aforementioned three cases, we can know that when $\beta>0$, we can always get $\xi'(\hat{\xi})=\alpha^s\hat{\xi}+(1-\alpha^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})\right]\le\xi$ in the equilibrium. The implication of this result is, two much capacity will not be useful for the retailer and over-reporting is morally costing, she will not over-report to make the capacity to surpass what she needs. Therefore, in the equilibrium we can always get $q_t=K_t=\xi'^*+\mu+G^{-1}\left(\frac{p-w}{p}\right)$. Hence, we can get the following result:

(1) For any $\xi\le\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)$, we can know that the retailer's expected profit is independent of α^s.

(2) For any $0\le\alpha_1^s<\alpha_2^s\le1$,we can show for any $\xi\ge\alpha_2^s\overline{\xi}+(1-\alpha_2^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]$, we can prove that the retailer's profit will strictly increase with trust level α^s when $\beta>0$. ie., . We will always have $\pi_t^r(\alpha_1^s)<\pi_t^r(\alpha_2^s)$. Since the retailer with private information $\xi\ge\alpha_2^s\overline{\xi}+(1-\alpha_2^s)\left[\underline{\xi}+\frac{w-c-c_k}{w-c}\left(\overline{\xi}-\underline{\xi}\right)\right]$ will always suffer from capacity insufficiency, we can get $\pi_t^r(\alpha_1^s)<\pi_t^r(\alpha_2^s)\iff q_t^r(\alpha_1^s)<q_t^r(\alpha_2^s)\iff\xi'(\alpha_1^s)<\xi'(\alpha_2^s)$. We can check $\xi'(\hat{\xi})=\alpha^s\hat{\xi}+(1-\alpha^s)\left[\underline{\xi}+\right.$

$\frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})]$ is always increasing with α^s, when $\hat{\xi} \geq \xi > \frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})+\underline{\xi}$. Therefore, we can get $\pi_t^r(\alpha_1^s) < \pi_t^r(\alpha_2^s)$.

(3) Similar to the reason in the previous case, For any $0 \leq \alpha_1^s < \alpha_2^s \leq 1$, we can show for any $\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi}) < \xi < \alpha_2^s\overline{\xi} + (1-\alpha_2^s)\left[\underline{\xi} + \frac{w-c-c_k}{w-c}(\overline{\xi}-\underline{\xi})\right]$, we can prove that the retailer's profit will increase with trust level α^s when $\beta > 0$. We will always have $\pi_t^r(\alpha_1^s) < \pi_t^r(\alpha_2^s)$.

Since $q_t = K_t = \xi'^* + \mu + G^{-1}(\frac{p-w}{p})$ in equilibrium, the supplier's profit is also increasing with order quantity, we can get the same result for the supplier and the total supply chain.

3.7.9 Proof of Proposition 20

In this subsection, we assume capacity cost $c_k = 0$, and the supplier can choose any capacity $K \geq \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$. By doing this, he can always ensure enough capacity for the retailer and maximize his profit. Thus, we don't need to consider the capacity constraint in this model. This allows us to capture the effect of trust and trustworthiness on the *DM* problem alone, without considering the *CM* problem.

In equilibrium of this game, retailer's reporting strategy $\hat{\xi}^*(\xi)$ is summarized in the following Lemma. We define $\xi_t \equiv DMI + \overline{\xi}$, and $\xi_t' \equiv \frac{\xi_t}{\alpha^s}$. In equilibrium of the trust embedded model, the optimal report strategy for the retailer will be as following:

(1) If $DMI \geq 0$, when $\beta \geq \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, the retailer with type ξ choose $\hat{\xi}^*(\xi) = \xi$

when $\beta < \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \underline{\xi}$

(2) If $2\underline{\xi} < DMI \leq \underline{\xi}$ and $\alpha^s \leq \frac{\xi_t}{\underline{\xi}}$, the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \xi$.

(3) If $2\underline{\xi} < DMI \leq \underline{\xi}$ and $\alpha^s > \frac{\xi_t}{\underline{\xi}}$,

for the retailer with type ξ, which satisfies $\underline{\xi} \leq \xi \leq \frac{\xi_t}{\alpha^s}$,

- When $\beta \geq \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \xi$
- When $\beta < \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \underline{\xi}$

for the retailer with type ξ, which satisfies $\xi > \frac{\xi_t}{\alpha^s}$,

- When $\beta \geq \frac{\xi_t'-\underline{\xi}}{\xi-\underline{\xi}} \cdot \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$,the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \xi$
- When $\beta < \frac{\xi_t'-\underline{\xi}}{\xi-\underline{\xi}} \cdot \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$ the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \underline{\xi}$

(4) If $DMI \leq 2\underline{\xi}$, the retailer with type ξ will choose $\hat{\xi}^*(\xi) = \xi$ when $\beta > 0$.

From Lemma 3.7.9, the optimal reporting strategy for the retailer with type ξ can be either $\hat{\xi}^*(\xi) = \xi$, or $\hat{\xi}^*(\xi) = \underline{\xi}$. On the one hand, the retailer always has an incentive to under-report her type, because under-reporting her type can make the supplier charge a lower wholesale price and thus increase her profit. On the other hand, retailer always has an incentive to tell the truth, because lying brings dis-utility to her. When the moral cost is very high, the incentive to tell the truth is stronger than the incentive to under-report, the retailer chooses to report the true type; On the contrary, when the dis-utility of deception is small, the retailer chooses to report the lowest type in order to get a low wholesale price.

The proof of Lemma 3.7.9 is as following.

In this case , we assume capacity cost $c_k = 0$, and the supplier can choose any capacity $K \geq \overline{\xi} + \mu + G^{-1}(\frac{p-w}{p})$. By doing this, he can always ensure enough capacity for the retailer and maximize his profit. Thus, we do not consider the capacity constraint in this model. This allows us to capture the effect of trust and trustworthiness on double marginalization problem, without considering the capacity misalignment problem.

Given the wholesale price w, retailer's best response is:

$$q^*(w, \xi) = \begin{cases} \xi + \mu + G^{-1}(\frac{p-w}{p}) & if\ w \leq p \\ 0 & if\ w > p \end{cases} \tag{3.16}$$

Given retailer's best response, the supplier optimizes his expected profit(with respect to ξ) $\Pi^s(w)$:

$$\max_{w} \quad \Pi^s(w) = (w-c)\mathbb{E}_{\xi} q^*(w, \alpha^s\hat{\xi} + (1-\alpha^s)\xi) \tag{3.17}$$

Given retailer's report $\hat{\xi}$, in the second stage of the game, supplier choose the wholesale price $w_t(\hat{\xi})$:

$$w_t(\hat{\xi}) = \min\big(p, \frac{(\mu + \alpha^s\hat{\xi} + \overline{\epsilon})p + (\overline{\epsilon} - \underline{\epsilon})c}{2(\overline{\epsilon} - \underline{\epsilon})}\big) \tag{3.18}$$

Given retailer's report $\hat{\xi}$, her order quantity $q_t(\hat{\xi})$ in the third stage of the game is:

$$q_t(\hat{\xi}) = \xi + \mu + G^{-1}\big(\frac{p - w_t(\hat{\xi})}{p}\big) \tag{3.19}$$

Therefore, in the first stage of the game, the retailer chooses an optimal $\hat{\xi}$ to maximize her expected profit:

$$\begin{aligned} \max_{\hat{\xi}} \quad & \Pi_t^r(\hat{\xi}) = p\mathbb{E}_{\epsilon} \min(\mu + \xi + \epsilon, q_t(\hat{\xi})) - w_t(\hat{\xi})q_t(\hat{\xi}) - \beta|\hat{\xi} - \xi| \\ s.t. \quad & \hat{\xi} \in [\underline{\xi}, \overline{\xi}] \end{aligned} \tag{3.20}$$

In the following analysis, we define $\xi_t \equiv DMI + \overline{\xi}$. When we consider different cases of ξ_t, it is equivalent to consider different cases of DMI.

(1) **First, we consider the case where $\xi_t \geq \overline{\xi}$ (i.e., $DMI \geq 0$).** Since we don't need to consider the constraint c_k, in the third stage of the game, retailer's best

response will be:

$$q(w,\xi) = \begin{cases} \xi + \mu + G^{-1}\left(\frac{p-w}{p}\right) & if\, w \leq p \\ 0 & if\, w > p \end{cases}$$

Given retailer's best response, supplier's expected profit function will be: $\Pi^s(w) = (w-c)\mathbb{E}_\xi q(w, \alpha^s\hat{\xi} + (1-\alpha^s)\xi)$. Thus, given retailer's report $\hat{\xi}$, and her best order quantity response, in the second stage of the game, we can get the optimal w for the supplier: $w_t(\hat{\xi}) = \min\left(p, \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right)$. Because $\xi_t \geq \overline{\xi}$, we can know $\frac{(\mu+\overline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \leq p$. Therefore, $\frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \leq p$. Therefore, we can know: $w_t(\hat{\xi}) = \min\left(p, \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$. Since $w_t(\hat{\xi}) \leq p$, we can get the retailer's optimal order quantity in the third stage of the game: $q_t(\hat{\xi}) = \xi + \mu + G^{-1}\left(\frac{p-w_t(\hat{\xi})}{p}\right) = \xi + \mu + \bar{\epsilon} - \frac{w_t(\hat{\xi})}{p}(\bar{\epsilon} - \underline{\epsilon}) \geq 0$. Therefore, in the first stage of the game, the retailer will choose a optimal $\hat{\xi}$ to maximize the expected profit:

$$\Pi_t^r(\hat{\xi}) = p\mathbb{E}_\epsilon \min(\mu + \xi + \epsilon, q(\hat{\xi})) - w(\hat{\xi})q(\hat{\xi}) - \beta|\hat{\xi} - \xi| \tag{3.21}$$

We define $\Pi^r(w,q) \equiv p\mathbb{E}_\epsilon \min(\mu+\xi+\epsilon, q) - wq$. Therefore, $\Pi_t^r(\hat{\xi}) = \Pi^r\left(q_t(\hat{\xi}), w_t(\hat{\xi})\right) - \beta|\hat{\xi} - \xi|$. According to the envelope theorem: When $\hat{\xi} > \xi$

$$\begin{aligned} \frac{\partial \Pi_t^r}{\partial \hat{\xi}} &= \frac{\partial \Pi_t^r}{\partial q_t} \cdot \frac{\partial q_t(\hat{\xi})}{\partial \hat{\xi}} + \frac{\partial \Pi_t^r}{\partial w_t} \cdot \frac{\partial w_t(\hat{\xi})}{\partial \hat{\xi}} - \frac{\partial \beta(\hat{\xi} - \xi)}{\partial \hat{\xi}} \\ &= 0 \cdot \frac{\partial q_t(\hat{\xi})}{\partial \hat{\xi}} + (-q_t) \cdot \frac{\partial w_t(\hat{\xi})}{\partial \hat{\xi}} - \frac{\partial \beta(\hat{\xi} - \xi)}{\partial \hat{\xi}} \\ &= -q_t(\hat{\xi})\frac{\alpha^s p}{2(\bar{\epsilon} - \underline{\epsilon})} - \beta < 0 \end{aligned}$$

When $\hat{\xi} \leq \xi$

$$\begin{aligned} \frac{\partial \Pi_t^r}{\partial \hat{\xi}} &= \frac{\partial \Pi_t^r}{\partial q_t} \cdot \frac{\partial q_t^(\hat{\xi})}{\partial \hat{\xi}} + \frac{\partial \Pi_t^r}{\partial w_t} \cdot \frac{\partial w_t(\hat{\xi})}{\partial \hat{\xi}} + \frac{\partial \beta(\hat{\xi} - \xi)}{\partial \hat{\xi}} \\ &= 0 \cdot \frac{\partial q_t(\hat{\xi})}{\partial \hat{\xi}} + (-q_t) \cdot \frac{\partial w_t(\hat{\xi})}{\partial \hat{\xi}} + \frac{\partial \beta(\hat{\xi} - \xi)}{\partial \hat{\xi}} \\ &= -q_t(\hat{\xi})\frac{\alpha^s p}{2(\bar{\epsilon} - \underline{\epsilon})} + \beta \end{aligned}$$

We can also get: $\frac{\partial^2 \Pi_t^r}{\partial \hat{\xi}^2} = \frac{(\alpha^s)^2 p}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$, Therefore, $\Pi_t^r(\hat{\xi})$ is convex. Because when $\hat{\xi} > \xi$, $\frac{\partial \Pi_t^r}{\partial \hat{\xi}} < 0$ the retailer will never over report $\hat{\xi} > \xi$, ie., any $\hat{\xi} > \xi$ will be dominated by $\hat{\xi} = \xi$. In addition, due to the convexity of $\Pi_t^r(\hat{\xi})$ when $\hat{\xi} \leq \xi$, the optimal $\hat{\xi}$ should be either $\underline{\xi}$,or ξ. We can get when $\hat{\xi} \leq \xi$:

$$\Pi_t^r(\hat{\xi}) = p(\mu+\xi) - w_t(\hat{\xi})q_t(\hat{\xi}) + \frac{p}{2(\bar{\epsilon} - \underline{\epsilon})}\left[-\underline{\epsilon}^2 + 2(q_t(\hat{\xi}) - \mu - \xi)\bar{\epsilon} - (q_t(\hat{\xi}) - \mu - \xi)^2\right] - \beta(\xi -$$

After some algebra, we can get:

$$\Pi_t^r(\hat{\xi}=\xi)-\Pi_t^r(\hat{\xi}=\underline{\xi})=(\xi-\underline{\xi})\left(\frac{\alpha^s\big[-\big(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\big)p+(\overline{\epsilon}-\underline{\epsilon})c\big]}{4(\overline{\epsilon}-\underline{\epsilon})}+\beta\right)$$
$$=(\xi-\underline{\xi})\left(\frac{q_t(\hat{\xi}=\underline{\xi})+q_t(\hat{\xi}=\xi)}{2}\frac{-\alpha^s p}{2(\overline{\epsilon}-\underline{\epsilon})}+\beta\right)$$

Therefore, for the retailer in the first stage, the optimal reporting strategy is as following:

- If $\beta \geq \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, we can get: $\Pi_t^r(\hat{\xi}=\xi)\geq\Pi_t^r(\hat{\xi}=\underline{\xi})$. The retailer with private type ξ will choose the optimal report $\hat{\xi}^*=\xi$
- When $\beta < \frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$.we can get: $\Pi_t^r(\hat{\xi}=\xi)<\Pi_t^r(\hat{\xi}=\underline{\xi})$, The retailer with private type ξ will choose the optimal report $\hat{\xi}^*=\underline{\xi}$

