# BRANCH-AND-CUT-AND-PRICE METHODS FOR LOGISTICS PROBLEMS WITH LIMITED RISK

by

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To my father

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**ABSTRACT**

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There exist many uncertainties in a logistics system, such as unknown demand, unsteady fuel cost, machine breakdowns, and accidents to name a few. Logistics management is difficult because logistics managers must solve a global optimization problem, which includes eliminating as much uncertainty as possible, finding effective methods of managing uncertainty, and operating the entire logistics system effectively. To cope with uncertainty, many efforts have been made for vehicle routing problems. On the contrary, traditional ship-scheduling models ignore uncertainty, even in highly volatile markets.

We present a set-packing model that limits risk using a quadratic variance constraint. After generating first-order linear constraints to represent the variance constraint, we develop a branch-and-cut-and-price (*delayed column and cut generation*, DCCG) algorithm for medium-sized ship-scheduling problems. Computational results show that we can significantly limit standard deviation of the profit with a small expected profit reduction. We also present a lagrangian decomposition method in which the set-packing model with a quadratic constraint can be reformulated as the sum of two integer problems by introducing linking variables and constraints. We explore a lagrangian-based heuristic method and a simple rounding heuristic. The heuristic methods are applicable throughout the branch-and-bound tree and can substantially improve the DCCG algorithm. By incorporating heuristic methods with the DCCG algorithm, we can find very good solutions effectively and reduce the CPU time significantly. Computational results are provided, and extensions are discussed.

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# CHAPTER 1

# INTRODUCTION

*Logistics* can be defined as getting the right quantity of products and services to the right locations, at the right time, and for the right price. The activities that take place in the logistics system include purchasing raw materials, manufacturing products, transporting them to warehouses for intermediate storage, and shipping finished goods to retailers or customers. *Logistics problems* can be viewed as "how these activities can be operated efficiently," and *logistics managers* organize and plan the activities by solving logistics problems so that there is a coordination of resources in an organization. These problems are the definition of *logistics management*, which "plans, implements, and controls the efficient, effective forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customers' requirements." (CSCMP, 2006)

Effective logistics management introduces three levels of decision-making problems; from the strategic through the tactical to the operations level. The strategic level decisions can be the number, location, and capacity of facilities, and the tactical level decisions may include purchasing, production, and fleet size determination. Lastly the operational level decisions are day-to-day decisions such as staffing, scheduling, routing, and truck loading. The last step in logistics activities is moving goods from distribution centers to customers on the most costly link of the logistics network (Bodin et al., 1983). If a logistics manager wants to manage distribution system effectively, he must approach the planning and execution of transportation activities in a rational manner. Thus the operational level decision-making problems, typically vehicle routing and scheduling problems, have drawn much attention from researchers.

Simchi-Levi et al. (2003) mentioned two general issues to describe the difficulty of logistics management. First, logistics consists of many systems, so finding the best systemwide strategy is challenging. It is often difficult to find the best design in a single facility so as to minimize costs while maintaining service level. When the whole system is being considered, the difficulty increases exponentially. Second, uncertainty is prevalent in logistics. Forecasting exact customer demand is almost impossible, travel times and fuel costs can vary, and machines and vehicles will break down. Thus logistics managers must be able to eliminate as much uncertainty as possible and to find effective methods of managing remaining uncertainty. In this dissertation, we focus on the operational level decision-making problems with uncertainty.

## 1.1 Uncertainty in Logistics Systems

The world is full of uncertainty and risk, e.g., natural disaster, unusual weather changes, threats of terrorism, depleted oil reserves, and war, which makes world economy unstable. In addition to them, there exist many uncertainties in logistics system, such as unknown demand, unsteady fuel cost, machine breakdowns, and accidents to name a few.

Major domestic airline carriers experience disruptions everyday. For example, pilot strikes, mechanical failures, extended in-flight delays and severe weather cause delays and traffic congestion. Average daily flight delays increased 20% from 1998 to 1999 and 16.5% from 1999 to 2000, which had been mostly caused by weather. Travel delays, runway delays, and flight cancellations increased 50%, 130%, and 68%, respectively, from 1996 to 2000 (Rosenberger, 2001). Historically, delays and flight cancellations cost multi-billion dollars per a year. Many efforts have been done to cope with these disruptions (see Lettovský et al., 1996; Cao and Kanafani, 1997; Luo and Yu, 1997; Lettovský et al., 2000; Schaefer, 2000; Rosenberger, 2001; Ehrgott and Ryan, 2003). In airline operations, typical standard deviations of the demand are 20-50% of the mean demand (Berge and Hopperstad, 1993), so inappropriate fleet utilization would result in heavy loss from either empty seats or seat shortage. On the contrary, slight improvement of flight utilization yields multi-million dollars profit increase in a year. For example, airline fleet assignment models yeild annual savings of $100 million at Delta, $15 million annually at USAirways, and a 1.4% improvement in operating margins at American Airlines (Pilla et al., 2005). Fleet management research is prevalent in academic literature (see Botimer and Belobaba, 1999; Weatherford and Belobaba, 2002; List et al., 2003; Bish et al., 2004; Lohatepanont and Barnhart, 2004; Listes and Dekker, 2005).

Numerous organizations operate fleets of trucks to move goods. It is no exaggeration to say that every single product is delivered by a truck. The number of large trucks was over 8 million in 2000, and truck-involved accidents was 28% of total crashes. The average costs of large truck-involved crashes and multiple combination truck-involved crashes were $59,153 and $88,483, respectively (Zaloshnja and Miller, 2004). We never experience a day without accidents that result in delays, changes in fuel costs, and vehicle breakdowns, which demonstrate the fact that trucking operations are very vulnerable to uncertainties. Some studies related to trucking under uncertainty are as follows. Fenga et al. (2004) solved the truck despatching problem for delivering mixed concrete to construction sites, which considers travel time, unknown demand, and job rates at sites. Powell et al. (2000) captured uncertainties such as customer demands, travel times, and user noncompliance using myoptic greedy method. Golshani et al. (1996) and Ghiani et al. (2003) considered real time traffic information to find optimal routes. Haghani and Jung (2005) and Jula and Dessouky (2006) considered travel times in urban areas due to a variety of factors, such as accidents, traffic conditions and weather conditions.

Maritime logistics problems optimize the transportation of commodities, so they are vital to world trade and military logistics. A ship requires a multi-million dollar capital investment, and the daily operating costs of a ship can be tens of thousands of dollars. Consequently, improved fleet utilization can yield significant financial benefit. However, maritime transportation problems have attracted less attention in comparison with inland vehicle routing and scheduling problems. Ronen (1983) explains the reason for the low attention drawn by maritime transportation problems in terms of (1) *low visibility* — the use of truck or rail is much larger than that of ships in the U.S.A., so most maritime transportation studies has been done in European countries, which depend more on ocean shipping, (2) *less structured* — problem structures and operating environments are full of variety, (3) *much more uncertainty in ship operations* — severe weather, mechanical problems, and strikes can cause delays, and expensive daily operating cost gives very little slack in their schedules, (4) *a volatile, international, capital intensive, and relatively free market* — different national laws and regulations can be applicable, and the shipping market is a perfect free market, (5) *long tradition* — the long tradition of conservative thinking in shipping industry does not welcome new ideas such as supportive optimization techniques.

Research for real shipping business cases, such as Chajakis (1997) and Chajakis (1999), have shown that significant savings can be achieved by managing demand uncertainty and optimizing ship operations. To the best of our knowledge, we can find little research on disruptions or uncertainties, e.g., market fluctuation, variable fuel cost, ship breakdowns, or unknown demand, in ship routing and scheduling problems. Some of them include Lo and McCord (1998), Azaron and Kianfar (2003), and Cheng and Duran (2004). This fact motivated us for further research on limiting risk of ship scheduling.

## 1.2 Dissertation Objectives and Outline

In this dissertation, we formulate a new ship-scheduling model as a set-packing problem with a quadratic constraint that limits the variance of shipping profit. Then we develop a branch-and-cut-and-price method that generates both columns and cuts in branch-and-bound trees, which is an exact algorithm incorporating Kelley's cutting plane method. We also propose an alternative model, which reformulates the quadratic constraint into general packing constraints based on lagrangian decomposition. Then we explore heuristic methods that include both a lagrangian heuristic utilizing subgradient optimization and a simple rounding heuristic. The lagrangian heuristic uses a modified column generation for finding promising columns. The heuristic methods are incorporated with the branch-and-cut-and-price method. We conduct computational experiments for medium-sized ship-scheduling problems by using the branch-and-cut-and-price method and the heuristic methods. Computational results are discussed, and future research topics are identified.

In Chapter 2, a brief background on vehicle routing and scheduling under uncertainty and solution methods is described. In Chapter 3, the need for managing risk in logistics problems, risk in ship scheduling, and the ship-scheduling models that limit variance are developed. Solution approaches and computational results based on a branch-and-cut-and-price algorithm are presented in Chapter 4. The heuristic methods incorporating with both a lagrangian-based heuristic and a simple rounding heuristc, and computational results are presented in Chapter 5. Finally, conclusions and future research topics are given in Chapter 6.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Vehicles Routing and Scheduling

The *vehicle routing problem* (VRP) is a model that determines a set of minimum cost routes, originating and terminating at a depot, for a fleet of vehicles that services a set of geographically dispersed cities or customers with known demands (Ahuja et al., 1993). The VRP arises naturally as a central problem in the fields of transportation, distribution and logistics. Practical examples of the VRP include the delivery of packages to customers, the delivery of end-user products to retail stores, the collection of money from banks, and the routes of school buses to ride students.

The VRP is $\mathcal{NP}$-hard and a well known integer programming problem. The VRP is defined as follows. Let $G = (N, A)$ be a network, where $A$ is the arc set and $N$ is the node set. Node 1 represents a common depot while the remaining nodes correspond to cities or customers. An arc represents a route segment, where each route starts and ends at the depot. Associated with each node $i \in V$ is a non-negative demand $d_i$. Associated with each arc $(i, j) \in A$ is a cost or distance $c_{ij}$. There is a fleet of capacitated vehicles based at the depot. These vehicles are homogeneous and have capacity $u$. The VRP determines the minimum cost set of routes for delivering the goods to the customers so that each customer is serviced exactly once by one vehicle and the total customer demand on any route does not exceed $u$. In real world VRPs, many side constraints appear, and there are different variants. For example, the fleet of vehicles may be nonhomogeneous, every customer may have to be supplied within a certain time window, several depots may exist, customers may be served by different vehicles, some values (such as number of customers, their demands, service time or travel time) are random.

VRPs decide the spatial configuration of vehicle movements and involve the specification of a sequence of locations that a vehicle may visit, while vehicle scheduling problems consider the times at which locations are visited. Therefore, *the vehicle scheduling problem* can be considered as a VRP with additional constraints related to the times when various activities take place. For general formulations and solution methods on vehicle routing and scheduling are described in Bodin et al. (1983), Laporte (1992), Tan et al. (2001), Teng et al. (2003), and Bräysy and Gendreau (2005a,b).

## 2.2 Ship-Scheduling

Shipping industries provide maritime transportation services between countries, which play an important role in international trade moving goods by sea. Growing international trade requires more and larger ships, and shipping industries enable suppliers to meet increasing demand thanks to the new technologies, which could build ultra-large crude carriers, huge container ships, roll-on/roll-off vessels, and ultramodern LPG/LNG ships. The success of a shipping company lies on the allocation and operation of its ships. Even if all of the strategic level decisions that are related to fleet size, trade routes, and ports and terminals to use are made, tactical and operational level decisions, such as ship scheduling and routing, remain challenging and critical.

Since the pioneering work of Dantzig and Fulkerson (1954), ship-scheduling problems have been studied extensively in academic literature, and Christiansen et al. (2004) surveys the literature prior to 2004. Since then, a few more studies on ship-scheduling have been conducted, e.g., Bhasi (2004), Fagerholt (2004), and Persson and Göthe-Lundgren (2005). There are three types of shipping operations—*industrial operators*, *tramp shippers*, and *liners*. Industrial operators deliver their own cargoes on their own ships at minimal cost, while tramp shippers transport cargoes for other companies. Tramp shippers often have some cargoes under contract that they must ship, *contracts of affreightment*, so general optimization models for industrial and tramp shippers are formulated similarly. Unlike industrial and tramp shippers, liners operate according to published schedules, so they differ significantly from the other two types of shipping operations. In this dissertation, we focus on the industrial and tramp-shipper problems.

A *cargo* is the entire content of a ship transported between two ports, and a *schedule* is a sequence of cargoes delivered by the same ship. Ship-scheduling problems are solved by generating a set of feasible delivery schedules for each ship and optimizing a set-packing (or set-partitioning) problem so that overall costs (or profits) are minimized (or maximized). (see Bausch et al., 1998; Kim, 1999; Fagerholt and Christiansen, 2000a,b; Fagerholt, 2001; Christiansen and Fagerholt, 2002; Bhasi, 2004; Fagerholt, 2004; Persson and Göthe-Lundgren, 2005). However, most traditional set-packing (or set-partitioning) models ignore uncertainty.

## 2.3 Vehicle Routing Under Uncertainty

Approaches to optimization under uncertainty include stochastic programming, fuzzy programming, and stochastic dynamic programming (Sahinidis, 2004). Because there are many uncertainties in vehicle routing such as, stochastic demands, stochastic travel times, and unknown customers, the *stochastic vehicle routing problem* (SVRP) has received considerable attention in the literature.

SVRPs are modeled as two-stage *stochastic programs* (SPs), which consist of two groups of uncertain decision variables according to the period when these decisions are taken. A number of decisions have to be taken without full information on some random events, which is called *first-stage decisions*. After the uncertainty of these random events is revealed, further information is available, and a number of decisions can be taken as *recourse actions*, which called *second-stage decisions*. An SP is modeled either as a *stochastic program with recourse* (SPR) or a *chance constrained program* (CCP). The goal of an SPR is to determine first-stage decisions so as to minimize the sum of the cost of first-stage decisions and the expected cost of second-stage decisions, and CCPs don't consider the cost of corrective actions in case of failure (Gendreau et al., 1996; Birge and Louveaux, 1997).

Vladimirou and Zenios (1997), Morton and Wood (1999), Beraldi et al. (2000), and Sherali and Fraticelli (2002) provided formulations and algorithms for SPRs, using parametric programs, restricted-recourse bounds, parallel algorithms, and *lift-and-project* cutting plane, respectively. Carøe and Tind (1998), Carøe and Schultz (1999), and Ahmed et al. (2004) developed branch-and-bound algorithms for stochastic integer programs. *L-shaped method* is an extension to the stochastic integer case of *Benders' decomposition* (Benders, 1962), and Laporte et al. (2002) studied an implementation of the integer L-shaped method (Laporte and Louveaux, 1993) for the exact solution of the SVRP. List et al. (2003) and Beraldi et al. (2004) presented formulations and solution procedures for *robust optimization* (Mulvey et al., 1995; Takriti and Ahmed, 2004) for fleet planning under uncertainty. Kleywegt et al. (2004) and Jula and Dessouky (2006) proposed approximate solution methods based on dynamic programming.

Gendreau et al. (1995), Hjorring and Holt (1999), and Secomandi (2001) presented SVRP formulations with probabilistic demands and/or customers. Gendreau and Laporte (1996) described a *tabu search* algorithm. Haughton (1998, 2000) discussed the benefits and learning requirements regarding the re-optimization of routes. Swihart and Papastavrou (1999) developed a stochastic and dynamic model for the pick-up and delivery problem. Some studies on the SVRP with random travel and service times include Laporte (1992) and Kenyon and Morton (2003).

Cheng and Duran (2004) captured stochastic inventory and transportation system in crude oil shipping by integrating discrete event simulation and stochastic optimal control. Azaron and Kianfar (2003) used stochastic dynamic programming to find shortest paths for the ship routing problem assuming that the environmental variables of the adjacent nodes are fully known in stochastic dynamic networks. Lo and McCord (1998) formulated a dynamic programming model to find optimal strategic routes for saving fuel costs.

