ACADEMIC INTEGRATION AND SELF-REGULATION STRATEGIES IN PRECALCULUS AND CALCULUS I

by

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Abstract

ACADEMIC INTEGRATION AND SELF-REGULATION STRATEGIES IN PREPARATION TO CALCULUS AND CALCULUS I

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This case study examines the self-regulation strategies and academic integration of first-semester undergraduate students enrolled in precalculus and first-semester calculus at a large urban university in the southwestern United States. I use a sequence of interviews to examine the relationship of mathematics self-efficacy and mathematics identity on students' use of these strategies. Five interviews occurred during the Fall 2021 semester with three precalculus and six calculus students, and I distributed initial surveys to thirteen first-semester calculus sections and five precalculus sections. To analyze the data, I integrated frameworks from Zimmerman and Pons (1986) and Wolters (1998) on the self-regulation strategies used in academic settings. I used quantitative and qualitative analysis to examine the data. This included analyzing the correlation between the use of self-regulation strategies versus both self-efficacy and mathematics identity. I also used content analysis to code instances of self-regulation strategy usage in the student interviews and how students adapt their strategy usage during the transition from high school mathematics courses to college mathematics courses. Through the analysis of these interviews and surveys, I present a multiple case study highlighting the transitional difficulties that the interviewed students faced, how they overcame these difficulties using self-regulation strategies, and how these results align with the larger context of the self-regulation strategies used by students as reported on the surveys. Findings suggest a positive correlation between students' self-regulation strategy usage versus their mathematics identity and self-efficacy. Furthermore, based on data analysis, three broad self-regulation strategies were frequently used by academically successful students during their transition to college-level mathematics courses: metacognitive self-regulation, time and study environment, and effort regulation. The use of these self-regulation strategies and others were present in academically successful students. Additional research is then needed on how to best foster the use of these beneficial self-regulation strategies throughout their academic careers.

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Chapter 1

Introduction

The transition to college from high school can be challenging for many students (Clark, 2005; Gueudet, 2008; Ulriksen et al., 2017). In mathematics, these challenges in STEM (Science Technology Engineering and Mathematics) gateway courses such as calculus and precalculus also affect career options (Seymour & Hunter, 2019 and Ellis et al., 2014). These challenges encompass foundational mathematics content, lack of self-regulation strategies, low degrees of self-efficacy or general confidence, social challenges of being in a new environment with new resources, and institutional challenges that may arise while interacting with the university (Hernandez-Martinez et al., 2011 and Seymour & Hunter, 2019). That is, these students face challenges in adapting to social and sociomathematical norms (See Yackel & Cobb, 1996) that have changed from their secondary school experience to their new undergraduate school experience. First-generation undergraduate students often experience these challenges in a higher concentration than more traditional students (Collier & Morgan, 2008).

The use of self-regulation strategies has been researched by Zimmerman and Pons (1986) at the high school level and by Wolters (1998) and Johns (2020) at the undergraduate level. Johns (2020) found a correlation between high strategy usage in an undergraduate mathematics class with higher performance, but further research is needed on how these students develop and acquire self-regulation strategies in academics. Cribbs et al. (2015) examined how undergraduates' mathematical identity is cultivated and found that competence and performance in mathematics courses was an indirect influencer and that interest in mathematics and recognition by others as a "math person" had a direct effect on identity formulation.

The purpose of this case study is to identify these strategies that successful undergraduate first-year students use in their mathematics classrooms and how students' mathematics identity and mathematics self-efficacy relate to this strategy usage. For this study, self-regulation strategies are defined as those actions used to overcome some obstacle when working on tasks associated with a mathematics classroom, whether that is a specific problem context or, more generally, in how students respond to tests and grades. I examine first-year students' strategy usage to see how these strategies develop during the transition to undergraduate study.

Pape, Bell, and Yetkin in 2003 explore the use of self-regulation strategies in a middle school mathematics context, and Wolters wrote in 1998 about self-regulation strategies at the university level with psychology students. Researchers such as Zimmerman and Pons note that fostering these strategies in students can promote academic success (Zimmerman & Pons, 1986). But little research exists that look at the development of these self-regulation strategies for students entering undergraduate mathematics courses from secondary school mathematics. This research aims to extend the work of self-regulation into an undergraduate mathematics context to explore how successful mathematics students develop and use these strategies. This research also seeks to link the ideas of sociomathematical norms or academic expectations to how students use self-regulation strategies. The research questions examined are:

- In what ways do students adapt self-regulation strategies from high school mathematics to college-level mathematics?
    - How does a student's mathematics identity and/or mathematics self-efficacy relate to their implementation of self-regulation strategies?

- How do students characterize differences and similarities between high school mathematics course expectations and college-level mathematics course expectations?
    - What role does mathematics identity and/or mathematics self-efficacy play in students' academic integration in precalculus and calculus?

Through exploring these questions, I hope to highlight ways in which undergraduate students adapt to the challenges of first-semester mathematics courses through the implementation of self-regulation strategies. Furthermore, by identifying which of these strategies are used by successful students I seek to generate a collection of beneficial strategies for undergraduate STEM students.

Chapter 2

Literature Review

The development of self-regulation as a means of controlling one's emotions has been a topic of discussion in psychology and childhood development as it pertains to parents demonstrating how to handle difficult situations for their children. Moving to an academic context in mathematics, it can be examined how students develop the control of their affective domains and motivational strategies through their mathematics courses. Difficulty may arise in two areas of their mathematics courses during the first year of undergraduate mathematics: in the broader context of transitioning into a new environment as it relates to how students manage expectational differences and identity changes, and in the specific mathematical content presented in the course (see Gueudet, 2008; Darragh, 2016; and Seymour & Hewitt, 1994). The use of self-regulation strategies to attend to these challenges is evident in the transition to undergraduate study (Clark, 2005) and in the field of problem-solving. Schoenfeld found that self-regulation was one of the "essential parts of sense making" in mathematics (Schoenfeld, 2014, p. 405).

In this chapter, I highlight the following areas which inform the research of self-regulation and identity: self-regulation strategies, mathematical identity, academic integration and expectations, self-efficacy, the transitional period from secondary to tertiary school, and several factors that affect undergraduate performance.

2.1 Self-Regulation Strategies

Self-regulation learning strategies are defined as those actions directed at acquiring information or skills that involve agency, purpose, and instrumentality self-perceptions by a learner by Zimmerman and Pons (1986, pg. 615). Self-regulation strategies in different settings, such as secondary school versus tertiary school, manifest similarities such as: seeking assistance, whether from another person or from a source such as a book or online resource, and self-evaluating or setting goals based on performance (Zimmerman & Pons, 1986; Wolters, 1998). Differences based on the settings involved recording and monitoring progress, organizing and transforming information presented in a secondary classroom (Zimmerman & Pons, 1986) versus affective strategies related to emotion, self-efficacy, interest, and providing rewards based on progress (Wolters, 1998). Beneficial self-regulation strategies tended to include seeking information, keeping records and monitoring, and organizing and transforming information (Zimmerman & Pons, 1986) in a high school setting. In an undergraduate mathematics course, high achieving students often used strategies associated with intrinsic motivation, task value, and self-efficacy (Johns, 2020). Unsuccessful students often reported less frequent use of self-regulation strategies (Johns, 2020), and students who did not set specific, timely goals often underperformed compared to those who had a firm schedule for tasks and goals (Bandura & Schunk, 1981).

Self-regulation can then be thought of as those skills or strategies used by students in an academic setting while attempting to synthesize and process new information, as well as maintaining agency in one’s learning. Zimmerman and Pons (1986) produced several categories of strategies primarily used by high achieving students from a middle-class suburban high school, where high achieving was based on test scores, GPA (GRADE POINT AVERAGE), and teacher and counselor recommendations. The categories they found are as follows: self-evaluation, organization and transformation of information, goal setting and planning, seeking information, keeping records and monitoring, environmental structuring, self-consequences, rehearsing and memorizing, seeking teacher assistance, seeking adult assistance, seeking peer assistance, reviewing tests, reviewing notes, reviewing textbooks, and other (see Figure 1). It is important to note that among these categories are the strategies of seeking help from others, which is the ability for students to understand when it is beneficial to seek help. Zimmerman and Pons found that the strategies most often used by high achieving students were seeking information, keeping records and monitoring, and organization and transformation of information, with the strategy of self-evaluation having the least positive relationship to high school achievement. In their study, they were able to create an instrument implemented during interviews that measured what strategies were used by students. The data was then blinded, and by analyzing the strategies used, a researcher was able to determine whether the student was a part of a high achieving or lower achieving group accurately up to 93% of the time. This analysis shows that self-regulation strategies can be used as a predictor of academic success. By highlighting students' self-regulation strategies, the authors show that further instruction could take place to improve the student's use of strategies as well as to inform them regarding what strategies are out there to use (Zimmerman & Pons, 1986).

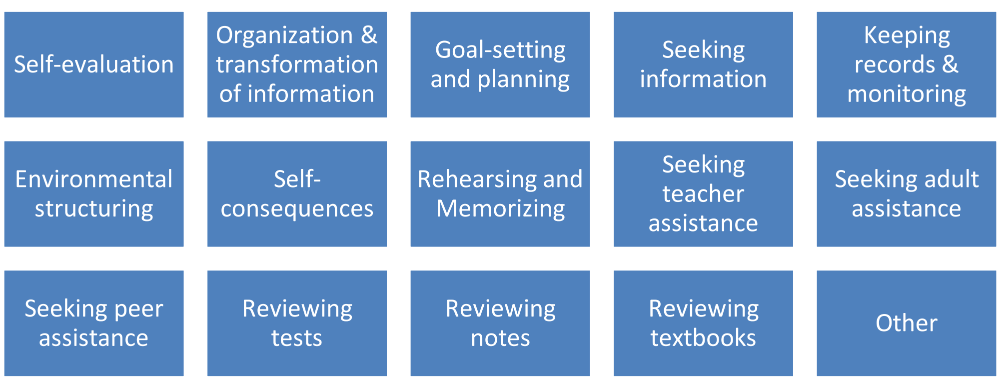


Figure 1: Self-regulation strategies identified in high school setting

This last point of making students aware of the available strategies aligns with Pape, Bell, and Yetkin's (2003) teaching experiment in a seventh-grade mathematics classroom. One of the authors taught in a seventh-grade classroom and performed a teaching experiment that looked at self-regulation strategies. While trying to teach with engaging materials and encouraging students to pursue their own lines of mathematical inquiry for a problem, the researchers gave students forms to fill out that outlined what strategy the student would use to study and whether the student felt that that strategy would help them while taking a quiz the following day. Students filled out another form after taking the quiz asking whether they found their strategy beneficial and whether they would try to improve upon any part of the strategy. They concluded that using multiple representations of solutions and procedures benefitted student learning while engaging in rich mathematical tasks. For this class, they found that discourse in the classroom centered around the strategies being employed by the students was critical to further students' use of strategies. These discussions seemed to be especially helpful as they allowed students to consider strategies they may not have encountered before. In fostering these strategies, the researchers saw improvements in students' awareness of choices. Perhaps due to the students' interest level, the researchers noticed students had increased awareness of strategies, but this does not necessarily correspond to the students implementing these strategies while studying (Pape et al., 2003). The effective use of these strategies presented by Pape, Bell, and Yetkin (2003) may have been further fostered by the rich interactions between the instructor and the students. Komarraju, Musulkin, and Bhattacharya (2010) reported on how faculty-student interactions affect academic self-concept, motivation, and achievement for 950 first-year undergraduates enrolled in a psychology course. They found that by instructors engaging students in formal and informal discussions and being approachable and respectful, the students reported higher levels of confidence in their academic abilities and higher motivation both intrinsically and extrinsically (Komarraju, Musulkin, & Bhattacharya, 2010).

Another researcher of self-regulation strategies, Wolters (1998), wrote about self-regulation strategies he observed while giving open-response questionnaires to groups of psychology students in a large midwestern university. During this process, the students reported on their behavior if they were to encounter a specific academic situation, categorized as boring/uninteresting, too difficult, or seemingly irrelevant to what they were learning. During this analysis of the 115 student responses, the research team found 14 separate categories of strategies, which were: performance goals, extrinsic rewards, task value, interest, mastery goals, efficacy, cognition, help seeking, environment, attention, willpower, emotion, other motivation, and other (Wolters, 1998). The classification of these strategies was more general than those of Zimmerman and Pons' (1986) mentioned above, as this looked at strategies not just exhibited by students during a mathematics problem or a problem relating to an English class. While analyzing these strategies, they grouped them into broader themes that seemed to be used more often to approach a specific type of problem. For irrelevant material, students used extrinsic regulation strategies, which included performance goals and extrinsic rewards, strategies that emphasize some outside influence on the importance of the task as leading to a reward. For material deemed too difficult, students reported using information-processing strategies, which include cognition and help seeking, both of which involve using additional materials or time in thought to address the problem. This process seems to require that the student does not have enough knowledge on their own to address the problem but acknowledges that the problem can still be solved with some help. Then for problems found to be boring, students reported mainly using volitional strategies, including environment, attention, willpower, and emotion. These strategies involve changing your environment to better focus, such as going to a quiet place to study. The other strategies are often involuntary, such as having the willpower to complete something. The last set of strategies, intrinsic strategies, were not linked to a specific type of problem situation. Intrinsic strategies included mastery goals, value, interest, and efficacy. These involve internal mindsets of the inherent value of learning the concept or one's beliefs about their ability to understand the concept. This group of strategies was not shown to be a significant indicator of course grade; rather, it was extrinsic regulation strategies that were most often found to be a significant predictor of course grade. This suggests that students are primarily motivated through a sense of future reward for a grade rather than an internal mindset of the information being valuable.

Figure 2: Self-regulation strategies identified in university setting

Similar to Zimmerman and Pons' (1986) study that examined achievement and performance differences on self-regulation strategies, Johns (2020) study looked at the regulation strategies of 424 consenting first-semester calculus students and tested whether there were differences relating to gender and academic achievement. She used the Motivated Strategies for Learning Questionnaire (MSLQ), which measures students' motivation and learning strategy use. From the MSLQ and by looking at final grades in first-semester calculus, Johns separated the students into four groups, low achievers (n = 167), overachievers (n = 73), underachievers (n = 67), and high achievers (n = 176). She found that high achievers and overachievers both reported greater intrinsic motivation, task value, and self-efficacy than low achievers. High achievers also reported greater use of self-regulation strategies than low achievers. Her research found a correlation between the use of self-regulation and motivational strategies with student achievement (Johns, 2020).

Elementary school students exhibit other self-regulation strategies in Bandura and Schunk's 1981 study. Students solved mathematics problems for several days after being given ideas of how to set goals. Bandura and Schunk (1981) report on the benefit of elementary school children having different types of goals to motivate them to finish a large number of problems. Forty elementary school students from 6 different schools were identified to have an arithmetic deficiency and low interest in mathematics. These students were separated into four groups, each given instruction about a different type of goal setting, except for the control group, which was given no instruction. Each student was given 42 pages of problems with varying levels of difficulty. A proctor of the first group explained how to use proximal goals to finish the work over seven days by completing six pages each day. Another group was told about distal goals by being told just to complete all 42 pages by the end of the seventh day. The third group was to try and complete as many problems as possible as they went along without any discussion of goal setting. Lastly, a control group was established that had no instructional material given to them.

Bandura and Schunk found that those students who received proximal goals suggestions benefited by increasing their perceived self-efficacy and showed gains with performance measured by a post-experiment test. Students given distal goals also showed improved self-efficacy, but showed a decline after the post-test. As a component of self-regulation, students using proximal goals demonstrated more persistence in solving problems and solved more problems during a post-assessment. They also reported more accurate self-efficacy scores when tested (Bandura & Schunk, 1981). These can be thought of as regulation strategies as they can relate to Wolter's (1998) self-regulation strategy category of volition because they would help focus the attention of the student for the time. The authors related the use of these goals to how children's self-efficacy and performance increased, which was shown to have the most positive effect from the proximal goal suggestions. This use of strategies was studied by Clark in 2005 as he discussed how undergraduates negotiate the transition to freshman year (Clark, 2005). This article presents obstacles that undergraduates face during this transition, relating to classes, professors, academic responsibilities, grades, extracurricular activities, studying, and relationships with family and peers. He argues that the best way to learn strategies to deal with challenges that arise in these domains is by using freshman orientation classes that can help student awareness of resources such as on-campus tutoring, office hours, or library resources.

Earlier studies examined the use of self-regulation strategies in different academic settings including undergraduate mathematics. This work looks to examine how the use of self-regulation strategies for the undergraduate setting is developed during the transition from secondary to post-secondary schooling. The categories established by Wolters and Zimmerman & Pons will be used while discussing undergraduates' use of these strategies. Changes and the development of these strategies will be analyzed as to how they relate to undergraduates' mathematics identity.

2.2 Mathematics Identity

Mathematics identity has been a widely discussed concept, with research having been done on its development and its effect on performance and persistence (Darragh, 2016). Darragh (2016) examines literature on the concept of identity, finding there are two primary ways to frame identity, one focusing on identity as an action and the other as an acquisition. By searching through 188 articles that she found to clearly identify mathematical identity, Darragh looked for common characteristics within the articles. She offers several broad categories of defining identity which include "participative, narrative, discursive, psychoanalytic or performative" (Darragh, pg. 24). With this mindset, we can see the themes of how mathematics identity is defined in the research and how the transition to university mathematics may influence the identity formula.

Mathematics identity has been defined as relating to "an individual's self-perceptions with respect to mathematics" by Cribbs, Hazari, Sonnert, and Sadler (Cribbs et al. 2015). In their articles, Cribbs et al. (2015) report on surveying over 10,000 calculus students across the U.S. with a questionnaire that gauged interest, recognition of how others perceive one's own mathematical ability, competence/performance in mathematics, and how students view themselves as mathematics learners. In looking at what factors influence identity the authors first report on the significance that a positive identity can have, "students' perceptions then have the potential to influence their behavior and choices" (Cribbs et al. pg. 1059). How students view themselves becomes a driving force for students to then continue in their mathematics studies. The factors that then influence this identity were found to be indirectly competence and performance, with interest and recognition by others to be direct influences (Cribbs et al. 2015). These results point to the idea that students are affected by how others see them. This speaks to the strong influence on students that being recognized as a math person has on their future studies or careers. Other researchers such as Gutiérrez (2013) view identity more from a sociopolitical frame which views identity from an action perspective, so what students do rather than something that they are. For this study, I used Cribbs et al.'s (2015) idea of identity being how students perceive themselves as mathematics learners. I believe that this identity can change based on social influences or transitions such as from secondary to post-secondary education when students must often discover more about themselves as learners.

Researchers in the last decade explored the transitional aspect of identity formation (Ulriksen et al., 2017; Komarraju et al. 2010). Ulriksen, Holmegaard, and Madsen's study explored the transition into a university STEM program. Interviews were done with 20 students that discussed their experiences with transitioning into a university STEM program. While studying how students experienced this transition, the researchers investigated how identity is affected by factors such as curriculum and university expectations for the students. They found that the classification and framing of a curriculum offer students an opportunity and conditions for the construction of their disciplinary identity, but the curriculum does not determine the identity. Rather it depends on students' engagement with the curriculum, as well as their prior knowledge and experiences. They also concluded that the "implied student," that is the model of a student that a university or school presupposes, would have interests and practices that the university deems necessary to function in the course. It is within this model that curriculum is built which can become a deterrent to students building their own identities. They argue that students try to fit into this mold or fail to fit this mold and become frustrated as these students were not active participants in creating their own identity.

What students are expected to do in the class is counterproductive to their identity formation, such as being passive, obedient, and patient students that often follows what the instructor says without considering how they interact personally with the materials (Ulriksen et al., 2017). Komarraju et al. (2010) also explored how identity is formed at the university level. They found that students' academic self-concept was more closely related to the respect they were shown by an instructor during faculty-student interactions within the first year of undergraduate, rather than pre-college academic content (Komarraju et al. 2010).

Students' mathematics identity can be interpreted by what expectations they put on themselves as well as how they understood what expectations instructors had of them in their mathematics courses. This identity formation starts in high school, but as discussed later by Hernandez-Martinez et al. (2011), during times of transition and challenges identities can be reformed and reinterpreted to meet the needs of the new situation.

2.3 Expectations/Academic Integration

Expectational themes are divided into two categories: first how undergraduate expectations of what is required of them in an undergraduate course may be different than what faculty expect of them; and second, how undergraduate's acceptance of faculty expectations can benefit undergraduates. Expectational differences have been examined through the lens of how the transition to undergraduate courses impacts students and how traditional students and first-generation students see these expectations (Ulriksen et al., 2017; Collier & Morgan, 2008) as well as differences arising between high school mathematics and university mathematics (Ghedamsi & Lecorre, 2021). Ulriksen et al. (2017) brought up expectational differences that the first-year university students held. The authors found that students generally believed that the mathematics courses they were required to take were too difficult and were not clearly connected to the material they needed to learn in their other STEM courses to be relevant (Ulriksen et al., 2017). The expectations of the individual course are not made explicit in this research study, unlike in studies such as Collier and Morgan's 2008 study and Cobb and Yackel's 1996 study. Collier and Morgan looked at answering the question "Why do some students succeed in college while others do not?" while Cobb and Yackel explored the social and sociomathematical norms that exist in mathematics classrooms. Collier and Morgan frame this question by looking at first-generation college students' and more traditional college students' understanding of faculty expectations within business or liberal arts and science courses. They divided 63 students into focus groups, either having first-generation college students or students who had at least one parent graduate from college. They also had two faculty focus groups with a total of 15 faculty members. The focus groups were organized around gathering information about college expectations that relate to classroom experiences, such as basic priorities about students' schoolwork, what students were supposed to know about these expectations, and the kinds of problems students encountered through not understanding expectations and how students should react to these problems.

Collier and Morgan (2008) were able to find clear expectational themes across the faculty focus groups. They found that faculty expected that undergraduate students would work for 2-3 hours each week outside of class time for each credit hour that the class gave. For example, a typical three-credit hour course would necessitate 6-9 hours of work outside of class dedicated to studying and working on homework for the class each week. Faculty also expected students to prioritize their time around getting an education, so extracurricular activities would be less important. Faculty expects their students to be able to gauge how much time individual projects or homework tasks will take, and then dedicate at least that amount of time to the task. Faculty expect students to understand that college coursework will be more demanding than their previous experience of high school coursework. For assignments that require some explanation or writing, faculty expects students to be able to demonstrate basic writing skills and to communicate proficiently while writing, as well as to know formatting conventions, especially when citing sources. When students encounter issues, it is expected by the instructor that the student will approach them to clarify and seek help, and that this period of communication would be focused on problem-solving, not just giving an answer. Faculty expects students to understand the purpose of office hours and to appropriately use that time to ask questions. Instructors also have individual expectations for specific courses. Finally, the majority of instructors believe that their expectations are clearly outlined in the course syllabus but observe that students often misinterpret or misunderstand the expectations.

Students also expressed expectations for coursework and classes that may differ from the faculty's expectations. Overall, they found that the expectations held by first-generation students differed more greatly from faculty expectations than those held by traditional students. This may be due in part to traditional students having more immediate relationships with people who can tell them about firsthand experiences of undergraduate classes. Where instructors gave a set amount of time expected to study outside of class, and that this amount of time would be prioritized by students, students reported that the time they spent working on a class reflected how much time they had available. This contrasts with faculty's expectations of taking the time to master the material, students viewed time spent studying for a class as being something added to their schedule, rather than a priority. Students reported that school should fit into the already established schedule of work, family demands, and students' general life. This is again in contrast to instructors' views that the class should be prioritized, and it should function as a job for students. Students believed that it was in their best interests to establish a relationship with the instructor by initiating conversations and by scheduling a visit during office hours to get acquainted with the instructor. For assignments, students want instructors to be as explicit as possible about what is expected from the assignment, and if it involves writing to state what format and style the assignment should be. Students expect instructors to be proficient at communicating, not overusing technical jargon but using what they felt was appropriate vocabulary. They also emphasized that instructors should be able to rephrase questions when students are confused. Regarding the syllabus, often students, especially first-generation students, did not understand the point of the syllabus. Students were fairly split between first-generation and traditional as to how they preferred the format of a syllabus though. The first-generation students wanted as many details as possible, while traditional students often wanted a simpler syllabus. Though in the use of the syllabus, traditional students saw the information presented by the instructor to be interchangeable with the information on the syllabus, they were comfortable relying on either source. The first-generation students on the other hand relied more on the instructor, choosing to mostly ignore information presented on the syllabus and expecting that the instructor would inform them of any essential information verbally. There was confusion on what the purpose of office hours was among the first-generation students, as it was not made clear to them that office hours were dedicated times that the instructor sets aside for students to ask questions (Collier & Morgan, 2008).

Overall, in looking at these expectational differences Collier and Morgan (2008) found that there was a greater gap between the faculty expectations of students and first-year students' expectations of themselves. This is likely due to the lesser degree of influence on these students as their parents would not have been able to express the faculty expectations. And due to this gap, the authors found that students' performance in class was more likely to suffer. Similarly, to Collier and Morgan, Ghedamsi and Lecorre looked at expectations between faculty and students from a high school and university perspective while focusing specifically on the mathematics content and skills that were expected. By examining the curricula being taught in high schools and universities in Tunisia along with information from university instructors teaching calculus, Ghedamsi and Lecorre (2021) found that in terms of conceptual understanding and critical definitions provided there were similarities between high school and university mathematics. The differences presented themselves in students' shortcomings in how to apply reason in a mathematics course as well as how to manipulate formal mathematics statements as proficiently as expected by instructors.

The extent to which students can accept these expectations set by their instructors is called academic integration. Academic integration relies on students accepting these expectations as legitimate and then behaving accordingly (Weidman, 1989). Although not all faculty expectations may be valid or beneficial to a student's learning, it will be assumed that those expectations as outlined by Collier and Morgan (2008) form a framework of positive expectations that would be beneficial for students to meet. These expectations allow for growth in students' academic careers and promote independent learning often. Hernandez-Martinez et al. (2011) argue that these types of collegiate expectations on students should not be lessened or adjusted, as these provide challenges that students can overcome (See also Collier & Morgan, 2008 and Weidman, 1989).

How these expectations relate to mathematics courses specifically will be explored in this current study. A collection of common expectational themes found among undergraduates will be formed from the undergraduate interviews that occur throughout a semester. The connection between these general undergraduate expectations as well as course-specific expectations and undergraduate's mathematics identity and self-efficacy will be drawn as to how students see themselves as capable mathematics students and what expectations they feel the need to meet from their instructors.

2.4 Self-Efficacy

Mathematics self-efficacy is examined from two major ideas through the following literature, first how self-efficacy is measured in students (Bandura, 1997; Hackett & Betz, 1989; and Usher & Pajares, 2009), and secondly how self-efficacy connects to performance and persistence (Chambers et al., 2006; Tinto, 2017).

Mathematics self-efficacy can be defined as a “situational or problem-specific assessment of an individual’s confidence in her or his ability to successfully perform or accomplish a particular task or problem” (Hackett & Betz, 1989, pg. 262). Students with high levels of self-efficacy have been linked to having a view of mathematics as being more useful, as well as lower levels of mathematics anxiety, and higher levels of confidence in mathematics in general. With information about self-efficacy of students’, Hackett and Betz (1989) claim that the ACT scores would be redundant as self-efficacy was a good predictor of performance and achievement. They then recommend that self-efficacy should be built up, especially among women, using a role model to combat messages of discouragement about being in a STEM field that happens as early as middle school. Others such as Usher and Pajares (2009) have extended Bandura’s 1997 work of looking at self-efficacy. Usher and Pajares (2009) worked with middle school students, teachers, parents, and faculty members to create measures for sources of mathematics self-efficacy. They created a 24-item scale to measure self-efficacy and the four sources of self-efficacy hypothesized by Bandura (1997) (mastery experience, vicarious experience, social persuasions, and emotional and physiological states). Usher and Pajares (2009) used information about these sources to create a survey that measured a middle school student's self-efficacy. The scale has been validated to work with middle school students, though I believe that it will be an effective baseline survey for undergraduate students that when paired with interviews will give a more complete picture of undergraduates' self-efficacy.

Chambers et al. (2016) looked at the connection between self-efficacy and performance and persistence of college enrollments of black women. The authors wanted to explore the relationship between mathematics self-efficacy and the enrollment of young black women in any postsecondary institution, as well as specifically a four-year institution. Data collected from the Educational Longitudinal Study looked at family socio-economic status, standardized mathematics achievement scores, mathematics self-efficacy scores, highest mathematics class completed, students' degree aspirations, and their plans for postsecondary enrollment. In analyzing this data, the authors found that in the transition from sophomore year to senior year in high school, the number of students aspiring to a bachelor's degree decreased, although the number of students aspiring to an associate degree increased. They found that the highest-level mathematics course taken had the largest positive effect on aspirations of attending postsecondary education. They mention that a critical component of the underrepresentation is due to the high report of Black women's highest mathematics class being algebra 2. They describe this as a necessary but not sufficient condition for college enrollment.

They also discuss how this underrepresentation could be affected by the lack of school counselors or the insufficient time spent with school counselors. They conclude that higher mathematics self-efficacy is associated with higher odds of attending a four-year college, but overall decreases the odds of attending any postsecondary institution.

This may be due to marginal effects outside of attending a four-year college. They mention several implications of this research: first that the change in the mathematics self-efficacy of young Black women in this sample over time is evident, and for the "average" students it may be overlooked as to how it may decrease. Secondly, attention is warranted to not only the racial but also gendered aspects of culturally relevant pedagogical techniques in teacher and counselor education preparation programs. Third, culturally relevant strategies are necessary to bridge the divide between young Black women's course-taking and examination performance. Lastly, further research on the disconnection between course-taking and examination performance is warranted (Chambers et al. 2016). Similarly, Tinto (2017) observed that a positive self-efficacy related to the attainment of goals and persistence while a weaker sense of self-efficacy negatively affected achievement. Tinto also found that the transition to college can weaken the self-efficacy of students, even those who previously had a high level of confidence coming into a university due to the challenges that occur in transition. To aid in this transition, Tinto recommends monitoring first-year student performance and providing academic and social support to aid in self-efficacy development thus aiding in the persistence of students (Tinto, 2017).

The use of self-efficacy in this study will inform how undergraduates see themselves as competent mathematics learners and how confident they are to rise to challenges or obstacles set before them in their first-year mathematics courses. Connections between self-efficacy and the use of self-regulation strategies will also be examined in this study.

2.5 Transition to Undergraduate Mathematics

Results following attend to two areas of transition which can then be broken down by emphasis: how students transition into a college course regardless of the subject matter (Ulriksen et al., 2017 & Clark, 2005), and how mathematics students handle the transition (e.g. Gueudet, 2008; Hernandez-Martinez et al., 2011; Sonnert et al., 2020; Hsu & Bressoud, 2015; and Seymour & Hunter 2019). When looking at transitional challenges related to mathematics courses, the major ideas that arise are lack of preparation from high school (Gueudet, 2008; Sonnert et al., 2020; Culpepper et al., 2010; and Seymour & Hunter 2019) and difference in pedagogical expectations (Gueudet, 2008; Hernandez-Martinez et al., 2011; and Ellis et al., 2014). The responses to these challenges have involved departmental efforts (Clark, 2005 & Johnson & Hanson, 2015) as well as an examination of remedial courses (Benken et al., 2015)

While looking at the challenges that occur during the transition to undergraduate study researchers such as Ulriksen et al. (2017) discuss the expectational differences that undergraduates experience in Denmark, especially how it related to the relationship between course curriculum and how mathematics identity is constructed. In their transition to undergraduate studies, the students often experienced difficulty because their expectations did not match their perception of the program (Ulriksen et al. 2017).

Gueudet in 2008 outlined several of the challenges that arise during the transition to tertiary school, especially related to individual, social, and institution phenomena. For the individual phenomena, Gueudet (2008) describes that undergraduates beginning the transition often lack flexibility to switch between different thinking modes, such as practical and theoretical thinking. Practical thinking can be thought of as using prototypical examples or schemes to complete tasks, whereas theoretical thinking involves organization of systems of concepts and reflection on how those concepts are represented. Adjusting to the demands of an undergraduate mathematics class is not a skill all students have. The researcher found that a common issue faced by students during this transition was related to how their knowledge of mathematics is organized.

This led to students being unable to handle complex problems, being limited to simple technical issues as what they knew was not being translated to more complex ideas. This may be due to not having enough experience or autonomy while solving non-routine problems in their past mathematics coursework. This is not always due to a student's lack of motivation or ability to communicate and prove mathematical statements. Rather it can start with the instructor not providing time or opportunities for students to develop these skills. The author found that early university instructors are not always precise about how they use mathematical language. Difficulties in communication can be compared to learning a new language, and it is then the instructors' job to teach this new language by example. Students tend to mirror the methods used by instructors which can lead to imprecise or unclear methods. Another difficulty some students may experience relates to how students perceive themselves as learners in general and, more specifically to whether they have a positive mathematics identity. Textbooks and instructors may present information in a specific method that students feel an obligation to follow to the letter without understanding the connections between the content. This may be due to learning to pass a test that relies on memorizing a set of information more often than having a complete understanding of a concept (Gueudet, 2008).

Sonnert et al. (2020) found that high school preparation was a significant factor in retention and performance for first-year students, but as the distance from high school grows the influences lessens. Though for those first-year students it was preparation in mathematics, and to a lesser extent attitudes about mathematics, which had the largest impact on performance in a calculus class. This is attributed to the highly cumulative nature of mathematics as calculus relies on earlier concepts (Sonnert et al. 2020). Hsu and Bressoud (2015) explored what influences calculus placement and the effectiveness of these measures. They found that institutions employ several different placement strategies, including department-created exams, system-wide created exams, Accuplacer, Compass, ALEKS, MAA Maplesoft, and entrance exams such as the ACT or SAT. Several of the institutions examined by Hsu and Bressoud also employed multiple measures, adding in high school GPA in addition to a computer-designed test. The authors recommended this strategy of using multiple methods such as an exam, experience, and even interview consultations with an enrollment adviser. Hsu and Bressoud also recommend that with any strategy being used careful monitoring is necessary, such as checking departmental-made exams for item discrimination or bias. An interesting result they found was that the scores of students who had precalculus at the institution did not differ greatly from those who chose not to take it based on their placement test recommending it. Many students did not enroll in calculus after taking precalculus even if their track would have needed it (Hsu & Bressoud, 2015). This leads to a perceived gap of students that were on a calculus track, but for some reason by taking precalculus decided to switch the track. Some of these reasons may relate to Seymour and Hunter's (2019) report on switching which is discussed further below.

One method of dealing with these challenges, as highlighted by Clark (2005), while going through this transition is using freshman orientation classes. These classes are usually designed to give freshmen information about campus resources as well as promote skills that may be useful for their academic careers. These skills included creating positive study habits, utilizing campus resources like libraries and tutoring services, managing one's time, how to deal with academic advisers, and being able to think critically. Importantly, these orientation classes aim to help students recognize challenges and improve communication between students and faculty so that students can have clear responsibilities as college students (Clark, 2005). Weidman is also a proponent of first-year student orientation classes, finding them critical for student success as they provide early opportunities to interact with faculty. This early exposure allows students to further meet those expectations that faculty have of students and promotes students' academic integration (Weidman, 1989). By interacting with faculty through these orientation classes, a mentoring relationship with faculty members may be produced which according to Komarraju et al. (2010) was found to be more beneficial to academic integration and performance than engaging in student peer groups.

Similarly, Johnson and Hanson (2015) report on some of the departmental and institutional support systems typically available to mathematics students. The study found that almost all institutions have a tutoring center, though the composition of the tutors differs, ranging from undergraduate students to full-time mathematics faculty serving in a tutoring position. Students working in study groups often prefer groups located in their dorms. They found that half of the students never attend office hours, and less than 15% go at least once a week. Support systems are either departmental or system-wide, often with system-wide tutoring services offering help with many topics and study habits. Some schools offer mentorship programs that are used to group students with similar degree tracks under a common mentor. Another support option is increased instructor courses. Some calculus courses offer additional time where students meet for homework help and to ask questions of the instructor in a smaller group of 12 to 20 persons. One of the institutions chose an option where pre-requisites were removed, allowing students to take whatever class, as well as having a no-penalty policy that eliminated D and F grades and replaced them with a No Record grade. Many institutions focus on fostering community among the students and building a space for students to work together. They found that students appreciated instructors encouraging them to work with others outside of class. The authors concluded that students are more likely to persist in college when they have both a social and an academic community. Though having the support systems alone is not enough, students must take advantage of the system, and this must be encouraged by instructors and administrators. Student engagement and involvement in focused study groups were found to be the single most key factor in student retention during their first year of college (Johnson & Hanson, 2015).

The other common strategy to deal with these transitional challenges is for students to take remedial, or developmental, mathematics courses. While conventional wisdom may say that further preparation in these remedial classes should benefit students, a study by Benken, Ramirez, Li, and Weterdorf (2015) challenges this idea. The researchers were able to analyze students' attitudes about mathematics by looking at surveys from students upon starting and ending a developmental mathematics course. These surveys asked questions about students' expectations, experiences, anxiety, attitudes, and course confidence before and after the semester. They found that most participants in the developmental course had taken more mathematics courses than were required for entrance to the university under study and that a quarter of the students took several years to earn a passing mark in an Algebra 2 course. Upon entering the class, students on average did not enjoy mathematics but did perceive that their mathematical skill was average and had confidence that they could pass the developmental course. They also found that students who had taken Algebra 2 as the highest mathematics course in high school did not have significantly lower self-reported skill levels than those that had taken more advanced courses in high school. In the post-surveys overall, participants reported positive changes in their self-perceptions regarding their mathematics skills, their enjoyment of mathematics, and their confidence and comfort working on mathematics tasks. Although there was a positive change, the reported means from this study were still in a mostly neutral range on Likert scale items. Approximately 60% of students also self-report that they spend three or fewer hours outside of class studying. In this study, 77.9% of the students passed with at least a C grade. A major discussion point the authors address is that almost two-thirds of the participants that required remediation had taken four years of mathematics courses in high school, with over 20% taking advanced courses such as calculus. This finding was troubling in that passing high school courses is a necessary but not sufficient criteria for being prepared for collegiate courses. They call for a reexamination of the criteria for having learned enough in a school course to be ready for the next course (Benken et al. 2015).

Hernandez-Martinez et al. (2011) expressed three transitional issues that have often been viewed as challenges for students. These issues relate to the social dimension of students acclimating to an unfamiliar environment with new people and resources, and how the student feels a sense of belonging in the new environment. The second issue deals with the continuity of curriculum and pedagogy. Where a perception of the instruction gap is present, research is being done to minimize the consequences of this gap. The third source of transitional issues as presented by the authors relates to how individuals progress as learners while interacting with the university. In interviewing students, the authors found that most undergraduate students when asked about transitional differences mentioned that there was an increased workload of college course requirements and that the concepts discussed were more difficult (Hernandez-Martinez et al., 2011). Relating this concept specifically to mathematics, when there is a shift in how students perceive mathematical objects, these shifts challenge students' confidence (Gray et al. 1999). These shifts include studying the rational numbers in middle school/high school, and then the concept of variables and solving algebraic equations occurring in late middle school and early high school. The shift occurring during calculus courses is the study of infinitesimals as well as proofs (Charalambous &

Pitta-Pantazi, 2007; Malisani & Spagnolo, 2009; Tall, 1992). In Hernandez-Martinez's (2011) study, the authors provide student quotes about how the newness of the mathematics taught was what made the mathematics difficult, and that the course material felt disconnected from what they had learned at earlier levels. The authors argue that this sense of newness being a challenge is also perceived as an opportunity for growth that allows students to obtain a new identity as a learner. Those who overcome the challenge often look back and appreciate that they were able to move forward in their learning. As such this challenge to transition should be viewed as an opportunity for growth and should not be lessened to make the experience more like high school (Hernandez-Martinez et al., 2011).

On the other side of the transitional gap, information about high school and middle school mathematics courses is analyzed to see how it affects college enrollment and performance. Culpepper et al. (2010) explored how standardized test scores predict performance in a freshman-level college algebra course, as well as how academic performance is affected by high school courses taken. When looking at the ACT mathematics scores, they found that for college algebra, performance was not as strong a predictor as high school mathematics courses completed. They found that high school trigonometry completion had the strongest impact, with students who completed high school trigonometry getting 1.5 grade levels higher in college courses than those who did not. With these results in mind, the authors give recommendations at the high school level to better align with post-secondary education as classes taken have a considerable influence on post-secondary performance. They also suggest a more careful monitoring process of middle-school students to identify those students who are college bound so that they can be put on a track that involves taking more mathematics courses to better their chances of success in college (Culpepper et. al. 2010).

Seymour and Hunter in the 2019 study of *Talking about Leaving Revisited* explored the issues that arose for students who switched out of a STEM major as they related to high school preparation and the transition to college. While many studies found that high school GPA, SAT/ACT scores, and enrollment in advanced classes predicted success and persistence in STEM majors, they also found that almost all the students in the study who switched out of a STEM major had previously excelled in these metrics and had taken advanced coursework in high school. In the earlier study, *Talking about Leaving,* almost 15% of students cited poor high school preparation as being a reason for switching, with minority students more often affect potentially due to attending under-resourced high school, especially during the time that study occurred (early 1990s). In the more recent revisited study, nearly 20% cited a lack of academic preparation as a part of their decision to switch. The students in this study were selected due to their high mathematics scores on relevant assessments to rule out preparation factors, but still, students found a lack of sufficient preparation to be a major reason for switching. The discipline of the students often affected whether the high school preparation was a factor in switching. The authors found that of the students who switched because of high school preparation issues 66% were life science majors, 22% were physical science majors, 11% were engineering, and 0% of them were mathematics or computer science majors. They found that the coursework most often cited as being difficult due to being underprepared was chemistry, and then calculus to a lesser extent. Many of the unprepared students found that their high school experiences were affected by poor instruction, lack of challenging math/science courses, and an emphasis on worksheets that promoted rote memorization rather than conceptual or abstract thinking that would be needed for college work. They found that women, students of color, and first-generation college students were more likely to cite preparation issues due to being placed in a low-ability or non-advanced mathematics track. This, however, does not represent most students who switched. Approximately two-thirds of the students who left a STEM major reported taking calculus in high school, and 61% of switchers had taken at least one AP or IB science course. In contrast to the unprepared undergraduates who switched, the authors discovered that those switchers who reported being well prepared also reported that college at times seemed easier compared to their rigorous high school experience. These students often mentioned other factors as being primary to the reasons they switched (Seymour & Hunter, 2019).