(2) **Second, we consider the case where $\xi_t\in(\underline{\xi},0]$ (i.e., $2\underline{\xi}<DMI\leq\underline{\xi}$).**

Note that any strategy $\hat{\xi}>\xi$ will be weakly dominated by $\hat{\xi}=\xi$, because over-reporting will always bring a higher whole sale price compared to reporting the true type, and at the same time bringing moral costs. We only consider $\hat{\xi}\leq\xi$ to find the optimal strategy of the supplier. When $\alpha^s\underline{\xi}\geq\xi_t$, i.e.,$\alpha^s\leq\frac{\xi_t}{\underline{\xi}}$, we can get $p=\frac{(\mu+\xi_t+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}\leq\frac{(\mu+\alpha^s\underline{\xi}+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}\leq\frac{(\mu+\alpha^s\hat{\xi}+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}$. Therefore, we can get, $w_t^*(\hat{\xi})=\min\left(p,\frac{(\mu+\alpha^s\hat{\xi}+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}\right)=p$. Thus the retailer's weakly dominating strategy will be $\hat{\xi}^*=\xi$, because reporting another types will not change the wholesale price, and also bring some moral costs. In addition, when $\alpha^s\underline{\xi}<\xi_t$, i.e.,$\alpha^s>\frac{\xi_t}{\underline{\xi}}$. We consider the following two cases:

① We consider the case where the retailer with type ξ, which satisfies $\underline{\xi}\leq\xi\leq\frac{\xi_t}{\alpha^s}<\xi_t$. Because we only consider $\hat{\xi}\leq\xi$. We can get: $\frac{(\mu+\alpha^s\hat{\xi}+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}\leq\frac{(\mu+\alpha^s\xi+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}\leq\frac{(\mu+\xi_t+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}=p$. Thus, with retailer's reporting $\hat{\xi}<\xi$, the supplier will always get $w_t(\hat{\xi})\leq p$. Thus the retailer will face the same optimal decision problem in the case where $\xi_t\geq\overline{\xi}$. Therefore, the retailer will choose the optimal reporting either $\hat{\xi}^*=\xi$, or $\hat{\xi}^*=\underline{\xi}$, depending on which report will bring more profit. Similar to the case where $\xi_t\geq\overline{\xi}$, for the retailer, the optimal reporting strategy is as following:

* When $\beta\geq\frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, we can get: $\Pi_t^r(\hat{\xi}=\xi)\geq\Pi_t^r(\hat{\xi}=\underline{\xi})$
  The retailer with private type ξ will choose the optimal report $\hat{\xi}^*=\xi$
* When $\beta<\frac{\alpha^s\left[\left(\mu+\overline{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\overline{\epsilon}-\underline{\epsilon})c\right]}{4(\overline{\epsilon}-\underline{\epsilon})}$, we can get: $\Pi_t^m(\hat{\xi}=\xi)<\Pi_t^m(\hat{\xi}=\underline{\xi})$, The retailer with private type ξ will choose the optimal report $\hat{\xi}^*=\underline{\xi}$

② We consider the case where the retailer with type ξ, which satisfies $\xi > \frac{\xi_t}{\alpha^s}$. If the retailer report $\frac{\xi_t}{\alpha^s} \le \hat{\xi} \le \xi$. We can get: $\frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \ge \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$. Therefore, any $\frac{\xi_t}{\alpha^s} \le \hat{\xi} < \xi$ will be weakly dominated by $\hat{\xi} = \xi$, because under reporting in this range will not lower the whole sale price, but may bring moral costs. If the retailer report $\underline{\xi} \le \hat{\xi} \le \frac{\xi_t}{\alpha^s}$. We can get: $\frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \le \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$. Similar to the case where $\xi_t > \bar{\xi}$, we can know that the supplier will either choose the optimal reporting strategy as either $\hat{\xi} = \frac{\xi_t}{\alpha^s}$ or $\hat{\xi} = \underline{\xi}$ in the range $\hat{\xi} \in [\underline{\xi}, \frac{\xi_t}{\alpha^s}]$. Because any $\frac{\xi_t}{\alpha^s} \le \hat{\xi} < \xi$ will be weakly dominated by $\hat{\xi} = \xi$, we can know that the retailer will either choose the optimal reporting strategy as either $\hat{\xi} = \xi$ or $\hat{\xi} = \underline{\xi}$. We can get when $\hat{\xi} \le \xi$:

$$\Pi_t^r(\hat{\xi}) = p(\mu+\xi) - w_t(\hat{\xi})q_t(\hat{\xi}) + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_t(\hat{\xi})-\mu-\xi)\bar{\epsilon} - (q_t(\hat{\xi})-\mu-\xi)^2\Big] -$$

Also, because $\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$, $w_t(\hat{\xi} = \xi) = \min(\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}, p) = p = \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$, thus $q_t(\hat{\xi} = \xi) = \mu + \xi + \underline{\epsilon} = \xi - \xi_t + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. Because $\frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} < \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$, we can get: $w_t(\hat{\xi} = \underline{\xi}) = \min(\frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}, p) = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$, thus $q_t(\hat{\xi} = \underline{\xi}) = \xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. After some algebra, we can get:

$$\begin{aligned}\Pi_t^r(\hat{\xi}=\xi) - \Pi_t^r(\hat{\xi}=\underline{\xi}) &= (\xi_t' - \underline{\xi})\left(\frac{\alpha^s\big[-\big(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\big)p+(\bar{\epsilon}-\underline{\epsilon})c\big]}{4(\bar{\epsilon}-\underline{\epsilon})}\right) + \beta(\xi - \\ &= (\xi_t' - \underline{\xi})\left(\frac{q_t(\hat{\xi}=\underline{\xi})+q_t(\hat{\xi}=\xi_t')}{2}\frac{-\alpha^s p}{2(\bar{\epsilon}-\underline{\epsilon})}\right) + \beta(\xi-\underline{\xi})\end{aligned} \tag{3.22}$$

Thus, in this case, the retailer's optimal reporting strategy in the first stage will be:

* If $\beta \ge \frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}}$, we can get: $\Pi_t^r(\hat{\xi}=\xi) \ge \Pi_t^r(\hat{\xi}=\underline{\xi})$
  The retailer with private type ξ will choose the optimal report $\hat{\xi}^* = \xi$
* When $\beta < \frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}}$,
  we can get: $\Pi_t^m(\hat{\xi}=\xi) < \Pi_t^m(\hat{\xi}=\underline{\xi})$, The retailer with private type ξ will choose the optimal report $\hat{\xi}^* = \underline{\xi}$

(3) **Third, we consider the case where $\xi_t \le \underline{\xi}$ ($DMI \le 2\underline{\xi}$).** Since we don't need to consider the capacity constraint, in the third stage of

the game, retailer's best response will be:

$$q(w,\xi) = \begin{cases} \xi + \mu + G^{-1}\left(\frac{p-w}{p}\right) & if\ w \le p \\ 0 & if\ w > p \end{cases}$$

Given retailer's best response, supplier's expected profit function will be: $\Pi^s(w) = (w-c)\mathbb{E}_\xi q(w, \alpha^s\hat{\xi} + (1-\alpha^s)\xi)$. Thus, given retailer's report $\hat{\xi}$, and her best order quantity response, in the second stage of the game, we can get the optimal w for the supplier: $w_t(\hat{\xi}) = \min\left(p, \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right)$. Because $\xi_t \le \underline{\xi}$, we can know $\frac{(\mu+\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \ge p$. Therefore, $\frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \ge \frac{(\mu+\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \ge p$. Therefore, we can know: $w_t(\hat{\xi}) = \min\left(p, \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = p$. Since the trust level cannot affect the wholesale price, which is always the fixed market price, trust cannot affect supply chain efficiency.

To make the model tractable and capture the essential insights, we use two special cases of β and investigate the effect of trust on supply chain efficiency. When the retailer is trustworthy,i.e., $\beta \ge \frac{(\mu+\bar{\epsilon}+3\bar{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$, in the equilibrium she reports $\hat{\xi}^*(\xi) = \xi$, according to Lemma 3.7.9. On the contrary, when retailer is fully untrustworthy, i.e., $\beta = 0$, she will report $\hat{\xi}^*(\xi) = \underline{\xi}$ to lower the wholesale price and get more profit. The role of trust and trustworthiness in supply chain efficiency is summarized in the following Lemma 3.7.9.

In equilibrium of the trust-embedded model when $c_k = 0$ and capacity is always enough, we investigate the effect of trust and trustworthiness on supply chain efficiency, and summarize the results in Table 1, and Table 2.

TABLE S7: The effect of trust and trustworthiness on Supply Chain Efficiency: $DMI \ge 0$

	$0 < \alpha^s < 1$	$\alpha^s = 0$	$\alpha^s = 1$
$\beta \ge \frac{(\mu+\bar{\epsilon}+2\bar{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = \frac{-2p\alpha^s}{8(\bar{\epsilon}-\underline{\epsilon})} var(\xi) < 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{s}(\xi)}{\partial \alpha^s} = \frac{p(1-\alpha^s)}{2(\bar{\epsilon}-\underline{\epsilon})} var(\xi) > 0$	$\mathbb{E}_\xi \pi_t^{s}(\xi) = \mathbb{E}_\xi \pi_{ii}^{s}(\xi)$	$\mathbb{E}_\xi \pi_t^{s}(\xi) = \mathbb{E}_\xi \pi_{ci}^{s}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{r}(\xi)}{\partial \alpha^s} = \frac{-(1-\frac{\alpha^s}{2})p}{2(\bar{\epsilon}-\underline{\epsilon})} var(\xi) < 0$	$\mathbb{E}_\xi \pi_t^{r}(\xi) = \mathbb{E}_\xi \pi_{ii}^{r}(\xi)$	$\mathbb{E}_\xi \pi_t^{r}(\xi) = \mathbb{E}_\xi \pi_{ci}^{r}(\xi)$
$\beta = 0$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = -\underline{\xi} \cdot \frac{(\mu+\bar{\epsilon}+\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) > \mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{s}(\xi)}{\partial \alpha^s} = -\frac{\alpha^s \underline{\xi}^2 p}{2(\bar{\epsilon}-\underline{\epsilon})} < 0$	$\mathbb{E}_\xi \pi_t^{s}(\xi) = \mathbb{E}_\xi \pi_{ii}^{s}(\xi)$	$\mathbb{E}_\xi \pi_t^{s}(\xi) < \mathbb{E}_\xi \pi_{ci}^{s}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{r}(\xi)}{\partial \alpha^s} = -\underline{\xi} \cdot \frac{(\mu+\bar{\epsilon}+2\xi-\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$	$\mathbb{E}_\xi \pi_t^{r}(\xi) = \mathbb{E}_\xi \pi_{ii}^{r}(\xi)$	$\mathbb{E}_\xi \pi_t^{r}(\xi) > \mathbb{E}_\xi \pi_{ci}^{r}(\xi)$

$\mathbb{E}_\xi \pi_t^r(\xi)$, $\mathbb{E}_\xi \pi_t^s(\xi)$, $\mathbb{E}_\xi \pi_t^{sc}(\xi)$, are the retailer, the supplier and the total supply chain's expected profit with respect to ξ in equilibrium of the trust-embedded model. $\mathbb{E}_\xi \pi_{ci}^r(\xi)$, $\mathbb{E}_\xi \pi_{ci}^s(\xi)$, $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$, are expected profit with respect to ξ when information is complete. $\mathbb{E}_\xi \pi_{ii}^r(\xi)$, $\mathbb{E}_\xi \pi_{ii}^s(\xi)$, $\mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$, are expected profit with respect to ξ when information is incomplete.

TABLE S8: The effect of trust and trustworthiness on Supply Chain Efficiency: $2\underline{\xi} < DMI \leq \underline{\xi}$

	$\frac{\xi_t}{\underline{\xi}} < \alpha^s < 1$	$0 \leq \alpha^s \leq \frac{\xi_t}{\underline{\xi}}$	$\alpha^s = 1$
$\beta \geq \frac{(\mu+\bar{\epsilon}+3\overline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}} -\frac{\xi}{2} \cdot \frac{(\mu+\bar{\epsilon}+\alpha^s\xi)p-(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} dF(\xi) > 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{s}(\xi)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}} \frac{\xi^2 p(1-\alpha^s)}{2(\bar{\epsilon}-\underline{\epsilon})} dF(\xi) > 0$	$\mathbb{E}_\xi \pi_t^{s}(\xi) = \mathbb{E}_\xi \pi_{ii}^{s}(\xi)$	$\mathbb{E}_\xi \pi_t^{s}(\xi) = \mathbb{E}_\xi \pi_{ci}^{s}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{r}(\xi)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}} -\xi \cdot \frac{(\mu+\bar{\epsilon}+2\xi-\alpha^s\xi)p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} dF(\xi) > 0$	$\mathbb{E}_\xi \pi_t^{r}(\xi) = \mathbb{E}_\xi \pi_{ii}^{r}(\xi)$	$\mathbb{E}_\xi \pi_t^{r}(\xi) = \mathbb{E}_\xi \pi_{ci}^{r}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\xi)}{\partial \alpha^s} = -\underline{\xi} \cdot \frac{(\mu+\bar{\epsilon}+\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) = \mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$	$\mathbb{E}_\xi \pi_t^{sc}(\xi) > \mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$
$\beta = 0$	$\frac{\partial \mathbb{E}_\xi \pi_t^{s}(\xi)}{\partial \alpha^s} = -\frac{\alpha^s \underline{\xi}^2 p}{2(\bar{\epsilon}-\underline{\epsilon})} < 0$	$\mathbb{E}_\xi \pi_t^{s}(\xi) = \mathbb{E}_\xi \pi_{ii}^{s}(\xi)$	$\mathbb{E}_\xi \pi_t^{s}(\xi) < \mathbb{E}_\xi \pi_{ci}^{s}(\xi)$
	$\frac{\partial \mathbb{E}_\xi \pi_t^{r}(\xi)}{\partial \alpha^s} = -\underline{\xi} \cdot \frac{(\mu+\bar{\epsilon}+2\xi-\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$	$\mathbb{E}_\xi \pi_t^{r}(\xi) = \mathbb{E}_\xi \pi_{ii}^{r}(\xi)$	$\mathbb{E}_\xi \pi_t^{r}(\xi) > \mathbb{E}_\xi \pi_{ci}^{r}(\xi)$

$\mathbb{E}_\xi \pi_t^r(\xi)$, $\mathbb{E}_\xi \pi_t^s(\xi)$, $\mathbb{E}_\xi \pi_t^{sc}(\xi)$, are the retailer, the supplier and the total supply chain's expected profit with respect to ξ in equilibrium of the trust-embedded model. $\mathbb{E}_\xi \pi_{ci}^r(\xi)$, $\mathbb{E}_\xi \pi_{ci}^s(\xi)$, $\mathbb{E}_\xi \pi_{ci}^{sc}(\xi)$, are expected profit with respect to ξ when information is complete. $\mathbb{E}_\xi \pi_{ii}^r(\xi)$, $\mathbb{E}_\xi \pi_{ii}^s(\xi)$, $\mathbb{E}_\xi \pi_{ii}^{sc}(\xi)$, are expected profit with respect to ξ when information is incomplete.

