PSEUDO-PROBABILISTIC USER EQUILIBRIUM ASSIGNMENT IN TRAVEL CORRIDOR NETWORKS CONTAINING MANAGED FACILITIES

by

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Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT ARLINGTON

December 2010

ACKNOWLEDGEMENTS

Throughout my research there were many people who offered support and insight, and though I cannot mention them all here, I would like to take this opportunity to acknowledge several who deserve special recognition.

First, I would like to express my most humble appreciation to Dr. Siamak Ardekani, my advisor and mentor, for teaching me the importance of constructive thinking in research. I also want to thank the other members of my dissertation committee: Dr. Stephen Mattingly, Dr. Ardeshir Anjomani, Dr. James Williams and Dr. Melanie Sattler. The time they spent revising my work and the care with which they made recommendations, significantly improved the final version of this thesis. For their many contributions I am sincerely grateful.

Thanks also to Carlos Contreras for his generosity in allowing me the use of the resources that made this work possible.

I want to thank Shahram Bohluli for his support, and Sasanka Pulipati, whose help throughout my research was always greatly appreciated.

My sincere gratitude also goes to Sahar Ghassemi for the kind assistance she provided when I needed it the most.

Finally, I wish to thank my brothers and sisters, as well as Yalda, Pedram and Nasim, for their support and encouragement.

Above all, my deepest gratitude goes to my dearest mother who gave me much needed love and support during my studies at UTA. This work is dedicated to her.

November 15, 2010

ABSTRACT

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Throughout the years, transportation organizations and agencies have been unable to keep up with increasing demand for roadway facilities. Similarly, traditional public-sector funding such as motor fuel taxes is falling short in meeting the growing demand for new transportation infrastructure. With deficit financing and congestion problems common to many highways throughout the United States, DOTs are turning to tolling the roadway facilities as a means of financing transportation improvements for inter-urban and urban facilities.

In turn, in order for managed facilities such as toll roads or managed lanes to be attractive and viable for the potential investors, the facilities must be predicted to generate sufficient revenue to cover the costs and also provide reasonable rates of return for debt servicing. This requires accurate revenue forecasting, which itself largely is based on an accurate traffic demand forecast. Therefore, the performance and reliability of models that

forecast traffic demand for toll roads are critical, and the likelihood that forecasted revenue matches the actual revenue is solely based on the performance and reliability of these travel demand models.

The purpose of this research is to evaluate the application of a pseudo-probabilistic route assignment method within a travel demand forecast model in order to forecast the diversion rate for a proposed tolled facility. This will result in an estimation of the future traffic of the tolled facility and its share of the total corridor demand. In addition, throughout this study, effort has been made to explore the existing toll road travel demand forecasting methods and address the technical modeling issues that affect the performance of such methods.

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CHAPTER 1

INTRODUCTION

1.1 Background

Throughout the United States, traditional public-sector funding sources for transportation projects are unable to meet the growing demand for new highway facilities and maintain an aging infrastructure. Motor fuel taxes—the primary source of transportation finance in the United States—have not kept pace with the demand for travel and, in turn, for capital investment owing to inflation, improved fuel efficiency, and increased alternative fuel vehicle usage [3].

The shortfall can lead to economic impacts on highway construction, as well as operation, maintenance, and expansion. As a result, some state departments of transportation (DOTs) and transportation authorities are relying increasingly on tolling as an alternative means of financing new and expanded highway infrastructure. A related problem is the need to improve the management and utilization of existing facilities because many of the urban centers that require additional capacity to relieve congestion have limited space for expansion [1].

With budget shortfalls and an increasing demand for roadway facilities, DOTs are turning to user-based fees or tolling as a means of financing roadway improvements and expansion and managing growing traffic demand for both inter-urban and urban facilities. The Texas DOT, for example, has determined that any new highway project in the state must be evaluated as a toll road [4].

As DOTs turn to tolling, increasing attention is being focused on the performance of the underlying revenue forecasts and the projected ability of the facility to service debt. This is because the performance of the revenue forecasts, which are derived largely from forecasts of traffic demand, has varied among projects. In some cases, the lower-than-anticipated revenues were addressed through alternative sources of revenue to pay debt service. These include sources such as other toll roads, gas taxes, or government guarantees, or, where available, through sufficient reserve funds. This ensured that no struggling project was entirely dependent on traffic revenues for debt payment. As a result, between 1985 and 1995, forecasting errors or inaccuracies, which were known to exist, did not result in a single default in payment or any serious payment difficulties with new toll road projects in the United States [5].

However, concern has since been expressed that these alternate sources of revenue may not be available to protect more recent projects or future projects. For example, the privately held Dulles Greenway in Virginia went into default in 1996 as a result of toll revenues being less than projected (achieving only 20% of projected revenues in 1995, its first year of operation, and still only 35% of projected revenues in its fifth year). Other toll roads have also struggled; for example, revenues from the Southern Connector in South Carolina have been sufficient to cover operating costs but only a portion of the debt service because traffic projections have not been met (just over half of the projected demand was realized in its third year of operation). Similarly, traffic on the Pocahontas Parkway located southeast of Richmond, Virginia, has realized just under half of the projected demand in its second year of operation. Consequently, the credit ratings for the bonds for both facilities were lowered [6]. The Foothill/Eastern toll road in Orange County, California, had to also be refinanced in 1999 [5].

Contributing factors to the financial problems of the Pocahontas Parkway and the Foothill/Eastern toll roads (and others) have been attributed to various inaccuracies in the demand and revenue forecasts, which included the unanticipated effects of a recession, actual ramp-up volumes being less than projected, and the failure to construct an expected extension of a connecting road [7].

For toll facilities to be financially viable and/or attractive to potential investors (public–private partnerships, etc.), the facility must be seen as able to generate sufficient revenue from operations to cover debt service cost and potentially other projects and maintenance costs over the lifetime of the facility, as well as providing a reasonable return on equity. This requires a reliable and credible forecast of the expected revenues, which are functions of the estimated traffic demand and toll rates for the facility. However, industry experience in tolling forecasts and the associated recoverable benefits historically have been quite varied in that demand (and the accompanying revenues) has ranged from frequently overestimated to occasionally underestimated. Also, the accuracy of *when* specific levels of demand are projected to occur has been mixed, with problems being particularly acute in the short-term facility ramp-up. The resultant variations have had significant impacts on both the actual revenue streams and on the facility's debt structuring and obligations [1].

This has led to concerns among facility owners and the financial community (which rates and insures and/or invests in the bonds that are issued for the facility's implementation) regarding the accuracy, reliability, and effectiveness of the demand forecasts upon which the revenue projections are based. In addition, the growing use of Intelligent Transportation Systems and other technologies provides DOTs with the ability to implement variable pricing, HOT (high-occupancy toll) lanes, and other innovations that require a greater level of accuracy in the projection of demand and revenues [1].

Accordingly, to maximize the prospects of a project's financial viability—that is, the likelihood that the forecasts match the actual revenues—it is necessary to look at the early stages of the process, specifically the models that are used to forecast travel (traffic) demand and their resulting successes or failures to improve the forecast results [1].

1.2 Problem Statement

While travel demand models (TDM) have been around for decades, traffic and revenue forecasts for managed facilities do not have such a long history. With evolving transportation technologies and a vast amalgamation of tolled facilities within the existing non-tolled transportation network, inaccuracy in traffic demand and revenue forecast for tolled facilities has become a big concern for investors. Studies show considerable variation in performance for the revenue forecast projected for such facilities, ranging from a low of 13.0% to a high of 152.2% overestimation in Year 1 [1].

Another study compared the traffic forecasts for 104 tolled facilities around the world. The comparison found considerable variability in the performance of the traffic forecasts for the first year (during ramp-up), ranging between 15% and 150% of actual performance. On average, the forecasts overestimated Year 1 traffic by 20%-30%. The mean projected versus actual performance ranged between 0.77 and 0.80 over the first 5 years of operation [1].

Table 1.1 summarizes the performance of 26 different toll highways throughout the United States. The table compares the actual revenue collected as a percentage of the revenue that was projected in traffic and revenue forecasts. The facilities are listed according to the year in which the facility opened (between 1986 and 2004) [1].