## 2.4 Branch-and-Cut-and-Price for Vehicle Routing

The *Branch-and-bound* (BB) algorithm is a divide-and-conquer approach for solving difficult integer programs (IPs). The basic structure of the algorithm is an enumeration tree, and a given IP starts on the root node. The algorithm divides the IP into simpler subproblems, and each of them is independently solved. The tree grows by using an appropriate *branching* scheme, which generates two or more child nodes of the parent node. Each subproblem at the child nodes is generally created by adding a bound on a single integer variable of the problem at the parent node. At each node of the tree, solving a *linear programming* (LP) *relaxation* (relaxing the integrality requirement on the variables) of the current problem yields a bound on the value of an optimal solution, which is called the *bounding* phase. If a better feasible solution is found, the current best solution is replaced with the new solution. If the subproblem is infeasible or the bound is not better than the value of the best feasible solution thus far, the node is pruned, otherwise branching occurs. The algorithm continues until an optimal solution is achieved.

If the IP contains huge number of decision variables (*columns*), then the *branch-and-price* (BP) algorithm considers only a small subset of the columns in the LP relaxation of the problem. To check the optimality of the LP solution, new columns that may enter the current basis are iteratively identified through the solution of a *pricing problem*, which is called *column generation* (Barnhart et al., 1998). If no columns price out to enter the current basis and the LP solution is not integral, branching occurs. Applying a standard BB strategy over the subset of the columns is unlikely to give an optimal solution to the IP. Thus the algorithm, a generalization of BB with LP relaxation, applies column generation throughout the BB tree.

In the *branch-and-cut* (BC) algorithm, the LP relaxation of the IP initially contains only a subset of the constraints of the original problem. An inequality (constraint or *cut*) that satisfies all feasible solutions is called a *valid inequality*. Valid inequalities that are not part of the current problem and violated by the optimal solution of the current LP relaxation are called *violated cuts*. The violated cuts are identified through the solution of the *separation problem*, which can be solved by exact or heuristic *separation procedures* (see Fischetti et al., 2001; Ralphs et al., 2003a). By adding violated cuts to the LP relaxation, which is called the *cutting plane* method (Gomory, 1958; Winston, 1994), the LP feasible region is tightened without changing the IP feasible region. Then the modified LP relaxation is resolved, and the procedure repeats until no violated cuts are found. The algorithm, another generalization of BB with LP relaxation, employs cutting planes throughout the BB tree.

The *branch-and-cut-and-price* (BCP) algorithm combines BP and BC methods, so at each node, both new variables and new cuts can be added. Additional difficulties arise since the algorithm must dynamically generate cutting planes without destroying the structure of the column generation subproblem. Developing efficient data structures for representing the objects as well as implementation of such methods are also very challenging.

References on effective modelling, preprocessing, implementation, and the methodologies for BCP include Barnhart et al. (2000), Johnson et al. (2000), Toth (2000), and Pangborn (2002).

Elhedhli and Goffin (2004) proposed adding cuts to recover primal feasibility and discussed the integration of an interior-point cutting plane method within a branch-and-price algorithm. Ralphs et al. (2003a) also described separation methodology based on decomposition. Jüenger and Thienel (2000) and Ralphs et al. (2003b) described the implementation and parallelization of BCP, respectively.

Barnhart et al. (2000) provided BCP solution strategy for origin-destination integer multicommodity flow problems and results for test problems arising from telecommunications. Longo et al. (2006) applied BCP to a capacitated vehicle routing problem that was transformed from a capacitated arc routing problem, which can be applicable to street garbage collection, postal delivery, and routing of electric meter readers. Verweij and Aardal (2003) formulated the pickup and delivery problem of the Dutch logistics company Van Gend & Loos as a merchant subtour problem (MSP) and used BCP as well as a tabu search algorithm. After finding a good feasible solution with the tabu search, they solved the MSP optimally with BCP. Kim et al. (1999) used BCP to solve an express package delivery problem formulated as a route-based model, which is one of the large scale transportation service network design problems with time windows that are prevalent in rail, airline, trucking, and intermodal industries (Kim and Barnhart, 1997).

## 2.5 Lagrangian Relaxation for Vehicle Routing

To find an optimal solution for a given minimization IP, a good algorithm should be able to discover good quality upper bounds as well as lower bounds. Lagrangian relaxation is available to find very good lower bounds for the IP. Upper bounds for the IP are found by searching for feasible solutions, for which some heuristics such as, interchange, tabu search, simulated annealing, and genetic algorithms, can be used.

Lagrangian relaxation involves; attaching lagrange multipliers to some of the hard constraints in a given IP formulation, adding these constraints into the objective function, and solving the resulting IP with remaining easy constraints. The quality of the lower bound depends upon determining values for the lagrangian multipliers, in which two general techniques are used, i.e., *subgradient optimization* and *multiplier adjustment*. Finding the values for the multipliers that provide maximum lower bound is called the *lagrangian dual problem*.

A lagrangian relaxation problem can be separated into the sum of several linear programs by introducing different artificial variables, adding all possible equality linking constraints between artificial variables and original variables, and relaxing the equality linking constraints, which is known as *lagrangian decomposition*. The solution of the lagrangian relaxation problem can be converted into a feasible solution for the original problem by proper adjustment, and this method is called *lagrangian heuristics*. Refer to Nemhauser and Wolsey (1988) and Beasley (1992) for more detailed information on lagrangian relaxation.

Ralphs and Galati (2006) provided a generic theoretical framework for generating dynamic cuts based on various decomposition methods, which could be extended to *decompose and cut*, a decomposition-based separation technique. Kallehauge et al. (2006) presented *lagrangian branch-and-cut-and-price* algorithm that used stabilized cuts (Jüenger and Thienel, 2000) and strong valid inequalities.

Ribeiro and Soumis (1994) and Huisman et al. (2005) combined column generation and lagrangian relaxation for multiple-depot vehicle scheduling problem and for integrated vehicle & crew scheduling (Freling et al., 2003), respectively. Fukasawa et al. (2006) combined BC and BP with lagrangian relaxation for capacitated VRP. Rana and

Vickson (1988, 1991) provided formulations and solution methods using lagrangian relaxation and decomposition for container ship operations.

# CHAPTER 3

# LIMITING RISK IN LOGISTICS PROBLEMS

Risk is prevalent in today's global economy. Threats of terrorism, depleted oil reserves, and war are examples of new sources of instability. Incomes in the United States have become more volatile, bankruptcies are more frequent, and households are shouldering more uncertainty (Gosselin, 2004). As economic fluctuation increases, logistics managers need to find methods of managing risk. We modify here a traditional set-packing problem for ship scheduling by adding a quadratic constraint to limit the variance of profit. The new model is formulated as

$$\max \mathbf{cx} \tag{3.1}$$

$$\text{s.t. } \mathbf{Ax} \leq \mathbf{1} \tag{3.2}$$

$$\mathbf{x^T Q x} \leq d \tag{3.3}$$

$$\mathbf{x} \in \{\mathbf{0}, \mathbf{1}\}^n, \tag{3.4}$$

where $A$ is a 0-1 matrix, and $Q$ is a symmetric positive definite matrix in which an entry in the matrix $[q_{ij}]$ is the covariance of profit for selecting both sets $i$ and $j$. Although the application considered in this dissertation is commercial ship-scheduling, the use of the set-packing problem is prevalent in industry. Some applications include air traffic flow management (e.g., Rossi and Smriglio, 2001), aircraft rescheduling (e.g., Andersson and Värbrand, 2004), and plant location (e.g., Cho et al., 1983; Canovas et al., 2002). Moreover, a set-packing problem can be easily transformed into a set-partitioning problem, and some set-partitioning applications include commercial airline crew scheduling (e.g., Vance et al., 1997; Klabjan et al., 2001), aircraft rerouting (e.g., Rosenberger et al., 2003), vehicle routing (e.g., Desaulniers, 2003; Sindhuchao, 2005), political redistricting (Mehrotra, 1998), and organ transplantation (Kong, 2005).

## 3.1 Risk in Ship-Scheduling

Both industrial and tramp shippers often transport additional cargoes from the *spot market* when capacity is available. When an entire ship is available, they will place it on the *spot charter market*, so other shipping companies can charter it. If an industrial or tramp shipper has a cargo that it cannot transport, this cargo is placed on the spot market. The importance of *spot rate costs* is addressed in an example of Fisher and Rosenwein (1989), *the Tanker Division of the Military Sealift Command of the U.S. Navy*, which is responsible for transporting bulk petroleum products world-wide with a fleet of approximately 20 tanker ships. They calculate total profit by the total spot rate costs that would have incurred if the cargo had been delivered by spot charters minus the operating costs of ships in their fleet.

Each year the *International Tanker Nominal Freight Scale Association Ltd. (ITN-FSAL)* calculates a set of values that estimate the cost of shipping between any combination of ports using a standard ship, called *Worldscale (WS) 100*. In addition to these values, *Worldscale* publishes the current market value of shipping freight in terms of a direct percentage of the WS 100 rates (Worldscale, 2000). The fluctuation of WS, or *spot tanker freight rates*, for the past five years (weekly: Jan.2001-Oct.2005) is well depicted in Figure 3.1.

As shown in the figure, the WS is highly variable, and, in OPEC data (OPEC, 2005), the fluctuation rate is as high as 116% of WS 100 within one month. When we convert WS into U.S. dollars, the maximum monthly fluctuation is $38.47/tonne, which is recorded in Gulf/West route during November and December, 2004. Considering the fact that a Very Large Crude Carrier (VLCC) is in the range of 150,000 to 300,000 tonne, the

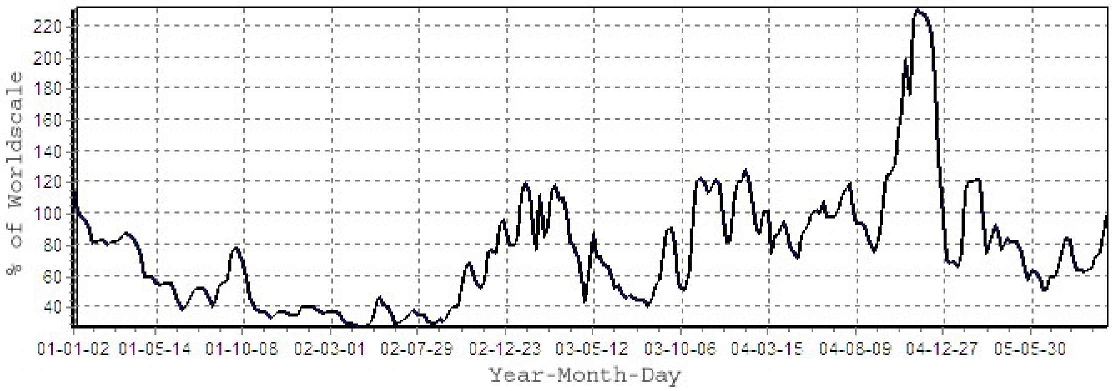


Figure 3.1. Spot Rates; VLCC-AG/WEST (Hanbada Corporation, 2003).

shipping cost increment of a VLCC could be \$5,770,500 to \$11,541,000 within a month. Even a small change of WS could easily increase shipping costs tens of thousands of dollars. Consequently, the cost of shipping on the spot market is extremely volatile, so managing these fluctuations is critically important for a shipper's success. In this dissertation, we limit volatility in ship-scheduling by constraining the variance of shipping profit.

## 3.2 New Ship-Scheduling Models

In this section, we formulate a new ship-scheduling model as a set-packing problem with a quadratic constraint that limits the variance of shipping profit.

Industrial and tramp shippers must transport contracted cargoes from origin to destination. In addition, they may rent out some of their ships, or they may deliver additional cargoes from the spot market when capacity is available. Conversely, they may also ship some contracted cargoes in the spot market. Most industrial and tramp-shipping problems are modeled as set-packing (or set-partitioning) problems (Christiansen et al., 2004), but we consider the variability of profit. The operating costs of the fleet are relatively constant and controllable compared with the randomness of the spot rate costs, so we focus on the volatility of the spot market.

Let $V$ be the set of ships to be scheduled, and let $K$ be the set of cargoes. Suppose set $K$ is divided into two sets of cargoes: $K_1$ is the set of cargoes in contracts of affreightment, and $K_2$ is the set of optionally shipped cargoes from the spot market. For each ship $v \in V$, let $F_v$ denote a set of candidate schedules, and let random variable $\tilde{g}_v$ be the spot rate cost if the company charters out ship $v$. Let $F$ be the set of all candidate schedules $F = \bigcup_{v \in V} F_v$. For each ship $v \in V$ and each schedule $f \in F_v$, let constant $c_{vf}$ be the cost of covering schedule $f$ with ship $v$, and let the binary variable

$$x_{vf} = \begin{cases} 1, & \text{if ship } v \text{ covers schedule } f; \\ 0, & \text{otherwise.} \end{cases}$$

For each ship $v \in V$, each schedule $f \in F_v$, and each cargo $k \in K$, the binary constant $a_{kvf}$ indicates whether ship $v$ delivers cargo $k$ in schedule $f$. For each cargo $k \in K$, let random variables $\tilde{r}_k$ and $\tilde{e}_k$ be the revenue and the spot rate cost of delivering cargo $k$ with a spot charter, respectively.

Then the industrial and tramp shippers ship-scheduling problem can be written as

$$\max \sum_{k\in K_1\cup K_2}\sum_{v\in V}\sum_{f\in F_v} E\left[\tilde{r}_k\right] a_{kvf}x_{vf} - \sum_{k\in K_1} E\left[\tilde{e}_k\right] s_k + \sum_{v\in V} E\left[\tilde{g}_v\right] u_v - \sum_{v\in V}\sum_{f\in F_v} c_{vf}x_{vf} \tag{3.5}$$

$$\text{s.t.} \sum_{v\in V}\sum_{f\in F_v} a_{kvf}x_{vf} + s_k = 1,\ \forall k \in K_1, \tag{3.6}$$

$$\sum_{v\in V}\sum_{f\in F_v} a_{kvf}x_{vf} \leq 1,\ \forall k \in K_2, \tag{3.7}$$

$$\sum_{f\in F_v} x_{vf} + u_v = 1,\ \forall v \in V, \tag{3.8}$$

$$x_{vf} \in \{0,1\},\ \forall f \in F_v, v \in V, \tag{3.9}$$

$$s_k \geq 0, \tag{3.10}$$

$$u_k \geq 0, \tag{3.11}$$

where, $E\left[\cdot\right]$ represents the expected value of $[\cdot]$, $s_k$ is a binary variable that is equal to one if cargo $k$ is serviced by a spot charter and zero otherwise, and $u_v$ is a binary variable that is equal to one if ship $v$ is chartered out on the spot market and zero otherwise. Variables $s_k$ and $u_v$ need not be defined as binary variables because of constraints (3.6) and (3.8). The profit of assigning ship $v$ to cover schedule $f$ is thus given by

$$\sum_{k\in K}\tilde{r}_k a_{kvf} + \sum_{k\in K_1}\tilde{e}_k a_{kvf} - \tilde{g}_v - c_{vf}.$$

Observe that $\tilde{e}_k$ represents the reduction in opportunity cost of having to ship cargo $k$ on the spot market, and similarly $\tilde{g}_v$ is the lost opportunity from using ship $v$ instead of selling it on the spot market.

To simplify notation, let

$$\tilde{r}_{vf} = \sum_{k\in K}\tilde{r}_k a_{kvf} \qquad \tilde{e}_{vf} = \sum_{k\in K_1}\tilde{e}_k a_{kvf}.$$

The ship-scheduling problem with limited profit variance (SPLPV) then becomes

$$\max \sum_{v\in V}\sum_{f\in F_v}\left(E\left[\tilde{r}_{vf}\right]+E\left[\tilde{e}_{vf}\right]-c_{vf}-E\left[\tilde{g}_v\right]\right)x_{vf} \tag{3.12}$$

$$\text{s.t.} \sum_{v\in V}\sum_{f\in F_v} a_{kvf}x_{vf} \leq 1,\ \forall k\in K, \tag{3.13}$$

$$\sum_{f\in F_v} x_{vf} \leq 1,\ \forall v\in V, \tag{3.14}$$

$$var\left(\sum_{v\in V}\sum_{f\in F_v}(\tilde{r}_{vf}+\tilde{e}_{vf}-c_{vf}-\tilde{g}_v)x_{vf}\right) \leq d, \tag{3.15}$$

$$x_{vf}\in\{0,1\},\ \forall f\in F_v, v\in V. \tag{3.16}$$

Here, objective function (3.12) maximizes expected profit. The constraints in set (3.13) ensure that cargoes in contracts of affreightment and profitable spot cargoes are serviced, while constraint set (3.14) implies that each ship in the fleet is assigned to exactly one schedule or chartered out on the spot market. Constraint (3.15) limits the variance of the profit to a fixed value $d > 0$, a measure traditional ship-scheduling models ignore. Finally set (3.16) represents the binary requirements on the variables.

