

MULTI-AREA UNIT COMMITMENT AND ECONOMIC DISPATCH
WITH MARKET OPERATION COMPONENTS

by

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In memory of my grandfather

Yongsuk Yingvivanapong

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ABSTRACT

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Unit commitment and economic dispatch, when combined together, is a useful tool to find the most economical generation schedule with which demand and spinning reserve requirement are supplied and all generating unit constraints, such as unit ramp rates, unit minimum and maximum generation capabilities, and unit minimum up-time and down-time, are satisfied over a time horizon. For systems consisting of multiple areas, area import/export capabilities and tie line capacity constraints also need to be taken into account. Adaptive Lagrangian relaxation, unit decommitment and lambda-iteration methods are applied in this dissertation to determine the optimal unit commitment and economic dispatch schedule for large-scale multi-area systems.

In addition, the market operation components are integrated into this newly developed multi-area UC and ED to optimize the cost of generation and meet the needs of the restructured power industry. These market operation components consist of bilateral contracts which are direct agreements between a power producer and either a load serving entity or an ISO outside of a centralized bid-based power market. The bilateral contracts considered in this dissertation include call options, put options, forward contracts and reliability must-run (RMR) contracts. Options and forward contracts are commonly used by generation companies and load serving entities to hedge against the risk of price volatility in the real-time market and increase their profit. The RMR contracts are bilateral agreements between an ISO and generation companies to operate designated generating units at specified MW for the local demand. These contracts are initiated to mitigate local area reliability problems when other market solutions such as transmission expansion are not available.

The multi-area unit commitment and economic dispatch program with important market operation components developed in this dissertation will provide the dispatchers a robust tool for planning both operation and market strategies with consideration of cost optimization, risk management, and resource adequacy.

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CHAPTER 1

INTRODUCTION

This chapter provides background of unit commitment (UC) and economic dispatch (ED), the impact of restructuring power industry on the development of the UC and ED, and the contribution of this dissertation. The dissertation organization is also described in this chapter.

1.1 Background of Unit Commitment and Economic Dispatch

Unit commitment and economic dispatch have been used to plan over a given time horizon the most economical schedule of committing and dispatching generating units to meet forecasted demand levels and spinning reserve requirements while all generating unit constraints are satisfied. For interconnected power systems which contain multiple areas and tie lines, area import/export capability and tie line capacity constraints are also needed to be considered in the multi-area UC and ED. The generating unit schedule that yields minimum total production cost, which consists of costs of fuel, operating crew members, maintenance, starting up generating units, and shutting down generating units, is the optimal solution of the UC and ED.

With better computer hardware and programming software as well as better methodologies, the UC and ED have been enhanced in terms of speed and optimization. There are three main factors involving in obtaining an optimal generation planning or

minimum total production cost. These factors are accurate forecasted demand, precise on-line and off-line schedule of generating units, and economic allocation of generation to available generating units, all of which are subject to demand and reserve requirements, generating unit constraints, area import/export capabilities and tie line capacity constraints.

Since the UC is a large non-linear, non-convex, and mixed-integer problem, it is a great challenge to obtain an optimal or sub-optimal UC solution. Mathematical formulations for solving a UC problem include conventional methods, i.e. Exhaustive Enumeration, Priority List, Dynamic Programming, Lagrangian Relaxation, Sequential method, Decolmitment method, Branch and Bound techniques, Mixed Integer Programming, etc., and new developing methods, i.e. Genetic Algorithms, Tabu Search, Artificial Neural Network, and Expert Systems, etc. The methodologies and analysis of these techniques are summarized in chapter 3.

Many methods are used to solve the ED problems. When considering only real power optimization with losses but without security constraints, methods for solving the ED include lambda-iteration, gradient method, Newton's method, dynamic programming, and artificial neural network. These methods are discussed in detail in chapter 3. When used with multi-area system, the ED determines the most economical dispatching schedule of on-line units and amount of power that can be economically generated in one area and transferred to other areas having higher marginal production cost with the consideration of area import/export constraints and tie line capacity constraints.

Prior to restructuring of the power system, the UC and ED were performed by vertically integrated utilities to minimize the production cost of supplying their customers and providing the spinning reserve. Occasionally, there were power exchanges or interchanges between utilities without sharing the necessary information such as the unit characteristics. To take more economical advantage on power interchange, power pools were formed by several interconnected utilities. The coordinating UC and ED were performed by a central dispatch office [1]. In addition to minimizing operating costs, the power pool minimizes the reserve requirement because power can be transferred from the pool transmission grid during emergencies or resource inadequacy. The pricing arrangements between the pool members may include take-or-pay contracted price, split saving, or fixed price [1-2].

1.2 Unit Commitment and Economic Dispatch in the Restructured Power Industry

This section provides the summary of restructuring U.S. power industry and the development of the UC and ED to meet the requirements of the competitive and centralized electricity market.

1.2.1 Restructuring U.S. Power Industry

In 1996, Federal Energy Regulatory Commission (FERC) of the United States of America has implemented the Energy Policy Act of 1992 by issuing Orders No. 888 and No. 889. Order 888 mandated all public utilities who own transmission to file Open Access non-discriminatory Transmission Tariffs (OATTs) and permitted public utilities and transmitting utilities to seek recovery of stranded costs. In order No. 888, FERC also recommended establishing Independent System Operators (ISOs) to monitor the

reliability of the power system and coordinate the supply of electricity in each region. Order No. 889 initiated the Open Access Same-Time Information System (OASIS) and Standards of Conduct, which required utilities to open information about their available transmission capacity, price, and access to transmission services non-discriminatorily. This ensures that transmission owners do not have an unfair advantage in accessing transmission over other transmission users.

Additionally, the historical structure of power system whereby a vertically integrated utility that owned generating plants, high voltage transmission system, and distribution lines as well as provided all electric services was required to be changed. Obligatorily, all vertically integrated utilities in the restructured regions needed to separate their assets and services into generation, delivery services including transmission and distribution, and retail sales. Therefore, a distinction can be made between generation companies (GENCOs), transmission companies (TRANSCO), distribution companies (DISTCOs), and load serving entities (LSEs). A load serving entity is an entity that provides electricity and related services to customers including wholesale customers. LSEs include retail electric providers (REPs), competitive retailers, and non-opt in entities that serve load. Market share of large vertically integrated utilities was also reduced as wholesale power markets were allocated to non-utility producers.

The power industry became more competitive and more regionalized. Some regions are organized by an Independent System Operator (ISO) or an Regional Transmission Organization (RTO) which was established by FERC in Order No. 2000.

The main roles of the ISOs and RTOs are to perform transmission planning, ensure wholesale power grid reliability as well as equal access to the grid, and economically balance electricity demand and supply. In 2002, FERC issued a notice of proposed rulemaking on standard market design (SMD) which established spot markets for energy, operating reserves, and transmission service as a bid-based, security-constrained, economic dispatch with locational marginal prices (LMPs). The commission also introduced congestion revenue rights (CRRs) which enable the right holders to receive the specified transmission congestion revenue to compensate the congestion charge. In addition, the commission proposed a resource adequacy requirement that complements bilateral contracting and long-term contracting.

In April 2003, the nine functioning ISOs and RTOs in North America formed an ISO/RTO council (IRC). The IRC is comprised of the Alberta Electric System Operator (AESO), ISO New England (ISO-NE), New York ISO (NYISO), PJM (covering Pennsylvania, New Jersey, Maryland, Delaware, Virginia, and the District of Columbia), Midwest ISO (MISO), California ISO (CAISO), Electric Reliability Council of Texas (ERCOT), the Independent Electricity System of Ontario (IESO), and Southwest Power Pool (SPP).

1.2.2 UC and ED after Restructuring Power Industry

After separation of generation from TRANSCOs, DISTCOs, and LSEs, the GENCOs pay more attention on how to maximize their profit and less on whether the demand and reserve are completely met [3]. Since reducing production cost of generation means more chance of competing with other suppliers in the regional market

whose transmission network is controlled by the ISO or RTO, many developers have been trying to increase the efficiency of the UC. All power producers who participate in the competitive electricity market must submit their supply bid curves which reflect the incremental cost curves of their generating units. Likewise, LSEs also have to submit their demand bid curves consisting of a set of quantities at certain prices.

The ISO or RTO then uses the supply and demand bid curves to determine the balancing energy market clearing price which is the intersection of the combined supply bid curve and the combined demand bid curve [4]. All suppliers who procure the bids are paid the same price – the market clearing price. This mechanism intends to encourage generators to submit the low supply bid curves. These curves are supposed to reflect generating units' marginal cost curve because theoretically, in perfectly competitive markets, profit is maximum when supply is sold at marginal cost.

Conventionally, UC and ED use generating units' monotonically increasing cost curves in either piecewise linear format or quadratic format to determine minimum production cost. Once, energy market became centralized and competitive, UC and ED have been applied to use with bid curves submitted by market participants. These bid curves are typically in piecewise linear format. The UC and ED, therefore, can determine which generating units owned by different firms will procure the bids or be committed and how much they will be dispatched.

1.2.3 UC and ED with Market Operation Components

In addition to participating in a bid-based market, the GENCOs and the LSEs can sell and buy power through the bilateral contracts. With the encouragement of SMD

issued by FERC and the benefits of long-term contracts, the bilateral contracts between the GENCOs and the LSEs and reliability must-run contracts between ISOs and GENCOs, began to play an important role in the generation planning. The details of these market operation components are discussed as follows.

1.2.3.1 Electricity Trading Contracts

Under uncertainty of competitive electricity market, electricity trading contracts or derivatives, such as *options*, *futures contracts*, and *forward contracts*, are used to hedge against the risk of price uncertainty in spot market and have better market liquidity due to providers being able to contract for energy for longer terms. These electricity derivatives, except futures contracts, are bilaterally exercised between two market participants over the phone, by electronic transactions, or through a broker. This is called bilateral contract in over-the-counter markets. On the other hand, electricity futures are traded in exchange markets. Therefore, the futures prices are more transparent to the public than the forward prices or settled option prices.

Both forward and futures contracts are agreements between two parties to buy or sell electricity at a certain future time, either short-term or long-term, for a certain price [5]. The *payoff* of these contracts is the difference between the contracted price and the spot price at the contracted time. However, futures contracts are often significantly smaller than that in forward contracts and more standardized in details as well as contain lower credit risks due to being paid out daily when compared to being paid out in a lump sum at maturity time as in forward markets [5, 6].

Although it does not cost anything to participate in the forward or futures markets but the holders must exercise the right. On the other hand, the holders of options are not obligated to follow the agreement but there is an expense for acquiring an option. The plain options can be categorized into two types which are *put option* and *call option*. A put option holder can sell the electricity when the price is equal to or greater than the settled price during a certain period. This settled price is known as *exercise price* or *strike price*. In contrast, a call option gives the holder the right to buy the electricity during a certain period when the price is equal to or below a strike price.

Oren in [7] claims that call and put options are the most effective tools available to power producers and power marketers for hedging the price risk. Options also create an incentive for investment in generation capacity and a solution for resource adequacy. Additionally, call options can be used as the option-based capacity mechanism to secure reserve capacity.

1.2.3.2 Reliability Must-Run Contracts

In addition to electricity trading contracts, there is another type of agreements, Reliability Must-Run (RMR) contracts, that affects the generation schedule of some generating units. These contracts are employed for the system reliability purpose. The RMR contract is established by ISOs or RTOs to ensure that power supply is adequate in certain areas to which power cannot be transferred from the cheaper source due to transmission constraint or voltage support issues. RMR participants can define a generating unit or a combination of units to be designated RMR unit(s). Generally, ISOs/RTOs have to contract for the entire capacity of each RMR unit and owners of

RMR units are paid in accordance with RMR contract. When the RMR units are requested to run less than its full capacity, the rest energy of RMR units, which are in must-run status, can be participated in the ED.

1.3 Objectives of Dissertation

The objectives of this dissertation are to improve in optimization and speed of multi-area UC and ED calculation as well as incorporate the market operation components into the conventional generation planning to maximize the profit, exercise the risk management, and maintain the reliability. Since the power industry has been restructured and the interconnected energy grids have been regionally operated, the efficient multi-area UC and ED which is able to handle large-scale systems and considers ramp rates of generating units, the area import/export capabilities and transfer limitation of tie lines is needed.

In this dissertation, lambda-iteration method through sorted lambda tables is applied for the ED and adaptive Lagrangian relaxation and unit decommitment methods are implemented for the UC. A DC power flow model with shift factors and the transfer limits is used to represent the multi-area transmission interconnections. Call and put options, forward contracts, and reliability must-run contracts are incorporated into the multi-area UC and ED decision as a derivative tool to procure the resource and supply the demand. The users have alternatives to implement some or all of these contracts. The comparison of the cost of generation schedule without implementing these contracts and the cost of that with implementing combinations of contracts will give users a guideline of the future generation plan.

The multi-area UC and ED with market operation components enable a firm who owns generating units in multiple areas are to optimize profit, manage its risk, and maintain the system reliability. In addition, an ISO or an RTO can further integrate security-constrained optimal power flow, a combination of a contingency analysis and an optimal power flow, into the multi-area UC and ED with market operation components for security-constrained unit commitment and economic dispatch to ensure the entire system reliability with resource adequacy and the fair electricity price for consumers.

1.4 Dissertation Organization

This dissertation consists of six chapters and three appendixes. Each chapter and appendix is described as follows.

The first chapter presents the introduction of UC and ED, the idea of UC and ED with market operation components in restructured power industry, and the objectives as well as organization of this dissertation.

The second chapter discusses models of thermal unit commitment and economic dispatch. It consists of notation used throughout this dissertation, an objective function of unit commitment subject to system, thermal unit, and other constraints, and an objective function of economic dispatch subject to power balance, generation limits of generating units, and area import/export capabilities.

The third chapter reviews UC and ED methodologies. It summarizes and analyzes several UC and ED methods.

The fourth chapter presents proposed algorithm for multi-area unit commitment and economic dispatch with market operation component.

System used and simulation results are presented in chapter five. A ten-unit system and a twenty-unit system are tested to compare the result, total production cost and computational time, with other UC methods presented in references. The multi-area system used for testing multi-area features contains seventy eight units which are located in six subareas and three areas. Market operation components are also tested in the single-area and multi-area systems. The comparison of simulation results of systems without and with market operation components are also illustrated.

The last chapter summarizes the proposed algorithms for the multi-area UC and ED with market operation components, the simulation results, and the contribution of this dissertation. Future studies are also suggested in chapter 6.

The first appendix includes class structures of the UC and ED. The second appendix consists of input data for the ten-unit system and simulation results of the ten-unit and twenty-unit systems. The last appendix consists of input data and simulation results of the multi-area system.

CHAPTER 2

MATHEMATICAL FORMULATION OF THERMAL UNIT COMMITMENT AND ECONOMIC DISPATCH

Each thermal generating unit consumes fuel, e.g. coal, gas, oil, or a combination of them, at a specific rate in million of Btu per hour (MBtu/Hr) to generate power in million of Watt (MW). This rate characteristic of a thermal generating unit can be plotted as an input-output or heat rate-power curve. The input-output curve when multiplied by cost of fuel in dollar per million of Btu (\$/MBtu) becomes a fuel cost curve of a generating unit.

The processes of determining a generating unit schedule that yields the minimum total production cost and satisfies all constraints are called unit commitment (UC) scheduling. The total production cost includes the fuel costs, operation and maintenance cost (O&M cost), start-up cost (cost of bringing units on-line), and shut-down cost (cost of shutting down units).

Based on the on-line/off-line schedule of generating units determined by unit commitment, economic dispatch (ED) uses the incremental cost curves of on-line generating units, which are the slope (the derivative) of their cost curves with respect to power output, to calculate for their most economical generation output level. Once the dispatch levels of on-line units are determined, their fuel cost curves and O&M cost curves are used to calculate for the total fuel cost and the total O&M cost.

Chapter 2 consists of three sections. All notations used in this dissertation are defined in section 2.1. The UC and ED mathematical formulations are described in section 2.2 and 2.3.

2.1 Notation

N	:	Number of all units in the system
N_k	:	Number of all units in area k
K	:	Number of areas in the system
T	:	Number of hours in the planning horizon
P_i^t	:	Real power of unit i generated at hour t [MW]
$P_{i,k}^t$:	Real power of unit i in area k generated at hour t [MW]
P_i	:	Real power of unit i generated at a given hour [MW]
P_T	:	Total real power of all on-line units generated at a given hour [MW]
$FC_i^t(P_i^t)$:	Fuel cost function with respect to P_i^t of unit i at hour t [\$]
$FC_i(P_i)$:	Fuel cost function with respect to P_i of unit i at a given hour [\$]
$OMC_i^t(P_i^t)$:	Operating and maintenance (O&M) cost function with respect to P_i^t of unit i at hour t [\$]
SC_i^t	:	Start-up cost of unit i at hour t [\$]
$SC_{i,k}^t$:	Start-up cost of unit i in area k at hour t [\$]

- SCD_i^t : Shut-down cost of unit i at hour t [\\$]
- $U_{i,k}^t$: State of unit i in area k at hour t , expressed by a binary variable (1 for on and 0 for off)
- $U_{i,k}^{t-1}$: State of unit i in area k at hour $t-1$, expressed by a binary variable (1 for on and 0 for off)
- D_k^t : Forecasted real power demand of area k at hour t [MW]
- D : Forecasted real power demand of the system at a given hour [MW]
- L_k^t : Real power losses of area k at hour t [kW, MW]
- L : Real power losses of the system at a given hour [kW, MW]
- R_k^t : Real power spinning reserve requirement of area k at hour t [MW]
- $P_{i,\min}$: Either plant minimum output limit specified by the manufacturer or economical minimum output limit [MW]
- $P_{i,k,\min}^t$: Minimum real power capacity of unit i in area k at hour t [MW]
- $P_{i,k,\min}$: Minimum real power capacity of unit i in area k at a given hour [MW]
- $P_{k,\min}$: Minimum real power capacity of area k at a given hour [MW]
- $P_{\text{sys},\min}$: Minimum real power capacity of the entire system at a given hour [MW]
- $P_{i,\max}$: Either plant maximum output limit specified by the manufacturer or economical maximum output limit. [MW]

- $P_{i,k,\max}^t$: Maximum real power capacity of unit i in area k at hour t [MW]
- $P_{i,k,\max}$: Maximum real power capacity of unit i in area k at a given hour [MW]
- $P_{k,\max}$: Maximum real power capacity of area k at a given hour [MW]
- $P_{\text{sys},\max}$: Maximum real power capacity of the entire system at a given hour [MW]
- $I_{k,\max}^t$: Real power import limit of area k at hour t [MW]
- $E_{k,\max}^t$: Real power export limit of area k at hour t [MW]
- I_k^t : Real power imported to area k [MW]
- I_k : Real power imported to area k at a given hour [MW]
- E_k^t : Real power exported from area k [MW]
- E_k : Real power exported from area k at a given hour [MW]
- $Intch_k$: Interchange of area k at a given hour [MW]
- $SF_k^{g,l}$: Shift factor or power flow from area g to area l due to the injection of 1 MW at area k and withdrawal of the 1 MW at the reference area.
- SF_k^f : Shift factor or power flow on flow f due to the injection of 1 MW at area k and withdrawal of the 1 MW at the reference area.
- $F^{g,l}$: Tie line capacity constraint of flow from area g to area l [MW]
- F^f : Tie line capacity constraint of flow f [MW]
- $Flow^{g,l}$: Actual tie line flow from area g to area l [MW]

F	:	Total number of flow in the system
P_i^{t-1}	:	Real power of unit i generated an hour prior to hour t [MW]
∇_i	:	Ramp-down rate of unit i [MW/minute]
Δ_i	:	Ramp-up rate of unit i [MW/minute]
x_i^{t-1}	:	Number of hours that unit i has consecutively been on-line (if it is a positive integer) or off-line (if it is a negative integer) from the hour that it was turned on or off to hour $t-1$ [Hour]
t_i^{up}	:	Minimum number of hours that unit i must remain on after it has been turned on, called minimum-up time [Hour]
t_i^{down}	:	Minimum number of hours that unit i must remain off after it has been turned off, called minimum-down time [Hour]
C_b	:	Banking-start constant [MBtu/Hr]
C_c	:	Cold-start constant [MBtu]
t_b	:	Number of hours the unit was banked [Hour]
γ	:	Thermal time constant for the generating unit
t_c	:	Number of hours the unit was cooled [Hour]
$Fprice_i$:	Price of fuel used for unit i [\$/MBtu]
C_f	:	Fixed start-up cost [\$]
m	:	Operation and maintenance cost coefficient of unit i at hour t per MWh [\$/MWh]

- FC_T : Total fuel cost of generating power to supply demand at a given hour [\\$]
- $F_i(P_i)$: Fuel function with respect to P_i of unit i [MBtu]
- a_i : Quadratic fuel coefficient of unit i [MBtu/MW²]
- b_i : Linear fuel coefficient of unit i [MBtu/MW]
- c_i : Minimum fuel used during no load of unit i [MBtu]
- PF_i : Penalty factor of unit i
- λ, λ_{sys} : Marginal cost of supplying energy to meet the system demand or incremental fuel cost rate of the entire system, also called *lambda* [\$/MWh]
- λ_i : Incremental fuel cost rate of unit i , also called unit lambda [\$/MWh]
- μ : Marginal cost of serving system spinning reserve requirement, also called *mu* [\$/MW]
- α_k : Marginal cost of serving spinning reserve requirement minus limited import capability of area k , also called *alpha* [\$/MW]
- β_k : Marginal cost of maintaining minimum capacity below demand plus limited export capability of area k , also called *beta* [\$/MW]
- θ_f : Marginal cost of maintaining power transferring on flow f below its transfer limit, also called *theta* [\$/MW]

- $P_i(\lambda)$: Power output of unit i with respect to λ or its incremental fuel cost rate [MW]
- A_i : Quadratic cost coefficient of unit i which is $a_i \cdot Fprice_i$ [\$/MW²]
- B_i : Linear cost coefficient of unit i which is $b_i \cdot Fprice_i$ [\$/MW]
- $\lambda[k]$: Incremental fuel cost rate at break point k of an incremental fuel cost curve [\$/MWh]
- $\lambda[k-1]$: Incremental fuel cost rate at break point $k-1$ of an incremental fuel cost curve [\$/MWh]
- $P[k]$: Total real power output at break point k of an incremental fuel cost curve [MW]
- $P[k-1]$: Total real power output at breakpoint $k-1$ of an incremental fuel cost curve [MW]
- δ, σ : Constant step size parameters, empirically determined, for updating Lagrange multipliers.
- j : Iteration number
- $\lambda^{t(j+1)}$: Lambda at hour t in iteration $j+1$ [\$/MWh]
- $\lambda^{t(j)}$: Lambda at hour t in iteration j [\$/MWh]
- $\mu^{t(j+1)}$: Mu at hour t in iteration $j+1$ [\$/MW]
- $\mu^{t(j)}$: Mu at hour t in iteration j [\$/MW]
- $\alpha_k^{t(j)}$: Alpha of area k in hour t for iteration j [\$/MW]

- $\beta_k^{t(j)}$: Beta of area k in hour t for iteration j [\$/MW]
- $\theta_f^{t(j)}$: Theta of flow f in hour t for iteration j [\$/MW]
- $pDiff^t$: Subgradient of power balance constraint function in hour t
- $rDiff^t$: Subgradient of spinning reserve constraint function in hour t
- $rDiff_k^t$: Subgradient of spinning reserve constraint function considering import limit of area k in hour t
- $mDiff_k^t$: Subgradient of minimum generation limit function considering export limit of area k in hour t
- $fDiff_f^t$: Subgradient of tie line capacity constraint function of flow f in hour t

2.2 Thermal Unit Commitment Formulation

The objective function of the unit commitment problem is minimizing the total operating cost of generating power over a planning horizon as follows:

$$\text{Min} \sum_{i=1}^N \sum_{t=1}^T \left\{ [FC_i^t(P_i^t) + OMC_i^t(P_i^t) + SC_i^t(1 - U_i^{t-1})] \cdot U_i^t + SCD_i^t(1 - U_i^t) \cdot U_i^{t-1} \right\} \quad (2.1)$$

This objective function is subject to following constraints.

2.2.1 System Constraints

System constraints include power balance, spinning reserve, area import/export, and tie line capacity constraints. These constraints involve with all generating units in the system, thus they are considered as coupling constraints. The detail of each constraint is described as follow.

2.2.1.1 Power Balance Constraint

Generation needs to be equal to power demand plus losses. Demand in the competitive market may mean the load ratio share, or load awarded, determined by ISOs and amount of total interchange (exported power minus imported power) to external systems. Power balance constraint can be written as equation 2.2.

$$\sum_{k=1}^K \sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) = \sum_{k=1}^K (D_k^t + L_k^t) \quad (2.2)$$

2.2.1.2 Spinning Reserve Constraint

Total maximum capacity of on-line units must be greater than demand plus losses and spinning reserve requirement. This spinning reserve is necessary to fulfill unexpected increase in demand or forced outages of generating unit. The amount of the required spinning reserve is usually determined by the maximum capacity of one of two largest generating units in the system or a given percentage of forecasted peak demand during interested time horizon. In the restructured power industry, some GENCOs may calculate the amount of the required spinning reserve from a given percentage of load ratio share determined by an ISO. Spinning reserve constraint formulation is shown in equations 2.3.

$$\sum_k \sum_{i=1}^{N_k} (P_{i,k,\max}^t \cdot U_{i,k}^t) \geq \sum_{k=1}^K (D_k^t + L_k^t + R_k^t) \quad (2.3)$$

2.2.1.3 Area Import/Export Constraints

With multiple areas and limited area import and export capabilities, multi-area unit commitment must consider spinning reserve requirement and capabilities of each

area as constraints in equations 2.4 and 2.5. These import and export limits are necessary for the multi-area unit commitment due to tie line limitation, area security considerations, fuel availability, and regulatory restrictions [2].

$$\sum_{i=1}^{N_k} (P_{i,k,\max}^t \cdot U_{i,k}^t) \geq D_k^t + L_k^t - I_{k,\max}^t + R_k^t \quad (2.4)$$

$$\sum_{i=1}^{N_k} (P_{i,k,\min}^t \cdot U_{i,k}^t) \leq D_k^t + L_k^t + E_{k,\max}^t \quad (2.5)$$

2.2.1.4 Tie Line Capacity Constraints

Allocating power from an area having cheaper generation cost to the other areas can contribute the total cost minimization. However, besides area import and export limits, tie line capacity constraints also limit the amount of power being transferred from areas having lower incremental cost rate to the other areas. With these constraints, the sum of power flows in direction from area g to area l must be within the transfer limit from area g to area l ($F^{g,l}$). The power flows on a specific tie line and specific direction contributed by power exported from areas can be determined by using shift factor when each area is assumed to be a bus.

The shift factor, also called power transfer distribution factor, is the power flow across a grid when one MW is injected at a source bus and withdrawn at a reference bus. This means that all shift factors must be between zero and one. Therefore, the power flow on any tie line on a specific direction due to the injection of MWs at an area and withdrawal of the equal amount of MWs at the reference area can be determined by applying the Kirchoff's voltage law and current law or using the shift factor. The shift

factor can be used with the following assumptions. First, a direct current (DC) power flow model is used in this study. Second, voltage at each node (area) is simply assumed to be equal to one per unit (1 p.u.). Finally, the effective impedance inside each area must be the same or power is injected at the same bus as that is withdrawn.

Therefore, tie line capacity constraints, as shown in equation 2.6, can be defined as the sum of the power flows from area g to area l contributed by the power exported from every area must be within the transfer limit of tie line from area g to area l . The power exported from any area is defined as the surplus generation from the demand and the losses of that area at a given hour as represented in equation 2.7. The number of tie line capacity constraints is equal to the number of tie lines times two because each line has two directions of power flow.

$$\sum_{k=1}^K (SF_k^{g,l} \cdot E_k^t) \leq F^{g,l} \quad (2.6)$$

$$E_k^t = \sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) - D_k^t - L_k^t \quad (2.7)$$

A simple three-area system with three tie lines is used as an example to explain how to obtain these tie line capacity constraints. In this example, area 1 is a reference bus, all tie lines are assumed to have the same impedance, and their flow limits are specified in table 2.1 and figure 2.1.

Table 2.1 Tie Line Capacity Limits of a Three-area System

g	l	$F^{g,l}$	$F^{l,g}$
1	2	1000	750
2	3	800	700
3	1	650	700

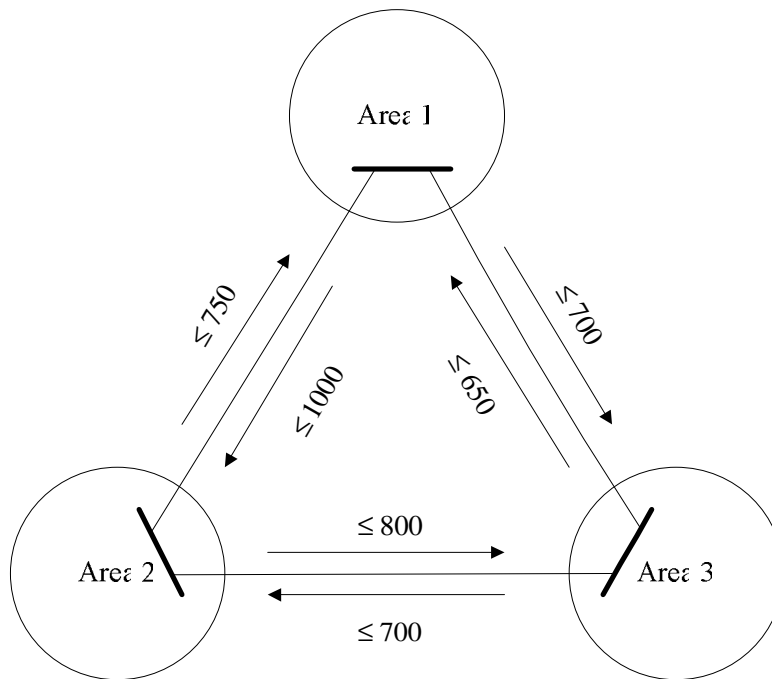


Figure 2.1 A sample of three-area system with three tie lines

With area 1 as the reference, the injection of 1 MW at:

- area 2, $2/3$ MW will flow from area 2 to area 1, $1/3$ MW will flow from area 3 to area 1, and $1/3$ MW will flow from area 2 to area 3, or at
- area 2, $-2/3$ MW will flow from area 1 to area 2, $-1/3$ MW will flow from area 1 to area 3, and $-1/3$ MW will flow from area 3 to area 2, or at
- area 3, $1/3$ MW will flow from area 2 to area 1, $2/3$ MW will flow from area 3 to area 1, and $-1/3$ MW will flow from area 2 to area 3, or at
- area 3, $-1/3$ MW will flow from area 1 to area 2, $-2/3$ MW will flow from area 1 to area 3, and $1/3$ MW will flow from area 3 to area 2, or at
- area 1, 0 MW will flow from area 2 to area 1, 0 MW will flow from area 3 to area 1, and 0 MW will flow from area 2 to area 3.

The shift factors, shown in table 2.2, are obtained by adding up the flow contributions over the lines.

Table 2.2 Shift Factors of a Three-area System with Area 1 As a Reference Bus

g	l	$SF_1^{g,l}$	$SF_2^{g,l}$	$SF_3^{g,l}$
1	2	0	-2/3	-1/3
2	1	0	2/3	1/3
2	3	0	1/3	-1/3
3	2	0	-1/3	1/3
3	1	0	1/3	2/3
1	3	0	-1/3	-2/3

Therefore, six constraints for this three-area and three-tie line system can be written as following six equations.

$$\text{Flow from area 1 to area 2 must be within 1000 MW, } 0 \cdot E_1^t - \frac{2}{3} \cdot E_2^t - \frac{1}{3} \cdot E_3^t \leq 1000$$

$$\text{Flow from area 2 to area 1 must be within 750 MW, } 0 \cdot E_1^t + \frac{2}{3} \cdot E_2^t + \frac{1}{3} \cdot E_3^t \leq 750$$

$$\text{Flow from area 2 to area 3 must be within 800 MW, } 0 \cdot E_1^t + \frac{1}{3} \cdot E_2^t - \frac{1}{3} \cdot E_3^t \leq 800$$

$$\text{Flow from area 3 to area 2 must be within 700 MW, } 0 \cdot E_1^t - \frac{1}{3} \cdot E_2^t + \frac{1}{3} \cdot E_3^t \leq 700$$

$$\text{Flow from area 3 to area 1 must be within 650 MW, } 0 \cdot E_1^t + \frac{1}{3} \cdot E_2^t + \frac{2}{3} \cdot E_3^t \leq 650$$

$$\text{Flow from area 1 to area 3 must be within 700 MW, } 0 \cdot E_1^t - \frac{1}{3} \cdot E_2^t - \frac{2}{3} \cdot E_3^t \leq 700$$

2.2.2 Thermal Unit Constraints

Each individual thermal unit has its own constraints which include initial condition, minimum and maximum generation output limits, maximum ramp-down and ramp-up rates, minimum up-time and down-time, and unit status restriction constraints. They are considered as non-coupling constraints. Each constraint is described as follows.

2.2.2.1 Initial Condition

Initial condition of a generating unit includes number of hours that it has consecutively been on-line ($x_i^0 > 0$) or off-line ($x_i^0 < 0$) and its generation output ($P_i^0 \geq 0$) at an hour before the scheduled time horizon starts. Both initial number of on-line/off-line hours and initial generation output, when associated with the other unit constraints, may limit the on-line/off-line status and the generation output of the unit in the scheduled time horizon.