Among various factors such as socioeconomic inputs, economic fluctuation, truck traffic pattern, values of time, unforeseen expansion of competing roads, inaccurate ramp-up estimation, and failure to anticipate network improvements that may alter the results of a traffic and revenue forecast, the modeling approach is one of the most important aspects. Within a travel demand forecast model, methodologies to utilize a system to determine the diversion rate for a proposed tolled facility solely will shape the outcome. Methods for modeling toll road demand or toll diversion models are the core of a travel demand model to determine the future share of a managed facility within a project corridor. Studies have shown that entities are failing to become more skilled in providing more accurate results after implementing several projects; therefore the results of traffic and revenue forecasts are not improving with newer facilities, which might have been expected given that the state of the practice of modeling generally is not improving [1].

Table 1.1 Actual Revenue as Percentage of Projected Results of Operation [1]

Authority/Facility	Year of Opening	Year 1	Year 2	Year 3	Year 4	Year 5
Florida's Turnpike Enterprise/Sawgrass Expressway [8]	1986	17.8%	23.4%	32.0%	37.1%	38.4%
North Texas Tollway Authority/Dallas North Tollway [8]	1987	73.9%	91.3%	94.7%	99.3%	99.0%
Harris County Toll Road Authority (Texas)/Hardy [8]	1988	29.2%	27.7%	23.8%	22.8%	22.3%
Illinois State Toll Highway Authority/Illinois North South Tollway [8]	1989	94.7%	104.3%	112.5%	116.9%	115.3%
Orlando-Orange Expressway Authority/Central Florida Greenway North Segment [8]	1989	96.8%	85.7%	81.4%	69.6%	77.1%
Harris County Toll Road Authority (Texas)/Sam Houston [8]	1990	64.9%	79.7%	81.0%	83.2%	78.0%
Orlando-Orange Expressway Authority/Central Florida Greenway South Segment [8]	1990	34.1%	36.2%	36.0%	50.0%	NA
Oklahoma Turnpike Authority/John Kilpatrick [5]	1991	18.0%	26.4%	29.3%	31.4%	34.7%
Oklahoma Turnpike Authority/Creek [5]	1992	49.0%	55.0%	56.8%	59.2%	65.5%
Mid-Bay Bridge Authority (Florida)/ Choctawhatchee Bay Bridge [16,17]	1993	79.8%	95.5%	108.9%	113.2%	116.7%
Orlando-Orange Expressway Authority/Central Florida Greenway Southern Connector [8]	1993	27.5%	36.6%	NA	NA	NA
State Road and Tollway Authority (Georgia)/GA 400 [5]	1993	117.0%	133.1%	139.8%	145.8%	141.8%
Florida's Turnpike Enterprise/Veteran's Expressway [5]	1994	50.1%	52.9%	62.5%	65.0%	56.8%
Florida's Turnpike Enterprise/Seminole Expressway [5]	1994	45.6%	58.0%	70.7%	78.4%	70.1%
Transportation Corridor Agencies (California)/Foothill North [5]	1995	86.5%	92.3%	99.3%	NA	NA
Osceola County (Florida)/Osceola County Parkway [5]	1995	13.0%	50.7%	38.5%	40.4%	NA
Toll Road Investment Partnership (Virginia)/Dulles Greenway [5]	1995	20.1%	24.9%	23.6%	25.8%	35.4%
Transportation Corridor Agencies (California)/San Joaquin Hills [5]	1996	31.6%	47.5%	51.5%	52.9%	54.1%
North Texas Tollway Authority/George Bush Expressway [5]	1998	152.2%	91.8%	NA	NA	NA
Transportation Corridor Agencies (California)/Foothill Eastern [5]	1999	119.1%	79.0%	79.2%	NA	NA
E-470 Public Highway Authority (Colorado)/E-470 [5]	1999	61.8%	59.6%	NA	95.4%	NA
Florida's Turnpike Enterprise/Polk [5]	1999	81.0%	67.5%	NA	NA	NA
Santa Rosa Bay Bridge Authority (Florida)/Garcon Point Bridge [18, 19]	1999	32.6%	54.8%	50.5%	47.1%	48.7%
Connector 2000 Association (South Carolina)/Greenville Connector [5]	2001	29.6%	NA	NA	NA	NA
Pocahontas Parkway Association (Virginia)/Pocahontas Parkway [20]	2002	41.6%	40.4%	50.8%	NA	NA
Northwest Parkway Public Highway Authority (Colorado)/Northwest Parkway [20, 21]	2004	60.5%	56%	NA	NA	NA

1.3 Scope and Objectives

There are several approaches to solve the problem of toll demand forecast for a proposed tolled facility. Among all these methods, programs, and components mounted within a four–step traditional travel demand model (TDM) are of interest throughout this study. The scope of this dissertation is toll demand models in line with a four-step framework. “Toll Diversion Model” and “Toll Diversion Methodology” are the terms that will be used throughout this literature to refer to such methods and model components.

The main objective of this dissertation, however, is to implement the application of a pseudo-probabilistic user equilibrium method in line with a four-step TDM and within the assignment step. Pseudo-probabilistic user equilibrium is a method within the urban transportation planning and travel demand forecast literature to approach the route choice dilemma when there is more than one feasible choice for trip makers for a given origin-destination pair. Nevertheless, there is less evidence that practitioners have widely adopted this methodology inside the traffic and revenue (T&R) forecast industry.

The application and implementation of a pseudo-probabilistic user equilibrium toll diversion methodology in this study involves adopting an existing pseudo-probabilistic approach (which was found in transportation literature), scripting the corresponding toll diversion program inside the model platform, and then exploring the possible advantages of this method compared to other existing toll diversion methodologies.

It is not within the scope of this study to deeply investigate and prove mathematical aspects of a pseudo-probabilistic user equilibrium algorithm. Establishing a sample travel demand forecast project, adopting a transportation network, adopting trip tables and other model inputs and components, inclusion of an existing toll diversion methodology, scripting and programming a pseudo-probabilistic toll diversion approach, and relevant analysis of the results in order to determine the advantages of the latest approach are the main objectives of this dissertation.

1.4 Overview

This section illustrates the organization of the chapters throughout this dissertation. Chapter two covers the components of a four-step travel demand model and provides a literature review of the existing methodologies for a tolled facility travel demand forecast and specific factors affecting the forecast performance.

Chapter three is devoted to the process of adopting the sample project area and model inputs such as network characteristics, trip tables by vehicle class and time of day, as well as a description of the adopted route choice models.

Chapter four describes the adopted methodology of a pseudo-probabilistic toll assignment program including corresponding stepwise assignment algorithm. Chapter five illustrates the implementation of the pseudo-probabilistic toll assignment algorithm in the framework of the Cube Voyager software. Chapter six presents the results of the pseudo-probabilistic toll assignment program, convergence, as well as other relevant analysis. Performance of this toll diversion model and its possible advantages will also be discussed in this chapter. Finally, the conclusions of the study and proposed future study extensions will be presented in Chapter seven.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview of Travel Demand Forecasting Models

This section briefly reviews the practice of travel demand forecasting models. The purpose is to provide a context for the discussion of toll road traffic forecasts at an appropriate level of detail. It should be noted that the materials found in the Transportation Research Board publications specifically, "Synthesis 364 of the National Cooperative Highway Research Program (NCHRP): Estimating Toll Road Demand and Revenue," had a major contribution to the development of the literature review for this research. Synthesis 364 explores the state of the practice for forecasting demand and revenues for toll roads in the United States. It looks at the travel demand forecasting models and their applications to project toll road revenues as a function of the estimated demand.

Travel demand forecasting models use demographic, socioeconomic, and land-use variables such as population, employment, and jobs to represent human activities. These activities include personal trips and goods movement. Travel demand also is shaped by and shapes the transportation network. The "supply" of transportation services determines how the demand uses the transportation network. Similarly, forecasts of demand define the required supply of transportation services such as how many lanes of road at what capacity are needed or where bus routes are needed, etc. [1].

Many medium—and most large-size urban areas in the United States and around the world use a travel demand forecasting model, albeit with various approaches and to varying degrees of detail and sophistication. In the United States, metropolitan planning organizations (MPOs) use models to develop long-range transportation plans [1].

2.1.1 The Four-Step Travel Demand Models

Within the rational planning framework, transportation forecasts have traditionally followed the sequential four-step model or urban transportation planning procedure, first implemented on mainframe computers in the 1950s at the Detroit Area Transportation Study and Chicago Area Transportation Study [10]. The process has been used for several decades in the United States and around the world. Figure 2.1 presents a general outline of the main inputs, processes, and outputs of this travel demand modeling.