We can rewrite constraint (3.15) as

$$\sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}} cov(\tilde{r}_{v_1f_1}+\tilde{e}_{v_1f_1}-\tilde{g}_{v_1}, \tilde{r}_{v_2f_2}+\tilde{e}_{v_2f_2}-\tilde{g}_{v_2})x_{v_1f_1}x_{v_2f_2} \leq d. \tag{3.17}$$

Variance of a random variable is positive, so the covariance matrix must be symmetric and positive definite (Wu, 2002). For each pair of ships $(v_1, v_2)\in V\times V$, we denote the covariance of costs from assigning ship $v_1$ to schedule $f_1\in F_{v_1}$ and assigning ship $v_2$ to schedule $f_2\in F_{v_2}$ as $q_{v_1f_1v_2f_2}$. Because the covariance matrix $Q=[q_{v_1f_1v_2f_2}]$ is symmetric and positive definite, the quadratic function $x^TQx$ is convex. By Kelley's cutting plane method (Kelley, 1960), we can replace the quadratic constraint (3.17) by an infinite set of first-order constraints given by

$$2\sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}} q_{v_1 f_1 v_2 f_2} w_{v_1 f_1} x_{v_2 f_2} \leq \ d + \sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}} q_{v_1 f_1 v_2 f_2} w_{v_1 f_1} w_{v_2 f_2},$$
$$\forall w \in \Re^{|F|}. \tag{3.18}$$

The formulations represented by (3.12)–(3.16) and (3.12)–(3.14), (3.16), and (3.18) are known to be equivalent in convex programming (Kelley, 1960).

## Tightening Constraints

Each constraint in set (3.18) can be tightened to

$$2w^T Qx \leq 2\sqrt{dw^T Qw} \qquad \forall w \in \Re^{|F|}. \tag{3.19}$$

By the triangle inequality, for each real vector $w \in \Re^{|F|}$,

$$2\sqrt{dw^T Qw} \leq d + w^T Qw,$$

so constraints in set (3.19) are at least as tight as those in set (3.18).

**Proposition 1.** *For all $w \in \Re^{|F|}$, the associated constraint in set (3.19) is a valid inequality.*

*Proof.* The constraints in both (3.18) and (3.19) for which $w = 0$ are redundant. For a vector $w \in \Re^{|F|} \setminus \{0^{|F|}\}$, $w^T Qw > 0$ because $Q$ is a positive definite matrix. Let $u = hw$, where $h$ is a positive constant equal to $\sqrt{\frac{d}{w^T Qw}}$. The constraint

$$2u^T Qx \leq d + u^T Qu$$

in the set (3.18) is a valid inequality. This implies

$$\begin{aligned} & 2w^TQx \leq \frac{d + h^2 w^T Q w}{h} \\ \Longrightarrow\ & 2w^TQx \leq \frac{\sqrt{w^TQw}}{\sqrt{d}} d + \frac{\sqrt{d}}{\sqrt{w^TQw}} w^TQw \\ \Longrightarrow\ & 2w^TQx \leq 2\sqrt{dw^TQw}. \end{aligned}$$

□

Note that the constraints in set (3.19) are tangent to the quadratic constraint (3.15).

### 3.3 Modeling Random Profits

Market shortages and surpluses may cause large increases and decreases on all chartering rates. Consequently, the spot rate cost $\tilde{e}_k$ will be given by

$$\tilde{e}_k = \alpha_k^e + \beta_k^e \tilde{M} + \tilde{\gamma}_k^e,$$

where $\tilde{M}$ is an independent random variable representing the fluctuation of spot market prices, $\alpha_k^e$ is the expected cost of chartering a ship on the spot market, $\beta_k^e$ is a constant rate for how the market random variable $\tilde{M}$ changes the spot rate, and $\tilde{\gamma}_k^e$ is an independent random variable for the fluctuation from $\alpha_k^e$. An implicit assumption is that shipping rates are linearly related to a single market chartering rate. Because of the dominance of WS on shipping rates, this assumption is reasonable. This type of random profit modeling can often be found in calculating the return on a portfolio (Sharpe, 1970).

The values $\alpha_k^e$, $\beta_k^e$, and $\tilde{\gamma}_k^e$ can be adjusted so that, without loss of generality, $E[\tilde{M}] = E[\tilde{\gamma}_k^e] = 0$, $E[\tilde{e}_k] = \alpha_k^e$, $var(\tilde{M}) = 1$, and $var(\tilde{e}_k) = \beta_k^{e2} + var[\gamma_k^e]$. For each cargo not under contract $k \in K_2$, we let $\alpha_k^e = \beta_k^e = \tilde{\gamma}_k^e = 0$. The random variables $\tilde{r}_k$ and $\tilde{g}_v$ are analogously defined; that is,

$$\tilde{r}_k = \alpha_k^r + \beta_k^r \tilde{M} + \tilde{\gamma}_k^r,$$

$$\tilde{g}_v = \alpha_v^g + \beta_v^g \tilde{M} + \tilde{\gamma}_v^g.$$

To simplify notation, let $\alpha_k = \alpha_k^r + \alpha_k^e$, $\beta_k = \beta_k^r + \beta_k^e$, $\tilde{\gamma}_k = \tilde{\gamma}_k^r + \tilde{\gamma}_k^e$, and $\beta_{vf} = \sum_{k \in K} \beta_k a_{kvf} + \beta_v^g$. For each pair of ships $(v_1, v_2) \in V \times V$, the covariance of costs for assigning ship $v_1$ to schedule $f_1$ and assigning ship $v_2$ to schedule $f_2$, $q_{v_1 f_1 v_2 f_2}$, is given by

$$q_{v_1 f_1 v_2 f_2} = \beta_{v_1 f_1} \beta_{v_2 f_2} + \sum_{k \in f_1 \cap f_2} var(\tilde{\gamma}_k) + var(\tilde{\gamma}_{v_1}^g) I_{v_1 = v_2}, \tag{3.20}$$

where the binary constant $I_{v_1=v_2}$ is defined as

$$I_{v_1=v_2} = \begin{cases} 1, & \text{if ships } v_1 \text{ and } v_2 \text{ are the same ship;} \\ 0, & \text{otherwise.} \end{cases}$$

# CHAPTER 4

# BRANCH-AND-CUT-AND-PRICE APPROACH

In Chapter 3, we described two formulations for ship-scheduling with constrained risk; one used a single quadratic constraint, while the other included an infinite set of first-order constraints. In this chapter, we develop a branch-and-cut-and-price to solve the latter model.

## 4.1 Delayed Column-and-Cut Generation

The simplest method to solve the continuous relaxation of SPLPV (CSPLPV) is Enumerated Kelley's Cutting Plane algorithm (EKCP), which is summarized in Algorithm 1.

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**Algorithm 1** Enumerated Kelley's Cutting Plane Algorithm (EKCP)

---

**Restricted Master Problem (RMP) Step:** Let $\mathcal{W} \subset \Re^{|F|}$ be a finite set, and solve the linear programming relaxation of (3.12)–(3.14), (3.16), and a subset of constraints (3.19) using $\mathcal{W}$ to obtain $x^*$.

**if** ${x^*}^T Q x^* > d + \varepsilon$, where $\varepsilon > 0$ is a very small constant **then**

  **Cut Generation Step:** $\mathcal{W} \leftarrow \mathcal{W} \cup \{x^*\}$ and return to the RMP Step.

**else**

  Return the optimal solution $x^*$.

**end if**

---

In this section, we develop a new delayed column-and-cut algorithm which combines delayed column generation with EKCP to solve CSPLPV. Let $\pi$ and $\rho$ be dual vectors for constraint sets (3.13) and (3.14), and (3.19), respectively. For any optimal solution $(x^*, \pi^*, \rho^*)$ of CSPLPV, the reduced cost $\overline{c}_{vf}$ of each variable $x_{vf}$ is non-positive; that is

$$\overline{c}_{vf} = E\left[\tilde{r}_{vf}\right] + E\left[\tilde{e}_{vf}\right] - c_{vf} - E\left[\tilde{g}_v\right] \quad - \sum_{k \in K} a_{kvf} \pi_k^* - \pi_v^* - 2 \sum_{w \in \mathcal{W}} \sum_{\tilde{v} \in V} \sum_{\tilde{f} \in F_{\tilde{v}}} \rho_w^* q_{vf\tilde{v}\tilde{f}} w_{\tilde{f}} \leq 0,$$
$$\forall f \in F_v, v \in V. \tag{4.1}$$

Consider the delayed column-and-cut generation algorithm (DCCG), represented by Algorithm 2, for solving CSPLPV. For the column generation step, we use a topological

**Algorithm 2** Delayed Column-and-Cut Generation Algorithm (DCCG)

Let $\mathcal{W} \leftarrow \emptyset$ be a subset of linear constraints from (3.19). Generate a subset of ship schedules $\overline{F}_v \subset F_v$, $\forall v \in V$.

**RMP Step:** Solve CSPLPV over the set of subsets $\overline{F} = \bigcup_{v \in V} \overline{F}_v$ and first-order constraint set $\mathcal{W}$ to get a solution $(x^*, \pi^*, \rho^*)$.

**if** ${x^*}^T Q x^* > d + \varepsilon$ **then**

  **Cut Generation Step:** Update the constraint set $\mathcal{W} \leftarrow \mathcal{W} \cup \{x^*\}$ and return to the RMP Step.

**else**

  Find a ship $v$ and a ship schedule $\overline{f} \in F_v \setminus \overline{F}_v$ that maximizes the *reduced cost* $\overline{c}_{v\overline{f}}$ from (4.1).

  **if** $\overline{c}_{v\overline{f}} \leq 0$ **then**

    Return the optimal solution $(x^*, \pi^*, \rho^*)$.

  **else**

    **Column Generation Step:** $\overline{F} \leftarrow \overline{F} \cup \left\{\overline{f}\right\}$ and return to the RMP step.

  **end if**

**end if**

sorting algorithm to find a new ship schedule with maximum reduced cost (4.1) on a directed acyclic graph described in Section 4.2.

Traditional EKCP assumes all columns are available, and it adds first-order linear constraints in place of convex constraints as in the cut generation step of DCCG. For large-scale problems, however, enumerating all of the ship schedules is often impractical. Even for medium sized SPLPV problems we need to generate the covariance matrix $Q$ which increases quadratically as the number of columns increases. The computations for constructing such problems grow exponentially, as noted in Section 4.4.1. Consequently, the use of DCCG is inevitable.

## 4.2 Simplified Reduced Cost

For each ship $v \in V$, each schedule $f \in F_v$, and a subset of schedules $\overline{F}_v$, the reduced cost from (4.1) is also given by:

$$\begin{aligned}\bar{c}_{vf} = &\sum_{k \in K} \left( \alpha_k - \pi_k^* - 2 \sum_{w \in \mathcal{W}} \sum_{\widehat{v} \in V} \sum_{\widehat{f} \in \overline{F}_{\widehat{v}}} \left( \beta_k \beta_{\widehat{v}\widehat{f}} + a_{k\widehat{f}\widehat{v}} var(\gamma_k) \right) \rho_w^* w_{\widehat{v}\widehat{f}} \right) a_{kvf} - c_{vf} \\ &+ \alpha_v^g - \pi_v^* - 2 \sum_{w \in \mathcal{W}} \sum_{\widehat{v} \in V} \sum_{\widehat{f} \in \overline{F}_{\widehat{v}}} \left( \beta_v^g \beta_{\widehat{v}\widehat{f}} + I_{v=\widehat{v}} var(\gamma_v^g) \right) \rho_w^* w_{\widehat{v}\widehat{f}}.\end{aligned} \tag{4.2}$$

The operating cost $c_{vf}$ is a linear function of the ship $v$ and each consecutive pair of cargo deliveries in the schedule $f$. Consequently, we can generate a directed network for each ship $v$, similar to the one in Kim and Lee (1997), to find the schedule $f$ with maximum reduced cost. Each node in the network represents the transportation of each cargo, and each arc represents a consecutive pair of cargo deliveries. The cost of an arc includes the operating cost component of $c_{vf}$ minus the coefficient of $a_{kvf}$ in (4.2) for the cargo $k$ at the head of the arc. Using this network, we can find a shortest path for each ship $v \in V$ and subtract the lower term in (4.2) to find the schedule with maximum reduced cost.

The coefficient for a constraint in (3.19) is given by

$$\begin{aligned} 2a_{wx_{vf}} =& 2\sum_{k\in K}\left(\sum_{w\in\mathcal{W}}\sum_{\widehat{v}\in V}\sum_{\widehat{f}\in\overline{F}_{\widehat{v}}}\left(\beta_k\beta_{\widehat{v}\widehat{f}}+a_{k\widehat{f}\widehat{v}}var(\gamma_k)\right)w_{\widehat{v}\widehat{f}}\right)a_{kvf} \\ &+2\sum_{w\in\mathcal{W}}\sum_{\widehat{v}\in V}\sum_{\widehat{f}\in\overline{F}_{\widehat{v}}}\left(\beta^g_v\beta_{\widehat{v}\widehat{f}}+I_{v=\widehat{v}}var(\gamma^g_v)\right)w_{\widehat{v}\widehat{f}}. \end{aligned} \tag{4.3}$$

Suppose

$$a_{wk} = \sum_{\widehat{v}\in V}\sum_{\widehat{f}\in\overline{F}_{\widehat{v}}}\left(\beta_k\beta_{\widehat{v}\widehat{f}}+a_{k\widehat{f}\widehat{v}}var(\gamma_k)\right)w_{\widehat{v}\widehat{f}}, \tag{4.4}$$

$$a_{wv} = \sum_{\widehat{v}\in V}\sum_{\widehat{f}\in\overline{F}_{\widehat{v}}}\left(\beta^g_v\beta_{\widehat{v}\widehat{f}}+I_{v=\widehat{v}}var(\gamma^g_v)\right)w_{\widehat{v}\widehat{f}}, \tag{4.5}$$

then the coefficient is simplified to

$$2a_{wx_{vf}} = 2a_{wv} + 2\sum_{k\in f} a_{wf}. \tag{4.6}$$

The reduced cost simplifies to the following:

$$\overline{c}_{vf} = \sum_{k\in K}\left(\alpha_k - \pi^*_k - \sum_{w\in\mathcal{W}} a_{wk}\rho^*_w\right)a_{kvf} + \alpha^g_v - \pi^*_v - \sum_{w\in\mathcal{W}} a_{wv}\rho^*_w - c_{vf}. \tag{4.7}$$

The arc costs in the network are now the operating cost component of $c_{vf}$ decreased by

$$\alpha_k - \pi^*_k - \sum_{w\in\mathcal{W}} a_{wk}\rho^*_w. \tag{4.8}$$

Similarly, the reduced cost of using a ship $v \in V$ is given by

$$\alpha^g_v - \pi^*_v - \sum_{w\in\mathcal{W}} a_{wv}\rho^*_w. \tag{4.9}$$

For implementation purpose, the cut structure may only include coefficients $a_{wk}$ and $a_{wv}$ instead of $a_{wx_{vf}}$. These coefficients are easier to manage, because the number of cargoes $|K|$ and ships $|V|$ are fixed, while the number of schedules in the subset $\overline{F}$ varies as DCCG is executed.

### 4.3 Follow-On Branching

Vance et al. (1997) showed that follow-on branching, which is a variant of Ryan-Foster branching (Ryan and Foster, 1981), improves the computational efficiency of the deterministic airline crew-scheduling problem. Considering their success, we use follow-on branching for ship-scheduling applications. One advantage to using follow-on branching is ease of applying the branching logic to the column-generation subproblem of DCCG. For the ship scheduling network, follow-on branching implies that we fix or delete certain edges representing a connection between two consecutive cargo deliveries or a deployment of a ship to the first cargo in a candidate schedule.

### 4.4 Computational Experiments

In this section, we present the computational results on SPLPV instances. For small problems, we tested EKCP. We implemented the first-order constraint set (3.19) and the DCCG method within a branch-and-bound tree using COIN/BCP (COIN-OR, 2005). CPLEX 9.120 was used as the LP engine to solve the CSPLPV. To generate columns that have the maximum reduced costs we used the topological sorting algorithm on the network described in Section 4.2. We branched on follow-on variables as explained in Section 4.3. Our experiments were conducted on a Dual 3.06-GHz Intel Xeon Workstation.