2.2.2.2 Minimum and Maximum Generation Output Limits

A range of unit power outputs is specified by either machine output limits or economical output limits. However, the minimum and maximum limits that bound the generation output of each generating unit in particular hour, as shown in equation 2.8, can be varied within the range of unit power outputs due to unit ramp rate constraints. These constraints are described in the following sections.

$$P_{i,\min}^t \leq P_i^t \leq P_{i,\max}^t \quad (2.8)$$

2.2.2.3 Maximum Ramp-down Rate

Maximum ramp-down rate is maximum generation output in a minute that unit i is able to decrease in an hour. This rate limits the minimum output capability of unit i at hour t ($P_{i,\min}^t$) as equation 2.10.

$$P_i^{t-1} - P_i^t \leq \nabla_i * 60 \quad (2.9)$$

$$P_{i,\min}^t = \max(P_{i,\min}, P_i^{t-1} - \nabla_i * 60) \quad (2.10)$$

2.2.2.4 Maximum Ramp-up Rate

Maximum ramp-up rate is maximum generation output in a minute that unit i is able to increase in an hour. This rate limits the maximum output capability of unit i at hour t ($P_{i,\max}^t$) as equation 2.12.

$$P_i^t - P_i^{t-1} \leq \Delta_i * 60 \quad (2.11)$$

$$P_{i,\max}^t = \min(P_{i,\max}, P_i^{t-1} + \Delta_i * 60) \quad (2.12)$$

2.2.2.5 Minimum Up-time and Minimum Down-time

Minimum up-time is minimum number of hours that a unit must be on-line once it has been turned on. Minimum down-time is minimum number of hours that unit must be off-line once it has been turned off.

$$U_i^t = \begin{cases} 1, & \text{if } 1 \leq x_i^{t-1} < t_i^{up} \\ 0, & \text{if } -1 \geq x_i^{t-1} > -t_i^{down} \\ 0 \text{ or } 1, & \text{otherwise} \end{cases} \quad (2.13)$$

2.2.2.6 Unit Status Restrictions

Unit status restrictions include must-run and must-off restrictions. Generating units that are under one of these restrictions are predefined and can be excluded from the unit commitment decision.

a) Must-run Units

Some units must be forced on-line due to the need of steam outside the steam plant, reliability, and/or economic purposes. Typically, the generation outputs of these units are fixed, thus these units can be excluded from the UC and ED decision unless the surplus generation capacity can be participated into the bid-based market. The must-run units include units in forward contracts, units in exercised call/put options, RMR units, nuclear power plants, some cogeneration units, and units with renewable resources such as wind-turbine units and some hydro power plants.

b) Must-off Units

Some units are required to be off-line due to maintenance schedule or forced outage. These units can be excluded from the UC decision.

2.2.3 Other Constraints

In addition to system and unit constraints, there are other constraints that need to be considered in the UC decision. They are described as follows.

2.2.3.1 Crew Constraints

Due to a limited number of crew members, there may be start-up time to bring two or more plants located in the same location on-line or shut-down time to shut them down.

2.2.3.2 Fuel Constraints

Due to the contracts with fuel suppliers, some power plants may have limited fuel or may need to burn a specified amount of fuel in a given time [1].

2.2.3.3 Emission Constraints

Emission, e.g. sulfur dioxide (SO_2), nitrogen oxides (NO_x), carbon dioxide (CO_2), and mercury, produced by fossil-fueled thermal power plants is required to comply with the allowance in the U.S. Clean Air Act Amendments of 1990 [8] and Clear Skies Act of 2003 [9]. The amount of emission depends on various factors such as the type of fuel used, level of generation output, and the efficiency of the unit [10, 11]. The production cost minimization may need to be compromised in order to have the generation schedule that meets the emission constraints.

2.2.4 Start-up, Shut-down, and O&M Costs

According to the objective function of the unit commitment problem, fuel cost as well as operation and maintenance (O&M) cost are calculated by the economic dispatch. Start-up and shut-down costs are determined in the unit commitment process. The formulation of fuel cost is described in following economic dispatch formulation section. The formulations of start-up, shut-down, and O&M costs are shown as follows.

2.2.4.1 Start-up Cost

If the unit is started-up when cooling, t_b in equation 2.14 is equal to zero and the start-up cost is an exponential function. On the other hand, if the operating temperature of the unit is maintained, called baking, t_c in equation 2.14 is equal to zero and the start-

up cost is a linear function. These cooling and baking start-up cost curves are continuous as illustrated in figure 2.2.

$$SC_i^t = (C_b \cdot t_b \cdot F_i) + C_c \left(1 - e^{-t_c/\gamma}\right) \cdot Fprice_i + C_f \quad (2.14)$$

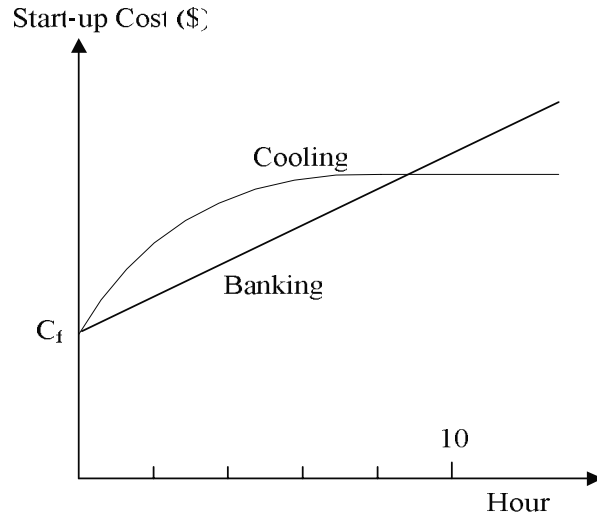


Figure 2.2 Continuous start-up cost curves

However, to compute the start-up cost, the ladder start-up cost curve is typically used. They have either two or three segments. Three-segment start-up cost curve consists of cold start-up cost, warm start-up cost, and hot start-up cost with cold start-up time, warm start-up time, and hot start-up time as illustrated in figure 2.3. For two-segment start-up cost curve, the warm start-up cost and time are omitted.

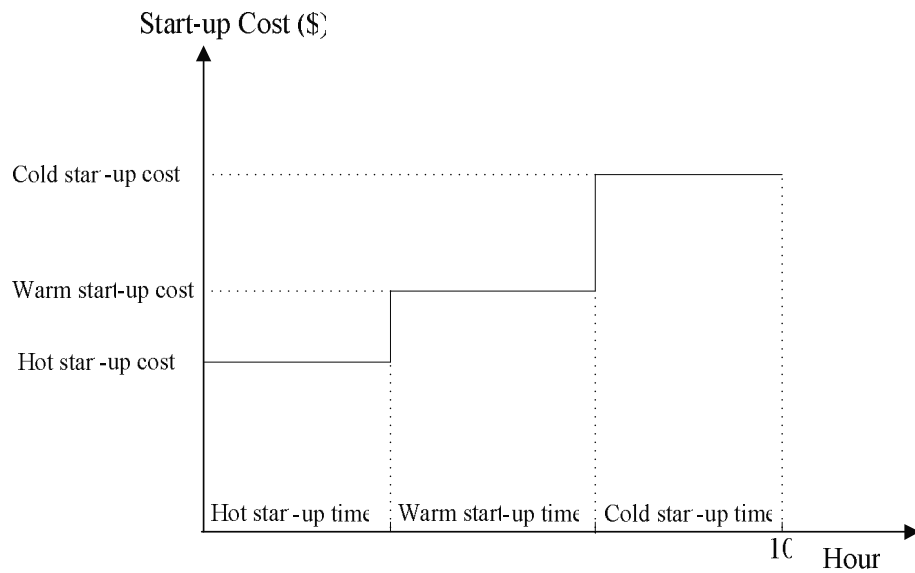


Figure 2.3 Ladder start-up cost curves

2.2.4.2 Shut-down Cost

Shut-down cost is incurred during shutting down generating units. In general, it is neglected from the unit commitment decision.

2.2.4.3 O&M Costs

Operation and maintenance cost is actually the labor cost of operating crews and the cost of plant maintenance. Typically, this cost depends on the amount of generating output.

$$OMC_i^t = mP_i^t \quad (2.15)$$

2.3 Economic Dispatch Formulation

The economic dispatch determines the output of all on-line units with an objective of a minimum total operating cost at a given hour, as shown in equation 2.16, which is subject to the power balance constraint in equation 2.17 and output limits in

equation 2.18. These constraints are the power balance constraint and unit output limits at a given hour.

$$\text{Min } FC_T = \min \sum_{i=1}^N FC_i(P_i) = FC_1(P_1) + FC_2(P_2) + \dots + FC_N(P_N) \quad (2.16)$$

subject to

$$P_T - D - L = 0, \quad (2.17)$$

$$P_{i,\min} \leq P_i \leq P_{i,\max}. \quad (2.18)$$

For multiple area system, the objective function of multi-area economic dispatch must also be subject to area import/export limits. Therefore, there are K additional constraints for K -area system as shown in equation 2.19. The detail of incorporating these constraints into the multi-area ED calculation is explained in chapter 4.

$$D_k + L_k - I_{k,\max} \leq \sum_{i=1}^{N_k} P_{i,k} \leq D_k + L_k + E_{k,\max} \quad (2.19)$$

In order to find the minimum cost with given constraints, the *Lagrangian function* (FC_T^*) is formulated by adding the power balance constraint function multiplied by an undetermined multiplier, called Lagrange multiplier or system lambda (λ_{sys}), into the objective function as equation 2.20 [1]. The derivative of equation 2.20 with respect to P_T is then set equal to zero in order to find the minimum FC_T as shown in equation 2.21.

$$FC_T^* = FC_T + \lambda_{\text{sys}}(D + L - P_T) \quad (2.20)$$

$$\frac{\partial FC_T^*}{\partial P_T} = \frac{dFC_T}{dP_T} - \lambda_{sys} \left(1 - \frac{\partial L}{\partial P_T}\right) = 0 \quad (2.21)$$

However, FC_T consists of the operating costs of all N units and they all are independent as Lagrange's assumption. Therefore, N equations of the derivative of Lagrangian functions with respect to unit power output, which contain the same Lagrange multiplier (λ_{sys}), can be represented as equation 2.22. With a different means of expression, equation 2.22 can be rewritten as λ_{sys} is equal to a unit lambda (λ_i) or the derivative of unit fuel cost multiplied by the unit penalty factor (PF_i) as shown in equation 2.23 and PF_i is defined as equation 2.24.

$$\frac{dFC_i}{dP_i} - \lambda_{sys} \left(1 - \frac{\partial L}{\partial P_i}\right) = 0, \quad (2.22)$$

$$\frac{dFC_i}{dP_i} \cdot PF_i = \lambda_i = \lambda_{sys}. \quad (2.23)$$

$$PF_i = \frac{1}{1 - \frac{\partial L}{\partial P_i}}, \quad (2.24)$$

Conclusively, to have the minimum total fuel cost, every unit should operate at the system incremental fuel cost rate (λ_{sys}). However, due to unit output limit inequality constraints, the operating power output and operating lambda of every unit are bounded by its minimum and maximum generation limits and the minimum and maximum lambda limits as equation 2.25. Therefore, some units may not operate at the system incremental fuel cost rate.

$$\left. \begin{aligned}
P_{i,\min} \leq P_i \leq P_{i,\max} \text{ and } \lambda_i = \lambda_{\text{sys}} & \quad \text{if } \lambda_{i,\min} \leq \lambda_{\text{sys}} \leq \lambda_{i,\max} \\
P_i = P_{i,\max} \text{ and } \lambda_i = \lambda_{i,\max} & \quad \text{if } \lambda_{\text{sys}} > \lambda_{i,\max} \\
P_i = P_{i,\min} \text{ and } \lambda_i = \lambda_{i,\min} & \quad \text{if } \lambda_{\text{sys}} < \lambda_{i,\min}
\end{aligned} \right\} \quad (2.25)$$

Each unit has its own input/output characteristic. This characteristic can be represented as an hourly fuel-consumption function in MBtu/Hr shown as equation 2.26 and known as an input-output function or a heat rate function. An hourly cost function in dollar per hour (\$/Hr), shown as equation 2.27, can be obtained by the product of the input-output function and the price of fuel used. Therefore, when the losses are considered, the unit incremental fuel cost function in dollar per MW per hour (\$/MWh) becomes equation 2.28. Some UC developers also include the effect of the unit efficiency factor in the lambda calculation as equation 2.29:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i, \quad (2.26)$$

$$FC_i(P_i) = (a_i P_i^2 + b_i P_i + c_i) \cdot Fprice_i, \quad (2.27)$$

$$\lambda_i = \frac{dFC_i(P_i)}{dP_i} \cdot PF_i = (2a_i P_i + b_i) \cdot (Fprice_i) \cdot \left(\frac{1}{1 - \frac{\partial L_i}{\partial P_i}} \right), \quad (2.28)$$

$$\lambda_i = \frac{dFC_i(P_i)}{dP_i} \cdot \frac{PF_i}{\text{Efficiency Factor}}. \quad (2.29)$$

Equations 2.26-2.27 are in polynomial quadratic format. However, the input-output function, the fuel cost function, the incremental heat rate function, and the incremental fuel cost function typically used are in the form of piecewise linear functions. These piecewise linear functions consist of single or multiple segments. For instance, a piecewise linear function of an incremental fuel cost rate of a generating unit

contains five intermediate break points, or four segments, of incremental fuel cost rates (lambda) and power outputs (P) as illustrated in figure 2.4. This figure shows that the minimum generation output and the minimum lambda are 250 MW and 1.8 \$/MWh and the maximum generation output and the maximum lambda are 700 MW and 3.2 \$/MWh.

When the system lambda is equal to 2.8 \$/MWh which is between the minimum and maximum unit lambda values according to the incremental fuel cost curve in figure 2.4, the power output is equal to 665 MW and the unit lambda is equal to the system lambda. However, with the same generating unit, when the system lambda is equal to 3.6 \$/MWh, the unit lambda is bounded by the maximum lambda 3.2 \$/MWh, thus the power output is also bounded by 700 MW.

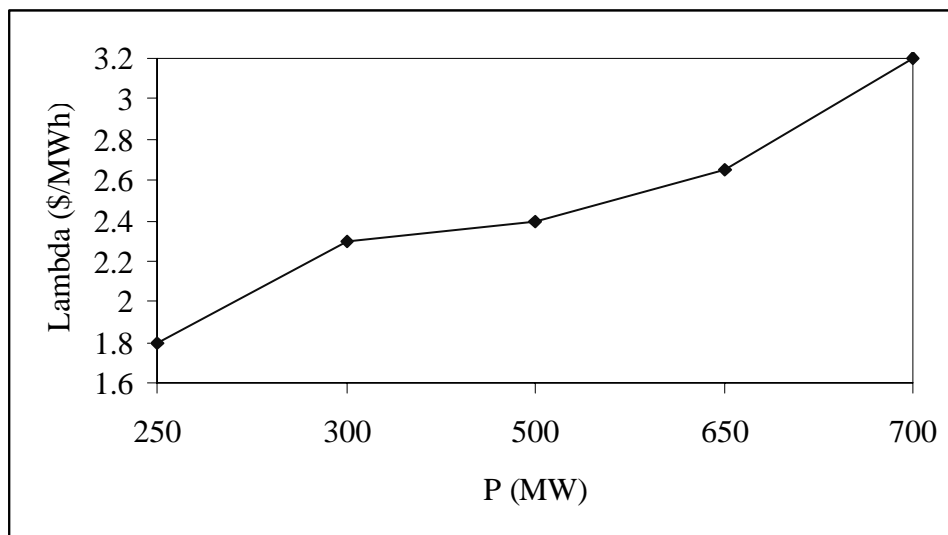


Figure 2.4 A sample of piecewise linear incremental fuel cost curve

It is shown that the system incremental fuel cost rate (λ_{sys}) is the key to find generation outputs of all units that yield the optimal fuel cost. The following chapters introduce various methodologies for economic dispatch and proposed method for multi-area economic dispatch.

CHAPTER 3

METHODOLOGY REVIEW

Various methodologies used to solve the unit commitment and economic dispatch problems are summarized and discussed as follows.

3.1 Unit Commitment Methodologies

Since unit commitment (UC) is a large, non-linear, non-convex, and mixed-integer problem, the attempt to receive the optimal schedule of committing generating units is challenging. Various methods have been developed to solve the UC problem with the same intention of minimizing the cost in the reasonable computational time. Several UC methodologies are summarized in the following sections.

3.1.1 Exhaustive Enumeration

The unit commitment problem, with exhaustive enumeration method, is basically solved by enumerating all possible on-line and off-line combinations of the generating units over the planning horizon. A unit commitment combination that yields the least operating cost is the unit commitment solution. This method is straightforward and provides an exactly optimal solution. However, it is a time-consuming method because it must search for the optimal solution from extensive unit commitment combinations. If there are N units, there will be $(2^N-1)^T$ possible commitment combinations for T hour planning. Although the unit ramp rates as well as unit up-time

and down-time constraints will lower the number of possible unit commitment combinations to that of feasible unit commitment combinations, it is still an enormous number. Due to the length of computational time this enumerative process and the size of computer memory required, presently this method is scarcely used.

3.1.2 Priority List

A simple unit priority list can be created by sorting generating units by average full-load cost (AFLC) in ascending order [1]. The AFLC is simply the generating unit average heat rate at full load, in MBtu/MWh, multiplied by the fuel cost, in \$/MBtu. The UC will commit units on the top of the list one unit at a time, if their unit minimum down-time constraints are not violated, until the demand and spinning-reserve are met. When there is excessive generation capacity in any hour, the last committed units, which have higher AFLC than the other on-line units, will be decommitted, if their unit minimum up-time constraints are not violated, until there is no excessive capacity or no further unit decommitment can be performed.

This method is simple and requires short computing time and small computer memory. However, the UC solution obtained from the priority list method may not be the optimal schedule because start-up cost and ramp rate constraints are not included in determining the priority commitment order and AFLC does not adequately reflect the operating cost of generating units when they do not operate at the full load [12-14]. Some other techniques are therefore incorporated into the priority list method to improve the UC solution.

Burns et al., in [15], proposed a dynamic priority list that varies with the system demand. Shoults et al. in [2] included the average start-up cost, in addition to the AFLC, in the determination of a unit priority list for multi-area unit commitment with area import/export constraints. Lee, *et al.* in [13-14] presented the priority list based on Commitment Utilization Factor (CUF) in association with the AFLC and claimed that the CUF can reflect the impact of multi-area transmission interconnection constraints and the priority list based on CUF and AFLC yields better results.

3.1.3 Dynamic Programming

Dynamic programming (DP) method hourly evaluates possible unit commitment schedules associated with decision made in the proceeding step by considering all constraints before searching for a schedule that yields the minimum cost. In the DP, the hour period of the planning horizon is typically called a *stage*. A set of unit commitment within a stage is known as a *state*. The cost incurred in each stage is affected by the decision made in the previous stage. The dynamic programming search can be in either forward or backward directions. The forward dynamic programming approach runs the unit commitment problem from the initial hour to the final hour and the backward dynamic programming approach does it conversely in time.

The forward DP approach is usually employed because the previous information of the unit can be used to compute the transition cost between hour $t-1$ and hour t such as the start-up cost as well as to check the unit constraints such as the unit minimum up-time and down-time. In the forward DP method, the minimum cost function of a state at a stage consists of the fuel cost and O&M cost obtained by running the ED of that state,

the start-up cost or shut-down cost if any is applicable, and the minimum cost function of the previous stage.

Although this approach tries to discard non-feasible combinations, the number of the feasible combinations still remains significant for the average to large size system. Therefore, Dynamic Programming - Truncated Combination (DP-TC), Dynamic Programming - Sequential Combination (DP-SC), and Dynamic Programming - Sequential Truncated Combination (DP-STC) were introduced. These methods truncate the number of states at each hour to reduce the solution search space. The DP-TC, introduced by Ouyang and Shahidehpour [16], and Pang and Chen [17], disregards the must-run units and unavailable units from the search range. The DP-SC evaluates the sequentially combined system states generated from a priority list but this method may inappropriately commit the last few units [18]. The DP-STC can solve this problem by generating a window to cover a set of available units whose commitment may violate the priority commitment order [18]. If the size of the window is small, the accuracy of the solution is reduced while the computational time is decreased.

3.1.4 Lagrangian Relaxation

Lagrangian relaxation (LR) method can eliminate the dimensionality problem encountered in the Dynamic Programming by temporarily relaxing coupling constraints and separately considering each unit. The LR method, based on the dual optimization theory, decomposes the unit commitment problem into one dual subproblem and one primal subproblem. The primal subproblem is as the objective function of the UC. The dual subproblem incorporates the objective function and the constraints multiplied with

Lagrange multipliers. The dual and primal subproblems are solved independently in an iterative process. Instead of solving the primal subproblem to receive the minimum cost, one can solve the dual subproblem to receive the maximum cost by maximizing the Lagrangian function with respect to the Lagrange multipliers, while minimizing with respect to the generation cost.

The convergence of the dual optimization method can be measured by the relative size of the duality gap between the primal and dual solutions [1]. As illustrated in figure 3.1, a simplified graphic representation of duality optimization, duality gap 1 is obtained by the difference between solutions of primal and dual subproblems. Duality gap 2 in figure 3.1 is the optimal duality gap, which is the difference between primal (global) minimum and dual (global) maximum. However, primal cost and dual cost functions in reality are not as smooth as the ones in figure 3.1 because they are non-differentiable and non-convex.

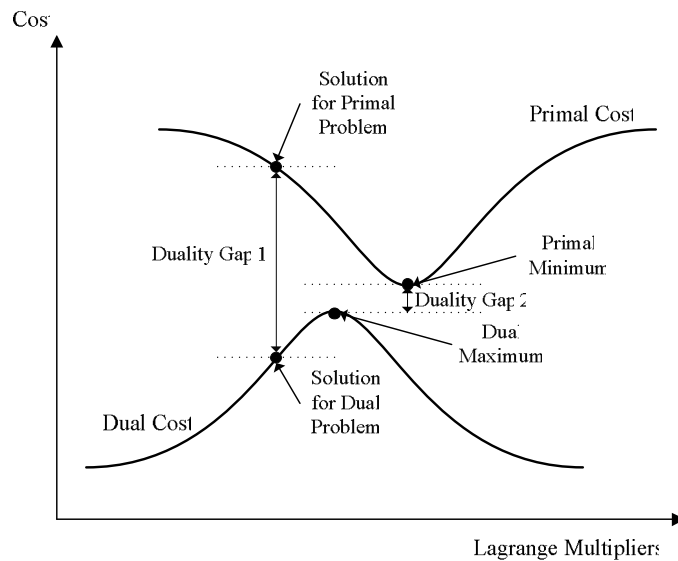


Figure 3.1 A simplified graphic representation of dual optimization

The solution of the LR greatly depends on the Lagrange multipliers. Therefore, setting the initial Lagrange multipliers and updating them are significant to the optimality of the solution. Inappropriate method of updating the Lagrangian multipliers may cause the solution adjustment to oscillate around the global optimum. With this reason, the unit commitment solution from this Lagrangian relaxation technique may not be optimal. There are three popular approaches to update Lagrange multipliers: (1) the subgradient method, (2) various versions of the simplex method implemented using column generation techniques, and (3) multiplier adjustment methods [19]. Since the subgradient method is easy to program and has worked well on many practical problems, it is the most popular method for updating Lagrange multipliers [19]. Mathematical formulations of the LR and subgradient methods are described in chapter 4.

The LR technique is suitable for large-scale power systems and both demand (energy) and spinning reserve requirement (capacity) are satisfied through Lagrange multipliers. It is also easy to incorporate other constraints, such as import/export limits and tie line capacity limits, into the dual subproblem. Therefore, the LR is one of the most widely used approaches for the UC problem and it is applied to this dissertation. However, the LR tends to over-commit generating units, which leads to the excessive spinning reserve and high production cost, when identical generating units with the same cost characteristics exist in the system. This is because the Lagrange multipliers are the key for the LR to determine the hourly unit commitment schedule.

Dekrajangpetch et. al. in [20] explained this problem with a simple system. In chapter 4 of this dissertation, a proposed algorithm for this over-commitment is described.

There are two main Lagrangian relaxation approaches, i.e. the classical Lagrangian relaxation (CLR) as described earlier and the augmented Lagrangian relaxation (ALR) [21]. The ALR method applies the quadratic penalty terms associated with the demand to the Lagrange function to improve the convergence of the dual optimization problem. This may help the unit commitment to reach the global optimum. However, the ALR method is more complicated and slower as well as the quadratic penalty terms are not separable and complicated. Beltran, *et al.*, in [21] proposes a two-phase approach which contains the CLR in the first phase and the ALR in the second phase. The CLR is to obtain the dual optimum and the ALR is to obtain a local optimal solution whose quality is assessed by the dual optimum obtained in the first phase.

3.1.5 Sequential Method

This sequential method was proposed in [22] to improve the speed and the solution quality of the short-term thermal unit commitment problem from the priority list, DP-SC, and DP-STC methods. The sequential unit commitment technique reduces the state space by doing decomposition. All units having similar unit capacity, minimum up time and minimum down time, or initial operating conditions are grouped together. Within each group, the generating units are sorted by their consistent cost index (CSC), which indicates relative operating cost per MWh of useful spinning capacity, in ascending order. The system marginal cost is the key factor in calculating for the CSC.

In each hour, the unit which has the comparatively least CSC among the units on the top of the list in each group is selected to commit. Next units are selected one at a time with the same criterion until the hourly demand and hourly spinning reserve requirement are fulfilled. Once, the unit commitment schedule is determined, the most economical output of on-line units, the total fuel cost, and the total O&M cost are determined by the ED.

Similar to the LR algorithm, the steps of calculating the unit relative operating costs, raking units, selecting units to commit, and performing the ED for the entire planning horizon are iteratively repeated with the different hourly system marginal costs until the convergence criteria are satisfied. The iteration that yields the least total production cost, which consists of the fuel cost, the O&M cost, the start-up cost, and the shut-down cost, is the UC result.

3.1.6 Decommitment Method

Unlike other methods, decommitment method determines the unit commitment schedule by decommitting the units from an initial state which all available units are brought on-line over the planning horizon. A unit having highest relative cost is decommitted at a time until there is no excessive spinning reserve or minimum up-time or ramp-down constraints prevent the rest of the units from decommitted. Tseng et. al. in [23] demonstrates that decommitment method is a reliable, efficient, and quick approach for solving the UC problem. However, their proposed method does not consider the unit ramp rate constraints. Tseng et. al. in [24], Ongsakul et. al. in [25], and Li et. al. in [26] applied decommitment method as a post unit commitment process to

reduce the excessive spinning reserve, caused by employing the Lagrangian relaxation method to determine the unit commitment schedule of a system containing generating units having similar or identical cost characteristics. This dissertation also incorporates the heuristic decommitment method into the Lagrangian relaxation method to enhance the cost minimization. The detail of this implementation is described in chapter 4.

3.1.7 Genetic Algorithm

A genetic algorithm is an optimization method based on a model of evolutionary adaptation and genetic mechanism in nature [27-28]. Binary numbers, 1 and 0, are used to indicate states, on and off, of a generating unit in the genetic algorithm. Initially, matrices, called genotypes, are generated. Each genotype contains N (number of generating units in the system) strings, each of which is a T -bit string to represent states of a unit from hour 1 to hour T , where T is a number of total hours in the planning horizon, as in [28-29]. However, some developers arrange each genotype as T strings, each of which is an N -bit string to represent states of units 1 to N in each hour, as in [30]. With both arrangements, each genotype has dimension equal to 1 by N times T ($N*T$). There are 2^{N*T} possible genotypes but a few genotypes are initially created to reduce the search dimension. Consequently, the size of the initial population is less than 2^{N*T} . The selection of initial genotypes can be either heuristically done or based on the knowledge based system [27].

Any genotype that yields an infeasible unit commitment schedule will be eliminated. The ED is performed during this evaluation process. Once the genotypes are evaluated, the remaining genotypes become parent genotypes and new offspring

genotypes are created by two basic genetic operators which are crossover and mutation [27-31]. The crossover operator probabilistically exchanges bits between two parent strings such that all the constraints such as spinning reserve requirement, power balance, and generation limits are also satisfied.

The mutation is a secondary operator. This operator toggles the binary bit from '0' to '1' and vice versa to ensure that no string will ever be the same through the UC process [31]. In addition to these two basic operators, some advanced genetic features, including elitism, fitness scaling, and adaptation of operator probabilities, are included to enhance the efficiency of the solution search [28]. Some offspring genotypes is then selected and become the parent genotypes of the new set of offspring. This iterative process is repeated until the convergence criterion is reached.

This genetic method is relatively complicated and involves extensive computational time. Therefore, this method is not suitable for large-scale systems. In addition, there is no guarantee that the search space of the genetic algorithm will lead to the optimal solution. Cheng et. al. in [32] incorporate the genetic algorithms into the Lagrangian relaxation method to update Lagrange multipliers and improve the unit commitment solution. There is a similar approach in the field of simulated evolution (SE) called evolutionary programming (EP). Both algorithms involve the random alterations and selection by the different means [33].

3.1.8 Branch and Bound Method

Branch and bound method tries to find the minimum value of a cost function over a feasible region in a search tree. This method consists of five steps – branching,

bounding, elimination, selection, and termination [34]. In branching step, a search tree where each node represents a problem is created. A problem is decomposed in several sub-problems. The sub-problems are bounded by lower and upper bounds in the bounding step. The lower bound, minimum cost, is typically solved by means of Lagrangian relaxation method or linear programming. Any sub-problem containing an infeasible unit commitment schedule is eliminated in the elimination step.

Any sub-problem having the lower bound less than the predetermined upper bound is selected in the selection step. Among selected sub-problems the least total cost becomes the predetermined upper bound for the next branching, bounding, elimination, and selection steps. These four steps are repeated until the termination criterion is satisfied. The criterion is that only one sub-problem having upper bound equal to lower bound remains. This sub-problem containing the lowest bound is assumed to be the optimal solution. Like several UC methods, the bunch-and-bound method is not suitable for large-scale systems due to the long computational time [35].

When combined with cutting plane method, the branch-and-bound method becomes the branch-and-cut algorithm which is a general algorithm used in mixed integer linear programming (MILP) method [36-39]. The MILP can be used to solve scheduling problem for thermal, combined cycle, hydro power and pump storage units without involving any heuristic method. However, the MILP is complex and requires long computational time when used with large-scale systems compared to the Lagrangian relaxation method [37-38].

3.1.9 Tabu Search

Tabu search technique applies *metaheuristic* algorithm [40] to iteratively search for a good solution among a set of feasible solutions. The neighborhood of the current good solution is then searched. If there is a better solution than the current one, the better solution will be stored as a current good solution instead. The information on the recent moves will be stored in a *Tabu List* to prevent the redundant search result. Borghetti et. al. in [40] suggested that, to explore all of the feasible solution space, the metaheuristic algorithm should occasionally accept the worse solution than the current one because the exploration of their neighborhood may lead to a better region and a better result. Once there is no better solution in the neighborhood of the current one, the Tabu search is terminated. There is no guarantee that the Tabu search will yield the global optimal result especially with large systems. There is a similar method named *Particle Swarm Optimization* proposed in [41-42].

3.1.10 Expert Systems / Artificial Neural Network

Expert systems, also known as knowledge based system (KBS), use the experience and knowledge of power system operators and programming developers to adjust or improve existing UC algorithms [43-45]. Similarly, artificial neural network (ANN) performs calculations in parallel based on the historical data and experience. As the process of human brain, the parallel computation gives the result quicker than traditional serial computation. However, there are some drawbacks of the ANN approach. First, since the wider the dimension of the historical data for neural network training, the better the quality of the result but the training time will be extensively long.

Secondly, neural network needs to be well trained such that the solution is reliable. Chung et. al. in [46] incorporates the genetic algorithms into ANN to evolve the weight and the interconnection of the neural network for the purpose of better ANN training. Ouyang et. al. in [47] and Daneshi et. al. in [48] presented that, with the ANN, the dynamic programming method can avoid unnecessary calculations for similar load profiles while the quality of the optimization is not deteriorated.

3.2 Economic Dispatch Methodologies

As mentioned in chapter 2, the system incremental fuel cost rate, called system lambda, is the key to find the most economical generation output of all on-line units. Wood et. al. in [1] describes several methods for the economic dispatch without security-constrained. These methods include lambda-iteration, gradient, reduced gradient, Newton's, and dynamic programming methods. In [49-53], Hopfield neural network method is proposed to solve the ED problem. Among these methods, the lambda-iteration method is simple, more favorable, and used in many commercial ED programs. The gradient, reduced gradient, Newton's, dynamic programming, and Hopfield neural network methods are either inefficient or too complicated.

However, when the cost function is more complex than a piecewise linear function or a quadratic function, gradient or reduced gradient method are more suitable than the lambda-iteration method [1]. Additionally, when the cost function is non-convex, it is necessary to use dynamic programming or Hopfield neural network methods for the ED [1], [51]. Hopfield neural network is claimed in [51-53] to

outperform the lambda-iteration method in terms of computational time on condition that the weighting factors are properly selected.