The Proof of Lemma 3.7.9 is as following.

(1) First, we consider the case where $\xi_t \geq \overline{\xi}$ (i.e., $DMI \geq 0$). Because we know that $q_t(\hat{\xi} = \underline{\xi}) \geq q_t(\hat{\xi} = \xi) \geq \mu + \underline{\xi} + \underline{\epsilon} > 0$, we can get: $\frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})} = \frac{q_t^*(\hat{\xi}=\underline{\xi})+q_t^*(\hat{\xi}=\xi)}{2}\frac{\alpha^s p}{2(\bar{\epsilon}-\underline{\epsilon})} \geq \frac{\alpha^s p}{2(\bar{\epsilon}-\underline{\epsilon})}(\mu+\underline{\xi}+\underline{\epsilon})$. Also note that $\frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})} \leq \frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\overline{\xi}-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})} \leq \frac{\left(\mu+\bar{\epsilon}+2\overline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$. Combining the aforementioned two conditions, we can get: $0 \leq \frac{\alpha^s p}{2(\bar{\epsilon}-\underline{\epsilon})}(\mu+\underline{\xi}+\underline{\epsilon}) \leq \frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})} \leq \frac{\left(\mu+\bar{\epsilon}+2\overline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$.

Therefore, when $\beta \geq \frac{\left(\mu+\bar{\epsilon}+2\overline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$. For any ξ and any α^s, we can always have $\beta \geq \frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}$. According to Lemma 3.7.9, the retailer with any private type ξ will choose the optimal report $\hat{\xi}^* = \xi$. Thus in the equilibrium, we can get the optimal w_t^*, and q_t^*: $w_t^* = \min\left(p, \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$. Since $w_t^*(\hat{\xi}) \leq p$, we can get the retailer's optimal order quantity in the third stage of the game: $q_t^* = \xi + \mu + G^{-1}\left(\frac{p-w_t^*(\hat{\xi})}{p}\right) = \xi - \frac{\alpha^s}{2}\xi + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. Therefore, in the equilibrium, $\pi_t^s(\xi) = (w_t^* - c) * q_t^* = \left(\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} - c\right)\left(\xi - \frac{\alpha^s}{2}\xi + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}\right)$. We can get the derivatives of $\pi_t^s(\xi)$ with α^s, we can get: $\frac{\partial \pi_t^s(\xi)}{\partial \alpha^s} = \frac{\xi^2 p(1-\alpha^s)}{2(\bar{\epsilon}-\underline{\epsilon})} \geq 0$. Specifically, when $\alpha^s = 0$, $\pi_t^s(\xi) = \pi_{ii}^s(\xi)$, when $\alpha^s = 1$, $\pi_t^s(\xi) = \pi_{ci}^s(\xi)$. In the equilibrium, for the total supply chain efficiency. $\pi_t^{sc}(\xi) = p(\mu+\xi) - cq_t^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_t^* - \mu - \xi)\bar{\epsilon} - (q_t^* - \mu - \xi)^2\Big]$, where $q_t^* = \xi - \frac{\alpha^s}{2}\xi + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. When $\alpha = 0$, the trust model will go back to the incomplete information case, $\pi_{ii}^{sc}(\xi) = p(\mu+\xi) - cq_{ii}^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_{ii}^*-\mu-\xi)\bar{\epsilon} - (q_{ii}^*-\mu-\xi)^2\Big]$,where $q_{ii}^* = \xi + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. After some algebra, we can get: $\mathbb{E}_\xi\pi_t^{sc}(\xi) - \mathbb{E}_\xi\pi_{ii}^{sc}(\xi) = \frac{-p}{8(\bar{\epsilon}-\underline{\epsilon})}\mathbb{E}_\xi\xi^2 = \frac{-p(\alpha^s)^2}{8(\bar{\epsilon}-\underline{\epsilon})}var(\xi) < 0$. Specifically, when $\alpha^s = 0$, $\mathbb{E}_\xi\pi_t^{sc}(\xi) = \mathbb{E}_\xi\pi_{ii}^{sc}(\xi)$ when $\alpha^s = 1$, $\mathbb{E}_\xi\pi_t^{sc}(\xi) = \mathbb{E}_\xi\pi_{ci}^{sc}(\xi)$. In the equilibrium, for the retailer,$\pi_t^r(\xi) = p(\mu+\xi) - w_t^*q_t^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_t^*-\mu-\xi)\bar{\epsilon} - (q_t^*-\mu-\xi)^2\Big]$. According to the envelope theorem, we can get: $\frac{\partial\mathbb{E}_\xi\pi_t^r(\xi)}{\partial\alpha^s} = \frac{-(1-\frac{\alpha^s}{2})p}{2(\bar{\epsilon}-\underline{\epsilon})}var(\xi) < 0$.

When $\beta = 0$, for any ξ and any α^s, we can always have $\beta \leq \frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}$. According to Lemma 3.7.9,the retailer with any private type ξ will choose the optimal report $\hat{\xi}^* = \underline{\xi}$. Thus in the equilibrium, we can get the optimal w_t^*, and q_t^*: $w_t^* = \min\left(p, \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$. Since $w_t^*(\hat{\xi}) \leq p$, we can get the retailer's optimal order quantity in the third stage of the game: $q_t^* = \xi+\mu+G^{-1}\left(\frac{p-w_t^*}{p}\right) = \xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. Thus $\pi_t^s(\xi) = (w_t^*-c)*q_t^* = \left(\frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} - c\right)\left(\xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}\right)$. We can get the derivatives of $\mathbb{E}_\xi\pi_t^s(\xi)$ with α^s, we can get:$\frac{\partial\mathbb{E}_\xi\pi_t^s(\xi)}{\partial\alpha^s} = -\frac{\alpha^s\underline{\xi}^2p}{2(\bar{\epsilon}-\underline{\epsilon})} \leq 0$. Specifically, when $\alpha^s = 0$, $\mathbb{E}_\xi\pi_t^s(\xi) = \mathbb{E}_\xi\pi_{ii}^s(\xi)$.In equilibrium, for the total supply chain, $\Pi_t^{sc}(\xi) = p(\mu+\xi) - cq_t^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2+2(q_t^*-\mu-\xi)\bar{\epsilon}-(q_t^*-\mu-\xi)^2\Big]$, where $q_t^* = \xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. For the retailer, $\pi_t^r(\xi) = p(\mu+\xi) - w_t^*q_t^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2+2(q_t^*-\mu-\xi)\bar{\epsilon}-(q_t^*-\mu-\xi)^2\Big]$. According to the envelope theorem, we can get: $\frac{\partial\mathbb{E}_\xi\Pi_t^r(\xi)}{\partial\alpha^s} = \int_{\underline{\xi}}^{\bar{\xi}} q_t^*\frac{-\xi p}{2(\bar{\epsilon}-\underline{\epsilon})}dF(\xi) > 0$. According to the incomplete information model: $\pi_{ii}^{sc}(\xi) = p(\mu+\xi) - cq_{ii}^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2+2(q_{ii}^*-\mu-\xi)\bar{\epsilon} - (q_{ii}^*-\mu-\xi)^2\Big]$,where $q_{ii}^* = \xi + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$ After some algebra, we can get: $\pi_t^{sc}(\xi) - \pi_{ii}^{sc}(\xi) = -\frac{\alpha^s\underline{\xi}}{2}\cdot\frac{(\mu+\bar{\epsilon}+\frac{\alpha^s}{2}\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > 0$. Also, we can get:$\frac{\partial\Pi_t^{sc}(\xi)}{\partial\alpha^s} = -\underline{\xi}\cdot\frac{(\mu+\bar{\epsilon}+\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})} > 0$. Specifically, when $\alpha^s = 0$, $\pi_t^{sc}(\xi) = \pi_{ii}^{sc}(\xi)$, when $\alpha^s = 1$, in equilibrium, we can compare the expected profit of the complete information model and the trust model, and get $\pi_t^s(\xi) < \pi_{ci}^s(\xi)$, $\pi_t^r(\xi) > \pi_{ci}^r(\xi)$, and $\pi_t^{sc}(\xi) > \pi_{ci}^{sc}(\xi)$.

(2) Second, we consider the case where $\xi_t \in (\underline{\xi}, 0]$ (i.e., $2\underline{\xi} < DMI \leq \underline{\xi}$). When $\xi_t' = \frac{\xi_t}{\alpha^s} > \underline{\xi}$, because we have $q_t^*(\hat{\xi} = \underline{\xi}) > q_t^*(\hat{\xi} = \xi_t') = \mu+\xi+\underline{\epsilon} > 0$. We can get: $\frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}} = \frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{q_t^*(\hat{\xi}=\underline{\xi})+q_t^*(\hat{\xi}=\xi_t')}{2}\cdot\frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}} > \frac{\alpha^s(\xi_t'-\underline{\xi})\left((\mu+\xi+\underline{\epsilon})\cdot\frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}} > 0$.Also because $\xi > \xi_t'$, $\xi-\underline{\xi} > \xi_t'-\underline{\xi} = \frac{\xi_t}{\alpha^s}-\underline{\xi}$, we

can get: $\frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}} < \frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\frac{\xi_t}{\alpha^s}-\underline{\xi}} =$ $\frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi_t'-\underline{\xi}} = \alpha^s\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right) < \frac{\left(\mu+\bar{\epsilon}+3\bar{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$.

According to Lemma 3.7.9, when $\beta \geq \frac{\left(\mu+\bar{\epsilon}+3\bar{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c}{4(\bar{\epsilon}-\underline{\epsilon})}$, we can get: for any ξ and any α^s, we can always have $\beta \geq \frac{\alpha^s\left[\left(\mu+\bar{\epsilon}+(2-\frac{\alpha^s}{2})\xi-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}$, and $\beta \geq \frac{\alpha^s(\xi_t'-\underline{\xi})\left(\frac{\left[\left(\mu+\bar{\epsilon}+2\xi-\frac{\alpha^s}{2}\xi_t'-\frac{\alpha^s}{2}\underline{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c\right]}{4(\bar{\epsilon}-\underline{\epsilon})}\right)}{\xi-\underline{\xi}}$ Therefore, the retailer with any private type ξ will choose the optimal report $\hat{\xi}^* = \xi$. Thus in the equilibrium, we can get the optimal w_t^*, and q_t^*: $w_t^* = \min\left(p, \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right)$,$q_t^* = \xi+\mu+G^{-1}(\frac{p-w_t^*}{p})$. We can easily get: when $\alpha^s = 0$, $w_t^* = w_{ii}^*$, $q_t^* = q_{ii}^*$, for any ξ; When $\alpha^s = 1$, $w_t^* = w_{ci}^*$, $q_t^* = q_{ci}^*$, for any ξ; For the ξ: $\xi > \frac{\xi_t}{\alpha^s} > \underline{\xi}$ We can get: $\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$. Thus, $w_t^* = \min\left(p, \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = p$, and $q_t^* = \mu+\xi+\underline{\epsilon}$. For the ξ: $\underline{\xi} \leq \xi < \frac{\xi_t}{\alpha^s}$, because $\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} < \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$,$w_t^* = \min\left(p, \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$,and $q_t^* = \xi+\mu+G^{-1}(\frac{p-w_t^*}{p}) = \xi-\frac{\alpha^s}{2}\xi+\frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. In equilibrium, the supplier's profit will be $\pi_t^s(\xi) = (w_t^*-c)*q_t^* = \left(\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}-c\right)\left(\xi-\frac{\alpha^s}{2}\xi+\frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}\right)$. Therefore, we can get: $\mathbb{E}_\xi\pi_t^s(\xi) = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}}\left(\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}-c\right)\left(\xi-\frac{\alpha^s}{2}\xi+\frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}\right)dF(\xi)+\int_{\frac{\xi_t}{\alpha^s}}^{\bar{\xi}}\left(p-c\right)\left(\mu+\xi+\underline{\epsilon}\right)dF(\xi)$. According to the Leibniz integral rule, we can get:$\frac{\partial\mathbb{E}_\xi\Pi_t^s(\xi)}{\partial\alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}}\frac{\xi^2p(1-\alpha^s)}{2(\bar{\epsilon}-\underline{\epsilon})}dF(\xi) > 0$. For the total supply chain, $\mathbb{E}_\xi\Pi_t^{sc}(\xi) = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}}\left(p(\mu+\xi)-cq_{t1}^*+\frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\left[-\underline{\epsilon}^2+2(q_{t1}^*-\mu-\xi)\bar{\epsilon}-(q_{t1}^*-\mu-\xi)^2\right]\right)dF(\xi)+\int_{\frac{\xi_t}{\alpha^s}}^{\bar{\xi}}\left(p(\mu+\xi)-cq_{t2}^*+\frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\left[-\underline{\epsilon}^2+2(q_{t2}^*-\mu-\xi)\bar{\epsilon}-(q_{t2}^*-\mu-\xi)^2\right]\right)dF(\xi)$, where $q_{t1}^* = \xi-\frac{\alpha^s}{2}\xi+\frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$, $q_{t2}^* = \mu+\xi+\underline{\epsilon} > 0$. According to the Leibniz integral rule, we can get:$\frac{\partial\mathbb{E}_\xi\pi_t^{sc}(\xi)}{\partial\alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}}-\frac{\xi}{2}\cdot\frac{(\mu+\bar{\epsilon}+\alpha^s\xi)p-(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}dF(\xi) > 0$. In the equilibrium, for the retailer:$\mathbb{E}_\xi\pi_t^r(\xi) = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}}\left(p(\mu+\xi)-w_{t1}^*q_{t1}^*+\frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\left[-\underline{\epsilon}^2+2(q_{t1}^*-\mu-\xi)\bar{\epsilon}-(q_{t1}^*-\mu-\xi)^2\right]\right)dF(\xi)+\int_{\frac{\xi_t}{\alpha^s}}^{\bar{\xi}}\left(p(\mu+\xi)-w_{t2}^*q_{t2}^*+\frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\left[-\underline{\epsilon}^2+2(q_{t2}^*-\mu-\xi)\bar{\epsilon}-(q_{t2}^*-\mu-\xi)^2\right]\right)dF(\xi)$, where $w_{t1}^* = \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$, $w_{t2}^* = p$ $q_{t1}^* = \xi-\frac{\alpha^s}{2}\xi+\frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$, $q_{t2}^* = \mu+\xi+\underline{\epsilon}$. According to the Leibniz integral rule, we can get:$\frac{\partial\mathbb{E}_\xi\Pi_t^r(\xi)}{\partial\alpha^s} = \int_{\underline{\xi}}^{\frac{\xi_t}{\alpha^s}}q_{t1}^*\frac{-\xi p}{2(\bar{\epsilon}-\underline{\epsilon})}dF(\xi) > 0$. When $0 < \alpha^s \leq \frac{\xi_t}{\underline{\xi}}$, we can know $\frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \geq \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$. Thus, $w_t^* = \min\left(p, \frac{(\mu+\alpha^s\xi+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = p$.Because

$\frac{(\mu+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$, we can get:$w_{ii}^* = \min\left(p, \frac{(\mu+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = p$. Therefore, when $0 < \alpha^s \leq \frac{\xi_t}{\underline{\xi}}$, we can always have $w_{ii}^* = w_t^*$, $q_{ii}^* = q_t^*$, for any ξ. Therefore, in equilibrium of the trust model, the retailer, supplier, and total supply chain expected profit will be the same as that in incomplete information condition.