#### 4.4.1 Problem Instances

In our computational analysis, we used modified instances of Kim and Lee (1997), which are similar to those in logistics for world-wide crude oil transportation of a major oil company. A set of cargoes and a set of ships are given for the planning period. In addition to the ship and cargo data, a distance matrix is given. There are two sets of cargoes. The first set of cargoes is contracts of affreightment. The second set of cargoes is from the spot market and may not be shipped depending on the schedule feasibility and/or profit. Each cargo is characterized by size, type, loading date, discharging date, loading port, discharging port, and revenue for lifting cargoes. A ship is assumed to carry only one cargo and can visit several ports in the planning period. Some ships may be chartered out if they have no feasible schedule, or they may also ship contracted cargoes in the spot market. Additional ships may be rented from the spot charter market. Each ship is characterized by size, permitted types of cargo, initial open position, initial open date, speed, fuel consumption, and the daily running costs.

Data sets from Kim and Lee (1997) include 96 ports, 30 ships, and 120 cargoes. Of course, we could create many additional combinations of port, ship, and cargo sets. However, we found that small-sized problems were trivial, so we focused only on medium-sized problems. We created SPLPV instances with the combinations of 30 ships and 30, 60, 90, 120 cargoes for the experiments. The number of variables increased exponentially with respect to the number of cargoes, and SPLPV instances have 1,409 variables, 4,561 variables, 414,369 variables, and 849,498 variables, respectively.

Each SPLPV instance was solved without constraint (3.15), which is equivalent to a traditional ship scheduling problem, and the variance of schedules in the optimal solution ($\overline{var}$) was calculated. Each instance was divided into six different levels of limited profit variability by setting the value of $\sqrt{d}$ to constraint (3.15) equal to 70, 75, 80, 85, 90, and 95 percent of standard deviation $\sqrt{\overline{var}}$. We solved SPLPV instances in all levels using both traditional EKCP and DCCG within branch-and-bound trees, and using a ten-hour time limit.

The variability of spot rates was decided by $\beta$ and $\gamma$. We randomly generated $\beta$ values between 0 and $\alpha/3$, and $\gamma$ values between $(0.05\alpha)^2$ and $(0.15\alpha)^2$. To see these values are practical, we calculated the mean squared percent error of OPEC data mentioned in Section 2, using an exponential growth trend and 6 months of prices to predict next months price. The mean squared percent error lies between 40% and 60%, and that is higher than that of our estimations, 17.7%, which suggested that our instances are conservative. Fifteen medium sized ship scheduling problems were constructed from the ship and cargo data sets, and these $\beta$ and $\gamma$ values.

To use EKCP, we had to generate all feasible schedules *a priori* and compare them with each other for constructing a covariance matrix. Though our instances are medium-sized problems, time consumed on constructing ship schedules and covariance matrices grew exponentially as the number of variables increased, which is shown in Figure 4.1. SPLPV instances with 90 and 120 cargoes could not be constructed within the time

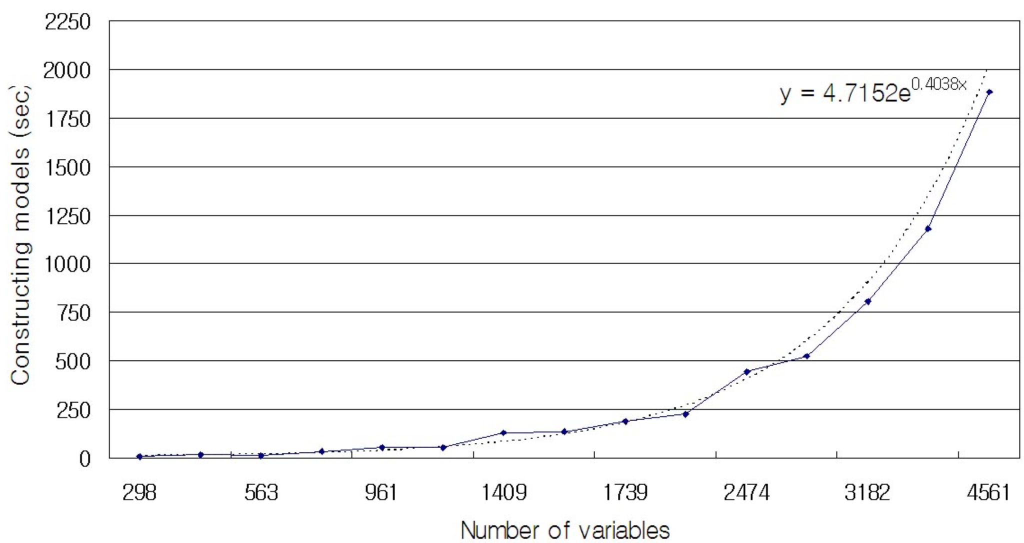


Figure 4.1. Time spent on constructing models.

limit, which have 414,369 and 849,498 variables, respectively. As a result, we did not use EKCP for these instances. Both EKCP and DCCG can reduce standard deviation to desired levels with reasonable costs, which is shown in Section 4.4.2.

### 4.4.2 Computational Results

Computational results using EKCP are shown in Table 4.1. The first column values are seven different levels of limited standard deviation $\sqrt{d}$. None of the instances with constrained variance solved to optimality, but in each instance very good solutions were found. The second column shows the standard deviation values of the best solution found within the time limit, while the third column displays the percentages of standard deviation reduction from $\sqrt{var}$. The fourth column values are the expected profit of the best solutions found, and the fifth column gives the proportions of the fourth column to the optimal solution found without the quadratic constraint. The column labeled "CPU BS" shows the time spent to the best solution in seconds, and the last column is the number of Kelley cuts generated.

Table 4.1. Enumerated Kelley's cutting plane method results

| $\sqrt{d}$ | SD | SD (%) | BS | Profit (%) | CPU BS | Cuts |
|---|---|---|---|---|---|---|
| 30 ships, 30 cargoes, 1409 vars x 60 constraints | | | | | | |
| 7434.02 | 7430.57 | 30.03 | 1560313 | 88.51 | 6530 | 88204 |
| 7965.02 | 7933.40 | 25.30 | 1627119 | 92.30 | 6612 | 73456 |
| 8496.02 | 8362.04 | 21.26 | 1665673 | 94.49 | 3568 | 15825 |
| 9027.02 | 8950.39 | 15.72 | 1721997 | 97.68 | 2490 | 9894 |
| 9558.03 | 9435.60 | 11.15 | 1760981 | 99.90 | 2 | 6 |
| 10089.03 | 9962.40 | 6.19 | 1762489 | 99.98 | 2 | 8 |
| 10620.03 | 10620.03 | 0.00 | 1762832 | 100.00 | 0 | 0 |
| 30 ships, 60 cargoes, 4561 vars x 89 constraints | | | | | | |
| 12767.97 | 12742.84 | 30.14 | 2528840 | 85.61 | 1744 | 361 |
| 13679.97 | 13608.75 | 25.39 | 2682384 | 90.81 | 34188 | 7483 |
| 14591.96 | 14454.00 | 20.76 | 2738172 | 92.70 | 14636 | 3315 |
| 15503.96 | 15389.90 | 15.63 | 2826678 | 95.69 | 19010 | 2699 |
| 16415.96 | 16376.45 | 10.22 | 2886632 | 97.72 | 6902 | 746 |
| 17327.96 | 17312.22 | 5.09 | 2920067 | 98.85 | 19629 | 1480 |
| 18239.96 | 18239.96 | 0.00 | 2953939 | 100.00 | 0 | 0 |

With a small expected profit reduction, we can significantly limit standard deviation. For example, with less than 5% profit reduction, we can decrease standard deviation by 15.63%, which is shown in the results with 30 ships and 60 cargoes. The relationship between standard deviation restriction and profit reduction is depicted by the efficient frontiers in Figure 4.2.

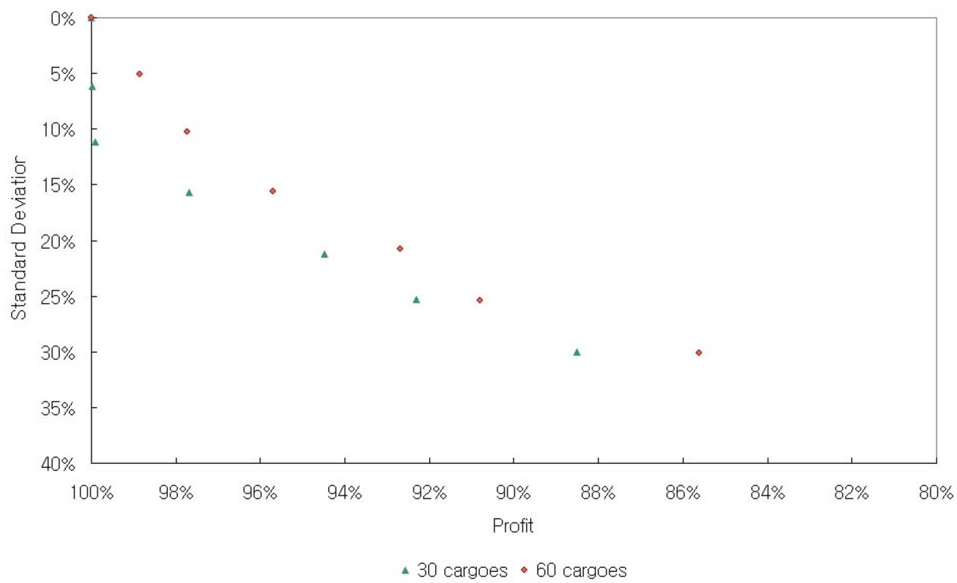


Figure 4.2. Enumerated Kelley's cutting plane method.

Table 4.2 presents computational results using DCCG for instances that have 30, 60, 90, and 120 cargoes. The first to seventh columns are the same as those in Table 4.1. The column labeled "Vars" displays the number of ship schedules generated to the best solution found, and "CPU Vars" shows the time spent generating ship schedules.

DCCG becomes the practical method as the problem size increases, although EKCP performs better than DCCG for small instances. DCCG can also significantly limit standard deviation with a small profit reduction. For example, with only 4.17% profit

Table 4.2. Delayed column-and-cut generation results

| $\sqrt{d}$ | SD | SD (%) | BS | Profit (%) | CPU BS | Cuts | Vars | CPU Vars |
|---|---|---|---|---|---|---|---|---|
| 30 ships, 30 cargoes, 1409 vars x 60 constraints | | | | | | | | |
| 7434.02 | 7367.90 | 30.62 | 1443665 | 81.89 | 2146 | 209606 | 97652 | 405 |
| 7965.02 | 7942.22 | 25.21 | 1507933 | 85.54 | 720 | 28200 | 27213 | 124 |
| 8496.02 | 8475.97 | 20.19 | 1553957 | 88.15 | 3201 | 431175 | 192270 | 658 |
| 9027.02 | 8981.53 | 15.43 | 1608842 | 91.26 | 76 | 3390 | 1850 | 11 |
| 9558.03 | 9516.31 | 10.39 | 1659654 | 94.15 | 57 | 2118 | 2303 | 10 |
| 10089.03 | 10044.60 | 5.42 | 1698373 | 96.34 | 8984 | 473007 | 509687 | 2079 |
| 10620.03 | 10620.03 | 0.00 | 1762832 | 100.00 | 2 | 0 | 90 | 0 |
| 30 ships, 60 cargoes, 4561 vars x 89 constraints | | | | | | | | |
| 12767.97 | 12728.12 | 30.22 | 2472711 | 83.71 | 21502 | 213467 | 198496 | 5163 |
| 13679.97 | 13679.66 | 25.00 | 2575115 | 87.18 | 28262 | 257952 | 249273 | 7095 |
| 14591.96 | 14551.91 | 20.22 | 2671811 | 90.45 | 61 | 232 | 670 | 18 |
| 15503.96 | 15454.84 | 15.27 | 2756273 | 93.31 | 73 | 151 | 627 | 22 |
| 16415.96 | 16383.86 | 10.18 | 2830737 | 95.83 | 31452 | 207321 | 294414 | 9052 |
| 17327.96 | 17326.48 | 5.01 | 2894907 | 98.00 | 22033 | 93115 | 290519 | 7426 |
| 18239.96 | 18239.96 | 0.00 | 2953940 | 100.00 | 11 | 0 | 190 | 3 |
| 30 ships, 90 cargoes, 414369 vars x 119 constraints | | | | | | | | |
| 22678.81 | 22667.33 | 30.04 | 4155973 | 81.15 | 35392 | 16541 | 17307 | 21182 |
| 24298.73 | 24235.86 | 25.19 | 4331288 | 84.58 | 5403 | 2597 | 3035 | 3794 |
| 25918.64 | 25892.53 | 20.08 | 4498549 | 87.84 | 33254 | 14105 | 19828 | 20356 |
| 27538.56 | 27534.32 | 15.01 | 4665335 | 91.10 | 24544 | 3764 | 15939 | 18780 |
| 29158.47 | 29149.92 | 10.03 | 4803991 | 93.81 | 35625 | 16975 | 19904 | 20004 |
| 30778.39 | 30664.88 | 5.35 | 4958686 | 96.83 | 4872 | 683 | 3009 | 3604 |
| 32398.30 | 32398.30 | 0.00 | 5121210 | 100.00 | 1059 | 0 | 550 | 1045 |
| 30 ships, 120 cargoes, 849498 vars x 148 constraints | | | | | | | | |
| 24989.61 | 24982.95 | 30.02 | 5116365 | 85.79 | 18805 | 3499 | 4661 | 14942 |
| 26774.58 | 26674.91 | 25.28 | 5315988 | 89.14 | 34197 | 5415 | 8262 | 27085 |
| 28559.55 | 28556.75 | 20.01 | 5489632 | 92.05 | 17923 | 2630 | 3926 | 15098 |
| 30344.52 | 30315.64 | 15.08 | 5591225 | 93.76 | 22301 | 1451 | 4919 | 18051 |
| 32129.50 | 32058.70 | 10.20 | 5748335 | 96.39 | 16705 | 1328 | 3975 | 14307 |
| 33914.47 | 33885.69 | 5.08 | 5847119 | 98.05 | 19288 | 703 | 4224 | 16643 |
| 35699.44 | 35699.44 | 0.00 | 5963505 | 100.00 | 3891 | 0 | 686 | 3735 |

reduction, we can restrict the deviation to 10.18% as shown in the instance with 30 ships and 60 cargoes instance. The efficient frontiers are depicted in Figure 4.3.

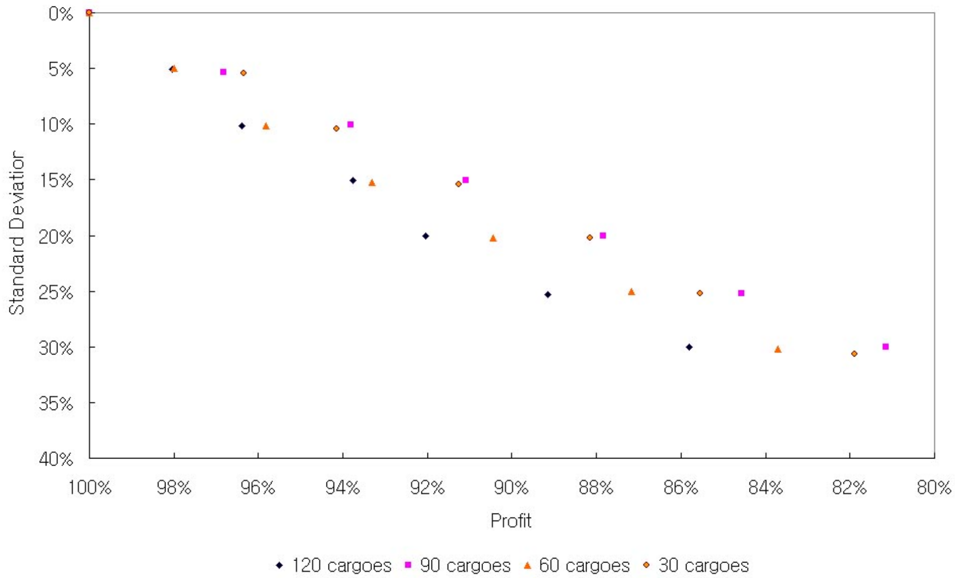


Figure 4.3. Delayed column and cut generation.