This dissertation uses the lambda-iteration method and focuses on only the fuel cost functions in quadratic and piecewise linear formats only. The detail of solving the ED problem by lambda-iteration method is described as follows.

3.2.1 Lambda-Iteration Method

Lambda-iteration method iteratively adjusts the system lambda and calculates for corresponding total power output contributed from all on-line units until one of the stopping criteria for iteration process is met. When the difference of the total generation output and the demand is within a specified tolerance or the number of times through the iteration loops reaches a predefined number, the lambda iteration process ends [1]. This method may slowly converge or yield an unacceptable result if the initial lambda and the method for updating the lambda are not properly selected.

Another approach is doing lambda-iteration through a table of data which consists of 1) the entire system incremental fuel cost rates, which are the system marginal costs of supplying energy (lambda) at different output levels, 2) the power output with respect to the lambda of each on-line unit, and 3) the sum of all unit power outputs. There is a limitation that all incremental fuel cost functions in quadratic format or piecewise linear format used in this method must be monotonically increasing.

There are two main steps in this lambda-iteration approach. The first step is creating the table of data which is called a sorted table. The second step is searching for

the system's operating lambda and each unit power output corresponding to the demand. These two steps are described as follows.

3.2.1.1 Sorted Table

The sorted table consists of sorted lambda values in ascending order, all on-line units' power output with respect to each lambda, and the sum of all units' power outputs corresponding to each lambda. Creating a sorted table of generating units having the incremental fuel cost functions or the bid price functions in piecewise linear format is slightly different from that of generating units having the incremental fuel cost functions in the format of the first derivative of the quadratic function as equation 2.28.

For units having the incremental fuel cost functions or bid price functions in piecewise linear format, their lambda values from every break point are included in the sorted table in ascending order. Corresponding to every lambda, the power outputs of all on-line units and the sum of them are calculated and included into the sorted table. When the lambda (λ) in the sorted table is equal to $\lambda_i[k]$ and unit i is scheduled to be on-line, the power output of unit i is equal to $P_i[k]$ where $\lambda_i[k]$, and $P_i[k]$ are lambda and power output of unit i at break point k . Otherwise, it can be evaluated by interpolating between two break points as shown in equation 3.1 when the lambda is in between two break points of a unit incremental fuel cost curve ($\lambda_i[k-1] < \lambda < \lambda_i[k]$) where λ is the lambda in the sorted table and $\lambda_i[k-1]$, $P_i[k-1]$ are lambda and power output of unit i at break point $k-1$.

$$P_i = P_i[k-1] + \left(\frac{\lambda - \lambda_i[k-1]}{\lambda_i[k] - \lambda_i[k-1]} \right) \cdot (P_i[k] - P_i[k-1]) \quad (3.1)$$

For units having the incremental fuel cost function in the format of the first derivative of the quadratic function, their minimum and maximum lambda values are included in the sorted table. The total power output is determined by summing all unit power outputs evaluated with respect to the lambda value. A unit power output with respect to a lambda, when losses are neglected and the fuel cost function is in the quadratic format, is calculated as shown in equation 3.2. Note that the power output of a unit is always in between its minimum and maximum generation limits.

$$P_i(\lambda) = \min(P_{i,\max}, \max(P_{i,\min}, \frac{\lambda - B_i}{2A_i})) \quad (3.2)$$

3.2.1.2 Determination of System's Operating Lambda and Unit Power Output

Demand is used to determine the system's operating lambda and units' operating output. If the demand is equal to the total power output of a break point in the sorted table, that break point contains data for operating lambda and units' operating power output. If the demand falls between two total power outputs of two break points in the sorted table, the linear interpolation needs to be performed to receive the system's operating lambda as shown in equation 3.3.

$$\lambda_{sys} = \lambda[k-1] + \left(\frac{D - P[k-1]}{P[k] - P[k-1]} \right) \cdot (\lambda[k] - \lambda[k-1]) \quad (3.3)$$

Table 3.1 shows a sample of a four-unit system. This table contains unit fuel cost function in quadratic format, the generation limits, and the lambda limits. From data in table 3.1, a sorted table contains 8 breakpoints, 7 segments, is created as shown in table 3.2.

Table 3.1 A Four-Unit System With Fuel Cost Function, Generation, and Lambda Limits

Unit	Fuel Cost (\$/h)	Generation Limits (MW)		Lambda (\$/MWh)	
		P_{min}	P_{max}	λ_{min}	λ_{max}
1	$0.001 \cdot P_1^2 + 1.2 \cdot P_1 + 25$	50	400	1.3	2.0
2	$0.003 \cdot P_1^2 + 1.8 \cdot P_1 + 29$	100	350	2.4	3.9
3	$0.002 \cdot P_1^2 + 1.3 \cdot P_1 + 29$	100	450	1.7	3.1
4	$0.004 \cdot P_1^2 + 2.1 \cdot P_1 + 30$	150	500	3.3	6.1

Table 3.2 Sorted Table of a Four-unit System

Breakpoint	λ_{sys} (\$/MWh)	Total P (MW)	Unit Generation			
			1	2	3	4
1	1.3	400	50	100	100	150
2	1.7	600	250	100	100	150
3	2.0	825	400	100	175	150
4	2.4	925	400	100	275	150
5	3.1	1216.67	400	216.67	450	150
6	3.3	1250	400	250	450	150
7	3.9	1425	400	350	450	225
8	6.1	1700	400	350	450	500

All of lambda and total power output values in table 3.2 are presented as a piecewise linear incremental fuel cost curve as illustrated in figure 3.2.

In this sample system, if the demand is equal to 1200 MW which falls in between breakpoints four and five, according to equations 3.1 and 3.2, the system lambda is approximately equal to 3.06 (\$/MWh), and the power outputs of units 1, 2, 3, and 4 are 400, 210, 440, and 150 MW, respectively.

For a multi-area system, the procedure in determining the operating lambda of the system, areas, and units as well as each unit generation output is more complicated. The details of the multi-area ED are described in chapter 4.

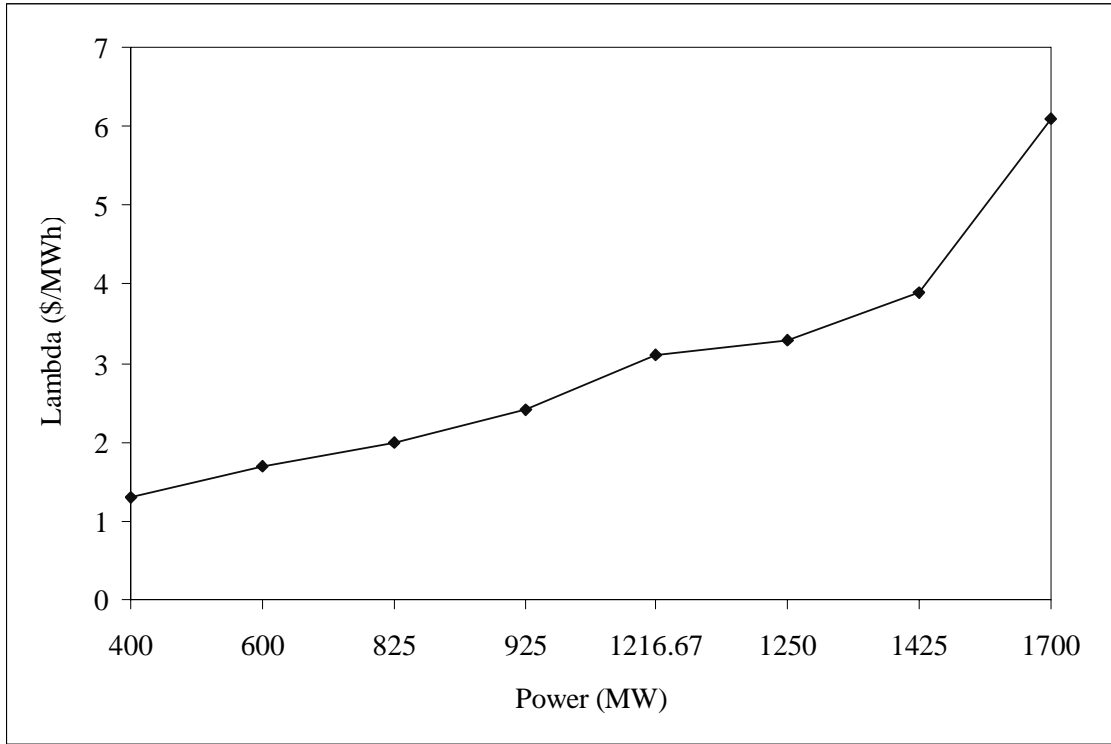


Figure 3.2 Equivalent incremental fuel cost of four-unit system

CHAPTER 4

PROPOSED METHODOLOGY

Multi-area unit commitment and economic dispatch with market operation components is designed to be a decision-making tool for both operation and market strategies. These strategies are to increase the profit, reduce the risk of price volatility in the real-time electricity market, and provide resource adequacy for both demand and spinning reserve requirement by searching for the optimal unit scheduling and taking into account bilateral contracts which include options, forward contracts, and reliability must-run contracts. In addition, improving speed of the UC and ED computation and providing the easy to use graphical user interface for system operators are taken into consideration. To achieve these objectives, adaptive Lagrangian relaxation, unit decommitment, and lambda-iteration methods are applied in the development of this dissertation.

4.1 Multi-area Unit Commitment

Adaptive Lagrangian relaxation is used to find the optimal unit commitment schedule and unit decommitment method is then applied to reduce the excessive spinning reserve and the total production cost by decommitting units which are not needed to be on-line. The algorithm is described in a flow chart diagram as illustrated in figure 4.1 and its details are explained in the sections 4.1.1-4.1.3.

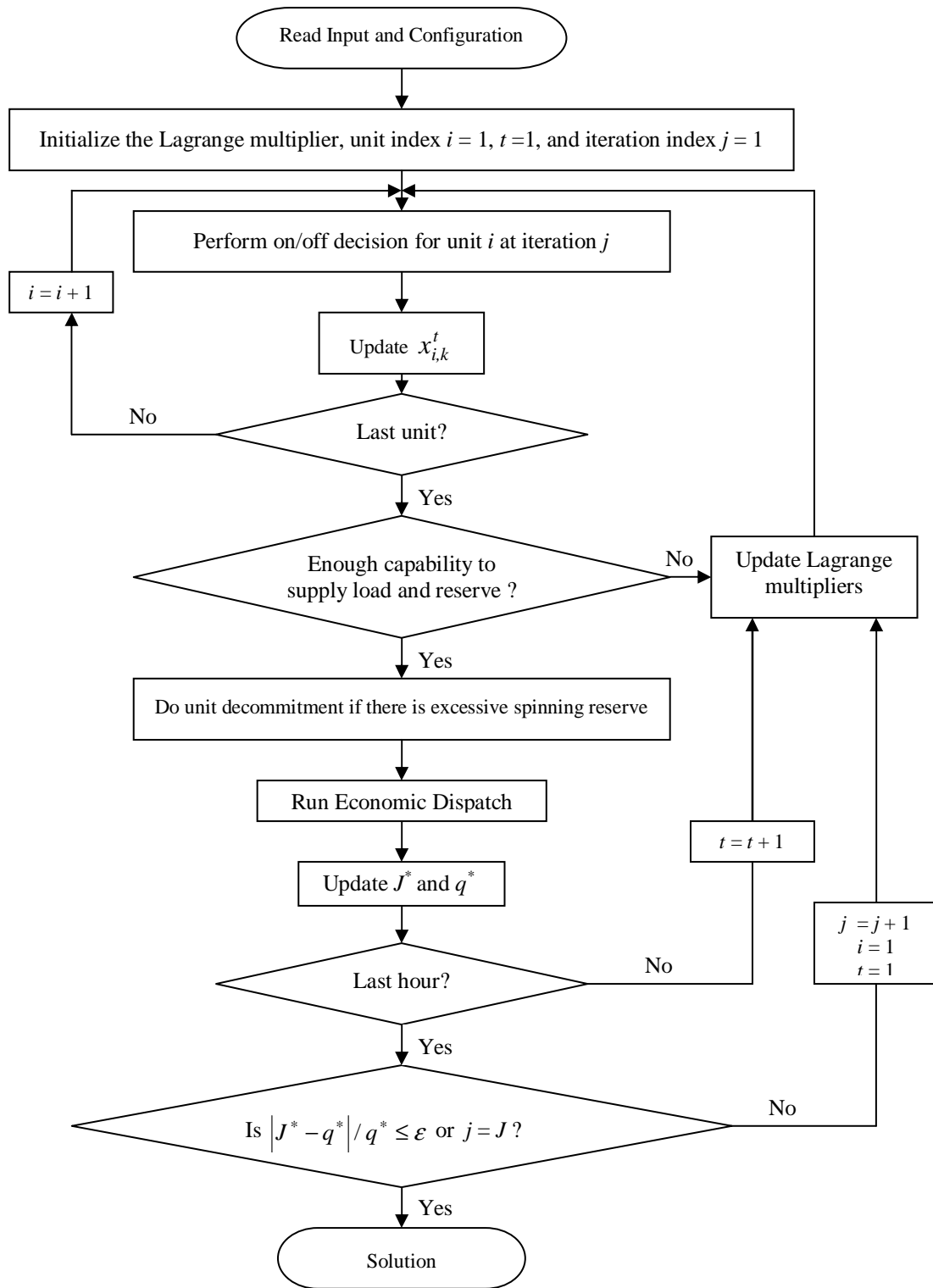


Figure 4.1 Unit commitment flow chart diagram

4.1.1 Lagrangian Relaxation

Lagrangian relaxation is an optimization technique that can solve problems subject to both equality and inequality constraints. This technique reduces the dimensionality of searching for the unit commitment schedule by temporarily relaxing coupling constraints. Therefore, it is suitable for large-scale systems. The unit commitment by the Lagrangian relaxation consists of two problems – a primal subproblem and a dual subproblem. The mathematical formulation and the procedures of Lagrangian relaxation optimization are described as follows.

4.1.1.1 Primal Subproblem

The primal subproblem, as shown in equation 4.1, is to find the minimum cost of committing and dispatching units, called primal cost J^* , subject to system, unit, and other constraints as discussed in chapter 2. The total fuel and O&M costs are determined by the ED. The start-up cost is determined by the UC and the shut-down cost is neglected. The losses are incorporated to the problem by the penalty factor during the incremental cost rate calculation as described in chapter 2.

$$J^* = \min \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^{N_k} \{ [FC_{i,k}^t (P_{i,k}^t) + OMC_{i,k}^t (P_{i,k}^t) + SC_{i,k}^t (1 - U_{i,k}^{t-1})] \cdot U_{i,k}^t \} \quad (4.1)$$

4.1.1.2 Dual Subproblem

The dual subproblem is to find the maximum cost received by maximizing the Lagrangian function with respect to the Lagrange multipliers, while minimizing with respect to the generation cost. All coupling constraints which are power balance constraint, system spinning reserve requirement, each area capability associated with its

area import/export limits, and each tie line capacity constraint, as shown in equations 4.2-4.6, are temporarily ignored from unit scheduling decision in the dual subproblem.

Note that constraint equations 4.4-4.6 do not exist for a single area system.

$$\sum_{k=1}^K (D_k^t) - \sum_{k=1}^K \sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) = 0 \quad (4.2)$$

$$\sum_{k=1}^K (D_k^t + R_k^t) - \sum_{k=1}^K \sum_{i=1}^{N_k} (P_{i,k,\max}^t \cdot U_{i,k}^t) \leq 0 \quad (4.3)$$

$$D_k^t - I_{k,\max}^t + R_k^t - \sum_{i=1}^{N_k} (P_{i,k,\max}^t \cdot U_{i,k}^t) \leq 0 \quad (4.4)$$

$$\sum_{i=1}^{N_k} (P_{i,k,\min}^t \cdot U_{i,k}^t) - D_k^t - E_{k,\max}^t \leq 0 \quad (4.5)$$

$$\sum_{k=1}^K \left\{ SF_k^t \left(\sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) - D_k^t \right) \right\} - F^t \leq 0 \quad (4.6)$$

To incorporate these relaxed constraints into the UC problem, the Lagrangian function is defined as equation 4.7. It contains the objective function, Lagrange multiplier λ multiplied by power balance constraint function, Lagrange multiplier μ multiplied with spinning reserve constraint function, Lagrange multipliers $\alpha_1, \alpha_2, \dots, \alpha_K$ multiplied by maximum capability constraint function of areas 1 to K , Lagrange multipliers $\beta_1, \beta_2, \dots, \beta_K$ multiplied by minimum capability constraint function of areas 1 to K , and Lagrange multiplier $\theta_1, \theta_2, \dots, \theta_F$ multiplied by tie line capacity constraint function of flows 1 to F over the time horizon T . The Lagrangian function is still subject to the non-coupling constraints, such as minimum up-time, minimum down-time, and ramp rate constraints.

$$L(P, U, \lambda, \mu, \beta, \alpha, \theta) = \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^{N_k} \left\{ [FC_{i,k}^t (P_{i,k}^t) + OMC_{i,k}^t (P_{i,k}^t) + SC_{i,k}^t (1 - U_{i,k}^{t-1})] \cdot U_{i,k}^t \right\}$$

$$\begin{aligned}
& + \sum_{t=1}^T \left[\lambda^t \cdot \sum_{k=1}^K \left(D_k^t - \sum_{i=1}^{N_k} (P_{i,k}^t U_{i,k}^t) \right) + \mu^t \cdot \sum_{k=1}^K \left(D_k^t + R_k^t - \sum_{i=1}^{N_k} (P_{i,k,\max}^t U_{i,k}^t) \right) \right] \\
& + \sum_{t=1}^T \sum_{k=1}^K \left[\alpha_k^t \cdot \left(D_k^t + R_k^t - I_{k,\max}^t - \sum_{i=1}^{N_k} (P_{i,k,\max}^t U_{i,k}^t) \right) \right] \\
& + \sum_{t=1}^T \sum_{k=1}^K \left[\beta_k^t \cdot \left(\sum_{i=1}^{N_k} (P_{i,k,\min}^t U_{i,k}^t) - D_k^t - E_{k,\max}^t \right) \right] \\
& + \sum_{t=1}^T \sum_{f=1}^F \left[\theta_f^t \cdot \left(\sum_{k=1}^K SF_k^f \left(\sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) - D_k^t \right) - F^f \right) \right] \quad (4.7)
\end{aligned}$$

Since all coupling constraints are temporarily relaxed, all generating units are separable and the Lagrangian function can be decomposed into N independent Lagrangian functions, where N is the total number of units in the system. The Lagrangian function of each unit is defined as equation 4.8.

$$\begin{aligned}
L(P_{i,k}, U_{i,k}, \lambda, \mu, \alpha_k, \beta_k, \theta) = & \sum_{t=1}^T \left\{ [FC_{i,k}^t(P_{i,k}^t) + OMC_{i,k}^t(P_{i,k}^t) + SC_{i,k}^t(1 - U_{i,k}^{t-1})] \cdot U_{i,k}^t \right\} \\
& + \sum_{t=1}^T \left[\lambda^t \cdot (D_k^t - P_{i,k}^t U_{i,k}^t) + \mu^t \cdot (D_k^t + R_k^t - (P_{i,k,\max}^t U_{i,k}^t)) \right] \\
& + \sum_{t=1}^T \left[\alpha_k^t \cdot (D_k^t + R_k^t - I_{k,\max}^t - (P_{i,k,\max}^t U_{i,k}^t)) \right] \\
& + \sum_{t=1}^T \left[\beta_k^t \cdot ((P_{i,k,\min}^t U_{i,k}^t) - D_k^t - E_{k,\max}^t) \right] \\
& + \sum_{t=1}^T \sum_{f=1}^F \left[\theta_f^t \cdot \left(\sum_{k=1}^K SF_k^f ((P_{i,k}^t \cdot U_{i,k}^t) - D_k^t) - F^f \right) \right] \quad (4.8)
\end{aligned}$$

The dual function, defined as equation 4.9, is determining a dual cost, called q^* . The value of q^* is received by maximizing equation 4.10 with respect to the Lagrange multipliers which must be equal to or greater than zero. The equation 4.10 is obtained by minimizing the Lagrangian function with respect to the generation cost.

$$q^* = \max_{\lambda, \mu, \beta, \alpha, \theta \geq 0} \left[\min_{P, U} L(P, U, \lambda, \mu, \alpha, \beta, \theta) \right] \quad (4.9)$$

where

$$\begin{aligned}
& \min_{P,U} L(P,U, \lambda, \mu, \alpha, \beta, \theta) \\
&= \sum_{k=1}^K \sum_{i=1}^{N_k} \min_{P_{i,k}, U_{i,k}} L(P_{i,k}, U_{i,k}, \lambda, \mu, \alpha_k, \beta_k, \theta) \\
&= \sum_{k=1}^K \sum_{i=1}^{N_k} \left\{ \min_{P_{i,k}^t, U_{i,k}^t} \sum_{t=1}^T \left\{ [FC_{i,k}^t(P_{i,k}^t) + OMC_{i,k}^t(P_{i,k}^t) + \beta_k^t P_{i,k,\min}^t + \sum_{f=1}^F (\theta_f^t SF_k^f P_{i,k}^t)] \cdot U_{i,k}^t \right\} \right\} \\
&\quad - \sum_{k=1}^K \sum_{i=1}^{N_k} \left\{ \max_{P_{i,k}^t, U_{i,k}^t} \sum_{t=1}^T \left[(\lambda^t P_{i,k}^t + \mu^t P_{i,k,\max} + \alpha_k^t P_{i,k,\max}^t) \cdot U_{i,k}^t \right] \right\} \tag{4.10}
\end{aligned}$$

4.1.1.3 Unit Status

Unit status is defined in the unit commitment process. The economic dispatch then determine the most economic power output of all generating units based on their statuses. However, some statuses are predetermined due to the unit status restrictions or agreements in bilateral contracts. There are four unit statuses described as follows.

2.1) Off-line units

These units are off-line due to the must-off restriction or the unit commitment decision.

2.2) Must-run units with predefined generation

These units must be on-line and their power output must be equal to predefined amount.

2.3) Must-run units with predefined minimum generation

These units must be on-line and their power output is determined by the economic dispatch but must be at least equal to predefined amount.

2.4) On-line units

These units are on-line due to the unit commitment decision and their power output is determined by the economic dispatch.

4.1.1.4 Solution of dual function

Dynamic programming is typically used to solve the dual function for the optimal unit commitment schedule over a planning horizon. However, this process is time-consuming although the dimensionality is reduced because the solution of each unit is independently searched. This dissertation enhances the Lagrangian relaxation method with a new technique, proposed by Ongsakul et. al. in [25], which is called on/off decision criterion to determine the unit commitment schedule. To determine on/off status of each unit, on/off decision criterion uses fuel cost and partial start-up cost (called reduced start-up cost in [25]), which is start-up cost divided by minimum up-time. Test results from [25] show that the production cost of schedule using partial start-up cost is less than that of schedule using full start-up cost. The procedures in determining on/off status of each unit are discussed as follows.

1) Unit i will be committed in the following conditions.

1.1) If it is in must-run status with predefined generation or with predefined minimum generation, it will be committed.

1.2) If it has been off-line longer than minimum down-time constraint and an inequality condition as shown in equation 4.11 is true, it will be committed. The power output, fuel cost, and O&M cost are determined as equation 2.25, 2.27, and 2.15 with a given λ^t .

$$FC_{i,k}^t(P_{i,k}^t) + OMC_{i,k}^t(P_{i,k}^t) + \frac{SC_{i,k}^t(1-U_{i,k}^t)}{t_{i,k}^{up}} \leq \text{LagrangePrice} \quad (4.11)$$

where Lagrange Price = $\lambda^t P_{i,k}^t + \mu^t P_{i,k,\max}^t + \alpha_k^t P_{i,k,\max}^t - \beta_k^t P_{i,k,\min}^t - \sum_{f=1}^F (\theta_f^t S F_k^f P_{i,k}^t)$.

1.3) If it has been on-line less than minimum up-time constraint, it will remain on-line regardless of the inequality condition as shown in equation 4.11.

1.4) If it cannot ramp down from the power output of previous hour to its minimum generation at the current hour due to the ramp down constraint, it must be on-line regardless of the inequality condition as shown in equation 4.11.

1.5) If it has been on-line and an inequality condition as shown in equation 4.11 is true, this unit will be committed.

2) Unit i will not be committed in the following conditions.

2.1) If it is not available due to scheduled maintenance or forced outage (must-off status), it must be off-line.

2.2) If it has been off-line less than minimum down-time constraint, it will not be committed.

2.3) If it has been on-line longer than its minimum up-time constraint and an inequality condition as shown in equation 4.11 is not true, this unit will be shut down. However, this unit should remain on-line only if it is more economical to operate it at the minimum generation limit to avoid the start-up and shut-down costs when more generation will be needed in next few hours. To do this cost assessment, the economic dispatch must be performed twice to compare the generation cost of having this unit on-line or off-line. If the first cost is less than the second cost plus the start-up and shut-

down costs or the number of hours from the current hour to the hour that this unit is needed again is less than its minimum down time, this unit should remain on-line.

4.1.1.5 Initial Lagrange Multipliers

The initial values of Lagrange multipliers have a significant impact on the computational time and the UC result [25]. Inappropriate initial values may cause the outcome to deviate from the cost optimum. The procedures used in this dissertation for initializing Lagrange multipliers are described as follows:

- 1) Set t equal to 1
- 2) Set initial $\alpha_k^{t(0)}$ and $\beta_k^{t(0)}$ of each area equal to zero.
- 3) Set initial $\theta_f^{t(0)}$ of each flow equal to zero.
- 4) All available units in hour t are set to be on-line if their minimum-down time constraint is not violated.
- 5) The economic dispatch is performed according to the unit schedule from previous step. The initial lambda ($\lambda^{t(0)}$) is the system marginal cost of supplying energy to meet the demand in hour t.
- 6) The initial $\mu^{t(0)}$ is calculated as following equation.

$$\mu^{t(0)} = \max \left(0, \frac{\sum_{k=1}^K \sum_{i=1}^{N_k} [FC_{i,k}^t (P_{i,k}^t)] - \lambda^{t(0)} D^t}{\sum_{k=1}^K \sum_{i=1}^{N_k} P_{i,k, \max}} \right) \quad (4.12)$$

- 7) If $t < T$, set $t = t+1$ and return to step 2. Otherwise, all Lagrange multipliers are initialized over the time horizon.

4.1.1.6 Updating Lagrange Multipliers

If the unit commitment problem has not converged in iteration j , all Lagrange multipliers must be updated for iteration $j+1$. The adjustment is in the direction that Lagrange multipliers in the next iteration will yield a feasible unit commitment schedule while all coupling constraints are satisfied. For instance, if the generation output is not sufficient to supply the demand in the iteration j , the Lagrange multiplier λ in the iteration $j+1$ must be greater than that in the iteration j to increase the number of on-line units. Adaptive subgradient method [19-20, 25, 54-57] is applied for updating Lagrange multipliers in this dissertation because it is simple and generally suitable for function that is piecewise smooth and non-differentiable as the Lagrangian function.

The essence of the adjustment is to know how much the Lagrange multipliers should be adjusted. If the Lagrange multipliers are gradually adjusted, the optimization process may consume a long computational time. If the Lagrange multipliers are rapidly adjusted, the results of iterations may oscillate around the global optimum and lead to undesirable solution. Therefore, the inconstant step size, which gets smaller as the number of iterations grows, is used in adjusting the Lagrange multipliers. Equation 4.13 defines the adaptive subgradient method for updating a Lagrange multiplier.

$$x^{t(j+1)} = \max \left(0, x^{t(j)} + \frac{S}{\|Diff\|} \cdot Diff^t \right) \quad (4.13)$$

where

$x^{t(j+1)}$: Lagrange multiplier $\lambda, \mu, \alpha_k, \beta_k$, or θ_f used in hour t at $j+1^{th}$ iteration

$x^{t(j)}$: Lagrange multiplier $\lambda, \mu, \alpha_k, \beta_k$, or θ_f used in hour t at j^{th} iteration

- S : Step size for subgradient, defined as equation 4.14
- $Diff^t$: Subgradient of Lagrangian function with respect to Lagrange multiplier x
- $\| Diff \|$: Norm of subgradients over the entire time horizon (T) as shown in equation 4.15

$$S = \frac{1}{\delta + \sigma(j+1)} \quad (4.14)$$

$$\| Diff \| = \sqrt{(Diff^1)^2 + (Diff^2)^2 + \dots + (Diff^T)^2} \quad (4.15)$$

The viability of the subgradient method depends critically on the step size parameters δ and σ . These parameters are empirically determined and they vary according to the system and constraint.

Subgradients of Lagrangian function with respect to Lagrange multipliers $\lambda, \mu, \alpha_k, \beta_k,$ and θ_f in hour t are defined as equations 4.16-4.20 which are the left side of the coupling constraint equations 4.2-4.6. If the unit commitment problem has not converged, these subgradients are calculated and used in updating Lagrange multipliers for the next iteration as defined in equation 4.13. Note that the unit generation output and the minimum and maximum capacity of the areas and the system used for the subgradient calculation are determined by the on/off decision in the dual subproblem.

$$pDiff^t = \sum_{k=1}^K (D_k^t) - \sum_{k=1}^K \sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) \quad (4.16)$$

$$rDiff^t = \sum_{k=1}^K (D_k^t + R_k^t) - \sum_k \sum_{i=1}^{N_k} (P_{i,k,\max}^t \cdot U_{i,k}^t) \quad (4.17)$$

$$rDiff_k^t = D_k^t - I_{k,\max}^t + R_k^t - \sum_{i=1}^{N_k} (P_{i,k,\max}^t \cdot U_{i,k}^t) \quad (4.18)$$

$$mDiff_k^t = \sum_{i=1}^{N_k} (P_{i,k,\min}^t \cdot U_{i,k}^t) - D_k^t - E_{k,\max}^t \quad (4.19)$$

$$fDiff_f^t = \sum_{k=1}^K \left\{ SF_k^f \left(\sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) - D_k^t \right) \right\} - F^f \quad (4.20)$$

4.1.1.7 Convergence check

The convergence of the dual optimization method can be measured, as defined in equation 4.21, by the relative size of the duality gap between the primal cost (J^*) and dual cost (q^*). The iterative process of searching for the optimal unit commitment based on Lagrangian relaxation technique stops when the relative duality gap is equal to or less than a predefined tolerance (ε), which is typically set to be a small number, or when the number of iterations reaches an allowed number.

$$\text{Relative duality gap} = \frac{|J^* - q^*|}{q^*} \quad 4.21$$

4.1.2 Unit Decommitment

Since the marginal cost is the key for Lagrangian relaxation method to determine unit statuses, units with the same cost characteristic are treated the same and may lead to over committing the generating units and result in high production cost. Virmani et. al. in [53] suggested that this problem can be solved by committing the units having identical cost characteristic as a group or slightly adjusting cost curves of identical units to make them distinct such that the Lagrangian relaxation problem will commit them separately.

However, it is more practical to reduce the excessive spinning reserve and consequently the total production cost by unit decommitment method. There are several

factors in selecting candidates for the unit decommitment. These factors include unit types, number of hour that units have been on-line, their minimum up-time constraints, etc. The following sections explain unit types and unit decommitment procedures.

4.1.2.1 Unit Types

Generating units can be classified into three types based on the unit characteristics, such as operation cost, start-up cost, minimum up-time and down-time constraints, as follows.

1) Base-load units

Base-load units have low operation cost, high start-up cost, and long minimum up-time and down-time constraints. If it is possible, these units should be on-line all the time.

2) Intermediate-load unit or cycling units

Intermediate-load units have medium operation cost, medium start-up cost, and medium minimum up-time and down-time constraints. These units can be either on-line or off-line as their minimum up-time and down-time constraints are observed. Generally, they are on-line when the capacity of all base-load units is not sufficient to supply the demand and spinning reserve requirement.

3) Peak-load units

Peak-load units have high operation cost, low start-up cost, and short minimum up-time and down-time constraints. In general, peak-load units can be quickly brought on-line, ramped up, ramped down, and shut down. These units are typically needed during peak load hours for short periods of time.

4.1.2.2 Unit Decommitment Procedures

When there is excessive spinning reserve in hour t , the following steps are performed.

1) List all on-line units in hour t excluding base-load units and units in must-run status. Only intermediate-load and peak-load units that are not in the must-run status are eligible for the decommitment.

2) If the list from previous step is not empty, count number of units in the list and keep it in the variable 'S'. Otherwise, exit the decommitment process.

3) Sort units in the list by their operating lambda, determined in the process of solving the dual subproblem, in descending order.

4) If the following conditions are all true for a unit on the top of the list, it can be decommitted. Otherwise, it must remain on-line.

4.1) The excessive spinning reserve is greater than the capacity of the unit.

4.2) If the unit is decommitted, its minimum up-time constraint is not violated.

4.3) Spinning reserve constraint of the area that the unit is located at is not violated if it is decommitted.

4.4) The demand plus spinning reserve requirement in the next $t^{down}-1$ hours (if the planning horizon ends before next $t^{down}-1$ hours, consider only the remaining hours) is less than or equal to current maximum capability where t^{down} is the minimum down-time constraint of the unit.

5) If the unit on the top of the list is decommitted, the excessive spinning reserve amount, the maximum system capability, cost incurred from the shut-down cost, and the

dual cost (q^*) in hour t must be updated. The excessive spinning reserve amount is reduced by the decommitted unit's maximum generation limit in hour t .

6) Delete the unit on the top of this list.