When $\beta = 0$, the retailer has a dominating strategy $\hat{\xi}^* = \underline{\xi}$. Obviously, when $\alpha^s = 0$, we can get $w_t^* = w_{ii}^*$, $q_t^* = q_{ii}^*$.Therefore, in equilibrium of the trust model, the retailer, supplier, and total supply chain expected profit will be the same as that in incomplete information condition. When $0 < \alpha^s \leq \frac{\xi_t}{\underline{\xi}}$, we can get $\alpha^s\underline{\xi} \geq \xi_t$. Therefore, $w_t^* = \min\left(p, \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = p$. We can get the retailer's order quantity in equilibrium in the third stage of the game: $q_t^* = \xi + \mu + G^{-1}\left(\frac{p-w_t^*}{p}\right) = \mu + \xi + \underline{\epsilon}$. Therefore, we can also get: we can get $w_t^* = w_{ii}^* = p$, $q_t^* = q_{ii}^* = \mu + \xi + \underline{\epsilon}$. When $\frac{\xi_t}{\underline{\xi}} < \alpha^s \leq 1$, we can get $\frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} < \frac{(\mu+\xi_t+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = p$. Therefore,$w_t^* = \min\left(p, \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\right) = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$.

In the equilibrium, the supplier's profit will be $\pi_t^s(\xi) = (w_t^* - c) * q_t^* = \left(\frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} - c\right)\left(\xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}\right)$. We can get the derivatives of $\mathbb{E}_\xi\pi_t^s(\xi)$ with α^s, we can get the derivatives of $\mathbb{E}_\xi\pi_t^s(\xi)$ with α^s, we can get:$\frac{\partial\mathbb{E}_\xi\pi_t^s(\xi)}{\partial\alpha^s} = -\frac{\alpha^s\underline{\xi}^2 p}{2(\bar{\epsilon}-\underline{\epsilon})} \leq 0$.
In equilibrium,the total supply chain expected profit will be $\Pi_t^{sc}(\xi) = p(\mu + \xi) - cq_t^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\left[-\underline{\epsilon}^2 + 2(q_t^* - \mu - \xi)\bar{\epsilon} - (q_t^* - \mu - \xi)^2\right]$, where $q_t^* = \xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. We have $\mathbb{E}_\xi\pi_t^{sc}(\xi) = \int_{\underline{\xi}}^{\bar{\xi}} \pi_t^{sc}(\xi)dF(\xi)$. Therefore, $\frac{\partial\mathbb{E}_\xi\pi_t^{sc}(\xi)}{\partial\alpha^s} = -\frac{\underline{\xi}}{2} \cdot \frac{(\mu+\bar{\epsilon}+\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > 0$. In equilibrium, the retailer's expected profit will be $\mathbb{E}_\xi\pi_t^r(\xi) = \int_{\underline{\xi}}^{\bar{\xi}}\left(p(\mu+\xi) - w_t^*q_t^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\left[-\underline{\epsilon}^2 + 2(q_t^* - \mu - \xi)\bar{\epsilon} - (q_t^* - \mu - \xi)^2\right]\right)dF(\xi)$, where $w_t^* = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$, $q_t^* = \xi - \frac{\alpha^s}{2}\underline{\xi} + \frac{(\mu+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$.
According to the Leibniz integral rule, we can get: $\frac{\partial\mathbb{E}_\xi\Pi_t^r(\xi)}{\partial\alpha^s} = q_t^*\frac{-\underline{\xi}p}{2(\bar{\epsilon}-\underline{\epsilon})} > 0$.

We can summarize Lemma 3.7.9, and get the result in Proposition 20.

3.7.10 Trust-embedded model: capacity cost is small

In this section, we consider the case where capacity cost is small, ie., $c_k \to 0$.[25] The following Lemma 3.7.10 (see proof in Appendix 3.7.11) summarizes the decisions in equilibrium of the trust embedded model in this case. If $c_k \to 0$, $DMI \geq 0$, $\beta = 0$,and $\frac{p-2c}{p}\frac{(\bar{\epsilon}-\underline{\epsilon})}{2} + \mu > 5\bar{\xi}$, in equilibrium of the trust embedded model:

(1) The report of the retailer with any type ξ is $\hat{\xi}^*(\xi) = \underline{\xi}$.

(2) The wholesale price supplier decides is $w_t^* = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$.

[25] When $c_k \to 0$, the decisions in equilibrium are approximate results. To keep the notation simple, we omit the approximation, and use equality instead. When $c_k = 0$, all the approximate results become exact value.

(3) The capacity decision the supplier decides is $K_t^* = \xi'^* + u + G^{-1}(\frac{p-w_t^*}{p})$, where $\xi'^* = \alpha^s \underline{\xi} + (1-\alpha^s)\overline{\xi}$

(4) The order quantity for the retailer is:

$$q_t^* = \begin{cases} q_{1t}^* = \xi + \mu + G^{-1}(\frac{p-w_t^*}{p}) & if\, \xi \leq \xi'^* \\ q_{2t}^* = \xi'^* + \mu + G^{-1}(\frac{p-w_t^*}{p}) & if\, \xi > \xi'^* \end{cases}$$

With the assumptions, retailer with any type ξ always reports the lowest type. In particular, $\beta = 0$ implies that retailer can manipulate her report without any disutility. Thus, the retailer always has incentive to under-report her type in order to lower the wholesale price and get more profit. On the other hand, the retailer can suffer from capacity insufficiency, if she has a high type ξ. Therefore, retailer with high type ξ has an incentive to not under-report, in order to get enough capacity.

However, when $\frac{p-2c}{p}\frac{(\overline{\epsilon}-\underline{\epsilon})}{2} + \mu > 5\overline{\xi}$, the profit margin and market uncertainty should be high, compared with the private information. This means the high type retailer has enough room the manipulate the wholesale price down, and thus the incentive to under-report is stronger than the incentive to not under-report. Therefore, the retailer chooses to report $\hat{\xi}^* = \underline{\xi}$. Then the supplier decides the wholesale price and capacity. On the one hand, $\xi'^* = \alpha^s \underline{\xi} + (1-\alpha^s)\overline{\xi}$ is decreasing with α^s. Higher trust level makes the retailer more likely to suffer from capacity insufficiency, which can harm the supply chain efficiency. On the other hand, $w_t^* = \frac{(\mu+\alpha^s\underline{\xi}+\overline{\epsilon})p+(\overline{\epsilon}-\underline{\epsilon})c}{2(\overline{\epsilon}-\underline{\epsilon})}$ is decreasing with α^s. Higher trust level leads to a lower wholesale price, which can improve the supply chain efficiency. Therefore, trust needs to trade off these two effects in terms of supply chain efficiency. The role of trust in supply chain efficiency is formally stated in the following Lemma 3.7.10 (see proof in Appendix 3.7.12). If $c_k \to 0$, $DMI \geq 0$, $\beta = 0$,and $\frac{p-2c}{p}\frac{(\overline{\epsilon}-\underline{\epsilon})}{2} + \mu > 5\overline{\xi}$, in equilibrium of the trust model, the following properties regarding the expected profit (with respect to ξ) of the total supply chain, i.e., $\mathbb{E}_\xi \pi_t^{sc}(\alpha^s)$, is satisfied:

(1)

$$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\xi'^*} \underbrace{\frac{\overline{\xi}}{2}\frac{p}{\overline{\epsilon}-\underline{\epsilon}}(\frac{(u+\xi+\overline{\epsilon})p-(\overline{\epsilon}-\underline{\epsilon})c}{p} - q_{1t}^*)}_{>0,\text{trust decreases the wholesale price}} dF(\xi)$$

$$+ \int_{\xi'^*}^{\overline{\xi}} \underbrace{\frac{-3\overline{\xi}}{2}\frac{p}{\overline{\epsilon}-\underline{\epsilon}}(\frac{(u+\xi+\overline{\epsilon})p-(\overline{\epsilon}-\underline{\epsilon})c}{p} - q_{2t}^*)}_{<0,\text{trust lowers the capacity}} dF(\xi)$$

(2) when $\alpha^s \to 0, \frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} > 0$; when $\alpha^s \to 1, \frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} < 0$;

(3) $\frac{\partial^2 \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^{s2}} < 0$, $\mathbb{E}_\xi \pi_t^{sc}(\alpha^s)$ is concave in α^s.

(4) There exists an unique trust level $\alpha^{s*} \in (0,1)$ s.t. $\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s}|_{\alpha^s=\alpha^{s*}} = 0$ and maximizes $\mathbb{E}_\xi \pi_t^{sc}(\alpha^s)$.

In Lemma 3.7.10, trust has to trade off between decreasing the wholesale price and lowering the capacity. There exists an trust level $\alpha^s \in (0,1)$ which maximizes total supply chain efficiency. On the one hand, higher trust level leads to lower wholesale price and thus higher supply chain efficiency. On the other hand, higher trust can lower the ξ'^* and the capacity decision, this makes retailer more likely to suffer from insufficient capacity and thus lower the supply chain efficiency. Combination of theses two effects will lead to an optimal trust level for the total supply chain efficiency.

3.7.11 Proof of Lemma 3.7.10

Given retailer's report $\hat{\xi}$ in the first stage, according to the assumption in trust model, supplier's posterior belief of ξ has the same distribution as $\alpha^s\hat{\xi} + (1-\alpha^s)\xi$, where $0 \le \alpha^s \le 1$ denotes the supplier's degree of trust. We denote this belief of ξ has distribution function $F_\alpha(\cdot)$. According to the second part of Lemma 17, the supplier will decide the capacity $K_t(\hat{\xi}) = \xi'_t(\hat{\xi}) + \mu + G^{-1}(\frac{p-w_t(\hat{\xi})}{p})$, where $\xi'_t(\hat{\xi}) = F_\alpha^{-1}(\frac{w_t(\hat{\xi})-c-c_k}{w_t(\hat{\xi})-c}) \in [\underline{\xi}, \overline{\xi}]$, and $w_t(\hat{\xi})$ is the decide wholesale price. Because $ck \to 0$, $\xi'_t(\hat{\xi}) = F_\alpha^{-1}(\frac{w_t(\hat{\xi})-c-c_k}{w_t(\hat{\xi})-c}) \to \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi}$, and $K_t(\hat{\xi}) \to \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi} + \mu + G^{-1}(\frac{p-w_t(\hat{\xi})}{p})$. In addition, the supplier decides the wholesale price $w_t(\hat{\xi}) = \min(w_r, p)$, where w_r is the unique real root satisfying $c + c_k < w_r < \frac{p(\mu+\bar{\epsilon})+(\bar{\epsilon}-\underline{\epsilon})(c+c_k)}{2(\bar{\epsilon}-\underline{\epsilon})}$ of equation $-2\frac{\bar{\epsilon}-\epsilon}{p}(w_r-c)^3 + \frac{p(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})-(\bar{\epsilon}-\underline{\epsilon})(c-c_k)}{p}(w_r-c)^2 + c_k^2(1-\alpha^s)\underline{\xi} = 0$. When $c_k \to 0$, $w_r \to \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$. With the assumption $\xi_t \ge \overline{\xi}$, $w_t(\hat{\xi}) \to \frac{(\mu+\alpha^s\hat{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$. [26]

First, we will show that for the retailer any report $\hat{\xi} > \xi$ will be dominated by $\hat{\xi} = \xi$. Note that when the retailer reports $\hat{\xi} = \xi$, the supplier will set the $\xi' = \alpha^s\xi + (1-\alpha^s)\overline{\xi} \ge \xi$. Therefore, reporting the true type will ensure the retailer get enough capacity. Since reporting lower type will also decrease the whole sale price and thus increase the profit, reporting $\hat{\xi} = \xi$ will dominate any $\hat{\xi} \ge \xi$. Therefore, we only consider $\hat{\xi} \ge \xi$.

For retailers with type $\xi \le \alpha^s\underline{\xi} + (1-\alpha^s)\overline{\xi}$, we can get $\xi \le \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi}$ for any $\hat{\xi}$. Therefore, they will always get enough capacity no matter what report is. Since lowering the report will decrease the wholesale price, and increase the profit, they will choose $\hat{\xi}^* = \underline{\xi}$, since lying will cost no utility for the retailer when $\beta = 0$.