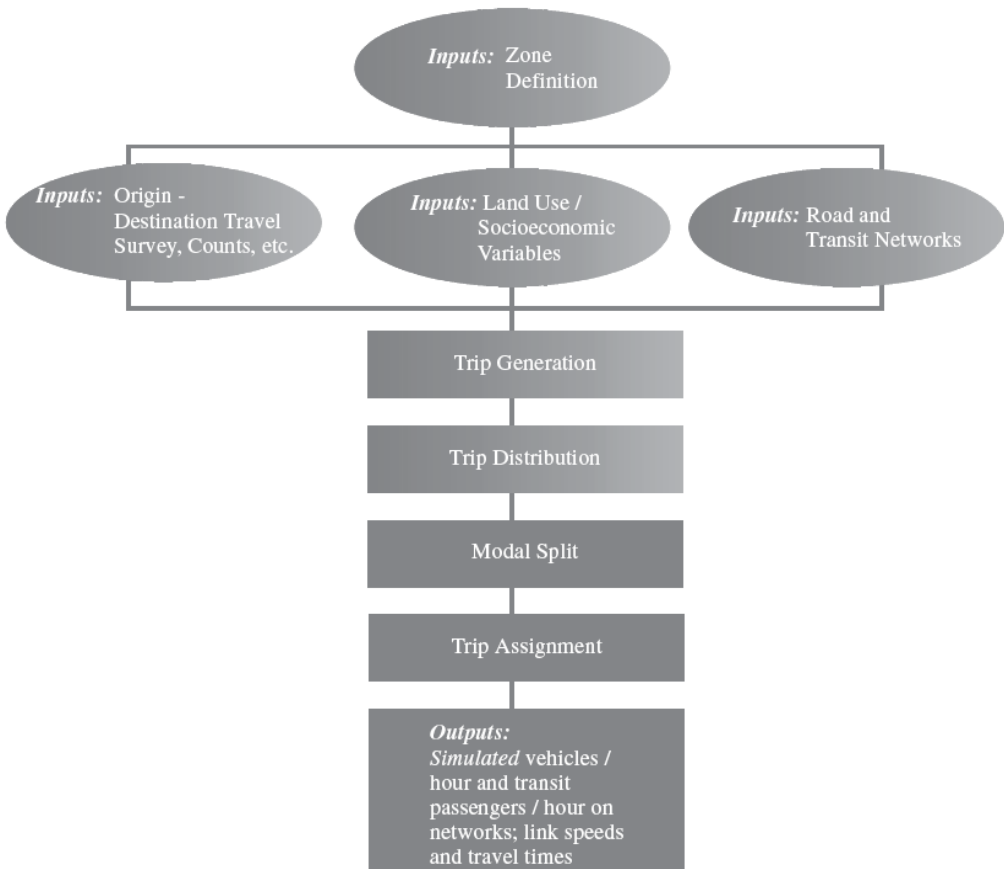


Figure 2.1 Outline of traditional “four-step” travel demand modeling process [11]

The individual elements are described as follows [11]:

1. Inputs
   - Zone definition—the urban area is divided into small spatial analytical areas, similar in concept to census tracts. Generally, traffic zones are defined by homogeneous land uses such as residential neighborhoods, central business districts, industrial areas, etc., major “traffic generators” such as universities, hospitals, shopping centers, and airports, or geographic boundaries such as rivers and railways.
   - Land-use inputs—these are defined for each traffic zone in terms of population, employment, floor space, etc.
   - Transportation network—this normally includes the major road and highway network, all roads except local streets, and the public transport network. These are defined in terms of a link-node network.
   - Observed travel characteristics—this is measured typically by origin-destination surveys. These surveys provide a quantitative portrait of travel characteristics in a city, typically on a weekday. Traditional origin-destination surveys are “revealed preference” surveys, conducted to observe how people actually behave, and “stated preference” surveys, which study the willingness to use a facility that currently is not in place.
2. Process—the four steps of the process consist of:
   - Trip generation—where the total numbers of trips that start and end in each zone are calculated as a function of the different land uses in each zone. The calculations take into account different trip purposes, which again are represented by land uses. For example, the daily home-to-work commute is commonly represented by population or dwelling units at the home end and by the number of jobs at the work end.

- Trip distribution—where the generated trip ends are distributed among all zones. The distribution is conducted as a function of the zonal land uses and the characteristics of the transportation network. The function is sensitive to the relative accessibility of a zone, which is measured as a function of travel time (congestion and cost), transit fares, parking charges, road tolls, etc. Different calculations are made for different trip purposes to take into account their different behaviors. The products of this step are expressed as matrices of trips for different purposes.
- Modal split—where the distributed trips are allocated to the different available travel modes. Typically, the allocation is between automobiles and public transport; however, some models further differentiate among public transport modes (including park and ride), between HOV (high occupancy vehicle) and SOV (single occupancy vehicle), and non-motorized modes (pedestrians and bicyclists). A common formulation in this step is the logit function, which simulates the traveler's utility according to out-of-pocket cost, door-to-door travel time, and other attributes of modal choice (such as trip distance, proximity of the transit stop to the workplace, in-vehicle comfort, the number of transfers required, and so on).
- Trip assignment—where the trips for each mode are loaded onto, or assigned to, the respective transportation network(s). This is a translation of demand, which is expressed as the number of trips by specific mode (for all purposes combined) between zone *i* and zone *j*, into automobile traffic volumes on a given road link and ridership on a bus route, etc.

3. Outputs

   Volumes by link and ridership numbers are the main outputs of the model, along with travel times and speeds across the transportation network by link. These outputs can be used in turn to identify costs, fuel consumption, and air pollutants, as well as revenues on a tolled facility.

2.2 Methods for Modeling Toll Road Demand

The purpose of this section is to review the models and methods of traffic demand forecast for a new toll project. As mentioned before, these models may also be referred to as "toll diversion models." This paper will use relevant publications including project reports and practitioners' experiences found in available articles, transportation journals, and online databases.

The review of the state of the practice for projects in several U.S. cities identified five categories of modeling procedures [1]. Although some of the reviewed projects address methods for specific managed facilities such as managed lanes, for the purpose of this study, all the projects were categorized under the general concept of managed facilities traffic demand forecast methods.

2.2.1 Models as Part of an Activity-Based Model

A combination of revealed and stated preference surveys could be used as the basis, with the stated preference data allowing for the modeling of choices that do not yet exist. Only Portland, Oregon, a pioneer in the development of activity-based models, has applied this type of model to the subject. The practical use of activity-based models in transportation planning is only now emerging and represents a significant effort [2].

Portland's methodology used in modeling may be classified as the state-of-the-art among all existing methodologies. Portland Metro developed the evaluation procedure as part of the Traffic Relief Options Study [27]. The study was initiated in mid-1996 with a pre-project grant from FHWA. The evaluation methodology was driven by Metro's tour-based activity model estimating the effects of various pricing options on mode, route, time of day, and destination choices, by income class and trip type [2].

Portland's activity-based travel models were developed as part of the Travel Model Improvement Program (TMIP) and were estimated using data from a two-day activity-based household travel survey conducted in the Portland area in 1994 and 1995. The revealed-preference estimation database was augmented by samples obtained from three stated-preference surveys: commute/non-commute pricing survey, auto acquisition survey, and urban design/residential location survey. The model is comprised of a series of disaggregate and nested logit discrete choice models presented in a hierarchy of model components [28].

A time of day model determines the timing of activities. A person's activity pattern is thus predicted in terms of frequency, timing, purpose, and complexity of the tours. A joint destination and mode choice model is applied at the primary home-based tour and secondary work-based tour levels. Within the model, each decision is highly conditional upon higher-level decisions, while those higher-level decisions have full information regarding lower level choices [2].

The model was applied using the regional 1,260 transportation analysis zones. One of the input requirements for the model is a set of auto travel times stratified by time of day, income (low, medium, and high), and auto occupancy (SOV and HOV). Overall, a total of 24 auto travel time skim tables are used by the model. Generation of these travel times requires several iterations of model runs. A time-equivalent toll is added to the skimmed auto travel time for scenarios involving a toll. Link-based travel times representing tolls are generated using a generalized cost multi-class equilibrium assignment. The equilibrium assignment with a special toll cost volume delay function was implemented using the EMME2 software platform. The assignment procedure uses two classes of demand: passenger cars and trucks (expressed in passenger car equivalents). A subsequent all-or-nothing assignment using the tolled volumes from the previous step but without the toll in the delay equation produces link times without tolls.