7) $S = S-1$

8) If $S > 0$ and excessive spinning reserve amount > 0 , return to step 4. Otherwise, the economic dispatch is performed with the new unit commitment schedule.

4.2 Multi-area Economic Dispatch

This dissertation implements the multi-area economic dispatch (ED) with the lambda-iteration method. The purpose of the multi-area ED is to find the most economical operating power output for the generating units while the demand is supplied as well as unit minimum and maximum generation constraints and area import/export capabilities are satisfied.

The following steps are the lambda-iteration algorithm for the multi-area economic dispatch. Since the multi-area economic dispatch takes into account area import/export limits, these steps are slightly more complex than the lambda-iteration algorithm for the single-area economic dispatch described in chapters 3.

4.2.1 Bounding Unit Incremental Fuel Cost Curves

Each unit incremental fuel cost curve (ICC) is bounded by its minimum and maximum generation limits as defined in equation 4.22 and shown in figure 4.2. Note that minimum and maximum generation limits are not constant every hour due to the ramp-up and ramp-down constraints as explained in chapter 2.

$$P_{i,k,\min} \leq P_{i,k} \leq P_{i,k,\max} \quad (4.22)$$

4.2.2 Binding Bounded Unit Incremental Fuel Cost Curves

Each area ICC is a combination of the bounded ICCs of all units located in the same area. This process is the same as creating a sorted table described in chapter 3. The combined curve is called unbounded area ICC.

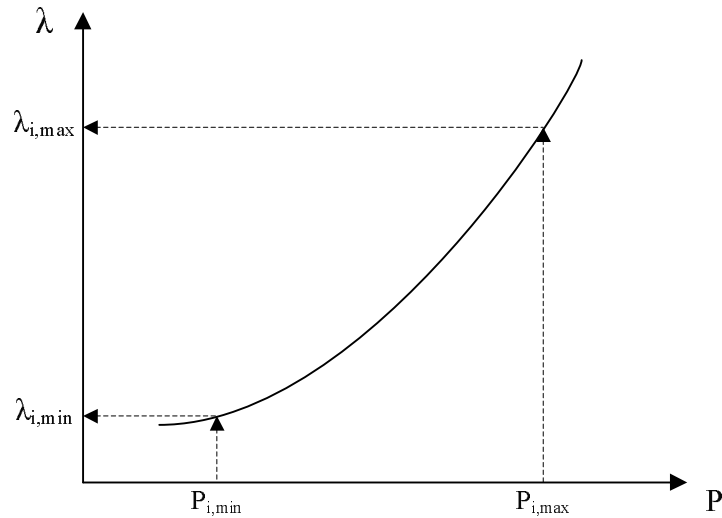


Figure 4.2 A sample of bounded unit incremental cost curve

4.2.3 Bounding Area Incremental Fuel Cost Curves

Each unbounded area ICC is bounded by area resource limits which are demand minus area import limit and area demand plus area export limit. Therefore, area minimum and maximum generation limits are defined as shown in equations 4.23-4.24. This will ensure that the area import/export limits are not violated. Figure 4.3 shows that minimum and maximum lambda (marginal cost) of bounded area ICC can be computed once the area minimum and maximum power limits are defined. When there are no area import/export constraints, the area unbounded ICC is the same as the area

bounded ICC and the area minimum and maximum generation limits are the sum of minimum and maximum generation limits of units located in the same area.

$$P_{k,\min} = D_k - I_{k,\max} \quad (4.23)$$

$$P_{k,\max} = D_k + E_{k,\max} \quad (4.24)$$

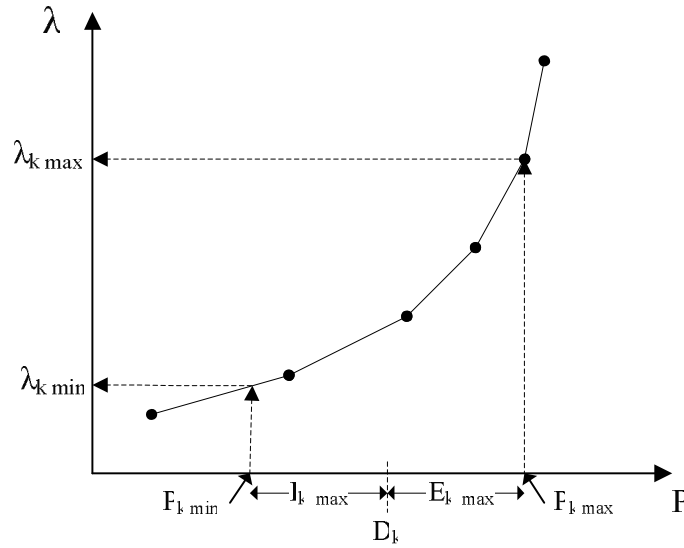


Figure 4.3 A sample of bounded area incremental cost curve

Some systems consist of areas and *subareas*. When there are multiple subareas in an area and each subarea has import/export limits, two additional steps have to be completed before creating unbounded area ICCs. The first additional step is for creating unbounded subarea ICCs from bounded ICCs of units located in the same subarea. The second additional step is bounding the unbounded subarea ICCs by subarea's demand and import/export limits as the process of bounding area unbounded ICCs. The unbounded area ICCs, then, can be created from combining the bounded subarea ICCs.

4.2.4 Binding Bounded Area Incremental Fuel Cost Curves

A system ICC is a combination of all bounded area ICCs. The process is also the same as creating the sorted table described in chapter 3. System minimum and maximum lambda limits are the incremental cost rates of the first break point and the last break point of the system ICC. The system minimum generation limit is the sum of every area minimum generation limits and the system maximum generation limit is the sum of every area maximum generation limits as shown in equations 4.25 and 4.26, respectively. When there are no subarea and area import/export constraints, the system minimum and maximum generation limits are the sum of every unit minimum and maximum generation limits.

$$P_{sys,min} = \sum_{k=1}^K (D_k - I_{k,max}) \quad (4.25)$$

$$P_{sys,max} = \sum_{k=1}^K (D_k + E_{k,max}) \quad (4.26)$$

4.2.5 Determination of System's Operating Lambda

The system's operating incremental fuel cost rate (system lambda) can be determined by the point that the system demand, the sum of all areas' demand, is located on the system ICC as illustrated in figure 4.4.

If the system demand is not within the system minimum and maximum generation limits, the unit commitment schedule provided by the unit commitment process is inadequate to supply the system demand.

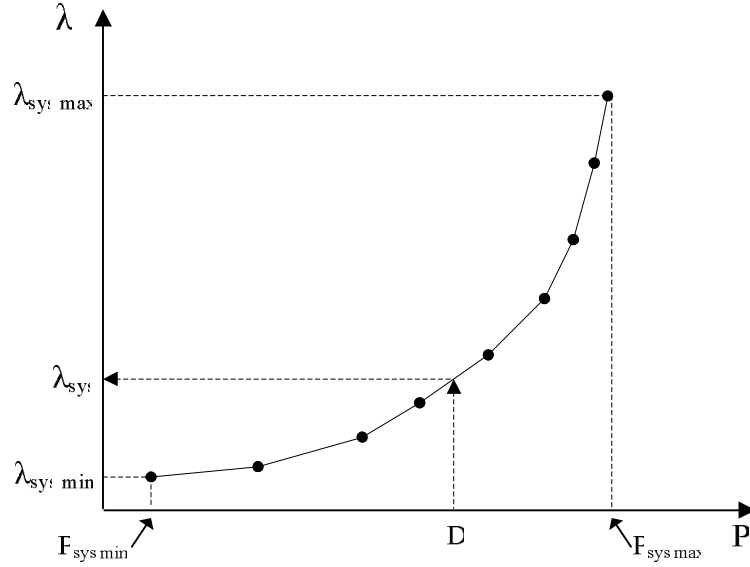


Figure 4.4 A sample of system incremental cost curve

4.2.6 Determination of Area's Operating Lambda and Generation

As shown in equation 4.27, an area's operating lambda is equal to the system's operating lambda if the system's operating lambda is within the area minimum and maximum lambda limits. Otherwise, the area's operating lambda is equal to the area minimum lambda or the area maximum lambda if the system's operating lambda is less than the area minimum lambda or greater than the area maximum lambda. The area generation is the point that the area's operating lambda is located on the area bounded ICC.

$$\left. \begin{array}{ll}
 P_{k,\min} \leq P_k \leq P_{k,\max} \text{ and } \lambda_k = \lambda_{\text{sys}} & \text{if } \lambda_{k,\min} \leq \lambda_{\text{sys}} \leq \lambda_{k,\max} \\
 P_k = P_{k,\max} \text{ and } \lambda_k = \lambda_{k,\max} & \text{if } \lambda_{\text{sys}} > \lambda_{k,\max} \\
 P_k = P_{k,\min} \text{ and } \lambda_k = \lambda_{k,\min} & \text{if } \lambda_{\text{sys}} < \lambda_{k,\min}
 \end{array} \right\} \quad (4.27)$$

Similarly, when there are multiple subareas in the same area, the area's operating lambda is used to find for subarea' operating lambda and generation if the area's operating lambda is within the subarea minimum and maximum lambda limits.

4.2.7 Determination of Unit's Operating Lambda, Generation, and Cost

Unit's operating lambda is equal to the operating lambda of the area (or subarea if there are multiple subareas in the same area) that unit is located at if the area's operating lambda (or subarea's operating lambda) is within the bounds of the unit bounded incremental cost curve. Otherwise, the unit's operating lambda is equal to the unit minimum lambda or the unit maximum lambda if the area's (or subarea's) operating lambda is less than the unit minimum lambda or greater than the unit maximum lambda. The unit generation is the point that the unit's operating lambda is located on the unit bounded ICC. The unit fuel cost and O&M cost can be calculated from the cost function and the O&M function once unit generation is determined.

4.2.8 Determination of Area Interchanges and Tie Line Flows

Interchange of an area at a given hour is an amount of real power exported from the area minus an amount of real power imported to the area at a given hour as defined in equation 4.28.

$$Intch_k = E_k - I_k \quad (4.28)$$

According to shift factors of tie lines and areas' exported MWs, the flow between areas in a direction can be calculated as defined in equation 4.29. If the power flow on any tie line exceeds its tie line capacity limit in any direction, the unit

commitment schedule provided by the unit commitment process is infeasible due to the violation of the tie line capacity constraint.

$$Flow^{g,l} = SF_k^{g,l} \left(\sum_{i=1}^{N_k} (P_{i,k}^t \cdot U_{i,k}^t) - D_k^t \right) \quad (4.29)$$

4.3 Market Operation Components

Market operation components are tools to maximize the profit in addition to having the optimal generation schedule. The market operation components considered in this dissertation includes general types of bilateral contracts. A bilateral contract is a direct contract between a power producer and either a load serving entity or an ISO outside a centralized bid-based market.

The bilateral contracts incorporated into the proposed program are categorized into three main types – forward contracts, options, and reliability must-run (RMR) contracts. Forward contract holders are obligated to buy or sell power at a predefined price for a specified period which can be from an hour to years [5]. Unlike forward contracts, options give their option purchasers the right, but not the obligation, to buy (for call option) or sell (for put option) a fixed amount of power at a predefined strike price during the option term which is usually from months to a couple of years [5]. This dissertation considers the option and forward contracts between internal generators and internal LSEs, between internal generators and external LSEs which is counted as sale transactions with the external system, or between external generators and internal LSEs which is counted as purchase transactions with the external system.

The RMR contracts are bilateral agreements between an ISO and GENCOs to operate designated generating units at specified MW for the local demand. These contracts are initiated to mitigate local area reliability problems when other market solutions such as transmission expansion are not available. The ISO will publicly post yearly, monthly, and week-ahead forecasts of the dispatch pattern it expects for each contracted RMR unit and will identify the owners of the RMR units the dispatch instruction a day ahead. The RMR agreements and the selection criteria for the RMR units vary between ISOs [58-60].

When one or a combination of market operation components from a list of available bilateral contracts and reliability must-run contracts, provided by the input data, are designated on the graphical user interface, the following procedures are taken place.

4.3.1 Forward Contracts

When the forward contracts are exercised, the following procedures must be completed before the multi-area UC and ED problems are solved.

4.3.1.1 Forward Contracts between Internal Generators and Internal LSEs

There are five necessary procedures when the forward contracts between the internal generators and the internal LSEs are exercised. First, the generation requirements of areas where the designated source generating units are located at are increased by the amounts of power and in the periods specified in the contracts. Second, the generation requirements of the designated sink load areas are decreased by the amounts of power and in the periods specified in the contracts. Consequently, the

adjusted generation requirements of the source areas are the demand of the source areas plus the contracted amounts of power and those of the sink areas are the demand of the sink areas minus the contracted amounts of power.

Third, the contracted amounts of power in the specified periods are counted against the export limits of the source areas and the import limits of the sink areas. Forth, the designated source generating units are assigned their status as must-run with predefined minimum generation equal to the amounts specified in the agreement. With this status, these units are excluded from the unit commitment due to predefined status and their minimum power outputs are counted towards supplying only the amounts of the generation requirement reduced from the sink areas. If the predefined minimum generation level of the source generating unit is less than its full capacity in any hour, the remaining capacity can participate in the economic dispatch. Finally, the tie line capacity limits must be adjusted with respect to flows contributed by contracted amounts of power.

4.3.1.2 Forward Contracts between External Generators and Internal LSEs

The forward contracts between the external generators and the internal LSEs are considered as the purchase contracts with the external systems. When they are exercised, the generation requirements of the designated sink load areas need to be decreased by the amounts of power and in the periods specified in the contracts. The contracted amounts of power are then counted against the import limits of the sink areas.

4.3.1.3 Forward Contracts between Internal Generators and External LSEs

The forward contracts between the internal generators and the external LSEs are considered as the sale contracts with the external systems. When they are exercised, the generation requirements of areas where the designated source generating units are located at are adjusted as the first step of section 4.3.1.3. The contracted amounts of power are counted against the export limits of the source areas. Meanwhile, the designated source generating units are arranged as the fourth step of section 4.3.1.3.

4.3.2 Options

The main difference between the forward contracts and options is that the holders of the forward contracts are obligated to buy and sell power while the holders of the options have the right to choose whether the contracts should be exercised. Once the call option or the put option is exercised, the procedures which must be completed before the multi-area UC and ED problems are solved are the same as described in the forward contract section.

4.3.3 Reliability Must-run Contracts

Generating units that are designated as RMR units are simply assigned their status as must-run with predefined minimum generation equal to the amounts requested from the ISO. With this status, these units are excluded from the unit commitment due to predefined status and their minimum power outputs are counted towards supplying only the local area. When the RMR units are requested to run at less than their full capacity, the remaining capacity can participate in the economic dispatch.

4.4 Program Structure of Multi-area UC and ED with Market Operation Components

Multi-area UC and ED with market operation components program presented in this dissertation is developed with Microsoft Visual Studio C# .NET which is an object-oriented programming language. Object-oriented programming (OOP) provides programming developers the flexibility and the ease to design, develop, and maintain the programs. The OOP is also suitable for large-scale programs.

The multi-area UC and ED with market operation components program, referred as MAUC-MC, consists of four main modules which are user interface, input/output, unit commitment, and economic dispatch. The relationships of these modules including external files are illustrated in figure 4.5. The user interface module is designed for the communication between the program and the users. The users can view input/output displays, execute commands, and edit input data. All possible interactions between the users and the program are illustrated in the figure 4.6.

The input/output module interacts with the other three modules and also external files such as input, output, and configuration files. In addition, the multi-area ED can be performed as a stand alone tool with UI, IO, and ED modules. The class diagram of multi-area UC and ED with market operation components and the class diagram of multi-area ED are shown in Appendix A.

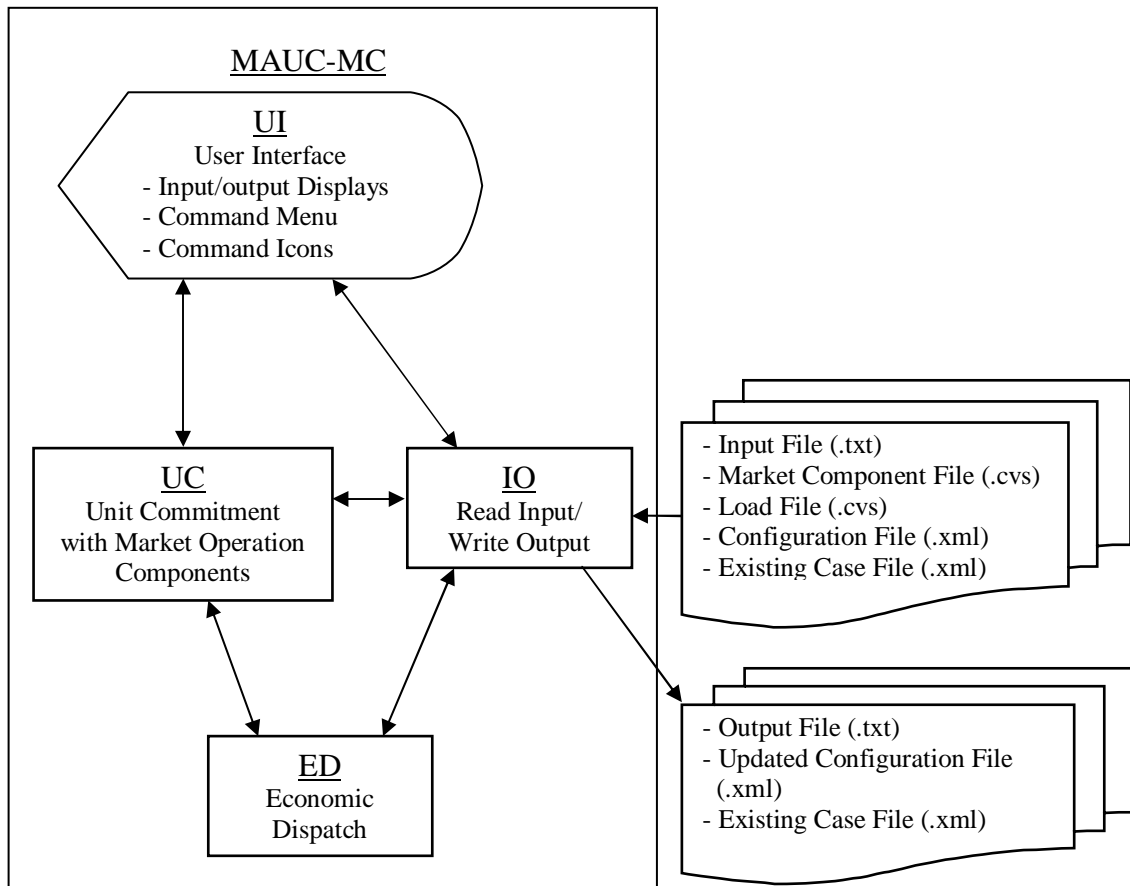


Figure 4.5 Structure of multi-area UC and ED with market operation components

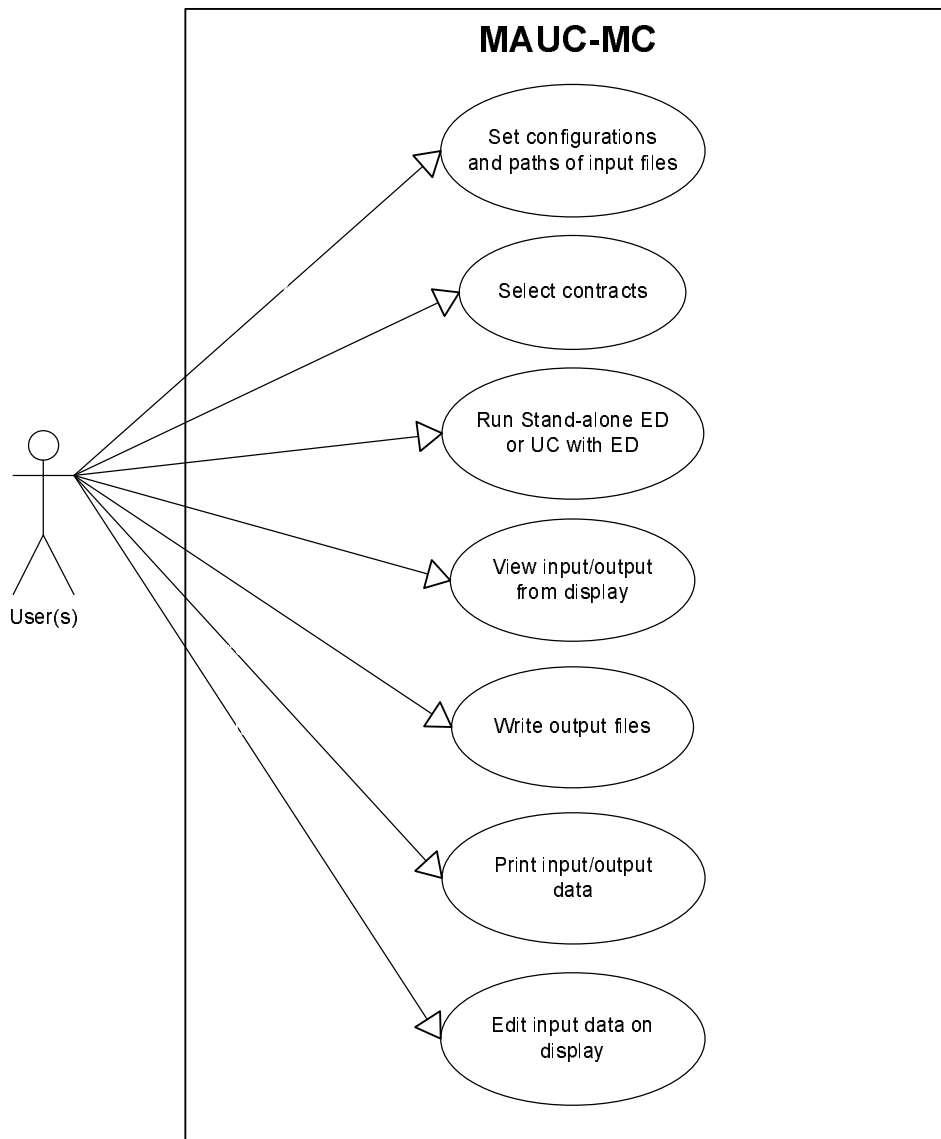


Figure 4.6 All possible interactions between user(s) and MAUC-MC

CHAPTER 5

TEST SYSTEMS AND SIMULATION RESULTS

The proposed algorithm for multi-area unit commitment and economic dispatch with market operation components described in chapter 4 is tested with two groups of simulations. The first group is testing the proposed algorithm with single-area systems which consist of ten and twenty generating units. The other group is testing the proposed algorithm with a multi-area system which consists of seventy-eight generating units, six subareas, three areas, and three tie lines. All simulations presented in this dissertation is performed on Pentium M processor 740 (1.73 GHz) and 512 MB of RAM.

5.1 Single-area System

Two single-area systems are used to test the speed and the solution quality of the proposed algorithm. In addition, the simulations illustrate the cost impact of incorporating the market operation components such as call option and forward contract into the generating unit scheduling.

5.1.1 Single-area Systems without Market Operation Components

Two single-area systems used in the test simulations consist of ten and twenty generating units. The characteristics of the generating units in the ten-unit system are shown in tables B.1 and B.2 in appendix B. The twenty-unit system consists of two

duplicated generating units of the ten-unit system. Therefore, the twenty-unit system contains ten pairs of identical generating units. Both systems require spinning reserve for ten percent of hourly demand.

In order to obtain the optimal solution, four groups of the step size parameters δ and σ empirically determined and used in the simulations for both ten-unit and twenty-unit systems are shown in table 5.1. First, when there are insufficient or barely sufficient total generation output and capacity to supply the demand and meet the spinning reserve requirement ($pDiff^t \geq 0$ and $rDiff^t \geq 0$), the step size parameters δ and σ equal to 0.02 and 0.05, respectively, are used to update both Lagrange multipliers λ and μ . Second, when the spinning reserve is excessive but the total generation output determined by the dual subproblem is insufficient ($pDiff^t > 0$ and $rDiff^t < 0$), the step size parameters δ and σ equal to 0.5 and 0.5, respectively, are used to update both Lagrange multipliers λ and μ .

Third, when the total generation output and capacity are excessive ($pDiff^t \leq 0$ and $rDiff^t < 0$), the step size parameters δ and σ equal to 0.5 and 0.5, respectively, are used to update both Lagrange multipliers λ and μ . Finally, when the total generation output is excessive but capacity is insufficient to meet the spinning reserve ($pDiff^t < 0$ and $rDiff^t > 0$), the step size parameters δ and σ equal to 0.02 and 0.02, respectively, are used to update only Lagrange multipliers μ . The parameters in the second and third cases are intentionally set to be larger than those in the first and fourth cases for the slower changes of Lagrange multipliers. The parameters in table 5.1 are slightly different than those presented in [25].

Table 5.1 Step Size Parameters for Updating Lagrange Multipliers λ and μ in a Single-area System

Case	Case Condition	δ	σ
1	$pDiff^t \geq 0$ and $rDiff^t \geq 0$	0.02	0.05
2	$pDiff^t > 0$ and $rDiff^t < 0$	0.5	0.5
3	$pDiff^t \leq 0$ and $rDiff^t < 0$	0.5	0.5
4	$pDiff^t < 0$ and $rDiff^t > 0$	0.02	0.02

5.1.1.1 Simulation Results of Ten-unit System

Table 5.2 shows an hourly demand profile for the ten-unit system and their unit commitment schedule obtained from the multi-area unit commitment and economic dispatch with market operation component program (MAUC-MC). Generating units 1 and 2 are base-load units, thus they remain on-line throughout the time horizon. Generating units 3, 4, 5, 6, and 7 are intermediate-load units and generating units 8, 9, and 10 are peak-load units. These units are brought on-line when the capacity of all base-load units is inadequate to supply the demand and meet the spinning reserve requirement. The peak-load units are on-line in peak-load hours and they can be quickly shut-down when the demand declines.

Hourly power output, fuel cost, and start-up cost of each generating unit for the twenty-four hour schedule are illustrated in tables B.3 and B.4 in appendix B. Since the incremental cost rate curve of the entire system is monotonically increasing, the hourly marginal costs of supplying power and serving spinning reserve requirement (λ and μ) determined in process of solving the dual subproblem are high when the hourly demand is high as shown in table B.5 in appendix B. The total production cost received from the simulation for the ten-unit system is 563,214 dollars

Table 5.2 Hourly Demand and UC Solution of Ten-unit System between Hours 1 and 24

Hour	Demand (MW)	Generating Unit									
		1	2	3	4	5	6	7	8	9	10
1	700	1	1	0	0	0	0	0	0	0	0
2	750	1	1	0	0	0	0	0	0	0	0
3	850	1	1	0	0	1	0	0	0	0	0
4	950	1	1	0	0	1	0	0	0	0	0
5	1000	1	1	0	1	1	0	0	0	0	0
6	1100	1	1	1	1	1	0	0	0	0	0
7	1150	1	1	1	1	1	0	0	0	0	0
8	1200	1	1	1	1	1	0	0	0	0	0
9	1300	1	1	1	1	1	1	1	0	0	0
10	1400	1	1	1	1	1	1	1	1	0	0
11	1450	1	1	1	1	1	1	1	1	1	0
12	1500	1	1	1	1	1	1	1	1	1	1
13	1400	1	1	1	1	1	1	1	1	0	0
14	1300	1	1	1	1	1	1	1	0	0	0
15	1200	1	1	1	1	1	0	0	0	0	0
16	1050	1	1	1	1	1	0	0	0	0	0
17	1000	1	1	1	1	1	0	0	0	0	0
18	1100	1	1	1	1	1	0	0	0	0	0
19	1200	1	1	1	1	1	0	0	0	0	0
20	1400	1	1	1	1	1	1	1	1	0	0
21	1300	1	1	1	1	1	1	1	0	0	0
22	1100	1	1	0	0	1	1	1	0	0	0
23	900	1	1	0	0	1	0	0	0	0	0
24	800	1	1	0	0	0	0	0	0	0	0

5.1.1.2 Simulation Results of Twenty-unit System

An hourly demand profile for the twenty-unit system which is twice as much as the demand for the ten-unit system is shown in tables 5.3 and 5.4. In the same tables, the unit commitment solution obtained from the MAUC-MC for the twenty-unit system is illustrated. Hourly power output, fuel cost, and start-up cost of each generating unit

for the twenty-four hour schedule are presented in tables B.6 - B.9 in appendix B. The hourly λ and μ used in the dual subproblem to determine the unit status for the twenty-unit system, as shown in table B.10, are similar to those for the ten-unit system due to the identical units.

Table 5.3 Hourly Demand and UC Solution of Twenty-unit System
Between Hours 1 and 12

		Hour											
		1	2	3	4	5	6	7	8	9	10	11	12
Demand (MW)		1400	1500	1700	1900	2000	2200	2300	2400	2600	2800	2900	3000
	Unit	1	1	1	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	1	1	1	1	
	3	0	0	0	0	0	0	0	1	1	1	1	
	4	0	0	0	0	1	1	1	1	1	1	1	
	5	0	0	0	0	0	1	1	1	1	1	1	
	6	0	0	0	0	0	0	0	0	1	1	1	
	7	0	0	0	0	0	0	0	0	0	1	1	
	8	0	0	0	0	0	0	0	0	0	1	1	
	9	0	0	0	0	0	0	0	0	0	0	1	
	10	0	0	0	0	0	0	0	0	0	0	1	
	11	1	1	1	1	1	1	1	1	1	1	1	
	12	1	1	1	1	1	1	1	1	1	1	1	
	13	0	0	0	0	0	1	1	1	1	1	1	
	14	0	0	0	1	1	1	1	1	1	1	1	
	15	0	0	1	1	1	1	1	1	1	1	1	
	16	0	0	0	0	0	0	0	0	1	1	1	
	17	0	0	0	0	0	0	0	0	1	1	1	
	18	0	0	0	0	0	0	0	0	0	1	1	
	19	0	0	0	0	0	0	0	0	0	0	1	
	20	0	0	0	0	0	0	0	0	0	0	1	

Table 5.4 Hourly Demand and UC Solution of Twenty-unit System
Between Hours 13 and 24

		Hour												
		13	14	15	16	17	18	19	20	21	22	23	24	
Demand (MW)		2800	2600	2400	2100	2000	2200	2400	2800	2600	2200	1800	1600	
Unit	1	1	1	1	1	1	1	1	1	1	1	1	1	
	2	1	1	1	1	1	1	1	1	1	1	1	1	
	3	1	0	0	0	0	0	0	1	1	1	1	0	
	4	1	1	1	1	1	1	1	1	1	0	0	0	
	5	1	1	1	1	1	1	1	1	1	0	0	0	
	6	1	1	1	0	0	0	0	0	1	1	1	0	0
	7	1	1	0	0	0	0	0	0	1	1	1	0	0
	8	1	0	0	0	0	0	0	0	1	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	1	1	1	1	1	1	1	1	1	1	1	1	1
	12	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	0	0	0	0
	14	1	1	1	1	1	1	1	1	1	1	0	0	0
	15	1	1	1	1	1	1	1	1	1	1	1	1	0
	16	1	1	1	0	0	0	0	0	1	1	1	0	0
	17	1	1	0	0	0	0	0	0	1	1	1	0	0
	18	1	0	0	0	0	0	0	0	1	0	0	0	0
	19	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	0	0	0	0	0	0	0	0	0	0	0	0	0

The total production cost received from the simulation for the twenty-unit system is 1,123,020 dollars. This cost is less than twice as much as the cost obtained from the ten-unit system because the number of on-line units in the twenty-unit system in some hours is not required to be twice as much as that of on-line units in the ten-unit system. Like the UC solution of ten-unit system, generating units 1, 2, 11, and 12 which are base-load units are on-line throughout the time horizon.

5.1.1.3 Total Production Cost and Computational Time Comparison

The total production costs calculated by the proposed algorithm (MAUC-MC) are the most economic when compared to the solution obtained by applying the conventional Lagrangian relaxation (LR) [28], the genetic algorithm (GA) [28], the evolutionary programming method (EP) [33], the Lagrangian relaxation with the genetic algorithm (LRGA) [32], the genetic algorithm based on characteristic classification (GAUC) [61], LR with Dynamic Programming (LRDP) [25], adaptive LR with on/off decision criteria (ALR) [25], and enhanced adaptive LR with on/off decision criteria (ELR) [25] as shown in table 5.5.

Computational times in solving the 10-unit system and 20-unit system without a market operation component are 0.5 and 1.2 seconds, respectively. The computational time of the proposed algorithm appears to be relatively low. However, different processors are used for the simulations in the references and this dissertation, thus the computational times presented are not comparable.

Table 5.5 Cost Comparison of Scheduling a Single-area System for 24 Hours

Total Production Cost (\$)		
Algorithm	Number of Generating Units	
	10	20
LR	565,825	1,130,660
GA	565,825	1,126,243
EP	564,551	1,125,494
LRGA	564,800	1,122,622
GAUC	563,977	1,125,516
DPLR	564,049	1,128,098
ALR	565,508	1,126,720
ELR	563,977	1,123,297
MAUC-MC	563,214	1,123,020

5.1.2 Single-area System with Market Operation Components

The ten-unit system in previous section is the base case for the simulations in this section. Case study 1 is performing the single-area unit commitment and economic dispatch with the base case.

Case study 2 is performing the case study 1 with a call option to purchase 50 MW from an external source for the entire time horizon which is a twenty four hour period at the rate of 23 dollars per MWh.