For retailers with type $\xi > \alpha^s\underline{\xi} + (1-\alpha^s)\overline{\xi}$. We will show that any $\hat{\xi} > \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$ will be dominated by $\hat{\xi} = \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$. Because when retailer reports $\hat{\xi} = \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$, we can get $\xi' = \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi} = \xi$. Thus reporting $\hat{\xi} = \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$ will ensure her to get sufficient capacity. Since reporting lower type will decrease the whole sale price, and thus increase the profit, $\hat{\xi} = \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$ will dominate any $\hat{\xi} > \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$. Therefore we only consider $\hat{\xi} \le \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$. When $\hat{\xi} \le \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$, we can know $\xi' = \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi} \le \xi$. Since we only need to consider $\hat{\xi} \le \frac{\xi-(1-\alpha^s)\overline{\xi}}{\alpha^s}$, we have: $q_t(\hat{\xi}) = \xi' + \mu + G^{-1}(\frac{p-w_t(\hat{\xi})}{p})$, where $\xi' = \alpha^s\hat{\xi} + (1-\alpha^s)\overline{\xi}$. With the assumption that ϵ follows uniform distribution, $q_t(\hat{\xi}) = \frac{\left(u+\bar{\epsilon}+2(1-\alpha^s)\overline{\xi}+\alpha^s\hat{\xi}\right)p-(\bar{\epsilon}-\underline{\epsilon})c}{2p}$. In the first stage

[26] We will omit the approaching symbol, and use equality in the following proof. This will give us approximate results, the exact results hold when the capacity cost $c_k = 0$.

of the game, the optimization problem of the retailer will be:

$$\max_{\hat{\xi}} \quad \Pi_t^r(\hat{\xi}) = p(\mu+\xi) - w_t^*(\hat{\xi})q_t^*(\hat{\xi}) + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_t^*(\hat{\xi}) - \mu - \xi)\bar{\epsilon} - (q_t^*(\hat{\xi}) - \mu - \xi)^2\Big]$$

$$s.t. \quad \hat{\xi} \in [\underline{\xi}, \frac{\xi - (1-\alpha^s)\overline{\xi}}{\alpha^s}]$$

$\frac{\partial \Pi_t^r(\xi)}{\partial \hat{\xi}} = \frac{-\alpha^s}{4(\bar{\epsilon}-\underline{\epsilon})}\big[\big(u+\bar{\epsilon}-2\xi+4(1-\alpha^s)\overline{\xi}+3\alpha^s\hat{\xi}\big)p - (\bar{\epsilon}-\underline{\epsilon})c\big]$. Because $\big(u+\bar{\epsilon}-2\xi+4(1-\alpha^s)\overline{\xi}+3\alpha^s\hat{\xi}\big)p - (\bar{\epsilon}-\underline{\epsilon})c \geq (u+\bar{\epsilon}-5\overline{\xi})p - (\bar{\epsilon}-\underline{\epsilon})c$ for any ξ, any α^s, and any $\hat{\xi}$. If $\frac{p-2c}{2p}(\bar{\epsilon}-\underline{\epsilon}) > \mu + 5\overline{\xi}$, we have $(u+\bar{\epsilon}-5\overline{\xi})p - (\bar{\epsilon}-\underline{\epsilon})c > 0$, we can get $\frac{\partial \Pi_t^r(\xi)}{\partial \hat{\xi}} < 0$ for any ξ, any α^s. Therefore the optimal reporting strategy will be $\hat{\xi}^* = \underline{\xi}$, if $(u+\bar{\epsilon}-5\overline{\xi})p - (\bar{\epsilon}-\underline{\epsilon})c > 0$. When $\beta = 0$, $\xi_t \geq \overline{\xi}$, and $(u+\bar{\epsilon}-5\overline{\xi})p - (\bar{\epsilon}-\underline{\epsilon})c > 0$, $\hat{\xi}^* = \underline{\xi}$ for all types of ξ, the decision in the equilibrium: $w_t^* = \min\big(p, \frac{(\mu+\alpha^s\hat{\xi}^*+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\big) = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}$, $K_t^* = \xi'^* + u + G^{-1}\big(\frac{p-w_t^*}{p}\big)$, where $\xi'^* = \alpha^s\underline{\xi} + (1-\alpha^s)\overline{\xi}$,

$$q_t^* = \begin{cases} q_{1t}^* = \xi + \mu + G^{-1}\big(\frac{p-w_t^*}{p}\big) & if\ \xi \leq \xi'^* \\ q_{2t}^* = \xi'^* + \mu + G^{-1}\big(\frac{p-w_t^*}{p}\big) & if\ \xi > \xi'^* \end{cases}$$

3.7.12 Proof of Proposition 22

From Lemma 3.7.10, the expected profit (on ξ) of the supply chain in the equilibrium will be:

$$\mathbb{E}_\xi \pi_t^{sc}(\alpha^s) = \int_{\underline{\xi}}^{\xi'^*} p(\mu+\xi) - cq_{1t}^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_{1t}^* - \mu - \xi)\bar{\epsilon} - (q_{1t}^* - \mu - \xi)^2\Big] dF(\xi)$$
$$+ \int_{\xi'^*}^{\overline{\xi}} p(\mu+\xi) - cq_{2t}^* + \frac{p}{2(\bar{\epsilon}-\underline{\epsilon})}\Big[-\underline{\epsilon}^2 + 2(q_{2t}^* - \mu - \xi)\bar{\epsilon} - (q_{2t}^* - \mu - \xi)^2\Big] dF(\xi)$$

Because $w_t^* = \min\big(p, \frac{(\mu+\alpha^s\hat{\xi}^*+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})}\big) = \frac{(\mu+\alpha^s\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > \frac{(\mu+\underline{\xi}+\bar{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} \geq \frac{(\bar{\epsilon}-\underline{\epsilon})p+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} > \frac{(\bar{\epsilon}-\underline{\epsilon})c+(\bar{\epsilon}-\underline{\epsilon})c}{2(\bar{\epsilon}-\underline{\epsilon})} = c$, $q_{1t}^* = \xi + \mu + G^{-1}\big(\frac{p-w_t^*}{p}\big) < \xi + \mu + G^{-1}\big(\frac{p-c}{p}\big) = \frac{(u+\xi+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p}$. When $\xi > \xi'^*$, $q_{2t}^* = \xi'^* + \mu + G^{-1}\big(\frac{p-w_t^*}{p}\big) < \xi + \mu + G^{-1}\big(\frac{p-w_t^*}{p}\big) < \xi + \mu + G^{-1}\big(\frac{p-c}{p}\big) = \frac{(u+\xi+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p}$. Therefore,

$$\frac{\partial \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^s} = \int_{\underline{\xi}}^{\xi'^*} \underbrace{\frac{\overline{\xi}}{2}\frac{p}{\bar{\epsilon}-\underline{\epsilon}}\Big(\frac{(u+\xi+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - q_{1t}^*\Big)}_{>0,\text{trust decreases the whole sale price}} dF(\xi)$$
$$+ \int_{\xi'^*}^{\overline{\xi}} \underbrace{\frac{-3\overline{\xi}}{2}\frac{p}{\bar{\epsilon}-\underline{\epsilon}}\Big(\frac{(u+\xi+\bar{\epsilon})p-(\bar{\epsilon}-\underline{\epsilon})c}{p} - q_{2t}^*\Big)}_{<0,\text{trust lowers the capacity}} dF(\xi)$$

$$\frac{\partial^2 \mathbb{E}_\xi \pi_t^{sc}(\alpha^s)}{\partial \alpha^{s2}} = \int_{\underline{\xi}}^{\xi'^*} \underbrace{-\frac{\overline{\xi}^2}{4}\frac{p}{\bar{\epsilon}-\underline{\epsilon}}}_{<0} dF(\xi) + \int_{\xi'^*}^{\overline{\xi}} \underbrace{-\frac{9\overline{\xi}^2}{4}\frac{p}{\bar{\epsilon}-\underline{\epsilon}}}_{<0} dF(\xi) + \underbrace{\frac{(u+\bar{\epsilon}+\alpha^s\underline{\xi})p-(\bar{\epsilon}-\underline{\epsilon})c}{(\bar{\epsilon}-\underline{\epsilon})}(-2\overline{\xi}^2)f(\xi'^*)}_{<0}$$

Because when $\alpha^s \to 0$, we have $\xi'^* \to \overline{\xi}$, then $\frac{\partial \mathbb{E}_\xi \Pi_t^{sc}(\alpha^s)}{\partial \alpha^s} \to \int_{\underline{\xi}}^{\overline{\xi}} \underbrace{\frac{\overline{\xi}}{2} \frac{p}{\overline{\epsilon} - \underline{\epsilon}} \left(\frac{(u + \xi + \overline{\epsilon})p - (\overline{\epsilon} - \underline{\epsilon})c}{p} - q_{1t}^* \right)}_{>0,\text{trust decreases the whole sale price}} dF(\xi)$.

When $\alpha^s \to 1$, we have $\xi'^* \to \underline{\xi}$, then $\frac{\partial \mathbb{E}_\xi \Pi_t^{sc}(\alpha^s)}{\partial \alpha^s} \to \int_{\xi'^*}^{\overline{\xi}} \underbrace{\frac{-3\overline{\xi}}{2} \frac{p}{\overline{\epsilon} - \underline{\epsilon}} \left(\frac{(u + \xi + \overline{\epsilon})p - (\overline{\epsilon} - \underline{\epsilon})c}{p} - q_{2t}^* \right)}_{<0,\text{trust lowers the capacity}} dF(\xi)$.

Because $\frac{\partial \mathbb{E}_\xi \Pi_t^{sc}(\alpha^s)}{\partial \alpha^s}$ is continuous, according to the intermediate value theorem, there exists an $\alpha^{s*} \in (0,1)$ s.t. $\frac{\partial \mathbb{E}_\xi \Pi_t^{sc}(\alpha^s)}{\partial \alpha^s}|_{\alpha^s = \alpha^{s*}} = 0$. Because $\frac{\partial^2 \mathbb{E}_\xi \Pi_t^{sc}(\alpha^s)}{\partial \alpha^{s2}} < 0$, we can know the α^{s*} is unique and maximizing $\mathbb{E}_\xi \Pi_t^{sc}(\alpha^s)$.

3.8 Figures for Numerical Analysis

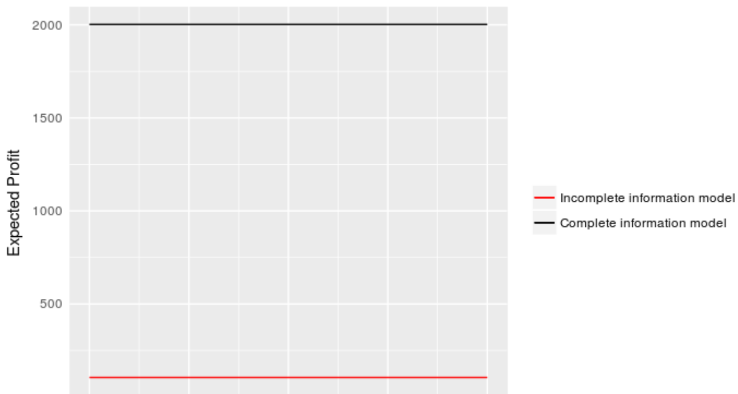


FIGURE S6: $p = 200, c = 10, c_k = 180, \mu = 400$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

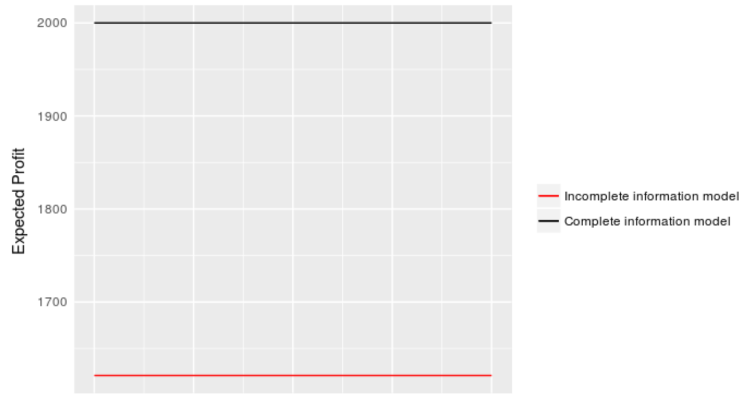


FIGURE S7: $p = 200, c = 10, c_k = 180, \mu = 400$. The market uncertainty is $\bar{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\bar{\epsilon} = 200$. The amount of incomplete information $\bar{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\bar{\xi} = 40$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

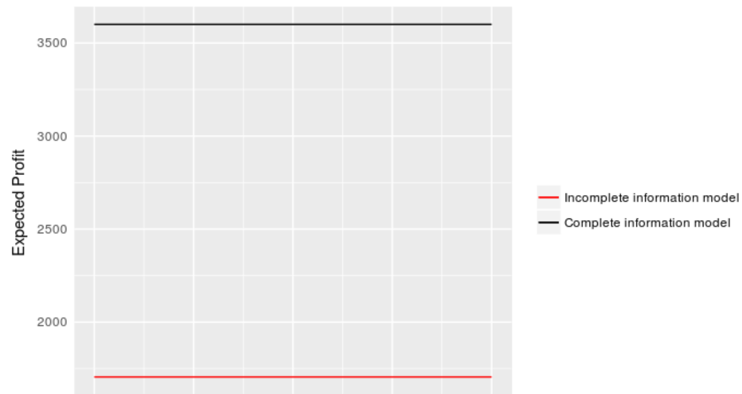


FIGURE S8: $p = 200, c = 10, c_k = 180, \mu = 400$. The market uncertainty is $\bar{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\bar{\epsilon} = 40$. The amount of incomplete information $\bar{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\bar{\xi} = 200$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

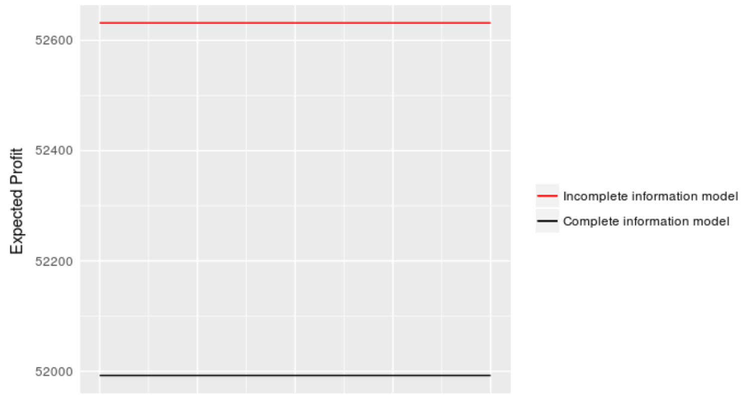


FIGURE S9: $p = 200, c = 10, c_k = 0.7, \mu = 400$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200, \overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

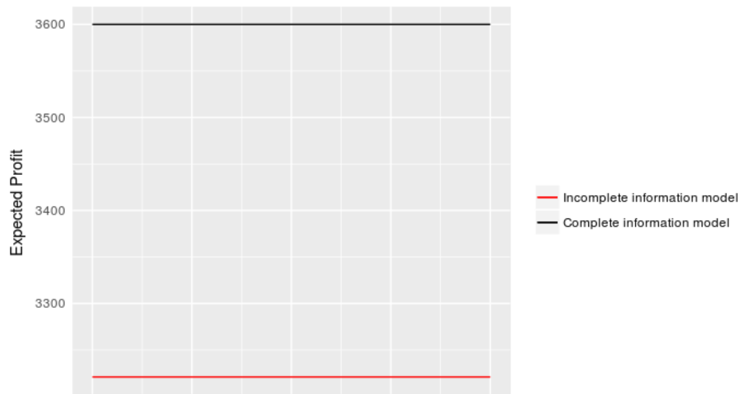


FIGURE S10: $p = 200, c = 10, c_k = 180, \mu = 400$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