The difference between the two travel times obtained under the first and second assignments are link tolls in minutes [2].

The aforementioned toll times and eight vehicle classes (SOV low, medium, and high income; HOV low, medium, and high income; external vehicles; trucks) are used in a seventy-five iteration assignment procedure. A toll weight derived from survey data and the base year demand is assigned to each vehicle class. The weights suggest that all else being equal, high-income travelers are twice as likely to use a tolled facility than those in lower income brackets. The procedure is designed to post a toll on each individual tolled link based on the supply and demand for each tolled link and the competing non-tolled links. The procedure is applied for each modeled time of day. Trip tables are generated by mode (drive alone, drive with passenger, auto passenger, bus/walk access, bus/auto access, LRT/walk access, LRT/auto access, walk, and bicycle), time of day, income, and trip purpose [2].

2.2.2 Models within the Modal Split Component of a Four-Step Model

Automobile trips on a tolled or non-tolled road are considered distinct modal choices, with separate modal split functions for work (or work-related) and non-work trip purposes (given the corresponding differences in values of time). The advantage of this approach is that out-of-pocket costs can be modeled explicitly because travelers' utilities are "directly affected by the value of tolls and so are the respective modal shares"; that is, the approach ensures "robustness" in the results. The approach also can be expanded to trip distribution modeling because the impedance incorporates the impact of tolls more explicitly. The ability to incorporate stated preference data into revealed preference data as a means to account for nonexistent facilities again was noted [2].

Phoenix, Arizona, and Sacramento, California, were cited as examples of urban areas that have used this approach. The Phoenix Maricopa Association of Governments (MAG) uses an iterative procedure to estimate demand and tolls for its network's managed lanes. Tolls are coded as per-mile costs on candidate freeway links. After an initial model run, congestion on candidate-managed lanes is calculated with the goal of maintaining a level of service D/E. Given the level of congestion, tolls are determined and adjusted in an iterative manner in order to determine the optimum value at which enough SOV traffic is diverted from freeway main lanes to take advantage of the managed lane's excess capacity while maintaining the desired level of service and preventing the build-up of traffic congestion to compromise faster travel times [2].

To illustrate the procedure in this method, Florida Standard Urban Transportation Modeling Structure (FSUTMS) will be discussed in more detail. The Florida Department of Transportation (FDOT) employs a statewide system of travel forecasting software known as the Florida Standard Urban Transportation Modeling Structure, or FSUTMS. First conceived and implemented over 20 years ago, FSUTMS has been subject to numerous updates and improvements designed to improve its effectiveness as the main transportation-planning tool for all urban and regional study areas within the state of Florida [31].

One of these improvements is recent toll mode choice model development activities undertaken by the FDOT Turnpike Enterprise. Within FSUTMS, toll modeling originated by establishing specific toll amounts for appropriate network links together with a coefficient of toll to convert tolls to travel time impedances. In order to address contemporary toll study issues, however, toll modeling innovations were desired that addressed trip makers' toll route decisions as a mode choice step sensitive to changes in service levels by time of day (such as change in toll and congestion levels during peak and off-peak periods), trip purpose, and socio-economic attributes of trip makers (such as income) [31].

The Florida Department of Transportation (FDOT) Turnpike Enterprise has relied on "best practice" toll-forecasting procedures for its periodic update of traffic and revenue forecasting analysis. This analysis is required for the existing toll facilities and for the planning, design, and economic feasibility assessment of proposed new toll facilities. In keeping with this tradition, a panel of travel demand modeling experts was formed in 1998 to advise Florida's Turnpike Enterprise on the short- and long-term "best practice" improvements that could be developed within three to ten years to enhance the existing toll modeling capabilities. The cornerstone of the expert panel's recommendations was the development of a multi-modal modeling system that has a discrete choice component for toll travel based on survey results and other observed data unique to each metropolitan area. As a result of this recommendation and because of the presence of an extensive network of toll facilities in the Orlando area, it was logical to launch the program in this region [31].

Figure 2.2 illustrates Florida's Turnpike Toll Mode Choice and Assignment Modeling Steps in contrast with traditional mode choice and assignment steps used in MPO model [31].

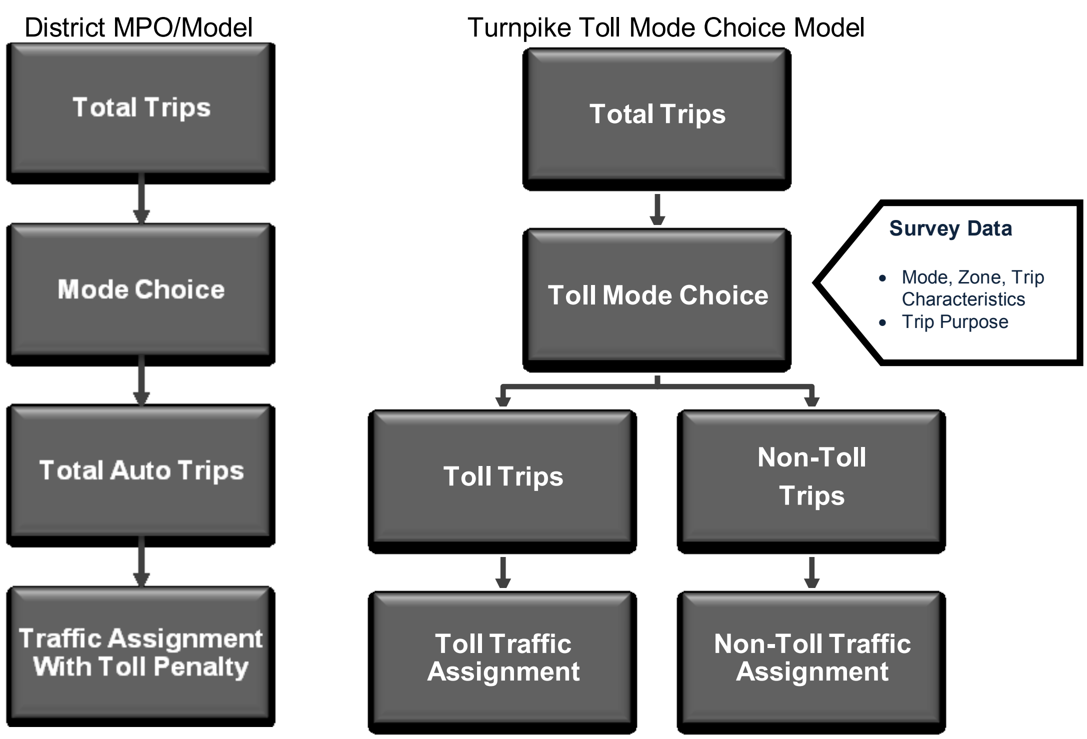


Figure 2.2 Comparison of the Toll Models

2.2.2.1 FSUTMS Toll Mode Choice Model development

This section describes the procedure of the development of the FSUTMS Toll Mode Choice Model at a level of detail found in related articles that is appropriate for the purpose of this research. The materials presented below are itemized directly based on toll model development. Other surrounding issues that have been considered less relevant to the issue of model development have been ignored.

2.2.2.2 Stated-Preference Survey

The stated preference survey provided quantitative information on the trade-offs that travelers make among travel time, cost, and other trip characteristics when they choose their mode, route, and time of travel. These data were used to statistically estimate the coefficients used to compute the share of travelers who choose a particular travel alternative given the characteristics of all of the available alternatives. The alternatives include auto drive-alone , auto shared-ride, and transit modes. For the auto modes, the choices between tolled and non-tolled routes and between different travel time periods (i.e., AM peak, mid-day, PM peak, and night) are represented in the model [31].

2.2.2.3 Origin-Destination Survey

The origin-destination survey provides details about travel times, purposes, vehicle occupancies, routes, and trip start/end locations. These data provide trip purpose by time-of-day factors and give a profile of the locations of internal trips (for use in the trip distribution). They were used along with the current travel information from the stated preference survey as “revealed preference” data for estimating route, time-of-day, and choice of mode [31].

2.2.2.4 Toll Mode Choice Estimation Analysis

This section describes the FSUTMS toll mode choice model estimation analyses. The model estimation used the origin-destination and stated-preference surveys that were conducted as part of the study and a 1996 transit (Lynx) on-board survey that provided revealed-preference data for transit passengers. Travel time and cost data from Orlando's regional four-step travel model were used to provide information about chosen and un-chosen alternatives for the revealed-preference modeling. The basic nested logit modeling approach focused on capturing travel behavior for choice of mode, route, and time-of-day [31]:

- Mode: auto drive-alone, auto with two occupants, auto with three or more occupants, bus, and rail
- Route: travel via toll road and travel via non-toll (free) road
- Time-of-day: choice between desired time of travel and time-shifted trip.