Case study 3 is performing the case study 1 with a forward contract to purchase 50 MW at the rate of 23 dollars per MWh from an external source between hours 8 and 22 which is considered as the peak load period.

Case study 4 is performing the case study 1 with a forward contract to purchase 50 MW at the rate of 26 dollars per MWh from an external source between hours 8 and 22.

Case study 5 is performing the case study 1 with a forward contract to sell 50 MW at the rate of 25 dollars per MWh from unit 1 to an external LSE between hours 14 and 18.

Case studies 2-5 are summarized in table 5.6. The results of simulating all case studies shown in table 5.7 illustrate that, in case studies 2, 3, and 5, bilateral contracts can increase the profit for generators by purchasing power from the external source to supply the internal demand or selling power to the external LSEs at certain rates.

Table 5.6 Descriptions of Case Studies 2-5 of Single-area System with Ten Units

Case Study	Contract	Source	Sink	MW	Rate (\$/MWh)	Period Hour
2	Call option	External system	Internal system	50	23	1-24
3	Forward contract	External system	Internal system	50	23	8-22
4	Forward contract	External system	Internal system	50	26	8-22
5	Forward contract	Unit 1	External system	50	25	14-18

Table 5.7 Cost Comparison of Single-area UC and ED with Market Operation Components

Case Study	UC and ED Cost Result (\$)	Cost of Purchase (\$)	Revenue of Sale (\$)	Total Cost (\$)
1	563,214	0	0	563,214
2	534,510	27,600	0	562,110
3	544,075	17,250	0	561,325
4	544,075	19,500	0	563,575
5	568,259	0	-6,250	562,090

5.2 Multi-area System

A sample multi-area system with three area, modified from data in [62], are used to test the algorithms for the import/export capability and tie line capacity constraints and the speed of solving the multi-area UC and ED with a large-scale system. All market operation components including call options, put options, forward contracts, and reliability must-run contracts are also integrated into the simulations.

In the sample multi-area system, each area consists of two subareas and twenty-six generating units. The total number of generating units is equal to seventy-eight and their characteristics are shown in tables C.1 and C.2 in appendix C. This system

contains three tie lines for area interconnections whose shift factors are shown in table C.3 in appendix C. There are four assumptions in order to use phase shifters with this sample multi-area system. First, a direct current (DC) power flow model is used. Second, voltage at each node (area) is simply assumed to be equal to one per unit (1 p.u.). Third, the effective impedance inside each area must be the same or power is injected at the same bus as that is withdrawn. Finally, two subareas in the same area share the same bus for interchanges and area 1 is the reference area.

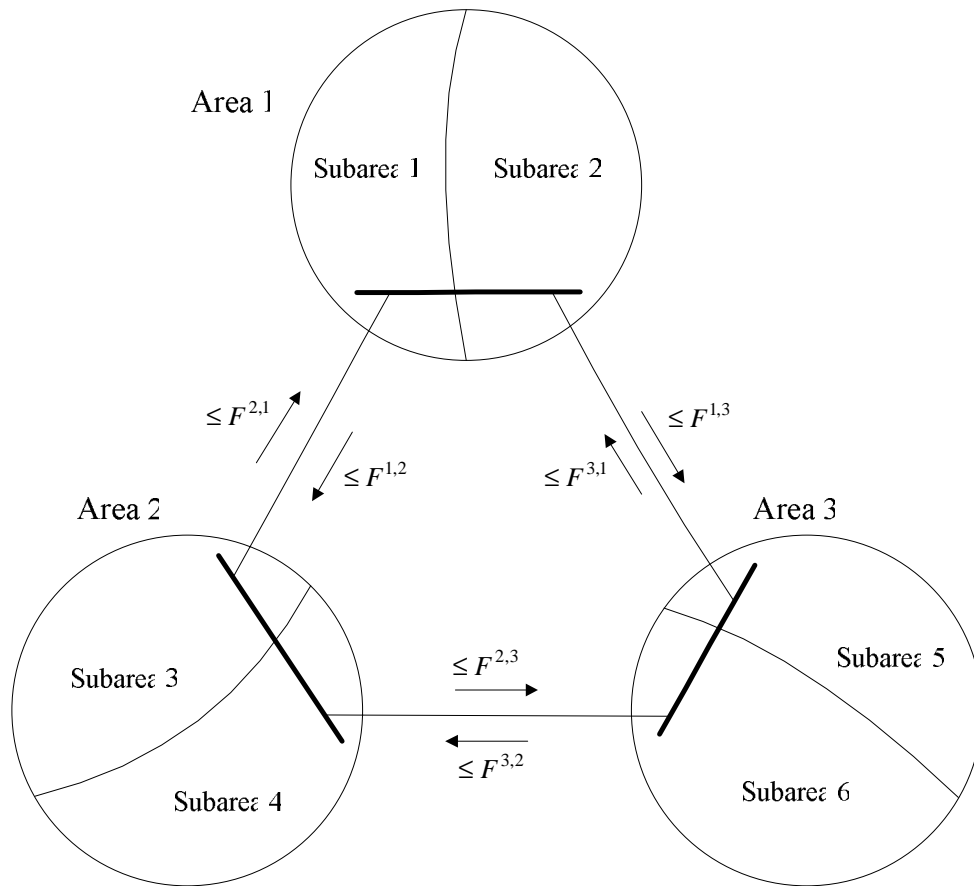


Figure 5.1 A simplified graphic representation of three-area, six-subarea system with three tie lines

A fictitious hourly demand profile for twenty-four hours as shown in table 5.8 is used for the multi-area UC and ED simulations. All simulations require the spinning reserve equal to ten percent of the hourly demand and use the step size parameters, empirically determined, as shown in tables 5.9 and 5.10 to update Lagrange multipliers λ , μ , α , β , and θ .

Table 5.8 Hourly Demand of Multi-area System

Hour	Demand (MW)									
	Sub-area1	Sub-area2	Area1	Sub-area3	Sub-area4	Area2	Sub-area5	Sub-area6	Area3	System
1	200	1279	1479	270	1264	1534	320	1319	1639	4652
2	130	1196	1326	187	1181	1368	237	1236	1473	4167
3	78	1017	1095	126	1002	1128	176	1057	1233	3456
4	69	938	1007	114	923	1037	164	978	1142	3186
5	94	876	970	90	861	951	140	916	1056	2977
6	138	1016	1154	208	1001	1209	258	1056	1314	3677
7	250	1178	1428	320	1163	1483	370	1218	1588	4499
8	430	1289	1719	500	1204	1704	550	1259	1809	5232
9	449	1350	1799	519	1235	1754	569	1390	1959	5512
10	458	1370	1828	528	1389	1917	578	1499	2077	5822
11	499	1438	1937	530	1479	2009	580	1557	2137	6083
12	560	1489	2049	550	1555	2105	600	1620	2220	6374
13	548	1474	2022	552	1420	1972	602	1565	2167	6161
14	537	1512	2049	560	1397	1957	610	1452	2062	6068
15	528	1547	2075	570	1386	1956	620	1423	2043	6074
16	517	1589	2106	587	1345	1932	637	1540	2177	6215
17	530	1642	2172	600	1479	2079	650	1729	2379	6630
18	558	1683	2241	628	1585	2213	678	1896	2574	7028
19	565	1700	2265	635	1774	2409	685	1886	2571	7245
20	557	1789	2346	627	1741	2368	677	1726	2403	7117
21	544	1700	2244	614	1688	2302	664	1629	2293	6839
22	499	1678	2177	569	1620	2189	619	1487	2106	6472
23	472	1587	2059	542	1565	2107	592	1432	2024	6190
24	360	1489	1849	430	1474	1904	480	1366	1846	5599

Table 5.9 Step Size Parameters for Updating Lagrange Multipliers λ and μ in a Multi-area System

Case	Case Condition	δ	σ
1	$pDiff^t \geq 0$ and $rDiff^t \geq 0$	0.02	0.02
2	$pDiff^t > 0$ and $rDiff^t < 0$	0.05	0.1
3	$pDiff^t \leq 0$ and $rDiff^t < 0$	0.09	0.09
4	$pDiff^t < 0$ and $rDiff^t > 0$	0.02	0.05

Table 5.10 Step Size Parameters for Updating Lagrange Multipliers α , β , and θ in a Multi-area System

Lagrange Multiplier	δ	σ
α, β for Every Area	0.05	0.06
α, β for Every Subarea	0.02	0.05
θ for Every Tie line	0.02	0.02

5.2.1 Multi-area System with Unlimited Import/Export Capabilities and Tie Line Capacities

A base case for simulations is the sample multi-area system with unlimited import/export capabilities and unlimited tie line capacities. Without area import/export and tie line flow limitations, the multi-area system is basically identical to the single-area system. Tables 5.11-5.16 show the unit commitment schedule of the base case obtained from the MAUC-MC program. The generation outputs and net interchanges of six subareas are illustrated in table C.4 in appendix C. The net interchange of a subarea (or an area) is the exported amount of power minus the imported amount of power. It is shown in table C.4 that subareas 1, 3, and 5, which contains units with high incremental cost rates, import relatively high amounts of the power to supply the internal demand. Table C.5 in appendix C shows the power flows on three tie lines and the net

interchanges of three areas when the area import/export capabilities and tie line capacities are not limited.

Hourly λ and μ used to determine the unit status in the dual subproblem, hourly system fuel costs, and hourly system start-up costs are shown in table C.6 in appendix C. The total production cost is 1,625,192.78 dollars. The MAUC-MC can solve the base case in 4.8 seconds which linearly increases with respect to the system size when compared to the computational time of solving the twenty-unit, single-area system.

Table 5.11 UC Solution of Subarea 1 for 24 Hours

Hour	Generating Unit										
	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	1	1	0	0	1	1	0	0	0
7	0	0	1	1	0	0	1	1	0	0	0
8	0	0	1	1	0	0	1	1	0	0	0
9	0	0	1	1	0	0	1	1	0	0	0
10	0	0	1	1	0	0	1	1	0	0	0
11	0	0	1	1	0	0	1	1	0	0	0
12	0	0	1	1	0	0	1	1	0	0	0
13	0	0	1	1	0	0	1	1	0	0	0
14	0	0	1	1	0	0	1	1	0	0	0
15	0	0	1	1	0	0	1	1	0	0	0
16	0	0	1	1	0	0	1	1	0	0	0
17	0	0	1	1	0	0	1	1	0	0	0
18	0	0	1	1	0	0	1	1	0	1	1
19	0	0	1	1	0	0	1	1	0	1	1
20	0	0	1	1	0	0	1	1	0	1	1
21	0	0	1	1	0	0	1	1	0	1	1
22	0	0	1	1	0	0	1	1	0	1	1
23	0	0	1	1	0	0	1	1	0	1	1
24	0	0	1	1	0	0	1	1	0	1	1

Table 5.12 UC Solution of Subarea 2 for 24 Hours

Hour	Generating Unit														
	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
4	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
7	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
10	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
16	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
17	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
18	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
19	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
20	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
21	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
23	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
24	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1

Table 5.13 UC Solution of Subarea 3 for 24 Hours

Hour	Generating Unit										
	27	28	29	30	31	32	33	34	35	36	37
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	1	1	0	0	1	1	0	0	0
7	0	0	1	1	0	0	1	1	0	0	0
8	0	0	1	1	0	0	1	1	0	0	0
9	0	0	1	1	0	0	1	1	0	0	0

Table 5.13 - *Continued*

Hour	Generating Unit										
	27	28	29	30	31	32	33	34	35	36	37
10	0	0	1	1	0	0	1	1	0	0	0
11	0	0	1	1	0	0	1	1	1	0	0
12	0	0	1	1	0	0	1	1	1	1	1
13	0	0	1	1	0	0	1	1	1	1	1
14	0	0	1	1	0	0	1	1	1	1	1
15	0	0	1	1	0	0	1	1	1	1	1
16	0	0	1	1	0	0	1	1	1	1	1
17	0	0	1	1	0	0	1	1	1	1	1
18	0	0	1	1	0	0	1	1	1	1	1
19	0	0	1	1	0	0	1	1	1	1	1
20	0	0	1	1	0	0	1	1	1	0	1
21	0	0	1	1	0	0	1	1	0	0	0
22	0	0	1	1	0	0	1	1	0	0	0
23	0	0	1	1	0	0	1	1	0	0	0
24	0	0	1	1	0	0	1	1	0	0	0

Table 5.14 UC Solution of Subarea 4 for 24 Hours

Hour	Generating Unit														
	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
4	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
7	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
10	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
16	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
17	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1

Table 5.14 - Continued

Hour	Generating Unit														
	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
18	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
19	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1
20	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1
21	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1
22	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1
23	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1
24	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1

Table 5.15 UC Solution of Subarea 5 for 24 Hours

Hour	Generating Unit											
	53	54	55	56	57	58	59	60	61	62	63	
1	0	0	0	0	0	0	0	0	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	1	1	0	0	1	1	0	0	0	
7	0	0	1	1	0	0	1	1	0	0	0	
8	0	0	1	1	0	0	1	1	0	0	0	
9	0	0	1	1	0	0	1	1	0	0	0	
10	0	0	1	1	0	0	1	1	0	0	0	
11	0	0	1	1	0	0	1	1	0	1	1	
12	0	0	1	1	0	0	1	1	1	1	1	
13	0	0	1	1	0	0	1	1	1	1	1	
14	0	0	1	1	0	0	1	1	1	1	1	
15	0	0	1	1	0	0	1	1	1	1	1	
16	0	0	1	1	0	0	1	1	1	1	1	
17	0	0	1	1	0	0	1	1	1	1	1	
18	0	0	1	1	0	0	1	1	1	1	1	
19	0	0	1	1	0	0	1	1	1	1	1	
20	0	0	1	1	0	0	1	1	1	1	1	
21	0	0	1	1	0	0	1	1	0	1	1	
22	0	0	1	1	0	0	1	1	0	0	0	
23	0	0	1	1	0	0	1	1	0	0	0	
24	0	0	1	1	0	0	1	1	0	0	0	

Table 5.16 UC Solution of Subarea 6 for 24 Hours

Hour	Generating Unit														
	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78
1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
4	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
7	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
10	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
11	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
12	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
13	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
14	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
15	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
16	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1
17	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1
18	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1
19	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1
20	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1
21	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1
22	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1
23	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1
24	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1

5.2.2 Multi-area System with Limited Import/Export Capabilities

The simulation of multi-area unit commitment and economic dispatch with unlimited import/export capabilities and unlimited tie line capacities in the previous section becomes case study 1 for the following case studies. The objective of simulating case studies 2-10 is to observe the impact of limited import/export capabilities of subareas and areas on the unit scheduling, interchanges, tie line flows, Lagrange multipliers and costs.

5.2.2.1 Limited Import Capabilities of Subareas

Case studies 2, 3, and 4 are performing the case study 1 but import capabilities of subareas 1, 3, and 5 are limited to 250, 300, and 300 MW, respectively. The solutions of these case studies are illustrated in table C.7-C.15 in appendix C. Tables C.7, C.10, and C.13 show that the amounts of imported power of subareas 1, 3, and 5 are limited by their import capabilities. Tables C.8, C.11, and C.14 show that the power flows on three tie lines and the net interchanges of three areas are affected by the limited import capabilities.

Tables C.9, C.12, and C.15 show the hourly system λ , hourly system μ , and hourly α of subareas 1, 3, and 5. The subarea alpha is the marginal cost of carrying the sufficient capacity to serve the spinning reserve requirement minus the import capability of the subarea. The alpha of a subarea is greater than zero when its economic dispatch is affected by the import limit.

The hourly fuel costs and start-up costs shown in table C.9, C.12, and C.15 are higher than those in the case study 1 because subareas 1, 3, and 5 must increase self-supply for their local demand when the import capabilities are limited. The total production cost of case study 4 is higher than that of case study 3 although the import capabilities of both subareas 3 and 5 are limited by 300 MW because the hourly demand of subarea 5 is higher than that of subarea 3. Therefore, limiting the import capability of subarea 5 has more effect on the increase in the system λ which leads to higher total production cost.

5.2.2.2 Limited Export Capabilities of Subareas

Case studies 5, 6, and 7 are performing the case study 1 but export capabilities of subareas 2, 4, and 6 are limited to 250, 300, and 300 MW. The solutions of these case studies are illustrated in table C.16-C.24 in appendix C. Tables C.16, C.19, and C.22 show that the amounts of exported power of subareas 2, 4, and 6 are limited by their export capabilities. Tables C.17, C.20, and C.23 shows the power flows on three tie lines and the net interchanges of three areas.

Tables C.18, C.21, and C.24 show the hourly system λ , hourly system μ , and hourly β of subareas 2, 4, and 6. The subarea β is the marginal cost of carrying the minimum capacity less than the demand plus the export capability of the subarea. The results illustrate that every subarea has β value equal to zero. The hourly fuel costs and start-up costs shown in table C.18, C.21, and C.24 are higher than those in the case study 1 because subareas 1, 3, and 5 can import less power due to the limited export capabilities of subareas 2, 4, and 6. However, as shown in the cost comparison in table 5.16, the impact of the limited export capabilities of subareas 2, 4, and 6 is less than that of the limited import capabilities of subareas 1, 3, and 5. The total production cost of case study 7, as shown in table 5.17, is higher than that of case studies 6 although the export capabilities of both subareas 2 and 4 are limited by 300 MW because the hourly demand of subarea 6 is higher than that of subareas 5 and 6.

5.2.2.3 Limited Import and Export Capabilities of Areas

Case studies 8, 9, 10 are performing the case study 1 but both import and export capabilities of area 1, 2, and 3 are limited to 50 MW. The solutions of these case studies

are illustrated in table C.25-C.33 in appendix C. Tables C.25, C.28, and C.31 show that the amounts of imported and exported power of areas 1, 2, and 3 are limited by their import and export capabilities. Tables C.26, C.29, and C.32 show the power flows on three tie lines and the net interchanges of three areas.

Tables C.27, C.30, and C.33 show the hourly system λ , hourly system μ , fuel cost, start-up cost, and hourly α and β of area 1, 2, and 3. Due to higher area demand, the total production cost of case study 10, as shown in table 5.17, is higher than that of case studies 8 and 9.

Table 5.17 Cost Comparison of Multi-area UC and ED with Limited Import/Export Capabilities

Case Study Number	Case Study Description	Total Production Cost (\$)
1	Unlimited import/export capabilities	1,625,192.78
2	Import capability of subarea 1 is 250 MW	1,643,077.63
3	Import capability of subarea 3 is 300 MW	1,641,452.41
4	Import capability of subarea 5 is 300 MW	1,665,562.39
5	Export capability of subarea 2 is 250 MW	1,625,883.93
6	Export capability of subarea 4 is 300 MW	1,626,230.35
7	Export capability of subarea 6 is 300 MW	1,629,792.89
8	Import and export capabilities of area 1 is 50 MW	1,637,522.49
9	Import and export capabilities of area 2 is 50 MW	1,645,475.25
10	Import and export capabilities of area 3 is 50 MW	1,894,134.50

In summary, import/export constraints limit interchanges of subareas and areas which limit transferring power from subareas or areas having low incremental cost rates to the other subareas or areas. Therefore, the total production costs of systems with import/export constraints are higher than those of system with unlimited import/export

constraints. Since Lagrange multipliers are the key to determine the unit schedule and the total cost, the same import/export limitation may differently affect on the results due to the different demand of each subarea or area.

5.2.3 Multi-area System with Limited Tie Line Capacities

Case study 11 is performing the case study 1 but tie line capacity constraints of flows between areas 1 and 2 in both directions are limited to 70 MW. This is to observe the impact of the limited tie line capacities on unit scheduling, interchanges, tie line flows, Lagrange multipliers, and costs. The solution of this case study is illustrated in table C.34-C.36 in appendix C. Tables C.34 shows generation outputs and interchanges of subareas. Tables C.35 shows that the amounts of power flows between areas 1 and 2 in both directions are within the limit which is lower than the power flows in the case study 1 shown in table C.6.

Tables C.36 shows the hourly system λ , hourly system μ , fuel cost, start-up cost, and hourly θ of flows between areas 1 and 2. The θ is the marginal cost of transferring power on the tie line. It is greater than zero when the tie line capacity limits transferring power from an economic source. With the power transferring limit, the total production cost of case study 11 is 1,632,092.90 dollars which is higher than that of case study 1.

Similar to the effect of import/export constraints, the tie line capacity constraints limit transferring power from generating units having low incremental cost rates. Therefore, the total production costs of systems with tie line capacity constraints are higher than or equal to those of system with unlimited tie line capacity constraints.

5.2.4 Multi-area System with Market Operation Components

Incorporating market operation components into the generation planning not only hedge the risk of both generators and LSEs from price volatility in the competitive electricity market but also increase the system reliability. In addition, in some cases, it may be more economic to buy or sell power through the bilateral contracts rather than through the centralized bid-based market. The following simulations present the capability of the proposed development in incorporating the market operation components into the UC and ED decision and the impact of bilateral contracts on the total production cost.

Case study 12 is performing the case study 1 with a put option to sell power 50 MW at the rate of 35 dollars per MWh from generating unit 23, Attlee, located in subarea 2 and 100 MW at the rate of 33 dollars per MWh from generating unit 52, Bloch3, located in subarea 4 between hours 2 and 16 to the external LSEs. Both generators are base-load units. The cost obtained from the MAUC-MC with this case study is 1,695,997.10 dollars and the total revenue of sale over the contracted period is 75,750.00 dollars. Therefore, the total cost of exercising this case study is 1,620,247.10 dollars which is 4,945.68 dollars lower than that of case study 1. This number reflects the saving for the entire system but not for the units that export the power. The saving of individual unit involves its own production cost, interchange cost, and sale revenue. Wood et. al. in [1] and Shoults et. al. in [2] explain the calculation methods of saving allocation for multi-area pool operation. However, if the cost information of generating

units is from the supply bid curves, the saving calculation becomes complicated and different from the pool operation.

Case study 13 is performing the case study 1 with a forward contract to sell power 100 MW at the rate of 34 dollars per MWh from generating unit 49, Behring, between hours 10 and 14 to an external LSE. Unit 49 is a base-load unit and located in subarea 4. The cost obtained from the MAUC-MC with this case study is 1,645,511.00 dollars and the total revenue of sale over the contracted period is 17,000.00 dollars. Therefore, the total cost of exercising this case study is 1,628,511.00 dollars which is 3,318.22 dollars greater than that of case study 1.

Case study 14 is as the case study 13 but 100 MW is contracted at the rate of 41 dollars per MWh. The cost obtained from the MAUC-MC with this case study is 1,645,511.00 dollars and the total revenue of sale over the contracted period is 20,500.00 dollars. Therefore, the total cost of exercising this case study is 1,625,011.00 dollars which is 181.78 dollars lower than that of case study 1. This illustrates that the saving exists when the sale price is within the certain range. With this contract, the price range should be between 40.636 dollars per MWh and higher.

Case study 15 is performing the case study 1 with a put option to sell power 20 MWh at the rate of 50 dollars per MW from generating unit 3, Abel3, between hours 8 and 11 to an external LSE. Unit 3 is a base-load unit and located in subarea 1. The cost obtained from the MAUC-MC with this case study is 1,631,175.00 dollars and the total revenue of sale over the contracted period is 3,000.00 dollars. Therefore, the total cost of exercising this case study is 1,628,175.00 dollars which is 2,982.22 dollars greater

than that of case study 1. The reason for this is that subarea 1 contains units with high marginal cost and it is more economic to import power from the other subareas. When demand is higher, it can either bring more expensive units on-line or import more powers each of which increase the entire system marginal cost.

Case study 16 is as the case study 15 but 20 MW is contracted at the rate of 100 dollars per MWh. The cost obtained from the MAUC-MC with this case study is the same as the previous case study and the total revenue of sale over the contracted period is 6,000.00 dollars. Therefore, the total cost of exercising this case study is 1,625,175.00 dollars which is 17.78 dollars lower than that of case study 1. With this contract, the price range should be between 74.775 dollars per MWh and higher.

Case study 17 is performing the case study 1 with a call option to purchase power 100 MW at the rate of 24 dollars per MWh from an external source to supply subarea 5 between hours 18 and 22. The cost obtained from the MAUC-MC with this case study is 1,589,048.58 dollars and the cost of purchase over the contracted period is 36,000.00 dollars. Therefore, the total cost of exercising this case study is 1,625,048.58 dollars which is 144.20 dollars lower than that of case study 1. This indicates that the saving exists when it is cheaper to purchase power to supply the demand if the purchase price is low enough.

Case study 18 is performing the case study 1 with a designated reliability must run unit 35, Baker 1, for 100 MW at the rate of 40 dollars per MWh between hours 8 and 22. Unit 35 is an intermediate-load unit located in subarea 3. The cost obtained from the MAUC-MC with this case study is 1,661,483.00 dollars but is compensated by

the revenue over the RMR contracted period is 60,000.00 dollars. Therefore, the total cost of exercising this case study is 1,601,483.00 dollars which is 23,709.78 dollars lower than that of case study 1. The generator in the areas having reliability issues can gain profit from the RMR contracts although RMR units are not efficient or base-load units while the ISO has to pay the higher price to maintain the system reliability.

Case study 19 is performing the case study 1 with a call option to purchase power 50 MW at the rate of 50 dollars per MWh from generating unit 78, Comte3, located in subarea 6 to a location in subarea 5 between hours 18 and 22. The cost obtained from the MAUC-MC with this case study is 1,641,401.55 dollars which is greater than that of case study 1. This illustrates that when the option designates 50 MW from the unit 78, Comte 3, firmly transfer to subarea 5, the total production cost is higher although in the case study 1 subarea 5 imports power contributed from generating units having low marginal cost in subarea 6 between hours 18 and 22 more than 50 MW every hour as shown in tables C.4 and C.5 in appendix C.

Case study 20 is performing the case study 1 with a put option to sell power 100 MW at the rate of 28 dollars per MWh from unit 40, Barlow3, located in subarea 4, area 2 to a location in subarea 2, area 1 between hours 18 and 22. The total production cost of this case study is 1,630,213.02 dollars which is greater than that of case study 1.

Case study 21 is as case study 20 but tie line capacity constraints of flows between areas 1 and 2 in both directions are limited to 70 MW as specified in case study 11. The total production cost of this case study is 1,630,213.02 dollars. With the shift factors in table C.3, the flow from area 2 to area 1 contributed from the contracted

amount of power for 100 MW is equal to 66.67 MW. Therefore, when the tie line capacity constraint of flow from area 2 to area 1 is less than 66.67 MW, this contract encounters the transmission congestion problem.

Study case 12-21 are summarized as shown in table 5.18 and the cost comparison of case studies 1 and 12-21 is shown in table 5.19.

Table 5.18 Descriptions of Case Studies 12 -21 of Multi-area System

Case Study	Contract	Source	Sink	MW	Rate (\$/MWh)	Period Hour
12	Put option	Unit 23 in subarea 2	External system	50	35	2-16
		Unit 52 in subarea 4	External system	100	33	2-16
13	Forward contract	Unit 49 in subarea 4	External system	100	34	10-14
14	Forward contract	Unit 49 in subarea 4	External system	100	41	10-14
15	Put option	Unit 3 in subarea 1	External system	20	50	8-11
16	Put option	Unit 3 in subarea 1	External system	20	100	8-11
17	Call option	External system	Subarea 5	100	24	18-22
18	RMR	Unit 35 in subarea 3	Subarea 3	100	40	8-22
19	Call option	Unit 78 in subarea 6	Subarea 5	50	50	18-22
20	Put option	Unit 47 in subarea 4, area 2	Subarea 2, area 1	100	28	18-22
21	Put option with 70 MW limited tie line flow between areas 1 and 2 in both directions	Unit 40 in subarea 4, area 2	Subarea 2, area 1	100	28	18-22

Table 5.19 Cost Comparison of Multi-area UC and ED
with Market Operation Components

Case Study	UC and ED Cost Result (\$)	Cost of Purchase (\$)	Revenue of Sale or RMR(\$)	Total Cost (\$)
1	1,625,192.78	0	0	1,625,192.78
12	1,695,997.10	0	-75,750.00	1,620,247.10
13	1,645,511.00	0	-17,000.00	1,628,511.00
14	1,645,511.00	0	-20,500.00	1,625,011.00
15	1,631,175.00	0	-3,000.00	1,628,175.00
16	1,631,175.00	0	-6,000.00	1,625,175.00
17	1,589,048.58	36,000.00	0	1,625,048.58
18	1,661,483.00	0	-60,000.00	1,601,483.00
19	1,641,401.55	12,500.00	-12,500.00	1,641,401.55
20	1,630,213.02	14,000.00	-14,000.00	1,630,213.02
21	1,630,213.02	14,000.00	-14,000.00	1,630,213.02

Case studies 12-21 illustrate that the feature of market operation components in this dissertation can enable the power producers and LSEs to establish a firm transaction to hedge against the price volatility in the spot market although some transactions may incur more total production cost. Integrating the market operation components into the multi-area UC and ED creates the flexibility in planning the operation and market strategies. In addition, the profit can be maximized through exercising the market operation components. For example, multiple bilateral contracts can be exercised in order to buy power at low prices and sell it at high prices.

CHAPTER 6

CONCLUSION AND RECOMMENDATION FOR FUTURE STUDIES

The accomplishment of this dissertation is summarized in the conclusion section. In the following section, future studies are recommended.

6.1 Conclusion

The multi-area unit commitment and economic dispatch with market operation components implemented with adaptive Lagrangian relaxation method, unit decommitment method, and lambda-iteration method yields the short-term unit scheduling result in the responsive seconds. In addition to constraints that conventional UC and ED programs consider, this dissertation includes the unit ramp rate constraints, the area import/export constraints, tie line capacity constraints, and requirements of market operation components in the formulations of the UC and the ED.

According to the simulation results, the computational time of the proposed algorithm appears to be relatively low in comparison with the computational times reported in the references. However, the times measured can not be directly comparable due to different processors used in this dissertation and the references. In addition, the computational times of the proposed algorithm linearly increase with respect to the increase in the number of generating units. Therefore, the proposed algorithm is suitable for large-scale systems.

The total production costs of the single-area systems computed by the proposed algorithm are optimal when compared to those presented in the references. The simulation results of multi-area UC and ED illustrates that the limited import/export capabilities and tie line capacity constraints of multiple areas have effects on the unit scheduling and the increase in the total production cost.

The Lagrange multipliers used to solve the unit commitment problem can represent the marginal cost of energy (λ), the marginal cost of system capacity (μ), the marginal cost of area maximum capacity with consideration of area import capability (α_k), the marginal cost of area minimum capacity with consideration of area export capability (β_k), and the marginal cost of power flows on a tie line (θ_f). The UC solution is significantly based on the values of λ and μ which are high when the system demand is peak. The value of α_k , β_k , or θ_f is greater than zero when the interchanges or tie line power flow reach the area import or export constraint or tie line capacity constraint.

With options and forward contracts, the generators and LSEs can hedge against the risk of price volatility and may be able to increase their profit. With reliability must-run contracts initiated by the ISOs, the reliability of local area is maintained although the RMR units are not economic resources. In addition, market operation components help to ensure that the system has resource adequacy. The multi-area UC and ED with market operation components, therefore, is a robust tool to provide the needs of the

competitive electricity market including the market economics, the electricity derivatives through bilateral contracts, and the system reliability.

The multi-area UC and ED with market operation components is developed with an object-oriented programming (OOP) language that supports the user-friendly graphical user interface, complicated calculation, and convenient maintenance. These bring benefits for large-scale systems and systems that require frequent alteration of the input data. Since restructuring power industry in some regions is still in an ongoing process, the operation protocols related to the market operation components of ISOs such as SPP and MISO are regularly modified. Therefore, the OOP that allows developers the flexibility to easily modify the program structure is suitable for the multi-area UC and ED with market operation components.

6.2 Recommendation for Future Studies

The proposed multi-area UC and ED can be incorporated with security-constrained optimal power flow, a combination of a contingency analysis and an optimal power flow, for security-constrained unit commitment (SCUC) and security-constrained economic dispatch (SCED) [63-66]. In addition, the locational marginal prices (LMPs) can be obtained from the nodal and zonal marginal prices of the security-constrained economic dispatch result. The integration of SCUC, SCED, and bilateral contracts under different ISOs can be further studied in [67-71].

Intimately associated to the LMPs and transmission congestion, congestion revenue rights (CRRs), also known as financial transmission rights (FTRs), are financial instruments in hedging against the transmission congestion price volatility [72]. The

implementation of CRRs/FTRs and bilateral contracts enable energy market participants to manage their market strategies for profit maximization.

In addition to, extra models for combined cycle units and the renewable resources such as hydro-thermal units, wind farms can be added into the proposed multi-area UC and ED to handle the flexible resources [73-77].