# CHAPTER 5

# HEURISTIC METHODS

Efficient optimal algorithms often find good upper bounds and lower bounds quickly. At each node of the branch-and-bound tree used for DCCG, the optimal solution values of the sub-problems of CSPLPV and the feasible solution values to SPLPV played the roles of upper bounds and lower bounds, respectively. Generating both cuts and columns, DCCG solves sub-problems of CSPLPV at each node, which is based on the continuous relaxation. According to the branch-and-bound algorithm, if the optimal solution value of a sub-problem is smaller than the current global upper bound, we can replace the upper bound with the solution value, and if the feasible solution value with respect to SPLPV is better than the current global lower bound, then the lower bound can be updated consequently.

Another well-known technique for finding upper bounds is lagrangian relaxation. With a careful examination on the ship-scheduling problem with a quadratic constraint, we found that a lagrangian decomposition method is well suited for dividing the hard problem (SPLPV) into the sum of easier problems. We may find better upper bounds by solving lagrangian relaxation problems instead of the continuous relaxation problems at each node of the branch-and-bound tree, and we can update lower bounds quickly by using a simple rounding heuristic method if solutions of lagrangian relaxation problems are not integral. The simple rounding heuristic rounds up the optimal solution values of a sub-problem that are greater than or equal to 0.5, and discards others. If the rounded-up values satisfy all the constraints of SPLPV, then they form a feasible solution, and we can utilize the solution to renew the lower bound. Lagrangian relaxation is prevalent in large-scale optimization. In this chapter, therefore, we explore a lagrangian-based heuristic method that can improve DCCG dramatically.

## 5.1 Reformulation of the New Ship-Scheduling Model

In this section, we revisit the new ship-scheduling model as a set-packing problem with a quadratic constraint (SPQC). Introducing linking variables, $\mathbf{y}$, we can replace the quadratic constraint of SPQC by the general packing constraints. Thus, SPQC can be reformulated as (RSPQC):

$$\max \ \mathbf{cx} \tag{5.1}$$

$$\text{s.t. } \mathbf{Ax} \leq \mathbf{1} \tag{5.2}$$

$$\mathbf{qy} \leq d \tag{5.3}$$

$$x_i + x_j - 1 \leq y_{ij}, \ \forall (i \times j) \in (F \times F) \tag{5.4}$$

$$\mathbf{x} \in \{\mathbf{0}, \mathbf{1}\}^{|F|} \tag{5.5}$$

$$\mathbf{y} \in \{\mathbf{0}, \mathbf{1}\}^{|F \times F|} \tag{5.6}$$

$$x_i \geq y_{ij}, x_j \geq y_{ij}, \ \forall (i \times j) \in (F \times F), \tag{5.7}$$

where $A$ is a 0-1 matrix; $F$ is the set of all candidate schedules $F = \bigcup_{v \in V} F_v$; entry $q_{ij}$ of the coefficient vector $\mathbf{q}$ for all $(i \times j) \in (F \times F)$ is the covariance of profit for selecting both sets $i$ and $j$; and $y_{ij}$ is the variable that links both variables $x_i$ and $x_j$. Constraints (5.4)—(5.7) ensure

$$y_{ij} = \begin{cases} 1, & \text{only if } x_i = x_j = 1; \\ 0, & \text{otherwise.} \end{cases}$$

In most practical cases, however, if the profits of delivering cargoes increase, the spot market rates also rise, which implies the profits are positively correlated to the spot market rates, i.e., $\mathbf{q} \geq 0$. As a result, we can drop constraints (5.7). Constraint set (5.6) can be relaxed as $0 \leq \mathbf{y} \leq 1$ because of constraint sets (5.4) and (5.5). Constraint sets (5.3), (5.4), and the relaxed constraint of (5.6) are standard means to linearize the quadratic constraint of SPQC. In fact we need only $\begin{pmatrix} |F|+1 \\ 2 \end{pmatrix}$ constraints in (5.4), but we leave them all in the formulation for clarification. It is obvious that the optimal solution value of SPQC is same as that of RSPQC. However, the continuous relaxation of SPQC (CR-SPQC) and the LP relaxation of RSPQC (LR-RSPQC) may not have the same optimal solution value. Let $\tilde{\mathbf{x}}^*$ and $(\hat{\mathbf{x}}^*, \hat{\mathbf{y}}^*)$ be the optimal solution of CR-SPQC and that of LR-RSPQC, respectively. The optimal solution value of LR-RSPQC is always greater than or equal to that of CR-SPQC, i.e., $\mathbf{c}\tilde{\mathbf{x}}^* \leq \mathbf{c}\hat{\mathbf{x}}^*$.

**Proposition 2.** *Let $\tilde{\mathbf{x}}^*$ and $(\hat{\mathbf{x}}^*, \hat{\mathbf{y}}^*)$ be the optimal solutions of CR-SPQC and LR-RSPQC, respectively. Then, $\mathbf{c}\tilde{\mathbf{x}}^* \leq \mathbf{c}\hat{\mathbf{x}}^*$.*

*Proof.* For any pair of variables $\tilde{\mathbf{x}}_i$ and $\tilde{\mathbf{x}}_j$ for both sets $i$ and $j$, $\tilde{\mathbf{x}} = \hat{\mathbf{x}}$ and $\hat{\mathbf{y}}_{ij} = \max\{0, \tilde{\mathbf{x}}_i + \tilde{\mathbf{x}}_j - 1\}$.

**Case 1:** Suppose $\tilde{\mathbf{x}}_i + \tilde{\mathbf{x}}_j - 1 > 0$, then $\hat{\mathbf{y}}_{ij} = \tilde{\mathbf{x}}_i + \tilde{\mathbf{x}}_j - 1$. This implies $q_{ij}\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j \geq q_{ij}\hat{\mathbf{y}}_{ij}$, because

$$\begin{aligned} &\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j - \tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_j + 1 \\ \Longrightarrow\ &(1 - \tilde{\mathbf{x}}_i)(1 - \tilde{\mathbf{x}}_j) \geq 0 \\ \Longrightarrow\ &q_{ij}(\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_j + 1) \geq 0 \\ \Longrightarrow\ &q_{ij}\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j \geq q_{ij}\hat{\mathbf{y}}_{ij}. \end{aligned}$$

**Case 2:** Suppose $\tilde{\mathbf{x}}_i + \tilde{\mathbf{x}}_j - 1 \leq 0$, then $\hat{\mathbf{y}}_{ij} = 0$. This implies $q_{ij}\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j \geq q_{ij}\hat{\mathbf{y}}_{ij}$, because

$$\begin{aligned} &\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j \geq 0 \\ \Longrightarrow\ &q_{ij}\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j \geq q_{ij}\hat{\mathbf{y}}_{ij}. \end{aligned}$$

Thus, $\sum_{(i\times j)\in(F\times F)} q_{ij}\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_j \geq \sum_{(i\times j)\in(F\times F)} q_{ij}\hat{\mathbf{y}}_{ij}$, which implies any feasible solution $\tilde{\mathbf{x}}$ to CR-SPQC is feasible to LR-RSPQC.

Therefore, $\mathbf{c}\tilde{\mathbf{x}}^* \leq \mathbf{c}\hat{\mathbf{x}}^*$. □

## 5.2 Lagrangian Relaxation Approach

We use lagrange multipliers on constraint set (5.4) and in the objective function to get following lagrangian upper bound problem (LUBP).

$$z(\lambda) = \max\ \mathbf{cx} + \sum_{(i\times j)\in(F\times F)} \lambda_{ij}(y_{ij} - x_i - x_j + 1) \tag{5.8}$$

$$\text{s.t. } \mathbf{Ax} \leq \mathbf{1} \tag{5.9}$$

$$\mathbf{qy} \leq d \tag{5.10}$$

$$\mathbf{x} \in \{\mathbf{0},\mathbf{1}\}^{|F|} \tag{5.11}$$

$$\mathbf{y} \in \{\mathbf{0},\mathbf{1}\}^{|F\times F|}, \tag{5.12}$$

where $\lambda_{ij} \geq 0, \forall(i\times j) \in (F\times F)$. Any solution to the continuous relaxation of LUBP provides an upper bound on the optimal solution to the original problem. A lower bound on the optimal solution can be found by the simple rounding heuristic. By tightening these bounds we can approach to the optimal solution of the original problem. Thus we are interested in finding the value for the multipliers that gives the minimum upper bound, which is called the lagrangian dual problem

$$\min_{\lambda\geq 0}\{z(\lambda)\}.$$

To decide lagrange multipliers, we may use (1) subgradient optimization, (2) multiplier adjustment, and (3) dual ascent. We use a subgradient optimization to determine lagrange multipliers, as it provides very good upper bounds and works better than others (Beasley, 1992). From an initial set of multipliers, the subgradient optimization iteratively generates further lagrange multipliers in a systematic fashion, which can be viewed a procedure to minimize the upper bound value obtained from LUBP with a suitable choice of multipliers. To solve the lagrangian dual problem, we used the subgradient optimization iterative procedure, which is described in Algorithm 3.

**Algorithm 3** Subgradient Optimization

Let the relaxed constraints be $x_i + x_j - y_{ij} \leq 1, \forall (i \times j) \in (F \times F)$.

**Step 1** Set $\theta^{Itr} = 2$; best upper bound $(Z_{UB}) = \infty$; best lower bound value $(Z_{LB}) = 0$; $Itr = 1$. Decide on an initial vector of multipliers, $\lambda^{Itr}$.

**Step 2** Solve the continuous relaxation of LUBP with current $\lambda^{Itr}$ to get a solution $(\overline{\mathbf{x}}^{Itr}, \overline{\mathbf{y}}^{Itr})$ and update $Z_{UB}$ according to the value of LUBP. Using the simple rounding heuristic, find a feasible solution $(\mathbf{x}^{*Itr}, \mathbf{y}^{*Itr})$ for SPLPV and update $Z_{LB}$, if possible.

**Step 3** Define the vector of subgradients, $\mathbf{G}$, for the relaxed constraints, and evaluate them at the current solution by:

$G_{ij}^{Itr} = \max\{0, \overline{y}_{ij}^{Itr} - \overline{x}_i^{Itr} - \overline{x}_j^{Itr} + 1\}, \forall (i \times j) \in (F \times F)$.

**Step 4** Define a step size $T$ by: $T = \theta^{Itr}(Z_{UB} - Z_{LB}) / \sum_{(i \times j) \in (F \times F)} {G_{ij}^{Itr}}^2$.

**Step 5** If termination rules are satisfied, go to Step 7. Otherwise, update $\lambda^{Itr+1}$ using $\lambda^{Itr+1} = \max\{0, \lambda^{Itr} + T\mathbf{G}\}$.

**Step 6** Update the profit vector with the new $\lambda^{Itr+1}$: i.e.,

$\mathbf{c}^{Itr+1} = \mathbf{c}^1 + \lambda^{Itr+1}(\overline{y}_{ij}^{Itr} - \overline{x}_i^{Itr} - \overline{x}_j^{Itr} + 1)$. Go to Step 2.

**Step 7** STOP. $(\overline{\mathbf{x}}^{Itr}, \overline{\mathbf{y}}^{Itr})$ is the best solution for $Z_{UB}$.

This procedure can terminate based on termination rules; for example, (1) limit the number of iterations or (2) reduce the value of $\theta$ during the course of the procedure and terminate when $\theta$ is small.

Observe that LUBP can be decomposed into the sum of two integer sub-problems and a constant:

$$(LD1) \qquad \max \ \mathbf{cx} - \sum_{(i \times j) \in (F \times F)} \lambda_{ij}(x_i + x_j) \tag{5.13}$$

$$\text{s.t. } \mathbf{Ax} \leq \mathbf{1} \tag{5.14}$$

$$\mathbf{x} \in \{\mathbf{0}, \mathbf{1}\}^{|F|} \tag{5.15}$$

$$+$$

$$(LD2) \qquad \max \sum_{(i \times j) \in (F \times F)} \lambda_{ij} y_{ij} \tag{5.16}$$

$$\text{s.t. } \mathbf{qy} \leq d \tag{5.17}$$

$$\mathbf{y} \in \{\mathbf{0}, \mathbf{1}\}^{|F \times F|}$$

$$+$$

$$\sum_{(i \times j) \in (F \times F)} \lambda_{ij} \tag{5.18}$$

Sub-problems LD1 and LD2 are a set-packing problem and a knapsack problem, respectively.

## 5.3 Alternative Ship-Scheduling Model

We revisit the ship-scheduling problem with limited profit variance (SPLPV), which was modeled as (3.12)-(3.16). With linking constraints and variables, SPLPV can be formulated as following mixed integer programming model (SPLPV-MIP).

$$\max \sum_{v\in V}\sum_{f\in F_v}(E\left[\tilde{r}_{vf}\right]+E\left[\tilde{e}_{vf}\right]-c_{vf}-E\left[\tilde{g}_v\right])x_{vf} \tag{5.19}$$

$$\text{s.t.} \sum_{v\in V}\sum_{f\in F_v} a_{kvf}x_{vf} \leq 1,\ \forall k \in K \tag{5.20}$$

$$\sum_{f\in F_v} x_{vf} \leq 1,\ \forall v \in V \tag{5.21}$$

$$\sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}} cov(\tilde{r}_{v_1f_1}+\tilde{e}_{v_1f_1}-\tilde{g}_{v_1}, \tilde{r}_{v_2f_2}+\tilde{e}_{v_2f_2}-\tilde{g}_{v_2})y_{v_1f_1v_2f_2} \leq d \tag{5.22}$$

$$x_{v_1f_1}+x_{v_2f_2}-y_{v_1f_1v_2f_2} \leq 1,\ \forall f_1 \in F_{v_1}, f_2 \in F_{v_2}, v_1, v_2 \in V \tag{5.23}$$

$$x_{vf} \in \{0,1\},\ \forall f \in F_v, v \in V \tag{5.24}$$

$$0 \leq y_{v_1f_1v_2f_2} \leq 1,\ \forall f_1 \in F_{v_1}, f_2 \in F_{v_2}, v_1, v_2 \in V. \tag{5.25}$$

SPLPV has the quadratic constraint that makes the problem harder than the traditional set-packing problems for ship scheduling. On the other hand, SPLPV-MIP contains only linear constraints, which seems to make the problem easier. However, we need $\begin{pmatrix} n+1 \\ 2 \end{pmatrix}$ constraints in set (5.23) and the same number of $y$ variables, where $n$ is the number of $x$ variables, which makes a huge mixed integer problem. Moreover, column generation is inevitable as enumerating all the columns is not practical. Unfortunately, there is not an easy way to accommodate column generation technique to SPLPV-MIP, so we explore a lagrangian decomposition method in following sections.