The statistical analysis of estimating logit choice models was performed using commercially available ALOGIT software. The statistical estimation was conducted in several stages, beginning with simple specifications and successively testing a wide range of specifications, segmentation schemes, and model structures. The initial modeling divided all travel time and cost variables into alternative-specific effects (assuming for example that transit travel time might be considered by travelers to be more or less onerous than auto travel times). In addition, all of the available demographic variables were included as alternative-specific variables in the models. These analyses were used to determine which mode-specific effects should be considered in the model specifications. The second estimation stage explored different segmentation schemes. Initially, two different segmentation approaches were evaluated: time-of-day and trip purpose (each having four segments):

- Time-of-day segments: AM peak, PM peak, midday, and night.
- Trip purpose segments: home-based work (HBW), home-based non-work (HBNW), non-home-based (NHB), and visitors.

The third estimation stage included tests of a wide range of nesting structures. The nesting structures were initially tested with the time-of-day segmentation and later tested with alternative segmentations. The fourth estimation stage incorporated revealed preference data into the estimation process. These data are useful for testing the match between the stated preference responses and actual observed behavior. In the final segmentation, combinations of time-of-day and trip purpose were evaluated.

2.2.2.5 Highway Assignments Procedure Using Generalized Costs

A generalized cost assignment procedure was developed for the purpose of achieving more realistic highway loading travel times. This process involved constructing two separate functions to reflect the generalized costs of traveling between two zones via toll road (GCtp) and via free road (GCfp) for time period (p). These two functions are mathematically expressed as follows [31]:

*GCtp = ap * [Time via toll road (minutes)] + bp * [Distance via toll road (miles) x (Tolls (cents/mile) +Operating Costs (cents/mile))]*

*GCfp = ap * [Time via free road (minutes)] + bp * [Distance via free road (miles) x (Operating Costs (cents/mile)]*

For the purpose of determining travel time and cost coefficients (represented, respectively, by “a” and “b” in the equations above for each time period, p, the time-period-specific route choice model was statistically estimated using the survey data.

The TRANPLAN software was modified to produce appropriate highway travel-time skim matrices. This permits the determination of congested travel-time matrices along the generalized cost paths for trip distribution and mode-choice applications. The initial results from using generalized costs in the highway equilibrium assignment process have shown improved loadings on toll facilities [31].

2.2.2.6 Feedback Procedure

The toll mode choice model application involves updating highway link times via toll and toll-free roads and feeding them back into the toll mode choice model through an iterative process. The purpose of this process is to produce more realistic modal travel demand estimation in each time period by using a more accurate representation of service levels for each time period. Highway travel times are calculated through a Mean Successive Averages COMbinations (MSACOM) program to achieve convergence on congested link times via toll and free roads. This program was implemented in the toll mode choice model application process. The process involves using loaded link volumes from successive model iterations "n" and "n+1" to calculate link volumes for iteration n+1 based on the following relationship [31]:

$$vol(n+1) = (1.0 - 1.0/n) \times vol(n-1) + (1.0/n) \times vol(n)$$

The resulting link volume, vol(n+1), is used in the volume-delay equation to determine the link time for cycle n+1. A generalized cost highway assignment process is used to produce loaded link volumes at each iteration.

The outcome of this process is a trip table for toll road users resulted from utility offered by the toll sub-model compared to utility offered by other modes, which is assigned to the network in the next step of toll assignment.

2.2.3 Models within the Trip Assignment Component of a Four-Step Model

This approach applies a diversion of trips within the trip assignment; that is, after (or in the absence of) demand modeling. It assumes that trip distribution and modal shares (not differentiating between tolled and non-tolled automobile trips) remain unchanged in the absence of feedback loops. There are two general methods for modeling traffic diversion in trip assignment: the first translates the monetary toll into a time equivalent through the use of values of time (generalized cost). The equivalent times are then incorporated into the model's volume-delay functions, which using the equilibrium assignment technique are used in turn to allocate trips among different paths according to travel time, capacity, and congestion. Queuing and service time at toll plazas similarly can be incorporated into the function (in essence, the tolls and plaza times are added as "penalties" to the modeling of actual travel time). Values of time can be derived for different trip purposes, income levels, etc. [12].

The second method uses diversion curves as the basis for toll forecasts. This commonly takes the form of a logit function, which calculates the propensity to use a tolled facility (the facility's share of traffic) as a function of the relative cost or travel time between the tolled and non-tolled route (i.e., for each origin-destination path that could use the tolled facility). The slope of the S-shaped diversion curve represents the elasticity of demand with respect to the relative cost or travel time using the tolled road. The elasticity of demand is related inversely to the value of time or willingness to pay. The shape of the curve can be determined in two ways: using observed data (in which case the value of time is implicit) or from a statistically estimated logit function based on revealed and/or stated preference survey data. The curves can be fitted according to different trip purposes and vehicle occupancies. The diversion curve is applied to the relevant trip table (for a given purpose, income group, automobile occupancy, time period, etc.) to derive tolled and non-tolled trip tables. These then are assigned to the network to yield both updated impedances, and the process is repeated until equilibrium is reached [12].

The difference between the two methods is the use of a dual minimum path (equilibrium) assignment, which develops two sets of paths for each origin-destination pair: one using the tolled facility (where applicable) and one without the tolled facility. A proportion of the total trips between each zonal pair is assigned to each network path according to the relative respective total costs, which can include vehicle operating costs as well as travel time costs and the costs of tolls [4].

The primary benefit of using diversion models to estimate toll road demand is that they can be applied to an existing four-step model without having to recalibrate it. However, the shape of the curve and the data upon which it is based generally are held as confidential or proprietary and are not made available to other users [13].

An example of the second method (diversion) is provided by a traffic and revenue forecasting study for construction of the 183A project, located in Williamson County north of Austin, Texas. For this study, the regional traffic model obtained from the Capital Area Metropolitan Planning Organization (CAMPO) and modified by Vollmer Associates for the 2002 Central Texas Turnpike Project (CTTP), was used as the basis for the traffic and revenue forecasts for this study [14].

A logit model was developed for several trip purposes based on a stated preference survey. The utility functions for work-related trip purposes were found to be sensitive to traveler income. The tolling diversion logit model was incorporated into the trip assignment component of an updated regional travel demand model. The model took into account different payment options (cash, cash plus electronic, and electronic only). The development of the logit model also accounted for toll road bias (the negative propensity to use a tolled road) and an electronic toll collection bias (the increased likelihood of using a tolled facility owing to the convenience associated with electronic toll collection). Both terms largely offset each other, with the toll road bias found to be common in regions that had no prior experience with tolling [14].

2.2.3.1 Assignment Toll Diversion Model Structure

The toll diversion model for the 183A project is structured as binary logit functions that partition trips between the toll and non-toll routes based on the travel time savings and associated toll costs. The logit model essentially determines the “probability” of selecting a toll road based on the time and cost tradeoffs. This probability reflects the share of trips between a given origin-destination zonal pair that will utilize the toll facility. The model has the following structure [14]:

Toll Share = (1 / (1+ e^U))

where:

Toll Share = Probability of using a toll road

e = Base of natural logarithm (Ln)

U (work) = a x (TimeTR-TimeFR) + b x (Cost)/Ln(Inc) + CTR + CETC

U (nonwork) = a x (TimeTR-TimeFR) + b x (Cost) + CTR + CETC

TimeTR = Toll road travel time in minutes

TimeFR = Non-toll road travel time in minutes

Cost = Toll in dollars

Inc = Annual income / 1000

CTR = Constant for toll road bias

CETC = Constant for ETC bias

a,b = Coefficients

These volumes (toll and non-toll) need to be fed back to the assignment procedure to determine the diversion with the new condition of the network after the first round, and this feedback (within the assignment program) will continue until the convergence is reached.

Pittsburgh, Pennsylvania, however, is another example of this approach to a toll diversion method that is slightly different from the previous method and can be considered an extension of the toll diversion model developed for Austin, Texas, since the feedback process to assignment program is omitted.