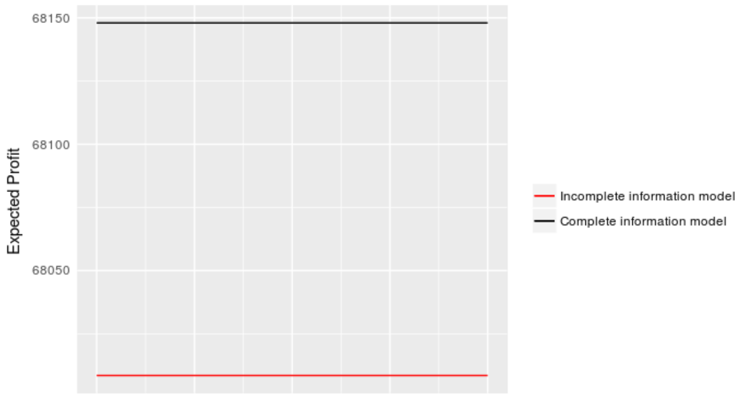


FIGURE S11: $p = 200, c = 10, c_k = 0.7, \mu = 400$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

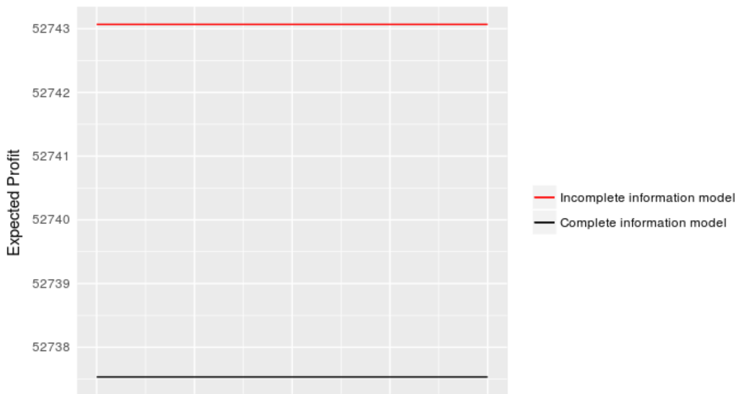


FIGURE S12: $p = 200, c = 10, c_k = 0.7, \mu = 400$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

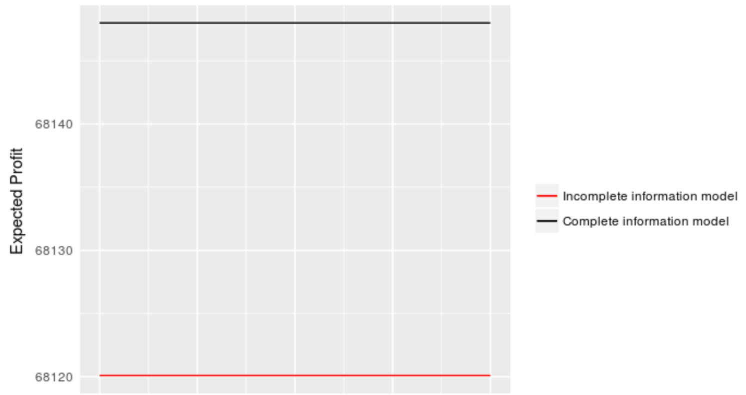


FIGURE S13: $p = 200, c = 10, c_k = 0.7, \mu = 400$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The vertical axis is the expected profit of the models.

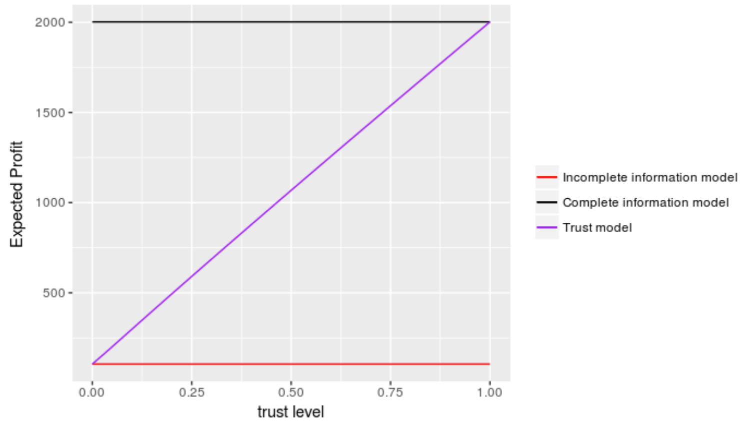


FIGURE S14: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

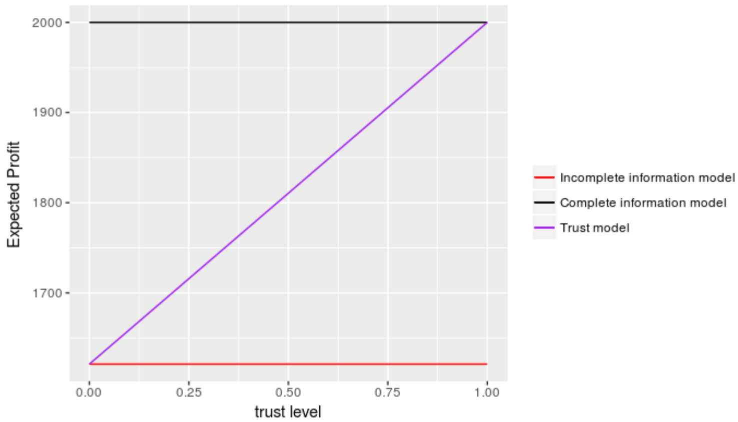


FIGURE S15: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

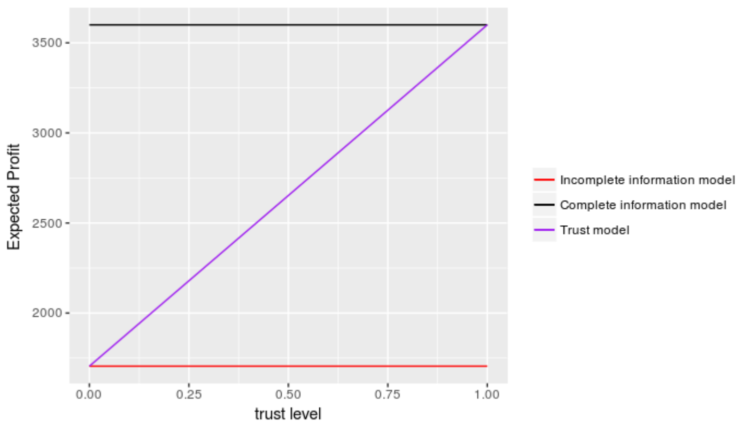


FIGURE S16: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

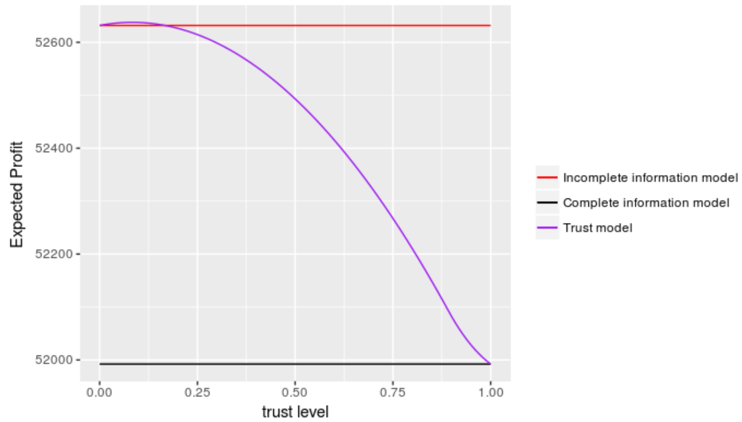


FIGURE S17: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

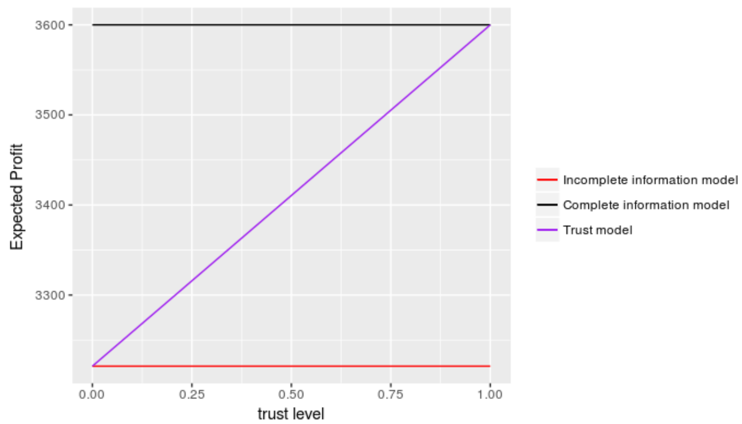


FIGURE S18: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

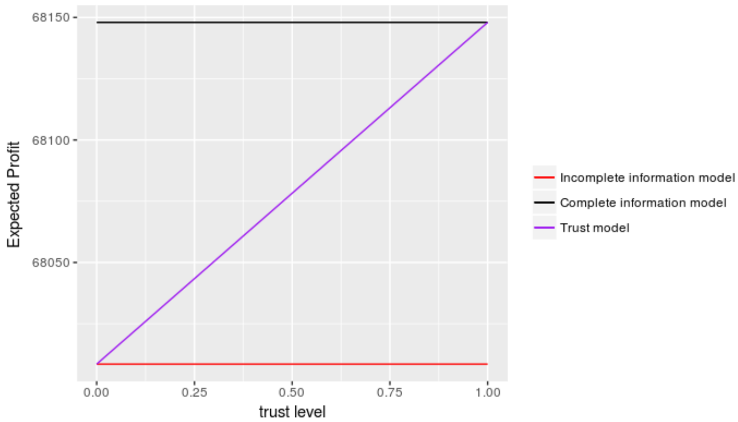


FIGURE S19: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40, \overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

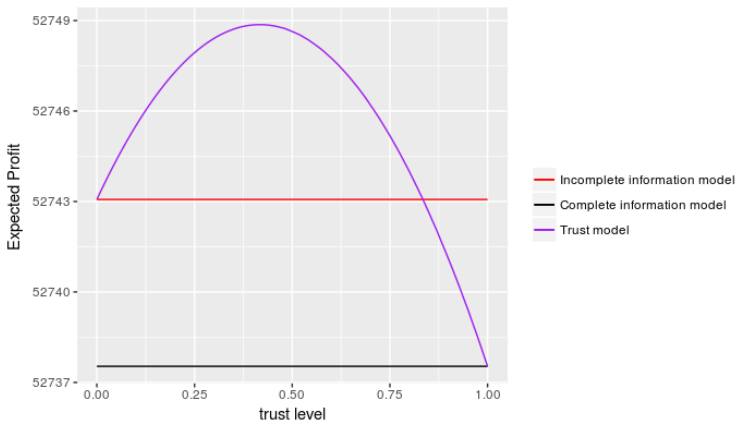


FIGURE S20: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

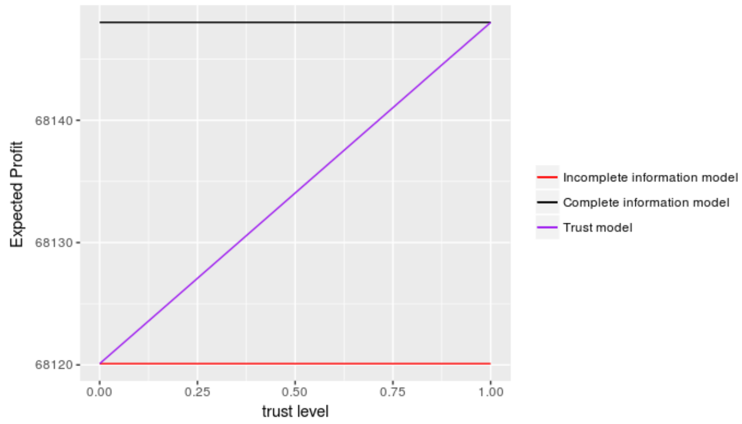


FIGURE S21: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 200$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40, \overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

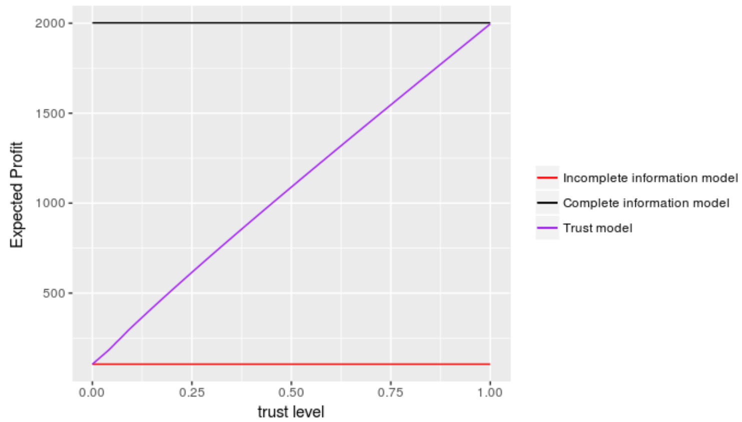


FIGURE S22: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 0.2$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

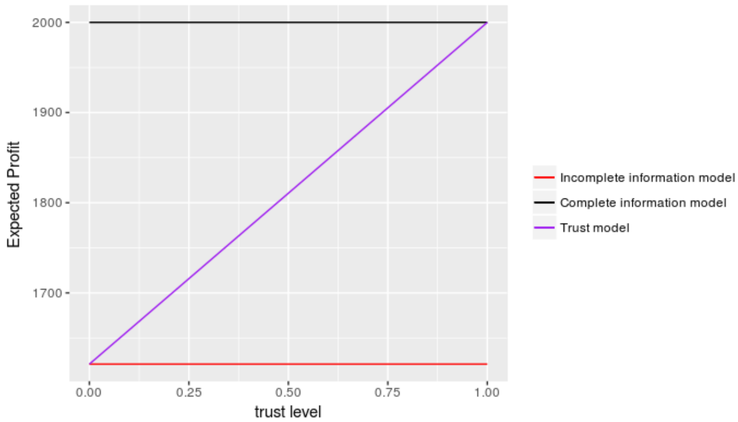


FIGURE S23: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 0.2$. The market uncertainty is $\bar{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\bar{\epsilon} = 200$. The amount of incomplete information $\bar{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\bar{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