Beyond specific mathematics content, Seymour and Hunter also found that students reported a lack of general preparation for college, especially regarding study skills. Students reported that those practices and strategies used to earn high grades in high school did not transition to college and that they did not develop study strategies often until their second year in college. One student reported that when taking high school calculus, it was sufficient to just show up to class and he could make an A on a test, but he experienced a "wakeup call" in college that he would need to spend time outside of class preparing and studying for exams (Seymour & Hunter, p. 142). Students also report difficulties in their early undergraduate experience as reasons for switching. These struggles produced a loss of confidence and academic identity. The struggles often presented themselves in gateway STEM courses, often attributed to alienation, isolation, and impersonal aspects as compared to high school experiences (Seymour & Hunter, 2019).

Similarly, Ellis, Kelton, and Rasmussen (2014) examined some of the factors that affect students' retention in undergraduate calculus. Rather than just categorizing the four groups of students who were examined as switchers and persisters, the authors added two categories of culminators and converters. Culminators are those students who began first-semester calculus without intending to take second-semester calculus, and converters are those who did not intend to take second-semester calculus but changed their minds at some point and intended to continue in the calculus sequence. The study examined how these groups report on several metrics relating to how often (or how little) instructors ask students to explain their thinking on a problem, hold a whole-class discussion, and lecture among other pedagogical activities. One component Ellis et al. (2014) found to be significant in persisting in calculus was how often instructors show students how to solve a problem like those that the student will be expected to work on their own for future assignments. Although students often see this occur, some students' perception of worked problems may be that the worked problem is not similar enough to homework or test problems they are expected to work on their own. Other differences the authors found related to how often instructors hold whole-class discussions, with persisters reporting a higher frequency of whole-class discussions. A difference in perception is again at times present where switchers do not perceive what was intended by the instructor as a whole class discussion. This may be because not all students are participating in the conversation, even if that was the instructor's intention. Overall, it was found that instructors often reported a higher frequency of pedagogical activities, while students reported a slightly lower frequency on average (Ellis et al. 2014).

In an examination of transitional challenges that undergraduates face in first-year courses such as precalculus and calculus, this study aims to identify what key self-regulation strategies are employed by undergraduates who had performed well in high school mathematics to see how their mathematics identity and self-efficacy influence their use of these strategies.

2.6 Performance in Undergraduate Mathematics

Research on performance has traditionally fit into two camps. The first explores what specific content knowledge is needed for a course (Carlson, 1998; Carlson et al. 2010; Tall & Vinner, 1981, and Frank & Thompson, 2021), and the second looks at non-content specific aspects such as social, institutional, and individual factors (Seymour & Hewitt, 1994; Seymour & Hunter, 2019; and Tinto, 2017).

To explore what specific concept knowledge is needed for first-year mathematics students to perform well in first-year mathematics courses authors several studies have been done (e.g. Carlson, 1998; Carlson et al. 2010; Tall & Vinner, 1981, and Frank & Thompson, 2021). Carlson in 1998 looked at when college algebra and second-semester calculus students acquired an understanding of the major aspects of the function concept, and what factors in these students' backgrounds have influenced their mathematical development and continued study of mathematics. A mix of quantitative and qualitative research designs was implemented that looked at students' responses to exam questions that were carefully chosen to gauge function concept images, as well as post-exam interviews that allowed for a more in-depth discussion about the questions. They analyzed several of the questions presented in the assessment and reported on which group performed better on average and what types of misconceptions students in particular groups were having, such as students in the college algebra group struggling to identify a function whose values are all equal, while the other group of calculus students answered the question correctly more frequently. They also found that the students in both groups did not employ strategies recently learned to solve the problems, meaning the calculus students did not often use calculus techniques to solve problems that could be solved with calculus. The college algebra students often relied more on the appearance of graphs presented in the problems rather than using the axis units correctly. Overall, they found that function constructs develop slowly, and even talented students have misconceptions about functions. Students in undergraduate programs struggle to represent real-world functional relationships and are not as confident making distinctions between zeros of functions and solutions of equations. For the second-semester calculus students, they had difficulty in the following areas: interpreting rate of change, covariational reasoning, using calculus in a real-world situation, using discontinuous functions, using dynamic graphical information, understanding function notation, translating from algebraic to graphic representations, and using calculus on a dynamic situation (Carlson, 1998).

In a 2010 study by Carlson, Oehrtman, and Engelke, the authors explore the benefits of using the Precalculus Concept Assessment in measuring key calculus concepts that students would need to know from precalculus. These concepts related to functions and the assessment included questions that looked at measuring these function concepts. Carlson et al. found that the assessment worked well in determining concept images of students, especially due to the post-interviews where students were given a chance to explain their reasoning. They found that precalculus students were proficient at composing functions algebraically but struggled graphically or numerically. They also found that the problems in the assessment were able to reveal weaknesses in students' process view of functions. They found that the PCA was a significant predictor of calculus readiness. They conclude with the idea that the construct of instrument validity must be rethought to focus on cognitive processes that are essential for students' continued learning. (Carlson et al., 2010).

Similarly, to Carlson (1998) and others, Tall and Vinner (1981) explored students' concept images and how these images affect definitions and potential conflict between the image and definition. Students' concept images of sequences, limits, and continuity are explored and analyzed from responses to The School Mathematics Project Advanced Level texts. Many students had conflicting concept images. Regarding sequences the major conflicting image dealt with the idea that they believed that the sequence could not equal the limit of the sequence. For limits of functions, they struggled with the idea that x did not need to be defined at the value it was approaching. And for continuity, many students' image of continuous functions having no gaps or being defined by a single expression persisted. Few students had strong understandings of the concept definition of these concepts. They found that many students have difficulty with manipulating the definitions of limits and continuity, as well as with quantifier meanings. They mentioned that at advanced levels it is harder to visualize concepts as mental pictures so there is a strong chance of potential conflicts with the concept definition (Tall & Vinner, 1981).

Similar to concept definitions, Frank and Thompson (2021) examined how US precalculus students construct mathematical meanings that would be productive to understanding calculus. They examined data related to variational reasoning, meanings for average rate of change, and representational use of function notation from 356 students' responses to a Calculus I Concept Inventory, secondary mathematics teachers' meanings from the Mathematical Meanings for Teaching secondary mathematics instrument, and how these ideas are treated in precalculus textbooks by authors who also wrote calculus textbooks. They conclude that the meanings presented in textbooks and held by secondary mathematics teachers are disconnected to the meanings that are productive to students' understanding of calculus. They then argue that reform needs to occur within the curricula of middle schools and high schools to better connect productive meanings of calculus ideas.

Other researchers examine non-content specific effects on performance and persistence of university students. In a study by Seymour and Hewitt in 1994 that explored factors attributed to undergraduates switching out of STEM majors, Seymour and Hewitt (1994) found that in mathematics, physical science, and biological science majors, around 52% of students switched to non-STEM majors. In the revisited study by in 2019 by Seymour and Hunter, the authors mention that the drop out/switching rate among women and people of color is higher than that of white men, and that specifically for women it is not a lack of ability or effort that is causing this loss of retention. They found the cause was a loss of confidence that was produced by the teaching and assessment methods that were designed to weed out students in gateway courses such as first semester calculus (see also Tinto, 2017). While performing the original study and the more recent revised study they found that students switched for similar reasons in both. These reasons included three major categories of factors: first are “push” factors that involve problems in a student’s experience that produces an obstacle for persisting; next are “pull” factors which are those factors that are perceived as more appealing to the student rather than persisting in the major; finally, “pragmatic” or instrumental factors that make persisting seem less feasible. A major finding in both studies was that often it was not a single factor that caused students to switch majors, but a collection of issues that often pushed them away from their intended major and pulled them towards a new, more feasible or appealing major.

They also found that these same categories that affect students who switch were found in persisting seniors of the same majors, just often in a lesser amount on average. In the first study it was found that those students who switched majors reported an average of 8.6 concerns that related to the three earlier categories (“push,” “pull” and “pragmatic”) Those who persisted reported an average of 5.4 concerns. In the later study, the averages were significantly increased, with switchers reporting an average of 64 concerns and those who persisted an average of 23. The factor they found was most present as a reason for switching involved students' learning experiences being negative, such as inferior quality of teaching, negative classroom culture, and difficulties in securing help with academic difficulties. Some of the individual factors they found in both studies that affected the choice to switch are: competitive, unsupportive STEM culture that makes it hard to belong; poor quality of STEM teaching; negative effects of weed-out classes; STEM curricular design problems such as pace, workload, labs, and alignment; conceptual difficulties with one or more STEM subject(s); problems related to class size; difficulties in seeking and getting appropriate timely help; poor teaching, lab or recitation support by TAs; language differences with foreign instructors or TAs; reasons for choice of STEM major prove inappropriate; difficult transition to college; inadequate high school preparation in subject and study skills; discouraged/lost confidence due to low grades in early years; and loss of incoming interest and motivation (Seymour & Hunter 2019). Other factors were identified in the study but related more to "pull" factors or financial factors that are not as relevant to this research project.

Chapter 3

Methodology

This multiple case study aimss to explore the development of self-regulation strategies among first-year undergraduate STEM students and analyze how this development is related to undergraduates' self-efficacy and mathematics identity. This study is bounded by first-semester calculus and precalculus courses with first-year undergraduates during the Fall 2021 semester. Each student defines a case, with the embedded units of analysis of this study being the three task-based undergraduate group interviews, two individual interviews occurring at the beginning and end of the semester, the surveys and questionnaires completed at the beginning of the semester, weekly post-lab surveys, as well as classroom observation notes.

3.1 Settings and Participants

I collected data at the beginning of the Fall 2021 semester in precalculus and first-semester calculus classes at a large, R1 university in the southwestern United States. The university is a Hispanic-serving institution, meaning that at least 25% of the population identifies as Hispanic. The total on-campus undergraduate and graduate student population was approximately 42,000 during the Fall 2021 semester. All the first-semester calculus courses and the preparation for calculus courses are coordinated by the Department of Mathematics and meet two (for 80-minute lecture periods) or three (for 50-minute lecture periods) times a week for lecture, then have two additional course meetings for a collaborative lab activity during one 50-minute class meeting and time of review and quiz during the other 50-minute class meeting during a 15-week semester. Each precalculus and calculus course has coordinated exams written by the course instructors and distributed to each of the courses. These exams are typically offered during the fourth full week of the semester, the ninth full week of the semester, and the final during the last week. A graduate teaching assistant often leads the lab sections in the mathematics department. In recent years, there have been approximately 14 first-semester calculus courses offered in the Fall semester and approximately 10 precalculus courses offered. Participants in this study included all consenting students who completed the initial survey or weekly post-lab surveys, as well as up to 6 undergraduates from each course for the individual and task-based interviews.

From course syllabi, the precalculus course learning outcomes include undergraduates being able to justify and explain their steps in problem-solving, demonstrate their ability to work with functions in multiple representations, identify characteristics of functions and their graphs, and interpret, define, and graph the six trigonometric functions while being able to use trigonometric identities. From the course syllabi, the course learning outcomes for the first-semester calculus course involve computing limits of various functions, computing derivatives and differentials of various functions connecting the idea of limit and derivative, finding the equation of a tangent line to a function by using the derivative, sketching functions by using critical points, extrema, and inflection points. Other outcomes include solving word problems that offer real-life examples of differentiation and optimization, being able to compute a Riemann sum, computing antiderivatives and applying the Fundamental Theorems of Calculus, and being able to justify and explain their problem-solving steps.

3.2 Data Collection

To obtain information regarding the first research question, data about participants' educational background, demographic background, mathematics identities and self-efficacy, use of self-regulation strategies, and expectations of college coursework was collected using an online questionnaire/demographic survey given to students with the informed consent document within the first week of the semester via the online course management system used by the university. Initial surveys were sent to thirteen of fourteen first-semester calculus sections and five of seven precalculus sections including approximately 1000 students. Of those invited, 213 students completed the initial survey (188 from the first-semester calculus course and 25 from precalculus). Included in this initial online survey were the Motivated Strategies for Learning Questionnaire (MSLQ) by Pintrich, Smith, García, and McKeachie (1991) (See Appendix A

Motivated Strategies for Learning Questionnaire), a survey to measure mathematical identity by Kaspersen (2017) (See Appendix B

Mathematics Identity Survey), select questions from the Factors Influencing College Success in Mathematics (FICSMath) by Sonnert et al. (2020) (See Appendix C

Factors Influencing College Success in Mathematics), and items from a survey about self-efficacy by Usher and Pajares (2009) (See Appendix D

Sources of Self-Efficacy). Information from the previously mentioned surveys was used to draw correlations between students' reported use of self-regulation strategies and their reported self-efficacy and mathematics identity scores.

The information from the initial survey was also used to choose up to 9 students from each course (up to 18 total students) to interview. I initially invited more than 18 students to get up to 18 participants for the initial interviews. I attempted to account for attrition between interviews to get at as many complete participant interviews as possible from each class. Undergraduates in these courses were invited, via email to their student email, to be interviewed five times during the semester for two different types of interviews. First, a brief individual interview occurred with each participant, then a series of three task-based group interviews, followed by an individual exit interview. I first acquired permission from most first-semester calculus and precalculus instructors to distribute links to online surveys and questionnaires via email or the university course website to their classes during the first week of class with their consent. Sampling for interviews occurred by looking first for first-year undergraduate students who attended high school within the last year based on survey responses. After that, to decide whom to invite for the interview process, students were chosen who had a strong performance in their past high school courses and had varying levels of self-regulation usage and mathematics identity/self-efficacy scores as computed by analysis of the initial survey.

To collect data on how students adapted their self-regulation strategies throughout the semester, the interviews occurred at three different points of the semester, within a week of each midterm. To address the second research question, participants were often asked in the individual interviews to compare or contrast their high school experiences and strategy usage with their view of their current mathematics course. The three sets of task-based group interviews occurred three weeks into the semester, three weeks after the first midterm, and three weeks after the second midterm with the students who consented to be a part of the interview process. Participants were placed initially in groups based on their scheduling preferences; and groups were adjusted to compensate for attrition between interviews and scheduling changes. Within a week before the first group interview, initial 15- to 30-minutes individual interviews occurred, then within a week after the third group interview, the individual exit interview occurred, also lasting between 15- to 30-minutes.

Completion of a pilot study in the Fall 2020 semester supported this distribution of interviews which allowed the researcher to see transitions in self-regulation strategy usage and mathematics self-efficacy based on factors such as lab experience and midterms. Group interviews lasted from 45-minutes to an hour. The interviews were audio and/or audio-video recorded from a video recorder (or, in the case of doing the interviews online, the recordings were done through a video conferencing software).

During the group interviews, reflection on the last three lab activities occurred by having interview participants look at a specific problem from each lab activity. The participants then explained what methods they used when solving the problem while attempting to solve it as a group. These problems focused on mathematics concepts that undergraduates often have misconceptions about, such as inverse functions for the precalculus course and the graphical relationship of a function with its first and second derivative for the first-semester calculus course. The interviewer asked what strategies each participant used at any point in the lab, given that he or she was stuck or needed assistance at any time. In the individual interviews, participants answered questions about their experiences in high school mathematics courses, how they used self-regulation strategies within and outside of a mathematics course, and their perception of how their high school mathematics course prepared them for undergraduate mathematics. I analyzed participants' reported self-regulation strategies, their changes in strategy usage, and what factors may have influenced those changes. For interview protocols view Appendix E

Individual Interview Protocol #1, Appendix F

Group Interview Protocol #1, Appendix G

Group Interview Protocol #2, Appendix H

Group Interview Protocol #3, and Appendix I

Individual Interview Protocol #2.

During the interviews, the following questions were presented to the participants to reflect on, provide insight regarding their self-regulation strategies, and work or rework the problem among the group of interviewed participants. Each group interview included up to three lab problems from selected worksheets given to the students each week during a collaborative lab activity from either the precalculus labs or the calculus labs respective of the group of participants being interviewed. For the complete lab activities that participants worked on in their classes, see Appendix K

Collaborative Precalculus Lab Activities and Appendix L

Collaborative Calculus Lab Activities.

3.2.1 Precalculus Group Interview Activities

The following figures show the problems examined in the three group interviews throughout the semester. Each interview included an exploration of three activities pulled from the collaborative lab activities used during the regular coursework of the precalculus section. The first three labs were all worked on prior to the first midterm and focused on linear equations, function notation and operations, and function composition and transformations. The following three problems (see Figures Figure 3-5) were selected as they involved some aspect of problem-solving and could be discussed as a group.

3. A closed box with a square bottom is three times high as it is wide.

    a) Express the surface area of the box in terms of its width.

    b) Express the volume of the box in terms of its width.

    c) Express the surface area in terms of the volume.

    d) If the box has a volume of 24 m^3, what is its surface area?

Figure 3: Precalculus Lab Activity 1, Problem #3 from University of Texas at Arlington

1. Determine whether the following statements are true, and give an explanation or counterexample.

a) If the domain of a function f is all real numbers, then the domain of the function $g(x) = f(x^2)$ is also all real numbers.

b) If $g(x) = x^3 - 1$, then $g(2) = g(2)^3 - 1$.

c) If $h(x) = 0$, then $h(x^2) = 0$.

d) The graph of a function can have symmetry with respect to the y-axis and with respect to the origin but cannot have symmetry with respect to the x-axis.

Figure 4: Precalculus Lab Activity 2, Problem #1 from University of Texas at Arlington

3. Consider the following transformations:

(v) vertical shift down by 2
(h) horizontal shift right by 3
(x) x-axis reflection
(y) y-axis reflection

Does it matter what order these transformations are applied? Apply these transformations to the graph of $y = \sqrt{x}$ in two different orderings. One ordering should be $(v), (h), (x), (y)$, and the other ordering should be $(x), (y), (v), (h)$. Which ordering produces the correct graph of $y = -\sqrt{-(x-3)} - 2$?

Figure 5: Precalculus Lab Activity 3, Problem #3 from University of Texas at Arlington

The next three problems (see Figures Figure 6-8) were pulled from the lab activities given between the first and second midterms. The problems on these labs focused on working with quadratic equations, inverse functions, and logarithmic and exponential equations and functions.

3. Suppose a freight company's shipping rates are \$8.50 for packages weighing less than 2 pounds and \$5.50 for each additional 2 pounds.

(a) How much does it cost to ship a 6.4-pound package?

(b) Write the shipping cost function in terms of the integer floor function.

(c) Graph the shipping cost function for weights between 0 and 20 pounds.

Figure 6: Precalculus Lab Activity 4, Problem #3 from University of Texas at Arlington

1. Consider $f(x) = \dfrac{-5-x}{3-2x}$.

a) Find $f^{-1}(7)$ **without finding** $f^{-1}(x)$
b) Find $f^{-1}(x)$

Figure 7: Precalculus Lab Activity 5, Problem #1 from University of Texas at Arlington

3. Let $m = \log_b M$ and $n = \log_b N$.

(a) Rewrite the above 2 equations using exponentials instead of logarithms.

(b) Using your equations from part (a), prove that $\log_b(MN) = \log_b M + \log_b N$.

Figure 8: Precalculus Lab Activity 6, Problem #3 from University of Texas at Arlington

The last three problems (see Figures Figure 9-11) used in the interviews were given between the second midterm and the final exam. These labs focused on working with trigonometry and the unit circle, transformations of trigonometric functions, and using trigonometric functions with right triangles.

4. Determine whether each function is even, odd, or neither.

(a) $f(x) = \cos x \tan x$ (b) $f(x) = \dfrac{\sin x - \cos x}{\tan x}$

Figure 9: Precalculus Lab Activity 7, Problem #4 from University of Texas at Arlington

2. Rewrite the function $g(x) = 4\cos x$ as a new function $h(x) = A\sin(Bx + C)$. Where A, B, and C are constants such that $g(x) = h(x)$ for all possible values of x.

Figure 10: Precalculus Lab Activity 8, Problem #2 from University of Texas at Arlington

2. Sketch a right triangle and label one of the acute angles as θ. If $\theta = \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)$, evaluate the following. Note that your answers will be in terms of x.

a) $\sin\theta$ b) $\cos\theta$ c) $\tan\theta$
d) $\csc\theta$ e) $\sec\theta$ f) $\cot\theta$

Figure 11: Precalculus Lab Activity 9, Problem #2 from University of Texas at Arlington

3.2.2 Calculus Group Interview Activities

As with the precalculus group interviews, the calculus group interviews explored three problems from three separate collaborative group activities that were a regular part of the calculus coursework. The timing of the labs relative to the midterms and final were also equivalent to the precalculus section as the midterms for calculus and precalculus occur during the same time. The first three lab activities (see Figures Figure 12-14) for calculus focused on a review of past material, working with limits, and continuity.

12. Find the exact value of $\sin^{-1}\left(\sin\frac{9\pi}{10}\right)$.

Figure 12: Calculus Lab Activity 1, Problem #12 from University of Texas at Arlington

2. Is it possible that $\lim_{x\to a} f(x)$ does not exist and $\lim_{x\to a} g(x)$ does not exist, but $\lim_{x\to a}[f(x) + g(x)]$ does exist? Consider the functions $f(x) = \frac{1}{x}$ and $g(x) = -\frac{1}{x}$.

   a. Sketch the graphs of $f(x)$ and $g(x)$ and find their limits (if they exist) as x approaches zero.
   b. Sketch the graph of $[f(x) + g(x)]$ and find $\lim_{x\to 0}[f(x) + g(x)]$.

Figure 13: Calculus Lab Activity 2, Problem #2 from University of Texas at Arlington

8. Let $f(x) = 2x^2 - 4x$.

a. Find the average rate of change of f over the interval $[3, 3 + h]$. How is this average rate of change related to the secant line passing through the points $(3,6)$ and $(3 + h, f(3 + h))$?

b. Use part (a) to find the instantaneous rate of change of f with respect to x at 3. How is this instantaneous rate of change related to the tangent line passing through the point $(3,6)$?

c. Find an equation of the tangent line to the graph of f at the point $(3,6)$.

Figure 14: Calculus Lab Activity 3, Problem #8 from University of Texas at Arlington

The next three lab activities (see Figures Figure 15-17) were from labs focused on using the product and quotient rule for derivatives, using derivatives for position, velocity, and acceleration functions, and using the chain rule for derivatives and implicit differentiation.

5. Use the product rule (twice) to find a formula for $\frac{d}{dx}\big(f(x)g(x)h(x)\big)$.

Figure 15: Calculus Lab Activity 4, Problem #5 from University of Texas at Arlington

3. The figure shows the graphs of three functions. One is the position of a car, one is the velocity of the car, and one is its acceleration. Identify each curve and explain your choices.

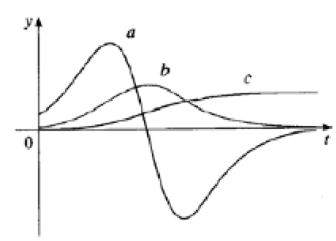


Figure 16: Calculus Lab Activity 5, Problem #3 from University of Texas at Arlington

(1) Functions f, g, and h are continuous and differentiable for all real numbers, and some of their values and values of their derivatives are given in the table below.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
0	1	-1	-1	4	1	-3
1	0	3	-7	2	3	6
2	3	0.5	4	-2	1	3

Show your work to justify the solutions to the following problems.

(a) Find $F'(1)$ if $F(x) = h(x) \cdot e^{-3f(x)}$

(b) Find $G'(2)$ if $G(x) = cos[\pi \cdot g(x)]$

(c) Find $H'(0)$ if $H(x) = e^{\left(\frac{h(x)}{f(x)}\right)^2}$

Figure 17: Calculus Lab Activity 6, Problem #1c from University of Texas at Arlington

The last three lab activities (see Figures Figure 18-20) used in the interviews were selected from labs focused on solving related rates problems, solving optimization problems, and working with linear approximations and differentials.

(5) The sides of a square baseball diamond are 90 feet long. When a player who is between second and third base is 60 feet from second base and heading towards third base at a speed of 22 feet per second, how fast is the distance between the player and home plate changing?

Figure 18: Calculus Lab Activity 7, Problem #5 from University of Texas at Arlington

(5) Minimizing Distance

(i) Find the point P on the line $y = 3x$ that is closest to the point $(50,0)$.

(ii) What is the least distance between P and $(50,0)$?

Figure 19: Calculus Lab Activity 8, Problem #5 from University of Texas at Arlington

(4) Consider the function $f(x) = \sqrt{2}\cos x$.

(i) Find the linear approximation L to the function f at $a = \frac{\pi}{4}$.

(ii) Graph f and L on the same set of axes.

(iii) Based on the graphs of part (ii), state whether linear approximations to f near a are underestimates or overestimates.

(iv) Compute $f''(a)$ to confirm your conclusion.

Figure 20: Calculus Lab Activity 9, Problem #4 from University of Texas at Arlington

During the group task-based interviews, participants at times wrote on the physical lab assignments on the scratch paper provided to show their thought process for solving a problem in the lab, producing a written artifact. In the case of the virtual interview, written work was shared through screen-sharing technology. In each of the three group interviews, participants examined the previous three labs. During the second group interview, a list of self-regulation strategies was given to the participants to determine which, if any, they regularly used during their mathematics courses from high school and what they were experiencing thus far in college mathematics. The first individual interview included questions about the participants' educational background and expectations for the semester. The last individual interview included questions about changes in self-efficacy and whether their expectations were met or unmet. Based on findings from the pilot study, information about students' preparation for midterms and specific mathematics concepts students felt needed more support was also obtained. Both individual and each task-based interviews also included a line of questioning about students' study habits for midterms and how those might have changed during the semester.

To address both research questions, additional data about students' use of self-regulation strategies and expectational differences from secondary to post-secondary mathematics courses was collected using a brief survey to be given to consenting students at the end of each lab session (once a week) for each of the twelve (precalculus) or thirteen (first-semester calculus) labs that were handed out. The post-lab survey included general questions about the difficulty of the lab, a specific question about one problem of the lab asking about self-regulation strategy usage, and a question about what self-regulation strategies were used throughout the lab, a question about mathematics self-efficacy from Usher and Pajares' (2009) survey. An electronic version of the survey was given either through a hyperlink provided to the course instructor, or the survey was embedded into the online course management system.

Observation data were also collected during the weekly lab section of one of the weekly lab sections of a selected precalculus and a selected first-semester calculus course each week during the semester, with consent of the instructors. I observed how calculus and precalculus students were working in groups to see what self-regulation strategies they appeared to be using during group work and to note how often these students consulted the instructor of record, the graduate teaching assistant, or any other teaching aides in the classroom. Observations occurred weekly when lab activities were done with one first-semester calculus course and one precalculus course (up to twenty-four observations overall; observations did not take place during the week of the midterms). No identifying information was collected, but just general comments about group activities. Observation data were used to triangulate strategy usage reported during interviews and identify habits used in a natural lab setting instead of an interview setting. For observation protocol, see Appendix J

Classroom Observation Form.

I transcribed each interview word-for-word. The interviewed subjects were given a pseudonym (see Table 1). Identifying information in the questionnaire, as well as in the audiovisual recordings and transcriptions of the interviews, was removed and replaced with this pseudonym.

Table 1: Interview Participants

Participant	Course	Major	Highest Mathematics Course Taken in Secondary school
Aria	Calculus 1	Civil Engineering	AP Calculus AB
Cyndy	Calculus 1	Biology	Precalculus
Frank	Calculus 1	Computer Engineering	AP Calculus AB
Jane	Calculus 1	Architectural Engineering	Calculus
Jay	Calculus 1	Computer Science	Trigonometry
Sunny	Calculus 1	Computer Science	AP Calculus BC
Bailey	Precalculus	Microbiology	AP Calculus AB
Rose	Precalculus	Mechanical Engineering	AP Calculus AB
Toby	Precalculus	Biology	Precalculus

3.3 Data Analysis

A multiple case study methodology was chosen to explore individuals' experiences of expectations and self-regulation strategies in-depth while trying to identify common themes among the cases. A mix of qualitative data from the interview process and quantitative data from the weekly surveys and initial surveys was used to get a more complete picture of the participants' backgrounds and how their high school experiences may have affected their early collegiate experiences. Using the data from the initial surveys, trends were analyzed to see if higher mathematics identity might be correlated with high use of self-regulation strategies. Responses to the surveys that gauge the use of self-regulation strategies and mathematics identity were used to select students that have differing levels of strategy usage and mathematics identity. This allowed me to invite a selection of students with high mathematics identity and low mathematics identity to interview to determine how these undergraduates might utilize self-regulation strategies. The case is defined to be the individual student with the units of analysis including the initial questionnaires (including the background questionnaire, the MSLQ, and questions from the FICS questionnaire), the five interviews, and the weekly post-lab surveys with the observation data used to validate the data from the other sources.

Mathematics identity was computed using Kaspersen's (2017) and a selection from Sonnert et al.'s (2020) surveys which were then put into a spreadsheet, and each participant's score was computed using an average of the individual survey scores and then categorized as low, low-medium, medium, medium-high, and high. Self-efficacy scores were similarly computed and categorized, using part of Sonnert et al.'s (2020) and Usher and Pajares' (2009) surveys. For self-efficacy, a low score would indicate that a student is not confident in their ability to complete mathematics tasks, while a high score indicates a strong confidence in that ability. The mathematics identity score relates to how strongly a student perceives themselves in relation to mathematics. A low score would indicate that the student does not see themselves as being particularly mathematically minded, while a high identity would indicate that students see themselves as being highly mathematically minded. The Motivated Strategies for Learning Questionnaire (1991) was used to determine students' use of motivation strategies, which included elements of self-regulation strategies. The categories that appeared on the MSLQ questionnaire are outlined in Table 2. Scores for each category were calculated using the scoring rubric provided with the MSLQ. Once self-efficacy, mathematics identity, and self-regulation strategy usage had all been scored, I plotted the scores calculated from the nine self-regulation strategies (rehearsal to help seeking) by identifying the number of scores each participant showed proficiency in (score above three). Each score for the nine strategies could range from 0 to 7, where 0 indicated that a participant never used the strategy and higher scores indicated a stronger, more frequent, use of the strategy. From the MSLQ the authors state that a score above 3 indicates an adequate use of the strategy. I then took a weighted average from the MSLQ scores and related it to both the self-efficacy scores and mathematics identity scores on a graph and calculated the correlation coefficients. I then calculated a two-tailed t-distribution to find how statistically significant the correlation coefficients were using the following equations for t, where n is the sample size and r is Pearson's correlation coefficient.

$$t = r * \frac{\sqrt{n-2}}{\sqrt{1-r^2}}$$

Table 2: MSLQ Categories and Definitions

Intrinsic Goal Orientation	The degree to which a student perceives themselves to be participating in a task for reasons such as challenge, curiosity, or mastery.
Extrinsic Goal Orientation	The degree to which a student perceives themselves to be participating in a task for reasons such as grades, rewards, performance, evaluation by others, and competition.
Task Value	Students' evaluation of how interesting, how important, and how useful a task is.
Control Beliefs	The belief that outcomes are contingent on one's own efforts, in contrast to external factors such as the teacher.
Self-Efficacy for Learning and Performance	A self-appraisal of one's ability to master a task.
Test Anxiety*	Students' negative thoughts and affective and physiological states that disrupt performance when testing.
Rehearsal	Reciting or naming items from a list to be learned.
Elaboration	Paraphrasing, summarizing, creating analogies, and generative note-taking to store information into long-term memory by building internal connections between items.
Organization	Clustering, outlining, and selecting the main ideas of material being read to construct connections among the information to be learned.
Critical Thinking	The degree to which students report applying previous knowledge to new situations to solve problems, reach decisions, or make critical evaluations with respect to standards of excellence.
Metacognitive Self-Regulation	The awareness, knowledge, and control of cognition, including planning, monitoring, and regulating motivational skills and actions.
Time and Study Environment	The management and regulation of time and study environments.
Effort Regulation	Students' ability to control their effort and attention in the face of distractions and uninteresting tasks.
Peer Learning	Collaboration with one's peers to help clarify course material and/or reach insights that may not have been attained on one's own.
Help Seeking	The recognition of one's lack of knowledge and being able to identify someone to provide assistance.

The interviews from a pilot study have been coded using content analysis coding schemes as outlined by Stemler (2000). Codes generated before data collection were identified by Zimmerman and Pons (1986) and Wolters (1998). Interviews were coded using the pre-established code book. Emergent codes did arise and were added to the code book. These codes related to what strategies participants would use to address issues that arose while working on homework, lab activities, or studying for assessments. A collection of codes used from the pilot study are included in the following table (

Table 3) and the emergent codes that appeared during the Fall 2021 interviews. Connections between the codes were identified as themes across the cases and are reported in Chapter 4. Data from the individual and group interviews were analyzed to see if there are any differences in expectations, whether undergraduates have unmet expectations and how they react to those transitions, as well as if they develop any new self-regulation strategies. Also, any changes in their mathematics identity were noted, as well as whether the participants have more congruent self-efficacy based on how they answer questions about their expectations of whether they can answer a mathematics question confidently. Codes from a subset of interviews were compared with another researcher to establish inter-coder reliability. The majority of codes were consistent between myself and the other researcher, and any differences were resolved between the codes.

Table 3: Codes and Descriptions

Code	Description
Self-Efficacy	Undergraduate's self-reported confidence of ability to pass the course
Identity	Undergraduate's report of how they view themselves as a mathematics learner
Env	Evidence of self-regulation strategy of changing environment
MG	Evidence of self-regulation strategy of setting mastery goals
OT	Evidence of self-regulation strategy of organizing information and transforming it into something useful
PG	Evidence of self-regulation strategy of setting performance goals
PR	Evidence of self-regulation strategy of providing extrinsic rewards
RM	Evidence of self-regulation strategy of keeping records and monitoring progress made in a course
SAA	Evidence of self-regulation strategy of seeking assistance from adult (other than an instructor)
SE	Evidence of self-regulation strategy of seeking information from an external source
SEU	Evidence of self-regulation strategy of self-evaluating own understanding
SPA	Evidence of self-regulation strategy of seeking peer assistance
STA	Evidence of self-regulation strategy of seeking teacher assistance
Value	Evidence of self-regulation strategy of reminder of the value of the task to learning
Examples/ Counterexamples	Evidence of self-regulation strategy of looking at a known example or counterexample to better understand a current problem.
Metacognition	Evidence of students' discussion of self-regulation strategy usage, especially considering changes or uses of a strategy.
Provide Time	Evidence of self-regulation strategy of providing time to process a problem while not actively working on the problem.
Routine	Evidence of self-regulation strategy of setting a weekly or daily routine of studying.
Worked Example	Evidence of self-regulation strategy of looking at a completed example to then mimic the process for a relevant exercise

Chapter 4

Findings

During the Fall 2021 semester I began analyzing the initial survey data as it was collected to identify those participants who would be invited to participate in interviews. After each set of interviews, I began work transcribing the interviews. Through the process described in the previous chapter, I identified the initial codes as well as the emergent codes reported below. This chapter includes findings regarding the quantitative data found from analyzing the initial surveys as they relate to students' self-regulation habits, mathematics self-efficacy, and mathematics identity. This chapter then includes the themes found in the sequence of participant interviews as they relate to their transitioning use of self-regulation strategies and academic integration.

4.1 Initial Survey Findings

Data analysis for the initial surveys included compiling the data from the online survey system into an Excel spreadsheet. Analysis of the initial surveys was first used to identify potential interview participants by selecting first-semester freshman who had varying levels of reported self-efficacy and mathematics identity. Using this information, I constructed the following scatterplots relating mathematics identity to self-regulation strategy usage and self-efficacy to self-regulation strategy usage using the entire population ($n = 213$), the first-semester calculus participants only ($n = 188$), and the precalculus participants only ($n = 25$) (See Figures Figure 21-26 below). The following scatterplots show the relationship between participants' reported self-regulation strategy usage versus their mathematics identity scores and mathematics self-efficacy scores. With each of the following figures I include Pearson's correlation coefficient, r, and the line of best fit to show the general trend of the data, as well as the results from a two-tailed t-test which provides information on how closely the means of the data samples are related.

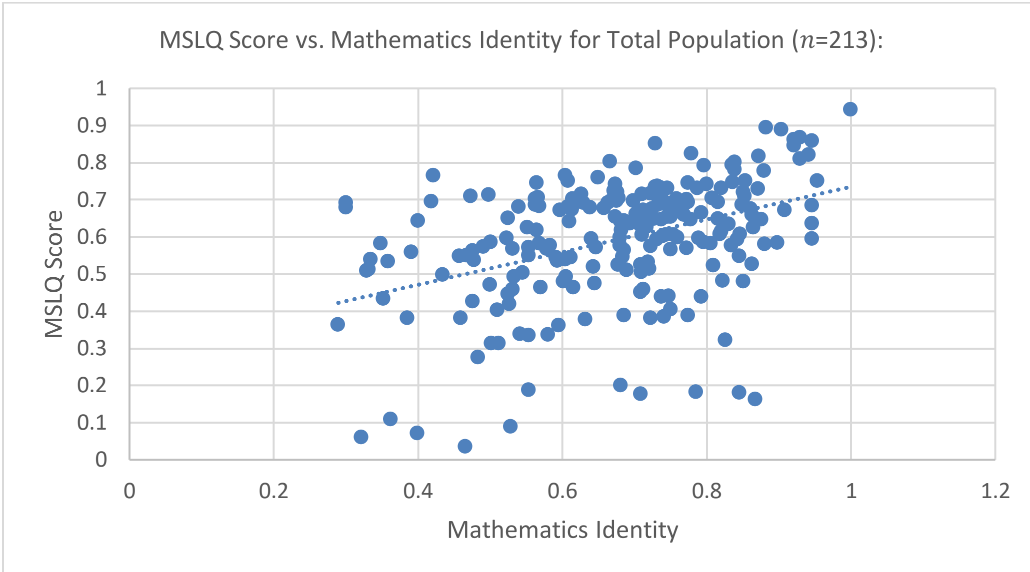


Figure 21: Total population of participants' ($n = 213$) score obtained from Motivated Strategies for Learning Questionnaire versus Mathematics Identity score. Pearson's $r = 0.4131$

In Figure 21 we see the total population of participants who completed the initial survey where the score obtained from the MSLQ is given as it relates to participants' mathematics identity which was computed using Kaspersen's Mathematical Depth Instrument (2017). The scatterplot above is moderately positively correlated, with the linear line of best fit given by $y = 0.4397x + 0.2954$. Calculating a two-tailed t-distribution using the method described in the previous chapter, I found that there is a significant increase in the use of self-regulation strategies as participants' mathematics identity score increases for $\alpha = 0.05$ with $p = 3.5 * 10^{-10}$.

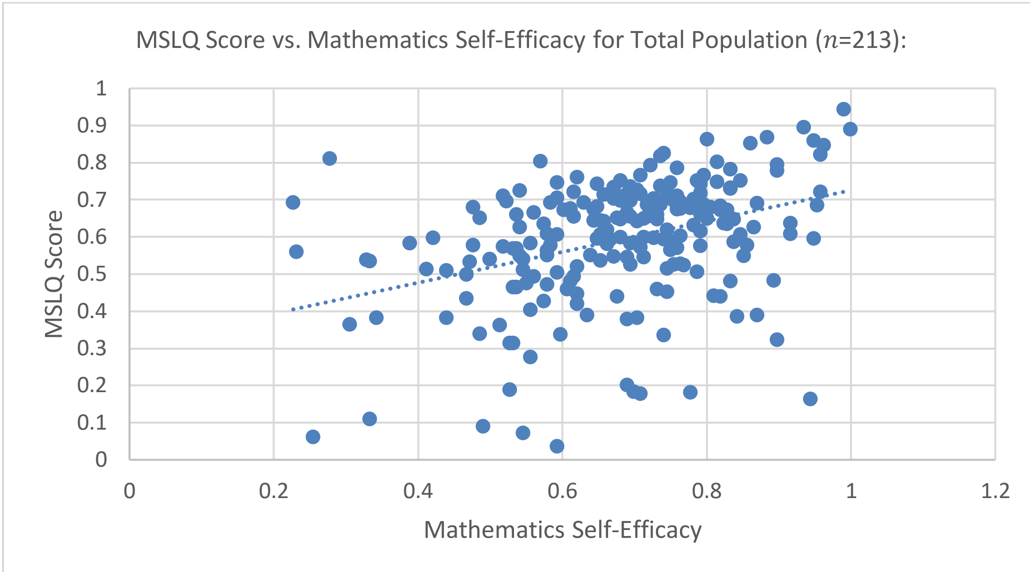


Figure 22: Total population of participants' ($n = 213$) score obtained from Motivated Strategies for Learning Questionnaire versus Mathematics Self-Efficacy score. Pearson's $r = 0.3774$

Similarly to what was found in Figure 21 with the same population of participants, we see a moderately positive correlation between participants' use of self-regulation strategies and their sense of self-efficacy as calculated by Usher and Pajares' Sources of Self-Efficacy Survey (2009). The line of best fit is given by $y = 0.4149x + 0.3108$ and I found that there is a significant increase in the use self-regulation strategies as participants' mathematics self-efficacy increase for $\alpha = 0.05$ with $p = 1.3 * 10^{-8}$.