As part of a managed lane feasibility study in Pittsburgh, the Pennsylvania Department of Transportation (PennDOT) has developed a customized assignment process to estimate demand for a series of HOV and HOT lane alternatives. The customized assignment process was structured to permit both fixed and variable tolls. The technique permits tolls to vary by time of day for individual peak and off-peak periods, as well as within a given time in response to demand. The technique, developed with TP+ travel forecast software, utilizes a customized function embedded in the highway equilibrium assignment to dynamically determine toll values appropriate to maintain an acceptable level of service on the managed lanes. A logit-based route choice model, also embedded in the assignment routine, is then utilized to partition the trips between tolled and non-tolled paths within each iteration of the equilibrium assignment [2].

The original process was developed in order to support investment-grade financing analysis. The approach to the project was to utilize the existing trip generation, trip distribution, and mode choice components of the regional model. The model's 24-hour highway assignment procedure was replaced with the customized time-of-day assignment process with the embedded route choice model. The use of an embedded route choice model in the assignment process is to provide consistency between trips predicted to use the toll road and the trips assigned to the toll road in an attempt to eliminate the need to "preload" toll trips and/or to use feedback procedures between mode choice and assignment to resolve any inconsistencies between predicted toll trips and assigned toll trips [2].

The process is fully contained within a conventional equilibrium assignment. Within each iteration of the assignment process, toll and non-toll paths are skimmed for each vehicle type. The logit-based route choice model then partitions the trips by trip purpose and vehicle type (SOV and HOV) based on the toll cost and times from the toll and non-toll paths along with the payment method (cash or ETC). The equilibrium assignment loads tolled and non-tolled trips from that iteration, skims the network with the current assigned traffic, and repeats the process. The assignment iterations continue until equilibrium reaches a convergence. The final tolled and non-tolled traffic is calculated as the weighted average of the iterations, as determined by the equilibrium assignment technique. For scenarios in which variable pricing is being analyzed, the assignment process provides a listing of the toll values and tolled traffic by vehicle type for each iteration, which permits the analyst to calculate the final weighted toll value predicted by the model [2].

The implementation of this modeling process inside of a regional model with a recursive feedback process is relatively straightforward since the technique is embedded directly in the highway assignment procedure. However, neither the Austin nor the Pittsburgh models utilize feedback loops to pass the congested travel times and costs back to trip distribution [2].

2.2.4 Models as a Post-Processor

This approach can be used either within the framework of a four-step model or exogenously using the output of the four-step model. Washington, D.C., and San Diego, California, provided examples of the former in which assigned volumes are diverted (i.e., after trip assignment) from general purpose lanes to managed lanes according to the excess capacity available in the latter [1].

The Metropolitan Washington Council of Governments (MWCOG) uses a post-processing methodology within the framework of its four-step model for developing managed lane forecasts. Currently under development, the methodology will be applied to evaluate a proposed HOT lane project in Northern Virginia. Managed lanes and general-purpose lanes are explicitly coded in MWCOG's highway network [2].

Highway pricing is addressed by converting tolls to equivalent travel time values for all pertinent i-j pairs, which are in turn added to travel times developed by skimming the highway network. Highway accessibility is, therefore, reduced when tolls are imposed. Equivalent travel times are developed by traveler's income to accommodate MWCOG's income-stratified trip distribution model, as well as by vehicle type for use in the mode choice and traffic assignment models. Through the application of a diversion method, traffic from general-purpose lanes is reallocated to the managed lane in a magnitude equivalent to the excess capacity of the managed lane while maintaining an acceptable level of service on the tolled facility [2].

To achieve this goal, an initial run of the four-step modeling process is applied to estimate the background HOV traffic on managed lanes. Next, an automated procedure is used to assess the amount of spare capacity available on each managed lane freeway segment. The ideal capacity for HOT lanes ranges from 1,400 to 1,800 vehicles per hour per lane. The spare capacity is, therefore, defined as the ideal capacity less the "background" hourly HOV volume as developed from the initial application of the regional travel demand model. Finally, traffic on the general purpose lanes is diverted to the managed lane in the amount of calculated residual capacity on a section-by-section basis [2].

2.2.5 Models as a Sketch Planning Method

Quick response analysis tools have also been used in the evaluation of value pricing projects. Decorla-Souza of FHWA has used the Spreadsheet Model for Induced Travel Estimation (SMITE) in a value pricing case study of the Capital Beltway in Northern Virginia. The study evaluated the proposal of financing the expansion of the Capital Beltway by charging SOV tolls on the proposed new lanes [29].

Estimation of induced traffic (e.g., traffic resulting from faster travel speeds, diverted traffic from other routes, destinations, or modes) in SMITE is based on the elasticity of demand with respect to travel time. An equilibration of price and demand is also part of the procedure. A modified version of SMITE (referred to as SMITE-Managed Lane or SMITE-ML) with a pivot point mode choice model was used in the case study. The pivot point logit model estimated changes in travel demand on different modes of travel resulting in changes to travel time, tolls, and improved transit service. A variety of performance statistics are generated for each tested alternative and used in the evaluation task. The model is relatively simple to implement and can be considered a reasonable tool for the initial screening of alternatives or in situations where the results of formal travel models are not readily available [29].

Another similar quick response analysis package, also developed by Patrick Decorla-Souza of FHWA, is the Sketch Planning for Road Use Charge Evaluation (SPRUCE) model [30]. Using a pivot point mode choice technique, the model estimates changes in a traveler's choice of mode and the associated revenues, costs, and travel time delays. The model is designed to address both HOT lanes and fast and intertwined regular (FAIR) lanes. The idea of FAIR lanes is a relatively new value pricing concept intended to overcome the equity issue normally associated with implementation of HOT lanes. Under a FAIR lane scenario, freeway lanes are separated into two sections, fast and regular lanes. Fast lanes are dynamically-priced (tolls are electronically charged) to ensure uncongested traffic movement under conditions close to free flow speeds [2].

Users of regular (non-tolled) lanes, on the other hand, would still be experiencing congestion but would be eligible to receive credits should their vehicles be equipped with electronic tags. The credits, set as a portion of the fast lane toll price, are meant to compensate the regular lane travelers for giving up the right to use the converted fast lanes. Accumulated credits could be used toward the use of fast lanes or transit and paratransit services. Using the estimated daily freeway and arterial traffic volumes under the base case scenario, estimates of vehicle demand and delays are prepared for the study corridor by hour of the day. A pivot point mode choice model estimates the change in mode share for each alternative (based on anticipated changes in travel cost and time within the corridor) and the resulting estimates of vehicle demand and delays for each hour and each alternative. The underlying assumption of the technique is that through the application of variable pricing, the entire capacity of the managed lane is utilized, in such way that, no delays are foreseen for those vehicles on the managed lane.

The model also considers the effects of spill-over demand on nearby arterials by calculating delays experienced on the corridor arterials resulting from the diversion of freeway traffic. Arterial network management and capacity enhancements are integral components of FAIR lanes value pricing scenarios. The model calculates measures of consumer surplus for the new carpoolers and transit riders as well as the single-occupant vehicles and previous carpoolers who continue to use freeway main lanes [2].

Table 2.1 summarizes and compares the modeling methodologies that have been described in this section.

Table 2.1 Summary and Comparison of the Toll Diversion Modeling Methodologies [2]

Modeling Methodology			Portland, OR	Pheonix, AZ	Atlanta, GA	Pittsburg, PA	San Diego, CA	Sacramento, CA	Washington, DC
Toll Categories	Vehicle Type (Car, Truck)		Y	Y	Y	Y			
	Occupancy Level (SOV, HOV)		Y	Y	Y	Y			
	Time Period (Peak, Off Peak)		Y	Y	Y	Y			
	Payment Method (Cash, ETC)					Y			
Tolls Influence	Trip Distribution		Y	Y				Y	Y
	Mode Choice		Y	Y	Y	Y		Y	Y
Diversion Modeled	Within Regional Model		Y	Y		Y		Y	Y
	Post Process – Model				Y				
	Post Process – Off Model						Y		
Toll Diversion Method	Mode Choice Model	Trip Purpose	Y	Y	Y			Y	
		Auto Occupancy	Y	Y	Y			Y	
		Payment Type							
		Feedback Loop	Y	Y				Y	
		HOV/Toll Trips Preloaded					Y		Y
	Highway Assignment (Route Choice Submodel)	Trip Purpose	Y			Y			
		Auto Occupancy	Y			Y			
		Vehicle Type	Y			Y			
		Payment Type				Y			
	Highway Assignment (Equivalent Time Penalties)	Trip Purpose							
		Auto Occupancy			Y		Y		Y
		Vehicle Type			Y			Y	
		Payment Type							
Diversion Sensitive to	Houshold Income		Y			Y		Y	
	Assignment Estimated Congestion		Y	Y	Y	Y	Y		Y
	Tolls Varied by Estimated Congestion		Y	Y	Y	Y			

2.3 Specific Factors Affecting Forecast Performance

Aside from modeling approaches and methodologies, there are other factors that can significantly affect the performance of a toll road traffic and revenue forecast. However, treatment of these factors is not the focus of this research; for completeness of the discussion and familiarity with the concepts around toll diversion models, they will be briefly discussed below.