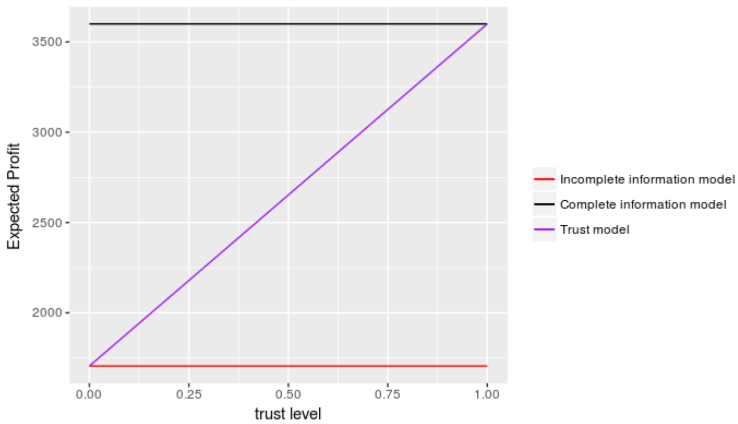


FIGURE S24: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 0.2$. The market uncertainty is $\bar{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\bar{\epsilon} = 40$. The amount of incomplete information $\bar{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\bar{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

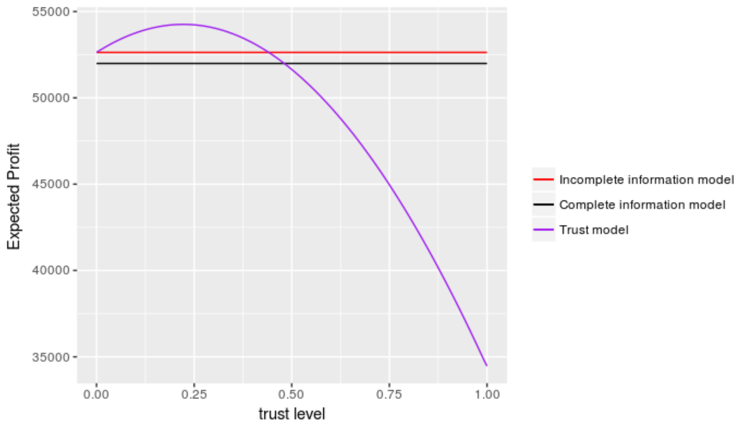


FIGURE S25: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 0.2$. The market uncertainty is $\bar{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\bar{\epsilon} = 200$. The amount of incomplete information $\bar{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\bar{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

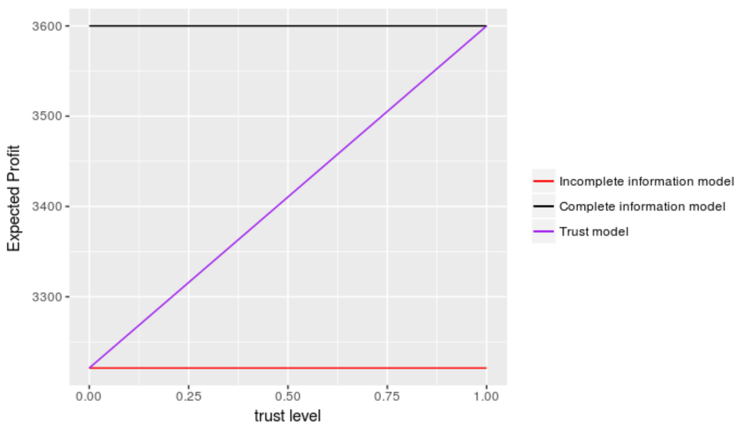


FIGURE S26: $p = 200, c = 10, c_k = 180, \mu = 400, \beta = 0.2$. The market uncertainty is $\bar{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\bar{\epsilon} = 40$. The amount of incomplete information $\bar{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\bar{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

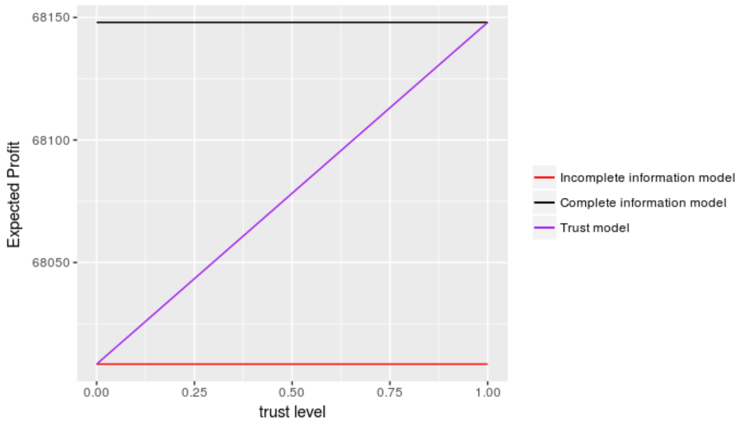


FIGURE S27: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 0.2$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 400$, where $\underline{\xi} = -200$, $\overline{\xi} = 200$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

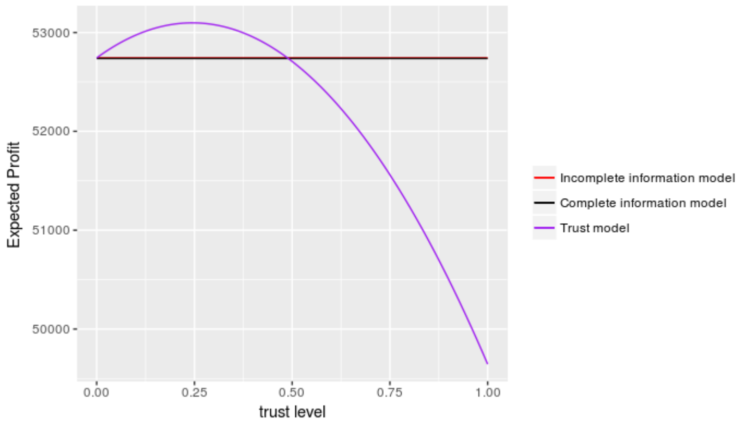


FIGURE S28: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 0.2$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 400$, where $\underline{\epsilon} = -200$, $\overline{\epsilon} = 200$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

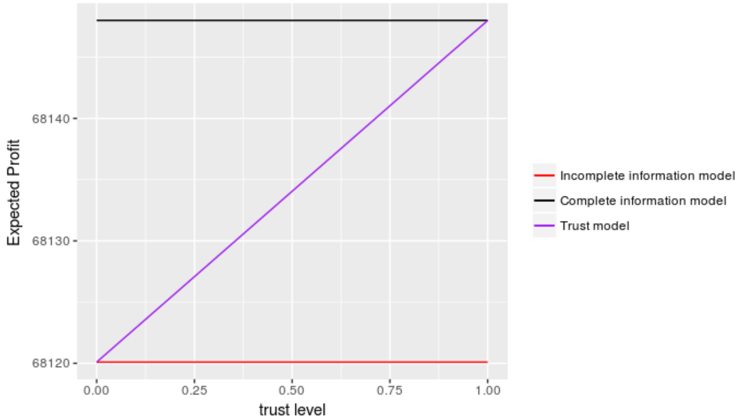


FIGURE S29: $p = 200, c = 10, c_k = 0.7, \mu = 400, \beta = 0.2$. The market uncertainty is $\overline{\epsilon} - \underline{\epsilon} = 80$, where $\underline{\epsilon} = -40$, $\overline{\epsilon} = 40$. The amount of incomplete information $\overline{\xi} - \underline{\xi} = 80$, where $\underline{\xi} = -40$, $\overline{\xi} = 40$. ϵ, and ξ follows uniform distribution. The horizontal axis is trust level, $\alpha^s \in [0, 1]$. The vertical axis is the expected profit of the models.

Bibliography

Alós-Ferrer, Carlos and Federica Farolfi (2019). “Trust games and beyond”. In: *Frontiers in Neuroscience* 13, p. 887.

Ashraf, Nava, Iris Bohnet, and Nikita Piankov (2006). “Decomposing trust and trustworthiness”. In: *Experimental economics* 9.3, pp. 193–208.

Asker, John and Estelle Cantillon (2008). “Properties of scoring auctions”. In: *The RAND Journal of Economics* 39.1, pp. 69–85.

— (2010). “Procurement when price and quality matter”. In: *The Rand journal of economics* 41.1, pp. 1–34.

Aviv, Yossi (2003). “A time-series framework for supply-chain inventory management”. In: *Operations Research* 51.2, pp. 210–227.

Bagnoli, Mark and Ted Bergstrom (2005). “Log-concave probability and its applications”. In: *Economic theory* 26.2, pp. 445–469.

Balzer, Benjamin and Antonio Rosato (2020). “Expectations-Based Loss Aversion in Auctions with Interdependent Values: Extensive vs. Intensive Risk”. In: *Management Science*.

Banerji, Abhijit and Neha Gupta (2014). “Detection, identification, and estimation of loss aversion: Evidence from an auction experiment”. In: *American Economic Journal: Microeconomics* 6.1, pp. 91–133.

Baron, Opher et al. (2015). “Newsvendor selling to loss-averse consumers with stochastic reference points”. In: *Manufacturing & Service Operations Management* 17.4, pp. 456–469.

Battermann, Harald L, Udo Broll, and Jack E Wahl (1997). *Constant relative risk aversion and form equivalence classes*. Tech. rep. Diskussionsbeiträge-Serie II.

Beer, Ruth, Hyun-Soo Ahn, and Stephen Leider (2018). “Can trustworthiness in a supply chain be signaled?” In: *Management science* 64.9, pp. 3974–3994.

Ben-Ner, Avner and Freyr Halldorsson (2010). “Trusting and trustworthiness: What are they, how to measure them, and what affects them”. In: *Journal of Economic Psychology* 31.1, pp. 64–79.

Ben-Ner, Avner and Louis Putterman (2001). “Trusting and trustworthiness”. In: *BUL Rev.* 81, p. 523.

Berg, Joyce, John Dickhaut, and Kevin McCabe (1995). “Trust, reciprocity, and social history”. In: *Games and economic behavior* 10.1, pp. 122–142.

Bernath, Michael S and Norma D Feshbach (1995). “Children’s trust: Theory, assessment, development, and research directions”. In: *Applied and Preventive Psychology* 4.1, pp. 1–19.

Bertsekas, Dimitri P et al. (1995). *Dynamic programming and optimal control*. Vol. 1. 2. Athena scientific Belmont, MA.

Bichler, Martin (2000). “An experimental analysis of multi-attribute auctions”. In: *Decision Support Systems* 29.3, pp. 249–268.

Bohnet, Iris and Rachel Croson (2004). “Trust and trustworthiness.” In:

Bohnet, Iris and Steffen Huck (2004). “Repetition and reputation: Implications for trust and trustworthiness when institutions change”. In: *American economic review* 94.2, pp. 362–366.

Bolton, Gary, Ben Greiner, and Axel Ockenfels (2013). "Engineering trust: reciprocity in the production of reputation information". In: *Management science* 59.2, pp. 265–285.

Bolton, Gary E, Elena Katok, and Axel Ockenfels (2004). "How effective are electronic reputation mechanisms? An experimental investigation". In: *Management science* 50.11, pp. 1587–1602.

Bowlby, John (1969). "Attachment and Loss. Vol 1: Attachment, Vol 2: Separation, Vol 3: Loss". In: *London: Hogarth Press* 1973, p. 1980.

Bowling, Michael et al. (2015). "Heads-up limit hold'em poker is solved". In: *Science* 347.6218, pp. 145–149.

Branco, Fernando (1997). "The design of multidimensional auctions". In: *The RAND Journal of Economics*, pp. 63–81.

Brown, Alexander L et al. (2021). "Meta-analysis of empirical estimates of loss-aversion". In:

Cachon, Gérard P (2003). "Supply chain coordination with contracts". In: *Handbooks in operations research and management science* 11, pp. 227–339.

Cachon, Gérard P and Martin A Lariviere (2001). "Contracting to assure supply: How to share demand forecasts in a supply chain". In: *Management science* 47.5, pp. 629–646.

Camerer, Colin F (2018). "Artificial intelligence and behavioral economics". In: *The Economics of Artificial Intelligence: An Agenda*. University of Chicago Press, pp. 587–608.

Campbell, Murray, A Joseph Hoane Jr, and Feng-hsiung Hsu (2002). "Deep blue". In: *Artificial intelligence* 134.1-2, pp. 57–83.

Charness, Gary, Ninghua Du, and Chun-Lei Yang (2011). "Trust and trustworthiness reputations in an investment game". In: *Games and economic behavior* 72.2, pp. 361–375.

Chaturvedi, Aadhaar and Victor Martínez-de Albéniz (2011). "Optimal procurement design in the presence of supply risk". In: *Manufacturing & Service operations management* 13.2, pp. 227–243.

Chaturvedi, Aadhaar, Elena Katok, and Damian R Beil (2019). "Split-award auctions: Insights from theory and experiments". In: *Management Science* 65.1, pp. 71–89.

Che, Yeon-Koo (1993). "Design competition through multidimensional auctions". In: *The RAND Journal of Economics*, pp. 668–680.

Chen, Fangruo (2007). "Auctioning supply contracts". In: *Management Science* 53.10, pp. 1562–1576.

Chen, Ningyuan and Javad Nasiry (2020). "Does Loss Aversion Preclude Price Variation?" In: *Manufacturing & Service Operations Management* 22.2, pp. 383–395.

Chen, Ying-Ju, Sridhar Seshadri, and Eitan Zemel (2008). "Sourcing through auctions and audits". In: *Production and Operations Management* 17.2, pp. 121–138.

Chen-Ritzo, Ching-Hua et al. (2005). "Better, faster, cheaper: An experimental analysis of a multiattribute reverse auction mechanism with restricted information feedback". In: *Management science* 51.12, pp. 1753–1762.

Chu, Leon Yang (2009). "Truthful bundle/multiunit double auctions". In: *Management Science* 55.7, pp. 1184–1198.

Cohen, Chen and Aner Sela (2005). "Manipulations in contests". In: *Economics Letters* 86.1, pp. 135–139.

Coller, Maribeth and Melonie B Williams (1999). "Eliciting individual discount rates". In: *Experimental Economics* 2.2, pp. 107–127.

Courty, Pascal and Javad Nasiry (2018). "Loss aversion and the uniform pricing puzzle for media and entertainment products". In: *Economic Theory* 66.1, pp. 105–140.

Crawford, Vincent P and Joel Sobel (1982). "Strategic information transmission". In: *Econometrica: Journal of the Econometric Society*, pp. 1431–1451.

Croson, Rachel and Nancy Buchan (1999). "Gender and culture: International experimental evidence from trust games". In: *American Economic Review* 89.2, pp. 386–391.

Davis, Andrew M, Elena Katok, and Anthony M Kwasnica (2014). "Should sellers prefer auctions? A laboratory comparison of auctions and sequential mechanisms". In: *Management Science* 60.4, pp. 990–1008.