### 5.4 Lagrangian-based Heuristic

As explained in Section 5.2, LUBP of SPLPV-MIP is separable into the sum of two sub-problems and a constant. One of the lagrangian decomposition sub-problems is a set packing problem (LDS-SPP), and the other sub-problem is a knap-sack problem (LDS-KP). With lagrangian multiplier $\lambda$, LUBP of SPLPV-MIP can be formulated as follows:

$$\begin{aligned}\text{(LDS-SPP)}\ \max \sum_{v\in V}\sum_{f\in F_v}\left(E\left[\tilde{r}_{vf}\right]+E\left[\tilde{e}_{vf}\right]-c_{vf}-E\left[\tilde{g}_v\right]\right)x_{vf} \\ +\sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}}\lambda_{v_1f_1v_2f_2}(x_{v_1f_1}+x_{v_2f_2})\end{aligned} \tag{5.26}$$

$$\text{s.t.} \sum_{v\in V}\sum_{f\in F_v} a_{kvf}x_{vf}\leq 1,\ \forall k\in K \tag{5.27}$$

$$\sum_{f\in F_v} x_{vf}\leq 1,\ \forall v\in V \tag{5.28}$$

$$x_{vf}\in\{0,1\},\ \forall f\in F_v, v\in V \tag{5.29}$$

$$+$$

$$\text{(LDS-KP)}\ \max \sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}}\lambda_{v_1f_1v_2f_2}y_{v_1f_1v_2f_2} \tag{5.30}$$

$$\text{s.t.} \sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}} cov(\tilde{r}_{v_1f_1}+\tilde{e}_{v_1f_1}-\tilde{g}_{v_1},\tilde{r}_{v_2f_2}+\tilde{e}_{v_2f_2}-\tilde{g}_{v_2})y_{v_1f_1v_2f_2}\leq d \tag{5.31}$$

$$0\leq y_{v_1f_1v_2f_2}\leq 1,\ \forall f_1\in F_{v_1}, f_2\in F_{v_2}, v_1, v_2\in V \tag{5.32}$$

$$+$$

$$\text{(LDS-CONST)} \sum_{v_1\in V}\sum_{f_1\in F_{v_1}}\sum_{v_2\in V}\sum_{f_2\in F_{v_2}}\lambda_{v_1f_1v_2f_2} \tag{5.33}$$

where, $\lambda_{v_1f_1v_2f_2}\geq 0,\ \forall f_1\in F_{v_1}, f_2\in F_{v_2}, v_1, v_2\in V$. The lagrangian relaxation of LDS-SPP may be solved by using a modified column generation technique, and LDS-KP can be easily solved by a greedy method. As we are solving the LP relaxation of SPLPV-MIP with a set of lagrangian multipliers, the solution values of linking variables in (5.32) can be fractional. By solving the LP relaxation of LUBP of SPLPV-MIP, we can get promising columns for SPLPV-MIP as well as an upper bound to SPLPV-MIP. Let $\delta$ be the dual vector for constraint sets (5.27) and (5.28). For any optimal solution $(x^*, \delta^*; \lambda^*)$ of the continuous relaxation of LDS-SPP, the reduced cost $\overline{c}_{vf}$ of each variable $x_{vf}$ is non-positive; that is

$$\overline{c}_{vf} = E\left[\tilde{r}_{vf}\right] + E\left[\tilde{e}_{vf}\right] - c_{vf} - E\left[\tilde{g}_{v}\right] + \sum_{\tilde{f} \in F_v} \lambda^*_{f\tilde{f}} - \sum_{k \in K} a_{kvf}\delta^*_k - \delta^*_v \leq 0, \quad \forall f \in F_v, v \in V. \tag{5.34}$$

If lagrangian multipliers are all zero, i.e., $\lambda = \mathbf{0}$, then the lagrangian relaxation of LDS-SPP turns to be a pure set-packing problem, which finds a schedule $f$ with the maximum reduced cost. If not, there exists difficulty in finding an exact method to incorporate lagrangian multipliers with column generation, which is regarded as a longest-path problem with side constraints and known as NP-hard. As a result, we generate a new schedule that has the most positive reduced cost using a heuristic method: We drop $\lambda^*$ in (5.34) and find a schedule with maximum reduced cost. Then, it is likely to generate the same schedule repeatedly. To prevent the same schedule from being regenerated, therefore, we maintain a list of schedules that have already been generated. The heuristic method is shown in Algorithm 4.

Now, we can solve SPLPV-MIP at each node in a branch-and-bound tree by the lagrangian-based heuristic algorithm (LHA) shown in Algorithm 5.

**Algorithm 4** Modified Column Generation Algorithm (MCGA)

Generate a subset of ship schedules $SL \subset F_v$, $\forall v \in V$ as a schedule list.

**Column Generation Step (CG):** Solve the continuous relaxation of the LDS-SPP sub-problem with both $SL$ and a set of non-negative $\lambda$. We drop $\lambda^*$ in (5.34) and find a ship schedule $\tilde{f} \in F_v$ with maximum reduced cost.

**if** CG found $\tilde{f}$ **then**

  **if** $\tilde{f} \in SL$ **then**

    Modify the graph so as not to generate $\tilde{f}$ again, i.e., disconnect an arc in $\tilde{f}$, and/or reconnect previously disconnected arc(s). Go to CG step.

  **else**

    $SL \leftarrow SL \cup \{\tilde{f}\}$. Update the set of $\lambda$. Go to CG step.

  **end if**

**else**

  The current sub-problem with $SL$ provides the optimal solution to the lagrangian relaxation of LDS-SPP. STOP.

**end if**

## 5.5 Computational Experiments

To test the heuristic methods within a branch-and-bound tree structured by COIN/BCP, we used the same machine, LP solver, problem instances and number of levels of limited profit variability as those described in Section 4.4. At first, we tried to apply LHA through the branch-and-bound tree. However, one LHA iteration would take more than one minute for larger instances. To compensate this expensive computation, we combined LHA with DCCG. To see the merit of the LHA and the simple rounding heuristic, we performed three different computational experiments. For the first experiment, we applied ten iterations of LHA at the root node and DCCG at the other nodes.

**Algorithm 5** Lagrangian-based Heuristic Algorithm (LHA)

Set the number of iteration, $Itr = 0$. Generate a subset of ship schedules $\overline{F}_v \subset F_v$, $\forall v \in V$, and solve optimally the continuous relaxation of a sub-problem of LDS-SPP using general column generation.

**Restricted Mater Problem (RMP) Step:** $Itr \leftarrow Itr + 1$. Determine the set of non-negative $\lambda$ for the variables appeared in the sub-problem of LDS-SPP (SLDS-SPP) at hand, i.e., $\lambda_{v_1 f_1 v_2 f_2}$, $\forall v_1, v_2 \in V, \forall f_1, f_2 \in \overline{F}_v$. Solve the continuous relaxation of SLDS-SPP to get a solution $(x^*, \delta^*)$.

**if** $\bar{c}_{vf} > 0$ **then**

  **Column Generation Step:** Find a ship schedule $\overline{f} \in F_v \setminus \overline{F}_v$ using MCGA. $\overline{F}_v \leftarrow \overline{F}_v \cup \{\overline{f}\}$. Return to the RMP step.

**else**

  **Sub Problem Step:** Solve a sub-problem of LDS-KP corresponding to RMP.

  **if** Stopping Criteria are met **then**

    Return the optimal solution $(x^*, \delta^*)$

  **else**

    With a positive constant $T$, update the set of $\lambda$ by:

    $\lambda_{v_1 f_1 v_2 f_2} \leftarrow [(\lambda_{v_1 f_1 v_2 f_2} + T(1 + y_{v_1 f_1 v_2 f_2} - x_{v_1 f_1} - x_{v_2 f_2})]^+, \ \forall v_1, v_2 \in V, \forall f_1, f_2 \in \overline{F}_v$.

    Return to RMP Step.

  **end if**

**end if**

For the second one, we solved optimally the continuous relaxation of SPLPV-MIP without the quadratic constraint at the root node, then used DCCG at the other nodes. The simple rounding heuristic is applied to both experiments. For the last one, we generated 30 columns at the root node and used only DCCG with the simple rounding heuristic. None of the instances with constrained variance solved to optimality within the ten-hour time limit, but in most instances the solution values are better than those reported in Section 4.4. However, there is not that much difference between solution values (less than 1%), which suggests that the best solutions at hand are close to the optimal solutions to SPLPVs. To compare these three different methods with DCCG, we measured CPU times to find good solution values as reported in Table 4.2.

All of three experiments showed that heuristic methods substantially improve DCCG with similar results. We calculated the average savings over different coefficients of variance in order to show overall performance. The average CPU times over different variance limits to the best solutions are depicted in Figure 5.1, which clearly shows that the heuristic methods significantly outperform DCCG. The average numbers of cuts and columns are also substantially reduced, which result in the significant CPU time reduction. The average CPU time reductions are 11,647, 10,358, and 12,402 seconds (3.24, 2.88, and 3.45 hours), respectively. The largest CPU time reductions are 33,837, 32,943, and 34,113 seconds (9.40, 9.15, and 9.48 hours), respectively, which were found in the instances with 90 cargoes. The average savings in generating cuts and columns as well as the average CPU times are listed in Table 5.1. The first column labeled “Cargoes” shows the number of cargoes in each problem instance. The columns “Cuts” and “Vars” display the average numbers of Kelley’s cuts and ship schedules generated to the best solutions, respectively. The columns “CPU Vars”, “CPU BS”, and “Max CPU saving” show the average CPU times spent on generating ship schedules and on finding the best solutions, and the largest average CPU times to the best solutions, respectively.

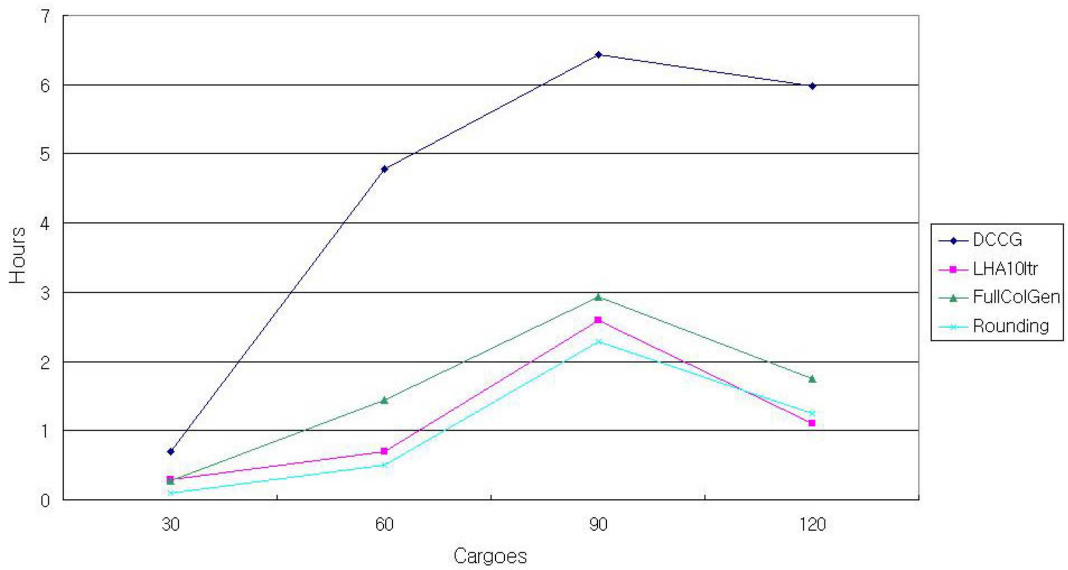


Figure 5.1. The average CPU times to the best solutions.

Table 5.1. Average savings over different variance limits. (CPU in hours)

| Cargoes | Cuts | Vars | CPU Vars | CPU BS | Max CPU Saving |
|---|---|---|---|---|---|
| LDA 10 Iterations at the root node | | | | | |
| 30 | 161,602 | 132,558 | 0.1 | 0.4 | 0.3 |
| 60 | 123,270 | 168,200 | 1.3 | 4.1 | 5.8 |
| 90 | 8,177 | 10,262 | 3.5 | 3.8 | 8.8 |
| 120 | 2,440 | 4,273 | 4.6 | 4.9 | 7.4 |
| Full column Generation at the root node | | | | | |
| 30 | 169,430 | 126,385 | 0.1 | 0.4 | 1.2 |
| 60 | 113,865 | 163,429 | 1.2 | 3.3 | 8.4 |
| 90 | 8,117 | 8,705 | 3.1 | 3.5 | 9.2 |
| 120 | 2,261 | 4,036 | 4.1 | 4.2 | 7.1 |
| Rounding Heuristic | | | | | |
| 30 | 135,175 | 130,463 | 0.1 | 0.6 | 2.1 |
| 60 | 14,907 | 149,924 | 1.2 | 4.3 | 8.6 |
| 90 | 6,615 | 3,569 | 2.1 | 4.2 | 9.5 |
| 120 | 2,113 | 4,018 | 3.7 | 4.7 | 7.9 |

Detailed computational results of the three experiments are summarized in Tables 5.2, 5.3, and 5.4, respectively. The first column shows the standard deviation values of the best solution found within the time limit, while the second column displays the percentages of standard deviation reduction from $\sqrt{var}$. The third column values are the expected profit values of the best solutions found within the time limit, and the fourth column gives the proportions of the third column to the optimal solution found without the quadratic constraint. The columns labeled “Vars” and “Cuts” display the number of Kelley’s cuts and the number of ship schedules generated to the best solution found, respectively. “CPU Vars”, “CPU BS”, and “CPU saved” show the time spent generating ship schedules, the time spent to find BS, and the CPU time saved to find BS compared with that of “CPU BS” in Table 4.2. The times are measured in seconds.

In general, as the number of cargoes increases, the CPU times to find the best solutions are greatly reduced. As we can see in the last two columns, the CPU times to find BS are significantly reduced, and most of the best solutions could be found within two hours. Both the number of Kelley’s cuts and the number of columns generated in DCCG procedures are considerably reduced, which may contribute to less CPU time. On the other hand, there are two cases in which the CPU time increased. The first and the second experiments with 90 cargoes experience the CPU time increments of 659 and 4,010 seconds (0.18 and 1.11 hours), respectively. Increased CPU time may imply that the selection of columns for sub-problems throughout the branch-and-bound tree are very important to reduce the search space. Therefore, we can conclude; (1) If we combine the heuristic methods with DCCG, good solutions can be found far earlier than when we use DCCG method only. In other words, the heuristic methods greatly accelerate DCCG. (2) We are able to identify promising columns in the column generation step of the lagrangian heuristic. (3) We can find good solutions at very reasonable computational expense. (4) We can limit standard deviation significantly with only a small reduction in expected profit. Although the method combined with LHA doesn't significantly outperform the others, the results draw meaningful interpretations. For example, if we could find a problem instance that can be formulated as a lagrangian decomposition model and solved really fast, then LHA may significantly improve the performance of DCCG; if we could generate as many promising columns as possible at the root node, then DCCG may identify near optimal solutions very quickly. Nonetheless, both manipulating a hard mixed integer problem to comprise decomposed sub-problems and finding a way to incorporate lagrangian multipliers into column generation technique are really challenging.

Table 5.2. Results of LHA 10 iterations at the root node with DCCG

| Variance | Deviation | BS | Profit ratio | Cuts | Vars | CPU Vars | CPU BS | CPU saved |
|---|---|---|---|---|---|---|---|---|
| 30 ships, 30 cargoes, 1409 vars x 60 constraints | | | | | | | | |
| 7368 | 30.62 | 1443508 | 81.89 | 19062 | 2986 | 102 | 921 | 1225 |
| 7942 | 25.21 | 1507933 | 85.54 | 5864 | 530 | 20 | 466 | 254 |
| 8374 | 21.15 | 1546066 | 87.70 | 111557 | 17042 | 384 | 2364 | 837 |
| 8982 | 15.43 | 1608844 | 91.26 | 2 | 0 | 0 | 5 | 71 |
| 9516 | 10.39 | 1659654 | 94.15 | 70 | 15 | 1 | 11 | 46 |
| 9949 | 6.32 | 1697321 | 96.28 | 41329 | 15053 | 321 | 2640 | 6344 |
| 10620 | 0.00 | 1762832 | 100.00 | | 345 | 2 | 2 | |
| 30 ships, 60 cargoes, 4561 vars x 89 constraints | | | | | | | | |
| 12730 | 30.21 | 2473756 | 83.74 | 4998 | 5511 | 320 | 3559 | 17943 |
| 13675 | 25.03 | 2572938 | 87.10 | 15559 | 9571 | 584 | 7541 | 20720 |
| 14552 | 20.22 | 2671811 | 90.45 | 31 | 19 | 0 | 29 | 32 |
| 15455 | 15.27 | 2756273 | 93.31 | 19 | 14 | 0 | 23 | 50 |
| 16384 | 10.18 | 2831017 | 95.84 | 282 | 220 | 6 | 149 | 31303 |
| 17327 | 5.01 | 2894397 | 97.98 | 11728 | 9467 | 273 | 3850 | 18182 |
| 18240 | 0.00 | 2953940 | 100.00 | | 562 | 8 | 9 | |
| 30 ships, 90 cargoes, 414369 vars x 119 constraints | | | | | | | | |
| 22666 | 30.04 | 4156930 | 81.17 | 2469 | 5957 | 4177 | 20038 | 15354 |
| 24236 | 25.19 | 4331283 | 84.58 | 530 | 1562 | 1129 | 6062 | -659 |
| 25890 | 20.09 | 4500339 | 87.88 | 29 | 268 | 252 | 1687 | 31567 |
| 27533 | 15.02 | 4663868 | 91.07 | 1733 | 7644 | 5774 | 22018 | 2526 |
| 29092 | 10.21 | 4808830 | 93.90 | 173 | 219 | 172 | 1788 | 33837 |
| 30692 | 5.27 | 4961844 | 96.89 | 669 | 1802 | 892 | 4430 | 443 |
| 32398 | 0.00 | 5121210 | 100.00 | | 2126 | 175 | 303 | |
| 30 ships, 120 cargoes, 849498 vars x 148 constraints | | | | | | | | |
| 24928 | 30.17 | 5117950 | 85.82 | 190 | 656 | 1893 | 6450 | 12354 |
| 26675 | 25.28 | 5315984 | 89.14 | 366 | 590 | 1944 | 7488 | 26709 |
| 28520 | 20.11 | 5490203 | 92.06 | 33 | 173 | 913 | 3674 | 14249 |
| 29775 | 16.59 | 5603400 | 93.96 | 55 | 445 | 1448 | 4533 | 17769 |
| 32054 | 10.21 | 5781226 | 96.94 | 155 | 436 | 1372 | 4968 | 11737 |
| 33881 | 5.09 | 5893962 | 98.83 | 13 | 160 | 638 | 2657 | 16631 |
| 35699 | 0.00 | 5963505 | 100.00 | | 2004 | 474 | 803 | |