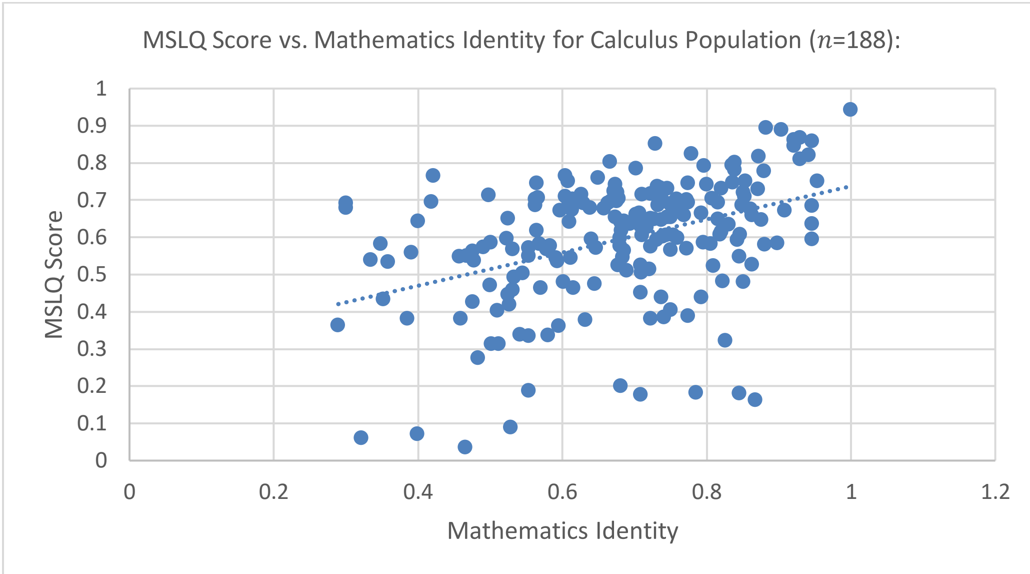


Figure 23: Population of Calculus participants' ($n = 188$) score obtained from Motivated Strategies for Learning Questionnaire versus Mathematics Identity score. Pearson's $r = 0.4101$

With the reduced sample of first-semester calculus participants, Figure 23, similarly to the previous figures, shows a moderately positive correlation between participants' use of self-regulation strategies and their measure of mathematics identity. The line of best fit for this data is given by $y = 0.4458x + 0.2917$ and the relationship in the smaller sample was found to be statistically significant for $\alpha = 0.05$ with $p = 5.1 * 10^{-9}$.

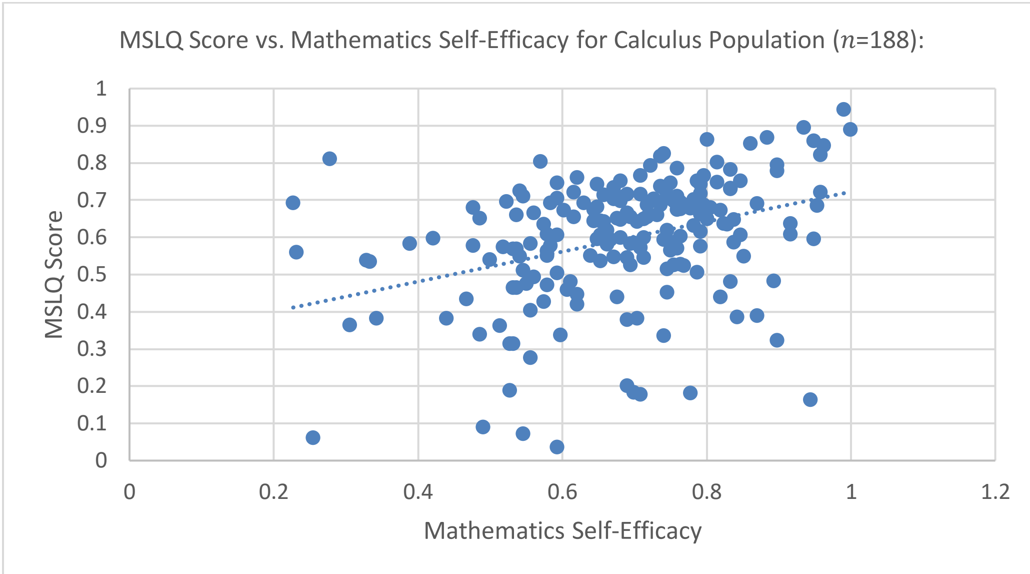


Figure 24: Population of Calculus participants' ($n = 188$) score obtained from Motivated Strategies for Learning Questionnaire versus Mathematics Self-Efficacy score. Pearson's $r = 0.3589$

Figure 24 shows the data related to the sample of first-semester calculus participants' use of self-regulation strategies and their self-efficacy in a mathematics course. The correlation is shown to be moderately positive, with the line of best fit given by $y = 0.4025x + 0.3201$, with this relationship still being statistically significant for $\alpha = 0.05$ with $p = 4.2 * 10^{-7}$.

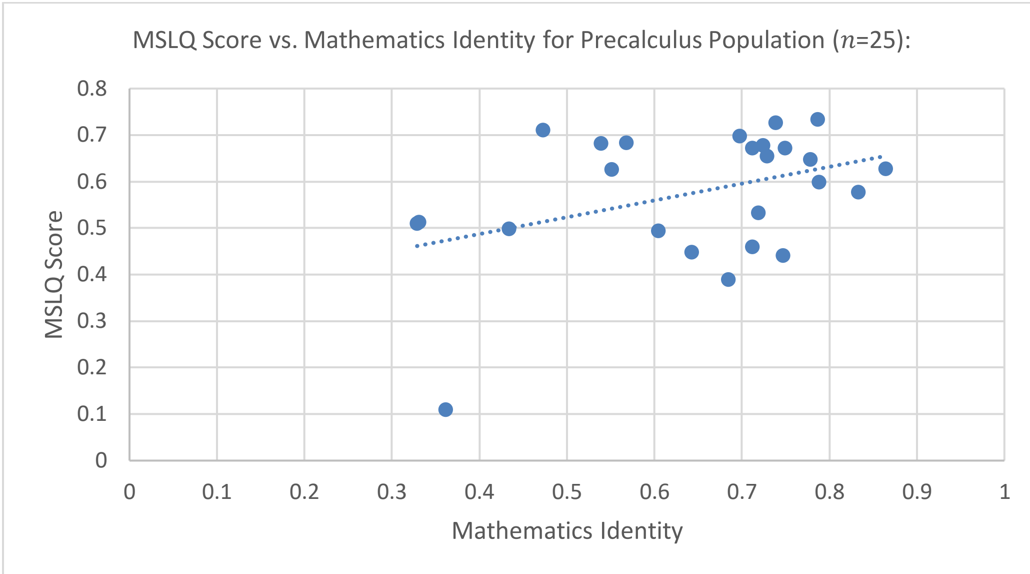


Figure 25: Population of Precalculus participants' $(n = 25)$ score obtained from Motivated Strategies for Learning Questionnaire versus Mathematics Identity score. Pearson's $r = 0.4021$

Figure 25 shows the smaller sample of participants enrolled in a preparation to calculus course who completed the initial survey. This scatterplot shows precalculus participants' use of self-regulation strategies as related to their mathematics identity. The line of best fit is given by $y = 0.3615x + 0.3425$, with this relationship of the small sample of precalculus participants being statistically significant for $\alpha = 0.05$ with $p = .046$.

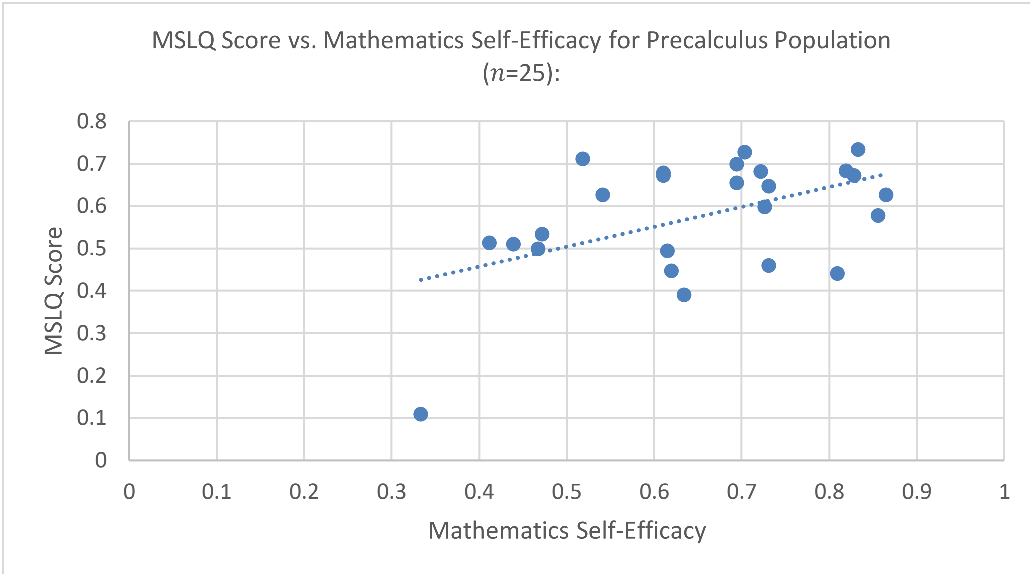


Figure 26: Population of Precalculus participants' ($n = 25$) score obtained from Motivated Strategies for Learning Questionnaire versus mathematics self-efficacy score. Pearson's $r = 0.5018$

Similarly to Figure 25, this scatterplot shows a smaller sample of participants, showing a moderately positive correlation. This shows the relation between the participants' scores from the MSLQ to their self-efficacy score. The line of best fit is given by $y = 0.4692x + 0.2695$, with this relationship being statistically significant for $\alpha = 0.05$ with $p = 0.011$.

The correlation for each of the relationships was found to be a weak to moderately positive correlation, as also indicated by the general shape of each of the scatterplots above. Each comparison between the increased use of self-regulation strategies versus the scores of mathematics identity and mathematics self-efficacy was found to be statistically significant using the two-tailed t-test with $\alpha = 0.05$.

With the data from the initial interview I was able to select participants with varying levels of mathematics self-efficacy and mathematics identity as interview participants. This selection process occurred before the statistical tests outlined above were performed. As shown in the following table, Table 4, the final participants that agreed to be interviewed had self-efficacy and identity scores that ranged from medium to high, but no participants with low scores agreed to be a part of the interviews. The examination of those participants with higher scores of mathematics self-efficacy and identity to identify what strategies and what frequency of strategy usage they employ provides a framework of self-regulation strategies that could benefit students in their transition to undergraduate mathematics courses. The individual scores related to mathematics identity and self-efficacy of the interviewed participants were identified and are presented in the table below.

Table 4: Interviewed Participants' Self-Efficacy and Mathematics Identity Scores

Participant	Self-Efficacy Score	Mathematics Identity Score
Aria	0.56 (Medium)	0.51 (Medium)
Cyndy	0.58 (Medium)	0.48 (Medium)
Frank	0.48 (Medium)	0.58 (Medium)
Jane	0.92 (High)	0.81 (High)
Jay	0.64 (Medium-High)	0.47 (Medium)
Sunny	0.78 (Medium-High)	0.77 (Medium-High)
Bailey	0.82 (High)	0.57 (Medium)
Rose	0.73 (Medium-High)	0.79 (Medium-High)
Toby	0.54 (Medium)	0.55 (Medium)

Along with the scores for self-efficacy and mathematics identity, aspects of participants' self-regulation strategies were analyzed using the initial survey. Scores for the interviewed participants based on their responses from the MSLQ are given in Table 5 below. Average scores for the MSLQ range between 1 (category does not apply to the student or skill is not used by student) to 7 (category strongly applies to the student or skill is strongly used by student) (see Table 2 for definitions of categories of MSLQ). In the following table, the first two categories are used as indicators for whether students have a stronger sense of intrinsic or extrinsic goal orientation. The categories of task value, control beliefs, and test anxiety are not discussed in this context. The remaining nine categories are included to give a picture of how students generally respond to testing conditions and what strategies they reported using in a general classroom setting. More detailed information about how these strategies arise in a mathematics specific context is given in the following section.

Table 5: Interviewed Participants' Motivated Strategies Scores

Categories	Aria	Cyndy	Frank	Jane	Jay	Sunny	Bailey	Rose	Toby
Intrinsic Goal Orientation	3.5	2.75	3.75	5.5	4.25	4.75	5.75	5	4.75
Extrinsic Goal Orientation	4.25	6	3.5	6.25	5.5	4.75	6.5	4	6.25
Task Value	4.17	3.67	3.67	5.67	4	6.17	5.67	6.33	3.5
Control Beliefs	5	5.75	4	7	4.5	6.5	6	5.75	6.5
Self-Efficacy for Learning and Performance	5	4.25	4.38	6.5	4.13	5	6	6.25	5.25
Test Anxiety*	2.4	6.4	3	5.4	5.2	5.4	5	3.8	3.6
Rehearsal	4.5	6	4.25	4.75	4.5	5.25	5.5	6.25	4
Elaboration	1.83	4.83	4	5.67	5	5.5	4.83	3.83	4.17
Organization	3.5	7	4.5	5	4.5	6.25	4.75	6.5	4.75
Critical Thinking	1.8	3.2	4	6	3.8	3	3.2	1	3.6
Metacognitive Self-Regulation	3.08	3.67	3.67	5.75	4	4.67	5.42	4.67	3.92
Time and Study Environment	4.75	5.38	4.13	5.125	5.88	5.75	5.25	5.5	4.88
Effort Regulation	5	4.5	3.5	5.25	6.5	6.25	4.75	6	5.5
Peer Learning	3.33	1.33	4.33	3.33	1.33	3.33	4.33	4.67	4.67
Help Seeking	5	4	4	2.25	3.5	4.25	5	4	4

*High score for Test Anxiety is viewed as worrying

4.2 Interview Findings

To address the first research question, "in what ways do students adapt their self-regulation strategies from high school mathematics to college level mathematics?" codes regarding participants' adaptations of self-regulation strategies as they relate to the course content and structural differences between a high school course and a college course are discussed. To discuss these codes, first a background of each of the interviewed participants is given, then data from each of the two individual interviews is presented and a portion of the second group interview focused on their individual use of self-regulation strategies, with the discussion of the specific content and structural difficulties given after. Within these sections, participants report how they went about adapting their usage of self-regulation strategies to adjust to the transition of entering a college mathematics course. I also discuss the self-regulation strategies used while actively working on collaborative activities throughout the semester by the interviewed participants. Lastly, for the calculus context I provide a larger sample of participants' self-regulation strategy usage and perception of lab difficulty through post-lab surveys collected throughout the Fall 2021 semester to provide a broader context of how these labs were viewed by participants.

4.2.1 Interviewed Students

Nine participants completed the interviews throughout the semester and provided insight as to how they viewed their transition to college-level mathematics courses and the challenges they experienced. In the following section, I present a brief overview of each participant's mathematics background, their intended major, and a summary of data obtained from the initial survey. I first present backgrounds for each of the three precalculus participants, Bailey, Rose, and Toby, and then the six first-semester calculus participants, Aria, Cyndy, Frank, Jane, Jay, and Sunny. All the interviewed participants were first-semester freshman and had completed high school in the United States.

4.2.1.1 Precalculus: Bailey

The first precalculus case I describe is Bailey. Bailey was a microbiology major but reported that she might possibly be switching to medical technology as her major. Bailey took AP Calculus AB in high school and reported doing "good" but her performance "went downhill towards the end of the year." Bailey's high school calculus course was online. Additionally, she reported that she was involved in an honors mathematics society, Mu Alpha Theta, where she tutored junior high students, which helped her to review older mathematics material she may have forgotten. Depending on whether Bailey switches majors, she reported potentially needing one additional mathematics course - either statistics or trigonometry - but otherwise she would be done with mathematics courses after precalculus. When asked how she felt about the precalculus course, and whether she was confident she could pass, Bailey stated that she was confident that she will pass the course, and with effort she will make an A in the course.

Bailey then spoke about her perception of herself as a mathematician, which is then related to her initial survey. She reported that she does not see herself as being good at mathematics, saying, “if you were to ask me a specific math problem and I didn’t know about it, then I wouldn’t help you or answer you. I only know math if I’m being currently taught in it.” She did report that she has had peers tell her she is good at mathematics, but often she dismisses it saying that she just studied the material more. Her definition of being good at mathematics was that “if anybody comes up to you and asks you about [a math] question, you would be able to answer it, not just being in that course, and being able to get the answer correctly.” As reported in Table 4, Bailey had a high sense of self-efficacy and a medium sense of mathematic identity to indicate that when working on mathematics tasks she would likely be confident in being able to solve the problem. However, she saw herself as being neither especially good nor bad at mathematics in general, which aligns with her statements, given that the problems she is tasked with solving were from a current course. Given the data from the MSLQ presented in Table 5 we also see that Bailey’s scores indicated that she had a higher sense of extrinsic goal orientation, meaning that she is more often motivated by external factors rather than internal factors of success, but has a strong use of both types of goal orientation.

4.2.1.2 Precalculus: Toby

Toby was biology major and expected to take up to first-semester calculus. He took IB Mathematics online in high school, stating that he did well especially since the extra time due to the lack of extracurriculars during the COVID-19 pandemic provided more “time to read the textbook." He reported doing well in his high school mathematics courses but stated that he did not view himself as being particularly good at mathematics, finding that he often compared his mathematics ability to other students around him, which he reported as being overwhelming when he sees other students do well without seeing improvements in his own ability.

When looking at his perceptions of himself as a mathematics student, Toby did see himself as being very confident in his mathematics ability, reporting that he felt he could pass the course. Toby was the only participant who completed all the interviews but ended up dropping out of the course. This decision is later explained, but was primarily due to wanting a higher GPA, although he was on track to pass the course. As reported on Table 4 and Table 5 Toby had a medium score for both mathematics self-efficacy and mathematics identity. Toby's scores on the MSLQ indicated a stronger sense of extrinsic goal orientation rather than intrinsic, but still with a good use of intrinsic motivational factors. This high sense of extrinsic goal orientation aligned with his decision to drop out of the course to get a higher GPA at a later time.

4.2.1.3 Precalculus: Rose

Rose was majoring in mechanical engineering and reported that even though she took AP Calculus AB in high school, she was not "prepared enough… most of the problems I had [in high school] were a little bit less complex, less wordy than they are now, so I kind of feel a little behind." She stated that she ended up with a B overall in the course, adding that if she were to rate her own understanding on a scale of one to ten it would be a three. She further stated that she "never understood what was going on… all the tests that we took, I would fail but since [the teacher] added a curve it was passing." Rose stated that she expects to take up to second semester calculus to meet her degree requirements. Rose reporting that she was "fairly confident" that she would pass the course, aiming for a B in the course overall.

While discussing her self-perceptions as a mathematics student, she stated that she viewed herself as being good at mathematics which was supported by parents and primary/secondary teachers. Based on the previous two tables (Table 4 and Table 5) we see that Rose had a medium-high score in both mathematics self-efficacy and mathematics identity. This would indicate that Rose felt confident in her mathematics ability and viewed herself as being relatively good at mathematics, which is consistent with her self-reported confidence during the interviews. Of the three precalculus participants interviewed, Rose is the only participant whose intrinsic goal orientation was higher than that of her extrinsic goal orientation, which indicates that she is more often motivated by internal factors such as mastery of a topic, but is still at times motivated by external factors like a letter grade.

4.2.1.4 Calculus: Aria

Moving to the six calculus participants' backgrounds we start with Aria. Aria, a civil engineering major, reported taking AP Calculus AB in high school, scoring an A in the course. She found that doing examples and memorizing formulas helped her be successful in her high school course, and that her exam preparation for high school included going over a test review or notes if no test review was given. Aria reported that she would need the full sequence of three semesters of calculus for her major. Aria felt confident in passing the course but was not as confident of earning an A or a high B due to a negative experience on a chemistry test and receiving a lower-than-expected grade, claiming that calculus will likely be similar.

When asked whether she views herself as a confident mathematics learner, she said that she is confident given there is someone to teach her mathematics, adding that she is not sure she could learn it on her own. She does see herself as being good at mathematics, stating that she was the student her peers would come to with questions, and she saw herself as better at mathematics than most of the students in her high school courses. Based on Table 4 and Table 5, we see that Aria, like Toby and several others, had medium scores in both mathematics self-efficacy and mathematics identity, which seems lower than her self-reported confidence and view of her mathematics identity. Her scores on the MSLQ indicated a higher sense of extrinsic goal orientation, but still employed intrinsic goal orientation at times.

4.2.1.5 Calculus: Cyndy

Cyndy a biology major, took precalculus in high school, but said that due to the online setting she "didn't understand as much as I would have liked to, but I did come out with a B in that class." She had help from peers in her high school course who had already taken the course, current classmates, as well as online resources that helped her be successful in the course. As a biology major, Cyndy would be done with her mathematics course requirements after completing the first-semester calculus course. Cyndy felt that she was "definitely confident enough" to pass the course and expected a high B or a low A in the course overall.

Although she had a strong sense of reported self-efficacy, she said that she "absolutely [did] not" see herself as being good at mathematics, continuing saying that "everybody says I'm pretty decent at math, but I'm always having to look for like 'how did you get this answer? How did you get this part of the question?'… at first glance if I look at a question, I would not know how to figure it out." Examining the earlier tables (Table 4 and Table 5) we see that Cyndy is one of the several participants with medium senses of both mathematics self-efficacy and mathematics identity. We also see the largest difference of the interviewed participants between her extrinsic goal orientation and intrinsic goal orientation, with her intrinsic goal orientation being the lowest of the nine interviewed participants. This indicates that she rarely uses internal goals to motivate herself to accomplish a task, relying on external factors to keep herself motivated.

4.2.1.6 Calculus: Frank

Frank, a computer engineering major, reported he felt his AP Calculus AB course adequately prepared him for college stating that "since I've already taken calculus one, and I'm in calculus one now in college, it's most likely going to be review." He reported passing the high school course, but not passing the AP exam with a high enough score to get the college credit. Frank was not sure what his highest level of mathematics course would be at the time of the interviews, but he did expect to complete the calculus sequence. Frank felt confident that he would end up passing the course but acknowledged that he would need to “study hard” to earn an A or a B in the course.

Frank saw himself as being good at mathematics, citing the fact that he got good grades in mathematics courses as a primary indicator. Based on Table 4 and Table 5, we see that Frank was among the group to have both medium scores in mathematics self-efficacy and mathematics identity. Frank’s intrinsic and extrinsic goal orientations were not significantly different, with intrinsic goal orientation just 0.25 points higher, which indicates that he employs internal and external factors to motivate himself almost equally, with his statements of viewing grades as an indicator of being good at mathematics being a strong external motivating factor.

4.2.1.7 Calculus: Jane

Jane, an architectural engineering major, who took calculus in high school. She reported that she performed “fairly well” in her high school course due to being “able to do my homework with a pretty good understanding of what I was doing and why it was working, and I was able to do decently well on tests.” Jane attended a STEM academy which had a higher emphasis on showing “real world examples… [and working on] a lot of projects and presentations” to better understand the mathematics topics. At the time of the interviews Jane was not sure what the highest mathematics course she would be taking but felt confident she would at least go through the calculus sequence. Jane reported feeling “pretty confident” that she would pass the course and earn an A or a B in the course. Jane stated that since she took calculus during the COVID-19 pandemic it required her to be more independent in her studying even though it “was a challenge at first.”

When asked about her own perceptions as a mathematics student, Jane had a high sense of self-efficacy in mathematics and viewed herself as being good at mathematics, citing that mathematics “just seems to sort of make sense to me, and it’s just always seemed like a very logical thought process that I’ve enjoyed, and I tend to generally understand.” Examining Table 4 and Table 5 we see that Jane was the only interviewed participant with both her mathematics self-efficacy and mathematics identity scores being labeled as high, indicated that she had a strong sense of confidence in her ability to solve problems and in her own perception as a mathematics student, aligning well with her own perceptions of her ability as reported in the interview. Jane had high scores in both intrinsic and extrinsic goal orientation, with extrinsic being a bit higher, indicating a higher use of external motivating factors.

4.2.1.8 Calculus: Jay

Jay was a computer science major. She completed trigonometry in high school, in which she received a B. In that course she believed that “reviewing constantly and using flash cards for definitions and formulas” helped her be successful in mathematics. Like Jane and several others, Jay was not sure of the highest level of mathematics course she will be taking but does expect to go through the calculus sequence. Jay reported feeling “pretty confident” that she would pass the course, and “not completely confident” of being able to get an A or a B in the course.

Jay viewed herself as being good at mathematics, saying that what makes her good at mathematics was that “math doesn’t necessarily change, like you do have different concepts, but it’s pretty much, you know, a formula, you plug in the things and then you get the answer. I personally don’t find math difficult. It's just a lot to remember in terms of formulas, and you know rules and stuff like that.” Looking at the data from Table 4 and Table 5 we can see that Jay had a medium-high score for mathematics self-efficacy and a medium score for mathematics identity, indicating that she was fairly confident in her ability to complete mathematics tasks, but did not strongly consider herself as being a mathematics minded person. We also see from her MSLQ sub-scores that she is more extrinsically oriented for her goals, but she does employ intrinsic goals at times.

4.2.1.9 Calculus: Sunny

Sunny, another computer science major, reported that she took AP Calculus BC in high school, but she did not feel prepared for college mathematics, giving this feeling as a reason to take a first-semester calculus course in college to “go back and really understand what’s happening” regarding various topics presented in calculus. Sunny did not believe she mastered the concepts in high school partially due to the online nature of her course due to the COVID-19 pandemic. While in high school, Sunny would make sure to take notes during the lectures or recorded lectures, since “if I didn’t take notes, I wouldn’t pay attention.” Just like Jane and Jay before her, Sunny expected to go through the calculus sequence, but was not sure of the highest level of mathematics course she would take. She did report that she is considering getting a minor in mathematics so she does expect to take further mathematics courses past the calculus sequence. Sunny felt confident about being able to pass her current calculus course, and was fairly confident of earning an A or a B.

Sunny viewed herself as being “relatively good” at mathematics. She felt stronger when she was younger but now she is “flattening the curve” due to the amount of study she used to do over the summer but has since stopped. She stated that a past teacher did not think she was good enough at mathematics to skip a mathematics course in elementary, which decreased her confidence. Based on the information in Table 4 and Table 5 we see that Sunny’s scores indicated a medium-high sense of mathematics self-efficacy and mathematics identity. She is the only interviewed participant who had equal scores for intrinsic and extrinsic goal orientation, indicating that she uses both strategies approximately equally, showing a strong use of intrinsic goal orientation in her decision to take a first-semester calculus course to better her own understanding.

4.2.2 Calculus and Precalculus Individual Interview Findings

With the profiles of these participants in mind, we move into the discussions that arose during the two individual interviews and part of the second group interview. Each of the individual interviews focused on participants’ own study habits and course expectations, with questions specifically about any changes made to study habits and why those changes may have occurred. During the interviews, two common themes arose as to why students might have adjusted their use of self-regulation strategies, content challenges related to learning the material presented in the course, and structural differences between their current course and their high school course. In the following sections, I outline the findings from each of the individual interviews for both calculus and precalculus and present findings related to the use of specific self-regulation strategies that students used to address transitional challenges. I then discuss the specific content challenges and structural differences that arose in those interviews.

4.2.2.1 Initial Individual Interview for Calculus and Precalculus

In the first set of individual interviews, participants provided information about how well their high school mathematics prepared them for college-level mathematics courses, how they studied for high school mathematics tests, and how they intend to study for college mathematics tests. In this following section, I present findings from the first set of individual interviews for both calculus and precalculus where participants discuss their experiences from high school and any perceived differences or challenges they faced while transitioning into college-level mathematics.

During this first interview, participants discussed how they felt their high school mathematics course prepared them for the college mathematics course they were enrolled in during their first semester of college. At the time of the first interview, Toby, Aria, Frank, Jane, and Jay all felt that they had adequate preparation from high school mathematics for what they expected to occur in college mathematics. Toby said that "I think that it [high school mathematics course] did prepare [me], just because it still gave a basic foundation, even if it wasn't at this level… I've had to work a lot harder to teach myself this [college level mathematics]." Jay expressed that having to work independently "taught me… some better ways to just sort of figure it out myself, work through it myself, ask for help from other people." Jane also reported that at the STEM academy she regularly wrote papers in her calculus course about various calculus topics which allowed her to review topics and refresh her understanding.

While several participants felt prepared for the content after finishing their high school courses, Rose, Cyndy, and Sunny reported feeling underprepared as seen above in section 4.2.1. Clarifying this feeling, Rose reported that in high school when she learned about special right triangles she was taught to memorize the unit circle, but that she stayed “away from the triangle because I didn’t really know how it worked.” She then emphasized that in her college course she felt the need to better understand the concept at a deeper level, so she sought additional help from the instructor. Rose reported feeling supported by instructors in her early undergraduate courses, saying that the office hours and having graduate teaching assistants as well as access to tutoring aided in her confidence in her mathematics ability. This appeared in contrast to her report of a high school course where she said that the teacher primarily focused on helping the “top students.” For Sunny, the act of going to a first-semester calculus course was to help her prepare for college mathematics course, as she intended to better understand the concepts. She also recognized that there would be a need to have a more in-depth knowledge of the topics. Both Sunny and Cyndy felt that had their high school courses been in person they would have had a better understanding of the topics, but it was primarily a structural difficulty that prevented them from fully understanding the material presented. Rose, Cyndy, and Sunny all were optimistic about the experiences in their college courses so far, Rose pointed out that there is additional assistance, while Cyndy and Sunny appreciated the in-person classes.

Interviewed participants provided descriptions of their study habits in high school mathematics courses, which informs us of the types of self-regulation strategies students would use in high school. Several participants reported that the extent of their studying was to do the test review packet that was given by the teacher. This included Toby and Sunny. Both Toby and Sunny had a similar view of noting that the reviews that were given before a test were, in Sunny’s words, “just the test with different numbers.” Aria, Cyndy, Rose, and Jay each used test reviews and added some other element of studying as well. Aria looked over notes that she had written in lecture. Cyndy added that she worked with peers, especially those classmates she knew who had already taken the course. Rose described that she “understood the material, so the night before [a test] I would do the review sheets.” Rose added that she had a study group in high school and would at times attend tutoring. Aside from the reviews, Jay added that she would make flash cards with definitions and formulas to memorize information. The use of these study habits would fall under the self-regulation strategy of self-evaluating one’s own understanding, as it involves a self-check of content knowledge. Similarly, Cyndy added onto her strategy usage by using the self-regulation strategy of seeking peer assistance. Jay’s habit of using flash cards could be identified with self-evaluating one’s own understanding or using Pintrich’s MSLQ categories it would be categorized as rehearsal, which involves the act of memorization through repetition. I categorize this as self-evaluation as Jay would base whether she was proficient in the material in her ability to correctly recall information, especially in terms of definitions and formula, that had been written on either side of a flash card.

The remaining three participants, Bailey, Frank, and Jane, did not mention using test reviews in high school; rather, they each had a different approach. Bailey described that she studied for mathematics tests in high school, saying that since the tests *were not cumulative* that by doing homework she was generally prepared enough for the test, with little additional studying needed. This, like the test reviews, falls under self-evaluating one’s own understanding. Frank, on the other hand, stated that his primary strategy to be successful in high school was reviewing past material through looking at notes. Frank then explained that his study habits when reviewing notes were to focus on memorizing formulas because “you don’t need to remember stuff except for the formulas, so I usually just hone in on [those].” This process would involve the self-regulation strategies of organizing and transforming information into something usable, by taking the lecture notes and focusing on what he viewed as vital he engaged in this organization process. This process, like Jay’s earlier, is also an example of rehearsal, which as seen in Table 5, was one of the highest scores for Frank in terms of the self-regulation strategies listed. As in Jay's use of flash cards, I identified Frank's use of memorization to be an act of self-evaluation as it relates to determining whether he had adequate understanding of the topic. Lastly, when studying for tests in her high school, Jane would make her own study guide which included written notes of important ideas, including "any definitions, any theorems, and then I would also… write example problems and go through the textbook and find problems I hadn't already done and do some of those." Jane did report studying with peers and talking to teachers when there was "anything I didn't quite understand." Jane demonstrated a strong use of the self-regulation strategy of organizing information and transforming it into something useful as well as the strategy of seeking peer assistance. She also demonstrated a use of the emergent code, examples and counterexamples, by identifying example problems and working through them to have a portfolio of examples to consider when looking at novel problems.

The self-regulation strategies reported as being used for exam preparation in a high school setting are shown in Table 6. Note that there were three distinct ways participants' study habits were classified involving self-evaluating their understanding (SEU). For the Homework subcategory that indicates that there were no strategies reported outside of working homework problems. For the Memorize category this could include several methods to attempt to memorize information, including reviewing notes and making flash cards.

Table 6: Self-Regulation Strategies for High School Mathematics Exam Preparation

Participant	SEU: Using Test Review	SEU: Memorize	SEU: Homework	SPA	STA: Tutoring	OT
Aria	X					
Cyndy	X			X		
Frank		X				X
Jane		X		X		X
Jay	X	X				
Sunny	X					
Bailey			X			
Rose	X			X	X	
Toby	X					

To contrast those self-regulation strategies used to study for high school mathematics tests, participants also discussed how they intended to study for their first midterm, which shows the first possible change of self-regulation strategies to adapt to college mathematics course content. Several participants believed that their strategies used in high school were sufficient and would continue to use them for the first midterm. This included Frank, Jane, and Jay, who each reported that they would start by studying for the first midterm using a similar strategy as they used in high school. Frank said he expected to spend less time studying mathematics in college from high school, going from an hour a day to less than an hour a day “since I've already taken calculus one… it’s most likely going to be a review.” Frank added that if his studying was not sufficient he would start to work and study with a peer group. Jane added briefly that she would work more example problems but would continue going through notes and self-evaluating her understanding and organizing information. Jay clarified that instead of using flash cards to memorize and review material, she would focus on reading the textbook to review formulas. This does present a small change in self-regulation strategies from self-evaluation to seeking external resources, which includes textbooks.

Each of the remaining six participants reported that they would need to adjust their study habits, with the recognition that what they did in high school would not be sufficient for college. First, I will present those participants who used a similar or exact strategy from high school, but adopted new strategies, then I present those participants who dropped the strategies used in high school and attempted a new way of studying altogether.

Aria, Rose, and Sunny each found that while their high school study habits were useful, they needed to add more studying strategies to adapt to a perceived difficulty, primarily arising from content or structural challenges. Aria reported that she would add in working through problems found in the textbook, citing the different structure and fast pace of the course as being a key factor in why she felt the need to add this strategy of seeking external information. Rose recognized that her high school study habits “weren’t the greatest” so she “won’t be relying on those for my college classes" but reported she would continue to attend tutoring, but at a higher frequency. The two tutoring services she planned to attend were offered through a university program as well as in a mathematics clinic supported by the university mathematics department. Additionally, she reported forming a study group of peers. While her strategy usage did not change significantly, Rose reported spending more time on studying through these tutoring and group study strategies, thus engaging in an emergent self-regulation strategy of setting a routine to better focus her study habits. Sunny believed that just working through the review would not be enough, but would continue using that strategy of self-evaluating, adding that as she goes through the review she would "stop if I find that I had an issue doing one of them [then] I would go back and reread my notes or re-watch a lecture." She also reported that she expected to spend more time in college studying, going from four to five hours each week to five to six hours in college. Sunny included the use of self-evaluating understanding while adding seeking external resources through recorded videos and organizing information and transforming it into something usable as she would reread her notes to study as well as setting a more rigorous routine for her study habits.

The remaining three participants, Bailey, Cyndy, and Toby, felt that their previous strategies were either insufficient or would not be available to use and thus developed new strategies to prepare for exams. Bailey reported not using the same strategies as in high school; rather, she primarily focused on looking through her notes to study shortly before the exam. Bailey's first individual interview occurred shortly after she took the first midterm, but before she got her graded exam back, due to scheduling conflicts. Due to this, Bailey was able to reflect that she did not find her study strategy to be successful. Also contributing to this sense of feeling unsuccessful for the first midterm, Bailey reported that she only spent about 20 minutes every other day working on mathematics homework, as opposed to the three hours a day that she would spend in high school, stating that a reason for this was that she didn't "really like the online homework problems." Cyndy said that due to the larger class size, and less individualized attention from the instructor, she would be more self-motivated to look for online practice problems to assist in her studying for exams, contrasting her high school experience of the teacher providing reviews for the students. Cyndy also expected that she would attend office hours to help her prepare for the exams. This demonstrated her use of seeking external resources as well as seeking teacher assistance. Toby was not sure if review packets would be given out in his precalculus class, so he reported that he would rely on reading the textbook and looking over the lecture notes he took, changing his strategy from primarily self-evaluating to seeking external sources for information and organizing information and transforming it into something usable.

The intended self-regulation strategies that were identified to be used by the participants for their college mathematics exams based on their reports are listed in Table 7. Comparing Table 6 and Table 7 we see that participants planned on using a review for the test, seeking peer and teacher assistance from high school to university mathematics. None of the participants reported that homework alone would be sufficient for studying for college exams, unlike high school. There was also a higher frequency of planning to seek external sources, organize information and transform it, and add time to studying through developing a routine.

Table 7: Self-Regulation Strategies Intended for College Mathematics Exam Preparation

Participants	SEU: Using Test Review	SEU: Memorize	SPA	STA: Tutoring	STA: Office Hours	OT	SE	Routine
Aria	X						X	
Cyndy					X		X	
Frank		X	X			X		
Jane		X	X			X		
Jay	X	X					X	
Sunny	X					X	X	X
Bailey		X						
Rose	X		X	X				X
Toby						X	X	

4.2.2.2 Final Individual Interview for Calculus and Precalculus

In the final set of individual interviews, participants were given time to reflect on their experiences during their first semester of college mathematics, with interviews occurring close to the end of the semester. Participants discuss any changes in study habits especially in relation to how they studied for the first two exams and planned to study for the final, confidence of their mathematics ability, or perceptions of themselves as mathematics students as they relate to the experiences of their first semester in college. I present the findings as they relate to these experiences in the following section to provide information on how students may have changed during their first semester in college mathematics.

While not every interviewed participant reported specific changes to their self-regulation strategies, those who did often discussed what caused the change and the efficacy of their changes. After taking the first midterm several participants changed their study habits, primarily due to receiving a lower-than-expected grade on a midterm, and then discuss their plans to adjust to attempt to do better on the second midterm. Aria, Frank, and Sunny each reported consistent strategy usage from what they intended for the first midterm, as mentioned above, for what they used on the first two midterms.

Each of the other six participants reported some change between how they intended to study for the first midterm and how they studied for the second midterm. Bailey founded that she needed to spend more time studying, specifying that she set a routine of working on homework daily and looking through notes her instructor uploaded to the online course management system. Bailey stated that this change came after she recognized she didn’t spend as much time as she needed to perform well on the first midterm. She came to this recognition shortly before taking the first midterm, and then identified some strategies she believed would help her. She later specified she went from about 30-minutes every other day studying to an hour or two each day after getting the lower-than-expected first midterm grade. Bailey did state she tried attending the on campus tutoring a couple times but did not find it especially helpful saying that when the tutors would attempt to answer her questions that she "felt like they didn't know [how to solve the problem]." As seen in Table 17 which is presented with the post-labs surveys in 4.3 Calculus Post-Lab Surveys, we do see an increase to Bailey's test scores since the addition of her self-regulation strategies of creating a routine and seeking external resources with instructor course notes, but found seeking peer assistance from on-campus tutoring to be unhelpful to her learning.

Cyndy felt similarly to Bailey in that she adjusted her study habits after her grade on the first midterm was not "as high of a grade as I had hoped." Along with her intended study strategies mentioned above for the first midterm, Cyndy reviewed the collaborative lab activity problems (see Figures 3-20 for examples in 3.2.1 Precalculus Group Interview Activities and 3.2.2 Calculus Group Interview Activities). Then after getting her grade back, she resolved to improve her strategy and she acquired old exams to practice from the instructor of the course as well as peers who had taken the course previously. With this addition of self-evaluating her own understanding through the use of practice tests she reported getting a score "a lot higher than I anticipated."

As with Bailey and Cyndy, Jane reported a change of study habits based on her first exam score. She realized that "my overall grade can fluctuate [easily] based on how well I do on exams." The first midterm for this course was weighed as 20% of the course grade, with the later exams weighed more heavily, 25% for the second midterm and 35% for the final exam. With this recognition, Jane spent additional time studying for the second midterm and worked through additional practice exams to prepare. She found it especially useful to keep careful records in her notes by putting stars next to concepts or topics that she is not fully sure of her understanding, so she could focus on reviewing those topics. She did report that she learned several of these study habits in high school but didn't feel the need to use them for her high school mathematics courses "because I did not usually study… so I've just been applying them as I think would best benefit me." Jane ended up spending less time per week than initially expected, reporting spending four hours outside of class each week studying instead of the two to three hours a day she stated in her first interview, with a slight increase during an exam week. For Jane, it was primarily an increased use of the self-regulation strategy of setting a routine and keeping records and monitoring her own progress in the course that worked for her. She also demonstrated use of setting performance goals as she attended to her scores on each exam and had a goal to improve her exam performance.

While not mentioning test scores like the previous three participants, Jay found that she adjusted her use of self-regulation strategies as well. She felt that the course was getting more difficult and she “wasn’t doing as well as [she was] predicting.” To adjust to this challenge with not understanding the content, Jay spent more time working through the textbook, working on practice tests provided by the course instructor, as well as use online resources like Khan Academy or video explanations. Jay spent slightly less time than expected studying each week. Instead of the three to four hours each day, she reported spending anywhere between seven to ten hours a week. Like several other participants, Jay recognized that she needed to spend more time on studying for her mathematics course. To that end she adapted her self-regulation strategies of self-evaluating her own understanding; instead of using flash cards to study, she used practice tests that were given by the instructor of the course.