APPENDIX A
CLASS DIAGRAMS

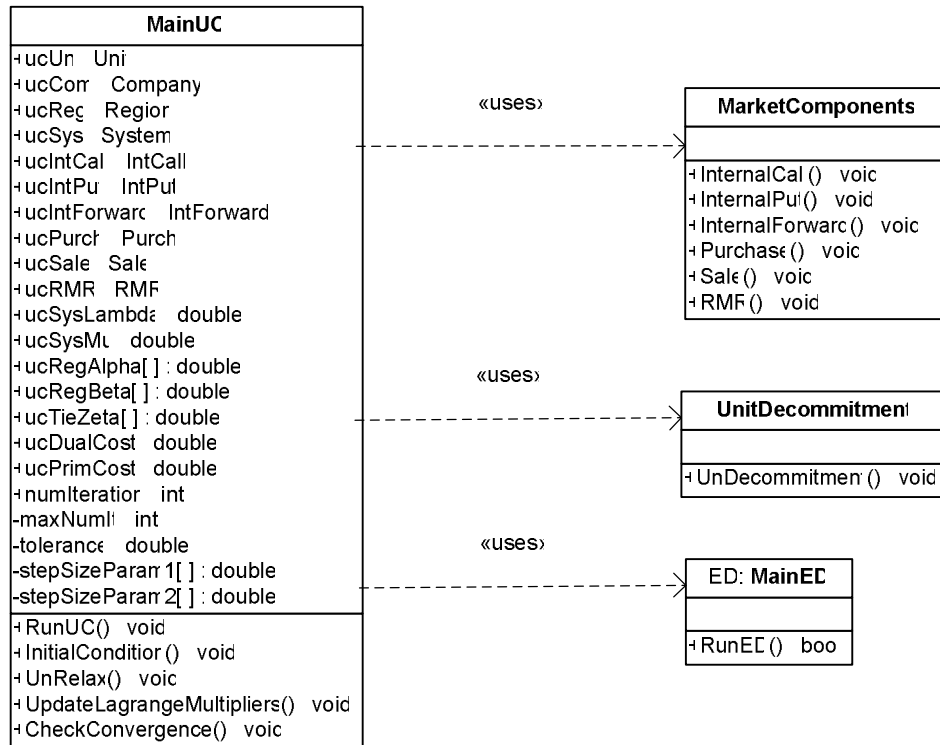


Figure A.1 Class Diagram of Multi-area Unit Commitment and Economic Dispatch with Market Operation Components

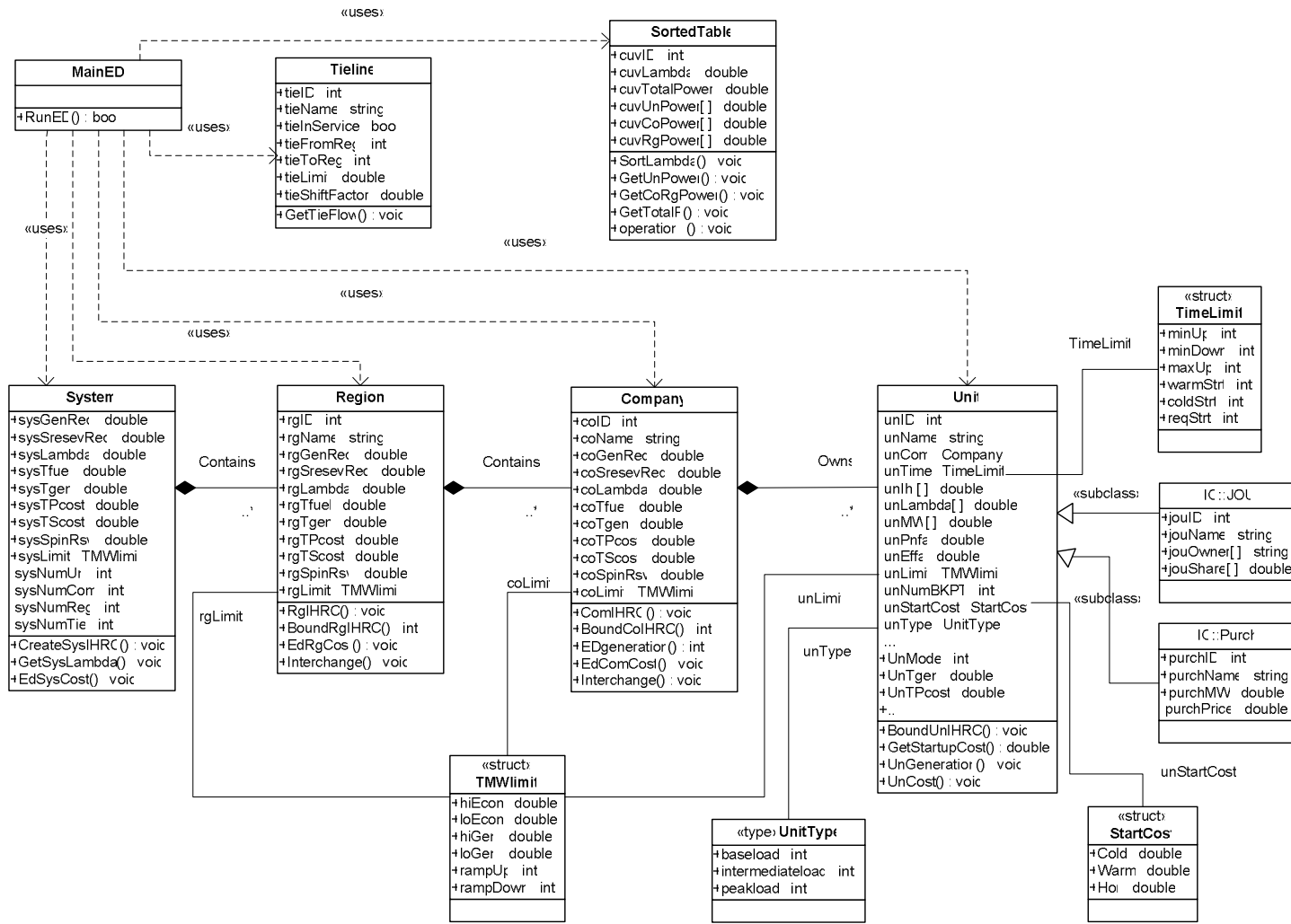


Figure A.2 Class Diagram of Multi-area Economic Dispatch

APPENDIX B

SINGLE-AREA SYSTEM

Table B.1 Characteristics of 10 Generating Units in a Single-area System

Unit ID	Unit Name	Econ High (MW)	Econ Low (MW)	Dep. Capacity (MW)	O&M Cost (\$/MW)	Min up Time (h)	Min down Time (h)	Ramp Up & Down (MW/s)	Cold Start		Hot Start		Req. Start Time (h)	Initial Time (h)	Initial Output (MW)
									Time (h)	Cost (\$)	Time (h)	Cost (\$)			
1	Unit1	455	150	455	0	8	8	8	5	9000	0	4500	0	8	320
2	Unit2	455	150	455	0	8	8	8	5	10000	0	5000	0	8	320
3	Unit3	130	20	130	0	5	5	5	4	1100	0	550	0	-5	0
4	Unit4	130	20	130	0	5	5	5	4	1120	0	560	0	-5	0
5	Unit5	162	25	162	0	6	6	5	4	1800	0	900	0	-6	0
6	Unit6	80	20	80	0	3	3	5	2	340	0	170	0	-3	0
7	Unit7	85	25	85	0	3	3	5	2	520	0	260	0	-3	0
8	Unit8	55	10	55	0	1	1	5	0	60	0	30	0	-1	0
9	Unit9	55	10	55	0	1	1	5	0	60	0	30	0	-1	0
10	Unit10	55	10	55	0	1	1	5	0	60	0	30	0	-1	0

Table B.2 Unit Types, Fuel Prices, and Cost Characteristics of 10 Generating Units in a Single-area System

Unit Name	Unit Type	Fuel Price (\$/MBtu)	a (MBtu/h)	b (MBtu/MWh)	c (MBtu/MW ² h)
Unit1	Base-load	1	1000	16.19	0.00048
Unit2	Base-load	1	970	17.26	0.00031
Unit3	Intermediate-load	5	140	3.32	0.0004
Unit4	Intermediate-load	5	136	3.3	0.000422
Unit5	Intermediate-load	5	90	3.94	0.000796
Unit6	Intermediate-load	5	74	4.452	0.001424
Unit7	Intermediate-load	5	96	3.94	0.000158
Unit8	Peak-load	5	132	5.184	0.000826
Unit9	Peak-load	5	133	5.454	0.000444
Unit10	Peak-load	5	134	5.558	0.000346

Table B.3 Generation Output, Fuel Cost, and Start-up Cost of Units 1-5 in Ten-unit System for 24 Hours

Hour	Unit														
	1			2			3			4			5		
	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost
1	455	8465.8	0	245	5217.3	0	0	0	0	0	0.0	0	0	0.0	0
2	455	8465.8	0	295	6088.7	0	0	0	0	0	0.0	0	0	0.0	0
3	455	8465.8	0	370	7398.6	0	0	0	0	0	0.0	0	25	945.0	1800
4	455	8465.8	0	455	8887.5	0	0	0	0	0	0.0	0	40	1244.4	0
5	455	8465.8	0	390	7748.6	0	0	0	0	130	2860.7	1120	25	945.0	0
6	455	8465.8	0	360	7223.8	0	130	2891.8	1100	130	2860.7	0	25	945.0	0
7	455	8465.8	0	410	8098.7	0	130	2891.8	0	130	2860.7	0	25	945.0	0
8	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	30	1044.6	0
9	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	25	945.0	0
10	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
11	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	155	3599.1	0
12	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	162	3745.9	0
13	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
14	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	25	945.0	0
15	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	30	1044.6	0
16	455	8465.8	0	310	6350.4	0	130	2891.8	0	130	2860.7	0	25	945.0	0
17	455	8465.8	0	260	5478.6	0	130	2891.8	0	130	2860.7	0	25	945.0	0
18	455	8465.8	0	360	7223.8	0	130	2891.8	0	130	2860.7	0	25	945.0	0
19	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	30	1044.6	0
20	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
21	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	25	945.0	0
22	455	8465.8	0	455	8887.5	0	0	0	0	0	0	0	85	2153.3	0
23	455	8465.8	0	420	8273.9	0	0	0	0	0	0	0	25	945.0	0
24	455	8465.8	0	345	6961.6	0	0	0	0	0	0	0	0	0	0

Table B.4 Generation Output, Fuel Cost, and Start-up Cost of Units 6-10 in Ten-unit System for 24 Hours

Hour	Unit														
	6			7			8			9			10		
	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	20	818.0	340	85	2160.2	520	0	0	0	0	0	0	0	0	0
10	20	818.0	0	85	2160.2	0	10	919.6	60	0	0	0	0	0	0
11	20	818.0	0	85	2160.2	0	10	919.6	0	10	937.9	60	0	0	0
12	53	1569.8	0	85	2160.2	0	10	919.6	0	10	937.9	0	10	948.1	60
13	20	818.0	0	85	2160.2	0	10	919.6	0	0	0	0	0	0	0
14	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	20	818.05	340	85	2160.2	520	10	919.6	60	0	0	0	0	0	0
21	20	818.05	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
22	20	818.05	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table B.5 Hourly System Demand, System λ , System μ , Fuel Cost, and Start-up Cost of a Single-area System with Ten Generating Units

Hour	Demand (MW)	λ (\$/MWh)	μ (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	700	17.412	2.734	13683.13	0
2	750	17.443	3.359	14554.50	0
3	850	17.862	4.962	16809.45	1800.00
4	950	20.034	3.796	18597.67	0
5	1000	19.368	4.646	20020.02	1120.00
6	1100	19.118	4.839	22387.04	1100.00
7	1150	19.539	5.695	23261.98	0
8	1200	20.000	6.486	24150.34	0
9	1300	21.473	6.282	27029.00	860.00
10	1400	23.672	13.523	29771.76	60.00
11	1450	24.367	13.718	31540.67	60.00
12	1500	27.043	13.476	33387.21	60.00
13	1400	23.544	12.449	29771.76	0
14	1300	20.975	5.089	27029.00	0
15	1200	19.557	5.328	24150.34	0
16	1050	19.044	4.333	21513.66	0
17	1000	18.555	3.692	20641.82	0
18	1100	19.654	6.611	22387.04	0
19	1200	20.289	6.559	24150.34	0
20	1400	23.672	13.523	29771.76	920.00
21	1300	20.975	5.089	27029.00	0
22	1100	18.452	3.741	22484.81	0
23	900	17.556	3.241	17684.69	0
24	800	17.474	3.017	15427.42	0

Table B.6 Generation Output, Fuel Cost, and Start-up Cost of Units 1-5 in Twenty-unit System for 24 Hours

Hour	Unit														
	1			2			3			4			5		
	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost
1	455	8465.8	0	245	5217.3	0	0	0	0	0	0	0	0	0	0
2	455	8465.8	0	295	6088.7	0	0	0	0	0	0	0	0	0	0
3	455	8465.8	0	382.5	7617.3	0	0	0	0	0	0	0	0	0	0
4	455	8465.8	0	417.5	8230.1	0	0	0	0	0	0	0	0	0	0
5	455	8465.8	0	402.5	7967.4	0	0	0	0	130	2860.7	1120	0	0	0
6	455	8465.8	0	425	8361.5	0	0	0	0	130	2860.7	0	25	945.0	1800
7	455	8465.8	0	455	8887.5	0	0	0	0	130	2860.7	0	45	1344.6	0
8	455	8465.8	0	455	8887.5	0	130	2891.8	1100	130	2860.7	0	30	1044.6	0
9	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	67.5	1797.9	0
10	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
11	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	155	3599.1	0
12	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	162	3745.9	0
13	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
14	455	8465.8	0	455	8887.5	0	0	0	0	130	2860.7	0	90	2255.2	0
15	455	8465.8	0	455	8887.5	0	0	0	0	130	2860.7	0	75	1949.9	0
16	455	8465.8	0	375	7486.1	0	0	0	0	130	2860.7	0	25	945.0	0
17	455	8465.8	0	325	6612.2	0	0	0	0	130	2860.7	0	25	945.0	0
18	455	8465.8	0	425	8361.5	0	0	0	0	130	2860.7	0	25	945.0	0
19	455	8465.8	0	455	8887.5	0	130	2891.8	1100	130	2860.7	0	30	1044.6	0
20	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
21	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	90	2255.2	0
22	455	8465.8	0	455	8887.5	0	130	2891.8	0	0	0	0	0	0	0
23	455	8465.8	0	367.5	7354.9	0	130	2891.8	0	0	0	0	0	0	0
24	455	8465.8	0	345	6961.6	0	0	0	0	0	0	0	0	0	0

Table B.7 Generation Output, Fuel Cost, and Start-up Cost of Units 6-10 in Twenty-unit System for 24 Hours

Hour	Unit														
	6			7			8			9			10		
	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	20	818.0	340	0	0	0	0	0	0	0	0	0	0	0	0
10	20	818.0	0	85	2160.2	520	10	919.6	60	0	0	0	0	0	0
11	20	818.0	0	85	2160.2	0	10	919.6	0	10	937.9	60	0	0	0
12	53	1569.8	0	85	2160.2	0	10	919.6	0	10	937.9	0	10	948.1	60
13	20	818.0	0	85	2160.2	0	10	919.6	0	0	0	0	0	0	0
14	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
15	20	818.0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	20	818.0	340	85	2160.2	520	10	919.6	60	0	0	0	0	0	0
21	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
22	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table B.8 Generation Output, Fuel Cost, and Start-up Cost of Units 11-15 in Twenty-unit System for 24 Hours

Hour	Unit														
	1			2			3			4			5		
	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost
1	455	8465.8	0	245	5217.3	0	0	0	0	0	0	0	0	0	0
2	455	8465.8	0	295	6088.7	0	0	0	0	0	0	0	0	0	0
3	455	8465.8	0	382.5	7617.3	0	0	0	0	0	0	0	25	945.0	1800
4	455	8465.8	0	417.5	8230.1	0	0	0	0	130	2860.7	1120	25	945.0	0
5	455	8465.8	0	402.5	7967.4	0	0	0	0	130	2860.7	0	25	945.0	0
6	455	8465.8	0	425	8361.5	0	130	2891.8	1100	130	2860.7	0	25	945.0	0
7	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	45	1344.6	0
8	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	30	1044.6	0
9	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	67.5	1797.9	0
10	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
11	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	155	3599.1	0
12	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	162	3745.9	0
13	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
14	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	90	2255.2	0
15	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	75	1949.9	0
16	455	8465.8	0	375	7486.1	0	130	2891.8	0	130	2860.7	0	25	945.0	0
17	455	8465.8	0	325	6612.2	0	130	2891.8	0	130	2860.7	0	25	945.0	0
18	455	8465.8	0	425	8361.5	0	130	2891.8	0	130	2860.7	0	25	945.0	0
19	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	30	1044.6	0
20	455	8465.8	0	455	8887.5	0	130	2891.8	0	130	2860.7	0	115	2768.1	0
21	455	8465.8	0	455	8887.5	0	0	0	0	130	2860.7	0	90	2255.2	0
22	455	8465.8	0	455	8887.5	0	0	0	0	0	0	0	40	1244.4	0
23	455	8465.8	0	367.5	7354.9	0	0	0	0	0	0	0	25	945.0	0
24	455	8465.8	0	345	6961.6	0	0	0	0	0	0	0	0	0	0

Table B.9 Generation Output, Fuel Cost, and Start-up Cost of Units 16-20 in Twenty-unit System for 24 Hours

Hour	Unit														
	6			7			8			9			10		
	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost	Output MW	Fuel Cost	Start-up Cost
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	20	818.0	340	85	2160.2	520	0	0	0	0	0	0	0	0	0
10	20	818.0	0	85	2160.2	0	10	919.6	60	0	0	0	0	0	0
11	20	818.0	0	85	2160.2	0	10	919.6	0	10	937.9	60	0	0	0
12	53	1569.8	0	85	2160.2	0	10	919.6	0	10	937.9	0	10	948.1	60
13	20	818.0	0	85	2160.2	0	10	919.6	0	0	0	0	0	0	0
14	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
15	20	818.0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	20	818.0	340	85	2160.2	520	10	919.6	60	0	0	0	0	0	0
21	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
22	20	818.0	0	85	2160.2	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table B.10 Hourly System Demand, System λ , System μ , Fuel Cost, and Start-up Cost of a Single-area System with Twenty Generating Units

Hour	Demand (MW)	λ (\$/MWh)	μ (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	1400	17.412	2.235	27366.26	0
2	1500	17.443	3.087	29109.00	0
3	1700	17.743	4.973	33111.24	1800.00
4	1900	19.522	4.207	37197.46	1120.00
5	2000	19.288	4.643	39532.69	1120.00
6	2200	19.528	4.675	44157.72	2900.00
7	2300	20.015	5.466	46008.84	0
8	2400	19.993	6.382	48300.68	1100.00
9	2600	21.579	6.258	53603.59	1200.00
10	2800	23.247	13.598	59543.53	640.00
11	2900	24.097	13.782	63081.34	120.00
12	3000	26.836	14.863	66774.41	120.00
13	2800	23.173	12.705	59543.53	0
14	2600	20.799	6.018	53786.71	0
15	2400	19.924	5.487	48855.59	0
16	2100	19.453	4.447	42406.92	0
17	2000	18.738	3.645	40659.22	0
18	2200	20.039	7.008	44157.72	0
19	2400	20.333	6.553	48300.68	1100.00
20	2800	23.247	13.598	59543.53	1840.00
21	2600	20.527	4.861	53786.71	0
22	2200	18.894	3.668	44799.28	0
23	1800	17.495	3.152	35478.27	0
24	1600	17.441	2.784	30854.84	0

APPENDIX C

MULTI-AREA SYSTEM

Table C.1 Generating Unit Characteristics of A Multi-area System

Unit ID	Unit Name	Com ID	Econ High (MW)	Econ Low (MW)	Dep. Cap. (MW)	O&M Cost (\$/MW)	Min up Time (h)	Min down Time (h)	Ramp Up & Down (MW/s)	Cold Start		Hot Start		Req. Start Time (h)	Initial Time (h)	Initial Output (MW)
										Time (h)	Cost (\$)	Time (h)	Cost (\$)			
1	Abel1	1	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-10	0
2	Abel2	1	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-10	0
3	Abel3	1	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	30	69
4	Abel4	1	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	30	69
5	Adams1	1	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-12	0
6	Adams2	1	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-12	0
7	Adams3	1	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	45	60
8	Adams4	1	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	45	60
9	Alder1	1	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-4	0
10	Alder2	1	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-4	0
11	Alder3	1	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-4	0
12	Ame1	2	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	25	100
13	Ame2	2	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	25	100
14	Ame3	2	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	25	100
15	Arthur1	2	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
16	Arthur2	2	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
17	Arthur3	2	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
18	Arthur4	2	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
19	Arthur5	2	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
20	Arthur6	2	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	10	120
21	Asser	2	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	10	120
22	Astor	2	400	100	400	0.8	48	48	20	24	8000	24	8000	24	120	380
23	Attlee	2	400	100	400	0.8	48	48	20	24	8000	24	8000	24	120	380
24	Austen1	2	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	45	124
25	Austen2	2	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	45	124
26	Austen3	2	350	140	350	0.8	24	48	4	13	5361.6	9	3829.8	1	50	320
27	Bach1	3	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-30	0

Table C.1 - *Continued*

Unit ID	Unit Name	Com ID	Econ High (MW)	Econ Low (MW)	Dep. Cap. (MW)	O&M Cost (\$/MW)	Min up Time (h)	Min down Time (h)	Ramp Up & Down (MW/s)	Cold Start		Hot Start		Req. Start Time (h)	Initial Time (h)	Initial Output (MW)
										Time (h)	Cost (\$)	Time (h)	Cost (\$)			
28	Bach2	3	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-30	0
29	Bach3	3	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	10	62
30	Bach4	3	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	10	62
31	Bacon1	3	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-29	0
32	Bacon2	3	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-29	0
33	Bacon3	3	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	15	70
34	Bacon4	3	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	15	70
35	Baker1	3	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-6	0
36	Baker2	3	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-6	0
37	Baker3	3	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-6	0
38	Barlow1	4	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	-5	0
39	Barlow2	4	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	-5	0
40	Barlow3	4	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	-5	0
41	Barton1	4	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
42	Barton2	4	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
43	Barton3	4	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
44	Barton4	4	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
45	Barton5	4	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-10	0
46	Barton6	4	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	32	138
47	Basov	4	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	32	138
48	Bayle	4	400	100	400	0.8	48	48	20	24	8000	24	8000	24	120	380
49	Behring	4	400	100	400	0.8	48	48	20	24	8000	24	8000	24	120	380
50	Bloch1	4	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	40	135
51	Bloch2	4	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	40	135
52	Bloch3	4	350	140	350	0.8	24	48	4	13	5361.6	9	3829.8	1	60	300
53	Cabeli1	5	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-40	0
54	Cabeli2	5	20	15.8	20	0.8	1	1	3	1	27.5	0	27.5	1	-40	0

Table C.1 - *Continued*

Unit ID	Unit Name	Com ID	Econ High (MW)	Econ Low (MW)	Dep. Cap. (MW)	O&M Cost (\$/MW)	Min up Time (h)	Min down Time (h)	Ramp Up & Down (MW/s)	Cold Start		Hot Start		Req. Start Time (h)	Initial Time (h)	Initial Output (MW)
										Time (h)	Cost (\$)	Time (h)	Cost (\$)			
55	Cabeli3	5	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	-38	70
56	Cabeli4	5	76	15.2	76	0.8	8	4	2	11	715.2	4	715.2	1	-38	70
57	Cabot1	5	20	15.8	20	0.8	1	1	3	1	27.5	0	209	1	-43	0
58	Cabot2	5	20	15.8	20	0.8	1	1	3	1	27.5	0	209	1	-43	0
59	Cabot3	5	76	15.2	76	0.8	8	4	2	13	715.2	4	715.2	1	30	72
60	Cabot4	5	76	15.2	76	0.8	8	4	2	13	715.2	4	715.2	1	30	72
61	Carew1	5	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-26	0
62	Carew2	5	100	25	100	0.8	8	8	7	8	3113	3	2244	1	-26	0
63	Carew3	5	100	25	100	0.8	8	8	7	8	3113	3	2244	1	30	80
64	Cecil1	6	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	15	180
65	Cecil2	6	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	-30	0
66	Cecil3	6	197	68.95	197	0.8	12	10	3	8	4262.5	5	3349.5	1	-30	0
67	Chase1	6	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-30	0
68	Chase2	6	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-30	0
69	Chase3	6	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-30	0
70	Chase4	6	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-30	0
71	Chase5	6	12	2.4	12	0.8	4	2	1	5	374	3	291.5	1	-30	0
72	Chase6	6	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	30	132
73	Chifa	6	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	30	132
74	Clark	6	400	100	400	0.8	48	48	20	0	8000	0	8000	1	120	380
75	Cobb	6	400	100	400	0.8	48	48	20	0	8000	0	8000	1	120	380
76	Comte1	6	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	49	140
77	Comte2	6	155	54.25	155	0.8	8	8	3	12	1143.6	4	727.8	1	49	140
78	Comte3	6	350	140	350	0.8	24	48	4	13	5361.6	9	3829.8	1	70	315

Table C.2 Generating Unit Types, Fuel Prices, and Piecewise Linear Incremental Heat Rate Characteristics of A Multi-area System

Unit Name	Unit Type	Fuel Price (\$/MBtu)	Min Fuel (MBtu)	Break-point	Power (MW)	IHR (MBtu/MWh)
Arthur1, Arthur2, Arthur3, Arthur4, Arthur5, Barton1, Barton2, Barton3, Barton4, Barton5, Chase1, Chase2, Chase3, Chase4, Chase5	Intermediate-load (Oil/Steam)	5.5	38.4408	1	2.4	10.179
				2	6	10.33
				3	9.6	11.668
				4	12	13.219
Abel1, Abel2, Adams1, Adams2, Bach1, Bach2, Bacon1, Bacon2, Cabeli1, Cabeli2, Cabot1, Cabot2	Peak load (Oil/CT)	5.5	237.9954	1	15.8	9.859
				2	16	10.139
				3	19.8	14.272
				4	20	14.427
Abel3, Abel4, Adams3, Adams4, Bach3, Bach 4, Bacon3, Bacon4, Cabeli3, Cabeli4, Cabot3, Cabot4	Base-load (Coal/Steam)	1.2	260.0264	1	15.2	9.548
				2	38	9.966
				3	60.8	11.576
				4	76	13.311
Alder1, Alder2, Alder3, Baker1, Baker2, Baker3, Carew1, Carew2, Carew3	Intermediate-load (Oil/Steam)	5.5	324.975	1	25	8.089
				2	50	8.708
				3	80	9.42
				4	100	9.877
Arthur6, Asser, Austen1, Austen2, Barton6, Basov, Bloch1, Bloch2, Chase6, Chifa, Comte1, Comte2	Base-load (Coal/Steam)	1.2	609.987	1	54.25	8.265
				2	93	8.541
				3	124	8.9
				4	155	9.381
Ame1, Ame2, Ame3, Barlow1, Barlow2, Barlow3, Cecil1, Cecil2, Cecil3	Intermediate-load (Oil/Steam)	5.5	741.2125	1	68.95	8.348
				2	118.2	8.833
				3	157.6	9.225
				4	197	9.62
Austen3, Bloch3, Comte3	Base-load (Coal/Steam)	1.2	1428	1	140	8.402
				2	227.5	8.896
				3	280	9.244
				4	350	9.768
Astor, Attlee, Bayle, Behring, Clark, Cobb	Base-load (Nuclear)	0.5	1275.1	1	100	8.848
				2	200	8.965
				3	320	9.21
				4	400	9.438

Table C.3 Shift Factors of Tie Line When Area 1 is The Reference

Tie Name	From Area	To Area	Shift Factor Area 1	Shift Factor Area 2	Shift Factor Area 3
Tie1	1	2	0	-0.66667	-0.33333
Tie2	2	3	0	0.333333	-0.33333
Tie3	3	1	0	0.333333	0.66667
Tie1	2	1	0	0.66667	0.333333
Tie2	3	2	0	-0.33333	0.333333
Tie3	1	3	0	-0.33333	-0.66667

Table C.4 Generation Output and Interchange of Subareas in Multi-area System with Unlimited Import/Export Capability Constraints and Unlimited Tie Line Capacity Constraints (Case Study 1)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	60.80	-369.20	1683.20	394.20	60.80	-439.20	1683.20	479.20	60.80	-489.20	1683.20	424.20
9	83.08	-365.92	1754.25	404.25	83.08	-435.92	1754.25	519.25	83.08	-485.92	1754.25	364.25
10	147.68	-310.32	1770.00	400.00	147.68	-380.32	1770.00	381.00	147.68	-430.32	1770.00	271.00
11	209.68	-289.32	1770.00	332.00	234.68	-295.32	1770.00	291.00	259.68	-320.32	1838.95	281.95
12	281.68	-278.32	1770.00	281.00	356.68	-193.32	1770.00	215.00	356.68	-243.32	1838.95	218.95
13	210.68	-337.32	1770.00	296.00	285.68	-266.32	1770.00	350.00	285.68	-316.32	1838.95	273.95
14	179.68	-357.32	1770.00	258.00	254.68	-305.32	1770.00	373.00	254.68	-355.32	1838.95	386.95
15	181.68	-346.32	1770.00	223.00	256.68	-313.32	1770.00	384.00	256.68	-363.32	1838.95	415.95
16	228.68	-288.32	1770.00	181.00	303.68	-283.32	1770.00	425.00	303.68	-333.32	1838.95	298.95
17	304.00	-226.00	1770.00	128.00	404.58	-195.42	1770.00	291.00	404.58	-245.42	1976.85	247.85
18	402.22	-155.78	1770.00	87.00	451.34	-176.66	1873.28	288.28	451.34	-226.66	2079.83	183.83
19	413.52	-151.48	1770.00	70.00	468.28	-166.72	2003.96	229.96	468.28	-216.72	2120.95	234.95
20	405.83	-151.17	1770.00	-19.00	405.83	-221.17	1985.43	244.43	456.75	-220.25	2093.15	367.15
21	390.66	-153.34	1770.00	70.00	304.00	-310.00	1947.47	259.47	390.66	-273.34	2036.20	407.20
22	328.73	-170.27	1770.00	92.00	278.73	-290.27	1907.90	287.90	278.73	-340.27	1907.90	420.90
23	234.73	-237.27	1770.00	183.00	184.73	-357.27	1907.90	342.90	184.73	-407.27	1907.90	475.90
24	110.80	-249.20	1696.93	207.93	60.80	-369.20	1834.83	360.83	60.80	-419.20	1834.83	468.83

Table C.5 Tie Line Flow and Area Net Interchange of Multi-area System with Unlimited Import/Export Capability Constraints and Unlimited Tie Line Capacity Constraints (Case Study 1)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	-5.00		35.00		-30.00		5.00		-35.00		30.00		25.00	40.00	-65.00
9	-15.00		68.33		-53.33		15.00		-68.33		53.33		38.33	83.33	-121.67
10	29.67		30.35		-60.02		-29.67		-30.35		60.02		89.68	0.68	-90.37
11	15.67		11.35		-27.02		-15.67		-11.35		27.02		42.68	-4.32	-38.37
12	-6.33		15.35		-9.02		6.33		-15.35		9.02		2.68	21.68	-24.37
13	-41.67		42.02		-0.35		41.67		-42.02		0.35		-41.32	83.68	-42.37
14	-55.67		12.02		43.65		55.67		-12.02		-43.65		-99.32	67.68	31.63
15	-64.67		6.02		58.65		64.67		-6.02		-58.65		-123.32	70.68	52.63
16	-83.00		58.68		24.32		83.00		-58.68		-24.32		-107.32	141.68	-34.37
17	-64.52		31.05		33.47		64.52		-31.05		-33.47		-98.00	95.58	2.42
18	-60.13		51.48		8.65		60.13		-51.48		-8.65		-68.78	111.61	-42.84
19	-48.24		15.01		33.24		48.24		-15.01		-33.24		-81.48	63.25	18.23
20	-64.48		-41.21		105.69		64.48		41.21		-105.69		-170.17	23.27	146.90
21	-10.94		-61.47		72.40		10.94		61.47		-72.40		-83.34	-50.53	133.87
22	-25.30		-27.67		52.97		25.30		27.67		-52.97		-78.27	-2.37	80.63
23	-13.30		-27.67		40.97		13.30		27.67		-40.97		-54.27	-14.37	68.63
24	-10.97		-19.33		30.30		10.97		19.33		-30.30		-41.27	-8.37	49.63

Table C.6 Hourly System Demand, System λ , System μ , Fuel Cost, and Start-up Cost of Multi-area System with Unlimited Import/Export Capability Constraints and Unlimited Tie Line Capacity Constraints (Case Study 1)

Hour	Demand (MW)	λ (\$/MWh)	μ (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	4652	10.63396	1.688133	40495.59	0
2	4167	8.175041	2.976198	34987.65	0
3	3456	6.233298	2.690917	27288.36	0
4	3186	7.723115	2.976259	25807.85	0
5	2977	6.99125	2.933588	24673.71	0
6	3677	9.531487	5.431916	31368.65	8582.4
7	4499	13.11032	10.95131	40411.56	0
8	5232	16.54423	11.62798	48912.86	0
9	5512	18.24676	13.16628	52318.12	0
10	5822	24.43436	15.65255	59416.79	4262.5
11	6083	23.83471	16.74651	67257.67	9339
12	6374	23.37847	17.1182	75833.43	9339
13	6161	19.87546	13.38978	72661.73	0
14	6068	17.39254	11.07712	71390.05	0
15	6074	17.43743	11.13159	71470.25	0
16	6215	18.49227	12.41041	73428.16	0
17	6630	24.66302	15.18225	87430.4	8525
18	7028	30.24544	15.31683	108330.1	10488.5
19	7245	33.04993	13.06441	119608.4	4262.5
20	7117	28.50456	12.79674	112819.5	0
21	6839	22.72439	12.22238	97749.33	0
22	6472	18.51296	9.759643	80772.75	0
23	6190	15.41402	7.693855	76673.6	0
24	5599	11.99069	4.233679	69287.45	0