Dirks, Kurt T and Donald L Ferrin (2001). "The role of trust in organizational settings". In: *Organization science* 12.4, pp. 450–467.

Eckel, Catherine C and Rick K Wilson (2004). "Is trust a risky decision?" In: *Journal of Economic Behavior & Organization* 55.4, pp. 447–465.

Eisenhuth, Roland (2010). "Auction design with loss averse bidders:The optimality of all pay mechanisms". In:

Engelbrecht-Wiggans, Richard, Ernan Haruvy, and Elena Katok (2007). "A comparison of buyer-determined and price-based multiattribute mechanisms". In: *Marketing Science* 26.5, pp. 629–641.

Engle-Warnick, Jim and Robert L Slonim (2004). "The evolution of strategies in a repeated trust game". In: *Journal of Economic Behavior & Organization* 55.4, pp. 553–573.

— (2006). "Learning to trust in indefinitely repeated games". In: *Games and Economic Behavior* 54.1, pp. 95–114.

Erikson, Erik H (1993). *Childhood and society*. WW Norton & Company.

Fan, Xiaoshuai, Ying-Ju Chen, and Christopher S Tang (2020). "To Bribe or Not in a Procurement Auction under Disparate Corruption Pressure". In: *Production and Operations Management*.

Fehr, Ernst, Urs Fischbacher, and Michael Kosfeld (2005). "Neuroeconomic foundations of trust and social preferences: initial evidence". In: *American Economic Review* 95.2, pp. 346–351.

Fu, Qiang, Jingfeng Lu, and Yuanzhu Lu (2012). "Incentivizing R&D: Prize or subsidies?" In: *International Journal of Industrial Organization* 30.1, pp. 67–79.

Fugger, Nicolas, Elena Katok, and Achim Wambach (2016). "Collusion in dynamic buyer-determined reverse auctions". In: *Management Science* 62.2, pp. 518–533.

— (2019). "Trust in procurement interactions". In: *Management Science* 65.11, pp. 5110–5127.

Gächter, Simon, Benedikt Herrmann, and Christian Thöni (2004). "Trust, voluntary cooperation, and socio-economic background: survey and experimental evidence". In: *Journal of Economic Behavior & Organization* 55.4, pp. 505–531.

Glaeser, Edward L et al. (2000). "Measuring trust". In: *The quarterly journal of economics* 115.3, pp. 811–846.

Gupta, Shivam et al. (2021). "Artificial intelligence for decision support systems in the field of operations research: review and future scope of research". In: *Annals of Operations Research*, pp. 1–60.

Ha, Albert Y and Shilu Tong (2008). "Contracting and information sharing under supply chain competition". In: *Management science* 54.4, pp. 701–715.

Hanazono, Makoto, Jun Nakabayashi, and Masanori Tsuruoka (2013). "Procurement auctions with general price-quality evaluation". In: *KIER Discussion Paper* 845.

Harrison, Glenn W, Morten I Lau, and Melonie B Williams (2002). "Estimating individual discount rates in Denmark: A field experiment". In: *American economic review* 92.5, pp. 1606–1617.

Haruvy, Ernan and Elena Katok (2013). “Increasing revenue by decreasing information in procurement auctions”. In: *Production and Operations Management* 22.1, pp. 19–35.

Heidhues, Paul and Botond Kőszegi (2008). “Competition and price variation when consumers are loss averse”. In: *American Economic Review* 98.4, pp. 1245–68.

Herweg, Fabian (2013). “The expectation-based loss-averse newsvendor”. In: *Economics Letters* 120.3, pp. 429–432.

Herweg, Fabian, Daniel Müller, and Philipp Weinschenk (2010). “Binary payment schemes: Moral hazard and loss aversion”. In: *American Economic Review* 100.5, pp. 2451–77.

Ho, Teck-Hua and Juanjuan Zhang (2008). “Designing pricing contracts for boundedly rational customers: Does the framing of the fixed fee matter?” In: *Management Science* 54.4, pp. 686–700.

Holmström, Jan et al. (2002). “Collaborative planning forecasting and replenishment: new solutions needed for mass collaboration”. In: *Supply Chain Management: An International Journal* 7.3, pp. 136–145.

Hong, Kessely and Iris Bohnet (2007). “Status and distrust: The relevance of inequality and betrayal aversion”. In: *Journal of Economic Psychology* 28.2, pp. 197–213.

Horton, John Joseph and Lydia B Chilton (2010). “The labor economics of paid crowdsourcing”. In: *Proceedings of the 11th ACM conference on Electronic commerce*, pp. 209–218.

Hu, Zhenyu and Javad Nasiry (2018). “Are markets with loss-averse consumers more sensitive to losses?” In: *Management Science* 64.3, pp. 1384–1395.

Inada, Ken-Ichi (1963). “On a two-sector model of economic growth: Comments and a generalization”. In: *The Review of Economic Studies* 30.2, pp. 119–127.

Jeffries, Frank L and Richard Reed (2000). “Trust and adaptation in relational contracting”. In: *Academy of management review* 25.4, pp. 873–882.

Johnson, Noel D and Alexandra A Mislin (2011). “Trust games: A meta-analysis”. In: *Journal of Economic Psychology* 32.5, pp. 865–889.

Kagel, John H (2020). *7. Auctions: A Survey of Experimental Research*. Princeton University Press.

Kagel, John H and Dan Levin (2008). “Auctions: A survey of experimental research, 1995-2008”. In: *Handbook of experimental economics* 2.2.

Kahneman, Daniel, Jack L Knetsch, and Richard H Thaler (1990). “Experimental tests of the endowment effect and the Coase theorem”. In: *Journal of political Economy* 98.6, pp. 1325–1348.

Kahneman, Daniel and Amos Tversky (1979). “Prospect Theory: An Analysis of Decision under Risk”. In: *Econometrica* 47.2, pp. 263–291. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1914185.

King-Casas, Brooks et al. (2005). “Getting to know you: reputation and trust in a two-person economic exchange”. In: *Science* 308.5718, pp. 78–83.

Klemperer, Paul (2014). “A survey of auction theory”. In:

Kosfeld, Michael et al. (2005). “Oxytocin increases trust in humans”. In: *Nature* 435.7042, pp. 673–676.

Kostamis, Dimitris, Damian R Beil, and Izak Duenyas (2009). “Total-cost procurement auctions: Impact of suppliers’ cost adjustments on auction format choice”. In: *Management Science* 55.12, pp. 1985–1999.

Koszegi, Botond (2014). “Behavioral contract theory”. In: *Journal of Economic Literature* 52.4, pp. 1075–1118.

Kőszegi, Botond and Matthew Rabin (2006). “A model of reference-dependent preferences”. In: *The Quarterly Journal of Economics* 121.4, pp. 1133–1165.

— (2007). “Reference-dependent risk attitudes”. In: *American Economic Review* 97.4, pp. 1047–1073.
Kovenock, Dan and Jingfeng Lu (2020). “All Pay Quality-Bids in Score Procurement Auctions”. In: *Available at SSRN 3523943*.
Kreps, David M (1996). “Corporate culture and economic theory”. In: *Firms, Organizations and Contracts, Oxford University Press, Oxford*, pp. 221–275.
Laffont, Jean-Jacques and Jean Tirole (1987). “Auctioning incentive contracts”. In: *Journal of Political Economy* 95.5, pp. 921–937.
Lange, Andreas and Anmol Ratan (2010). “Multi-dimensional reference-dependent preferences in sealed-bid auctions–how (most) laboratory experiments differ from the field”. In: *Games and Economic Behavior* 68.2, pp. 634–645.
Li, Lode (2002). “Information sharing in a supply chain with horizontal competition”. In: *Management Science* 48.9, pp. 1196–1212.
Li, Lode and Hongtao Zhang (2008). “Confidentiality and information sharing in supply chain coordination”. In: *Management science* 54.8, pp. 1467–1481.
Lichtenberg, Frank R (1988). “The private R and D investment response to federal design and technical competitions”. In: *The American Economic Review* 78.3, pp. 550–559.
Liu, Yong and Shulin Liu (2019). “Effects of risk aversion on all-pay auction with reimbursement”. In: *Economics Letters* 185, p. 108751.
Long, Xiaoyang and Javad Nasiry (2015). “Prospect theory explains newsvendor behavior: The role of reference points”. In: *Management Science* 61.12, pp. 3009–3012.
Lynkova, Darina (2021). *Women in Technology Statistics: What’s New in 2021?* URL: https://techjury.net/blog/women-in-technology-statistics/.
Mahajna, Ahmad et al. (2008). “Subjective discount rates among israeli arabs and israeli jews”. In: *The Journal of Socio-Economics* 37.6, pp. 2513–2522.
Matros, Alexander and Daniel Armanios (2009). “Tullock’s contest with reimbursements”. In: *Public Choice* 141.1-2, pp. 49–63.
Minchuk, Yizhaq (2018). “Effect of reimbursement on all-pay auction”. In: *Economics Letters* 172, pp. 28–30.
Mnih, Volodymyr et al. (2015). “Human-level control through deep reinforcement learning”. In: *Nature* 518.7540, pp. 529–533.
Murray, Martin (2018 (accessed December 5, 2018)). *Small Business Supply Chain: Vendor Managed Inventory (VMI)*. https://www.thebalancesmb.com/vendor-managed-inventory-vmi-2221270.
Myerson, Roger B (1981). “Optimal auction design”. In: *Mathematics of operations research* 6.1, pp. 58–73.
Nasiry, Javad and Ioana Popescu (2011). “Dynamic pricing with loss-averse consumers and peak-end anchoring”. In: *Operations research* 59.6, pp. 1361–1368.
Oh, Sechan and Özalp Özer (2013). “Mechanism design for capacity planning under dynamic evolutions of asymmetric demand forecasts”. In: *Management Science* 59.4, pp. 987–1007.
Okada, Erica Mina (2001). “Trade-ins, mental accounting, and product replacement decisions”. In: *Journal of Consumer Research* 27.4, pp. 433–446.
Özer, Özalp and Wei Wei (2006). “Strategic commitments for an optimal capacity decision under asymmetric forecast information”. In: *Management Science* 52.8, pp. 1238–1257.
Özer, Özalp, Yanchong Zheng, and Kay-Yut Chen (2011). “Trust in forecast information sharing”. In: *Management Science* 57.6, pp. 1111–1137.

Özer, Özalp, Yanchong Zheng, and Yufei Ren (2014). "Trust, trustworthiness, and information sharing in supply chains bridging China and the United States". In: *Management Science* 60.10, pp. 2435–2460.

Rempel, John K, John G Holmes, and Mark P Zanna (1985). "Trust in close relationships." In: *Journal of personality and social psychology* 49.1, p. 95.

Riedl, René and Andrija Javor (2012). "The biology of trust: Integrating evidence from genetics, endocrinology, and functional brain imaging." In: *Journal of Neuroscience, Psychology, and Economics* 5.2, p. 63.

Riedl, René et al. (2014). "Trusting humans and avatars: A brain imaging study based on evolution theory". In: *Journal of Management Information Systems* 30.4, pp. 83–114.

Ring, Peter Smith and Andrew H Van de Ven (1994). "Developmental processes of cooperative interorganizational relationships". In: *Academy of management review* 19.1, pp. 90–118.

Rosato, Antonio and Agnieszka A Tymula (2019). "Loss aversion and competition in Vickrey auctions: Money ain't no good". In: *Games and Economic Behavior* 115, pp. 188–208.

Rousseau, Denise M et al. (1998). "Not so different after all: A cross-discipline view of trust". In: *Academy of management review* 23.3, pp. 393–404.

Saha, Atanu (1993). "Expo-power utility: a 'flexible'form for absolute and relative risk aversion". In: *American Journal of Agricultural Economics* 75.4, pp. 905–913.

Schweitzer, Maurice E and Gérard P Cachon (2000). "Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence". In: *Management Science* 46.3, pp. 404–420.

Shachat, Jason and Lijia Tan (2015). "An experimental investigation of auctions and bargaining in procurement". In: *Management science* 61.5, pp. 1036–1051.

Shang, Weixin, Albert Y Ha, and Shilu Tong (2015). "Information sharing in a supply chain with a common retailer". In: *Management Science* 62.1, pp. 245–263.

Shin, Hyoduk and Tunay I Tunca (2010). "Do firms invest in forecasting efficiently? The effect of competition on demand forecast investments and supply chain coordination". In: *Operations research* 58.6, pp. 1592–1610.

Shrestha, Ajay and Ausif Mahmood (2019). "Review of deep learning algorithms and architectures". In: *IEEE Access* 7, pp. 53040–53065.

Silver, David et al. (2016). "Mastering the game of Go with deep neural networks and tree search". In: *nature* 529.7587, p. 484.

Silver, David et al. (2017). "Mastering the game of go without human knowledge". In: *nature* 550.7676, pp. 354–359.

Simon, Carl P. and Lawrence Blume (1995). "Simon, Carl P. and Lawrence Blume, Mathematics for Economists, 1994, WW Norton & Company". In: 32.1, p85–89.

Spengler, Joseph J (1950). "Vertical integration and antitrust policy". In: *Journal of political economy* 58.4, pp. 347–352.

Sugiyama, S. (2019). *Human Behavior and Another Kind in Consciousness: Emerging Research and Opportunities: Emerging Research and Opportunities*. Advances in Human and Social Aspects of Technology (2328-1316). IGI Global. ISBN: 9781522582182. URL: https://books.google.com/books?id=9CqQDwAAQBAJ.

Sutton, Richard S, Andrew G Barto, et al. (1998). *Introduction to reinforcement learning*. Vol. 135. MIT press Cambridge.

Taylor, Terry A and Wenqiang Xiao (2010). "Does a manufacturer benefit from selling to a better-forecasting retailer?" In: *Management Science* 56.9, pp. 1584–1598.

Thaler, Richard (1980). "Toward a positive theory of consumer choice". In: *Journal of economic behavior & organization* 1.1, pp. 39–60.

The Economist, Website (2012). *The planemaker struggles to fulfil a rush of orders*. URL: http://www.economist.com/node/21543555 (visited on 2017).

Van Mieghem, Jan A (2003). "Commissioned paper: Capacity management, investment, and hedging: Review and recent developments". In: *Manufacturing & Service Operations Management* 5.4, pp. 269–302.

Wang, Charles X and Scott Webster (2009). "The loss-averse newsvendor problem". In: *Omega* 37.1, pp. 93–105.

Wang, Hao-nan et al. (2020). "Deep reinforcement learning: a survey". In: *Frontiers of Information Technology & Electronic Engineering*, pp. 1–19.

Warner, John T and Saul Pleeter (2001). "The personal discount rate: Evidence from military downsizing programs". In: *American Economic Review* 91.1, pp. 33–53.