Rose discussed a minimal change to her self-regulation strategy, focusing more on how she might better the strategies she already used, rather than adopting completing new techniques to study. Rose indicated that she felt good about her first midterm score, but “definitely want[ed] to get a higher grade” for the second midterm, which is why she added to her study habits. Rose changed her environment by going to the university library to study the day of the exam, sought assistance by attending tutoring before exams, and self-evaluated by working through review and practice tests that were given to the class. She found this strategy helpful, as seen by her stating that she got a “pretty good grade” for the first midterm and stated that she plans on continuing this strategy in future mathematics courses but added that she would be attending tutoring more regularly to get a higher grade. Rose planned to at least double her time spent studying and working on homework in her mathematics classes from two hours in high school to four hours in undergraduate courses. To improve her study habits, she said that she would work on homework daily with more consistently and establish a better routine for study. Rose reported that her study habits were self-developed, stating that in high school she could “take the B or… C or whatever grade” but that these college level courses were “fundamental classes that I need to take, so I actually have to work hard to get a good grade in this class, because if I don’t understand it now, I’m not going to understand it later.” This demonstrated a use of the self-regulation strategy of setting mastery goals, as she attempted to master the material.

Toby, like Jay, felt a need to change his self-regulation strategies due to the overall difficulty of the course and new concepts being learned. He would review his lecture notes shortly after the lecture occurred and that if he came “across a concept [he] was not familiar with… [he would] imagine that there’s someone else that [he’s] teaching it to.” This was done to self-evaluate his understanding to then determine if he needed to further study a concept by utilizing the textbook or an online resource. His strategy for approaching the online homework assigned in the course was to attempt it without reviewing the notes, but if the material seemed too difficult, he would review notes for half an hour and then reattempt the homework, spending roughly an hour working on each assignment. Later in the interview Toby claimed that his reliance on using notes while working on homework problems was becoming unhelpful as looking at the notes is “not going to work on the exam.” While several participants discuss the efficacy of a strategy at some point, Toby explicitly discussed how a strategy he often relied on, using notes for homework, was hurting his understanding, thus demonstrating the emergent self-regulation strategy of metacognition, the act of discussing or considering how a strategy may be beneficial or harmful to one’s own learning.

Three participants also spoke of their intention to change or add some self-regulation strategy it to improve their overall course grade and score for the final. Aria and Jay both added that they intended to look through notes and spend more time studying and working through practice exams. Cyndy planned to re-watch certain recorded lectures to prepare for the final, adding that she is adding more study habits for the final due to the “pressure of taking finals.” The other six participants reported that they would continue using the strategy they developed for the second midterm, as the majority saw improvements in how they studied.

Table 8 below shows my classification of self-regulation strategies based on participants’ reports of study habits for the first two midterms and their plans for the final exam. As shown in the table we see that each participant employed some facet of self-evaluating their understanding, though this could take several forms. We also see an increase of overall strategy usage from their high school strategy usage as shown in Table 6. Most participants reported increasing time spent studying or improving the weekly study routines.

Table 8: Self-Regulation Strategies for College Mathematics Exam Preparation

Participants	SEU: Using Test Review	SEU: Memorize	SEU: Explaining	SPA	STA: Tutoring	STA: Office Hours	OT	SE	Routine	RM	PG	ENV	MG
Aria	X							X	X				
Cyndy	X					X		X					
Frank		X		X			X						
Jane		X		X			X		X	X	X		
Jay	X							X	X		X		
Sunny	X						X	X	X				
Bailey		X			X			X	X				
Rose	X			X	X				X			X	X
Toby			X				X	X	X				

Along with a change in some participants' self-regulation strategies that manifested through their study habits, several participants also reported a change in their overall confidence in mathematics after having gone through a semester of college mathematics. The primary reasons found for a change in confidence were identified by participants as being related to specific content knowledge and changes in the structure of the course, with some participants mentioning both factors. Cyndy, Frank, Jane, Jay, Rose, Sunny, and Toby found that their confidence shifted due to the grades or understanding of the material. Aria, Bailey, Cyndy, Jay, and Sunny reported that the structure of the course and especially of the lab activities were what affected their overall confidence in the course.

Cyndy, Jay, and Toby each reported that their confidence in their ability to do mathematics tasks was lower than when beginning the course. Cyndy felt confident of getting "a good grade on the final" but when asked how she felt with her confidence in a mathematics class in general she said her "confidence level is pretty low on the scale, I wasn't great at math to begin with, so coming into this class I knew I was gonna have to take a little bit extra time to be able to work problems." This aligns with Cyndy's earlier statements of not seeing herself as being particularly good at mathematics in general but having a higher sense of confidence in her performance. Jay found that it was the exams, and the stress that exams bring, that lowered her confidence the most, as well as the fast-paced nature of the course. She added that, "it's not necessarily a bad thing, 'cause I did learn from [the course] I do think I would probably be maybe 50 to 60% sure that I would pass now." This lowering of confidence was reported to lead to the realization that she would need to self-evaluate her understanding and review her notes more frequently. Toby also reported his confidence as being lowered, generalizing it to "not having done well in precalculus." This lowering of confidence was also affected by having to drop the course due to not being able to get the high-grade Toby wanted to keep his GPA up.

Frank, Jane, Rose, and Sunny each reported ending the semester with more confidence than when they started. Frank reported that his confidence relied on how easily he can learn a subject “and apply it, if I recently learned it.” He did not report any times where his confidence decreased during the semester. While Jane still felt confident about passing the course, she was not sure what her overall course grade would be but said that she felt” like I understood the material well.” When asked if her overall confidence in the course had changed since her first interview Jane reported that

> I actually feel slightly more confident, which is funny, ‘cause I have a lower grade than I used to have, but I feel more confident in my ability to do mathematics now because I did have a lower grade and so I was really putting in a lot of effort and really working hard to sort of raise that… that’s not something I’ve really ever had to do before, it’s all been really easy to me naturally, and so I feel confident that if I had to, you know, put the nail to the grindstone, I’d be able to understand concepts that maybe I struggle with.

Rose found that her confidence of being able to pass the course rose and fell during the semester but ended being confident that she would earn a high B in the course. But in general, she found that she was “very confident” in her understanding of the course material and believed it to be a “good foundation” for future mathematics courses. Similar to Rose, Sunny reported her confidence in mathematics was higher saying that “I understand more of the way things work and if I see something I can actually derive it instead of just having to memorize it.”

The other primary factor that participants discussed that affected their overall confidence was the structure of their collaborative lab activities. Two participants reported the lab activities as being detrimental to her confidence, Bailey and Cyndy, while the other three participants who discussed structural factors found their conditions to be helpful in growing their confidence. While confident that she would pass the course, Bailey felt that the collaborative lab activities negatively affected her confidence and that working in a group “detracted from my learning” due to a lack of contribution from other group members. She reported at that some point in the middle of the semester her group changed after she spoke to the instructor to request a new group. Likewise, Cyndy shared that doing the collaborative lab activities lowered her confidence as it felt “a little like an unnecessary approach” that took up her time but did not seem to help her answer homework questions. Cyndy was in a calculus section that required the group lab activities to be turned in to the instructor during the 50-minute class period, she also reported that if she were given more time that “getting more time to look at the problems and referring to other examples would definitely help me.” Although, she also said having more time alone would not have positively changed her overall confidence in mathematics.

Aria, Jay, and Sunny each discussed how the lab structure aided in their understanding of the material and confidence in the course. All three participants started in a class that had them submit the collaborative lab activity during the 50-minute class period. Shortly after the first midterm, the structure of the course was changed to allow up to a week for students to work outside of class collaboratively on the lab problems. Sunny reported this extension of time “made me a little bit more confident in my labs ‘cause I had more time to work on it, and then I felt like doing them actually helped me understand the content a little bit more than just rushing it in one class.” Similarly, Aria reported that this change in when the lab activity was due “helped more because now I don’t stress as much [about] getting it done, now I think about [whether] I truly understand what this is, what the concept is of the lab.” Jay echoed these earlier reports, saying that this change allowed her time to “review and go over our answers and make sure what we’re turning in is correct.” Participants were more able to use self-regulation strategies, including organizing information and transforming it into something useful and self-evaluating their understanding given the extra time.

While not as prevalent to changes in confidence, some participants reported on how they viewed themselves differently as mathematics students after having completed a semester of college mathematics. Like confidence, there were participants whose view of themselves as being good at mathematics was affected positively, negatively, or not at all. Both Jay and Toby said that they do not consider themselves to be good at mathematics, while at the time of their first interview they both considered themselves to be good at mathematics. Only one participant reported that she felt better in her perception of herself as a mathematics student, which was Sunny, who already said she felt she was good at mathematics, but added that she "feel[s] a lot better now" after taking this course. The remaining six participants did not report any changes in how they viewed themselves as mathematics students.

As mentioned, Jay and Toby felt they were good at mathematics, but they ended the semester claiming otherwise. Toby reported that he had dropped out of his precalculus course due to grade concerns after taking the second midterm. He reported that given his test performance the best he could get in the course was a B, which was not what he wanted. He decided to drop the course and instead take calculus at a local community college over the summer then transfer the credit to his undergraduate university. He said this was to aid in getting into graduate school where a high GPA would be needed, and he would only need the calculus credit for his biology degree. He felt that his overall confidence was lower than at the beginning of the semester since "all I saw was the relatively easy problems I saw in high school." But even though he experienced difficulty he did report that he could see himself improving in his mathematical ability given time. Jay elaborated on her statement of no longer finding herself good at mathematics, saying "I don't think I'm terrible at math, but I do think I could be better with practice, but I don't think right now the level is where I want to be." Jay also added by taking this course, even though she stated she would not likely pass the course, that she has realized "what I need to learn and what I need to review, even though I was going into it thinking I'll do fine… I think it's showing me that I have some things to work on." Both participants faced content challenges during the course which caused them to lose some confidence in themselves, but also recognized that there was the potential for growth in their mathematics ability.

Sunny found that her more positive self-perception of herself as a mathematics student was due to being able to review the material and "being able to see how things work… how the math works and then also being able to do well." Sunny also said that her she would compare herself to her peers and that “whenever I hear people say that they didn’t do well on a test [that] I did, that’s one of the things that made me feel a lot more confident this year.” Sunny was able to recognize that the material she was learning was difficult, but as she was performing well, it added to her mathematics identity as being a well-performing student.

The remaining participants did not report any changes to their mathematics identity. Jane found that in part this consistency of her mathematics identity was the support she received from past teachers and instructors, saying that she has “been lucky to always have amazing teachers and they’ve always sort of taught everyone that even if you don’t think you’re good at math, you probably are, it’s just kind of something you just get into a pattern of… you get better at it, you keep working at it… it’s just a way of thinking.”

4.2.3 Content Challenges

Having examined both the initial and final individual interviews for all nine precalculus and calculus participants, I report on common content challenges that were identified in the interviews. The most prevalent challenge that participants experienced was the increased difficulty of exams as compared to their high school mathematics courses. With each participant either changing or adding some self-regulation strategy to better adapt to the difficulty of the exams we see that this is a primary influencer on the development of self-regulation strategy usage.

The most common self-regulation strategies used to attempt to overcome this obstacle were self-evaluating one’s own understanding, creating or modifying a routine for study, and seeking external resources. Self-evaluation of understanding was commonly demonstrated with working through practice exams before a test as well as working out example problems presented in lecture or a textbook. This strategy was in part used by most participants, before the transition to high school and after, but its efficacy was increased by using multiple means of self-evaluation as well as spending more time self-evaluating. The means of self-evaluation was found to be significant, as more participants found that working through instructor provided practice tests was more beneficial than online practice problems. For the self-regulation strategy of routine, participants reported an increase of time spent each day working through homework or studying, as well as dedicating a time or space to study. This links closely to the self-regulation strategy of changing one’s environment but adds the dimension of setting aside specific time to study. Lastly, participants used external resources to further their understanding or clarify misconceptions they have had had in lecture or on homework. This involved re-watching recorded lectures, watching additional videos produced by other sources, and using textbooks along with lectures.

Another content challenge participants faced was the increased depth and complexity of course material being studied. Participants commonly reported an expectation that they would need to understand the course topics in more depth to perform well in the course. To adjust to this challenge participants relied on seeking assistance, from either peers or instructors, and through organizing information and transforming it into something usable. Seeking assistance included attending tutoring, often through a free on-campus tutoring service, forming study groups of peers, and attending office hours with an instructor. This allowed participants to get individualized help for concepts that were deemed too difficult to process on one’s own. The organization and transformation of information often involved reviewing notes and identifying areas where participants did not understand or had questions on.

The last content challenge participants faced was how to adjust to differences in course homework. Participants reported on the differences in how they worked on homework problems in high school and what changes they may have made to their use of strategies since entering a college mathematics course. First, I report on frequent strategies participants used in high school, and then I present the strategies used to adjust to the demands of a college course.

When completing homework tasks in high school, participants reported frequent use of using external resources and seeking assistance. Aria, Bailey, Cyndy, Rose, and Sunny each mentioned using external resources when getting stuck on homework tasks. For the participants this generally looked like searching for the topic online or using a textbook to further their understanding of a topic. For Bailey and Cyndy this strategy was used initially, and then followed using a different strategy if there was still confusion. Aria, Rose, and Sunny on the other hand reported seeking assistance from peers or tutors first and then using online resources if there was still confusion. Aria, Bailey, Cyndy, Rose, Jay, and Sunny all reported seeking peer or teacher assistance. Often this was given in a set order, with participants approaching peers first and then their teacher if there was still confusion.

When discussing how they went about college homework assignments participants reported some changes in how they approached the tasks. The frequent strategies used to address homework difficulty were continued use of seeking assistance and using external resources with the addition of setting a routine and the emergent self-regulation strategy of looked at a worked solution of a task. Those participants who sought assistance in high school continued to seek assistance in college, this now included Frank who worked with a roommate often. What changed for this strategy was the order in which help was sought. Participants reported commonly going to the instructor first and then reaching out to peers. Aria reported that for her this was due to the unfamiliarity she had with her peers that she did not have a set group to go to for posing questions. Several participants reported increased time each week studying and working on homework. This included Aria adding two to three hours each week on homework, and several participants reported needing to increase the time spent studying in general for their mathematics course. While Rose did not explicit report adding time she reported that having gone through her precalculus course taught her to “manage my time better” by “trying to work on [homework] problems ahead of time.” For both calculus and precalculus homework is given through an online system, which allowed several participants to use a feature of the online system which allowed them to see a similar problem and see explanations of how to work each step of the problem. Aria, Frank, Jane, Sunny, and Toby each reported using this feature regularly, and once they worked out one or two problems using this feature they generally felt confident to complete the remaining problems independently.

4.2.4 Structural Differences

While there were specific and general content issues that participants addressed, other factors that appeared throughout the interviews was noting how college courses may be structured differently than the high school courses or the professor of a college course had a different teaching style than a high school teacher. These differences in structure at times prompted participants to adjust their self-regulation strategies. The structural or course differences are highlighted below, and at times followed by how some participants changed their self-regulation strategy usage to adapt to perceived differences or expectations.

During the precalculus interviews each participant discussed some structural difference with either the types of mathematics tasks being presented or how lectures took place. Toby discussed how in his high school classes he would primarily work with multiple choice questions, but when given problems on the lab activities he was unfamiliar with some of the requirements, especially having to produce an accurate sketch by hand. Rose felt like Toby with what types of problems were required to be solved, saying that she may have seen similar problems to the labs as “problems that we saw as a class… but I don’t remember receiving a test or quiz or any homework that look like [the lab activities].” While Toby felt prepared for these types of differences because of the “basic foundation” of the concepts that he was given in high school, Rose reported that “everything [being] free response” was unfamiliar and she did not “feel like I was prepared in high school to do these types of questions.” Bailey when relating what she saw as being different emphasized the online nature of her high school mathematics courses, that occurred due to the COVID-19 pandemic. She found that all her homework was online so having more open-ended free response questions was “pretty unexpected coming to college.” Along with this, Bailey reported feeling “stranded” when working on the collaborative lab activities due to feeling like she had to “carry the group” since she was not sure how mathematically strong her other group members were. She also reported that working in groups was not a common practice in her high school, so she had to learn how to navigate working with peers on mathematics tasks.

This sentiment from Bailey also arose during half the calculus interviews. Cyndy, Sunny, and Jay all spoke about differences between the online nature of their high school course due to the COVID-19 pandemic and the in-person activities that were required in college. Jay reported that for her high school courses her teacher offered “a slightly guiding, like ‘I’ll tell you the basics that you need to know and then I’ll let you do it on your own.’” Contrasting this with her college mathematics course, she experienced instructors being “more hands on” but also holding students to be “responsible to do certain things on your own.” To adjust for this Jay reported using more external resources such as textbooks saying that the concepts are often “easier for me to understand with seeing it explained in a written form.” Sunny reported that lectures were a significant difference, as in her high school online mathematics course the lectures would be pre-recorded but she at times had difficulty paying attention saying that “if I didn’t take notes, I wouldn’t pay attention, so I had to make sure I wrote things down...” Later she reported that the pre-recorded lectures were “more difficult for me because I’d find myself not paying attention and be like I’ll just skip back a little bit so I can rewrite everything, or I just like make the video go faster.” Sunny reported that due to this difficulty she takes “better notes than I used to, I have a new strategy… of adding in things that the teacher will say” rather than writing “exactly what’s on the board.” Cyndy reported a similar perceived difference to Bailey in that most of her homework was online and did not require anything physically written out, while the work required in her college course often did. Cyndy also reported that due to this online format “you could find the answers online by using a secondary source.”

The remaining three calculus participants spoke of only slight changes in structure but did not view it as being an obstacle to their learning when differences occurred. Aria reported that she did not often get a chance to work in a group during her high school experience, comparing that to the collaborative lab activities where "we actually do have that time to talk about it [with the group members]." While Frank reported many of the structural components as being similar, his one reported difference was in how time had to be managed during the college course. He said that while in high school the teacher would "give me a strict timeline of things to do… but with college life… I have a little more time to fully understand concepts and adapt to them." Jane did not report any significant differences, saying that as she went to a STEM academy that it was structured similarly to what was anticipated in a college course, so she felt well prepared for the structure of a college course.

4.2.5 Self-Regulation Strategies used to Adapt to Transitional Challenges

At the end of the second group interview for both precalculus and calculus groups a list of self-regulation strategies as outlined by Zimmerman and Pons (1986) was shown to participants (see Appendix G

Group Interview Protocol #2) and each participant was asked to explain whether they used each of the self-regulation strategies, and if so, how in a mathematics course. Below I present participant's use of each self-regulation strategies when viewed generally, as compared to the self-regulation strategies cited above that were used to adapt to content or structural challenges and the self-regulation strategies discussed in the following section which looks at specific mathematic tasks. In the following sections I organize the information by the self-regulation strategy discussed and bring in commonalities and differences present in the findings. Appendix N

Participant Transcript Excerpts from Group Interview #2 has the exact transcripts of participants when discussing how they use the self-regulation strategies outlined by Zimmerman and Pons (1986).

4.2.5.1 Seek Information from an External Source (SE)

Students exhibiting this self-regulation strategy would demonstrate using resources, excluding from other people, outside of themselves to gain additional information about a subject. As participants discussed how they used this strategy, common themes arose including watching videos on specific topics from online sources such as YouTube or Khan Academy, using online resources like Desmos to visualize information, looking at worked solutions online, and using online resources such as Google and Paul's Online Math notes to find relevant formulas or written explanations. The prevalence of using online resources was common across all interviews, as it is an easily accessed resource for most participants, especially being familiar with learning from online resources during their high school courses during the COVID-19 pandemic. Participants' own written notes would not be included in this category as those would not be considered as an external source of information.

4.2.5.2 Keep Records and Monitor Progress (RM)

Students exhibiting this self-regulation strategy would demonstrate self-initiated actions to record their experiences and check on their progress in the course. This included the following common practices from interviewed participants: identifying questions or concepts that are not understood to later review, keeping records of quiz or assignment grades and reviewing problems based on low scores, keeping records of formulas and theorems used in the course, and monitoring overall grade average in the course to identify overall understanding in the course. This strategy was aided by online grade management systems which allowed participants to track their progress without self-calculating expected scores.

4.2.5.3 Organize Information and Transform it into Something Usable (OT)

Students exhibiting this self-regulation strategy would demonstrate self-initiated actions to take information they have collected and organize it in a way they can easily use it for a task. As participants discussed this strategy, the following themes arose: highlighting important information in notes, summarizing ideas found in notes, making flash cards with information, creating a study guide based on notes for an upcoming exam. The use of this strategy as discussed by participants is different than that which is identified later when going over problem specific strategies. Participants at times reference simply going over their notes to study, as this was vague as to how they used notes those instances were not coded as OT. This will be discussed in the following section of the self-regulation strategies used in lab activities.

4.2.5.4 Seek Teacher Assistance (STA)

Students exhibiting this self-regulation strategy would demonstrate seeking help from the teacher of the course. For those participants who do seek teacher assistance it was either during the collaborative lab period or during office hours, with seeking assistance during the lab period being mentioned significantly more than attending office hours.

4.2.5.5 Seek Peer Assistance (SPA)

Students exhibiting this self-regulation strategy would demonstrate seeking help from peers who are taking or have recently taken a similar course. Similarly to STA, participants primarily mention whether they seek peer assistance. For those that regularly do, several mentioned that the randomly assigned lab groups were often the groups they worked with outside of class.

4.2.5.6 Seek Assistance from Adult (SAA)

Students exhibiting this self-regulation strategy would demonstrate seeking help from an adult who is not the instructor or a peer. Just as with the previous two strategies, participants discussed not how they used this strategy but whether they used it. Only two interviewed participants reported using this at times. Jane and Toby said that it would be their father they would ask for help. For a couple of the participants who reported not using this strategy, they explained that it was due to not having family members that would have the relevant knowledge to be helpful.

4.2.5.7 Self-Evaluate your own Understanding (SEU)

Students exhibiting this self-regulation strategy would demonstrate self-initiated evaluations of their work and the progress they have made. The themes identified as participants discussed their use of this strategy included reworking quizzes and assignments with low scores, identify what concepts are not fully understood and working through practice exams before a course exam to review material. A primary feature of this strategy is that it does not rely on input from others but can be done individually.

4.2.5.8 Provide Goals for yourself based on Performance (PG)

Students exhibiting this self-regulation strategy would demonstrate setting goals based on how well they perform in the course or on course tasks and how to accomplish those goals. For the participants who used this strategy they reported setting a certain grade as a goal, setting subgoals for attaining grades on specific exams or assignments, setting goals for amount of time spent studying in a week and setting a goal to improve exam grade by a certain percentage from the previous exam.

4.2.5.9 Provide Extrinsic Rewards (PR)

Students exhibiting this self-regulation strategy would demonstrate providing rewards based on accomplishing some task. Some examples of extrinsic rewards provided by participants were times of rest, time spent with friends, and eating some favorite food or going out to eat at a restaurant after some task is complete.

4.2.5.10 Remind yourself of the Value of the Task to your Learning (Value)

Students exhibiting this self-regulation strategy would demonstrate would relate the course or task to some valuable aspect of their life. Ways in which the interviewed participants reminded themselves of the value of the course or task included considering how the course is needed for their degree plan or future courses and thinking of the task as an obstacle that needs to be overcome to continue with career or future plans. One participant discussed how they reminded themselves that calculus did not have value or use for her future career, which aided her in not becoming stressed about low grades.

4.2.5.11 Provide Goals for yourself based on Mastering the Concepts (MG)

Students exhibiting this self-regulation strategy would demonstrate setting goals based on how well they understand course material and how to accomplish those goals. Like the performance goals, participants described the following theme of identifying content areas that need improvement and working on problems related to those areas. This strategy was infrequently used, with Bailey, Rose, Jane, and Sunny being the only interviewed participants to report using this strategy.

4.2.5.12 Change your Environment (ENV)

Students exhibiting this self-regulation strategy would demonstrate changing physical environment factors to better focus or attend to course objectives. This strategy was demonstrated by participants through setting a physical location to study such as the campus library and playing music to block out distractions. At times participants would discuss having multiple locations set to study, that way if they felt unmotivated or distracted in one location they could move to the other location to remotivate them.

4.2.6 Self-Regulation Strategies used during Lab Activities

During the task-based group interviews, participants examined a problem from a previously worked collaborative lab activity (see Figure 3-20). While working these problems, participants reported what techniques were used during the class period if they had gotten stuck at any point which are then linked to self-regulation strategies, and then participants were given the chance to work through the problems again, utilizing any such strategies during the interview. Below I outline how the interviewed participants responded to each of the selected problems from the lab activities for both precalculus and calculus. In the following sections, the use of self-regulation strategies is examined for more local problem contexts rather than the larger context of studying for exams or for a course in general as mentioned above. For the calculus activities, I provide quantitative results of how participants in two sections of calculus reacted to the labs in general as well as how they thought the lab workload and difficulty compared to that of a typical high school assignment. This is provided to give a more complete picture of ways in which participants respond to the collaborative lab activities using self-regulation strategies.

4.2.6.1 Precalculus Group Interviews: Labs 1-3

The first precalculus group only included Rose and Toby as Bailey was unable to participate due to a scheduling conflict. In the first group interview both Rose and Toby reported that they had seen similar types of problems as presented in Figure 3-5 in their high school coursework, with Rose adding that the problems in high school may not have been as “wordy” as the lab activities they were presented in college.

While looking at the first lab activity which focused on setting up linear relationships and solving linear equations, Rose and Toby both explain how they worked as a group in their original lab setting. Toby stated that his group worked primarily independently and then they would review the answer after each problem as a group. Rose said her group worked similarly, adding that they would ask questions of the group as they came up while working. Both found this strategy of seeking peer assistance to be successful, explaining that this still requires each group member to do each problem, but having the added support of their peers. Toby then went through his process of what he would do to get unstuck given some difficulty. He said that he would first ask his group for help, and if that did not work the group would “sit in silence for a few seconds trying to figure it out and then either move on to a different question” or eventually ask the instructor for help if they were available, presenting a hierarchy seeking peer assistance, self-evaluating, and then seeking instructor assistance. Rose said her group had the same strategy but added that before going to the instructor they would attempt to draw some picture of the problem situation to “see if that would help,” thereby adding the strategy of organizing information and transforming it into something usable.

While discussing the specific problem in the first lab, Rose and Toby were observed to use several self-regulation strategies. To illustrate her use of organizing information and transforming it, Rose was asked what picture she might draw for the first lab problem (See Figure 3). She drew the following image, Figure 27, labeling the side of the square base as X and the height of the box as 3X.

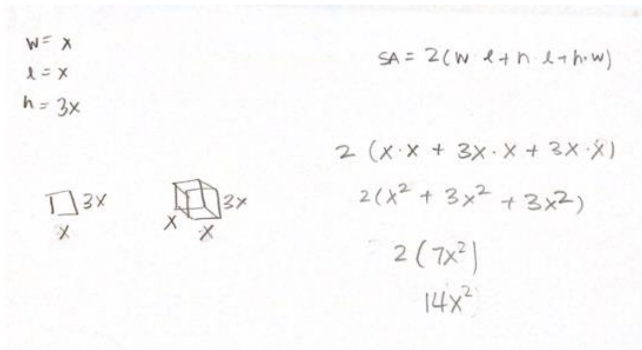


Figure 27: Rose’s Written Work for Lab 1 Problem #3

She was not sure how to then start on part (a) of the problem, which involved setting up an equation for surface area of the box. Upon Rose drawing up the sketch as shown in Figure 27, Toby mentioned that he often also tried to draw a sketch for problems like this to organize the information. He then stated that when looking for the surface area, before asking for help from the instructor, he would look up a formula for surface area using an external online resource. Rose and Toby then engaged in some discussion about what the formula for surface area could be as shown in the following transcript:

Rose: Well, the. Area is length times width. So, would the surface area just be length times width times height?

Toby: No, that’s volume OK. Something I believe it’s… I’m trying to...

Rose: We need the area of both the bottom square and the rectangle.

Toby: Well, at this point I would probably look it up on my phone. I wouldn’t want… Yeah, I would not ask the teacher in a real situation. Check for a formula if I can look it up.

Here Rose attempted to find the formula using what she already knew about area, while Toby attempted to rely on seeking external information. The primary strategies observed while participants worked the problem were seeking external resource, seeking peer assistance, and organizing information and transforming it into something usable.

For the second lab activity, which focused on function notation and operations, both Rose and Toby spoke about their experiences with this lab activity. Rose found that her group felt more comfortable working together on the problems, finding the familiarity of the group as being helpful for conversations to flow. Although when her group got stuck, she reported that they sat in silence until the instructor was able to assist them, still relying on seeking instructor assistance for these activities. Toby reported a similar outcome with his group being more open to talk about the methods being used to solve problems, and he said that his group members "were more OK with being wrong or asking for help from [other] group members."

When looking at the specific problem (see Figure 4) both participants worked through the problem and provided insight into what their self-regulation strategies were for the problem. Both Toby and Rose were quick to claim the statement in part (a) as true, with Toby appealing to an argument of when the situation would be false, saying "the only way that it would not be true is it was $f(\sqrt{x})$, because then you can't square root a negative… I believe it is true because any number can be squared," this demonstrated use of the emergent strategy of using examples or counterexamples to justify claims or provide further insight. Rose also agreed the statement was true but looked at the domain of the function being "like a parabola" than appeals to using an example to help justify her answer saying, "because I mean if you get a negative one in the square, it's just positive one." When looking at part (b) the participants at first believed the statement to be true, with a brief discussion Rose pointed out the extra element of the equation, g, which then caused some confusion. Toby then reported that if this were to happen in a lab he would ask his group members and then the lab instructor if needed. After some time to work out the problem carefully by computing each side of the equation, both participants eventually agreed it was false. For these problems where students had to identify if a claim was true or false, we see a similar strategy as above, seeking assistance, but see the use of an emergent self-regulation strategy of using examples or counterexamples to aid in the justification of their claim.

For the third lab, which focused on function composition and transformations, the participants reported on their experiences for the problem. This problem was more familiar to both participants as less time had passed between the participants initially working on this lab activity and the group interview. Both participants quickly agreed that in general, the order of transformations matters but had forgotten parts of how to do the actual transformations. Rose stated that for her group when they were working on this problem, they "picked a random point and went through the list as to which [transformation order] would be correct." This demonstrated use of the examples and counterexamples self-regulation strategy as she intended to use a few known quantities to then find a more general relationship. Toby said that he would "check myself along the way, I would use a graphing calculator to see what each equation looks like and then compare it to the correct graph." Here Toby used the self-regulation strategies of self-evaluation as well as organizing the information and transforming it, in this case into a graphical representation.

For these three problems Rose and Toby were identified to be using the following self-regulation as shown in Table 9. As a note for the following tables which show identified self-regulation strategies, the frequency of the strategy is not noted, only whether the strategy was employed at any time while discussing the specific problem presented.

Table 9: Self-Regulation Strategies Identified in First Group Interview – Precalculus

	Rose	Toby
Lab 1, problem #3	OT, MG	OT, SPA, SE, MG
Lab 2, problem #1	Examples/Counterexamples, STA	Examples/Counterexamples, SPA
Lab 3, problem #3	OT, Examples/Counterexamples	OT, SEU

4.2.6.2 Precalculus Group Interviews: Labs 4-6

In the second round of group interviews, all three precalculus participants were able to participate in the interviews and discuss their experiences working on these three lab activities.

Due to scheduling conflicts Rose participated on her own and Toby and Bailey participated together. The second group interview examined the lab activities given shortly after the first midterm, see Figures Figure 6-8 for the selected problems from each lab activity. When looking at each of the labs, Toby reported that Lab 4 (see Figure 6) was more familiar to what was done in high school because of the algebra focus. Toby also reported that he didn't remember inverses from high school, and only a small amount about logarithms. Bailey reported that she felt similarly to Toby but did remember inverse functions from high school. Rose reported that Lab 5 (see Figure 7) was more familiar to what was done in her high school. Rose also reported that she did not work the Lab 4 class activity, so discussion of that lab was omitted from her interview.

Bailey and Toby discussed how they approached problems in Lab 4 (see Figure 6) and their self-regulation strategies during the discussion were noted. For this lab activity Bailey reported that working as a group on each problem of the lab she was able to "look back in the notes or ask for help" from other group members when any of them got stuck, demonstrating the use of keeping records of learned information as well as seeking assistance from peers. Toby found that his previous strategy of working primarily independently, but asking questions as needed, was not helping him learn the concepts. Rather he said that he would not often go back and ask questions of his group if he "ended up getting [a problem] wrong" but would prefer if they "talked through it the whole way through and then someone would say something that may be missed." Toby had not yet implemented this new strategy in his labs at the time of the interview, but he intended to bring it up with his group sometime soon.

When looking at the specific problem for this lab, participants reported using the following techniques and methods to attempt to solve the problem. The first thing Bailey reported was that she was "bad at word problems." Similarly, Toby reported that "what's tripping me up is the 2 pounds." Both participants reported being confused by the wording of the problem, Bailey then reported that when this she tries "to break it up… but then I'm not really confident with inequalities, that's why I'm not sure where to start." Bailey attempted to make sense of the problem but was not confident enough with the topic under discussion to be able to accurately determine what the question was asking for. Bailey and Toby then reported on what strategies they might use when getting stuck on these types of problems. Bailey said that she would review notes or previously worked homework examples, while Toby said he would ask the lab instructor while in class.

After a brief discussion between Toby and Bailey about some examples of certain weights, Toby drew a number line with intervals marked for every two pounds, with appropriate prices over each interval, to organize the information. Further discussion arose over what happened when the package was exactly two pounds, which both participants reported confusion on, but felt confident about the price when the package was strictly less than two pounds and strictly greater than two pounds but less than four pounds. When asked to consider the problem statement of 6.4 pounds, Toby extended his sketch of the interval, adding $5.50 for each two-pound interval. After this discussion, with the addition of Toby's sketch, Bailey began defining a piecewise function, but was unsure of how to state the function given each domain. After further discussion trying to define the function, the interviewer asked if they were familiar with the integer floor function, as mentioned in the problem. Neither participant was confident of how the floor function worked. Before moving to the next problem, both Toby and Bailey reported that they often try to sketch some picture or diagram if they do not immediately understand the problem, and as shown above participants engaged in seeking peer assistance, and using examples to help generalize the problem.

The second problem for the group interview came from Lab 5 which focused on inverse functions. When looking specifically at the first problem, participants reported confusion, often trying to start with part (b), finding the inverse function, before part (a) finding $f^{-1}(7)$. Bailey reported that when looking at this problem that she was "confused on how it works." She provided a description of what her instructor used to help students visual the domain and range relationship between a function and its inverse saying there is “a set of domains and then a set of ranges… [and] he drew a picture [of this]… and what we got confused was I explained it as they want to know what the value of the range is when the domain is seven… but [the instructor] said that if you’re finding the inverse, they’re actually saying what’s the domain when the range is seven.” Bailey in her statement realized that the work she started, which was to calculate $f(7)$, was backwards of what was needed. Toby reported that he “just found the inverse first.” Which Bailey said her group also did, finding that process easier to understand. Rose also reported a similar obstacle so Rose graphed the original function on Desmos and with recognized that “it was just, instead of [having] the domain, we [have] the range.”

After talking about their initial confusion of the problem, the participants then begin to work the problem, but encountered some difficulty. Both Bailey and Rose reported that the domain and range were switched in this problem, but their first step when working the problem was to compute $f(7)$. Bailey reported that she knew “the answer was $x = 2$, [but] I forgot how it got there… you have to get 7 into the range…” After a couple of minutes, Bailey then stated, “I think I did it wrong.” She then crossed out her previous work and continued saying “I think it’s supposed to be $f(x) = 7$ and then you solve for x.” Bailey reported that the image presented by the instructor (see Figure 28) of the set of domain and range helped to make sense as to why the domain and range were switched.

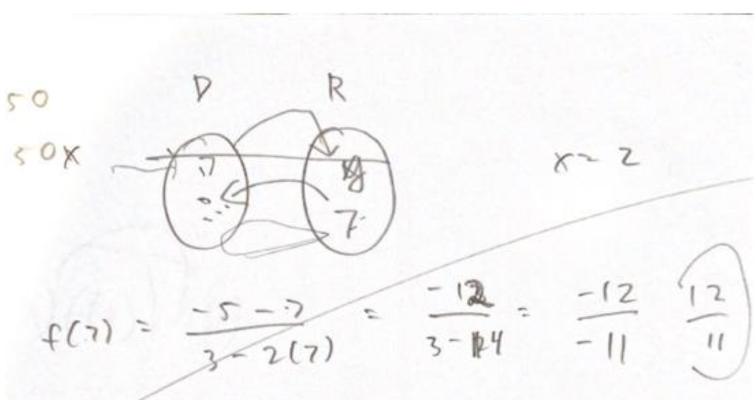


Figure 28: Bailey’s Written Work for Lab 5 Problem #1

During this period Toby reported that Bailey's process made sense, but he did not articulate his own understanding of the problem. Rose, after graphing the function on Desmos but still reported confusion began to solve for f^{-1} using a method of isolating the x variable. Reporting feeling stuck at this part, Rose stated that she would generally review notes, ask a group member for assistance, and then lastly ask for the lab instructor for help. After identifying how to set up the equation to find the inverse of seven, both Bailey and Rose were able to work out the algebra to get the correct answer. Due to the confusion that the participants still had with the problem during the interview, several self-regulation strategies were observed, including seeking peer assistance, organizing information and transforming it with a graph, and consulting external resources such as lecture notes.

The third problem worked in this set of interviews came from a lab exploring logarithmic and exponential functions and equations. Participants first expressed how their groups worked together on this lab, then began work on the specific problem situation. Toby said that he remembered needing help from a group member for the problem number three, but "forgot now what we did." Bailey reported needing help on the same problem, and asking the instructor to explain the problem She was then shown how to change a logarithmic equation to an exponential equation using an example. Rose also reported needing help to start, but with problem number 2, which explored function transformations with exponential functions, she stated that the lab instructor worked the problem for the class on the board.

When looking at problem three each participant expressed their thought process and strategies used to solve the task. Rose was able to articulate the process for how to convert a logarithmic equation into an exponential equation, but when she began trying to show that $\log_b(MN) = \log_b(M) + \log_b(N)$ she began to get stuck and report that what she had written felt wrong, which was that $\log_b(MN) = \log_b(b^m = M) + \log_b(b^n = N)$. After some time, Rose restarted the problem crossing out her previous work, supplying the correct substitution, and continued explaining the process of how the exponential function and the logarithmic function cancel. Rose was able to self-evaluate her understanding while working to help her identify errors in her work.

Both Bailey and Toby had difficulty starting on the problem, similarly to Rose. They stated that when this happens they would review notes to go over any logarithmic properties they might have written previously. After being given the property that $\log_b a = x$ is equivalent to $b^x = a$, Bailey stated she used the example that $\log_3 9 = 2$ so $3^2 = 9$ to remember the property. Although she had this previous understanding, when working on the general case she wrote incorrectly that $\log_b(M) = m$ is equivalent to $b = M^m$, stating that was what she remembered the instructor telling her. Toby produced the correct conversion, writing $b^m = M$, but Bailey was not at first convinced. After a minute, and with Toby's explanation of how the bases would interact, Bailey responded that she preferred Toby's form. After consulting her notes she realized that she had misremembered her instructor's explanation. After this discussion, both participants were able to solve part (b) of the problem to show the linearity property of logarithms. Later in the interview Bailey reported that she would often try working backwards on these types of problems, starting with the answer and trying to figure out how to get to the original equation.

As with the first interview, the self-regulation strategies identified during participants' discussion of each of the lab problems is summarized in the following table. As seen below, and expanded upon by the descriptions above, we see different types of self-regulation strategies for specific types of problems, while common strategies can be identified from participants' reports, the use of self-regulation strategies seems to vary based on the specific problem.

Table 10: Self-Regulation Strategies Identified in Second Group Interview - Precalculus

	Rose	Toby	Bailey
Lab 4, problem #3	N/A	STA, OT, SPA	OT, SE, Worked Examples, SPA
Lab 5, problem #1	OT, SE, STA, SPA, RM	PG, Examples/Counterexamples	OT, PR, SEU, Metacognition, STA
Lab 6, problem #3	MG, STA, SEU, Metacognition	SPA, SE, RM, SEU, Worked Examples	STA, SE, Examples/Counterexamples, SPA, Worked Examples

4.2.6.3 Precalculus Group Interviews: Labs 7-9

In the final round of group interviews all three precalculus participants were able to participate in the same interview. Before speaking of the specific problems on each lab, the participants discussed how prepared they felt for these labs and their overall perceptions of the lab activities. See Figures Figure 9-11 for the specific problems addressed in the interview. All three participants agreed that the lab activities presented during this interview were less familiar compared with what was done in their high school courses, citing primarily a difference in format, such as not having multiple choice as well as an emphasis on sketching a graph by hand, which aligns with participants' individual interview findings as reported above. When looking at the labs, each participant reported that the way they worked as a group was consistent to the previous labs. Bailey reported that she did not remember working on lab 7. At this point in the semester Toby had already dropped out of his precalculus course, so he did not necessarily work each of these remaining labs, but his comments are still recorded as to how he worked the problems as novel mathematics problems.

The first lab explored was given after the second midterm, Lab 7, which focused on exploring angles on the unit circle and trigonometric functions. The group of participants discussed their experiences with this lab and how they went about problem 4. Both Rose and Toby reported being confused about problem 4 (see Figure 9) and how to determine if a function is even or odd. To address this confusion, Bailey reported that she would consult the notes that her instructor uploads to the online class management system as well as seek assistance from peers. All three participants were quick to speak about the problem together with Rose considering known examples stating that she "know[s] cosine is even, [but I] don't know if tangent is even or odd or neither." The group then worked together to determine what it means for a function to be even or odd. Rose appealed to the graph of such functions, graphing sine and cosine as an odd and even function respectively, using those functions as examples to determine the general properties of even and odd functions. Toby looked up the process of finding if a function is even or odd using an external online resource. Bailey used the notes she found from her lecture material, also relying on external resources. With a few minutes of discussion between the participants, they eventually concluded that $f(x) = \cos x \tan x$ was odd. After concluding the function was odd, Rose suggested that another method they could use to help, apart from the graphical argument she previously made, was to evaluate $f(-x)$ to see how that than relates to $f(x)$. The self-regulation strategies most frequently observed involved peer assistance, seeking external resources, and organizing information and transforming it using a graphing tool.