Table C.7 Generation Output and Interchange of Subareas in Multi-area System
with 250 MW Import Capability of Subarea 1 (Case Study 2)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	180.00	-250.00	1643.47	354.47	60.80	-439.20	1643.47	439.47	60.80	-489.20	1643.47	384.47
9	199.00	-250.00	1730.47	380.47	60.80	-458.20	1730.47	495.47	60.80	-508.20	1730.47	340.47
10	208.00	-250.00	1770.00	400.00	177.00	-351.00	1770.00	381.00	127.00	-451.00	1770.00	271.00
11	249.00	-250.00	1770.00	332.00	287.00	-243.00	1770.00	291.00	237.00	-343.00	1770.00	213.00
12	310.00	-250.00	1770.00	281.00	333.05	-216.95	1770.00	215.00	283.05	-316.95	1907.90	287.90
13	298.00	-250.00	1770.00	296.00	232.55	-319.45	1770.00	350.00	182.55	-419.45	1907.90	342.90
14	287.00	-250.00	1770.00	258.00	191.55	-368.45	1770.00	373.00	141.55	-468.45	1907.90	455.90
15	278.00	-250.00	1770.00	223.00	199.05	-370.95	1770.00	384.00	149.05	-470.95	1907.90	484.90
16	267.00	-250.00	1770.00	181.00	275.05	-311.95	1770.00	425.00	225.05	-411.95	1907.90	367.90
17	341.36	-188.64	1770.00	128.00	416.09	-183.91	1770.00	291.00	341.36	-308.64	1991.19	262.19
18	352.58	-205.42	1770.00	87.00	401.17	-226.83	2075.83	490.83	352.58	-325.42	2075.83	179.83
19	358.05	-206.95	1885.26	185.26	412.09	-222.91	2115.78	341.78	358.05	-326.95	2115.78	229.78
20	354.32	-202.68	1876.29	87.29	354.32	-272.68	2088.87	347.87	354.32	-322.68	2088.87	362.87
21	344.78	-199.22	1852.32	152.32	304.00	-310.00	2016.95	328.95	304.00	-360.00	2016.95	387.95
22	249.00	-250.00	1838.95	160.95	215.18	-353.83	1976.85	356.85	215.18	-403.83	1976.85	489.85
23	222.00	-250.00	1832.21	245.21	97.79	-444.21	1970.11	405.11	97.79	-494.21	1970.11	538.11
24	110.00	-250.00	1743.17	254.17	60.80	-369.20	1881.07	407.07	60.80	-419.20	1743.17	377.17

Table C.8 Tie Line Flow and Area Net Interchange of Multi-area System
with 250 MW Import Capability of Subarea 1 (Case Study 2)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	34.73		35.00		-69.73		-34.73		-35.00		69.73		104.47	0.27	-104.73
9	31.07		68.33		-99.40		-31.07		-68.33		99.40		130.47	37.27	-167.73
10	40.00		70.00		-110.00		-40.00		-70.00		110.00		150.00	30.00	-180.00
11	11.33		59.33		-70.67		-11.33		-59.33		70.67		82.00	48.00	-130.00
12	10.98		9.03		-20.02		-10.98		-9.03		20.02		31.00	-1.95	-29.05
13	5.15		35.70		-40.85		-5.15		-35.70		40.85		46.00	30.55	-76.55
14	1.15		5.70		-6.85		-1.15		-5.70		6.85		8.00	4.55	-12.55
15	-13.35		-0.30		13.65		13.35		0.30		-13.65		-27.00	13.05	13.95
16	-60.68		52.37		8.32		60.68		-52.37		-8.32		-69.00	113.05	-44.05
17	-55.91		51.18		4.73		55.91		-51.18		-4.73		-60.64	107.09	-46.45
18	-127.47		136.53		-9.06		127.47		-136.53		9.06		-118.42	264.00	-145.58
19	-46.85		72.02		-25.16		46.85		-72.02		25.16		-21.69	118.87	-97.18
20	-63.53		11.67		51.86		63.53		-11.67		-51.86		-115.39	75.19	40.19
21	-21.95		-3.00		24.95		21.95		3.00		-24.95		-46.91	18.95	27.95
22	-30.69		-27.67		58.36		30.69		27.67		-58.36		-89.05	3.03	86.03
23	11.44		-27.67		16.23		-11.44		27.67		-16.23		-4.79	-39.10	43.90
24	-11.23		26.63		-15.40		11.23		-26.63		15.40		4.17	37.87	-42.03

Table C.9 System λ , System μ , and α of Subarea 1, Fuel Cost, and Start-up Cost of Multi-area System
When Its Import Capability Is Limited to 50 MW (Case Study 2)

Hour	λ (\$/MWh)	μ (\$/MW)	α of Subarea 1 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.64969	1.46581	0	40495.59	0
2	9.53485	2.18258	0	34987.65	0
3	7.02900	2.51798	0	27288.36	0
4	7.78466	2.25111	0	25807.85	0
5	6.83307	2.55764	0	24673.71	0
6	10.51036	4.75957	0	31368.65	8582.40
7	12.68852	12.34897	0	40411.56	0
8	17.26426	14.89180	0	49001.97	0
9	19.83384	16.98629	0	52382.75	0
10	26.42724	15.78577	0	59193.14	6226.00
11	26.22913	15.94613	0	67022.95	9339.00
12	23.07037	15.30684	0	77438.46	8525.00
13	20.41295	13.33853	0	74442.37	0
14	16.73103	10.26775	0	73231.62	0
15	16.77873	10.32320	0	73283.77	0
16	18.10861	11.77639	0	75104.55	0
17	25.07809	14.07838	0	86819.90	4262.50
18	30.59995	13.93429	0	107587.93	12787.50
19	35.37285	12.58699	0	118857.25	4262.50
20	29.33304	12.22887	0	112074.79	0
21	23.71504	12.16580	0.13956	97590.54	0
22	19.41352	11.35060	0	87076.08	0
23	15.61512	8.48834	0	83367.04	0
24	12.20070	4.13682	0	69584.27	0

Table C.10 Generation Output and Interchange of Subareas in Multi-area System
with 300 MW Import Capability of Subarea 3 (Case Study 3)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	60.80	-369.20	1636.80	347.80	200.00	-300.00	1636.80	432.80	60.80	-489.20	1636.80	377.80
9	60.80	-388.20	1723.80	373.80	219.00	-300.00	1723.80	488.80	60.80	-508.20	1723.80	333.80
10	108.14	-349.86	1769.59	399.59	228.00	-300.00	1769.59	380.59	108.14	-469.86	1838.54	339.54
11	209.68	-289.32	1770.00	332.00	234.68	-295.32	1770.00	291.00	259.68	-320.32	1838.95	281.95
12	260.72	-299.28	1770.00	281.00	285.72	-264.28	1770.00	215.00	310.72	-289.28	1976.85	356.85
13	171.08	-376.93	1770.00	296.00	252.00	-300.00	1770.00	350.00	221.08	-380.93	1976.85	411.85
14	120.58	-416.43	1770.00	258.00	260.00	-300.00	1770.00	373.00	170.58	-439.43	1976.85	524.85
15	118.58	-409.43	1770.00	223.00	270.00	-300.00	1770.00	384.00	168.58	-451.43	1976.85	553.85
16	180.58	-336.43	1770.00	181.00	287.00	-300.00	1770.00	425.00	230.58	-406.43	1976.85	436.85
17	304.00	-226.00	1770.00	128.00	341.44	-258.56	1843.92	364.92	378.88	-271.12	1991.76	262.76
18	402.22	-155.78	1770.00	87.00	451.34	-176.66	1873.28	288.28	451.34	-226.66	2079.83	183.83
19	413.52	-151.48	1770.00	70.00	468.28	-166.72	2003.96	229.96	468.28	-216.72	2120.95	234.95
20	405.83	-151.17	1770.00	-19.00	405.83	-221.17	1985.43	244.43	456.75	-220.25	2093.15	367.15
21	385.73	-158.27	1770.00	70.00	385.73	-228.27	1935.07	247.07	344.86	-319.14	2017.61	388.61
22	303.73	-195.27	1770.00	92.00	303.73	-265.27	1907.90	287.90	278.73	-340.27	1907.90	420.90
23	193.60	-278.40	1770.00	183.00	242.00	-300.00	1907.90	342.90	168.60	-423.40	1907.90	475.90
24	110.80	-249.20	1711.50	222.50	130.00	-300.00	1849.40	375.40	85.80	-394.20	1711.50	345.50

Table C.11 Tie Line Flow and Area Net Interchange of Multi-area System
with 300 MW Import Capability of Subarea 3 (Case Study 3)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	-51.40		81.40		-30.00		51.40		-81.40		30.00		-21.40	132.80	-111.40
9	-67.73		121.07		-53.33		67.73		-121.07		53.33		-14.40	188.80	-174.40
10	-10.29		70.30		-60.02		10.29		-70.30		60.02		49.73	80.59	-130.32
11	15.67		11.35		-27.02		-15.67		-11.35		27.02		42.68	-4.32	-38.37
12	10.33		-38.95		28.62		-10.33		38.95		-28.62		-18.28	-49.28	67.57
13	-43.64		6.36		37.28		43.64		-6.36		-37.28		-80.93	50.00	30.93
14	-77.14		-4.14		81.28		77.14		4.14		-81.28		-158.43	73.00	85.43
15	-90.14		-6.14		96.28		90.14		6.14		-96.28		-186.43	84.00	102.43
16	-93.47		31.52		61.95		93.47		-31.52		-61.95		-155.43	125.00	30.43
17	-68.12		38.24		29.88		68.12		-38.24		-29.88		-98.00	106.36	-8.36
18	-60.13		51.48		8.65		60.13		-51.48		-8.65		-68.78	111.61	-42.84
19	-48.24		15.01		33.24		48.24		-15.01		-33.24		-81.48	63.25	18.23
20	-64.48		-41.21		105.69		64.48		41.21		-105.69		-170.17	23.27	146.90
21	-35.69		-16.89		52.58		35.69		16.89		-52.58		-88.27	18.80	69.47
22	-41.97		-19.33		61.30		41.97		19.33		-61.30		-103.27	22.63	80.63
23	-46.10		-3.20		49.30		46.10		3.20		-49.30		-95.40	42.90	52.50
24	-34.03		41.37		-7.33		34.03		-41.37		7.33		-26.70	75.40	-48.70

Table C.12 System λ , System μ , and α of Subarea 3, Fuel Cost, and Start-up Cost of Multi-area System
When Its Import Capability Is Limited to 50 MW (Case Study 3)

Hour	λ (\$/MWh)	μ (\$/MW)	α of Subarea 3 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.64700	1.53034	0	40495.59	0
2	9.25920	2.39581	0	34987.65	0
3	7.03720	2.98372	0	27288.36	0
4	7.90669	2.80685	0	25807.85	0
5	6.69541	2.40639	0	24673.71	0
6	9.98794	4.70548	0	31368.65	8582.40
7	12.21865	11.60973	0	40411.56	0
8	16.59645	13.54163	0	49036.84	0
9	18.94321	15.41352	0	52419.35	0
10	25.54539	15.07705	0	59488.69	4262.50
11	25.16217	15.94700	0	67257.67	9339.00
12	23.29734	15.77138	0	77640.29	8525.00
13	19.54129	13.21497	0	74608.29	0
14	16.91699	10.82242	0	73432.79	0
15	16.98188	10.88983	0	73528.73	0
16	18.28143	12.31148	0	75377.77	0
17	25.01602	14.25680	0	86451.90	4262.50
18	30.80000	14.57997	0	108330.06	15565.00
19	34.65832	12.54334	0	119608.38	4262.50
20	29.14613	12.20250	0	112819.47	0
21	23.41717	12.17415	0.22706	98326.45	0
22	18.92265	10.84170	0.24832	84973.85	0
23	15.42069	8.18463	0	81062.04	0
24	11.76666	3.68217	0	67257.58	0

Table C.13 Generation Output and Interchange of Subareas in Multi-area System
with 300 MW Import Capability of Subarea 5 (Case Study 4)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	60.80	-139.20	1500.00	221.00	60.80	-209.20	1500.00	236.00	30.40	-289.60	1500.00	181.00
2	60.80	-69.20	1338.33	142.33	60.80	-126.20	1338.33	157.33	30.40	-206.60	1338.33	102.33
3	60.80	-17.20	1101.33	84.33	60.80	-65.20	1101.33	99.33	30.40	-145.60	1101.33	44.33
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	0.00	-138.00	1215.53	199.53	0.00	-208.00	1215.53	214.53	30.40	-227.60	1215.53	159.53
7	0.00	-250.00	1476.33	298.33	0.00	-320.00	1476.33	313.33	70.00	-300.00	1476.33	258.33
8	0.00	-430.00	1652.33	363.33	25.00	-475.00	1652.33	448.33	250.00	-300.00	1652.33	393.33
9	60.80	-388.20	1698.80	348.80	85.80	-433.20	1698.80	463.80	269.00	-300.00	1698.80	308.80
10	106.56	-351.44	1768.63	398.63	131.56	-396.44	1768.63	379.63	278.00	-300.00	1768.63	269.63
11	209.00	-290.00	1770.00	332.00	284.00	-246.00	1770.00	291.00	280.00	-300.00	1770.00	213.00
12	267.03	-292.97	1770.00	281.00	342.03	-207.97	1770.00	215.00	317.03	-282.97	1907.90	287.90
13	168.05	-379.95	1770.00	296.00	243.05	-308.95	1770.00	350.00	302.00	-300.00	1907.90	342.90
14	117.55	-419.45	1770.00	258.00	192.55	-367.45	1770.00	373.00	310.00	-300.00	1907.90	455.90
15	115.55	-412.45	1770.00	223.00	190.55	-379.45	1770.00	384.00	320.00	-300.00	1907.90	484.90
16	190.05	-326.95	1770.00	181.00	240.05	-346.95	1770.00	425.00	337.00	-300.00	1907.90	367.90
17	304.00	-226.00	1770.00	128.00	378.72	-221.28	1770.00	291.00	416.09	-233.91	1991.19	262.19
18	304.00	-254.00	1770.00	87.00	401.17	-226.83	2075.83	490.83	401.17	-276.83	2075.83	179.83
19	412.79	-152.21	1770.00	70.00	412.79	-222.21	2118.31	344.31	412.79	-272.21	2118.31	232.31
20	405.23	-151.77	1770.00	-19.00	354.61	-272.39	2090.96	349.96	405.23	-271.77	2090.96	364.96
21	381.36	-162.64	1770.00	70.00	304.00	-310.00	2001.14	313.14	381.36	-282.64	2001.14	372.14
22	239.65	-259.35	1770.00	92.00	189.65	-379.35	1976.85	356.85	319.00	-300.00	1976.85	489.85
23	134.48	-337.52	1755.11	168.11	84.48	-457.52	1961.96	396.96	292.00	-300.00	1961.96	529.96
24	110.80	-249.20	1657.20	168.20	60.80	-369.20	1864.05	390.05	180.00	-300.00	1726.15	360.15

Table C.14 Tie Line Flow and Area Net Interchange of Multi-area System
with 300 MW Import Capability of Subarea 5 (Case Study 4)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		45.13		-63.47		-18.33		-45.13		63.47		81.80	26.80	-108.60
2	14.00		45.13		-59.13		-14.00		-45.13		59.13		73.13	31.13	-104.27
3	11.00		45.13		-56.13		-11.00		-45.13		56.13		67.13	34.13	-101.27
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		24.87		-43.20		-18.33		-24.87		43.20		61.53	6.53	-68.07
7	18.33		11.67		-30.00		-18.33		-11.67		30.00		48.33	-6.67	-41.67
8	-13.33		-40.00		53.33		13.33		40.00		-53.33		-66.67	-26.67	93.33
9	-23.33		7.27		16.07		23.33		-7.27		-16.07		-39.40	30.60	8.80
10	21.33		4.52		-25.85		-21.33		-4.52		25.85		47.19	-16.81	-30.37
11	-1.00		44.00		-43.00		1.00		-44.00		43.00		42.00	45.00	-87.00
12	-6.33		0.70		5.63		6.33		-0.70		-5.63		-11.97	7.03	4.93
13	-41.67		-0.62		42.28		41.67		0.62		-42.28		-83.95	41.05	42.90
14	-55.67		-50.12		105.78		55.67		50.12		-105.78		-161.45	5.55	155.90
15	-64.67		-60.12		124.78		64.67		60.12		-124.78		-189.45	4.55	184.90
16	-74.67		3.38		71.28		74.67		-3.38		-71.28		-145.95	78.05	67.90
17	-55.91		13.82		42.09		55.91		-13.82		-42.09		-98.00	69.72	28.28
18	-143.67		120.33		23.33		143.67		-120.33		-23.33		-167.00	264.00	-97.00
19	-68.10		54.00		14.10		68.10		-54.00		-14.10		-82.21	122.10	-39.90
20	-82.78		-5.20		87.99		82.78		5.20		-87.99		-170.77	77.58	93.19
21	-31.93		-28.79		60.71		31.93		28.79		-60.71		-92.64	3.14	89.50
22	-48.28		-70.78		119.07		48.28		70.78		-119.07		-167.35	-22.50	189.85
23	-36.28		-96.84		133.12		36.28		96.84		-133.12		-169.41	-60.56	229.96
24	-33.95		-13.10		47.05		33.95		13.10		-47.05		-81.00	20.85	60.15

Table C.15 System λ , System μ , and α of Subarea 5, Fuel Cost, and Start-up Cost of Multi-area System
When Its Import Capability Is Limited to 50 MW (Case Study 4)

Hour	λ (\$/MWh)	μ (\$/MW)	α of Subarea 5 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	12.18382	2.41083	0	41869.85	0
2	10.58634	2.76300	0	36419.05	0
3	8.91905	2.35725	0	29573.08	0
4	9.08632	1.71973	0	25807.85	0
5	5.86287	2.95245	0	24673.71	0
6	10.56460	5.55964	0	29888.25	1430.40
7	14.06175	14.35067	0	39058.42	0
8	21.34837	16.47537	0	53979.30	9339.00
9	22.71620	16.10944	0	56878.49	0
10	26.98616	14.99608	0	60698.41	0
11	28.67788	16.79202	0	67025.05	6226.00
12	22.81440	14.96797	0	77430.16	0
13	20.26552	12.97415	0	74392.73	0
14	16.29649	9.58759	0	73226.15	0
15	16.36400	9.66070	0	73330.27	0
16	17.98793	11.41486	2.17790	73737.70	0
17	25.12876	13.75253	2.49997	86819.90	7375.50
18	30.41744	13.84277	2.10246	107587.90	12787.50
19	38.97305	14.97387	1.11485	119232.60	6226.00
20	31.51937	14.24398	0.91665	112447.00	0
21	25.74677	13.14221	0.59459	98548.66	0
22	19.85692	12.46233	0.61891	88360.34	0
23	16.0130	8.86155	1.30134	84684.23	0
24	11.92098	4.59163	0	70831.80	0

Table C.16 Generation Output and Interchange of Subareas in Multi-area System
with 300 MW Export Capability of Subarea 2 (Case Study 5)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1529.00	250.00	0.00	-270.00	1561.50	297.50	0.00	-320.00	1561.50	242.50
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1428.00	250.00	60.80	-259.20	1444.30	281.30	60.80	-309.20	1444.30	226.30
8	67.75	-362.25	1539.00	250.00	67.75	-432.25	1744.87	540.87	67.75	-482.25	1744.87	485.87
9	124.00	-325.00	1600.00	250.00	124.00	-395.00	1770.00	535.00	124.00	-445.00	1770.00	380.00
10	197.68	-260.32	1620.00	250.00	197.68	-330.32	1770.00	381.00	197.68	-380.32	1838.95	339.95
11	237.02	-261.98	1688.00	250.00	262.02	-267.98	1770.00	291.00	287.02	-292.98	1838.95	281.95
12	292.02	-267.98	1739.00	250.00	367.02	-182.98	1770.00	215.00	367.02	-232.98	1838.95	218.95
13	226.02	-321.98	1724.00	250.00	301.02	-250.98	1770.00	350.00	301.02	-300.98	1838.95	273.95
14	182.35	-354.65	1762.00	250.00	257.35	-302.65	1770.00	373.00	257.35	-352.65	1838.95	386.95
15	181.68	-346.32	1770.00	223.00	256.68	-313.32	1770.00	384.00	256.68	-363.32	1838.95	415.95
16	228.68	-288.32	1770.00	181.00	303.68	-283.32	1770.00	425.00	303.68	-333.32	1838.95	298.95
17	304.00	-226.00	1770.00	128.00	404.58	-195.43	1770.00	291.00	404.58	-245.43	1976.85	247.85
18	402.22	-155.78	1770.00	87.00	451.34	-176.66	1873.28	288.28	451.34	-226.66	2079.83	183.83
19	413.52	-151.48	1770.00	70.00	468.28	-166.72	2003.96	229.96	468.28	-216.72	2120.95	234.95
20	405.83	-151.17	1770.00	-19.00	405.83	-221.17	1985.43	244.43	456.75	-220.25	2093.15	367.15
21	390.66	-153.34	1770.00	70.00	304.00	-310.00	1947.47	259.47	390.66	-273.34	2036.21	407.21
22	328.73	-170.27	1770.00	92.00	278.73	-290.27	1907.90	287.90	278.73	-340.27	1907.90	420.90
23	234.73	-237.27	1770.00	183.00	184.73	-357.27	1907.90	342.90	184.73	-407.27	1907.90	475.90
24	110.80	-249.20	1696.93	207.93	60.80	-369.20	1834.83	360.83	60.80	-419.20	1834.83	468.83

Table C.17 Tie Line Flow and Area Net Interchange of Multi-area System
with 300 MW Export Capability of Subarea 2 (Case Study 5)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	7.50		35.00		-42.50		-7.50		-35.00		42.50		50.00	27.50	-77.50
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	12.90		35.00		-47.90		-12.90		-35.00		47.90		60.80	22.10	-82.90
8	-73.62		35.00		38.62		73.62		-35.00		-38.62		-112.25	108.62	3.62
9	-71.67		68.33		3.33		71.67		-68.33		-3.33		-75.00	140.00	-65.00
10	-20.33		30.35		-10.02		20.33		-30.35		10.02		-10.32	50.68	-40.37
11	-11.67		11.35		0.32		11.67		-11.35		-0.32		-11.98	23.02	-11.03
12	-16.67		15.35		1.32		16.67		-15.35		-1.32		-17.98	32.02	-14.03
13	-57.00		42.02		14.98		57.00		-42.02		-14.98		-71.98	99.02	-27.03
14	-58.33		12.02		46.32		58.33		-12.02		-46.32		-104.65	70.35	34.30
15	-64.67		6.02		58.65		64.67		-6.02		-58.65		-123.32	70.68	52.63
16	-83.00		58.68		24.32		83.00		-58.68		-24.32		-107.32	141.68	-34.37
17	-64.52		31.05		33.47		64.52		-31.05		-33.47		-98.00	95.58	2.42
18	-60.13		51.48		8.65		60.13		-51.48		-8.65		-68.78	111.61	-42.84
19	-48.24		15.01		33.24		48.24		-15.01		-33.24		-81.48	63.25	18.23
20	-64.48		-41.21		105.69		64.48		41.21		-105.69		-170.17	23.27	146.90
21	-10.94		-61.47		72.40		10.94		61.47		-72.40		-83.34	-50.53	133.87
22	-25.30		-27.67		52.97		25.30		27.67		-52.97		-78.27	-2.37	80.63
23	-13.30		-27.67		40.97		13.30		27.67		-40.97		-54.27	-14.37	68.63
24	-10.97		-19.33		30.30		10.97		19.33		-30.30		-41.27	-8.37	49.63

Table C.18 System λ , System μ , and β of Subarea 2, Fuel Cost, and Start-up Cost of Multi-area System
When Its Export Capability Is Limited to 50 MW (Case Study 5)

Hour	λ (\$/MWh)	μ (\$/MW)	β of Subarea 2 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.59573	1.61219	0	40496.62	0
2	8.74890	2.69849	0	34987.65	0
3	7.31184	2.71390	0	27288.36	0
4	7.51009	2.63807	0	25807.85	0
5	6.98573	2.46860	0	24673.71	0
6	9.88163	4.93657	0	31368.65	8582.40
7	13.21151	11.20255	0	40411.76	0
8	17.50814	12.76933	0	48966.38	0
9	19.76839	14.62564	0	52399.35	0
10	23.96919	15.57034	0	59590.94	4262.50
11	24.68303	17.39564	0	67428.58	9339.00
12	23.49388	16.09622	0	75951.34	9339.00
13	18.72206	12.94385	0	72746.82	0
14	17.26058	10.65768	0	71397.16	0
15	17.37180	10.74564	0	71470.25	0
16	17.36821	12.08128	0	73428.16	0
17	24.84029	14.52060	0	87430.40	8525.00
18	30.95378	14.58219	0	108330.06	10488.50
19	34.22978	12.37116	0	119608.38	4262.50
20	29.10815	12.39862	0	112819.47	0
21	24.02053	12.41211	0	97749.33	0
22	19.24770	10.66357	0	80772.75	0
23	15.52820	7.77523	0	76673.60	0
24	11.71845	3.96544	0	69287.45	0

Table C.19 Generation Output and Interchange of Subareas in Multi-area System
with 300 MW Export Capability of Subarea 4 (Case Study 6)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	76.04	-353.96	1749.94	460.94	76.04	-423.96	1504.00	300.00	76.04	-473.96	1749.94	490.94
9	145.67	-303.33	1770.00	420.00	145.67	-373.33	1535.00	300.00	145.67	-423.33	1770.00	380.00
10	174.68	-283.32	1770.00	400.00	174.68	-353.32	1689.00	300.00	174.68	-403.32	1838.95	339.95
11	209.68	-289.32	1770.00	332.00	234.68	-295.32	1770.00	291.00	259.68	-320.32	1838.95	281.95
12	281.68	-278.32	1770.00	281.00	356.68	-193.32	1770.00	215.00	356.68	-243.32	1838.95	218.95
13	227.35	-320.65	1770.00	296.00	302.35	-249.65	1720.00	300.00	302.35	-299.65	1838.95	273.95
14	204.02	-332.98	1770.00	258.00	279.02	-280.98	1697.00	300.00	279.02	-330.98	1838.95	386.95
15	209.68	-318.32	1770.00	223.00	284.68	-285.32	1686.00	300.00	284.68	-335.32	1838.95	415.95
16	270.35	-246.65	1770.00	181.00	345.35	-241.65	1645.00	300.00	345.35	-291.65	1838.95	298.95
17	304.00	-226.00	1770.00	128.00	404.58	-195.43	1770.00	291.00	404.58	-245.43	1976.85	247.85
18	402.22	-155.78	1770.00	87.00	451.34	-176.66	1873.28	288.28	451.34	-226.66	2079.83	183.83
19	413.52	-151.48	1770.00	70.00	468.28	-166.72	2003.96	229.96	468.28	-216.72	2120.95	234.95
20	405.83	-151.17	1770.00	-19.00	405.83	-221.17	1985.43	244.43	456.75	-220.25	2093.15	367.15
21	390.66	-153.34	1770.00	70.00	304.00	-310.00	1947.47	259.47	390.66	-273.34	2036.21	407.21
22	328.73	-170.27	1770.00	92.00	278.73	-290.27	1907.90	287.90	278.73	-340.27	1907.90	420.90
23	249.03	-222.97	1770.00	183.00	199.03	-342.97	1865.00	300.00	199.03	-392.97	1907.90	475.90
24	110.80	-249.20	1727.35	238.35	60.80	-369.20	1774.00	300.00	60.80	-419.20	1865.25	499.25

Table C.20 Tie Line Flow and Area Net Interchange of Multi-area System
with 300 MW Export Capability of Subarea 4 (Case Study 6)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	76.98		-46.98		-30.00		-76.98		46.98		30.00		106.98	-123.96	16.98
9	63.33		-10.00		-53.33		-63.33		10.00		53.33		116.67	-73.33	-43.33
10	56.67		3.35		-60.02		-56.67		-3.35		60.02		116.68	-53.32	-63.37
11	15.67		11.35		-27.02		-15.67		-11.35		27.02		42.68	-4.32	-38.37
12	-6.33		15.35		-9.02		6.33		-15.35		9.02		2.68	21.68	-24.37
13	-25.00		25.35		-0.35		25.00		-25.35		0.35		-24.65	50.35	-25.70
14	-31.33		-12.32		43.65		31.33		12.32		-43.65		-74.98	19.02	55.97
15	-36.67		-21.98		58.65		36.67		21.98		-58.65		-95.32	14.68	80.63
16	-41.33		17.02		24.32		41.33		-17.02		-24.32		-65.65	58.35	7.30
17	-64.52		31.05		33.47		64.52		-31.05		-33.47		-98.00	95.58	2.42
18	-60.13		51.48		8.65		60.13		-51.48		-8.65		-68.78	111.61	-42.84
19	-48.24		15.01		33.24		48.24		-15.01		-33.24		-81.48	63.25	18.23
20	-64.48		-41.21		105.69		64.48		41.21		-105.69		-170.17	23.27	146.90
21	-10.94		-61.47		72.40		10.94		61.47		-72.40		-83.34	-50.53	133.87
22	-25.30		-27.67		52.97		25.30		27.67		-52.97		-78.27	-2.37	80.63
23	1.00		-41.97		40.97		-1.00		41.97		-40.97		-39.97	-42.97	82.93
24	19.45		-49.75		30.30		-19.45		49.75		-30.30		-10.85	-69.20	80.05

Table C.21 System λ , System μ , and β of Subarea 4, Fuel Cost, and Start-up Cost of Multi-area System
When Its Export Capability Is Limited to 50 MW (Case Study 6)

Hour	λ (\$/MWh)	μ (\$/MW)	β of Subarea 5 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.59286	1.61119	0	40495.59	0
2	8.76502	2.69487	0	34987.65	0
3	7.35538	2.70601	0	27288.36	0
4	7.30004	2.65663	0	25807.85	0
5	7.05019	2.48381	0	24673.71	0
6	9.93633	4.94896	0	31368.65	8582.40
7	13.27855	11.23330	0	40411.56	0
8	17.55256	12.79699	0	48995.87	0
9	19.81231	14.63663	0	52468.08	0
10	24.24335	15.69181	0	59479.16	4262.50
11	25.96279	16.82233	0	67257.67	9339.00
12	23.96690	16.18750	0	75833.43	9339.00
13	18.82292	12.94104	0	72755.83	0
14	16.00358	10.59679	0	71491.61	0
15	16.08932	10.64147	0	71596.55	0
16	17.65618	11.81565	0	73784.78	0
17	25.18919	14.73960	0	87430.40	8525.00
18	30.32275	14.26970	0	108330.06	10488.50
19	33.98967	12.72569	0	119608.38	4262.50
20	29.28484	12.42271	0	112819.47	0
21	23.39592	12.06925	0	97749.33	0
22	19.32201	10.52932	0	80772.75	0
23	15.79656	7.84058	0	76728.31	0
24	11.76197	4.00525	0	69296.39	0

Table C.22 Generation Output and Interchange of Subareas in Multi-area System
with 300 MW Export Capability of Subarea 6 (Case Study 7)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	63.02	-366.98	1741.97	452.97	63.02	-436.98	1741.97	537.97	63.02	-486.98	1559.00	300.00
9	98.29	-350.71	1763.56	413.56	98.29	-420.71	1763.56	528.56	98.29	-470.71	1690.00	300.00
10	161.00	-297.00	1770.00	400.00	161.00	-367.00	1770.00	381.00	161.00	-417.00	1799.00	300.00
11	209.68	-289.32	1770.00	332.00	234.68	-295.32	1770.00	291.00	259.68	-320.32	1838.95	281.95
12	281.68	-278.32	1770.00	281.00	356.68	-193.32	1770.00	215.00	356.68	-243.32	1838.95	218.95
13	210.68	-337.32	1770.00	296.00	285.68	-266.32	1770.00	350.00	285.68	-316.32	1838.95	273.95
14	208.67	-328.33	1770.00	258.00	283.67	-276.33	1770.00	373.00	283.67	-326.33	1752.00	300.00
15	220.33	-307.67	1770.00	223.00	295.33	-274.67	1770.00	384.00	295.33	-324.67	1723.00	300.00
16	228.68	-288.32	1770.00	181.00	303.68	-283.32	1770.00	425.00	303.68	-333.32	1838.95	298.95
17	304.00	-226.00	1770.00	128.00	404.58	-195.43	1770.00	291.00	404.58	-245.43	1976.85	247.85
18	402.22	-155.78	1770.00	87.00	451.34	-176.66	1873.28	288.28	451.34	-226.66	2079.83	183.83
19	413.52	-151.48	1770.00	70.00	468.28	-166.72	2003.96	229.96	468.28	-216.72	2120.95	234.95
20	417.21	-139.79	1770.00	-19.00	417.21	-209.79	2012.78	271.78	473.81	-203.19	2026.00	300.00
21	414.66	-129.34	1770.00	70.00	304.00	-310.00	2006.69	318.69	414.66	-249.34	1929.00	300.00
22	381.80	-117.20	1770.00	92.00	304.00	-265.00	1925.20	305.20	304.00	-315.00	1787.00	300.00
23	293.37	-178.63	1770.00	183.00	243.37	-298.63	1907.90	342.90	243.37	-348.63	1732.00	300.00
24	130.09	-229.91	1752.42	263.42	80.09	-349.91	1890.32	416.32	80.09	-399.91	1666.00	300.00