2.3.1 Demographic and Socioeconomic Inputs

There are two relevant issues regarding the demographic and socioeconomic inputs. The first concerns the use of long-range demographic and socioeconomic forecasts (so-called land use inputs to the model) that may reflect an MPO's planning policy (i.e., as the source for these inputs) as opposed to market trends. Recent toll road demand and revenue forecasts have responded to these concerns by modifying these assumptions to account for input scenarios that were more conservative and that took into account historical trends and a more realistic assessment of likely future growth [7].

The second issue is the lack of consideration of the impact of short-term economic fluctuations on travel demand. The impact of optimistic economic projections on traffic projections was noted in several studies. The national recession of 1990-1991 affected the use of the first two segments of the Central Florida Greenway, which had opened in 1989 with first-year projections just slightly below actual, but with poorer results for the next two years (over the course of the recession). A "drag" from the recession was considered to have affected toll roads in Oklahoma City and Tulsa, Oklahoma, which opened just after the recession. Local economic impacts, such as the collapse in oil prices and the subsequent sharp regional economic downturn of 1986, left economic growth in the Houston area well below projections, with corresponding impacts on the Hardy and Sam Houston toll road revenues. Even when regional economic activity was close to the original projections, the performance of some tolled facilities still fell short because economic activity within the immediate corridor did not meet projections (e.g., the Sawgrass Expressway in Florida) or the expected build-out of residential areas was slower than expected (e.g., the Seminole Expressway toll road, also in Florida) [8]. Practitioners have begun to consider the impact of short-term economic changes.

The aforementioned Transportation Corridor System forecast took into account a "recession scenario," which considered a "double dip" of below average job gains in the immediate term, followed by job losses for the next two years, then by a modest recovery and a recessionary dip in the seventh year. These inputs were used as part of a sensitivity test of the demand and revenue forecasts [23].

Although the analytical horizon for toll road demand and revenue forecasts clearly cannot be shortened, a current-year or very-short-term toll demand forecast based on a hypothetical immediate opening of the facility would allow analysts and users to differentiate the demand that would result from the network improvement and from assumed demographic or economic growth. Further analyses could test the impact of the facility, with and without tolls, again in the short term, to isolate the impacts of tolls. In other words, although these short-term forecasts might have limited use in the development of absolute estimates of revenues, they would be valuable in grounding and interpreting the long-range forecasts (i.e., to provide a reference against which to compare that proportion of forecasted long-term facility traffic that would use the facility whether or not the toll is in place or independent of assumed growth) [1].

2.3.2 Travel Characteristics

The availability of appropriate data and the quality of these data are among major sources of potential forecasting inaccuracies. These data include such variables as traffic counts, network characteristics, travel costs, land use, and employment. Inappropriate base-year data can result in model validation errors, which in turn affect all subsequent applications and forecasts. In practice, these data, which are the foundation of the forecasts, were found to be subject to substantial numbers of measurement and processing errors [12].

Current data collection practices are well-established. They include origin-destination surveys, trip diaries, activity-based surveys, stated preference surveys, traffic counts, travel time data, and speed surveys. Surveys of existing socioeconomic and transportation system characteristics are required for calibration. However, some potential sources of error can be introduced by sampling (in surveys) [1].

On the other hand, the appropriate categorization of travel markets in terms of their individual values of time and willingness to pay are important, with income levels and time sensitivity (i.e., trip purpose) being important determinants. The ability to save time is the most important determinant of whether or not a private automobile driver chooses to use a toll road, whereas truck drivers also take into account the impact on vehicle operating costs (i.e., that a toll road's "competitive advantage" for trucks must be measured both in terms of time savings and the ability to save on fuel costs and reduce vehicle wear and tear). Other important factors in this regard are assumptions related to land use forecast, future network configurations, public and political influences in different scenarios, environmental or economic development considerations, and economic climate [1].

2.3.3 Value of Time and Willingness to Pay

The treatment of the ability and willingness of potential users to pay is cited as a key performance factor both in the literature and by practitioners. Values of time can be differentiated by purpose, mode, and/or vehicle class. Willingness to pay is a variation of value of time that accounts for how much travelers value different attributes of the toll facility, such as safety and reliability [1]. Some drivers in fact perceive managed facilities to be safer. This would in turn affect their willingness to pay beyond their actual value of time.

The valuation of travel time is based on two underlying principles [24]. The first principle states that time is valuable because people can associate it directly with results such as making money or participating in a leisure activity—that is, the time spent in travel could be devoted instead to other activities. The second principle assumes that time can have an additional cost over and above that associated with the first principle; for example, travelers might find it undesirable to have to walk, wait for transit, travel on a crowded bus, be late for work and other appointments, or drive in congested conditions. As a result, the value of saving time may vary depending on both the purpose of travel, which affects the possible alternative uses of time, and the conditions under which it occurs.

The measurement of the perceived value of a driver's travel time yields the value of time. This influences a driver's decision to use a toll road. Values of time vary from region to region, and what is assumed for one forecast may not be transferable to another forecast. The value of time is a function of a driver's purpose (where work trips are more valuable than discretionary trips), income, and personality. The value of time is used to convert the monetary toll to time. This allows the monetary value to be incorporated into the model's generalized cost function. As described earlier in Methods for Modeling Toll Road Demand, this is incorporated into the calculation of route diversion (within or post trip assignment). Most of the toll road traffic and revenue forecast studies incorporate value of time in their model, and some use willingness to pay. However, there are studies that use both [1].

The choice and derivation of the values used for this determination are the subject of considerable debate in the literature so that the U.S.DOT has developed a guidance document on the subject. The guidance reviewed the factors that are associated with the value of travel time. For trips made during work or when the traveler could vary his or her work hours, the guidance noted that "the wage paid for the productive work that is sacrificed to travel" could be used to represent value of time. The value of time for other (personal) purposes can be represented by some fraction of the wage rate. Thus, the hourly income (before-tax wage rates, including fringe benefits) could be used as a “standard against which their estimated value of time is measured.” In addition, higher income has been associated with higher values of time, meaning that toll roads that operate in higher-income areas should experience greater patronage (and support higher toll rates) [5]. The guidance document developed tables that expressed the values of time (and “plausible ranges”) as percentages of hourly incomes, categorized by local and intercity travel, business (work-related) or personal trip purposes, and mode (surface modes taken together and air travel). A separate category was developed for truck drivers [1].

Recently, however, stated preference surveys as the basis of valuating time has been used widely in toll demand and revenue forecasts. Stated preference surveys attempt to measure the value of time by presenting hypothetical options to respondents to quantify how toll rates would affect driver behavior. Stated preference surveys for toll demand forecasting are generally in three parts: (1) background information on a recent trip in the study corridor, (2) a set of stated preference experiments, and (3) demographic information. The background information provides revealed preference data about an actual trip, as well as baseline data to customize the stated preference scenarios. The variables of interest are determined and ordered into a series of scenarios that is presented to respondents as part of the experiments. The scenarios are designed so as to allow the subsequent estimation of the respondents’ relative preferences for each of the tested variables [1].

Diversion (multinomial logit) models and values of time in turn are calculated from these estimates. Travel time, toll cost, and income typically are included as attributes, with values of time calculated by trip purpose (work and non-work) and separately for automobiles and trucks. Stated preference surveys in areas that do not have tolled facilities have tended to result in low values of time because respondents express their “anti-toll road sentiment.” In areas that have existing toll roads and severe peak period congestion, respondents have tended to overestimate their values of time. In either case, the calculated values of time may have to be recalibrated to reflect actual conditions more reasonably. The availability of electronic toll collection (as opposed to cash collection) may also influence the value of time in that electronic toll collection users may be less aware of and therefore less sensitive to the total toll paid on a trip, at least in the short term [1].