Table 5.3. Results of full column generation at the root node with DCCG

| Variance | Deviation | BS | Profit ratio | Cuts | Vars | CPU Vars | CPU BS | CPU saved |
|---|---|---|---|---|---|---|---|---|
| 30 ships, 30 cargoes, 1409 vars x 60 constraints | | | | | | | | |
| 7368 | 30.62 | 1443509 | 81.89 | 19522 | 5610 | 93 | 698 | 1448 |
| 7942 | 25.21 | 1507933 | 85.54 | 1101 | 148 | 3 | 86 | 634 |
| 8374 | 21.15 | 1546515 | 87.73 | 5904 | 2005 | 38 | 244 | 2957 |
| 8982 | 15.43 | 1608844 | 91.26 | 13 | 5 | 0 | 3 | 72 |
| 9516 | 10.39 | 1659654 | 94.15 | 86 | 70 | 1 | 13 | 44 |
| 9949 | 6.32 | 1698314 | 96.34 | 104290 | 64829 | 858 | 4829 | 4156 |
| 10620 | 0.00 | 1762832 | 100.00 | | 345 | 2 | 2 | |
| 30 ships, 60 cargoes, 4561 vars x 89 constraints | | | | | | | | |
| 12730 | 30.21 | 2473667 | 83.74 | 11010 | 13074 | 622 | 5685 | 15817 |
| 13647 | 25.18 | 2576341 | 87.22 | 74381 | 37609 | 2165 | 24165 | 4097 |
| 14552 | 20.22 | 2671811 | 90.45 | 29 | 22 | 1 | 22 | 40 |
| 15455 | 15.27 | 2756273 | 93.31 | 18 | 18 | 0 | 17 | 56 |
| 16384 | 10.18 | 2830993 | 95.84 | 3561 | 2673 | 71 | 1243 | 30209 |
| 17238 | 5.49 | 2893511 | 97.95 | 50 | 29 | 1 | 29 | 22004 |
| 18240 | 0.00 | 2953940 | 100.00 | | 562 | 8 | 9 | |
| 30 ships, 90 cargoes, 414369 vars x 119 constraints | | | | | | | | |
| 22640 | 30.12 | 4154578 | 81.12 | 2822 | 7022 | 5988 | 21330 | 14061 |
| 24236 | 25.19 | 4331283 | 84.58 | 23 | 436 | 600 | 2393 | 3010 |
| 25893 | 20.08 | 4501282 | 87.89 | 384 | 3219 | 2555 | 7140 | 26114 |
| 27533 | 15.02 | 4663593 | 91.06 | 2340 | 15675 | 11659 | 28554 | -4010 |
| 29092 | 10.21 | 4811698 | 93.96 | 291 | 272 | 411 | 2682 | 32943 |
| 30776 | 5.01 | 4970536 | 97.06 | 101 | 168 | 276 | 1416 | 3457 |
| 32398 | 0.00 | 5121210 | 100.00 | | 2126 | 175 | 303 | |
| 30 ships, 120 cargoes, 849498 vars x 148 constraints | | | | | | | | |
| 24762 | 30.64 | 5118114 | 85.82 | 583 | 3869 | 9021 | 16422 | 2383 |
| 26675 | 25.28 | 5316024 | 89.14 | 569 | 814 | 3004 | 8568 | 25629 |
| 28520 | 20.11 | 5490031 | 92.06 | 52 | 309 | 1571 | 3614 | 14310 |
| 29941 | 16.13 | 5621132 | 94.26 | 26 | 450 | 1843 | 3789 | 18513 |
| 31840 | 10.81 | 5775933 | 96.85 | 219 | 181 | 1052 | 3597 | 13108 |
| 33896 | 5.05 | 5902297 | 98.97 | 11 | 126 | 705 | 1737 | 17551 |
| 35699 | 0.00 | 5963505 | 100.00 | | 2004 | 474 | 803 | |

Table 5.4. Results of DCCG with the simple rounding heuristic

| Variance | Deviation | BS | Profit ratio | Cuts | Vars | CPU Vars | CPU BS | CPU saved |
|---|---|---|---|---|---|---|---|---|
| 30 ships, 30 cargoes, 1409 vars x 60 constraints | | | | | | | | |
| 7423 | 30.11 | 1450055 | 82.26 | 164 | 147 | 2 | 7 | 2140 |
| 7942 | 25.21 | 1507933 | 85.54 | 1810 | 477 | 4 | 24 | 696 |
| 8476 | 20.19 | 1555077 | 88.21 | 52549 | 19467 | 167 | 606 | 2595 |
| 8982 | 15.43 | 1608844 | 91.26 | 186 | 92 | 0 | 4 | 71 |
| 9516 | 10.39 | 1659652 | 94.15 | 1089 | 1290 | 8 | 28 | 29 |
| 10045 | 5.42 | 1697516 | 96.29 | 280651 | 26723 | 339 | 1269 | 7716 |
| 10620 | 0.00 | 1762832 | 100.00 | | 345 | 2 | 2 | |
| 30 ships, 60 cargoes, 4561 vars x 89 constraints | | | | | | | | |
| 12730 | 30.21 | 2473260 | 83.73 | 27554 | 57716 | 1799 | 5123 | 16379 |
| 13647 | 25.18 | 2575719 | 87.20 | 42713 | 56285 | 1433 | 4388 | 23873 |
| 14552 | 20.22 | 2671811 | 90.45 | 143 | 188 | 4 | 14 | 47 |
| 15455 | 15.27 | 2756273 | 93.31 | 117 | 188 | 4 | 14 | 59 |
| 16384 | 10.18 | 2830865 | 95.83 | 5928 | 7028 | 143 | 518 | 30934 |
| 17238 | 5.49 | 2893510 | 97.95 | 6343 | 13051 | 253 | 764 | 21269 |
| 18240 | 0.00 | 2953940 | 100.00 | | 562 | 8 | 9 | |
| 30 ships, 90 cargoes, 414369 vars x 119 constraints | | | | | | | | |
| 22676 | 30.01 | 4156242 | 81.16 | 7384 | 19150 | 13708 | 16195 | 19197 |
| 24282 | 25.05 | 4332432 | 84.60 | 1548 | 4330 | 3824 | 4486 | 916 |
| 25893 | 20.08 | 4501257 | 87.89 | 257 | 648 | 996 | 1096 | 32158 |
| 27533 | 15.02 | 4664119 | 91.07 | 4372 | 30242 | 19870 | 23149 | 1395 |
| 29092 | 10.21 | 4810160 | 93.93 | 643 | 889 | 1276 | 1513 | 34113 |
| 30776 | 5.01 | 4970523 | 97.06 | 774 | 2351 | 2455 | 2943 | 1929 |
| 32398 | 0.00 | 5121210 | 100.00 | | 2126 | 175 | 303 | |
| 30 ships, 120 cargoes, 849498 vars x 148 constraints | | | | | | | | |
| 24928 | 30.17 | 5119972 | 85.86 | 370 | 1007 | 4180 | 4413 | 14392 |
| 26747 | 25.08 | 5319667 | 89.20 | 914 | 1422 | 5287 | 5743 | 28455 |
| 28520 | 20.11 | 5490171 | 92.06 | 239 | 720 | 3738 | 3931 | 13992 |
| 30308 | 15.10 | 5652738 | 94.79 | 227 | 759 | 3796 | 3997 | 18304 |
| 32040 | 10.25 | 5780781 | 96.94 | 447 | 1242 | 4678 | 5043 | 11662 |
| 33911 | 5.01 | 5902291 | 98.97 | 152 | 711 | 3763 | 3965 | 15323 |
| 35699 | 0.00 | 5963505 | 100.00 | | 2004 | 474 | 803 | |

# CHAPTER 6

# CONCLUSIONS AND FUTURE RESEARCH

This dissertation focussed both on a way to limit risk in logistics problems and on solution methods to solve them. Logistics problems are difficult not only because they consist of many systems, but because they involve many uncertainties such as, natural disaster, unusual weather changes, threats of terrorism, depleted oil reserves, and war. As a result, logistics managers have to resolve global optimization problems and manage the risk. As the world economy becomes vulnerable with increasing risk, many efforts of coping with uncertainties have been made, e.g., stochastic vehicle routing. Contrary to the stochastic vehicle routing problems that have drawn much attention, few traditional ship-scheduling models consider uncertainty, which motivated our research on ship-scheduling with limited risk.

Contributions of this dissertation include:

- We presented a new set-packing model for ship-scheduling problems, which limits the risk (variance) of the fluctuation in the spot market by using a quadratic constraint.
- We developed a branch-and-cut-and-price algorithm (DCCG) to solve the model, in which the quadratic constraint is represented by first-order constraints.
- We proposed an alternative model, which reformulates the quadratic constraint into general packing constraints.
- We explored heuristic methods not only to solve the alternative model, but to improve the DCCG; The methods incorporate a lagrangian heuristic and a simple rounding heuristic with DCCG.

We used traditional Kelley's cutting plane algorithm and DCCG on medium-sized ship-scheduling problems with restricted variance. To use Kelley's cutting plane algorithm, we enumerated all feasible schedules *a priori* with the covariance matrix. As the number of schedules increased, the time for constructing instances grew exponentially and using Kelley's cutting plane algorithm became impractical, a fact that motivated us to develop DCCG. In each iteration of DCCG, we added either a new Kelley's cut or a new schedule with maximum reduced cost. The new schedule was found by using a topological sorting algorithm on a directed acyclic graph. Computational experiments with instances similar to those in logistics for world-wide crude oil transportation of a major oil company showed that both Kelley's cutting plane algorithm and DCCG can reduce variance significantly with reasonable expected profit reduction. Even though neither method could optimize medium-sized instances within a ten-hour time limit, very good solutions were found. With a careful examination of the model, we could utilize a lagrangian-based heuristic, and other heuristics are employed, i.e., a simple rounding heuristic and a column generation heuristic. The current lagrangian-based heuristic becomes not useful as the size of the problem increase. To overcome this difficult, we incorporated the heuristic with DCCG. Computational experiments showed that heuristic methods substantially improved DCCG. CPU times to find best solutions within the time limit are significantly reduced, and the numbers of Kelley's cuts and columns generated in the DCCG procedures are considerably reduced.

Future research topics are identified throughout this dissertation:

- Developing methods to add multiple cuts and columns in each iteration of DCCG, as adding multiple cuts and columns in each iteration of branch-and-price-and-cut often improves computational efficiency.

- Generating as many promising columns as possible at early algorithm stages, because computational experience indicated that current methods work better with meaningful columns not with just many columns.
- Manipulating the hard mixed integer problem (P2) to encompass lagrangian relaxation and developing efficient solution methods to handle lagrangian sub-problems.
- Finding methods to incorporate lagrangian multipliers into column generation.
- Explorign different approaches, e.g., stochastic programming, stochastic dynamic programming.

## REFERENCES


S. Ahmed, M. Tawarmalani, and N. V. Sahinidis, "A Finite Branch-and-Bound Algorithm for Two-Stage Stochastic Integer Programs," *Mathematical Programming Ser. A* 100, 355–377 (2004).

R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows*, Prentice Hall (1993).

T. Andersson and P. Värbrand, "The Flight Perturbation Problem," *Transportation Planning & Technology* 27, 91–117 (2004).

A. Azaron and F. Kianfar, "Dynamic Shortest Path in Stochastic Dynamic Networks: Ship Routing Problem," *European Journal of Operational Research* 144, 138–156 (2003).

C. Barnhart, C. A. Hane, and P. H. Vance, "Using Branch-and-Price-and-Cut to Solve Origindestination Integer Multicommodity Flow Problems," *Operations Research* 48, 318–326 (2000).

C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. Savelsbergh, and P. Vance, "Branch-and-Price: Column Generation for Solving Huge Integer Programs," *Operations Research* 46, 316–329 (1998).

D. O. Bausch, G. G. Brown, and D. Ronen, "Scheduling Short-Term Maritime Transport of Bulk Products," *Maritime Policy and Management* 25, 335–348 (1998).

J. Beasley, *Lagrangean Relaxation*, The Management School Imperial College, London SW7 2AZ, England (1992).

J. F. Benders, "Partitioning Procedures for Solving Mixed Variables Programming Problems," *Numerische Mathematics* 4, 238–252 (1962).

P. Beraldi, M. Bruni, and D. Conforti, "Designing robust emergency medical service via stochastic programming," *European Journal of Operational Research* 158, 183–193 (2004).

P. Beraldi, L. Grandinetti, R. Musmanno, and C. Triki, "Parallel algorithms to solve two-stage stochastic linear programs with robustness constraints," *Parallel Computing* 26, 1189–1908 (2000).

M. E. Berge and C. A. Hopperstad, "Demand Driven Dispatch: A Method for Dynamic Aircraft Capacity Assignment, Models and Algorithms," *Operations Research* 41, 153–168 (1993).

M. Bhasi, "A Simulation Based Model for Scheduling of Barges for Ship Lightering," *Bulk Solids Handling* 24, 248–251 (2004).

J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*, Springer, New York, New York (1997).

E. K. Bish, R. Suwandechochai, and D. R. Bish, "Strategies for Managing the Flexible Capacity in the Airline Industry," *Naval Research Logistics* 51, 654–685 (2004).

L. Bodin, B. Golden, A. Assad, and M. Ball, "Routing and Scheduling of Vehicles and Crews: The State of the Art," *Computers & Operations Research* 10, 63–211 (1983).

T. Botimer and P. Belobaba, "Airline Pricing and Fare Product Differentiation: A New Theoretical Framework," *Journal of the Operational Research Society* 50, 1085–1097 (1999).

O. Bräysy and M. Gendreau, "Vehicle Routing Problem with Time Windows, Part I: Route construction and local search algorithms," *Transportation Science* 39, 104–118 (2005a).

O. Bräysy and M. Gendreau, "Vehicle Routing Problem with Time Windows, Part II: Metaheuristics," *Transportation Science* 39, 119–139 (2005b).

L. Canovas, M. Landete, and A. Marin, "On the Facets of the Simple Plant Location Packing Polytope," *Discrete Applied Mathematics* 124, 27–54 (2002).

J.-M. Cao and A. Kanafani, "Real-Time Decision Support for Integration of Airline Flight Cancellations and Delays, Part I & II," *Transportation Planning and Technology* 20, 183–199, 201–217 (1997).

C. C. Carøe and R. Schultz, "Dual decomposition in stochastic integer programming," *Operations Research Letters* 24, 34–75 (1999).

C. C. Carøe and J. Tind, "L-shaped decomposition of two-stage stochastic programs with integer recourse," *Mathematical Programming* 83, 451–464 (1998).

E. Chajakis, "Sophisticated Crude Transportation," *OR/MS Today* 24, 30–34 (1997).

M. Chajakis, "Reflections on a Marine Vessel Affair: A Management Science Sea Story," *OR/MS Today* 26, 32–37 (1999).

L. Cheng and M. A. Duran, "Logistics for World-Wide Crude Oil Transportation using Discrete Event Simulation and Optimal Control," *Computers and Chemical Engineering* 28, 897–911 (2004).

D. C. Cho, E. L. Johnson, M. Padberg, and M. R. Rao, "On the Uncapacitated Plant Location Problem. I: Valid Inequalities and Facets," *Mathematics of Operations Research* 8, 579–589 (1983).

M. Christiansen and K. Fagerholt, “Robust Ship Scheduling with Multiple Time Windows,” *Naval Research Logistics* 49, 611–625 (2002).

M. Christiansen, K. Fagerholt, and D. Ronen, “Ship Routing and Scheduling: Status and Perspectives,” *Transportation Science* 38, 1–18 (2004).

COIN-OR, “The Computational Infrastructure for Operations Research,” (2005), http://www.coin-or.org/.

CSCMP, “Council of Supply Chain Management Professionals,” (2006), http://www.cscmp.org//.