Moving to the second lab activity for the final interview, Lab 8, which focused on transformations of trigonometric functions the three participants discuss their approaching this lab activity. Toby and Rose both stated that sketching/graphing functions, especially trigonometric functions, is difficult for them. Bailey said that she is "hesitant to approach cosine and sine in general, 'cause it's hard for me to memorize what they look like and then applying transformations to them." When asked about problem 2 (see Figure 10) specifically, after Bailey expressed some concern about transforming the cosine function into a sine function, Rose stated that "sine and cosine functions are basically the same thing, except that they're shifted…" While Rose didn't remember the exact type of shift involved, with a conversation between the three participants and the use of online resources such as Desmos and lecture notes, the participants claimed it could either be a left or right shift with an adjustment of the radian measure. To verify this, Rose suggested to type each possible answer the group came up with into Desmos to verify which were correct by comparing it with the original function's graph. While coming up with this conclusion there were several instances of seeking peer assistance, as well as self-evaluation to verify they were considering the problem correctly, as well as organizing the information into a graph.

While discussing the last problem of the interview from Lab 9 (see Figure 11), which focused on using right triangle trigonometry to solve problems, participants were asked to discuss how they started the problem. Bailey reported first drawing a triangle while organizing the information given onto the triangle. Both Toby and Rose also agreed that this approach would be what they would have done. The participants worked as a group and discussed the trigonometric relationships of the sides of the right triangle, citing the use of the mnemonic device SOH-CAH-TOA to remember which ratio corresponds to which trigonometric function. Bailey also brought in that the Pythagorean Theorem would be needed to solve for the adjacent side to their angle θ. Toby reported that the visualization of the triangle was helpful in solving the problem, while Rose reported that while the visualization was helpful, she did not at first see how the Pythagorean Theorem would be helpful, saying she "wouldn't want to use that approach because of the square root and the square." She was not able to come up with another approach, but due to what she saw for the expressions of the hypotenuse and opposite sides of θ she was hesitant. She later said she would ask for help from the instructor, but that Bailey's written work did make sense to her.

As with the previous group interviews, the summary of self-regulation strategies identified while working the individual problems is recorded in the following table.

Table 11: Self-Regulation Strategies Identified in Final Group Interview - Precalculus

	Rose	Toby	Bailey
Lab 7, problem #4	OT, STA, Examples/Counterexamples	STA, SE, SEU, SPA,	SE, SPA, OT,
Lab 8, problem #2	OT	OT, SE	SPA
Lab 9, problem #2	OT	OT	OT

4.2.6.4 Calculus Group Interviews: Labs 1-3

The first round of interviews for the calculus groups had the following groups of participants: Frank and Aria, Jay and Jane, and Sunny, Cyndy, and Participant A. Participant A did not participate in the final group or individual interviews so their work will not be discussed unless it directly relates to the work of another participant. The first round of interviews explored labs presented before the first midterm, see Figure 12-14. While looking at each of the first three labs Aria, Frank, and Cyndy reported not having seen worksheets like the labs in high school, while Jay, Jane, and Sunny reported having worked on similar activities while in high school.

While looking at the first lab specifically (see Figure 12), each participant first explained how their group worked on the individual problems and then focused on the specific problem on the lab. Aria and Sunny reported that their groups worked primarily independently and then Aria added that her group “compared answers if we got different answers.” Both Jay and Jane reported a similar strategy where the problems were divided amongst the group, and after completing their assigned problems would be open to help other group members. Frank and Cyndy reported that their groups worked as Frank put it “collectively… if we got stuck on something someone else would come up and explain it.” While each participant reported some use of seeking peer assistance, we can see there are varying degrees of use for certain groups.

When looking at the problem presented in Figure 12 specifically, Frank immediately stated that he “was never good at sine negative one exponent” and then asked Aria for help, employing seeking assistance from a peer. Aria reported that she recognized that “it represents arcsine” but was not sure how to go from there. When both participants reported they were confused, they reported they would ask for instructor assistance. With no other strategies mentioned for this problem neither Frank nor Aria was able to complete the problem during the interview. Jane on the other hand initially suggested that “the inverse sine would undo the sine in the problem but to confirm I plugged in inverse sine of sine of two and got the answer to be two.” Jane used an example to clarify how she understood the problem. Jay reported a similar strategy of using a calculator to evaluate the expression. Neither participant reported considering the domain or range of $\arcsin x$, but focused primarily on the inverse property. Sunny and Cyndy had a similar strategy to Jay and Jane in that they talked about the “inverse … just cancels each other out and it’s just whatever is inside [of the function].” Again, neither participant attended to the domain or range of the functions. For this problem we see instances of seeking peer assistance and using examples or counterexamples.

Moving to the second lab, participants first reported any changes they saw with how their group worked together and spoke about the lab problem and their solution paths. Aria reported that she was in a different group, and her group worked more collaboratively during this second lab activity, which she found helpful. Frank reported that his group members changed and this time they “mainly went separate and then came back whenever we’re finished” which he did not find as being any more or less helpful to his first group’s approach of working collaboratively. Jay and Jane reported that although their individual groups may have changed slightly, their strategy of dividing the problems among the group was the same. Sunny reported that her group worked in the same manner, and Cyndy reported that her group generally worked together, and that since they were close to a whiteboard it was convenient for one member to work on the whiteboard while the others gave feedback and comments.

For the specific problem in this lab (see Figure 13), participants report on how they attempted to solve the problem and it is noted what self-regulation strategies were observed. Aria and Frank both reported that their groups believed that the functions would add together to, and as Aria put it, “be zero because [it] seemed like they would cancel out” but they were not certain so both groups consulted the lab instructor to confirm their intuition. Both Aria and Frank reported organizing the given information by sketching the graphs of each function which was reported to have been helpful for their understanding of the problem. In general, Frank reports that he does not often draw a sketch first, wanting to “approach it analytically” first, though not expanding on how this looked in action. Aria reported that she will only draw a sketch “if it’s like a very simple graph.”

For Jay and Jane’s group, each had a separate method of looking at the problem. Jay first wanted to draw a sketch of the functions, while Jane reported that she would not have sketched the graphs had the problem not stated to do so. Instead, Jane would “just try and think of the ways that these functions – what they look like and then kind of see what direction they’re each heading.” When asked to work through the problem, both Jay and Jane sketched out the functions, but when asked to also sketch the sum of the functions, Jay produces the following graph (Figure 29) where she overlaid the functions onto the same graph, rather than adding the functions together.

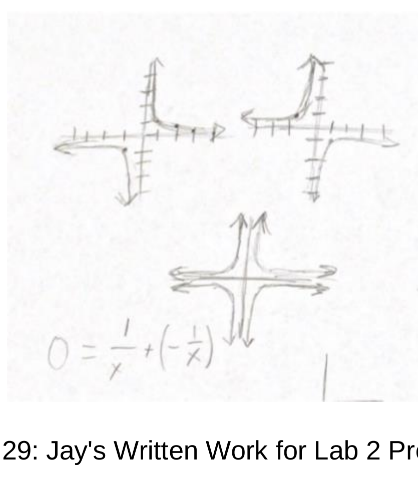


Figure 29: Jay's Written Work for Lab 2 Problem #2

With some discussion between Jay and Jane, they agreed that the sum of the functions should eventually be zero but were not able to reconcile the graph that Jay produced. Given this difficulty, Jay reported that she would reference an online resource to check formulas, as well as use a graphing calculator to see if her graphs were accurate. Jane reported that she would look for a similar problem online, and if unable to find any she would ask a friend for assistance. After using an online graphing calculator, both Jay and Jane were able to recognize the earlier error.

The last group with Cyndy and Sunny both reported starting with different strategies. Sunny started similarly to Jay in first sketching each function individually, while Cyndy reported she would first “plug it in” explaining that she meant to calculate each limit individually first, without sketching a graph. After writing out the limits, Cyndy paused for a short time and began sketching the graph of the functions. Sunny’s work is shown in Figure 30.

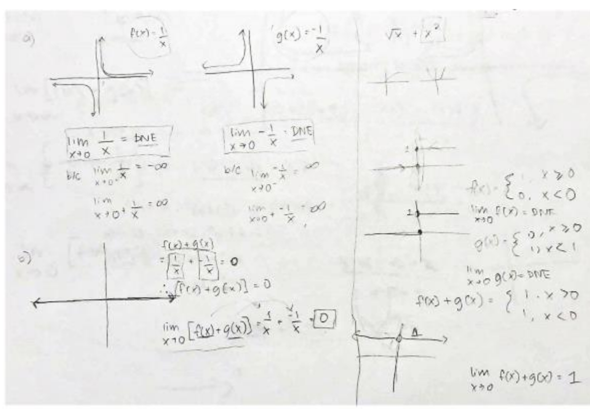


Figure 30: Sunny's Written Work Lab 2 Problem #2

She provided a clear explanation of her process saying:

> I started by graphing these two [functions] and then I just looked at the graph and to see what the limit was doing at zero, and this one [limit of $f(x)$ as x approaches zero] does not exist and this one [limit of $g(x)$ as x approaches zero] does not exist. Then I wrote down a little bit of why just to help myself justify it in my head, and then for the second part where it talks about $f(x)$ plus $g(x)$, I also started with graphing it, and then I

figured out how to graph it by just adding these two functions together which just gets 0. So, it's just a straight line at 0, and then the limit as this graph $[f(x) + g(x)]$ goes to 0.

After Sunny's explanation, Cyndy reported that she had gotten stuck, not being able to determine if the limit of the sum should be zero or whether it does not exist at all, but that she would ask a peer for help at this point. Cyndy and Sunny then discussed further and used Desmos to also show the graphs more accurately. Cyndy then agreed with Sunny's process. As shown in Sunny's work in Figure 30, she also produced another example of piecewise functions whose individual limits do not exist, but the limit of the sum does exist to aid in the understanding of her group members.

Moving to the third lab, which was the last lab discussed during the first group interview, participants mentioned how their groups worked together and then, while they looked at problem three (see Figure 14), explained how they attempted to solve the problem. Aria, Jay, Jane, Sunny, and Cyndy reported to be in either the same groups or use the same strategy as the second lab as to how they worked together. Frank was the only participant who reported that his group changed, which produced a mix of working independently and regularly checking in to work together on some problems.

When starting on the problem in Figure 14, participants first explained how their group got started, and then how the group addressed any possible confusion. Frank started by writing out the function and taking its derivative, but then asked Aria if she knew the formula for average rate of change. Aria provided the appropriate formula, after which Frank wrote out $y - (3 + h) = 2(x - 2)$. He then asked Aria whether she thought he was going about this process correctly, Aria replied that the formula he had was "for slope… I guess if you're thinking about it in that format, I guess it's kind of the same thing." Aria then explained how she went through it, saying that she set up the equation

$$\frac{[2(3+h)^2 - 4(3+h)] - [2 * 3^2 - 4 * 3]}{(3+h) - 3}$$

Aria noted that this was like the definition of the derivative. Frank then agreed with this set-up, and both participants then responded that for the next part of the problem they would take the limit as h approaches zero. Aria was not observed to show any self-regulation strategies here as she did not report encountering any challenges with this problem.

As the second group approached the problem, we see that each approached the problem differently. As Jay and Jane looked at the problem, Jay also stated she did not remember the formula so she would “go back to either the [lecture notes] or the online resources like Paul’s Online notes or something.” Jane did remember the average rate of change formula, saying that a way to think of it to help her remember was to consider it as just the slope from one point to another. After producing the formula, both Jane and Jay were able to set up the problem well and set up how to adjust it to find the instantaneous rate of change. Jane, like Aria, was not observed to use any self-regulation strategies.

Both Cyndy and Sunny were unsure of the formula for the average rate of change. Sunny was not sure whether to use the limit notation or not with the average rate of change. After a discussion between Cyndy and Sunny regarding whether the quotient needed the limit, it was mentioned that the second part of the problem requires a limit, so the first part likely did not need it. After a few minutes Sunny was able to clearly articulate how she used the average rate of change formula for this function, while Cyndy had a slightly different process, but found that Sunny’s explanation sounded good. After working this problem, Cyndy reported that she “need[ed] to study more” due to her misunderstanding early on. She reported that she would review notes, go to the university tutoring center, and speak with the lab instructor.

For each of the calculus labs examined in the group interviews, a summary of self-regulation strategies is shown in the table following the discussion of participants’ methods. As with the precalculus interviews, the strategies identified in the table do not report on the frequency of use of any strategy, only whether a participant was observed using the strategy.

Table 12: Self-Regulation Strategies Identified in First Group Interview - Calculus

	Aria	Cyndy	Frank	Jane	Jay	Sunny
Lab 1, problem #12	SPA, STA	OT	SPA, STA	Examples/ Counterexamples	Examples/ Counterexamples	OT
Lab 2, problem #2	STA, OT	Examples/ Counterexamples, SPA	STA, OT	OT, SEU, SE, SPA	OT, SE	OT, Metacognition, SE, Examples/ Counterexamples, MG
Lab 3, problem #8	None identified	SPA	SPA, STA	None identified	SE	SPA, Metacognition, SEU

4.2.6.5 Calculus Group Interviews: Labs 4-6

During the second group interview, the groups of participants looked at activities from the three labs given after the first midterm, Lab 4, 5, and 6 (see Figures Figure 15-17) and reported on their overall perceptions of each lab. Due to some scheduling difficulty, there were two participants who could not participate as a group but did participate as individuals. The groups of participants were as follows: Cyndy and Jane, Sunny and Jay, Aria, and Frank, with Aria and Frank both participating on their own. While looking at all three of the labs together Jay, Jane, and Frank found that these labs were “pretty similar” to what they saw in high school, with Jay stating that it was “a list of problems that we’re allowed to work in a group.” Sunny though found it more similar to homework problems that were “mostly easy” in high school. Cyndy reported these activities were not familiar to what she did in high school, saying that she primarily had online activities. Aria said it depended on the labs, noting that the problems in high school were “more straightforward and [you had to] just do the math.” She then said some of the more conceptual based problems in Lab 5 were less familiar, and some of the problems in Lab 6 were more familiar.

Looking at Lab 4 (see Figure 15), the participants first described how their in-class groups worked through the lab and then began to discuss the specific problem. Aria reported that her group would each work out the problem then compare answers and if at any point anyone needed help they would ask other group members. Jane, Cyndy, Jay, and Sunny continued their strategies as before when working as a group. Frank's group further divided from a group of four into two groups of two, where two students would work on each problem to compare answers, but divided the problem set between the two groups of two. As seen here an adjustment of seeking peer assistance can be observed to show that students adjust their methods to find how their strategies are most productive.

Looking at the specific problem in the lab, which was to use the product rule twice on the product of three functions, each group explained how their started the problem and addressed any confusions. Sunny and Jay both wrote out what they believed would be the solution without using the product rule twice, with Sunny producing the correct result and Jay using a product instead of a sum [$f'(x)g(x)h(x) \cdot f(x)g'(x)h(x) \cdot f(x)g(x)h'(x)$], which was corrected through a conversation by Sunny and Jay. Sunny reported that she observed "the pattern, I know this is like the derivative, [it] will move to the next one." Jay also reported that she recognized the pattern, just misplaced the product instead of addition for each of the three terms. When asked to reread the question, Sunny noted that she "skipped a step" by not using the product rule twice, so then corrects her work by first writing out $(f'(x)g(x) + f(x)g'(x))'h(x)$ but after being asked to explain how she got that recognized her mistake and wrote the product rule considering the product $f(x)g(x)$ as one function, writing $\left(f(x)g(x)\right)'h(x) + \left(f(x)g(x)\right)h'(x)$. After that she was able to finish the problem appropriately. While working this problem both Sunny and Jay reported they would first ask other group members for help, but if the whole group was stuck, they would then ask the instructor for help. As seen above Sunny also displayed use of self-evaluation by looking back at her work and correcting errors, although it was prompted by discussion with a peer and the interviewer.

Frank had a similar start to Jay and Sunny. He began by writing what he believed to be the answer but misunderstood what the question was asking. Frank wrote out the following equations in succession:

$$\big(f(x)g'(x) + g(x)f'(x)\big)h(x)$$

$$h(x)f'(x)g''(x) + h'(x)f(x)g'(x) + h(x)g'(x)f''(x) + h'(x)g(x)f''(x)$$

Frank continued after writing and said, “I went through the product rule like having these [f & g] combined and this will create [$\big(f(x)g'(x) + g(x)f'(x)\big)h(x)$], but it also asks you [to use] the product rule twice.” So, he then attempted to take the derivative of the earlier function a second time. He then reported that he would check his work with a group member, and if they did not agree he would ask the instructor for help.

When Cyndy and Jane began the problem, each participant first wrote out the general product rule for the product of two functions as reference. Cyndy employed a similar strategy to Sunny and Jay earlier and attempted to start with pattern recognition writing out $\frac{d}{dx}\big(f(x)g(x)h(x)\big) = f'(x)g(x)\_ + f(x)g'(x)\_ + \_\_\_\_$ where the blanks would be where h is involved. Jane suggested considering $f(x)g(x)$ as its own function then allowing her to write $\big(f'(x)g(x) + f(x)g'(x)\big)h(x) + f(x)g(x)h'(x)$ after providing an explanation of considering the product as a single function. When asked how she might continue after getting stuck, Cyndy reported that she would ask her lab members, and then the lab instructor if needed. Cyndy then asked Jane for help, Jane then provided an explanation that used “redefining [the function] terms” writing out the following equation:

$$\frac{d}{dx}\big(a(x)b(x)\big) = a'(x)b(x) + a(x)b'(x)$$

She then stated that the a function was $f(x)g(x)$ and the b function was $h(x)$. After that, Cyndy reported that she understood the process now, being able to better organize the information given in a generalized form.

Aria approached the problem as Jane had above, grouping the functions f and g and applying the product rule similarly. She did report that during the lab period when working this problem out she looked up how to use the product rule, as at that point it was still unfamiliar, she also stated that if she was not able to figure it out by just looking up information, she would have asked her team members and her instructor if needed.

Lab 5, which examined the relationship between position, velocity, and acceleration in terms of derivatives, was then examined by each of the groups during the second group interview. All six interviewed participants reported using their same strategies as in earlier labs when working as a group.

For problem 3 on the lab (see Figure 16), participants were asked to determine which of three functions represented position, velocity, and acceleration for a specific context. As Aria started this problem, she stated that she "looked at the zeros of each graph" and then marked on the paper the location of any local extreme points of the graphs a and b. Aria then started to consider what the derivative of a would need to look like, considering where the graph has positive slope and negative slope. This helped her identify that neither b nor c is the derivative graph of a, so she made the claim that the graph of a represents the "third derivative." She then turned her attention to the graph of b, and compared where the local extreme value of b and noticed that the x value producing the local extreme of b was a zero for a, then making the claim that "a is the derivative of b." She also at this point organized her information by writing in the margins that $b' = a$. She was then able to identify that b is the derivative of c by acknowledging that the slope of c is always positive, but tends towards zero, and b is always positive and tends towards zero. She then correctly organized the information writing $c'' = b' = a$, but then produced an incorrect conclusion that a is the position, and c is the acceleration. She was able to accurately state that "the derivative of the position is the velocity" but mixed up the interpretation during the interview.

As Jane and Cyndy started this problem, Jane asserted that a was the position, b was velocity, and c was acceleration. Cyndy was not sure which graph was which but was able to correctly state the relationship between position, velocity, and acceleration in general. When asked how she came up with her hypothesis, Jane stated that

> I had suggested the ideas that I had just from knowing that the exponent of this [a], based on this graph would be one higher than the exponent of this [b], which would be one higher than that [c]… I was looking at it based on the logic that I know, like generally a cubic function would have, is sort of similar like down [then] up, and then a quadratic which is one less exponent, and that would just sort of have one curve, like one bump, whereas the derivative [of] that would be linear, which was more of a straight line, and this just better seemed to match the image in my head for these.

After listening to Jane's explanation, Cyndy proposed that she was considering b as the position function "'cause I know the position of the car would either go up, and then get to a point where this stops." Cyndy then stated she was considering it from a physics-based perspective, not elaborating further as to what that meant except that she did not understand Jane's process. When it came up that they had different approaches, they stated they would likely ask another group member had they been in a lab setting, with Jane saying, "it's majority rules in the end." It was then suggested if the other group member had a completely different answer that Cyndy would ask the instructor, and Jane would look online for the problem or looked online for position, velocity, and acceleration graphs.

Frank had a similar start to Aria, where he looked at the zeros, whether the slopes of the functions were positive or negative, and local extrema. From this he made the claim that a was the derivative of b, as well as noting that c is the acceleration since "it's constantly increasing." He then further this line of thinking claiming b was the position but noted "it could be wrong." He did not speak to any strategies he might use to check his work or what could be done if he were stuck.

For Sunny and Jay, when asked to look at the problem in question, Sunny mentioned that she “remember[ed] the answers ‘cause I got this one wrong.” She then supplied that c would be the position, stating it had something to do with the extreme values relating to the zeros, but she was “not sure what this point [extreme point of b] has to do with this.” Sunny then later added that if c is position, then b would have to be velocity because “position [is] always going forward… it would never have a negative velocity.” Jay then added that a would be the acceleration bringing in the context that the car is “going faster, and then if you hit your brakes here [local maximum of c] and then your acceleration would drop rapidly.” A conversation between Sunny and Jay then continued confirming their ideas of how the context of a car traveling would make sense given their assumption of which graph was position, velocity, and acceleration. After both Sunny and Jay agreed with their conclusion, Jay suggested that to check her work she might look in her lecture notes or online for “a similar graph that has acceleration and velocity, and comparatively see if it seems similar.” These participants used the strategies of seeking peer assistance when discussing the problem with each other, and Jay suggested she would seek external sources to verify her work.

Lab 6 was the final lab explored in the second group interview, which focused on using the chain rule for several examples of composite functions. As before each group of participants first discussed how their original lab groups worked together, and then discuss the problem and their solution paths. All the participants except for Cyndy continued to use their previously mentioned group strategies. Cyndy had her lab activity done online so she reported that the students all worked individually that day.

When discussing the specific problem (see Figure 17) participants were asked how they might begin working on problem (c). Aria began by attempting to take a derivative of the function $H(x)$, but remarked that she did not remember how to work with exponential functions. When asked how she might attend to this, she reported she would go through her notes. The interviewer provided the formula $\frac{d}{dx}e^x = e^x$, but Aria reported she would need more than just that formula. Aria recognized that she would use the power rule and quotient rule at some point. From this Aria wrote out the derivative as being

$$e^{\left(\frac{h(x)}{f(x)}\right)^2} \cdot 2e^{\frac{h(x)}{f(x)}} \cdot \left(\frac{f(x)h'(x) - f'(x)h(x)}{f(x)^2}\right)$$

While Aria did not explicitly mention needing the chain rule, it is clear that she did attempt to apply the chain rule while writing out her work. She then reported that she would just need to plug in $x = 0$ for each function to complete the problem. Aria's work and discussion of this problem did not indicate the use of any self-regulation strategies.

As Cyndy and Jane started this problem, both participants began by rewriting the function while discussing possible avenues to take the derivative of the function. Jane at first misread the problem thinking the function was $e \cdot \left(\frac{h(x)}{f(x)}\right)^2$, rather than being an exponential function, but was able to quickly correct when that was mentioned. While attempting to find the derivative function, Cyndy and Jane had separate approaches, with Jane attempting to simplify the problem by first considering the derivative of e^{x^2} which she wrote as $e^{x^2} * e^{2x}$, while Cyndy started by trying to find the derivative of the quotient, writing $\frac{h'(x)}{f'(x)} = \frac{2h(x)}{2f(x)}$. Cyndy was unsure of her work, saying that when looking at Jane's work, she would "probably trust [Jane's] work more than [her own] work" because she was not confident of what she had done so far. Cyndy reported shortly after that she was stuck and would ask a group member or the instructor for help, but before that she would try to "just plug in a number" to see if she could get a reasonable answer. Jane worked for a few more minutes and came up with her "best guess" which was the following, which she then reported she would finish the problem by evaluating when $x = 0$:

$$\frac{d}{dx}\left(e^{\left(\frac{h(x)}{f(x)}\right)^2}\right) = e^{\left(\frac{h(x)}{f(x)}\right)^2} \cdot 2\left(\frac{h(x)}{f(x)}\right)\frac{\left(h'(x)f(x) - h(x)f'(x)\right)}{f(x)^2}$$

Frank started the problem recognizing that he would need quotient rule, and wrote out the following equation:

$$H'(x) = 2\left(\frac{f(x)h'(x) - h(x)f'(x)}{\left(f(x)\right)^2}\right)e^{\left(\frac{h(x)}{f(x)}\right)^2}$$

Then after writing this plugged in $x = 0$. Frank reported that the only difficulty this problem posed was having to remember the quotient rule, but during the interview did not employ any self-regulation strategies while working out this problem.

While Sunny and Jay started the problem, each went about the problem in their own direction. Sunny employed u-substitution, letting $u = \left(\frac{h(x)}{f(x)}\right)^2$, while Jay began by evaluating $H(0)$. Sunny was able to then produce the correct derivative of $H(x)$, utilizing the u-substitution method. Jay came up with an answer claiming that she used values in the table to "just plug it into [the function] and then use the power rule and put two in front of [the function]." After her explanation, Sunny noted that Jay may have misread the problem, saying that the quotient is in the exponent of e, to which Jay then responded that the "formatting might have messed me up a little bit." Once Jay saw the correct format of the problem, she claimed she would use the chain rule for the problem. After working through the problem, Sunny suggested that she might use logarithmic differentiation to solve this, which at the time of the lab had not been taught but had been taught by the time of the interview. Sunny seems to be setting mastery goals for better understanding the concepts when attempting to identify multiple solution pathways. Once they were both convinced of Sunny's solution, Jay reported that in a lab setting she would have "probably checked with another group member… or refer[ed] back to our notes to see if we can find a similar problem."

The following table summarizes the self-regulation strategies I identified after analyzing the interviews. Each participant's identified strategy usage changed based on the specific problem at hand, and the strategies used between participants are not always persistent.

Table 13: Self-Regulation Strategies Identified in Second Group Interview - Calculus

	Aria*	Cyndy	Frank*	Jane	Jay	Sunny
Lab 4, problem #5	OT, SE, SPA, STA	OT, Examples/ Counterexamples, SPA, STA	STA	MG	SPA, Examples/ Counterexamples, STA	Examples/ Counterexamples, STA
Lab 5, problem #3	OT	STA	SEU	Examples/ Counterexamples, SPA, SE	OT, SE	OT
Lab 6, problem #1(c)	SE	SPA, STA	None identified	SPA	SPA, SE	SPA, MG

*Participated in interviews individually

4.2.6.6 Calculus Group Interviews: Labs 7-9

In the final round of group interviews for the calculus participants there were two groups of three that were interviewed. The first group included Sunny, Aria, and Jane while the second group included Frank, Cyndy, and Jay. Analysis for this set of interviews examined the labs given after the second midterm, Lab 7, Lab 8, and Lab 9. For the problems explored during the interviews see Figure 18-20. When asked if the three lab activities were similar to what was done in high school, Frank, Jay, and Jane all found these labs to be at least somewhat similar to what they had done in high school. The other participants, Aria, Cyndy, and Sunny said they had not seen activities like these in high school. Participants in these interviews stated that the collaborative groups worked together as they had done as previously reported in the second group interview.

Lab 7, looking at related rates problems, was the first lab examined during the final group interview. The specific problem worked by the participants is shown in Figure 18. When starting this problem in the first group, Aria began by first sketching “a picture of the scenario” while organizing given information and writing out the Pythagorean Theorem. Both Sunny and Jane use the same process to get started, with Aria explaining her process:

> I first drew out the information that the problem gave me so I could understand better visually. So, I wrote down the information, and from that we can conclude that I had to use the Pythagorean Theorem, and the distance between the player and third base was 30 feet. I used the Pythagorean Theorem to solve what C equals, because later on when you… take the derivative, we need to know what C equals… I took the derivative of A^2 and C^2, then I substituted the information I knew and then add my variable.

As she was explaining this process, Aria noticed an issue in her differentiation in that she did not correctly differentiate the constant length to be zero, which affected her arithmetic moving forward. She was able to correct this during the explanation and self-evaluation of her work and move forward smoothly after. Sunny found that her work agreed with Aria's once Aria fixed her mistake, and Jane reported that is what she was working on, not having written out all the arithmetic at that point. As reported by Aria and confirmed by the others, the self-regulation strategies observed included organizing the information and transforming it into a sketch and self-evaluating one's work.

The second group had a similar start with each participant first sketching the problem situation. Cyndy began by finding the area of the right triangle formed in the problem but stated that she does not "know how to utilize [the rate of change, 22ft/s]." She referenced lecture notes to review and asked her group members for help. Upon asking her group members for help, Jay reported being confused about some aspects of the sketch, since at first Cyndy did not accurately mark the bases or where the player was running on her sketch. After this interaction the participants began working separately, with Frank setting up the Pythagorean Theorem to find the length of the hypotenuse of the triangle formed by the bases and the runner, while Jay looked at a proposed relationship between rate of change and time and set up a ratio $\frac{22}{1} = \frac{60}{t}$. Cyndy asked Jay about her process to which Jay responded that she was looking for

> how fast is the distance changing, we know their speed equals distance over time, and they gave us a speed which is 22 feet per second... From second or third base will be

30 [feet], already traveling 60 [feet], but we've actually traveled 90 [feet], 120 [feet], since we've already gone from first base to second base and each side is 90 feet long.

Jay and Cyndy then spoke for a few minutes about this, but eventually got stuck again, with Cyndy going back to her notes and Jay working independently. While this was happening Frank had tried using implicit differentiation on the Pythagorean Theorem but ended up scratching out the work "because I don't think it's how the problem should go, I don't think that's the formula… the numbers that I have don't fit in with the formula." After all participants reported they were stuck, they each discussed how they might try to resolve their confusion if given more time. Cyndy reported she would try to find a similar problem on Google, Jay said she would review her notes and if needed use another resource that explain the concept differently than what was explained in her lecture, and Frank reported he would use a similar process to Cyndy and Jay of using external resources.

Moving to Lab 8, which is an exploration of optimization problems (see Figure 19), the groups each explained their process to solve this problem. From the first group, both Sunny and Jane began by plotting a graph and writing out the distance formula to organize their information. Aria, after a few minutes of consideration, drew out the graph to start. Sunny explained how she went about this problem after each participant individually worked for several minutes:

> The way I started was I drew the graph and I drew the line $y = 3x$ and then I also plotted the point $(50,0)$ and then I tried to think about, at first, I thought what kind of shape is that, I didn't think that would really help, and I was thinking 'cause it's not a right triangle and I was like, well we're trying to find the point that's closest, if we can't find the smallest distance from any point on here to this one [$(50,0)$]. So, with that I just decided to write down the distance equation so I could look at it… so I just plug [the point $(50,0)$], and expanded it a bit to make it a little bit easier to think about, 'cause I knew that I would end up having to do the derivations at some point, [and] it would be

> easier to derive something with the polynomial… I remember there was one constraint function and one objective function… I knew I wanted to make the distance the smallest because it is the closest, and then I knew that this [$y = 3x$] would end up being my constraint because it has to be a point on this line. So, I plugged in the $3x$ wherever y was, then I end up with this
>
> $$d = \sqrt{x^2 - 100x + 2500 + 9x^2}$$
>
> Then I simplified it a bit… then I did this derivation because I remember that wherever it's zero, that's where something happens, something like this, and it's like – it's weird. What do we call it, the slope changes from negative to positive.

After hearing her explanation, Jane reported going about the process in the same way. Aria reported she knew she needed the distance formula but was unsure how to use it without knowing two points, saying that she was stuck with how to work with the distance formula and the constraint. She then reported she would speak with her peers when this type of confusion came up. Aria still reported some confusion after hearing Sunny's explanation, but then listened to Sunny further explain how the constraint function can be used as the second point, providing examples of points on the line that could be tested to find their distance. After discussing the problem all three participants reported they would refer to their notes if they were stuck on a similar problem.

From the second group each group member started differently. Cyndy began by rewriting the problem, Frank drew a sketch of the graph, and Jay wrote out the distance formula. After writing out the problem, Cyndy then drew a sketch of the graph. While working, both Frank and Jay referred to notes they had taken during lecture. Cyndy proposed using the Pythagorean Theorem to help her saying that "I tried to visualize [the graph] and then I know you have to do Pythagorean Theorem to find the least distance" but Cyndy was unsure of what point she would use on the constraint function to then use the Pythagorean Theorem, so she stated she would ask her group members for help. Jay looked in her notes for the distance formula, while Frank was not sure how to go about the problem but did state that using the distance formula sounded like a useful starting place. None of the participants were sure of where to go from here. Cyndy reported she would look for a similar problem, while Frank said he would at this point move on to the next problem. The three participants reported that given more time and extra resources they would eventually be able to figure out the rest of the problem.

Lab 9 was the last lab examined during the group interviews, which covered linear approximations and differentials. While looking at this lab the participants began to explain or work through problem 4(i) (see Figure 20). For the first group, each participant began by rewriting the function and writing out the general equation for a linear approximation, $L(x) = f(a) + f'(a)(x - a)$. Sunny and Jane also organized the given information by stating what a equaled. To help her calculate $f(a)$, Aria drew part of the unit circle. Shortly after writing the linear approximation formula, Sunny erased instances of a from her previously correct equation saying that "I think I did this wrong because we're approximating it at this point [$a = \frac{\pi}{4}$], so I need to choose another [point] that's close to this, but not exactly it to plug in [as a]…. Oh wait… I always get mixed up right here." Here Sunny was wrestling with the correct form, so she turned to her group members, Jane then explained that the a value should be used in the equation because she thought "they're trying to find the line that it's similar to there [at a]." Sunny then recognized what she at initially was correct, and to then confirm said she read later parts of the problem to see if she could gather any hints about earlier part. Aria reported she was still confused about setting up the initial equation, after which Sunny began explaining her work to Aria.

From the second group the participants explained how they started the problem but did not finish working through the task. Jay and Frank first attempted to write out the linear approximation formula. Frank was able to correctly write it out, while Jay wrote out what she remembered, which was $L = f'(x) + (x - a)$. When asked to compare their equations, Jay reported she was unsure of her equation, but when she saw Frank's work, she recognized it as being correct. Cyndy did not report on any of her work for this problem but stated that with time and the formula she believed she could finish the problem. Due to time constraints during the interview the three participants did not progress further than recognizing what formula to use. Other than the organizational strategy employed by Jay and Frank, there were no self-regulation strategies that were identified during the discussion of this problem.

As before, the following table summarized the strategies identified while working through specific lab problems, noting whether the strategy was used not the frequency of use.

Table 14: Self-Regulation Strategies Identified during Final Group Interview - Calculus

	Aria	Cyndy	Frank	Jane	Jay	Sunny
Lab 7, problem #5	OT, SEU	OT, SE, SPA, Worked Example	OT, SE	OT	OT, SE, Provide Time	OT
Lab 8, problem #5	OT, SPA, SE	OT, SPA, SE	OT, SE, Provide Time	OT, SE	OT, SE	OT, MG, SE
Lab 9, problem #4(i)	OT	None identified	OT	OT, Examples/ Counterexamples	OT	OT, STA, Provide Time

Looking at specific instances of participants working on these lab activities we see a more in-depth picture of how students are reacting to these problems and what self-regulation strategies they are using. Each of the nine participants interviewed from calculus and precalculus reported that they would ask group members for help when stuck on a problem at least once during the group interviews, with several of them mentioning throughout the interviews that they would ask a peer for help. Cyndy reported this most frequently at six times during discussion of the nine lab problems. This result aligns with what was captured in the post-lab survey that participants often seek peer assistance for these activities as seen in Section 4.3 Calculus Post-Lab Surveys. Several of the participants, including Aria and Jane, reported that they rarely seek peer assistance outside of these lab settings. These participants, along with each participant at some point, reported they would often look up the information using an external source, such as Google or MathWay. The participants also commonly reported that they would use a feature in their online homework system that allowed them to work through a scaffolded problem, then reattempt the initial problem, listed as "see similar example." Through identifying these strategies used by participants we can begin to build a framework that models those strategies used by successful calculus students and see how those strategies are used during the transition to undergraduate mathematics.

4.3 Calculus Post-Lab Surveys

Aside from the self-regulation strategies reported and observed during the group interviews, survey data was collected from two sections of calculus over the strategies participants used throughout the entire lab activity, as well as their perception of how difficult the lab was compared to what was experienced in high school (See Appendix M Calculus Post-Lab Survey Excerpt). In

Table 15 a summary of the data collected from the calculus sections is presented, where n is the number of participants surveyed, and r is the number of participant responses. Note that each participant had the option of selecting more than one choice, so there are often more responses than the number of participants, though there were times when a participant did not select any strategies on the post-lab survey. In the following table, I present the relative percentage of each strategy in terms of the overall number of responses, was well as the frequency of each strategy. Labs 10, 11, and 12 are not discussed in interviews, but their data is presented here. Those labs covered L'Hôpital's Rule, Riemann sums, and definite integrals respectively.

Table 15: Self-Regulation Strategies Reported on Calculus Post-Lab Surveys

	Reflect on a similar problem that was worked either by the instructor or from homework.	Consult a group member for help about the problem.	Consult the TA/Instructor/ SI for help about the problem.	Moved on to another problem to come back to the problem later.	Gave up and watched my group members proceed with the problem.
Lab 1: $n = 113$ $r = 225$	17.33% (39)	39.11% (88)	17.78% (40)	23.11% (52)	2.67% (6)
Lab 2: $n = 52$* $r = 48$	16.67% (8)	68.75% (33)	6.25% (3)	8.33% (4)	0% (0)
Lab 3: $n = 101$ $r = 238$	20.59% (49)	36.13% (86)	19.75% (47)	21.43% (51)	2.10% (5)
Lab 4: $n = 74$ $r = 158$	25.32% (40)	34.81% (55)	20.25% (32)	18.99% (30)	0.63% (1)
Lab 5: $n = 87$ $r = 204$	25.98% (53)	32.84% (67)	20.59% (42)	19.12% (39)	1.47% (3)
Lab 6: $n = 86$ $r = 203$	24.14% (49)	34.98% (71)	21.67% (44)	17.73% (36)	1.48% (3)
Lab 7: $n = 88$ $r = 214$	29.91% (64)	32.71% (70)	19.16% (41)	17.76% (38)	0.47% (1)
Lab 8: $n = 83$ $r = 199$	26.63% (53)	34.17% (68)	22.11% (44)	16.58% (33)	0.50% (1)
Lab 9: $n = 79$ $r = 183$	27.87% (51)	36.61% (67)	21.31% (39)	14.21% (26)	0% (0)
Lab 10: $n = 67$ $r = 163$	30.67% (50)	32.52% (53)	22.09% (36)	14.11% (23)	0.61% (1)
Lab 11: $n = 76$ $r = 193$	31.61% (61)	30.05% (58)	21.24% (41)	16.06% (31)	1.04% (2)
Lab 12: $n = 66$ $r = 167$	29.94% (50)	31.14% (52)	21.56% (36)	17.37% (29)	0% (0)

*Post-Lab survey was not distributed to one of the sections

Using the frequencies of self-regulation strategies as well as participants' perceptions of how difficult the labs were in relation to high school mathematics activities, a chi-square test of independence was performed. I compiled the frequencies of each strategy presented in each post-lab survey and linked those frequencies to the participant's perception of the lab difficulty. Those frequencies are presented Table 16. When looking at all strategies used across all twelve labs and testing the relation between the perceived difficulty, the relation between these variables was not found to be significant, X^2 (8, N = 2191) = 8.38, p = .40. A relationship between the use of these self-regulation strategies and the perceived difficulty of lab activities was not able to be established.

Table 16: Perceived Difficulty of Lab Activities versus Frequency of Self-Regulation Strategies

Labs 1-12	Reflect on a similar problem that was worked either by the instructor or from homework.	Consult a group member for help about the problem.	Consult the TA/Instructor/SI for help about the problem.	Moved on to another problem to come back to the problem later.	Gave up and watched my group members proceed with the problem.	Total
Less work than a typical HS assignment	18	28	12	16	0	74
Approximately same amount of work as typical HS assignment	247	328	181	158	5	919
More work than a typical HS assignment	301	411	251	217	18	1198
Total	566	767	444	391	23	2191

As shown in

Table 15 the use of self-regulation strategies across calculus courses throughout the Fall 2021 semester had some slight variation. The most commonly reported strategy used on all but one lab was to consult a group member to help with the problem whenever a student had gotten stuck. This activity was a collaborative group activity, and considering this, the high frequency of this strategy is not surprising. This strategy was encouraged by the calculus and precalculus lab instructors, often reinforced by instructors advising students to discuss problems together before asking the instructor for help. Late in the semester, during the second to last lab, the strategy of reflecting on a similar worked by an instructor or through homework was the most common strategy. That lab, while not examined in any interviews, tasked students with using the definition of the definite integral to evaluate several definite integrals, due to the procedural nature of that lab it is likely most students had an example problem they had felt comfortable following. In most labs, nine out of the remaining eleven, this strategy of looking at a previously worked problem was the second most common strategy, growing in relative use throughout the semester, this strategy changed the most throughout the semester, not considering the outlier of the second lab as the surveys were only completed by one of the two calculus sections for that lab.