Table C.23 Tie Line Flow and Area Net Interchange of Multi-area System
with 300 MW Export Capability for Subarea 6 (Case Study 7)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	-5.00		95.99		-90.99		5.00		-95.99		90.99		85.99	100.99	-186.98
9	-15.00		92.85		-77.85		15.00		-92.85		77.85		62.85	107.85	-170.71
10	29.67		43.67		-73.33		-29.67		-43.67		73.33		103.00	14.00	-117.00
11	15.67		11.35		-27.02		-15.67		-11.35		27.02		42.68	-4.32	-38.37
12	-6.33		15.35		-9.02		6.33		-15.35		9.02		2.68	21.68	-24.37
13	-41.67		42.02		-0.35		41.67		-42.02		0.35		-41.32	83.68	-42.37
14	-55.67		41.00		14.67		55.67		-41.00		-14.67		-70.33	96.67	-26.33
15	-64.67		44.67		20.00		64.67		-44.67		-20.00		-84.67	109.33	-24.67
16	-83.00		58.68		24.32		83.00		-58.68		-24.32		-107.32	141.68	-34.37
17	-64.52		31.05		33.47		64.52		-31.05		-33.47		-98.00	95.58	2.42
18	-60.13		51.48		8.65		60.13		-51.48		-8.65		-68.78	111.61	-42.84
19	-48.24		15.01		33.24		48.24		-15.01		-33.24		-81.48	63.25	18.23
20	-73.59		-11.61		85.20		73.59		11.61		-85.20		-158.79	61.98	96.81
21	-22.68		-13.99		36.67		22.68		13.99		-36.67		-59.34	8.69	50.66
22	-21.80		18.40		3.40		21.80		-18.40		-3.40		-25.20	40.20	-15.00
23	-13.30		30.97		-17.67		13.30		-30.97		17.67		4.37	44.27	-48.63
24	-10.97		55.44		-44.47		10.97		-55.44		44.47		33.51	66.41	-99.91

Table C.24 System λ , Sstem μ , and β of Subarea 6, Fuel Cost, and Start-up Cost of Multi-area System
When Its Export Capability Is Limited to 50 MW (Case Study 7)

Hour	λ (\$/MWh)	μ (\$/MW)	β of Subarea 6 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.59415	1.61338	0	40495.59	0
2	8.71838	2.70304	0	34987.65	0
3	7.24016	2.71978	0	27288.36	0
4	7.98842	2.67191	0	25807.85	0
5	6.87547	2.50129	0	24673.71	0
6	9.80010	4.96733	0	31368.65	8582.40
7	13.26931	11.23712	0	40411.56	0
8	17.43388	12.80772	0	48951.78	0
9	19.68737	14.68749	0	52336.98	0
10	24.77750	15.19381	0	59435.87	4262.50
11	25.62403	16.87335	0	67257.67	9339.00
12	23.38528	16.36212	0	75833.43	9339.00
13	19.71489	13.07184	0	72661.73	0
14	17.02557	10.65114	0	71518.67	0
15	17.02358	10.67975	0	71666.34	0
16	17.16335	12.08389	0	73428.16	0
17	24.83276	14.57903	0	87430.40	8525.00
18	30.63605	14.87847	0	108330.06	10488.50
19	33.56513	12.73407	0	119608.38	4262.50
20	28.91848	12.43286	0	112885.08	0
21	23.83897	12.34376	0	99565.17	0
22	18.54941	10.39352	0	82646.54	0
23	14.44377	7.69341	0	77039.62	0
24	11.72499	3.99396	0	69364.72	0

Table C.25 Generation Output and Interchange of Subareas in Multi-area System
with 50 MW Import and Export Capability of Area 1 (Case Study 8)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	60.80	-139.20	1468.20	189.20	60.80	-209.20	1515.90	251.90	30.40	-289.60	1515.90	196.90
2	60.80	-69.20	1315.20	119.20	60.80	-126.20	1349.90	168.90	30.40	-206.60	1349.90	113.90
3	0.00	-78.00	1145.00	128.00	0.00	-126.00	1155.50	153.50	0.00	-176.00	1155.50	98.50
4	0.00	-69.00	1057.00	119.00	0.00	-114.00	1064.50	141.50	0.00	-164.00	1064.50	86.50
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	0.00	-138.00	1204.00	188.00	0.00	-208.00	1221.30	220.30	30.40	-227.60	1221.30	165.30
7	0.00	-250.00	1478.00	300.00	0.00	-320.00	1495.30	332.30	30.40	-339.60	1495.30	277.30
8	60.80	-369.20	1683.20	394.20	60.80	-439.20	1683.20	479.20	60.80	-489.20	1683.20	424.20
9	83.08	-365.92	1754.25	404.25	83.08	-435.92	1754.25	519.25	83.08	-485.92	1754.25	364.25
10	108.30	-349.70	1769.70	399.70	227.00	-301.00	1770.00	381.00	177.00	-401.00	1770.00	271.00
11	217.00	-282.00	1770.00	332.00	303.00	-227.00	1770.00	291.00	253.00	-327.00	1770.00	213.00
12	329.00	-231.00	1770.00	281.00	358.03	-191.98	1770.00	215.00	308.03	-291.98	1838.95	218.95
13	260.68	-287.32	1770.00	296.00	285.68	-266.32	1770.00	350.00	235.68	-366.32	1838.95	273.95
14	229.68	-307.32	1770.00	258.00	254.68	-305.32	1770.00	373.00	204.68	-405.32	1838.95	386.95
15	255.00	-273.00	1770.00	223.00	245.03	-324.98	1770.00	384.00	195.03	-424.98	1838.95	415.95
16	286.00	-231.00	1770.00	181.00	300.03	-286.98	1770.00	425.00	250.03	-386.98	1838.95	298.95
17	314.10	-215.90	1907.90	265.90	433.04	-166.96	1770.00	291.00	347.01	-302.99	1857.94	128.94
18	383.10	-174.90	1907.90	224.90	428.04	-199.96	1770.00	185.00	366.02	-311.98	2172.94	276.94
19	407.10	-157.90	1907.90	207.90	437.40	-197.60	1915.47	141.47	370.70	-314.30	2206.42	320.42
20	405.23	-151.77	1983.98	194.98	405.23	-221.77	1876.99	135.99	354.61	-322.39	2090.96	364.96
21	378.10	-165.90	1915.90	215.90	304.00	-310.00	1869.25	181.25	304.00	-360.00	2067.75	438.75
22	266.10	-232.90	1907.90	229.90	241.10	-327.90	1838.95	218.95	241.10	-377.90	1976.85	489.85
23	172.10	-299.90	1907.90	320.90	147.10	-394.90	1838.95	273.95	147.10	-444.90	1976.85	544.85
24	85.80	-274.20	1813.20	324.20	60.80	-369.20	1754.73	280.73	60.80	-419.20	1823.68	457.68

Table C.26 Tie Line Flow and Area Net Interchange of Multi-area System
with 50 MW Import and Export Capability of Area 1 (Case Study 8)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	2.43		45.13		-47.57		-2.43		-45.13		47.57		50.00	42.70	-92.70
2	2.43		45.13		-47.57		-2.43		-45.13		47.57		50.00	42.70	-92.70
3	7.50		35.00		-42.50		-7.50		-35.00		42.50		50.00	27.50	-77.50
4	7.50		35.00		-42.50		-7.50		-35.00		42.50		50.00	27.50	-77.50
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	12.57		24.87		-37.43		-12.57		-24.87		37.43		50.00	12.30	-62.30
7	12.57		24.87		-37.43		-12.57		-24.87		37.43		50.00	12.30	-62.30
8	-5.00		35.00		-30.00		5.00		-35.00		30.00		25.00	40.00	-65.00
9	-15.00		68.33		-53.33		15.00		-68.33		53.33		38.33	83.33	-121.67
10	-10.00		70.00		-60.00		10.00		-70.00		60.00		50.00	80.00	-130.00
11	-4.67		59.33		-54.67		4.67		-59.33		54.67		50.00	64.00	-114.00
12	8.99		32.02		-41.01		-8.99		-32.02		41.01		50.00	23.03	-73.03
13	-25.00		58.68		-33.68		25.00		-58.68		33.68		8.68	83.68	-92.37
14	-39.00		28.68		10.32		39.00		-28.68		-10.32		-49.32	67.68	-18.37
15	-36.34		22.68		13.66		36.34		-22.68		-13.66		-50.00	59.03	-9.03
16	-62.67		75.35		-12.67		62.67		-75.35		12.67		-50.00	138.03	-88.03
17	-24.68		99.36		-74.68		24.68		-99.36		74.68		50.00	124.04	-174.04
18	21.65		6.69		-28.35		-21.65		-6.69		28.35		50.00	-14.96	-35.04
19	35.37		-20.75		-14.63		-35.37		20.75		14.63		50.00	-56.12	6.12
20	43.00		-42.79		-0.21		-43.00		42.79		0.21		43.20	-85.78	42.58
21	59.58		-69.17		9.58		-59.58		69.17		-9.58		50.00	-128.75	78.75
22	35.32		-73.63		38.32		-35.32		73.63		-38.32		-3.00	-108.95	111.95
23	47.32		-73.63		26.32		-47.32		73.63		-26.32		21.00	-120.95	99.95
24	46.16		-42.32		-3.84		-46.16		42.32		3.84		50.00	-88.48	38.48

Table C.27 System λ , System μ , α and β of Area 1, Fuel Cost, and Start-up Cost of Multi-area System When Its Import and Export Capabilities Are Limited to 50 MW (Case Study 8)

Hour	λ (\$/MWh)	μ (\$/MW)	α of Area 1 (\$/MW)	β of Area 1 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	12.19194	2.41456	0.31899	0	41871.60	0
2	10.98797	2.71780	0.19323	0	36419.82	0
3	8.26124	2.26195	0.00336	0	27288.39	0
4	8.92108	1.81547	0	0	25807.87	0
5	7.04852	2.49046	0	0	24673.71	0
6	10.56701	4.85510	0	0	29888.46	1430.40
7	13.73850	13.33524	0.08833	0	39015.48	0
8	19.53071	16.12355	2.46485	0	48912.86	7152.00
9	22.10504	16.49650	3.02513	0	52318.12	0
10	25.74155	15.52890	2.61831	0	59175.05	6226.00
11	28.10136	16.05186	3.16183	0	67031.09	9339.00
12	23.20500	14.62848	1.58835	0	75833.62	7375.50
13	19.02183	12.24679	0.06805	0	72661.73	0
14	16.21869	9.78520	0.13297	0	71390.05	0
15	16.28431	9.84832	0.70522	0	71478.89	0
16	18.00404	11.37256	1.78419	0	73429.01	0
17	25.37691	13.23528	1.24931	0	88690.98	8525.00
18	31.54330	13.49405	0.97996	0	108239.60	11638.00
19	49.84344	5.63192	0.50471	0	119557.62	4262.50
20	27.64833	11.54783	2.35444	0	112447.02	0
21	23.67778	11.30373	1.41810	0	97411.73	0
22	17.30344	8.83918	0.84829	0	85415.38	0
23	14.32510	6.80272	0	0	81565.22	0
24	11.75986	2.65204	0	0	71050.82	0

Table C.28 Generation Output and Interchange of Subareas in Multi-area System
with 50 MW Import and Export Capability of Area 2 (Case Study 8)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	60.80	-369.20	1683.20	394.20	60.80	-439.20	1683.20	479.20	60.80	-489.20	1683.20	424.20
9	93.42	-355.58	1760.58	410.58	62.41	-456.59	1741.59	506.59	93.42	-475.58	1760.58	370.58
10	157.50	-300.50	1770.00	400.00	197.00	-331.00	1770.00	381.00	157.50	-420.50	1770.00	271.00
11	242.00	-257.00	1770.00	332.00	289.00	-241.00	1770.00	291.00	242.00	-338.00	1770.00	213.00
12	292.03	-267.97	1770.00	281.00	342.03	-207.97	1770.00	215.00	292.03	-307.97	1907.90	287.90
13	230.55	-317.45	1770.00	296.00	252.00	-300.00	1770.00	350.00	230.55	-371.45	1907.90	342.90
14	191.55	-345.45	1770.00	258.00	237.00	-323.00	1770.00	373.00	191.55	-418.45	1907.90	455.90
15	195.05	-332.95	1770.00	223.00	236.00	-334.00	1770.00	384.00	195.05	-424.95	1907.90	484.90
16	277.55	-239.45	1770.00	181.00	212.00	-375.00	1770.00	425.00	277.55	-359.45	1907.90	367.90
17	347.34	-182.66	1770.00	128.00	359.00	-241.00	1770.00	291.00	347.34	-302.66	2036.31	307.31
18	375.85	-182.15	1770.00	87.00	286.15	-341.85	1976.85	391.85	375.85	-302.15	2243.29	347.29
19	364.00	-201.00	1899.50	199.50	397.39	-237.61	2061.61	287.61	364.00	-321.00	2158.50	272.50
20	304.00	-253.00	1893.39	104.39	388.80	-238.20	2029.20	288.20	361.44	-315.56	2140.17	414.17
21	304.00	-240.00	1858.14	158.14	304.00	-310.00	2034.43	346.43	304.00	-360.00	2034.43	405.43
22	226.45	-272.55	1838.95	160.95	226.45	-342.55	1976.85	356.85	226.45	-392.55	1976.85	489.85
23	132.45	-339.55	1838.95	251.95	132.45	-409.55	1976.85	411.85	132.45	-459.55	1976.85	544.85
24	60.80	-299.20	1761.70	272.70	60.80	-369.20	1893.20	419.20	60.80	-419.20	1761.70	395.70

Table C.29 Tie Line Flow and Area Net Interchange of Multi-area System
with 50 MW Import and Export Capability of Area 2 (Case Study 9)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	-5.00		35.00		-30.00		5.00		-35.00		30.00		25.00	40.00	-65.00
9	1.67		51.67		-53.33		-1.67		-51.67		53.33		55.00	50.00	-105.00
10	16.50		66.50		-83.00		-16.50		-66.50		83.00		99.50	50.00	-149.50
11	8.33		58.33		-66.67		-8.33		-58.33		66.67		75.00	50.00	-125.00
12	2.00		9.03		-11.03		-2.00		-9.03		11.03		13.03	7.03	-20.07
13	-23.82		26.18		-2.37		23.82		-26.18		2.37		-21.45	50.00	-28.55
14	-45.82		4.18		41.63		45.82		-4.18		-41.63		-87.45	50.00	37.45
15	-53.32		-3.32		56.63		53.32		3.32		-56.63		-109.95	50.00	59.95
16	-36.15		13.85		22.30		36.15		-13.85		-22.30		-58.45	50.00	8.45
17	-34.89		15.11		19.77		34.89		-15.11		-19.77		-54.66	50.00	4.66
18	-48.38		1.62		46.76		48.38		-1.62		-46.76		-95.15	50.00	45.15
19	-17.17		32.83		-15.67		17.17		-32.83		15.67		-1.50	50.00	-48.50
20	-66.20		-16.20		82.41		66.20		16.20		-82.41		-148.61	50.00	98.61
21	-39.43		-3.00		42.43		39.43		3.00		-42.43		-81.86	36.43	45.43
22	-41.97		-27.67		69.63		41.97		27.67		-69.63		-111.60	14.30	97.30
23	-29.97		-27.67		57.63		29.97		27.67		-57.63		-87.60	2.30	85.30
24	-25.50		24.50		1.00		25.50		-24.50		-1.00		-26.50	50.00	-23.50

Table C.30 System λ , System μ , α and β of Area 2, Fuel Cost, and Start-up Cost of Multi-area System When Its Import and Export Capabilities Are Limited to 50 MW (Case Study 9)

Hour	λ (\$/MWh)	μ (\$/MW)	α of Area 2 (\$/MW)	β of Area 2 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.69185	1.47819	0	0	40495.59	0
2	9.36546	2.10179	0	0	34987.65	0
3	7.83036	2.97454	0	0	27288.36	0
4	8.10006	2.77541	0	0	25807.85	0
5	7.02810	2.45392	0	0	24673.71	0
6	10.12683	4.82297	0	0	31368.65	8582.40
7	14.13005	13.47156	0.77288	0	40411.56	0
8	20.67449	16.09195	1.63253	0	48912.86	0
9	23.25754	16.78843	1.30032	0	52320.96	0
10	29.96557	14.81649	1.36510	0	59157.19	6226.00
11	30.60586	18.34326	2.51678	0	67022.00	9339.00
12	24.20965	14.70278	1.04840	0	77430.16	8525.00
13	20.12987	12.46140	0	0	74348.21	0
14	15.71621	9.04887	0	0	73099.78	0
15	15.77500	9.11982	0	0	73178.55	0
16	16.65485	10.63392	0	0	75185.78	0
17	24.79070	13.97602	0	0	87485.21	4262.50
18	30.97845	13.37356	0	0	110308.84	12787.50
19	33.98173	12.99722	1.86015	0	118917.59	4262.50
20	30.11236	13.32187	1.25425	0	112150.47	0
21	24.54343	12.62181	0.80007	0	97012.51	0
22	18.64410	10.77368	1.00691	0	87067.78	0
23	14.69057	7.42732	0.55395	0	83296.71	0
24	11.61916	3.41643	0	0	69562.38	0

Table C.31 Generation Output and Interchange of Subareas in Multi-area System with 50 MW Import and Export Capability of Area 3 (Case Study 10)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1531.50	252.50	0.00	-270.00	1531.50	267.50	0.00	-320.00	1589.00	270.00
2	0.00	-130.00	1372.00	176.00	0.00	-187.00	1372.00	191.00	0.00	-237.00	1423.00	187.00
3	0.00	-78.00	1136.50	119.50	0.00	-126.00	1136.50	134.50	0.00	-176.00	1183.00	126.00
4	0.00	-69.00	997.00	59.00	0.00	-114.00	997.00	74.00	0.00	-164.00	1192.00	214.00
5	0.00	-94.00	935.50	59.50	0.00	-90.00	935.50	74.50	0.00	-140.00	1106.00	190.00
6	60.80	-77.20	1095.70	79.70	60.80	-147.20	1095.70	94.70	60.80	-197.20	1303.20	247.20
7	280.50	30.50	1150.00	-28.00	280.50	-39.50	1150.00	-13.00	60.80	-309.20	1577.20	359.20
8	304.00	-126.00	1150.00	-139.00	304.00	-196.00	1347.00	143.00	110.80	-439.20	1748.20	489.20
9	504.00	55.00	1150.00	-200.00	404.00	-115.00	1347.00	112.00	123.01	-445.99	1885.99	495.99
10	604.00	146.00	1150.00	-220.00	504.00	-24.00	1347.00	-42.00	211.90	-366.10	1915.10	416.10
11	601.00	102.00	1150.00	-288.00	601.00	71.00	1544.00	65.00	271.90	-308.10	1915.10	358.10
12	604.00	44.00	1150.00	-339.00	604.00	54.00	1741.00	186.00	285.95	-314.05	1984.05	364.05
13	158.72	-389.28	1754.65	280.65	158.72	-393.28	1961.50	541.50	158.72	-443.28	1968.70	403.70
14	140.98	-396.02	1743.78	231.78	140.98	-419.02	1950.63	553.63	140.98	-469.02	1950.63	498.63
15	142.41	-385.59	1744.66	197.66	142.41	-427.59	1951.51	565.51	141.84	-478.16	1951.16	528.16
16	175.60	-341.40	1764.98	175.98	175.60	-411.40	1971.83	626.83	150.15	-486.85	1976.85	436.85
17	264.58	-265.43	1770.00	128.00	289.58	-310.43	1976.85	497.85	340.95	-309.05	1988.05	259.05
18	304.00	-254.00	1770.00	87.00	391.32	-236.68	2038.68	453.68	363.30	-314.70	2160.70	264.70
19	304.00	-261.00	1980.45	280.45	353.89	-281.11	2085.67	311.67	362.94	-322.06	2158.06	272.06
20	304.00	-253.00	2000.93	211.93	358.13	-268.87	2000.93	259.93	354.64	-322.36	2098.36	372.36
21	304.00	-240.00	1958.12	258.12	304.00	-310.00	1958.12	270.12	349.45	-314.55	1965.32	336.32
22	272.42	-226.58	1907.90	229.90	272.42	-296.58	1907.90	287.90	272.42	-346.58	1838.95	351.95
23	201.40	-270.60	1907.90	320.90	201.40	-340.60	1838.95	273.95	201.40	-390.60	1838.95	406.95
24	72.56	-287.44	1885.71	396.71	72.56	-357.44	1747.81	273.81	72.56	-407.44	1747.81	381.81

Table C.32 Tie Line Flow and Area Net Interchange of Multi-area System with 50 MW Import and Export Capability of Area 3 (Case Study 10)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		15.83		-34.17		-18.33		-15.83		34.17		52.50	-2.50	-50.00
2	14.00		18.00		-32.00		-14.00		-18.00		32.00		46.00	4.00	-50.00
3	11.00		19.50		-30.50		-11.00		-19.50		30.50		41.50	8.50	-50.00
4	10.00		-30.00		20.00		-10.00		30.00		-20.00		-10.00	-40.00	50.00
5	-6.33		-21.83		28.17		6.33		21.83		-28.17		-34.50	-15.50	50.00
6	18.33		-34.17		15.83		-18.33		34.17		-15.83		2.50	-52.50	50.00
7	18.33		-34.17		15.83		-18.33		34.17		-15.83		2.50	-52.50	50.00
8	18.67		-34.33		15.67		-18.67		34.33		-15.67		-265.00	-53.00	50.00
9	-14.67		-17.67		32.33		14.67		17.67		-32.33		-145.00	-3.00	50.00
10	27.33		-38.67		11.33		-27.33		38.67		-11.33		-74.00	-66.00	50.00
11	-107.33		28.67		78.67		107.33		-28.67		-78.67		-186.00	136.00	50.00
12	-176.67		63.33		113.33		176.67		-63.33		-113.33		-295.00	240.00	50.00
13	-85.62		62.60		23.02		85.62		-62.60		-23.02		-108.63	148.22	-39.58
14	-99.62		35.00		64.62		99.62		-35.00		-64.62		-164.23	134.62	29.62
15	-108.62		29.31		79.31		108.62		-29.31		-79.31		-187.92	137.93	50.00
16	-126.95		88.47		38.47		126.95		-88.47		-38.47		-165.43	215.42	-50.00
17	-108.28		79.14		29.14		108.28		-79.14		-29.14		-137.43	187.43	-50.00
18	-128.00		89.00		39.00		128.00		-89.00		-39.00		-167.00	217.00	-50.00
19	-3.70		26.85		-23.15		3.70		-26.85		23.15		19.45	30.55	-50.00
20	-10.71		-19.64		30.36		10.71		19.64		-30.36		-41.07	-8.93	50.00
21	19.33		-20.55		1.22		-19.33		20.55		-1.22		18.12	-39.88	21.76
22	4.00		-4.68		0.68		-4.00		4.68		-0.68		3.32	-8.68	5.37
23	38.98		-27.67		-11.32		-38.98		27.67		11.32		50.30	-66.65	16.35
24	64.30		-19.33		-44.97		-64.30		19.33		44.97		109.27	-83.63	-25.63

Table C.33 System λ , System μ , α and β of Area 3, Fuel Cost, and Start-up Cost of Multi-area System
When Its Import and Export Capabilities Are Limited to 50 MW (Case Study 10)

Hour	λ (\$/MWh)	μ (\$/MW)	α of Area 3 (\$/MW)	β of Area 3 (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	11.07089	1.78443	0	0	40498.82	0
2	10.43944	1.70961	0	0	34989.64	0
3	9.79534	1.53431	0.10248	0	27424.53	0
4	9.94991	1.23084	0.03685	0	23154.29	0
5	6.76489	2.44483	0	0	21185.05	0
6	11.01898	5.42195	0.01869	0	30406.96	8582.40
7	14.98829	15.77095	2.10001	0	40762.89	0
8	22.97614	15.86186	2.63214	0	64151.64	19013.50
9	23.58957	16.02473	4.92145	0	82626.52	9339.00
10	30.31422	14.33097	5.59208	0	97237.11	10461.00
11	34.63007	18.33870	5.69344	0	113756.57	7375.50
12	25.26589	14.23046	1.31874	0	128862.83	8525.00
13	20.32655	11.52317	0	0	93537.05	5822.40
14	14.34960	7.43251	0	0	91844.44	0
15	14.42957	7.52114	0	0	91918.18	0
16	16.28968	9.44551	0	0	90713.42	0
17	25.60921	12.67354	1.81623	0	92795.43	0
18	29.86540	13.36585	6.14639	0	107413.89	874.50
19	35.10421	13.66915	5.24518	0	118789.39	8525.00
20	29.82759	12.26079	0.09344	0	111842.96	0
21	24.69100	12.31186	0	0	97106.17	0
22	19.40480	13.03323	0	0	80975.29	0
23	14.89304	8.43166	0	0	73780.34	0
24	11.90140	4.16485	0	0	59842.78	0

Table C.34 Generation Output and Interchange of Subareas in Multi-area System with 70 MW
Flow Limits between Areas 1 and 2 in Both Directions (Case Study 11)

Hour	Subarea 1		Subarea 2		Subarea 3		Subarea 4		Subarea 5		Subarea 6	
	$\Sigma P_{i,1}$	Intch ₁	$\Sigma P_{i,2}$	Intch ₂	$\Sigma P_{i,3}$	Intch ₃	$\Sigma P_{i,4}$	Intch ₄	$\Sigma P_{i,5}$	Intch ₅	$\Sigma P_{i,6}$	Intch ₆
1	0.00	-200.00	1550.67	271.67	0.00	-270.00	1550.67	286.67	0.00	-320.00	1550.67	231.67
2	0.00	-130.00	1389.00	193.00	0.00	-187.00	1389.00	208.00	0.00	-237.00	1389.00	153.00
3	0.00	-78.00	1152.00	135.00	0.00	-126.00	1152.00	150.00	0.00	-176.00	1152.00	95.00
4	0.00	-69.00	1062.00	124.00	0.00	-114.00	1062.00	139.00	0.00	-164.00	1062.00	84.00
5	0.00	-94.00	992.33	116.33	0.00	-90.00	992.33	131.33	0.00	-140.00	992.33	76.33
6	60.80	-77.20	1164.87	148.87	60.80	-147.20	1164.87	163.87	60.80	-197.20	1164.87	108.87
7	60.80	-189.20	1438.87	260.87	60.80	-259.20	1438.87	275.87	60.80	-309.20	1438.87	220.87
8	60.80	-369.20	1683.20	394.20	60.80	-439.20	1683.20	479.20	60.80	-489.20	1683.20	424.20
9	83.08	-365.92	1754.25	404.25	83.08	-435.92	1754.25	519.25	83.08	-485.92	1754.25	364.25
10	154.00	-304.00	1770.00	400.00	204.00	-324.00	1770.00	381.00	154.00	-424.00	1770.00	271.00
11	241.00	-258.00	1770.00	332.00	291.00	-239.00	1770.00	291.00	241.00	-339.00	1770.00	213.00
12	363.00	-197.00	1770.00	281.00	363.00	-187.00	1770.00	215.00	338.00	-262.00	1770.00	150.00
13	292.00	-256.00	1770.00	296.00	292.00	-260.00	1770.00	350.00	267.00	-335.00	1770.00	205.00
14	261.00	-276.00	1770.00	258.00	261.00	-299.00	1770.00	373.00	236.00	-374.00	1770.00	318.00
15	263.00	-265.00	1770.00	223.00	263.00	-307.00	1770.00	384.00	238.00	-382.00	1770.00	347.00
16	310.00	-207.00	1770.00	181.00	310.00	-277.00	1770.00	425.00	285.00	-352.00	1770.00	230.00
17	405.29	-124.71	1770.00	128.00	405.29	-194.71	1770.00	291.00	371.53	-278.48	1907.90	178.90
18	451.34	-106.66	1770.00	87.00	402.22	-225.78	1873.28	288.28	451.34	-226.66	2079.83	183.83
19	468.28	-96.72	1770.00	70.00	413.52	-221.48	2003.96	229.96	468.28	-216.72	2120.95	234.95
20	405.83	-151.17	1770.00	-19.00	405.83	-221.17	1985.43	244.43	456.75	-220.25	2093.15	367.15
21	304.00	-240.00	1770.00	70.00	347.33	-266.67	1947.47	259.47	433.99	-230.01	2036.21	407.21
22	264.08	-234.92	1770.00	92.00	264.08	-304.92	1907.90	287.90	289.08	-329.92	1976.85	489.85
23	170.08	-301.92	1770.00	183.00	170.08	-371.92	1907.90	342.90	195.08	-396.92	1976.85	544.85
24	60.80	-299.20	1682.28	193.28	60.80	-369.20	1820.18	346.18	85.80	-394.20	1889.13	523.13

Table C.35 Tie Line Flow and Area Net Interchange of Multi-area System with 70 MW
Flow Limits between Areas 1 and 2 in Both Directions (Case Study 11)

Hour	Tie Line Flow (MW)												Net Interchange (MW)		
	From Area 1	To Area 2	From Area 2	To Area 3	From Area 3	To Area 1	From Area 2	To Area 1	From Area 3	To Area 2	From Area 1	To Area 3	Area 1	Area 2	Area 3
1	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
2	14.00		35.00		-49.00		-14.00		-35.00		49.00		63.00	21.00	-84.00
3	11.00		35.00		-46.00		-11.00		-35.00		46.00		57.00	24.00	-81.00
4	10.00		35.00		-45.00		-10.00		-35.00		45.00		55.00	25.00	-80.00
5	-6.33		35.00		-28.67		6.33		-35.00		28.67		22.33	41.33	-63.67
6	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
7	18.33		35.00		-53.33		-18.33		-35.00		53.33		71.67	16.67	-88.33
8	-5.00		35.00		-30.00		5.00		-35.00		30.00		25.00	40.00	-65.00
9	-15.00		68.33		-53.33		15.00		-68.33		53.33		38.33	83.33	-121.67
10	13.00		70.00		-83.00		-13.00		-70.00		83.00		96.00	57.00	-153.00
11	7.33		59.33		-66.67		-7.33		-59.33		66.67		74.00	52.00	-126.00
12	18.67		46.67		-65.33		-18.67		-46.67		65.33		84.00	28.00	-112.00
13	-16.67		73.33		-56.67		16.67		-73.33		56.67		40.00	90.00	-130.00
14	-30.67		43.33		-12.67		30.67		-43.33		12.67		-18.00	74.00	-56.00
15	-39.67		37.33		2.33		39.67		-37.33		-2.33		-42.00	77.00	-35.00
16	-58.00		90.00		-32.00		58.00		-90.00		32.00		-26.00	148.00	-122.00
17	-31.00		65.29		-34.29		31.00		-65.29		34.29		3.29	96.29	-99.58
18	-27.39		35.11		-7.72		27.39		-35.11		7.72		-19.66	62.50	-42.84
19	-11.73		-3.25		14.98		11.73		3.25		-14.98		-26.72	8.49	18.23
20	-64.48		-41.21		105.69		64.48		41.21		-105.69		-170.17	23.27	146.90
21	-54.27		-61.47		115.73		54.27		61.47		-115.73		-170.00	-7.20	177.20
22	-41.97		-58.98		100.95		41.97		58.98		-100.95		-142.92	-17.02	159.93
23	-29.97		-58.98		88.95		29.97		58.98		-88.95		-118.92	-29.02	147.93
24	-27.63		-50.65		78.28		27.63		50.65		-78.28		-105.92	-23.02	128.93

Table C.36 System λ , System μ , Fuel Cost, Start-up Cost of Multi-area System, and θ of 70 MW Limited Flow between Areas 1 and 2 in Both Directions (Case Study 11)

Hour	λ (\$/MWh)	μ (\$/MW)	θ of Flows between Areas 1 and 2 in Both Directions (\$/MW)	Fuel Cost (\$)	Start-up Cost (\$)
1	10.58364	1.55378	0	40495.59	0
2	9.04966	2.48513	0	34987.65	0
3	7.05289	3.13037	0	27288.36	0
4	8.01594	3.00010	0	25807.85	0
5	6.87632	2.63571	0	24673.71	0
6	10.03579	4.95935	0	31368.65	8582.40
7	12.66760	11.72004	0	40411.56	0
8	16.91918	13.52502	0	48912.86	0
9	19.16798	15.49432	0	52318.12	0
10	25.62063	16.11558	0	59156.61	6226.00
11	25.26490	16.10505	0	67021.94	9339.00
12	23.41920	15.98002	0	75636.91	9339.00
13	19.63408	13.21097	0	72426.40	0
14	17.02799	10.82603	0	71142.27	0
15	17.08082	10.88516	0	71223.28	0
16	18.32241	12.27358	1.03629	73200.05	0
17	24.84508	14.50740	0	87798.83	8525.00
18	30.75690	14.72248	0	108330.06	11638.00
19	34.54435	12.76695	0	119608.38	4262.50
20	29.06911	12.31784	0	112819.47	0
21	23.65985	12.53967	0	97749.33	0
22	19.20606	10.69397	0	82373.93	0
23	15.55697	8.19789	0	78378.49	0
24	11.92633	3.82826	0	71050.71	0

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