G. B. Dantzig and D. R. Fulkerson, “Minimizing the Number of Tankers to Meet a Fixed Schedule,” *Naval Research Logistics Quarterly* 1, 217–222 (1954).

G. Desaulniers, “Dispatching and Conflict-Free Routing of Automated Guided Vehicles: An Exact Approach,” *International Journal of Flexible Manufacturing Systems* 15, 309–331+357 (2003).

M. Ehrgott and D. M. Ryan, “Constructing Robust Crew Schedules with Bicriteria Optimization,” (2003), to appear in *Journal for Multi-Criteria Decision Analysis*.

S. Elhedhli and J.-L. Goffin, “The Integration of an Interior-Point Cutting Plane Method within a Branch-and-Price Algorithm,” *Mathematical Programming* 100, 267–294 (2004).

K. Fagerholt, “Ship Scheduling with Soft Time Windows: An Optimisation Based Approach,” *European Journal of Operational Research* 13, 559–571 (2001).

K. Fagerholt, “A Computer-Based Decision Support System for Vessel Fleet Scheduling: Experience and Future Research,” *Decision Support Systems* 37, 35–47 (2004).

K. Fagerholt and M. Christiansen, “A Combined Ship Scheduling and Allocation Problem,” *Journal of the Operational Research Society* 51, 834–842 (2000a).

K. Fagerholt and M. Christiansen, “A Travelling Salesman Problem with Allocation, Time Window and Precedence Constraints. An Application to Ship Scheduling,” *Internat. Trans. Oper. Res.* 7, 231–244 (2000b).

C.-W. Fenga, T.-M. Cheng, and H.-T. Wu, “Optimizing the Schedule of Dispatching RMC Trucks Through Genetic Algorithms,” *Automation in Construction* 13, 327–340 (2004).

M. Fischetti, A. Lodi, S. Martello, and P. Toth, “A Polyhedral Approach to Simplified Crew Scheduling and Vehicle Scheduling Problems,” *Management Science* 47, 833–850 (2001).

M. L. Fisher and M. B. Rosenwein, “An Interactive Optimization System for Bulk-Cargo Ship Scheduling,” *Naval Research Logistics* 36, 27–42 (1989).

R. Freling, D. Huisman, and A. P. M. Wagelmans, “Models and Algorithms for Integration of Vehicle and Crew Scheduling,” *Journal of Scheduling* 6, 63–85 (2003).

R. Fukasawa, H. Longo, J. Lysgaard, M. P. De Aragao, M. Reis, E. Uchoa, and R. F. Werneck, “Robust Branch-and-Cut-and-Price for the Capacitated Vehicle Routing Problem,” *Mathematical Programming* 106, 491–511 (2006).

M. Gendreau and G. Laporte, “A Tabu Search Heuristic for the Vehicle Routing Problem with Stochastic Demands and Customers,” *Operations Research* 44, 469–478 (1996).

M. Gendreau, G. Laporte, and R. Séguin, “An Exact Algorithm for the Vehicle Routing Problem with Stochastic Demands and Customers,” *Transportation Science* 29, 143–155 (1995).

M. Gendreau, G. Laporte, and R. Séguin, "Stochastic Vehicle Routing," *European Journal of Operational Research* 88, 3–12 (1996).

G. Ghiani, F. Guerriero, G. Laporte, and R. Musmanno, "Real-Time Vehicle Routing: Solution Concepts, Algorithms and Parallel Computing Strategies," *European Journal of Operational Research* 151, 1–11 (2003).

F. Golshani, E. Cortes-Rello, and T. H. Howell, "Dynamic Route Planning with Uncertain Information," *Knowledge-Based Systems* 9, 223–232 (1996).

R. Gomory, "Outline of an Algorithm for Integer Solutions to Linear Programs," *Bulletin of the American Mathematical Society* 64, 275–278 (1958).

P. G. Gosselin, "THE NEW DEAL; How Just a Handful of Setbacks Sent the Ryans Tumbling Out of Prosperity," *Los Angeles Times* A1 (2004).

A. Haghani and S. Jung, "A Dynamic Vehicle Routing Problem with Time-Dependent Travel Times," *Computers and Operations Research* 32, 2959–2986 (2005).

Hanbada Corporation, "Welcome to Hanbada-Sinwoo Shipping Co., Ltd." (2003), http://www.okhanbada.co.kr/.

M. A. Haughton, "Performance of Route Modification and Demand Stabilization Strategies in Stochastic Vehicle Routing," *Transportation Research, Part B: Methodological* 32B, 551–566 (1998).

M. A. Haughton, "Quantifying the Benefits of Route Reoptimization Under Stochastic Customer Demands," *Journal of the Operational Research Society* 51, 320–332 (2000).

C. Hjorring and J. Holt, "New Optimality Cuts for a Single-Vehicle Stochastic Routing Problem," *Annals of Operations Research* 86, 569–584 (1999).

D. Huisman, R. Freling, and A. P. M. Wagelmans, "Multiple-Depot Integrated Vehicle and Crew Scheduling," *Transportation Science* 39, 491–502 (2005).

E. L. Johnson, G. L. Nemhauser, and M. W. Savelsbergh, "Progress in Linear Programming-Based Algorithms for Integer Programming: An Exposition," *INFORMS Journal on Computing* 12, 2–23 (2000).

M. Jüenger and S. Thienel, "ABACUS System for Branch-and-Cut-and-Price Algorithms in Integer Programming and Combinatorial Optimization," *Software - Practice and Experience* 30, 1325–1352 (2000).

H. Jula and M. Dessouky, "Truck Route Planning in Nonstationary Stochastic Networks With Time Windows at Customer Locations," *IEEE Transactions on Intelligent Transportation Systems* 7, 51–62 (2006).

B. Kallehauge, J. Larsen, and O. B. Madsen, "Lagrangian Duality Applied to the Vehicle Routing Problem with Time Windows," *Computers and Operations Research* 33, 1464–1487 (2006).

J. E. Kelley, "The Cutting-Plane Method for Solving Convex Programs," *SIAM Journal on Applied Mathematics* 8, 703–712 (1960).

A. S. Kenyon and D. P. Morton, "Stochastic Vehicle Routing with Random Travel Times," *Transportation Science* 37, 69–82 (2003).

D. Kim and C. Barnhart, "Multimodal Express Shipment Service Design: Models and Algorithms," *Computers & Industrial Engineering* 33, 685–688 (1997).

D. Kim, C. Barnhart, K. Ware, and G. Reinhardt, "Multimodal Express Package Delivery: A Service Network Design Application," *Transportation Science* 33, 391–407 (1999).

S. H. Kim, *Optimization-Based Decision Support System For Maritime Transportation Problems In Tramp Shipping*, Ph.D. thesis, Industrial Engieering, Pusan National University, Pusan, South Korea (1999).

S. H. Kim and K. G. Lee, "An Optimization-Based Decision Support System for Ship Scheduling," *Computers and Industrial Engineering,* 33, 689–692 (1997).

D. Klabjan, E. L. Johnson, G. L. Nemhauser, E. Gelman, and S. Ramaswamy, "Solving Large Airline Crew Scheduling Problems: Random Pairing Generation and Strong Branching," *Computational Optimization and Applications* 20, 73–91 (2001).

A. J. Kleywegt, V. S. Nori, and M. W. P. Savelsbergh, "Dynamic Programming Approximations for a Stochastic Inventory Routing Problem," *Transportation Science* 38, 42–70 (2004).

N. Kong, *The Optimal Design of Regions for Organ Transplantation*, Ph.D. thesis, University of Pittsburgh, Department of Industrial Engineering (2005).

G. Laporte, "Vehicle Routing Problem: An Overview of Exact and Approximate Algorithms," *European Journal of Operational Research* 59, 345–358 (1992).

G. Laporte and F. V. Louveaux, "Integer L-Shaped Method for Stochastic Integer Programs with Complete Recourse," *Operations Research Letters* 13, 133 (1993).

G. Laporte, F. V. Louveaux, and L. Van Hamme, "An Integer L-Shaped Algorithm for the Capacitated Vehicle Routing Problem with Stochastic Demands," *Operations Research* 50, 415–423 (2002).

L. Lettovský, E. L. Johnson, and G. L. Nemhauser, "Airline Crew Recovery," *Transportation Science* 34, 337–348 (2000).

L. Lettovský, E. L. Johnson, G. L. Nemhauser, and M. Reynolds, "Airline Crew Recovery," *proceedings, AGIFORS Operations Control Study Group meeting in Sun City, South Africa* (1996).

G. F. List, B. Wood, L. K. Nozick, M. A. Turnquist, D. A. Jones, E. A. Kjeldgaard, and C. R. Lawton, "Robust Optimization for Fleet Planning under Uncertainty," *Transportation Research Part E* 39, 209–227 (2003).

O. Listes and R. Dekker, "A Scenario Aggregation-Based Approach for Determining a Robust Airline Fleet Composition for Dynamic Capacity Allocation," *Transportation Science* 39, 367–382 (2005).

H. K. Lo and M. R. McCord, "Adaptive Ship Routing through Stochastic Ocean Currents: General Formulations and Empirical Results," *Transportation Research, Part A: Policy and Practice* 32, 547–561 (1998).

M. Lohatepanont and C. Barnhart, "Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment," *Transportation Science* 38, 19–32 (2004).

H. Longo, M. P. De Aragao, and E. Uchoa, "Solving Capacitated Arc Routing Problems using a Transformation to the CVRP," *Computers and Operations Research* 33, 1823–1837 (2006).

S. Luo and G. Yu, "On the Airline Schedule Perturbation Problem Caused by the Ground Delay Program," *Transportation Science* 31, 298–311 (1997).

A. Mehrotra, "An Optimization Based Heuristic for Political Districting," *Management Science* 44, 1100 – 1114 (1998).

D. P. Morton and R. K. Wood, "Restricted-Recourse Bounds for Stochastic Linear Programming," *Operations Research* 47, 943–956 (1999).

J. M. Mulvey, R. J. Vanderbei, and S. A. Zenios, "Robust Optimization of Large-Scale System," *Operations Research* 43, 264–281 (1995).

G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, John Wiley and Sons, New York, NY (1988).

OPEC, "Organization of the Petroleum Exporting Countries," (2005), http://www.opec.org/.

G. M. Pangborn, *A Branch-and-Cut-and-Price Implementatin for Airline Crew Scheduling*, Ph.D. thesis, Cornell University, Operations Research (2002).

J. Persson and M. Göthe-Lundgren, "Shipment Planning at Oil Refineries using Column Generation and Valid Inequalities," *European Journal of Operational Research* 163, 631–652 (2005).

V. L. Pilla, J. M. Rosenberger, V. Chen, and B. C. Smith, "A Statistical Computer Experiments Approach to Airline Fleet Assignment," Technical Report COSMOS-05-03, The University of Texas at Arlington, 2005.

W. B. Powell, M. T. Towns, and A. Marar, "On the Value of Optimal Myopic Solutions for Dynamic Routing and Scheduling Problems in the Presence of User Noncompliance," *Transportation Science,* 34, 67–85 (2000).

T. Ralphs, L. Kopman, W. Pulleyblank, and L. Trotter, "On the Capacitated Vehicle Routing Problem," *Mathematical Programming* 94, 343–359 (2003a).

T. Ralphs, L. Ladanyi, and M. Saltzman, "Parallel Branch, Cut, and Price for Large-Scale Discrete Optimization," *Mathematical Programming* 98, 253–280 (2003b).

T. K. Ralphs and M. V. Galati, "Decomposition and Dynamic Cut Generation in Integer Linear Programming," *Mathematical Programming* 106, 261–285 (2006).

K. Rana and R. Vickson, "Model and Solution Algorithm for Optimal Routing of a Time-Chartered Containership," *Transportation Science* 22, 83–95 (1988).

K. Rana and R. Vickson, "Routing Container Ships using Lagrangean Relaxation and Decomposition," *Transportation Science* 25, 201–214 (1991).

C. C. Ribeiro and F. Soumis, "A Column Generation Approach to the Multiple-Depot Vehicle Scheduling Problem," *Operations Research* 42, 41–52 (1994).

D. Ronen, "Cargo Ships Routing and Scheduling: Survey of Models and Problems," *European Journal of Operational Research* 12, 119–126 (1983).

J. M. Rosenberger, *Topics in Airline Operations*, Ph.D. thesis, Georgia Institute of Technology, School of Industrial and Systems Engineering (2001).

J. M. Rosenberger, E. L. Johnson, and G. L. Nemhauser, "Rerouting Aircraft for Airline Recovery," *Transportation Science* 37, 408–421 (2003).

F. Rossi and S. Smriglio, "A Set Packing Model for the Ground Holding Problem in Congested Networks," *European Journal of Operational Research* 131, 400–416 (2001).

D. M. Ryan and B. A. Foster, "An Integer Programming Approach to Scheduling," in *Computer Scheduling of Public Transport Urban Passenger Vehicle and Crew Scheduling*, A. Wren (ed), 269–280, North-Holland, Amsterdam (1981).

N. V. Sahinidis, "Optimization under Uncertainty: State-of-the-Art and Opportunities," *Computers and Chemical Engineering* 20, 971–983 (2004).

A. J. Schaefer, *Airline Crew Scheduling Under Uncertainty*, Ph.D. thesis, Georgia Institute of Technology (2000).

N. Secomandi, "A Rollout Policy for the Vehicle Routing Problem with Stochastic Demands," *Operations Research* 49, 796–802 (2001).

W. F. Sharpe, *Portfolio Theory and Capital Markets*, McGraw-Hill (1970).

H. D. Sherali and B. Fraticelli, "A modification of Benders' decomposition algorithm for discrete subproblems: An approach for stochastic programs with integer recourse," *Journal of Global Optimization* 22, 319–342 (2002).

D. Simchi-Levi, P. Kaminsky, and E. Simchi-Levi, *Designing and Managing the Supply Chain*, McGraw-Hill (2003).

S. Sindhuchao, "An Integrated Inventory-Routing System for Multi-Item Joint Replenishment with Limited Vehicle Capacity," *Journal of Global Optimization* 32, 93–118 (2005).

M. R. Swihart and J. D. Papastavrou, "Stochastic and Dynamic Model for The Single-Vehicle Pick-Up and Delivery Problem," *European Journal of Operational Research* 114, 447–464 (1999).

S. Takriti and S. Ahmed, "On Robust Optimization of Two-Stage Systems," *Mathematical Programming* Ser. A99, 109–126 (2004).

K. Tan, L. Lee, Q. Zhu, and K. Ou, "Heuristic Methods for Vehicle Routing Problem with Time Windows," *Artificial Intelligence in Engineering* 15, 281–295 (2001).

S. Teng, H. L. Ong, and H. Huang, "A Comparative Study of Metaheuristics for Vehicle Routing Problem with Stochastic Demands," *Asia-Pacific Journal of Operational Research* 20, 103–119 (2003).

P. Toth, "Optimization Engineering Techniques for the Exact Solution of NP-Hard Combinatorial Optimization Problems," *European Journal of Operational Research* 125, 222–238 (2000).

P. H. Vance, A. Atamturk, C. Barnhart, E. Gelman, E. L. Johnson, A. Krishna, D. Madidhara, G. L. Nemhauser, and R. Rebello, "A Heuristic Branch-and-Price Approach for the Airline Crew Pairing Problem," Technical Report TLI/LEC-97-06, Georgia Institute of Technology, Atlanta, GA, 1997.

B. Verweij and K. Aardal, "The Merchant Subtour Problem," *Mathematical Programming* 94, 295–322 (2003).

H. Vladimirou and S. Zenios, "Stochastic Linear Programs with Restricted Recourse," *European Journal of Operational Research* 101, 177–192 (1997).

L. Weatherford and P. Belobaba, "Revenue Impacts of Fare Input and Demand Forecast Accuracy in Airline Yield Management," *Journal of the Operational Research Society* 53, 811–821 (2002).

W. L. Winston, *Operations Research: Applications and Algorithms*, International Thomson Publishing, 3rd edition (1994).

Worldscale, "New Worldwide Tanker Nominal Freight Scale," (2000), http://www.worldscale.co.uk/Worldscale/.

D. O. Wu, "Dapeng Oliver Wu's Homepage," (2002), http://www.wu.ece.ufl.edu/books/math/algebra/LinearAlgebra.html/.

E. Zaloshnja and T. R. Miller, "Costs of Large Truck-Involved Crashes in the United States," *Accident Analysis and Prevention* 36, 801–808 (2004).

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