The range of some strategies is given in the following section to show where the most and least variation occurs. The strategy of looking at previously worked problems ranged in percentage use from 17% of participants to 32% of participant usage. The strategy of consulting the instructor when stuck remained consistent as being used by about 20% of the participants for each lab, except for Lab 2. The least change occurred with the strategy of giving up, where only a few participants reported the use of this strategy throughout the surveys. There was not much change to the overall strategy usage of participants on these labs throughout the semester, except for the increase of reflecting on a similar problem.

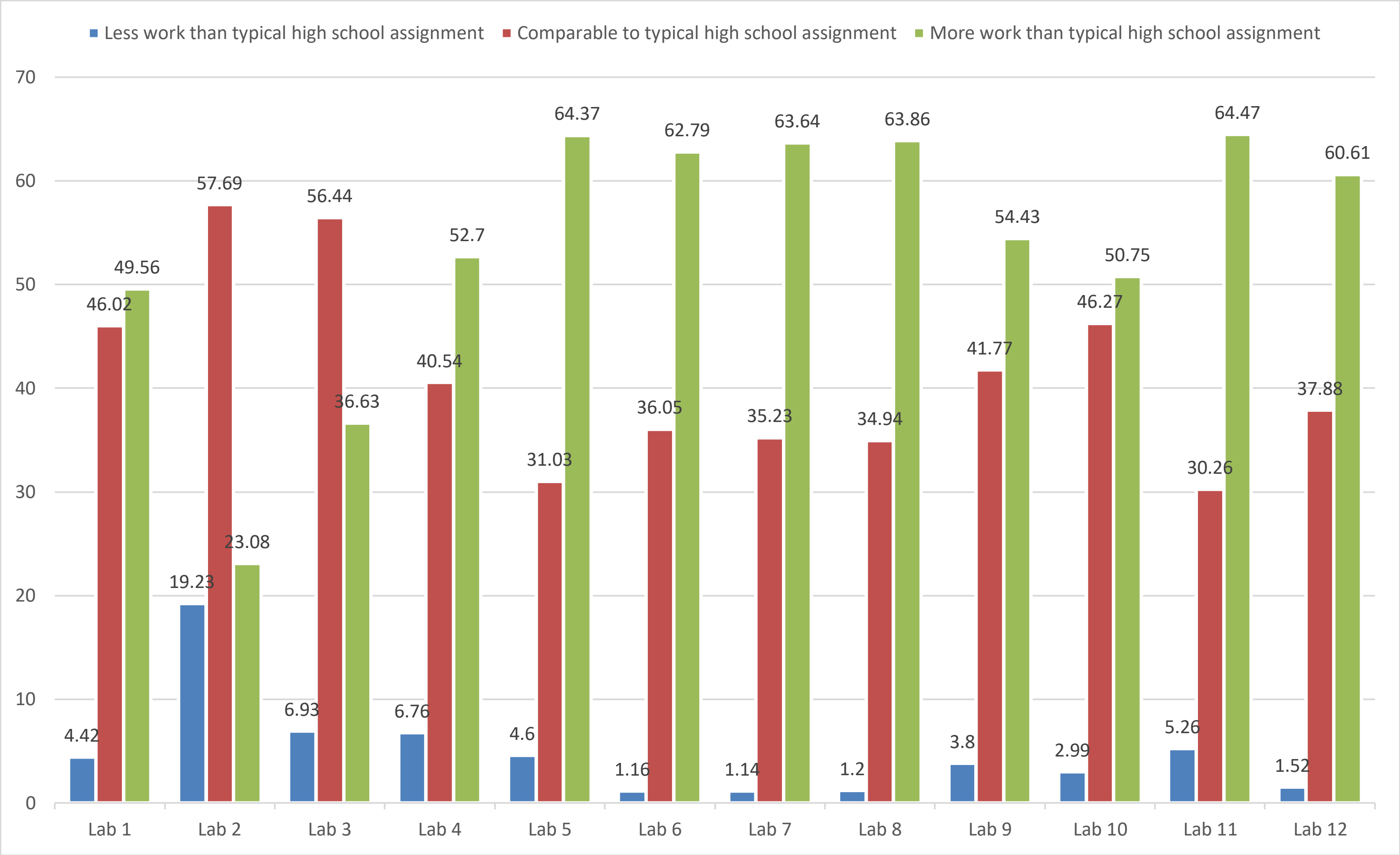


Figure 31: Participant Perceptions of Calculus Lab Difficulty as Compared to Typical High School Assignment

Figure 31, above, displays the results from the post-lab surveys regarding participants' perception of the difficulty of each lab activity. As noted previously Lab 2 is considered an outlier as only one calculus section received the post-lab surveys instead of the two sections that responded to the other surveys. As seen below there is variation in how these labs are viewed, with the labs in the middle of the semester and at the very end being viewed by most participants as more difficult. Early lab activities were more often viewed as being less work than a traditional high school activity, with some decrease in the number of participants who held that view as the semester progressed.

During the final two post-lab surveys, participants reported not only their self-regulation strategy usage, but also their expected grade upon completing the course. In the following table (Table 17) I present the interviewed participants self-efficacy scores, reported midterm grades, and projected final grade for those interviewed participants who also completed the surveys.

Table 17: Self-Efficacy, Exam Scores, and Projected Course Grade (Interviewed Participants)

Participant	Self-Efficacy Score	Midterm 1 Percentage	Midterm 2 Percentage	Projected Course Grade
Bailey	0.73	77	84	B
Rose	0.73	81	82	B
Aria	0.56	86	88	A
Cyndy	0.58	45	51	D
Frank	0.48	96	74	B
Jane	0.92	77	65.5	Not Reported
Jay	0.64	41	38	D
Sunny	0.78	96	95	A

4.4 Self-Regulation Strategies used by Successful Students

During the final individual interview as well as the final post-lab surveys the following participants reported feeling confident they would perform well in the course, which they marked as earning an A or a B for their overall course grade: Aria, Bailey, Frank, Rose, and Sunny. Jane did not report her expected course grade in the survey, and when asked what she expected in the course during an interview she reported uncertainty of whether it would be a B or a C. While Jane did show strong self-regulation strategy usage, in this section I will only report on participants who were confident they earned at least a B in the course overall. As stated previously Toby dropped out of the course after the second midterm, and both Cyndy and Jay reported in both the survey and interview that they expected to pass the course with a D. In this following section I discuss what strategies were commonly used or are commonly absent from those participants who projected their grades as being A's or B's (see Appendix O Self-Efficacy, Exam Scores, and Project Course Grade (Surveyed Participants [excluding interviewed participants]) as performing successfully in the course, without regard to their self-efficacy or mathematics identity.

Based on the group interviews we see a high use of OT (organize and transform information into something usable) usage among these participants. Except for one participant, Rose, we also see a tendency to employ SE (seeking external resources) and SPA (seeking peer assistance). Rose in these situations more often used STA (seeking teacher assistance). The participants in this group were the only participants to speak metacognitively about their self-regulation strategies while discussing the lab problems. These participants were not observed to use PG (setting performance goals) or, except for Rose, use RM (keep records and monitor progress) when working on the lab problems. From the initial survey, we see a high use of the following strategies among these five participants: rehearsal, organization, metacognitive self-regulation, time and study environment, effort regulation, peer learning, and help seeking. There were not any strategies from the MSLQ that were weak across those five participants. It is important to note that participants who did not perform as well also had some strong use of a subset of these self-regulation strategies, such as Jane, Jay, and Cyndy who each had good scores based on the MSLQ for all but one of the nine strategies listed from the MSLQ (help seeking for both Jane and Jay and peer learning for Cyndy). Both Jane and Jay entered the interviews reporting a high level of confidence, and as mentioned earlier in reference to Jane, this may have been what kept them from seeking help early in the course which otherwise might have aided in their performance.

Looking at the larger sample of calculus and precalculus participants, we can examine the use of self-regulation strategies as reported from the MSLQ and relate it to participants' projected grades, assuming that their projections were accurate. We examine the strategies generally used by participants who projected getting an A or a B overall in the course. From a total of 52 participants who completed the survey and projected a high grade, all the participants had at least adequate use of the following two self-regulation strategies: time and study environment and effort regulation. These are participants who understood and used their resources effectively, especially time, to focus on content that they needed to pass the course. 50 out of the 52 participants had scores showing adequate use of metacognitive self-regulation. The least used strategies from this group were peer learning with 41 of the 52 participants' scores showing adequate use. The next least frequent strategies reported were critical thinking and rehearsal with 43 and 44 participants respectively having scores that showed adequate use.

As we look at the three most often used strategies by successful participants from the initial survey (metacognitive self-regulation, time and study environment, and effort regulation), I will discuss ways in which participants reported using these strategies as they relate to Zimmerman and Pon's categorization of strategies as identified during the group interviews. During the second group interview, each participant discussed how they used each of the self-regulation strategies as categorized by Zimmerman and Pons (1986), while there is not a one-to-one relationship that maps Zimmerman and Pons' categorization to that of the MSLQ, below I report on the facets that most closely align with the three MSLQ strategies identified above.

For metacognitive self-regulation, which is the awareness, knowledge, and control of cognition, including planning, monitoring, and regulating motivation skills and actions, we look at participants' discussion of RM (keep records and monitor progress) and the emergent strategy of metacognition. Bailey's reported strategies included marking homework problems she did not fully understand to review them later. She also spoke several times about how useful she viewed her strategy usage while considering her performance on earlier exams. Rose reported a similar strategy to Bailey but focused on reviewing quiz questions that were missed previously instead of homework problems. She also regularly spoke about how useful certain strategies are in specific situations, such as when to use general variables to solve a problem or when to use specific numbers to aid in the understanding of a problem (see Figure 3). Like Rose, Aria kept track of mistakes she made on quizzes and reviewed those problems to study to monitor her progress. Aria also regularly discussed how she viewed her strategy usage, giving insight into why she prefers to work more independently, even when she does have questions. She found primarily, which is likely common to other students, that it is at times inconvenient to ask for others' help. For Aria this was especially prevalent as she felt she did not know many of her peers well enough yet to ask for help. Like Rose and Aria, Frank also kept track of quiz grades and mistakes, adding that he also monitored his work on labs, to see what needed additional time spent studying. Frank spoke about his study habits, comparing certain strategies that he found particularly useful. He found that the practice tests given out before a midterm exam were especially helpful for him, which aligns with his earlier discussion of working through a problem until he memorized the method to solve it. Lastly, Sunny had a broader way of monitoring her progress, reporting that she would just look at her average grade in the course using the online course management system and if the grades looked low to her, she would "start studying a little bit more." Sunny also regularly discussed how her strategies of solving a problem were beneficial, often incorporating the benefits of using multiple strategies or problem-solving techniques while working a problem, such as noticing patterns and sketching a picture or graph of the problem.

For the time and study environment self-regulation strategy, which involves the management and regulation of students' time and environments while studying, I discuss participant interactions that were coded as ENV (changing the environment) from Zimmerman and Pons (1986) and the emergent code Routine, which were instances when participants discuss setting aside time to study on a regular basis. From the interviews, Bailey, Rose, Frank, and Sunny all mention moving to a different physical location than their home to study best, saying the university library or within the mathematics department on campus are beneficial for focus. Aria on the other hand explained that she adjusts her environment by changing the background music or getting rid of distractions. All five of the participants also explained their study routine for preparing for exams and studying in general. Themes between the participants included increased time spent studying during an exam week, spending a dedicated amount of time ranging from one to three hours a day working on homework, reviewing notes with a peer, or taking the day before an exam to focus on studying past homework or practice exams.

The last of the three most prevalent strategies from the MSLQ was effort regulation, which involves students' ability to control their effort and attention in the face of distractions and uninteresting tasks. There is not a clear correlation between Zimmerman and Pons' categorization of self-regulation strategies and this idea, but participants did report on how they stay motivated in certain situations, especially in the situation when they feel unmotivated to continue working. This could include students reminding themselves of the value of the topic or course they are in (coded as Value) or providing performance goals (PG) and rewards (PR) for themselves based on completion of a task. It is important to note that not all three coded strategies (Value, PG, and PR) are be used by every participant, and it is not exhaustive, but these were the three primary strategies the five successful interviewed participants used when facing lack of motivation in their studies. This is similar to Wolters' (1998) findings that found extrinsic motivators, which in this case would be PG and PR, are used for problem situations found irrelevant and identified other types of strategies for other problem situations. Bailey and Sunny both reported that they remind themselves of the value of the course. Bailey said that she needed to finish the course to further her microbiology major, and Sunny reported that as she intends to study mathematics further, so she needs to fully understand the basics of the course to do well in higher levels of mathematics. Bailey and Rose both provide goals based on performance, with Bailey setting course grade goals of trying to get an A in the course, while Rose sets performance goals on the exams and to do that she sets an amount of time to study in hopes to achieve a certain grade. Bailey, Frank, and Sunny also discuss providing extrinsic rewards based on performance to keep themselves motivated. In Bailey and Sunny's cases this looks like providing times of rest after continued study, for Frank, he rewards himself with going out to eat after exams. As a part of her effort regulation, Aria reported that she does not use the strategies the other four participants at times discussed, but rather emphasized that she changes her environment to refocus and ignore distractors.

Chapter 5

Discussion and Conclusion

In the previous chapter, I reported on the self-regulation strategies identified in participant interviews and surveys. Participants reported on their study and homework habits and overall confidence in the course. Mathematics self-efficacy and mathematics identity scores were computed and compared to the use of self-regulation strategies to identify whether a relationship exists between these variables. This statistical analysis furthers the conversation of self-regulation strategies used in academics by Zimmerman and Pons (1986), Wolters (1998), and Johns (2020). Zimmerman, Pons, and Wolters created categorizations I used to identify self-regulation strategies, focusing on those used in a mathematics context. Whereas, Johns established that the use of self-regulation strategies in mathematics is linked to performance in a mathematics classroom.

We begin this chapter with a discussion of the key research questions and how key evidence from the surveys and interviews informs them. Then, I explore three additional ideas including (1) the relationship of students' attitudes about motivation and their self-regulation strategy implementation, (2) the strategies used by successful students, and (3) suggestions for how instructors might foster the use of these strategies for incoming mathematics students.

5.1 Research Questions

5.1.1 RQ 1

The first research question was, "In what ways do students adapt self-regulation strategies from high school mathematics to college-level mathematics?". From examining a large sample of precalculus and calculus student survey responses related to how university mathematics students react to problems upon getting stuck throughout the semester, as well as a more in-depth analysis of nine interviewed participants at three different times during the semester, several ideas emerged as to why students adapt or adopt self-regulation strategies, as well as to what strategies are chosen during this transition to college mathematics. I use the strategies participants used at the beginning of the semester as a proxy for those strategies they likely used in high school mathematics courses. I compare these strategies to those used later or throughout the semester for individual participants and those strategies reported by the larger group of precalculus and first-semester calculus participants on the initial questionnaire. I examined individual interviews and surveys to describe how typical calculus and precalculus students adapt their strategy usage during their first semester in university mathematics. Further, I use the interview data to provide evidence regarding why these adaptations occurred and how they are related to each participant's success in the course.

Common self-regulation strategies found among participants included seeking peer assistance and seeking external resources. Seeking peer assistance has been viewed as an invaluable approach to learning mathematics, as seen in the Emerging Scholars Program model, which creates small, diverse communities of learning to work on mathematics collaboratively (Hsu et al. 2008). The lab setting of the first-semester calculus and precalculus courses investigated in this study naturally promoted this strategy of seeking peer assistance by creating small groups of students that work together on weekly assignments. This setting allowed students to create social networks of peers that can be used when challenges arise in the course. Participants in this study often reported that were in contact with their lab group members outside of class to seek assistance. Furthermore, some students, like Frank, reported that they were able to seek assistance from peers outside of their calculus class, in his case in the form of a roommate that happened to be taking calculus the same semester.

Another common strategy was that of seeking external resources. Seeking external resources frequently appeared in the context of online homework, in which participants often viewed a similar worked-out example. This result aligns with Dorko's (2021) findings that students use the "similar examples" in online homework to copy and paste answers, use the example as a template for their own work, as a way of troubleshooting their work to find an error, to see the form of the answer, to verify that their methods were appropriate and on the right track, to understand better the reasoning in a step or a solution, or lastly for extra practice. Toby found this method to be problematic in his overuse of viewing the similar examples and was not able to transfer the knowledge into his own work on later problems. While this strategy is helpful for students to improve a grade, students may over-rely on this in terms of exam preparation. Toby's extrinsic motivation may have influenced this over-reliance, just as Dorko found that students were often more focused on the goal of homework completion rather than content comprehension when using these similar examples.

Seeking external resources also appeared through finding online sources that offer alternative explanations to mathematics topics. These sites, including Khan Academy and Paul's Online Math Notes, offer additional practice and explanation of various topics to allow students to learn material independently of a lecture or to reinforce concepts learned in a lecture. Participants reported using online resources due to the online nature of their high school mathematics courses during the COVID-19 pandemic and reported being familiar with several websites for reviewing course related material.

In individual and group interviews, participants were able to more clearly express how they used different study habits related to self-regulation strategies in high school. Participants often reported speaking with peers for help, and then if they were still confused, they sought assistance from their teacher. This hierarchy of assistance was often reported in reverse order by college students. While this result may be viewed as an encouragement that participants, like Jay, find their instructors to be highly supportive and available, I believe it can also speak to the discomfort or unfamiliarity of working with new peers. Hernandez-Martinez et al. (2011) reported on this as a social challenge for students entering an unfamiliar environment and developing their sense of belonging, and Seymour and Hunter (2019) found there was a feeling of social isolation in these situations. These social dimension challenges likely apply to Aria when she reports that she did not know her peers well enough to ask questions.

Furthermore, Bailey's sentiment of feeling "stranded" in her group in college because she was not sure "where they are in math, how much they learn" may highlight this feeling of discomfort of working with unfamiliar peers whose mathematical competence is unknown. This difficulty in lab conditions led Bailey to speak with the instructor about getting a new group, which she reported as beneficial to her learning. While not explicitly asking for assistance for the content of the course, Bailey did show use of the self-regulation strategy of seeking assistance from an instructor about the structure of her course. This use of self-regulation strategies can then aid in multiple avenues of the transitional challenges, specifically the social dimension as mentioned above, as outlined by Gueudet (2008), Hernandez-Martinez et al. (2011), and Seymour and Hunter (2019). As mentioned previously, the lab setting for these courses is meant as a bridge to alleviate some of these social challenges. By grouping students, it allows peer study groups to form, with the goal of students then interacting outside of class to aid in their understanding of the course topics.

As for the participants that reported seeking instructor assistance, it seems in contrast to Johnson and Hanson's (2015) finding that most students never attend office hours. Although, those participants often referred to seeking assistance while in a lab setting, reporting infrequent attendance of office hours if at all. This result is consistent with earlier findings of Johnson and Hanson (2015) as well as Cotten and Wilson (2006) who found that freshman did not often interact with faculty outside of a class period. The lab setting for this research did seem to promote and foster the use of seeking teacher assistance as it gave a time and place where students were expected to ask questions. This setting served as a bridge for students to connect with their instructor outside of the traditional lecture period, which promotes the use of positive study habits, which Komarraju et al. (2010) recommend by using orientation courses, but the faculty-student interactions occur here during weekly lab periods.

As in Bailey's situation, it was often a negative factor that triggered the most significant change or adoption of strategy usage. Performing poorly on the first exam of the semester comprised the most common change-inducing factor for the interviewed students. This functioned as the “wakeup call” described by Seymour and Hunter (2019) that allowed students to recognize that the strategies that they used in high school, which may have worked well then, may not be well suited for the college course. This difference in course requirements and expectations is also discussed in Frank and Thompson’s (2021) report finding that US students in precalculus held concept meanings that the authors deemed unproductive in the learning of calculus.

Even in cases where the participant had previously taken the same content course in high school, there are reports of adding or changing study habits. This adaptation showed itself primarily in how much time participants spent studying, with participants adding an hour or more to their weekly routine for their mathematics class or creating a routine to study in the first place. This added time studying took three primary forms: (1) increased time reviewing notes from a lecture or rewatching recorded lectures where the participant was not fully confident in their understanding; (2) increased time in a tutoring center or, although rare, attending office hours with an instructor, recognizing that there are topics not fully understood and the student needed help from an expert source; and (3) increased time working on homework problems or practice problems found online or in the textbook. The first two aspects of this increased time can classified as additional time spent seeking information from various sources, which is discussed by Zimmerman and Pons (1986) as one of the strategies that high achieving students often use. The faculty-student interactions that are promoted by the second aspect of setting a routine are encouraged by Komarraju et al. (2010) when students seek instructor input. These adaptations came after a negative experience (i.e., poor performance on a midterm) that increased awareness of the insufficiency of previous practices. However, in the interest of improving success rates in gateway mathematics courses, more work needs to be done to find ways that students can come to this realization and make changes before a high-stakes adverse event. Clark (2005) and Weidman (1989) found that this freshmen orientation courses aided students in their preparation for college courses, especially those that encouraged faculty-student interactions. Setting clear expectations in these courses allowed students to become more academically integrated into the expectations of the university and develop effective strategies to better performance in mathematics courses.

Cases like Toby's present an issue where a content-based difficulty is compounded by a non-content-based obstacle. He did not understand the material presented in his course well enough to earn a high grade, but he needed to keep a high GPA for future academic studies. This finding is not surprising as research like Gueudet (2008) and Hernandez-Martinez et al. (2011) expand on transitional difficulties that include social and institutional factors. Gueudet describes the difficulty undergraduates have switching between practical and theoretical thinking, so while Toby was able to use practical thinking practices of looking at example problems on homework, he was unable to transition that process into his theoretical knowledge of the topic and how to organize the information for exams. This difficulty led to the institutional challenges of needing to maintain a certain GPA to further his education. Toby's performance in precalculus indicated some proficiency with the material, reflected by his expected grade of C in the course. Although he expressed confidence that he could pass the course, he withdrew from his college precalculus course because he wanted to maintain a high GPA. This non-content-based or external reason influenced him to withdraw with plans to take the mathematics course over the summer.

Just as Toby needed more time and additional instruction to meet his goal of raising his GPA, students developing and implementing self-regulation strategies require time and resources. Due to changes in instructor expectations for the collaborative lab activity, students reported a positive change in confidence and use of self-regulation strategies. At the beginning of the Fall 2021 semester, several instructors collected the collaborative lab activity at the end of the 50-minute period, but shortly after the first midterm there was a change to allow students up to a week to finish the activities. This change in instructor expectations seems to have allowed students to demonstrate more self-regulation strategies of seeking assistance and self-evaluating their work. This change also raised the confidence of several participants as they no longer felt rushed and unable to perform well under an arbitrary time restriction. Given this, I suggest that special attention be paid to the time given to mathematics assignments, as students in this study present a case of significant improvement in performance and confidence after being given time and resources to process the mathematics concepts. As reported by Mesa et al. (2015), when discussing fair assessments, this would not mean to lessen the difficulty but to provide challenging problems and allow students enough time to work through the problems. Furthermore, as shown in research into problem-solving (see Carlson et al., 2008, Schoenfeld, 2014, and Dawkins and Epperson, 2014), there are multiple stages to solving problems which again requires that students be allowed adequate time to work through these types of problems, allowing students to reflect on their work.

In summary, these findings support the first research question by revealing various techniques students use to adapt to college-level mathematics. This evolution of strategies begins with a continuation of high school strategies as seen in the lab surveys, but after some negative experience, students often resolve to adjust their habits to attempt to overcome the obstacle, which could include disengaging from the course to re-attempt the course at a different time. The persistence of strategy usage for the collaborative lab activities, as shown in

Table 15, could be due to participants feeling comfortable in their routine, especially for assignments that are not as high stakes as exams. For example, lab activities for these coordinated courses comprise 5% of the participants' overall course grade, while each exam comprises at least 25% of the overall grade. There was an increased frequency of participants reflecting on similar problems during the lab activities. This practice could be due to their familiarity with looking at similar examples as they did during online homework problems (see Dorko, 2021), or participants had a more extensive portfolio of problems to review as the semester progressed. Using this strategy allows students to build connections between tasks they had previously faced and tasks they were at the time facing. Surprisingly, the perceived difficulty of the lab did not have any noticeable effect on the use or variation of self-regulation strategies, given that more than half of the participants viewed several labs as being more difficult than a typical high school assignment. No significant change in strategies was used to adapt to this perceived difficulty, validated further using the χ^2 test for independence. This lack of adaption based on perceived lab difficulty might speak to students' consistency in strategy usage after establishing a pattern, while for exams students recognized the need to adjust their strategy due to the significant impact on their overall course performance.

The sub-question for the first research question was "How does a student's mathematics identity and/or mathematics self-efficacy influence their implementation of self-regulation strategies?". From the statistical analysis comparing self-regulation strategies versus mathematics identity and self-regulation strategies versus mathematics self-efficacy, we see a weak to moderately strong positive trend where the higher the mathematics identity or mathematics self-efficacy the more likely it is that a student will have a strong use of several self-regulation strategies (see Figure 21 and Figure 22). We can also see from the interviews that individual participants such as Jane, Rose, and Sunny, who had some of the higher scores for self-efficacy and mathematics identity (See Table 4), also had a strong use of self-regulation strategies, especially in terms of organization, elaboration, and effort regulation (See Table 5).

Of these, the strategy most often used successfully by participants with high degrees of self-efficacy and identity was being able to organize information and transform it into something useful. Zimmerman and Pons (1986) found that high achieving students often used this strategy in their academic coursework. This strategy appeared in several interviews with participants writing out information given in the problem, graphing to see its visual components, and sketching out a problem situation and listing out important information from the problem.

Several cases in this research demonstrate that participants' high scores in mathematics self-efficacy and mathematics identity may have hindered their development of the self-regulation strategy of seeking assistance. Jane had a high degree of self-efficacy and identity, as indicated by the initial survey, which, based on her report, also contributed to her low score in help-seeking. At one point in her interviews, she reported that her "pride" stopped her from asking questions of the instructor or peers at times. While a high sense of self-efficacy and identity is often viewed positively, in Jane's case, it likely was related to her avoidance to seek help even if help would benefit her learning of the topic. At several points in the group interviews, participants, including Jane, Frank, and Aria, seemed to work on their own, stopping and attempting to correct their work without asking for help even when another participant was nearby with a valid approach. This absence of help seeking could be influenced by the lack of group work in high school courses that often emphasize individual work. While it is critical that students can understand and perform individually, when learning new topics studying and learning from peers can enhance understanding (see Fullilove and Treisman, 1990). Seeking help and studying in peer groups should be encouraged as a productive strategy for students wanting to excel in mathematics courses.

When examining those interviewed participants whose score on the initial questionnaire indicated a high degree of mathematics self-efficacy and identity compared to those with a lower score of mathematics self-efficacy and identity, we see certain strategies appear in the first group. Interviewed participants with high self-efficacy and high degrees of mathematics identity were Jane, Rose, and Sunny. From their interviews, they showed a higher frequency of setting mastery goals (MG) and considering examples or counterexamples (Examples/ Counterexamples) to solve a problem which was an emergent code I listed as a sub-category to seeking external information (SE), while a generally lower frequency of seeking peer and teacher assistance. Given the limitation of not being able to interview any students with low scores of self-efficacy or mathematics identity, it is not possible in this report to expand on how a low sense of self-efficacy or mathematics identity may affect the development of self-regulation strategies from the interviews. While the initial survey does contain correlation data between a broader range of participants, it does not track the use of those strategies throughout the semester, and the post-lab surveys do not keep track of self-efficacy or mathematics identity scores. Overall, though, while there are some fluctuations and outliers, we do see a positive trend in the use of self-regulation strategies versus both mathematics identity and self-efficacy based on the initial survey. This is also supported in the interviews with participants with medium to high senses of identity and self-efficacy. We do not see a significant difference between precalculus and first-semester calculus participants' use of strategies based on self-efficacy or mathematics identity from the interviews, and from the initial survey, the correlation of those variables is weaker for the precalculus participants, but the small sample size can likely be the cause of this. Tinto (2017) found that having a high degree of self-efficacy was related to the attainment of goals and persistence in academics. The implication for the positive correlation between self-regulation strategies versus mathematics identity and mathematics self-efficacy is then to promote the growth of students' mathematics identity and self-efficacy at a young age, as Hackett and Betz (1989) suggested. As Tinto recommends, this development of self-efficacy can be done through careful monitoring of first-year college students to provide social and academic support when facing difficulties.

Given this information about the positive relationship between self-regulation strategies and both mathematics identity and self-efficacy, a goal of early mathematics education should be the development and encouragement of a positive mathematics identity and confidence. As shown in the correlation data, those students with higher senses of mathematics identity and mathematics self-efficacy are likely to use productive self-regulation strategies at a higher rate. Johns (2020) connects this increase in self-regulation strategies to performance, where she showed that those students with higher usage of self-regulation strategies performed better in a first-semester calculus course. When the participants of this study discussed what influenced their mathematics identity, those whose scores on the initial questionnaire indicated a high degree of mathematics identity often reported having a teacher encourage them. The opposite can also be seen, as a teacher who did not recognize the participant's mathematics ability was reported to have a negatively impact on how participants saw themselves. In Cotten and Wilson's (2006) study, several students reported that a singular negative experience with an instructor was damaging to how they interacted with future instructors, with one student remarking that instructor interactions made them feel unimportant and "like an idiot" (Pg. 500). With this understanding it is then vital that early mathematics educators build up students' mathematics identity and self-efficacy just as they attempt to build up mathematics content knowledge. Furthermore, it is vital for faculty at all academic levels to be cognizant of how they are interacting with students to promote an environment where academic growth is possible.

5.1.2 RQ 2

We use the nine participants' interviews to address the second research question, "How do students characterize differences and similarities between high school mathematics course expectations and college-level mathematics course expectations?". When looking at the collaborative lab activities, most participants reported that some aspect of the lab was unfamiliar to them. The structure of the activity, the work required to be done as a group, or the types of problems posed were causes of this unfamiliarity. Specifically, regarding the types of problems posed, the questions in the activity were often free response and engaged students in problem-solving. A significant difference between what participants expected from their high school courses and college-level courses was the difficulty of the work. Participants that had taken calculus in high school, in general, reported a high degree of familiarity with the course content. Some participants, like Sunny, found a stronger emphasis on understanding how and why certain equations or theorems worked rather than focusing on memorization and utilization of formulae. Participants also expressed unfamiliarity with the format and difficulty of the exams in their courses, often saying they were unprepared and did not study as much as they should have. This is consistent with Sonnert et al.'s 2020 findings that high school preparation in mathematics was a significant influencer on performance and connects to Hernandez-Martinez et al. (2011), who found that although these challenges were present, they could be viewed as opportunities for growth. This growth is frequently reported in this study as the development or refinement of self-regulation strategies.

It was common for several of the interviewed participants to report minimally studying in high school for exams, expecting a high grade on high school exams without additional studying. Participants, like Toby, reported that the tests given in his high school were often the same structure and questions as a review, only with a slight variation in numbers used. Furthermore, participants reported that after the first college mathematics exam they began to adjust their self-regulation strategies, as mentioned previously, which included self-evaluating their understanding by carefully and frequently working through practice exams before the exams. Participants attempted to become familiar with the expectations that the instructors had for them to perform well on the exams, by understanding the format of the exams, attending to the academic integration as outlined by Weidman (1989) to recognize the course expectations and exam expectations as being legitimate. Several of the interviewed participants reported doing poorly on the first exam, resolving to prepare better for the second exam, but still did not have a strong method of how to prepare, only citing a loose strategy of spending more time studying, but did not have any concrete strategies. Considering this, it may be beneficial to incorporate a lower-stakes exam earlier in the semester to help students confront possible weaknesses in their study habits and then encourage the use of self-regulation strategies to improve their understanding through explicit instruction, especially with those students that may have performed particularly poorly. Simzar et al. (2015) found that for mathematics courses, lower-stakes test performance is often predicted by motivational strategies, as some students may not put in the same level of effort as for higher stakes exams. As such, it is important to recognize that these exams may not measure retention of content knowledge but rather students' motivational habits.

Students come from high schools experiencing varying structures. Due to the unique timing of this research, each interviewed participant attended high school during the COVID-19 pandemic. This structural difference of primarily online schooling has highlighted several critical influences of high school education on students' preparation for college-level mathematics courses. Students experienced limited means of direct communication with teachers and peers, which led to more individualized learning, including seeking external sources. While there are ubiquitous online resources for mathematics, students reported feeling unprepared, often citing the online format as a major factor in this. One participant found that the recorded nature of her lectures in high school allowed her to lose focus, reporting that she could just rewatch it later. This could be fixed by encouraging an active-learning format in online courses, which keeps students' attention more and promote engagement during instruction (see Mesa et al. 2015).

This acts as a cautionary report for shifting to primarily online education for mathematics. The expectations established by the university in the study were for students to be able to work in groups and engage in active problem solving during the collaborative lab periods. The conditions that the participants in the study reported as being unhelpful to their learning and preparation for college are not unique to the online learning that occurred during the pandemic. A lack of collaborative group work, problem-solving activities, active-learning experiences, and rich interactions between teachers and students is not restricted to online high school courses but may have been more prevalent in them. This further underscores the importance of emphasizing active-learning experiences centered on rich mathematics problems in high school mathematics. Mesa et al. (2015) identify the use of active learning as having a positive impact on undergraduates' learning of mathematics. They also report on the K-12 mathematics literature that using active learning tasks in high school produces positive performance gains and affective gains in mathematics.

While academic integration has positive benefits for students (see Weidman, 1989), interviewed participants reported variations on how they might achieve academic integration. This is consistent with Weidman's conclusions that individuals have varying ways to accomplish this, especially as different instructors have different course expectations. Some interviewed participants reported they would attend more tutoring, and some reported they would spend more time working through homework or rewatching lectures. A consistent approach from most participants was the expectation that their college mathematics coursework would require more time than their high school mathematics coursework. However, from these reported strategies, participants reported not having significant direction to aid their academic integration. While instructors may have known some best practices for students and potentially informed select students of these ideas, interviewed participants did not seem to have been influenced by instructor input into how they might study better.

Some have recommended (see Weidman, 1989 and Tinto, 2017) that orientation courses that focus on providing early access for students to interact with instructors encourage the formation of academic integration. From a selection of mathematicians discussing what aspects of their schooling aided in their rich understanding of mathematics, one of the major influences was having a supportive teacher at some point promoting their continued study of mathematics (Riley, 2022). Komarraju et al. (2010) also found that this interaction between students and instructors was highly influential in developing motivational strategies and academic achievement. The question of further research then becomes how can instructors best prepare students to become academically integrated and support their students in the attainment of beneficial self-regulation strategies, such as seeking assistance and organizing and transforming information into something usable.

When trying to address the sub-question of the second research question, “What role does mathematics identity and/or mathematics self-efficacy play in students’ academic integration in precalculus and calculus?” I did not find a strong indicator of students’ academic integration during the interviews. Most participants at some point reported adjusting their exam study habits to improve their scores on exams, but whether it was to match the perceived instructors’ expectations or more personal course expectations is unclear. With this and the lack of strong variation in mathematics self-efficacy and mathematics identity, further research would be needed to address this question. In the same way, academic integration was not measured on any quantitative survey, so correlation data could not be analyzed. To better track the development of academic integration, instructor expectations could have been compiled before the start of the semester, and then student expectations could be measured on how much they agree with the instructor statements using a Likert scale. This measurement tool could be given as a pre/post survey to see if there is any change in student expectations and academic integration after having gone through the course.

5.2 Emergent Themes

Several themes emerged that provide insight into how participants use self-regulation strategies. These include the relationship between intrinsic and extrinsic motivational factors, what strategies academically successful students use, and how to foster these strategies in incoming students.

5.2.1 Intrinsic and Extrinsic Motivation

When analyzing a subset of scores from the MSLQ, specifically how intrinsic and extrinsic motivation correlates to the use of the self-regulation strategies presented from the MSLQ, we see the following trends. For all but two strategies, rehearsal and effort regulation, the correlation between intrinsic motivation and the remaining strategies is higher than that of extrinsic motivation. The highest correlation between intrinsic motivation and self-regulation strategies included critical thinking and metacognitive self-regulation, and to a lesser degree the strategies included elaboration and peer learning.

This research did not examine participants' critical thinking ability, but while working out the collaborative lab problems, several students demonstrated critical thinking and problem-solving skills. Critical thinking and problem-solving techniques have been discussed in relation to achievement in calculus (see Dawkins & Epperson, 2014). We see that students that were able to problem-solve using algebraic fluency and conceptual understanding of basic calculus were more successful in solving non-routine problems. In this study, we saw evidence of problem-solving strategies related to self-regulation strategies, including seeking assistance, looking for external resources when stuck, and considering examples or counterexamples to problems to generalize a solution path.

The second category identified as correlating to high intrinsic motivation was metacognitive self-regulation. This strategy, as defined in Table 2, is also seen in interviews, marked as Metacognition while discussing them in Chapter 4, and occurs as participants discuss how they see their own strategy usage as helping or hindering their academic progress. This often occurred when participants explained why they adjusted their strategy usage between midterms, noting that some strategies may not have been as successful as they had hoped. While successful participants examined different self-regulation strategies while speaking metacognitively, the process of identifying a strategy as being beneficial or detrimental was viewed positively to students' academic integration and development, as also found in Pape, Bell, and Yetkin (2003). This identification included participants discussing the advantages of strategies, such as the utility of graphing a function using a resource like Desmos or breaking a problem into several parts to simplify the work required. As well as the detrimental aspects, such as Toby's discussion of how his use of viewing similar examples was unhelpful for his exam preparation. As a student with a high intrinsic motivation would be more inclined to gain understanding to further an internal goal, it follows that this sense of motivation would be more closely related to a metacognitive aspect of examining strategy usage as being advantageous or not. This result is consistent with Johns’ (2020) work that connected high-performing students with a higher sense of intrinsic motivation and use of self-regulation strategies. We see this further through the in-depth look at how these strategies manifest in participants during group interviews as described above.

Extrinsic motivators for students can include high grades or awards for performance. While this can lead to the use of strong self-regulation strategies, we see mixed expressions of this motivation. Having an extrinsic motivation is not negative in these situations but does not seem to produce as strong self-regulation strategies as intrinsic motivation. In Toby’s case, it led to dropping out of the course to get a higher grade at a different time, or participants like Cyndy who, when studying, focused heavily on homework exercises expecting that to be enough to perform well in the course. The most common extrinsic motivator for participants was their performance on midterms in the course, as those grades tended to influence whether participants used the same study strategies or made some adaptations. Seymour and Hunter (2019) found that a common factor for students that contributed to them leaving a STEM degree was a loss of motivation and discouragement from low grades in early semesters. They also found that intrinsic interest in the subject matter was one of the best predictors of persistence for STEM students. Further research is then needed on how to foster students’ intrinsic motivation.

5.2.2 Self-Regulation Strategies used by Academically Successful Students

In the following section, I discuss those specific strategies commonly used by participants identified as academically successful from their self-projected course grades. From the initial survey and the interviewed participants, we see a high use of self-regulation strategies for successful students, consistent with Johns’ (2020) findings. Analyzing individual student’s usage of self-regulation strategies, we see that the strategies that emerged as most prevalent from the initial survey were metacognitive self-regulation, time and study environment, and effort regulation. This suggests that the effort and thought behind each student's study habits have more of an impact over whether they use a particular study habit or not. Johns (2020) found that the strategy of metacognitive self-regulation was more highly reported in high-achieving calculus students compared to low-achieving calculus students. She also found that the other two strategies, time and study environment and effort regulation, were more highly reported for high-achievers and overachievers. These two categories can be linked to Tinto's (2017) ideas about persistence in that it is those students who can regulate their effort, or persist, that often perform better academically.

It is worrying that peer learning would rank so low in the courses as that is highly encouraged by instructors of both calculus and precalculus at the institution where the research occurred, as well as mathematics education researchers such as Fullilove and Treisman (1990). However, it is possible that if this survey had been given out at the end of the semester, the use of these strategies would look different. This result seems contrary to Zimmerman and Pons (1986) findings on what self-regulation strategies were used by high-achieving students. However, we did see a high use of each of the strategies they found earlier (seeking information, keeping records and monitoring, and organizing and transforming information), but as they looked at high school students within several courses it does make sense that in a mathematics specific domain there would be slight variation. We also see a high frequency of those strategies used in the group interviews from the high achieving students' work and discussion of the lab activities.

5.2.3 Fostering Self-Regulation Strategies

Identifying what strategies are used by successful participants then becomes a point of discussion as to how these strategies might be fostered within incoming students. In the following section, I provide suggestions on which to better the usage of these strategies. First a general suggestion is given from the MSLQ as written by Pintrich et al. (1991), and then given more insight into how this fits into a mathematics specific domain.

The MSLQ manual gives suggestions for the development of each strategy assessed and scored on the initial survey we look at the three most prevalent strategies used by academically successful participants: metacognitive self-regulation, time and study environment, and effort regulation. For the metacognitive self-regulation they recommend to:

> Skim the reading material before you begin to see how it is organized. Look at headings and subheadings of the text to give yourself an idea of how things are related to each other. While reading, ask yourself questions about the paragraph you have just read and scribble key words in the margins of the book or in a notebook. Try to determine which concepts you don't understand well.

A common theme from the five participants interviewed who were recognized as being academically successful, Aria, Bailey, Rose, Frank, and Sunny, was the regular review of graded work to "determine which concepts [they] don't understand well." And while the earlier part of the recommendation is not mathematics specific, it could be further specified for mathematics students to look over material in the textbook prior to lecture to "see how it is organized" and work through the example problems while "ask[ing] yourself questions" about the methods used or any aspect of the problem that is not fully understood. Beyond this, I believe that the act of students talking with peers or instructors about their strategies to solve a problem or methods to study for a test is highly beneficial, consistent with Komarraju, Musulkin, and Bhattacharya (2010). In these conversations insights may be provided about what methods have been successful in the past or perhaps ways to refine current study methods if viewed as ineffective.

As before, the MSLQ manual gives suggestions on how to better the use of time and study environment:

> Keep track of what you do with your study time for a week. Write down your goals for each study period and then write down what you actually accomplished during that study period. Analyze the chart at the end of the week. You may want to change the place where you study, or the times when you study, or who you study with.

While the participants did not discuss using a goal chart when studying, this more metacognitive aspect could certainly be helpful to some students. The five participants mentioned previously used the later parts of the suggestions, which were to change their space, time, or study group to best focus on the task of studying. Due to the numerous distractions that are around students, especially in their residences, the strategy of changing location to a space that they feel comfortable studying in seems an especially beneficial strategy.

As with the previous two strategies, the MSLQ provides suggestions on how to better the use of effort regulation:

> Keep a list of the topics that you find yourself procrastinating instead of studying for. Try to analyze why you postpone studying these topics by discussing them with other students. Talking with them may lead you to consider an approach that may help you act more quickly instead of delaying studying the material.

As with the previous suggestion the MSLQ manual focuses on a metacognitive approach of being aware of how students' study and then encourages processing the strategy usage with peers. This range of strategy usage and metacognitive approach is consistent with Wolters (1998) findings that certain problem situations, i.e., a task being perceived as too difficult, irrelevant, or boring, have particular types of self-regulation strategies associated with overcoming these challenges. This result is more focused on how students view solving mathematics problems and studying in mathematics courses but does seem to align with Wolters findings. This strategy recommendation also includes seeking peer assistance to better understand problems or solution methods.

In each of these instances of metacognitive strategy usage, there will naturally be variation amongst students, participants like Rose reported that other peers have suggested she not study immediately before a test but recognized that her strategy has seemed successful for the second midterm she had in precalculus and intended to continue with that strategy. Frank and Aria found some success without seeking much peer assistance, even though it would likely benefit them to engage in more peer-to-peer dialogue. This process of fostering beneficial self-regulation strategies is aided by looking at general trends to see what most successful students employ, but without the knowledge of the individual students, instructors may not be well equipped to provide support to their students. It at times requires a supportive instructor, who is willing to interact and engage with their students about these strategies, to better foster productive skills in their students.

5.3 Limitations

This research was sometimes limited by the sample size and availability of participants. While the initial survey population was enough to produce appropriate quantitative results for calculus, the relatively few precalculus participants $(n = 25)$ did not allow for strong quantitative results to show if there are any significant differences between the calculus and precalculus populations. Along with this there was not a significant variation in the mathematics self-efficacy and mathematics identity scores for the interviewed participants. Originally the goal was to compare students with more variability in their self-efficacy and mathematics identity scores. Although this was not able to be done, we did see a high variability in performance for the interviewed students. We were then able to see how medium to high scores of self-efficacy and mathematics identity related to self-regulation strategies and performance in a first-semester college mathematics course. Focusing on students with higher scores in those categories allowed us to show more common strategies used by successful students. It is also worth noting that the population of interviewed participants is unique in that their last year of high school occurred during the COVID-19 pandemic, and the online format of their high school courses may have influenced their use of self-regulation strategies and study habits. Further, this research took place during the COVID-19 pandemic after many lock-down restrictions had been lifted, yet many students and instructors had lingering fatigue from the stress of the pandemic.

5.4 Conclusion

In conclusion, as presented from the quantitative data from the surveys and the qualitative data from the interviews, we see that self-regulation strategies have a positive correlation with both mathematics self-efficacy and mathematics identity and that these self-regulation strategies have positive benefits on the academic success of participants entering college mathematics courses. By identifying successful participants' self-regulation strategies, we can begin to consider how to best foster these strategies and support academic excellence and retention in these gateway courses for all incoming students. Findings suggest that by developing and encouraging a positive mathematics identity and a high degree of mathematics self-efficacy in students that mathematics students will be better prepared for the transition to college and the development of useful self-regulation strategies. Much of the self-regulation research has focused on middle school and high school education, with few studies examining strategy usage in undergraduate mathematics. Further research is needed on how students' self-regulation strategies may adapt later in their academic careers and whether instructors can successfully foster self-regulation strategies in calculus and precalculus.

Appendix A

Motivated Strategies for Learning Questionnaire

This is the user manual for the MSLQ. The MSLQ is in the public domain, and so you do not need permission to use the instrument. We do ask that you simply cite it appropriately (Pintrich, P.R., Smith, D.A.F., García, T., & McKeachie, W.J. (1991). A manual for the use of the motivated strategies questionnaire (MSLQ). Ann Arbor, MI: University of Michigan, National Center for Research to Improve Postsecondary Teaching and learning.)

Part A. Motivation

The following questions ask about your motivation for and attitudes about this class. **Remember there are no right or wrong answers, just answer as accurately as possible**. Use the scale below to answer the questions. If you think the statement is very true of you, circle 7; if a statement is not at all true of you, circle 1. If the statement is more or less true of you, find the number between 1 and 7 that best describes you.

1	2	3	4	5	6	7
not at all true of me						very true of me

1.	In a class like this, I prefer course material that really challenges me so I can learn new things.	1	2	3	4	5	6	7
2.	If I study in appropriate ways, then I will be able to learn the material in this course.	1	2	3	4	5	6	7
3.	When I take a test I think about how poorly I am doing compared with other students.	1	2	3	4	5	6	7
4.	I think I will be able to use what I learn in this course in other courses.	1	2	3	4	5	6	7
5.	I believe I will receive an excellent grade in this class.	1	2	3	4	5	6	7
6.	I'm certain I can understand the most difficult material presented in the readings for this course.	1	2	3	4	5	6	7
7.	Getting a good grade in this class is the most satisfying thing for me right now.	1	2	3	4	5	6	7
8.	When I take a test I think about items on other parts of the test I can't answer.	1	2	3	4	5	6	7

		not at all true of me						very true of me
9.	It is my own fault if I don't learn the material in this course.	1	2	3	4	5	6	7
10.	It is important for me to learn the course material in this class.	1	2	3	4	5	6	7
11.	The most important thing for me right now is improving my overall grade point average, so my main concern in this class is getting a good grade.	1	2	3	4	5	6	7
12.	I'm confident I can learn the basic concepts taught in this course.	1	2	3	4	5	6	7
13.	If I can, I want to get better grades in this class than most of the other students.	1	2	3	4	5	6	7
14.	When I take tests I think of the consequences of failing.	1	2	3	4	5	6	7
15.	I'm confident I can understand the most complex material presented by the instructor in this course.	1	2	3	4	5	6	7
16.	In a class like this, I prefer course material that arouses my curiosity, even if it is difficult to learn.	1	2	3	4	5	6	7
17.	I am very interested in the content area of this course.	1	2	3	4	5	6	7
18.	If I try hard enough, then I will understand the course material.	1	2	3	4	5	6	7
19.	I have an uneasy, upset feeling when I take an exam.	1	2	3	4	5	6	7

	not at all true of me						very true of me
20. I'm confident I can do an excellent job on the assignments and tests in this course.	1	2	3	4	5	6	7
21. I expect to do well in this class.	1	2	3	4	5	6	7
22. The most satisfying thing for me in this course is trying to understand the content as thoroughly as possible.	1	2	3	4	5	6	7
23. I think the course material in this class is useful for me to learn.	1	2	3	4	5	6	7
24. When I have the opportunity in this class, I choose course assignments that I can learn from even if they don't guarantee a good grade.	1	2	3	4	5	6	7
25. If I don't understand the course material, it is because I didn't try hard enough.	1	2	3	4	5	6	7
26. I like the subject matter of this course.	1	2	3	4	5	6	7
27. Understanding the subject matter of this course is very important to me.	1	2	3	4	5	6	7
28. I feel my heart beating fast when I take an exam.	1	2	3	4	5	6	7
29. I'm certain I can master the skills being taught in this class.	1	2	3	4	5	6	7
30. I want to do well in this class because it is important to show my ability to my family, friends, employer, or others.	1	2	3	4	5	6	7

31. Considering the difficulty of this course, the teacher, and my skills, I think I will do well in this class.	1	2	3	4	5	6	7

Part B. Learning Strategies

The following questions ask about your learning strategies and study skills for this class. **Again, there are no right or wrong answers. Answer the questions about how you study in this class as accurately as possible**. Use the same scale to answer the remaining questions. If you think the statement is very true of you, circle 7; if a statement is not at all true of you, circle 1. If the statement is more or less true of you, find the number between 1 and 7 that best describes you.

1	2	3	4	5	6	7
not at all true of me						very true of me

32. When I study the readings for this course, I outline the material to help me organize my thoughts.	1	2	3	4	5	6	7
33. During class time I often miss important points because I'm thinking of other things.	1	2	3	4	5	6	7
34. When studying for this course, I often try to explain the material to a classmate or friend.	1	2	3	4	5	6	7
35. I usually study in a place where I can concentrate on my course work.	1	2	3	4	5	6	7
36. When reading for this course, I make up questions to help focus my reading.	1	2	3	4	5	6	7
37. I often feel so lazy or bored when I study for this class that I quit before I finish what I planned to do.	1	2	3	4	5	6	7

38.	I often find myself questioning things I hear or read in this course to decide if I find them convincing.	1	2	3	4	5	6	7
39.	When I study for this class, I practice saying the material to myself over and over.	1	2	3	4	5	6	7

		not at all true of me						very true of me
40.	Even if I have trouble learning the material in this class, I try to do the work on my own, without help from anyone.	1	2	3	4	5	6	7
41.	When I become confused about something I'm reading for this class, I go back and try to figure it out.	1	2	3	4	5	6	7
42.	When I study for this course, I go through the readings and my class notes and try to find the most important ideas.	1	2	3	4	5	6	7
43.	I make good use of my study time for this course.	1	2	3	4	5	6	7
44.	If course readings are difficult to understand, I change the way I read the material.	1	2	3	4	5	6	7
45.	I try to work with other students from this class to complete the course assignments.	1	2	3	4	5	6	7
46.	When studying for this course, I read my class notes and the course readings over and over again.	1	2	3	4	5	6	7
47.	When a theory, interpretation, or conclusion is presented in class or in the readings, I try to decide if there is	1	2	3	4	5	6	7

good supporting evidence.

48.	I work hard to do well in this class even if I don't like what we are doing.	1	2	3	4	5	6	7
49.	I make simple charts, diagrams, or tables to help me organize course material.	1	2	3	4	5	6	7

		not at all true of me						very true of me
50.	When studying for this course, I often set aside time to discuss course material with a group of students from the class.	1	2	3	4	5	6	7
51.	I treat the course material as a starting point and try to develop my own ideas about it.	1	2	3	4	5	6	7
52.	I find it hard to stick to a study schedule.	1	2	3	4	5	6	7
53.	When I study for this class, I pull together information from different sources, such as lectures, readings, and discussions.	1	2	3	4	5	6	7
54.	Before I study new course material thoroughly, I often skim it to see how it is organized.	1	2	3	4	5	6	7
55.	I ask myself questions to make sure I understand the material I have been studying in this class.	1	2	3	4	5	6	7
56.	I try to change the way I study in order to fit the course requirements and the instructor's teaching style.	1	2	3	4	5	6	7
57.	I often find that I have been reading for this class but don't know what it was all about.	1	2	3	4	5	6	7

58.	I ask the instructor to clarify concepts I don't understand well.	1	2	3	4	5	6	7
59.	I memorize key words to remind me of important concepts in this class.	1	2	3	4	5	6	7
60.	When course work is difficult, I either give up or only study the easy parts.	1	2	3	4	5	6	7

		not at all true of me						very true of me
61.	I try to think through a topic and decide what I am supposed to learn from it rather than just reading it over when studying for this course.	1	2	3	4	5	6	7
62.	I try to relate ideas in this subject to those in other courses whenever possible.	1	2	3	4	5	6	7
63.	When I study for this course, I go over my class notes and make an outline of important concepts.	1	2	3	4	5	6	7
64.	When reading for this class, I try to relate the material to what I already know.	1	2	3	4	5	6	7
65.	I have a regular place set aside for studying.	1	2	3	4	5	6	7
66.	I try to play around with ideas of my own related to what I am learning in this course.	1	2	3	4	5	6	7
67.	When I study for this course, I write brief summaries of the main ideas from the readings and my class notes.	1	2	3	4	5	6	7
68.	When I can't understand the material in this course, I ask another student in this class for help.	1	2	3	4	5	6	7

69.	I try to understand the material in this class by making connections between the readings and the concepts from the lectures.	1	2	3	4	5	6	7
70.	I make sure that I keep up with the weekly readings and assignments for this course.	1	2	3	4	5	6	7
71.	Whenever I read or hear an assertion or conclusion in this class, I think about possible alternatives.	1	2	3	4	5	6	7

		not at all true of me						very true of me
72.	I make lists of important items for this course and memorize the lists.	1	2	3	4	5	6	7
73.	I attend this class regularly.	1	2	3	4	5	6	7
74.	Even when course materials are dull and uninteresting, I manage to keep working until I finish.	1	2	3	4	5	6	7
75.	I try to identify students in this class whom I can ask for help if necessary.	1	2	3	4	5	6	7
76.	When studying for this course I try to determine which concepts I don't understand well.	1	2	3	4	5	6	7
77.	I often find that I don't spend very much time on this course because of other activities.	1	2	3	4	5	6	7
78.	When I study for this class, I set goals for myself in order to direct my activities in each study period.	1	2	3	4	5	6	7
79.	If I get confused taking notes in class, I make sure I sort it out afterwards.	1	2	3	4	5	6	7

80.	I rarely find time to review my notes or readings before an exam.	1	2	3	4	5	6	7
81.	I try to apply ideas from course readings in other class activities such as lecture and discussion.	1	2	3	4	5	6	7

Appendix B

Mathematics Identity Survey

RE: Mathematics Identity Research Survey

EK **Eivind Kaspersen <eivind.kasper...** Saturday, October 31, 2020 at 11:00 AM

To: Turner, Kyle

You replied to this message.

This message is flagged for follow up.

Hi Kyle,

I am happy if you find the items useful. You can use the survey in any way you like. There is really no objective criterion for saying that a person has a "low" mathematical identity. In a few situations, I have used the term, but my suggestion is that you avoid it if you can.

You can also amend the instrument as you prefer. E.g., since you conduct a correlation study, you might want to consider to add some items; more items tend to increase reliability with the effect that the reported correlations advance towards the "true" correlations.

Best wishes,
Eivind

Never/almost never(1), Sometimes(2), Often(3), Always/almost always(3), Don't know(9)					
1. I take the initiative to learn more about math than what is required at school/work.	1	2	3	4	9
2. When I learn a new method, I take time to find out if I can find a better method.	1	2	3	4	9
3. When I learn a new method, I try to think of situations when it wouldn't work.	1	2	3	4	9
4. I struggle with putting math problems aside.	1	2	3	4	9
5. If I forget a formula or method, I try to derive it myself.	1	2	3	4	9
6. I get engaged when someone starts a mathematical discussion.	1	2	3	4	9
7. When I learn something new, I make my own problems.	1	2	3	4	9
8. Math ideas that I hear or learn about help me inspire new trains of thoughts.	1	2	3	4	9
9. When I learn a new method, I like to be told exactly what to do.	1	2	3	4	9
10. When I try to use a method that doesn't work, I spend time to find out why it didn't work.	1	2	3	4	9
11. When I learn a new formula/algorithm, I try to understand why it works.	1	2	3	4	9
12. When I face a proof, I study it until it becomes meaningful.	1	2	3	4	9
13. When I face a math problem, I consider different possible ways I can solve it.	1	2	3	4	9
14. When I work with a math problem, I move back and forth between various strategies.	1	2	3	4	9
15. When I learn something new, it makes me want to learn more things.	1	2	3	4	9
16. When I work with a problem, I pause along the way to reflect on what I am doing.	1	2	3	4	9
17. If I get stuck on a problem, I try to visualize it.	1	2	3	4	9
18. I can explain why my solutions are correct.	1	2	3	4	9
19. I try to connect new things I learn to what I already know.	1	2	3	4	9
20. If I immediately do not understand what to do, I keep trying.	1	2	3	4	9

Appendix C

Factors Influencing College Success in Mathematics

Re: Mathematics Attitudes Survey for College Calculus

SG

Sonnert, Gerhard <gsonnert@cf... Thursday, November 5, 2020 at 9:47 AM

To: Turner, Kyle; **Cc:** Sadler, Philip

283447-1vb-2.pdf
858.1 KB

Download All · Preview All

You replied to this message.

Dear Kyle,

Thank you for your interest in our paper. I am glad you found it helpful.

You have our permission to use our math attitudes survey questions for your research. Please acknowledge their origin in any publications/presentations that will result from your work.

Wishing you best of luck and much success with your project,

Gerhard Sonnert

CONCERNING YOUR PERSONAL THOUGHTS AND FEELINGS TOWARDS MATHEMATICS:

44. Do you agree or disagree with the following statements?

	Agree	Disagree		Agree	Disagree
I enjoy learning math.	○	○	I can do well on math exams.	○	○
Math is interesting.	○	○	I look forward to taking math.	○	○
Math makes me nervous.	○	○	I wish I did not have to take math.	○	○
Math is relevant to real life.	○	○	I understand the math I have studied.	○	○
Setbacks do not discourage me.	○	○			

45. Do the following people see you as a mathematics person?

	No, not at all					Yes, very much
Yourself	1	2	3	4	5	6
Parents/Relatives/Friends	1	2	3	4	5	6
Mathematics teacher	1	2	3	4	5	6

59. Please DO NOT bother to attempt to solve these problems. We would like to know whether or not your high school teacher(s) spent time teaching how to solve these types of problems in any of your high school mathematics courses.

	Yes	No
Find all solutions to the equation $2\sin\theta = -1$ on the interval $0 \le \theta \le 2\pi$.	○	○
Evaluate $\ln\left(\sqrt[3]{e}\right)$.	○	○
Find $\lim_{x \to +\infty} \frac{x^3 - 5}{4x^3 + x + 1}$.	○	○
Find $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ where $f(x) = x^2 + 4x + 9$.	○	○
Prove that $g(x) = \begin{cases} 1 & -\infty < x < -1 \\ -x & -1 \le x < 1 \\ x - 2 & 1 \le x < +\infty \end{cases}$ is continuous at $x = -1$ and $x = 1$.	○	○
Can the function $y=x^3-x$ be equal to zero in the interval $0<x<1$? Why or why not?	○	○
Give an epsilon-delta proof for the existence of the limit $\lim_{x \to 2}(3x - 1)$.	○	○
True or False? $\tan(hx) = \tanh(x)$	○	○
Find the Taylor series expansion of $\sin(x^2)$.	○	○
Given $f(x,y) = x^4 + x^3y - 3x^2y^2 + y^4$, find $\frac{\partial}{\partial x} f(x,y)$ and $\frac{\partial}{\partial y} f(x,y)$.	○	○
Evaluate $\iint_D [xy^2 + x^2y + 3]\,dydx$ where D is the region $0 \le x \le 1, 2 \le y \le 4$.	○	○

Appendix D

Sources of Self-Efficacy

RE: Sources of Self-Efficacy Survey

UE **Usher, Ellen L.** <ellen.usher@u... Wednesday, November 4, 2020 at 9:51 AM

To: Turner, Kyle

You replied to this message.

Hi Kyle,

Certainly you have my permission. That sounds like an interesting study. I look forward to hearing about your findings.

Best wishes,

Ellen

Ellen Usher
Professor and Program Chair, Educational Psychology
Director, P20 Motivation and Learning Lab

Table 4
Standardized factor pattern loadings for final sources of self-efficacy items by subgroup

	Item	Full Sample	Girls	Boys	African American	White	On Level	Above Level
1.	I make excellent grades on math tests (ME-1)[2]	.783 (.622)	.791 (.611)	.772 (.635)	.648 (.762)	.804 (.594)	.786 (.618)	.781 (.625)
2.	I have always been successful with math (ME-3)[2]	.740 (.672)	.743 (.669)	.736 (.677)	.740 (.673)	.723 (.691)	.722 (.692)	.756 (.654)
3.	Even when I study very hard, I do poorly in math (ME-6)[*1]	.677 (.736)	.698 (.716)	.652 (.759)	.611 (.792)	.711 (.703)	.705 (.709)	.643 (.766)
4.	I got good grades in math on my last report card (ME-8)[1]	.668 (.744)	.664 (.748)	.679 (.734)	.564 (.826)	.667 (.745)	.672 (.740)	.649 (.761)
5.	I do well on math assignments (ME-9)[1M]	.827 (.562)	.810 (.586)	.854 (.520)	.801 (.599)	.831 (.556)	.818 (.575)	.815 (.580)
6.	I do well on even the most difficult math assignments (ME-12)[3]	.793 (.610)	.812 (.584)	.766 (.643)	.724 (.690)	.827 (.562)	.775 (.632)	.841 (.542)
7.	Seeing adults do well in math pushes me to do better (VA-4)[2]	.699 (.716)	.720 (.694)	.683 (.731)	.705 (.709)	.682 (.731)	.683 (.731)	.730 (.683)
8.	When I see how my math teacher solves a problem, I can picture myself solving the problem in the same way (VA-6)[2]	.745 (.667)	.756 (.654)	.737 (.676)	.766(.643)	.753 (.658)	.739 (.674)	.745 (.668)
9.	Seeing kids do better than me in math pushes me to do better (VP-1)[1]	.627 (.779)	.596 (.803)	.657 (.753)	.669 (.743)	.614 (.789)	.620 (.785)	.637 (.771)
10.	When I see how another student solves a math problem, I can see myself solving the problem in the same way (VP-9)[2]	.681 (.732)	.639 (.770)	.718 (.696)	.619 (.786)	.697 (.718)	.696 (.719)	.635 (.773)
11.	I imagine myself working through challenging math problems successfully (VS-4)[1]	.714 (.700)	.761 (.649)	.670 (.742)	.710 (.704)	.719 (.695)	.701 (.714)	.724 (.690)
12.	I compete with myself in math (VS-5)[3]	.631 (.776)	.563 (.827)	.700 (.714)	.691 (.723)	.582 (.813)	.655 (.756)	.669 (.744)
13.	My math teachers have told that I am good at learning math (P-4)[1M]	.704 (.710)	.680 (.733)	.728 (.686)	.643 (.766)	.711 (.703)	.702 (.712)	.751 (.660)
14.	People have told me that I have a talent for math (P-5)[3]	.741 (.672)	.740 (.673)	.739 (.673)	.723 (.691)	.744 (.668)	.752(.660)	.717 (.697)
15.	Adults in my family have told me what a good math student I am (P-7)[2]	.741(.671)	.737 (.676)	.746 (.666)	.675 (.738)	.761 (.648)	.754 (.657)	.697 (.717)
16.	I have been praised for my ability in math (P-13)[1M]	.812 (.584)	.830 (.557)	.790 (.613)	.790 (.614)	.815 (.579)	.819 (.573)	.781 (.625)
17.	Other students have told me that I'm good at learning math (P-14)2M	.792 (.610)	.829 (.559)	.765 (.644)	.743 (.669)	.816 (.578)	.797 (.604)	.835 (.551)
18.	My classmates like to work with me in math because they think I'm good at it (P-16)[1M]	.715 (.700)	.762 (.647)	.667 (.745)	.666 (.746)	.718 (.696)	.736 (.677)	.699 (.715)
19.	Just being in math class makes feel stressed and nervous (PH-2)[*1M]	.779 (.626)	.827 (.562)	.722 (.691)	.644 (.765)	.815 (.579)	.784 (.621)	.805 (.593)
20.	Doing math work takes all of my energy (PH-3)[*2]	.612 (.791)	.617 (.787)	.607 (.795)	.449 (.893)	.672 (.740)	.604 (.797)	.633 (.774)
21.	I start to feel stressed-out as soon as I begin my math work (PH-5)[*1]	.823 (.568)	.843 (.538)	.797 (.604)	.799 (.601)	.837 (.547)	.824 (.567)	.843 (.538)
22.	My mind goes blank and I am unable to think clearly when doing math work (PH-7)[*1]	.693 (.721)	.715(.699)	.668 (.744)	.657 (.754)	.725 (.689)	.663 (.748)	.757(.653)
23.	I get depressed when I think about learning math (PH-9)[*1M]	.694 (.720)	.724 (.690)	.660 (.751)	.635 (.773)	.729 (.684)	.683 (.731)	.696 (.718)
24.	My whole body becomes tense when I have to do math (PH-12)[*1]	.777 (.630)	.785 (.620)	.767 (.642)	.753 (.658)	.807 (.591)	.783 (.622)	.784 (.621)

Note: All item loadings are statistically significant. Error variances are presented in parentheses to the right of each standardized estimate. Numeric superscripts denote the study phase in which each item was first introduced. Items that were modified in subsequent phases are followed by the superscript "M."

ME, Mastery Experience; VA, Vicarious Experience from Adults, VP, Vicarious Experience from Peers; VS, Vicarious Experience from Self; P, Social Persuasions; PH, Physiological State.

[*] Reverse-scored item.

Appendix E

Individual Interview Protocol #1

- (2-3minutes) Introduction Questions: “You provided some of the following information on the survey you completed in class, but I’ll ask you again to verify your responses.”
    - What is your major?
    - What was the highest level of mathematics course you took in high school?
        - How did you do in that class?
            - If they did well: What things did you do to be successful?
            - If they did not do well: What do you think the reason was?
    - What is the highest level of mathematics course you expect to take in college?
    - Were you enrolled in any after school math supplement classes during high school? If so, how often did you go?
        - Can you describe what activities you would do in these supplementary classes?
- (7 minutes) Expectations
    - How did you study for high school mathematics tests?
        - How do you expect to study for college mathematics tests?
    - How much time did you spend outside of class working on mathematics in high school? (HW/Study/projects)
        - How much time do you expect to spend outside of class in college working on mathematics?
    - Are there any experiences in your high school mathematics background that you remember as being particularly impactful for your learning of mathematics?
    - When assigned homework in high school, did you work independently, or with others? Did you ever talk to your teacher when getting stuck, or do you typically try to work through it or give up?
        - What are your opinions on these methods?
        - How do you expect to work on homework in college?
    - Thinking back to HS: When you were listening to a lecture or an explanation from the instructor, did you question what they are saying or how they are solving the problem, or did you accept it as being true?
        - Can you explain why you say that?
- (5 minutes) Preparedness
    - How confident do you feel about this class?
        - In other words, how confident are you that you will earn a passing mark?
    - Describe the structure of your high school class, do you think it prepared you for college math class?
        - Did you have modeling projects?
        - Were homework/tests just exercise problems or did they require critical thinking?
        - What type of structure do you prefer?
- (7minutes) Self-Efficacy
    - Tell me about your confidence level in mathematics in high school?
        - Why do you say that?
    - Do you consider yourself to be good at math?
        - Can you tell me more about why you said yes (or no)?
    - Describe any experiences in a math class, or from teachers or peers, that have affected your confidence in your ability to do math?

- To preserve your confidentiality, we like to use a pseudonym instead of your real name. Do you have a preferred pseudonym you would like to use, or would you like me to come up with a name for you?

“Thank you for participating in this interview, the group interview will occur shortly (give time of group interview).”

Appendix F

Group Interview Protocol #1

- Exploration of Lab activities:
    - Looking at this first three labs, did you do activities like this in high school?
        - If not, did you have any activities or projects in a high school mathematics class that required working in groups?
        - If yes, then in what ways were they similar or different?

    Ask this sequence of questions for each lab n, n=1,2,3.
    - For lab (n) how did you divide the work among the group? Do you think this strategy was successful for you learning the material?
    - If you were to get stuck on any part of lab (n), what was your process of problem solving? In other words, how did you respond to getting stuck on a problem?
    - Did you ask the TA for help at any point during the lab (n)? Was that helpful?
    - In high school, if you experienced problems like on this lab (n), what would you do if you got stuck on a problem?
    - For lab n, I would like your group to work out the following problem and work together just like you would in a lab setting:
        - N=1, #3 (precalculus); #12 on 1-B (calculus)
        - N=2, #1 (precalculus); #2 (calculus)
        - N=3, #3 (precalculus); #8 (calculus)

    Looking at all three labs:
    - Do you think these labs help you learn the material? Are they relevant to the topics you are discussing?
    - Do you think doing well on these labs will help you be a better mathematics learner? Explain.
        - Do you think doing on these labs will prepare you for the midterms in this class?

"Thank you for participating in this interview, I will email you within the next couple of weeks to schedule a time for a second interview if you are willing to continue in this process."

Appendix G

Group Interview Protocol #2

- Exploration of Lab activities:
    - Looking at this first three labs, did you do activities like this in high school?
        - If not, did you have any activities or projects in a high school mathematics class that required working in groups?
        - If yes, then in what ways were they similar or different?

Ask this sequence of questions for each lab n, n=4,5,6.

- For lab (n) how did you divide the work among the group? Do you think this strategy was successful for you learning the material?
- If you were to get stuck on any part of lab (n), what was your process of problem solving? In other words, how did you respond to getting stuck on a problem?
- Did you ask the TA for help at any point during the lab (n)? Was that helpful?
- In high school, if you experienced problems like on this lab (n), what would you do if you got stuck on a problem?
- For lab n, I would like your group to work out the following problem and work together just like you would in a lab setting:
    - N=4, #3 (precalculus); #5 (calculus)
    - N=5, #1 (precalculus); #3 (calculus)
    - N=6, #3 (precalculus); #1c (calculus)

Looking at all three labs:

- Do you think these labs help you learn the material? Are they relevant to the topics you are discussing?
- Do you think doing well on these labs will help you be a better mathematics learner? Explain.
    - Do you think doing on these labs will prepare you for the midterms in this class?
- While working on labs or homework problems for this class do you use any of the following strategies?
    - Seek information from an external source
    - Keep records and monitor progress
    - Organize information and transform it to something usable
    - Seek teacher assistance
    - Seek peer assistance
    - Seek assistance from adult (other than teacher)
    - Self-evaluate your own understanding
    - Provide goals for yourself based on performance
    - Provide extrinsic rewards
    - Remind yourself of the value of the task to your learning
    - Provide goals for yourself based on mastering the concepts
    - Change your environment
- Do you think any of the above strategies would be useful to you? Explain why.
- Have there been any concepts introduced in this course so far that made you adopt new strategies or adapt old strategies?
    - If so, what are those and what about them made you change your self-regulation strategies?

"Thank you for participating in this interview, I will email you within the next couple of weeks to schedule a time for a second interview if you are willing to continue in this process."

Appendix H

Group Interview Protocol #3

- Exploration of Lab activities:
    - Looking at this first three labs, did you do activities like this in high school?
        - If not, did you have any activities or projects in a high school mathematics class that required working in groups?
        - If yes, then in what ways were they similar or different?

  Ask this sequence of questions for each lab n, n=7,8,9.
    - For lab (n) how did you divide the work among the group? Do you think this strategy was successful for you learning the material?
    - If you were to get stuck on any part of lab (n), what was your process of problem solving? In other words, how did you respond to getting stuck on a problem?
        - For n = 7: Examine problem 5, how would you go about this problem? If at any point you were to get stuck on this problem, what would you do?
        - For n = 8: Examine problem 3, how would you go about this problem? If at any point you were to get stuck on this problem, what would you do?
        - For n = 9: Examine problem 1, how would you go about this problem? If at any point you were to get stuck on this problem, what would you do?
    - Did you ask the TA for help at any point during the lab (n)? Was that helpful?
    - In high school, if you experienced problems like on this lab (n), what would you do if you got stuck on a problem?
    - For lab n, I would like your group to work out the following problem and work together just like you would in a lab setting:
        - N=7, #4 (precalculus); #5 (calculus)
        - N=8, #2 (precalculus); #5 (calculus)
        - N=9, #2 (precalculus); #4 (calculus)

  Looking at all three labs:
    - Do you think these labs help you learn the material? Are they relevant to the topics you are discussing?
    - Do you think doing well on these labs will help you be a better mathematics learner? Explain.
        - Do you think doing on these labs will prepare you for the midterms in this class?

- Have there been any concepts introduced in this course so far that made you adopt new strategies or adapt old strategies?
    - If so, what are those and what about them made you change your self-regulation strategies?

"Thank you for participating in each of these interviews, your feedback has been valuable, and I appreciate you taking the time to help with this, there will be one more brief exit interview for each of you within the next few days."

Appendix I

Individual Interview Protocol #2

- (2 minutes) Introduction Questions:
    - What is the highest level of mathematics course you expect to take in college, has it changed since taking this class?
    - Describe how confident you are in being able to do well in that class?
- (10 minutes) Expectations
    - How did you study for college mathematics tests?
        - Is it different than high school?
        - Did it change during the semester at all? If so, was there something that caused this change?
    - How much time did you spend outside of class working on mathematics in this class? (HW/Study/projects)
        - How much time do you expect to spend outside of class in college working on mathematics in future classes?
    - When assigned homework in this class, did you work independently, or with others? How did you respond if you got stuck on a homework problem?
        - How do you expect to work on homework in college in future classes?
        - Has this strategy changed due to any experiences in this class?
            - Can you describe those experiences?
    - When listening to a lecture or an explanation from the instructor or TA, do you question what they are saying or how they are solving the problem, or do you accept it as being true?
        - Can you explain why you say that?
- (5 minutes) Preparedness
    - How confident did you feel about this class?
    - Describe the structure of your college math class, do you think high school prepared you for college math class?
    - How are the roles of teacher and professor similar or different?
- (5 minutes) Self-Efficacy
    - Describe your confidence level in mathematics since having taken this class?
        - Describe any events or experiences that may have affected this confidence.
    - Do you consider yourself to be good at math?
    - Describe any experiences in a math class, or from teachers or peers, that have affected your confidence in your ability to do math?

"Thank you for participating in each of these interviews, your feedback has been valuable, and I appreciate you taking the time to help with this."

Appendix J

Classroom Observation Form

Classroom Observation Outline:

Date ________ Class _________ Number of groups (2-4 people per group) ______

Group N regulation strategies observed:

- Number of interactions with TA/SI/Instructor ____
  - Types of interactions (asking single question, reporting they are stuck, clarifying comment, or other)

____

____

____

____

____

____

____

  - Division of work strategy (i.e., is everyone working independently on separate problems, are they taking each problem together, or some mix)

____

____

____

____

Appendix K

Collaborative Precalculus Lab Activities

1. A water barrel initially contains S liters of water. Water leaks out of the barrel at a rate of x liters of water per hour. Let W be the amount of water in the barrel after t hours have passed.

   a) Find an equation for the amount of water that is in the barrel after t hours, *i.e.* write W in terms of S and t.

   b) If $S = 200$ and $x = 4$, what is the value of W after $t = 6$ hours?

   c) If $S = 300$ and the barrel is empty after 12 hours, what is the value of x?

   d) If $x = 7$ and $W = 100$ after $t = 9$ hours, find the value of S.

2. If a 25 m tall tree casts a 15 m long shadow, how tall is a tree that casts an 18 m long shadow at the same time of day?

3. A closed box with a square bottom is three times high as it is wide.

   a) Express the surface area of the box in terms of its width.

   b) Express the volume of the box in terms of its width.

   c) Express the surface area in terms of the volume.

   d) If the box has a volume of 24 m^3, what is its surface area?

4. Two fishing boats depart from a harbor at the same time, one traveling east, and the other traveling west. The eastbound ship travels at a speed of 3 km/hr faster than the westbound ship. After 2 hours, the boats are 30 km apart.

   a) Find the speed of both boats.

   b) Repeat the question if the faster boat travels east and the slower boat travels south.

[1]

[1] UT-Arlington Precalculus Lab 1, Fall 2021

1. Determine whether the following statements are true, and give an explanation or counterexample.

   a) If the domain of a function f is all real numbers, then the domain of the function $g(x) = f(x^2)$ is also all real numbers.

   b) If $g(x) = x^3 - 1$, then $g(2) = g(2)^3 - 1$.

   c) If $h(x) = 0$, then $h(x^2) = 0$.

   d) The graph of a function can have symmetry with respect to the y-axis and with respect to the origin but cannot have symmetry with respect to the x-axis.

2. Given a function f, the difference quotient for f is the expression $\dfrac{f(x+h) - f(x)}{h}$. Find and simplify the difference quotient for the function $f(x) = \dfrac{1}{2x}$.

3. When walking at a constant pace, the relationship between distance traveled, rate (speed), and time is $d = rt$.

   a) Find the time it takes to walk 4 miles at a constant rate of 3 miles per hour.

   b) Find a function t that gives the time it takes to walk 4 miles at a constant rate of r miles per hour and state the natural domain of t.

   c) Find a function r that gives the walking speed when 4 miles are covered in t hours and state the natural domain of r.

   d) Find a function d that gives the distance walked in 3 hours at a constant rate of r miles per hour and state the natural domain of d.

2

[2] UT-Arlington Precalculus Lab 2, Fall 2021

b. $(f \circ f)(8)$

$f(x)$	8	7	6	5	4

c. $(g \circ g)(6)$

x	5	6	7	8	9
$g(x)$	4	5	7	6	8

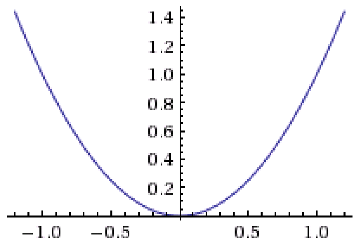
d. $(f \circ g)(6)$

e. $(g \circ f)(6)$

1. Let $f(x) = \sqrt{x}$ and $g(x) = x^2$. Is it true that $f(g(x))$ and $g(f(x))$ are the same function? Why or why not?

2. The graph of the function $y = f(x)$ is shown below. Use it to sketch the graph of the function $g(x) = -\frac{1}{2} f(2x + 6) + 1$.

8. The graph of the function $y = f(x)$ is show below. Graph the function $g(x) = \frac{1}{2} f(x + 3) - 1$.



3. Consider the following transformations:

(v) vertical shift down by 2

(h) horizontal shift right by 3

(x) x-axis reflection

(y) y-axis reflection

Does it matter what order these transformations are applied? Apply these transformations to the graph of $y = \sqrt{x}$ in two different orderings. One ordering should be (v), (h), (x), (y), and the other ordering should be (x), (y), (v), (h). Which ordering produces the correct graph of $y = -\sqrt{-(x - 3)} - 2$?

Contributing Writers: Cavender Campbell, Alicia Prieto, Julie Sutton, & Padmini Veerapen
AURAS is funded by the National Science Foundation, NSF STEP grant #0856796. Dr. Lynn Peterson (CSE) is the principal investigator, and the co-investigators are Dr. James Epperson (Math), Dr. Ramon Lopez (Phys), Dr. Kevin Schug (Chem/Biochem), and Dr. Carter Tiernan (CSE) in this $2M grant e_ort. NSF support for this program is gratefully acknowledged.

3

[3] UT-Arlington Precalculus Lab 3, Fall 2021

1. Consider the general quadratic equation $ax^2 + bx + c = 0$. Derive the quadratic formula by completing the square and solving for x.

2. Consider the general quadratic function $f(x) = ax^2 + bx + c$.

   (a) Write down the x-intercepts of the graph of f in terms of a, b, and c. Call one of these values x_1 and the other x_2.

   (b) Let (h, k) be the vertex of the parabola. On the x-axis, how does the position of h relate to x_1 and x_2? Use your answer to write h in terms of x_1 and x_2.

   (c) Plug the expressions from part (a) into your equation from (b) and simplify to obtain an equation for h in terms of the constants a, b, and c.

3. Suppose a freight company's shipping rates are $8.50 for packages weighing less than 2 pounds and $5.50 for each additional 2 pounds.

   (a) How much does it cost to ship a 6.4-pound package?

   (b) Write the shipping cost function in terms of the integer floor function.

   (c) Graph the shipping cost function for weights between 0 and 20 pounds.

4

[4] UT-Arlington Precalculus Lab 4, Fall 2021

1. Consider $f(x) = \dfrac{-5-x}{3-2x}$.

   a) Find $f^{-1}(7)$ **without finding** $f^{-1}(x)$
   b) Find $f^{-1}(x)$

2. a) Does the function $g(x) = 2x^2 - 9$ have an inverse? If it does find it. If it does not then explain why this is so.

   b) Does the function $p(x) = 3\sqrt{2x+1}$ have an inverse? If it does find it. If it does not then explain why this is so.

3. Recall the function from 2(a) above.
   a) Restrict the domain of g such that the function has an inverse. Explain why you chose the domain that you did. Is this unique? How do you know?

   b) Find g^{-1} using the domain restriction you stated in (a). Is this unique? Explain your answer.

5

[5] UT-Arlington Precalculus Lab 5, Fall 2021

1. Determine whether the following statements are true or false, and give an explanation or a counterexample.

   (a) $\log_3 y < \log_2 y$ for $y > 1$.

   (b) $\log_3 y < \log_2 y$ for $0 < y < 1$.

   (c) The domain of $f(x) = \ln(x^2)$ is $x > 0$.

2. Let $f(x) = e^x$ and $g(x) = \left(\frac{1}{2}\right)^x$.

   (a) Find A satisfying $g(x) = f(Ax)$.

   (b) Describe how the graph of f is transformed to be identical to the graph of g.

3. Let $m = \log_b M$ and $n = \log_b N$.

   (a) Rewrite the above 2 equations using exponentials instead of logarithms.

   (b) Using your equations from part (a), prove that $\log_b(MN) = \log_b M + \log_b N$.
   Hint: Start with $\log_b(MN)$ *and replace both* M *and* N *using your answers to part (a), then use the properties of exponentials to arrive at* $\log_b M + \log_b N$.

   (c) Using your equations from part (a), prove that $\log_b(M/N) = \log_b M - \log_b N$.

   (d) Using one of your equations from part (a), prove that $\log_b(M^p) = p \log_b M$.

6

[6] UT-Arlington Precalculus Lab 6, Fall 2021

1. Sketch the following angles in standard position on a unit circle.

   (a) $-\frac{3\pi}{2}$ (b) $\frac{35\pi}{6}$ (c) $-\frac{5000\pi}{2}$ (d) $\frac{101\pi}{4}$

2. The International Space Station orbit at an altitude of 220 miles above the Earth's surface. It completes one orbit every 1.5 hours. Assuming that the Space Station's orbit is circular and that the radius of the Earth is 3960 miles, approximate the distance the Space Station travels each hour.

3. Find an angle in $[0, 2\pi)$ that is coterminal to the given angle θ, then evaluate $\sin\theta$, $\cos\theta$, and $\tan\theta$. Do not use a calculator.

   (a) $\theta = \frac{35\pi}{6}$ (b) $\theta = -\frac{5000\pi}{2}$ (c) $\theta = \frac{101\pi}{4}$

4. Determine whether each function is even, odd, or neither.

   (a) $f(x) = \cos x \tan x$ (b) $f(x) = \frac{\sin x - \cos x}{\tan x}$

7

[7] UT-Arlington Precalculus Lab 7, Fall 2021

1. Describe (in words) how you can obtain the graph of $y = \sin\left(3t + \frac{7\pi}{4}\right)$ from the graph of $y = \sin(3t)$

2. Rewrite the function $g(x) = 4\cos x$ as a new function $h(x) = A\sin(Bx + C)$. Where A, B, and C are constants such that $g(x) = h(x)$ for all possible values of x.

3. Given the function $f(x) = -2\cos\dfrac{\pi x}{4}$:

   a) What is the amplitude?
   b) What is the period?
   c) Find two x-intercepts in the interval $0 \le x \le 8$.
   d) Graph the function f on 2 consecutive periods using the information you found in (a)-(c).

4. a) Sketch two complete periods of the graph of $f(\theta) = 2\sin(\theta + \pi) + 1$.
   b) Sketch two complete periods of the graph of $g(\theta) = 2\csc(\theta + \pi) + 1$.
   c) What do you notice about these two graphs?

8

[8] UT-Arlington Precalculus Lab 8, Fall 2021

1. A surveyor observes that point A is located on level ground 25 feet from a flag pole. The base of the flagpole represents point B. Approximate the height of the flagpole to the nearest tenth of a foot when the angle between the ground and the top of the flagpole is:

   a) $30°$ b) $38°$ c) $45°$

2. Sketch a right triangle and label one of the acute angles as θ. If $\theta = \sin^{-1}\left(\dfrac{x}{\sqrt{x^2+1}}\right)$, evaluate the following. Note that your answers will be in terms of x.

   a) $\sin\theta$ b) $\cos\theta$ c) $\tan\theta$
   d) $\csc\theta$ e) $\sec\theta$ f) $\cot\theta$

3. One of the following statements is true for every x in its domain; the other is true for some values of x in its domain and not for others. Which is which? Justify your answer with an example.

   a) $\sin^{-1}(\sin x) = x$ b) $\sin(\sin^{-1} x) = x$

9

[9] UT-Arlington Precalculus Lab 9, Fall 2021

Appendix L

Collaborative Calculus Lab Activities

The following problems are a review of precalculus topics.
Write clearly and coherently and include ALL steps needed to justify your solution.

1. Find the zeros of $-3x^2e^{-2x} + 3xe^{-2x}$.

2. Solve $e^x + 2 = 8e^{-x}$ for x.

3. Solve $e^{\ln(x+1)} = 3$ for x.

4. Solve $\log_7(2x+3) = \log_7 11 + \log_7 3$ for x.

5. Express the expression below as one logarithm.

$$\log\left(\frac{x^2}{y^3}\right) + 5\log y - 6\log\sqrt{xy}$$

6. Solve $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ for x in terms of y.

7. Solve $|3x-2| + 3 = 7$ for x.

8. Solve $\sqrt{7-5x} = 8$ for x. Give only real number solutions.

9. Find the exact value of $\cos\theta$ when $\sin\theta = -\frac{3}{5}$ and $\tan\theta > 0$.

1

[10]

[10] UT-Arlington Calculus Lab 1 Page 1, Fall 2021

10. Find the exact value of $\csc\alpha$ when $\cot\alpha = -\frac{12}{5}$ and $\cos\alpha = \frac{12}{13}$.

11. Find the exact solutions to the equation $2\sin^2 t - \cos t - 1 = 0$ for t in radians.

12. Find the exact value of $\sin^{-1}\left(\sin\frac{9\pi}{10}\right)$.

13. Find $\sin\theta$ and $\cos\theta$ as exact fractions given that $0 \le \theta < \frac{\pi}{2}$ and $\tan\theta = \frac{14}{11}$.

14. Find the exact solutions to the equation $\sin 2\theta = \sin\theta$ in radians.

15. Find the exact solutions to the equation $\sqrt{3} - \tan 3\theta = 0$ in radians.

16. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ if $f(x) = 4x - 1$.

17. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ if $f(x) = 5x^2 - 3x$.

18. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ if $f(x) = x^{-1}$.

11

[11] UT-Arlington Calculus Lab 1 Page 2, Fall 2021

1. Evaluate the following limits. Show work to justify your solutions.

a. $\lim_{x \to 1^+} \frac{x^4-1}{x-1}$

b. $\lim_{x \to 1} \frac{x^3-x^2+x-1}{x^2-1}$

c. $\lim_{x \to 0} \frac{\sqrt{4-x^2}-2}{x^2}$

d. $\lim_{h \to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h}$

2. Is it possible that $\lim_{x \to a} f(x)$ does not exist and $\lim_{x \to a} g(x)$ does not exist, but $\lim_{x \to a} [f(x) + g(x)]$ does exist? Consider the functions $f(x) = \frac{1}{x}$ and $g(x) = -\frac{1}{x}$.

   a. Sketch the graphs of $f(x)$ and $g(x)$ and find their limits (if they exist) as x approaches zero.

   b. Sketch the graph of $[f(x) + g(x)]$ and find $\lim_{x \to 0} [f(x) + g(x)]$.

3. Sketch the graphs of the piece-wise defined functions f and g and find the following limits.

$$f(x) = \begin{cases} 2 & \text{if} \quad x < -2 \\ x^2 & \text{if} \quad -2 \le x < 1 \\ -\frac{1}{2}x^2 + 1 & \text{if} \quad 1 < x < 3 \end{cases}$$

$$g(x) = \begin{cases} 3 & \text{if} \quad x < -2 \\ -x - 1 & \text{if} \quad -2 \le x < 1 \\ x - 5 & \text{if} \quad 1 \le x < 3 \end{cases}$$

a. $\lim_{x \to -2^-} f(x)$

b. $\lim_{x \to -2^+} f(x)$

c. $\lim_{x \to -2} f(x)$

d. $\lim_{x \to -2^-} g(x)$

e. $\lim_{x \to -2^+} g(x)$

f. $\lim_{x \to -2} g(x)$

g. $\lim_{x \to -2^-} [f(x) + g(x)]$

h. $\lim_{x \to -2^+} [f(x) + g(x)]$

i. $\lim_{x \to -2} [f(x) + g(x)]$

j. $\lim_{x \to 1} [f(x) + g(x)]$

k. $\lim_{x \to -2} [f(x) \cdot g(x)]$

l. $\lim_{x \to 1} [f(x) \cdot g(x)]$

12

[12] UT-Arlington Calculus Lab 2, Fall 2021

(1) Sketch the graph of a function f with all the following properties.

$\lim_{x\to-2^-} f(x) = \infty$ $\quad\lim_{x\to-2^+} f(x) = -\infty$ $\quad\lim_{x\to 0} f(x) = \infty$

$\lim_{x\to 3^-} f(x) = 2$ $\quad\lim_{x\to 3^+} f(x) = 4$ $\quad f(3) = 1$

(2) Let $f(x) = \frac{x^2-5x+6}{x^2-2x}$.

(a) Analyze $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$.

(b) Does the graph of f have vertical asymptotes? Explain.

(3) Let $f(x) = \frac{1+x-2x^2-x^3}{x^2+1}$.

(a) Analyze $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

(b) Does f have a slant asymptote? If so, write the equation of the slant asymptote.

(4) Sketch the graph of a function f that is continuous for every real number x, except $x = -2$, at which point f is continuous from the left.

(5) Sketch the graph of a function f that is not continuous at $x = 1$, but if we redefine f at 1 so that $f(1) = 2$, then f becomes continuous at $x = 1$.

(6) Give an example of a function f that is not continuous at $x = 1$, but if we redefine f at 1 so that $f(1) = 2$, then f becomes continuous at $x = 1$.

(7) Determine whether the following functions are continuous at a. Use the continuity checklist to justify your answers.

(i) $h(x) = \sqrt{x^2-9}$; $a = 3$ $\qquad$ **(ii)** $g(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ 8 & \text{if } x = 4 \end{cases}$; $a = 4$.

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[13] UT-Arlington Calculus Lab 3 Page 1, Fall 2021

(8) Let $f(x) = 2x^2 - 4x$.

(a) Find the average rate of change of f over the interval $[3, 3 + h]$.
How is this average rate of change related to the secant line passing through the points $(3, 6)$ and $(3 + h,\ f(3 + h))$?

(b) Use part (a) to find the instantaneous rate of change of f with respect to x at 3.
How is this instantaneous rate of change related to the tangent line passing through the point $(3, 6)$?

(c) Find an equation of the tangent line to the graph of f at the point $(3, 6)$.

(9) Use the definition of the derivative to show that:

(i) $f'(x) = \frac{1}{2\sqrt{x-3}}$ when $f(x) = \sqrt{x-3} + 1$

(ii) $f'(x) = -\frac{1}{x^2} - 2$ when $f(x) = \frac{1}{x} - 2x$

14

[14] UT-Arlington Calculus Lab 3 Page 2, Fall 2021

1. Use the product rule: $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$ and the fact that

   $e^{2x} = e^x \cdot e^x$ to <u>show</u> that $\frac{d}{dx}(e^{2x}) = 2e^{2x}$.

2. Use the quotient rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x)-f(x)g'(x)}{\left(g(x)\right)^2}$, the derivatives $\frac{d}{dx}(\sin x) = \cos x$

   and $\frac{d}{dx}(\cos x) = -\sin x$, and the fact that…

   (a) … $\tan x = \frac{\sin x}{\cos x}$ to <u>show</u> that $\frac{d}{dx}(\tan x) = \sec^2 x$.

   (b) … $\csc x = \frac{1}{\sin x}$ to <u>show</u> that $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

   (c) … $\sec x = \frac{1}{\cos x}$ to <u>show</u> that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

   (d) … $\cot x = \frac{\cos x}{\sin x}$ to <u>show</u> that $\frac{d}{dx}(\cot x) = -\csc^2 x$.

3. Find the derivative of $f(x) = \frac{x^2+3x+2}{x}$ using two different methods.

   (a) Apply the quotient rule to the function as written.

   (b) Break f into three separate fractions, simplify, then apply the power rule to each.

4. Find the derivative of $f(x) = \frac{x^2-2ax+a^2}{x-a}$, with a constant, using two different methods.

   (a) Apply the quotient rule to the function as written.

   (b) Simplify the function first, then find the derivative.

5. Use the product rule (twice) to find a formula for $\frac{d}{dx}\big(f(x)g(x)h(x)\big)$.

6. Use the formula you obtained in #5 above to find $\frac{d}{dx}\big(e^{2x}(x-1)(x+3)\big)$.

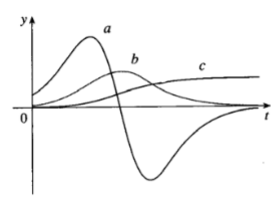
15

[15] UT-Arlington Calculus Lab 4, Fall 2021

1. The formula $h(t) = -16t^2 + 480t$, describes a model rocket's height. $h(t)$ is the rocket's height measured in feet from the ground, t seconds after take-off.
   a. Sketch the graph of $h(t)$ for $t \geq 0$. After how many seconds does the rocket impact the ground?
   b. Find the velocity function $v(t)$ of the rocket.
   c. Sketch the graph of $v(t)$ and interpret it based on the rocket's actual motion.
   d. Compute the rocket's velocity at $t = 0$, $t = 15$, and $t = 18$. How do you interpret the signs of the values you obtained?
   e Sketch the graph of $s(t)$, the speed function. Find the rocket's speed at $t = 0$, $t = 15$, and $t = 18$.
   f. After how many seconds will the rocket start falling? Use differentiation to find the maximum height attained by the rocket.
   g. After how many seconds will the rocket hit the ground? What is its velocity at that moment?
   h. Determine the rocket's acceleration, $a(t)$. Why does it make sense that $a(t)$ is constant and negative? Explain.

2. You are traveling down a country road at a rate of 95 feet/sec when you see a large cow 300 feet in front of you and directly in your lane. The cow is fearless, and staring you down. (Or perhaps she does not understand the gravity of the situation.) Regardless, if you simply hit your brakes, after t seconds, the car will be j(t)=95t-9t^2 feet from the point where the brakes were first applied.
   (i) Must you steer to avoid the cow, or can you rely solely on your brakes? Explain.
   (ii) Graph j(t), j'(t), and j''(t) and interpret their meanings in context.
   (iii) Is there a time t after which the graphs in part (ii) probably do not accurately model the path of the car?
   (iv) Write a piece-wise function using j(t) that would more accurately model the path of the car on the interval [0,10], assuming the car did not move after it stopped. Sketch this graph and its derivative.

3. The figure shows the graphs of three functions. One is the position of a car, one is the velocity of the car, and one is its acceleration. Identify each curve and explain your choices.



For problem 4, use the position function $p(t) = -16t^2 + v_0 t + p_0$, where *s* is measured in feet, *t* in seconds and v_0 and p_0 are the initial velocity and position respectively.

4. A professional baseball pitcher was capable of throwing a baseball 160 ft/s. During his career, he had the opportunity to pitch in the Houston Astrodome. The Astrodome was an indoor stadium with a ceiling 208 ft. high.
   a. Could the pitcher have hit the ceiling of the Astrodome if he were capable of giving a baseball an upward velocity of 100 ft/s from a height of 7.5 ft?
   b. How fast would the pitcher have to throw a ball upward from a height of 7.5 feet in order to hit the ceiling of the Astrodome?

16

[16] UT-Arlington Calculus Lab 5, Fall 2021

(1) Functions $f, g,$ and h are continuous and differentiable for all real numbers, and some of their values and values of their derivatives are given in the table below.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
0	1	-1	-1	4	1	-3
1	0	3	-7	2	3	6
2	3	0.5	4	-2	1	3

Show your work to justify the solutions to the following problems.

(a) Find $F'(1)$ if $F(x) = h(x) \cdot e^{-3f(x)}$

(b) Find $G'(2)$ if $G(x) = cos[\pi \cdot g(x)]$

(c) Find $H'(0)$ if $H(x) = e^{\left(\frac{h(x)}{f(x)}\right)^2}$

(d) Find $I'(2)$ if $I(x) = \ln[f(x) \cdot g(x)]$

(e) Find $J'(0)$ if $J(x) = \frac{\frac{f(x)}{g(x)}}{h(x)}$

(2) Functions $f, g, h,$ and j are continuous and differentiable for all real numbers, and some of their values and values of their derivatives are given in the table below.

x	$f(x)$	$g(x)$	$h(x)$	$j(x)$	$f'(x)$	$g'(x)$	$h'(x)$	$j'(x)$
0	1				4	1		8
1		3	0	-1			6	

If you know that $h(x) = f(x)g(x)$ and $j(x) = g(f(x))$, fill in the correct numbers for the missing values in the table. Show your work to justify the solutions.

(3) Find $\frac{dy}{dx}$. Show your work.

(a) $y = \sqrt{\frac{x^2-5}{x^2+4}}$

(b) $y = sin\left(cos(\ln(2x))\right)$

(c) $cos^2 y = y - 2x^2$

(d) $x^3 - x^2y - 2y^3 = 4x^2y^2$

(4) Find an equation of the line tangent to the graph of $2x^3y^3 = 2y - 4x$ at the point $(-1, -1)$. Show your work.

17

[17] UT-Arlington Calculus Lab 6, Fall 2021

MATH 1426 **Problem Solving Lab 7** **Fall 2021**

Show all your work as you solve each related rates problem.

(1) Let $\frac{xy^3}{1+y^2} = \frac{8}{5}$ model the path of a particle. If the x-coordinate is increasing at 6 units per second when the particle is at the point $(1,2)$, what is the rate of change of the y-coordinate at that moment? Is the particle rising or falling then?

(2) Oil spilled from a ruptured tanker spreads in a circle. (Three separate, yet related, problems.)
(i) If the area of the circle increases at a constant rate of 4 miles squared per hour, how fast is the radius of the spill increasing when the area is 15 miles squared?
(ii) If the radius is increasing at a rate of approximately .2913 miles per hour, how fast is the area of the circle increasing when the area is 15 miles squared?
(iii) How large is the circle (in terms of area) if the area of the circle is increasing at 4 miles squared per hour and simultaneously the radius is increasing at approximately .2913 miles per hour?

(3) At a certain instant of time, a cube has side length 2 inches and its volume is increasing at a rate of 0.5 inches cubed per hour. How fast is its surface area increasing at that instant?

(4) Wheat is poured through a chute at the rate of 20 feet cubed per minute in a conical pile whose bottom radius is always half the altitude.
(i) How fast will the area of the base be changing when the pile is 6 feet high?
(ii) How fast will the circumference of the base be changing when the pile is 6 feet high?
Recall, the volume of a cone is given by the formula: $V = \frac{1}{3}\pi r^2 h$.

(5) The sides of a square baseball diamond are 90 feet long. When a player who is between second and third base is 60 feet from second base and heading towards third base at a speed of 22 feet per second, how fast is the distance between the player and home plate changing?

18

[18] UT-Arlington Calculus Lab 7, Fall 2021

Show all work for the following Optimization problems.

(1) Maximizing Area

(i) Of all rectangles with a fixed perimeter of P feet, which one has the maximum area A? (Give the dimensions in terms of P.)

(ii) Let $P = 40$ feet and give the dimensions using your solution in part (i).

(iii) Sketch the area function A that was maximized in part (i). Use a reasonable domain. Label axes appropriately, including units.

(2) Minimizing Perimeter

(i) Of all rectangles with a fixed area of A inches squared, which one has the smallest perimeter P? (Give the dimensions in terms of A.)

(ii) Let $A = 100$ inches squared and give the dimensions using your solution in part (i).

(iii) Sketch the perimeter function P that was minimized in part (i). Use a reasonable domain. Label axes appropriately, including units.

(3) Minimizing Surface Area

(i) Of all boxes with a square base and a fixed volume V, which one has the minimum surface area A_S? (Give its dimensions in terms of V.)

(ii) Let $V = 1000$ meters cubed and give the dimensions using your solution in part (i).

(iii) Sketch the surface area function A_S that was minimized in part (i). Use a reasonable domain. Label axes appropriately, including units.

(4) Maximizing Volume

(i) Squares with sides of length x are cut out of each corner of a rectangular piece of cardboard measuring 3 ft by 4 ft. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.

(ii) Suppose that in part (i) the original piece of cardboard is a square with sides of length l. Find the volume of the largest box that can be formed in this way.

(5) Minimizing Distance

(i) Find the point P on the line $y = 3x$ that is closest to the point $(50,0)$.

(ii) What is the least distance between P and $(50,0)$?

(6) Minimizing Cost.

A builder wants to construct a cylindrical barrel with a capacity of $32\pi\ ft^3$. The cost per square foot of the material for the side of the barrel is half that of the cost per square foot for the top and bottom. Determine the dimensions of the barrel that can be constructed at a minimum cost in terms of material used.

19

[19] UT-Arlington Calculus Lab 8, Fall 2021

Linear Approximation and Differentials

(1) True or False. Explain why or why not.
Also, for each of (i)-(iv), graph f and L (if it exists) on one set of axes.
(i) The linear approximation to $f(x) = x^2$ at $x = 0$ is $L(x) = 0$.
(ii) Linear approximation at $x = 0$ provides a good approximation to $f(x) = |x|$.
(iii) If $f(x) = mx + b$, then the linear approximation to f at any point is $L(x) = f(x)$.
(iv) When linear approximation is used to estimate the value of $f(x) = \ln x$ near $x = e$ the approximations are underestimates of the true value.

(2) Use linear approximation to estimate $f(5.1)$ given that $f(5) = 10$, and $f'(5) = -2$.

(3) Given a function $f(x) = (1 + x)^n$, show that $L(x) = 1 + nx$ is the linear approximation of f at 0.

(4) Consider the function $f(x) = \sqrt{2}\cos x$.
(i) Find the linear approximation L to the function f at $a = \frac{\pi}{4}$.
(ii) Graph f and L on the same set of axes.
(iii) Based on the graphs of part (ii), state whether linear approximations to f near a are underestimates or overestimates.
(iv) Compute $f''(a)$ to confirm your conclusion.

(5) Use linear approximations to estimate the following quantities.
Choose a suitable function f and a value of a that produces a small error.
(i) $\sqrt[3]{-7.97}$
(ii) $e^{0.02}$

(6) Differentials. Consider the function $f(x) = \ln(1 - x)$.
(i) Express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)dx$.
(ii) Use your answer in part (i) to approximate the change in f when x changes from $x = -1$ to $x = -1.02$.

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[20] UT-Arlington Calculus Lab 9, Fall 2021

Appendix M

Calculus Post-Lab Survey Excerpt

UNIVERSITY OF TEXAS ARLINGTON

By completing this survey you consent to allow this data to be used in a graduate research study conducted by Kyle Turner. All identifying information will be removed when presenting data.

Name (First and Last)

When working on this lab, if at any point you did not know how to proceed on a problem which (if any) of the following steps did you ***initially*** take? (Check all that apply)

- [ ] Reflect on a similar problem that was worked either by the instructor or from homework.
- [ ] Consult a group member for help about the problem.
- [ ] Consult the TA/Instructor/SI for help about the problem.
- [ ] Moved on to another problem to come back to the problem later.
- [ ] Gave up and watched my group members proceed with the problem.
- [ ] Other (please give brief description)

How did the workload required by this lab compare to assignments that were given to you in high school?

- ◯ This lab required more work than a typical high school assignment.
- ◯ This lab required about the same amount of work that a typical high school assignment would require.
- ◯ This lab required less work than a typical high school assignment.

Appendix N

Participant Transcript Excerpts from Group Interview #2

Aria

Interviewer: So if you can just go down this list and maybe then like. Can you tell me if you use this strategy or not, and if so, how you Use that strategy.

Aria: So yes, I would seek information from external source. Because we do it on Pearson and if there wasn't an option to show me an example and I didn't know how to do the problem I would kind of look up how to do it on Google and see if there's any websites or videos if it would help me. When it's asking for keep records and monitor progress, does that mean like you write down every problem on a sheet paper.

Interviewer: It can be. I mean, is that something you do or I guess how do you interpret that like?

Aria: I interpret it as like keeping everything you like, working out all your problems on a sheet of paper, I do, work on my problems on paper, but it's more like scrap paper to me than it is we're like looking back at it.

Interviewer: So do you ever also look back at? Maybe the grades you received and think like am I improving in my quizzes/ labs/ homework?

Aria: No, 'cause usually homework is based on completion. OK so I know what I've gotten. And labs I sometimes look back at them, but they usually like a nine out of 10 or a 10 out of 10 so. And quizzes, yeah, we do get them back. So I do look at them. And see what I got and what mistakes I made. I don't keep up or I don't organize my information. I just use scrap paper so I don't look back at it. And I don't seek any assistance from a teacher or peer, or any other adult… Self-evaluating my own understanding. To an extent, like I understand that I need more help on a homework, especially if it's taking me more time to. So I'll know if I need to study more about this, especially before a midterm. But the home works don't help me that much with my own understanding… I don't give myself any goals or any rewards. I don't remind myself of a value 'cause it's like it's basically completion. So… Yeah I just need to get this done. And I really don't have goals when it comes to mastering content, I know we need help, but it's not really a certain goal. And I'll change my environment, kind of I'll. Uhm, like kind of put away some distractions or anything and like play some background music but not that much, I still like do it in my room like any other homework.

Bailey

Interviewer: Do you seek information from an external source?

Bailey: Yeah, sometimes I search up YouTube videos so it's like have another person explain it to me so it's easier to understand if I, I guess, have a preference like to choose which one I like better.

Interviewer: OK. And so just going down the list, do you keep records and monitor progress?

Bailey: Sometimes I write down which questions I don't understand, so I'm like this question was hard, go back and review it before an exam or something.

Interviewer: And just yeah, keep going down the list.

Bailey: On organize information, I mean my notes, I guess sometimes some highlighting formulas and then like summarizing it. Teacher assistance, asking in labs for help. Partner and peers, yeah, same thing. And assistance from adult other than a teacher, I don't think I do that. I think I would ask more from people outside of my class or like people who are higher math level. And self-evaluating, I did that from quizzes like if I get my quizzes back and I see that I don't like the grade that I got that then maybe I don't understand it as much, I then work on it. And then goals, for performance, I aim, I guess, aim for an A, but realistically like pass. And then rewards I just give myself like breaks from studying, basically, yeah… Remind yourself of the value, yeah, 'cause I need to pass the class 'cause it says you remind yourself the value of the task to your learning. I need to pass the class and you understand it and then like… Provide goals for yourself based on math. Wait, provide goals for yourself based on mastering concepts… I think like basically like recently, I'm looking at the review for the upcoming midterm and I would see which ones I don't understand. Without looking at the answers I try to go through the whole test just in one go, and if I don't understand it, that's probably like, oh, that's what you would probably miss if this was exam, so I need to study right now, before it is actually like that. And change my environment, depending on where I am, I study differently at home, I'm kind of more chill or like less harsh on myself, so I think studying on campus is better. So, I do like math on campus and then sometimes little less at home But yeah.

Cyndy

Interviewer: So, if you can go through kind of explain one. If you use these strategies.

Cyndy: So for the first one [seek information from an external source], we're not allowed to do this for lab, so I only do it for homework problems and usually the homework will guide like have a way for me to answer the problems without having to use Google or the Internet. So I'll do that and then if I still don't understand it, I'll go through my notes and then last like sort of help I'll go ask the Internet. I do keep record of all my homework problems and like all the papers I could do homework on. But I don't monitor my progress, mainly it's just if I finish the homework then that's finishing the homework. It organized information, organize it by whatever we're learning in chronological order. As for transforming it, I don't technically know how to transform it into something that I know other than a study guide, so only a study guide.

Interviewer: And just to. What do you mean by? A study guide like what? Would that look like?

Cyndy: Usually it's a lot of theorems, something the professor might have said in class that is useful on the exam, or I'll look through the exam and I'll create a study guide based on the exam past exams. I do seek teacher assistance only on labs. Not necessarily homework problems. I do seek more peer assistance than I do teacher assistance, mainly from students that have already taken the course or students that are currently taking the course. Seeking assistance from another adult, I guess, it's kind of similar to peer assistance. Like former students asking former students, yeah. Self-evaluating, I know my own understanding is like at a very low level for calculus, so I know that much, but I don't know how to technically self-evaluate myself to a point where I understand more calculus. Providing goals, I think providing goals and providing extrinsic rewards are kind of similar. So like if I do really well on like a homework exam or like a lab then I'll go, I'll go out with my friends or something. Or like I'll provide like myself, like a reward. To be honest, this next one [remind myself of the value of the course], I don't need calculus… but it was just on the degree plan. So I was like, OK, I'll take calculus and we'll see how it goes. 'cause I was never good at math now and I'll probably never be good at math so. That one is not that important to me and… Like it's basically reminding myself that calculus is important for the degree plan, but it's necessarily not going to be important for the career that I want to be in. OK, providing goals for myself based on mastering the concepts. I think that's similar to providing goals for myself based on the performance. So if I do really well, I'll go out. If I don't, I'll just study more and then I don't change my environment. I'll study in the library or study at home. It's usually one of those two places.

Frank

Interviewer: Do you use any of these strategies or study habits, and if so, if we can just go down the list, could you tell me how you use them? So, for example, do you ever seek information from an external source, and if so, how?.

Frank: Uh, yes I would say I'd seek information by you the looking up formulas from a computer, or like asking a friend how like they they went about it.

Interviewer: OK, do you keep your records and monitor your progress in the course?

Frank: So, I'd say I keep all of the papers that I've been given and like just see how see what I've done well with and see what I've needs more studying on.

Interviewer: Alright do you ever organize information of course information and then transform it into something like that you find usable?

Frank: Let's say because I usually write down everything that goes on in like the class as in notes and like tie it all together into like a whole area of like a subject.

Interviewer: And do you seek teacher assistance? So again, this could be in lab or doing her homework problems, or just in general.

Frank: Usually I only need assistance in labs. But usually I figure out on my own or asking a friend.

Interviewer: OK, and then do you ever seek peer assistance.

Frank: Yeah, I would…Usually whenever I'm working with a peer, it's like at like at in the afternoon. After all my classes are done and we're both working on homework and like either a study area, the great hall in our dorms. And just like seeing if they what they've got on like question 17 and how they went about it, usually that's how it goes.

Interviewer: And do you ever seek assistance from an adult other than the instructor?

Frank: No.

Interviewer: Do you self-evaluate your own understanding of the course?

Frank: Yes. By evaluating to see if I even understand it by looking at it at foot at first glance, or if I need to like stop and having to write down like the question and the way of going about it.

Interviewer: OK, thank you. And do you provide goals for yourself based on how you're performing in the class.

Frank: No, I usually just go along with it.

Interviewer: Sure, do you provide extrinsic rewards based on maybe how you're performing or what you're doing in the class?

Frank: I usually just treat myself with food.

Interviewer: Do you remind yourself of the value of the task that you're learning?

Frank: No, I don't think so.

Interviewer: Do you provide goals for yourself based on how you're mastering the concepts?

Frank: I don't usually provide goals.

Interviewer: And as you're studying or working on problems, do you ever change your environment to help you maybe refocus?

Frank: Yeah… Either I'm not confident, or like unmotivated, to like do a task, so I would have to like either go do something or just be in a different area too feel motivated.

Jane

Interviewer: So, while working on math problems homework, studying, labs, do you use any of these following strategies, and if so, how? So you can literally just go down The list say yes or no and then how.

Jane: Yep, so for the first one I do all the time, seek information from an external source, especially if it's like a concept I felt like I didn't quite grasp very well. I'll go and see if I can find a different way of looking at it online in videos. I don't really keep like physical records and monitor progress. I kind of just more mentally keep track of where I am and what I know and stuff like that. I do, generally, before an exam, I'll go through homework problems and labs and stuff and I'll try and take all that information and write down any like theorems that I use or like break down any like big problems that I know I'd struggled with just to kind of review how I did that and make like a little study guide. Don't often seek teacher assistance. I do occasionally seek peer assistance. I'll ask people in my group or even people that were in my calculus class… I don't often seek assistance from adults other than the teacher. On a very rare occasion, I will ask my dad but generally not anyone else. I do generally keep like a pretty good idea of what materials I need to study in my notes I take or any problems like homework problems I do. I will write down if there's like a concept I didn't quite understand, like I got the answer. But I didn't quite understand how the problem broke down and I will go back and review it another time, especially when I'm studying for a test. I do set goals for myself generally just to my goal is almost always just to make sure I understand all of the material I'm looking at instead of just going through the motions of doing it and getting the grade. I don't provide rewards. I do always try and keep, I always remind myself that even if the math seems difficult at times, it's setting me up towards success for something I really do enjoy doing and something I want to learn more about. Again, I sort of try and keep those goals for myself on the learning, and I don't usually change my environment, but I haven't ever had a problem with it.

Jay

Interviewer: Do you use these strategies and two? If so, how do you use these strategies.

Jay: For the first, I definitely use this often I use other sources like Khan Academy or Paul’s Online notes, or even just you know the textbook to you know, make sure I completely understand, or if I don't understand, find a different way to have explained. For the keeping records, I do, basically, you know just use [university online homework system] to monitor my grades and my progress, I guess, in the class. For the organizing information, for this class I'll say I do this in that I like to make flash cards, especially of the formulas. For seeking teacher assistance, maybe a little. I don't know. And for peer assistance I do do it sometimes, but personally I don't like to - I like to work out things on my own. Seeking assistance from an adult. Probably not. I don't have a whole lot of people in my life that would a understand calculus and be willing to, you know, work that out with me. I do self-evaluate my own understanding based on. You know, practice exams and labs and things trying to see how well I'm mastering the content. For providing goals. And the extrinsic rewards, I don't think I do this often because, you know, in my head I'm like, why you know wait for the reward when I can, you know, have it now. But I do tend to remind myself, you know, the value of the task and that really keeps me motivated. You know, to remind myself why I'm doing this in the first place. And provide goals based on mastering the concepts. I do like to provide goals for, you know grades I earn, but mastering the concepts, I don't think that's, not necessarily as important, but as necessary I guess. And for changing my environment, I do that often. See where I best study and seeing where I can you know best focus. On what I have to do I can get done.

Rose

Interviewer: List and tell me when do you use this and two, if so, how do you use this in your mathematics course?

Rose: So for the first one, seek information from an external source, I try not to do it. The only source that I actually use is Desmos, just because I like having a visual and usually when I'm graphing things I wanna make sure that like I'm right. But what are the other ones like mathway or Chegg or whatever other sources, I don't like to use them, I used to use them in high school, but then I came to the realization that that's not going to help me on the exam. Like I'm not going to have my phone with me and I won't be able look it up online and it just doesn't help. So I try to stay away from those other sources other than Desmos because it's very useful in visualization and I'm a very visual person. I do keep records and do try to monitor my progress by [when] we get our lab quizzes back in our lab assignments, so I always keep my lab quizzes even though sometimes I get bad grades when I don't know what I'm doing. But I do look back on them later, like this week [before the second midterm] I've been looking at what I've missed in lab quizzes and like what I can do to get a better grade. So I have my folder full of my papers that I've had. So I might come back to that one [organize information]. Seek teachers assistance, yes I do. I have been asking my professor for help and then I do ask the lab assistant for help sometimes. I do seek peer assistance. I do ask my group mates from my lab class and there are some people that I know, they're like in my other classes that are also in precal, so sometimes I ask them for help. Seek assistance from adult other than teacher. No, I don't think I've done this one before, just because I don't know an adult who's taking precal or has taken precal that will help. Mainly just usually referred to teachers or my peers. And I do self-evaluate my understanding, I try to see what it is that I don't understand, and I make notes of them and then I would go back and. That's kind of what helps me study. Provide goals for yourself based on performance. Yes, absolutely. I have a goal of what I want to make on this midterm, so I do know like. I do know what I know and what I don't know, so that's what I'm trying to focus more on and I know that if I set a goal for myself like studying like my goal is to study like this much for this week, or my goal is to study and get this grade, I will probably do better. So I have been setting goals. I'll come back to this one [provide extrinsic rewards]. Remind yourself of the value the task for learning, I don't think I really do that, remind myself of the value of the task I'm learning. I'm gonna go with no…Yeah, provide goals, first so, I guess, I do kind of set a goal for myself based on what I the concepts I already know, yeah, and I do change my environment. I know that if I go to my room right now and I say I'm going to study, I'm not. So I'm definitely keep in mind like where it is that I work better. If the library is a good place for. Me uhm, sometimes going outside helps and sometimes being in [the math department] helps as well, but it really just depends on my mood. Yeah, so I absolutely changed my environment and the music that I'm listening to usually I just put In my earbuds. Just to block out like outside noise, music obviously does help, what I'm listening to obviously affects me as well. So I said I'm going to come back to…Organize information, transform it into something useful…I do organize information and make it into something more usable, I do use that flash card method, I don't make flash cards for like everything but I have one flash card that I write down things that I don't know how to do. So then at the end, like when I'm in my dinner break later on today, I'll be looking at that flash card and say, OK, this is how I'm supposed to work this problem out memorize that kind of thing. So I don't make flash cards as like how do you transform an exponent into a log, and vice versa. But I do use a flash card method, which is a little different. And provide rewards for myself, I say, I would say no, not really. Yeah, I just I kind of just do whatever feels right, so I don't really for me, providing a reward for myself after doing something is not really something that I do so no.

Sunny

Interviewer: So, can you go down the list and. One, do you use this strategy and two, how so?

Sunny: OK, so uh for me, I do the first one a lot. Seeking information from external source 'cause I like to be able to check my answers and I also like to be able to like have some with some videos that will go through step-by-step just in case I didn't understand it from the lecture. And records and monitor progress. I also do the same thing with [university online homework system]. I just like still see my grades and see like, oh well, if they're going down, maybe I need to start studying a little bit more. And then organizing information. I think I use this one the most out of all of them because I like to keep - I like to write notes and keep notes because I like to be able to have somewhere to go back and have all my information in one place or I can access it and use it very easily. And teacher assistance probably mostly just in labs, like whenever I get stuck and same for peers for when we're doing labs and there's something I don't understand. This one [seek assistance from adults] not so much for this one. Mostly 'cause I don't have a lot of people in my family who would understand the calculus. And then for this one [self-evaluate own understanding] I did this when I do homework, like if I do a homework problem and I see that I'm not really getting it or I gets it wrong, then I'll try to go back and see like OK, well, what did I do wrong like or what are the steps that I need to take to understand this problem more. And for providing goals, sometimes I'll do this like if I finish my homework early, then I'll have like a free day tomorrow or something. And same with this [provide extrinsic rewards], like if I if I get it done before this period of time then I'll have like all these hours for free time and this one. This one is also one I use a lot because I know that I want to like do math further and so like I keep having to remind myself that like calculus one is like very basic and I need to understand the basics before I can go further into higher level classes and then this one [set goals based on mastering the content], I think kind of goes with this one [set goals based on performance] a little bit or it's like oh well, if I know I know how to do derivatives really well then I feel like that's good and I mastered that one and then I can move on to the next one without getting stuck. And I also do this, changing my environment a lot, mostly 'cause if I stay in one place sometime I'll like I'll start getting distracted with something that's just like sitting in front of me so I have to like move somewhere where there's less things to get distracted by.

Toby

Interviewer: Alright, so then, same idea that I want you to do is kind of go. Through down the list, tell me if you do them and if. So how you do this?

Toby: OK, for the seek information from an external source I also like watch YouTube videos, and like the Khan Academy style is pretty good explanations. So yeah, I sometimes look up from external sources. To keep records and monitor progress, mostly I check my grades. And I keep my quizzes in like certain folders and in my backpack, but I don't like monitor which ones I'm getting wrong, but that's a good idea. I should do that. Organize information and transform it to something usable. I don't think so. I don't do that. Seek teacher assistance. Yeah, whenever my peers - so I guess that ties into the next one. Whenever I don't get it from up here [peers], then I'll ask the teacher [in a lab setting]. OK, yeah and then from adults other than my teacher may. My dad's an engineer, so I sometimes ask him, he's good with math. Self-evaluate my understanding. Same thing with the quizzes, like if I don't like my grade, I check what I'm not understanding and what I am understanding and I try to look up practice questions just to make sure I'm getting it or not. Goals based on performance. Yeah, so like based on my last test grade I'll try to do like 10 points higher or 15 points higher or something like that. Extrinsic rewards, not really. Yeah, I also have the, not the best strategy of studying until I just get tired and I'm. Like, OK, I'll take a break and then my. I feel guilty and then go back. Remind myself of the value of subject in my learning yes. I always try to like, OK if I want to do this in life, I need to be able to get this over with and finish it. Provide goals for myself based on mastering the concepts. No, I don't do that. And then change my environment. I have a like a set place at home that I study and then sometimes if I'm already on campus, I study in the library.

Appendix O

Self-Efficacy, Exam Scores, and Project Course Grade (Surveyed Participants [excluding interviewed participants])

Self-Efficacy Score	Midterm 1 (%)	Midterm 2 (%)	Projected Course Grade	Self-Efficacy Score	Midterm 1 (%)	Midterm 2 (%)	Projected Course Grade
0.72	89	34.5	B	0.9	96	90	A
0.5	40	72.5	C or D	0.69	100	88	A
0.57	84	75	B	0.96	99	90	A
0.83	92	94.5	A	0.68	80	79	B
0.65	67	16	F	0.79	66	49	D
0.8	95	94	A	0.75	94	89	A
0.42	58	61	C	0.8	92	96	A
0.59	65	46.5	C	0.76	100	95	A
0.79	92	86	A or B	0.84	100	98	A
0.76	69.5	57.5	C	0.74	47	55	C*
0.69	93	77	B	0.59	71	51.5	C
0.71	95	82	A	0.67	63	52.5	D
0.69	38	15	B*	0.68	74	32.5	D
0.9	92	92	A	0.84	95	63	B
0.79	40	39.5	D or F	0.76	92	91	A
0.59	98	68	C	0.53	74	57.5	B
0.55	99	92	A	0.89	92	82	A
0.75	88	73	B	0.25	92	47.5	C
0.75	98	89	A	0.81	95	77	A or B
0.75	79	90	A or B	0.54	63	53	C
0.69	88	52.5	B or C	0.39	86	89	B
0.82	92	87	A	0.55	69.5	65.5	C

0.55	80	76	B or C	0.81	58	24	F
0.60	78	42.5	C	0.47	57	28	D
0.58	81	55	C	0.70	99	79	B
0.82	72	75	C	0.76	69	49	C
0.74	38	48.5	F	0.74	63	60	D
0.75	96	81	B	0.67	80	54	B
1.0	93	96	A	0.75	100	94	A
0.79	95.5	94	A	0.58	95	94	A
0.73	71	44	F	0.79	92	82	A
0.54	54	60	C or D	0.62	87	78.5	B
0.99	81	76	B	0.58	72	46.5	C
0.78	96	95	A	0.82	88	97	A
0.53	82	90	A	0.60	61.5	68	C
0.74	84	73	C	0.83	48	82	C
0.81	68	43	B	0.65	92	68	B or C
0.65	50	50	C	0.60	83	57	C
0.53	86	56.5	B or C	0.75	87.5	57.5	B
0.75	88	51.5	B	0.56	57	39	D
0.34	80	66	C	0.54	88	82	B
0.71	31	71	C	0.79	87	84	A
0.59	82	56	C				

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