Finally, in relevant studies it has been noted that travelers value reliability as an important factor in their trip making decision. Reliability generally reflects the day-to-day variability in expected journey times owing to non-recurrent congestion such as incidents, weather, construction, and so on. Reliability is considered important in variable priced managed lane applications, where tolls are adjusted according to traffic volumes, to maintain a specified level of service. Reliability can be critical for travelers with fixed schedules (such as individuals with daycare pick-ups or those going to the airport) and is not necessarily correlated with the traveler's general value of time. However, mainly due to a general lack of data, there are few (if any) operational demand models that account for reliability in traveler values of time or the value of travel time reliability [13].

2.3.4 Tolling Culture, Ramp-Up, Truck Forecasts, and Time Choice Modeling

An international review of toll road traffic and revenue forecasts demonstrated better performance in countries that had a "history" of toll roads compared with those where road tolling was new. In countries with a history of tolling, consumers can be observed making choices about route selection, effectively trading off the advantages against the costs of using tolled highway facilities. The consumer response can therefore be more readily understood by forecasters preparing predictions for new or extended facility use. In contrast, in countries where tolling is new, there are no revealed preference data on consumer behavior, which leaves forecasters more reliant on theoretical survey techniques and assumptions about how drivers may respond to tolls [25].

The ramp-up period reflects a toll facility's traffic performance during its early years of operation. This period may be characterized by unusually high traffic growth. The end of the ramp-up period is marked by annual growth figures that have (or appear to have) stabilized and that are closer to traffic patterns that have been observed on other, similar facilities. The ramp-up period reflects the users' unfamiliarity with a new highway and its benefits ("information lag"), as well as a community's reluctance to pay tolls (if there is no prior tolling culture) or to pay high tolls (if there is a history). The performance of the facility during ramp-up is particularly important to the financial community because the probability of default is typically at its highest during the early project years [9].

Truck traffic is another important input in toll road traffic and revenue forecasts. Although truck traffic typically comprises a relatively small portion of the traffic mix, they commonly pay two to five times and sometimes as high as ten times the respective car tariff, so their contribution to total revenues can be significant. However, it should be mentioned that the choice by trucking firms to use toll roads in turn was found to be "very sensitive" to tolls [22].

The need for models that more accurately capture the differences in travel patterns by time of day, day of week, and even season was identified by the financial community. The object is to account more explicitly for the temporal variation in composition of trip purposes, origins and destinations, and vehicle types, including (in addition to peak-hour travel) off-peak, midday, night, and weekend [9].

2.3.5 Risk

Many assumptions and variables must be interpreted and relied on to complete a traffic and revenue study. The ability to ensure exactness and accuracy in all of these is limited for representations of existing conditions as well as forecasts. A common treatment has been to address uncertainty through the simple use of conservative assumptions or ranges [1]. The literature indicated that sensitivity analyses on key variables was common practice, such as the area growth rate, value of travel time, planned toll rates, and other variables that were region-specific or had shown a high degree of variability in the past [1].

However, it is important to make the distinction between the assessment of risk and other indirect treatments of uncertainty (such as judgment or sensitivity analysis). The first is related to the inclusion of an appropriate and complete set of assumptions and inputs. The second requires a proper understanding of the roles of the different treatments of uncertainty. For example, it is mentioned in the literature that sensitivity analysis does not adequately reveal the range of possible outcomes in a toll road forecast [26]. Instead, a range of possible outcomes could be explored based on Monte Carlo simulation and the probability not only of the variables acting as individual occurrences but in combination with each other based on their respective probability of occurrence [26]. Another treatment is offered through “reference class forecasting,” which uses the experiences of past projects to help statistically identify the probability of given inputs occurring at a particular value [15].

Toll road demand and revenue forecasts have given little or no consideration to the possibility of a series of events occurring simultaneously, for example if economic growth recedes, oil prices spike, and a large development that was scheduled to be in place at the time of the opening of the toll road is cancelled. Traditional sensitivity analysis typically took each of these assumptions and varied them one at a time; however, these assumptions often varied by arbitrary amounts [26].

The National Federation of Municipal Analysts (NFMA) is comprised mainly of research analysts who are responsible for evaluating credit and other risks with respect to municipal securities. NFMA has worked with non-analyst professionals in various sectors to develop recommended best practice guidelines for certain markets, including the toll road demand and revenue forecasts. In these guidelines, NFMA has attempted to account for the likelihood that there are many possible outcomes if future events do not follow the projected assumptions that are predicted in the model. Given the large number of input variables required in the modeling process, NFMA found that the results of forecasts can be significantly influenced by changes in these inputs [26]. By applying the appropriate background data inputs to the toll forecast, a model could produce a traffic and revenue forecast that is most likely to occur, often called a base scenario [1].

Throughout the literature, it was also noted that the determination of risk should not focus on a single outcome but should explore a range of possible outcomes. This process first determines the degree of risk in each input variable by developing probability distributions for all variables. Risk analysis is carried out by allowing all the underlying variable estimates to vary simultaneously, which can be done using simulation techniques such as the Monte Carlo technique. The risk and uncertainty in the underlying input variable is then translated into a probabilistic, risk-adjusted forecast of output variables such as traffic levels, toll rate, revenue, and debt service coverage. Finally, the variables that drive risk—that is to say, the variables that have the greatest influence on the forecast—are identified [1].

CHAPTER 3

METHODOLOGY

3.1 General Description of the Project

As mentioned in previous chapters, this study focuses on path choice models in presence of road pricing. Generally, in the literature, two types of path choice models are employed in network equilibrium assignment models for road pricing applications. The first type implements deterministic path choice models based on a generalized path cost (time) function in which path travel time (path cost) is weighted by a trip-maker's value of time (VOT) representing how much money the trip-maker is willing to trade off for unit time saving. The other type is characterized by probabilistic discrete path choice (e.g. logit- or probit-based) models consisting mainly of travel time and out-of-pocket cost. Those discrete choice models can be constructed and calibrated from revealed or stated preference survey analyses to determine the probability that a trip-maker will use paths that include tolled facilities [38].

Empirical studies have found that path choice models with random coefficients have better goodness of fit than those with constant coefficients that assume homogeneous perception of tolls for all trip-makers. This means that every trip-maker is willing to trade off the same amount of money for a unit of time saving corresponding to the constant coefficients associated with the path travel time and travel cost in the generalized path cost function. Other studies have suggested that the VOT varies significantly across individuals because of different socio-economic characteristics, trip purposes, attitudes, and inherent preferences. This user heterogeneity is cleared by the fact that some trip-makers take slower paths to avoid tolls while others choose toll roads to save time. Therefore, it is essential to recognize explicitly and represent heterogeneous users in modeling users' response to toll charges for toll road applications. This is especially important in assessing the feasibility of a proposed toll facility and its financial viability from the standpoint of the public or private entity that will be operating it [38].

In this study, as will be discussed throughout this chapter, different factors (vehicle class, trip purpose, and time of day) will be considered in order to determine the toll road users based on their willingness to pay and value of time.

It is the main objective of this chapter to build the foundation of the research by adopting a series of data as inputs to the toll assignment model, namely a discrete choice model that solves for the aforementioned probability. This chapter generally will establish the framework of the research by using numerical examples for an existing network in Atlanta, Georgia.

This chapter is organized in two main parts. The first part introduces inputs to a regular assignment sub-model as part of a 4-step travel demand model. This data collection is done through adopting existing traffic and revenue studies and projects. For this particular prerequisite, the I-20 Managed Lanes project in Atlanta, Georgia was selected. It is well-suited to the purpose of this study based on its level of detail and complexity. The second part of the chapter presents the characteristics of the adopted discrete choice model and its underlying assumptions.

3.1.1 Study Area

The project study area is defined such that the majority of the trips have their origin and destination inside it. The study area is divided into smaller internal zones called Traffic Analysis Zones (TAZs).

TAZs are represented in the computer models as if all their attributes and properties were concentrated in a single point called the zone centroid. This reference point can be best thought of as floating in space and not a physical location on a map holding traffic information pertaining to the socioeconomic characteristics of each zone. The region external to the study area is normally divided into a number of external zones. In some cases, it might be enough to consider each external zone as representing “the rest of the world” in a particular direction of the transport links feeding into the study area [33].

The selected study area for this research is 20 counties in and around Atlanta, Georgia. Figure 3.1 depicts the study area along with the corresponding TAZs.

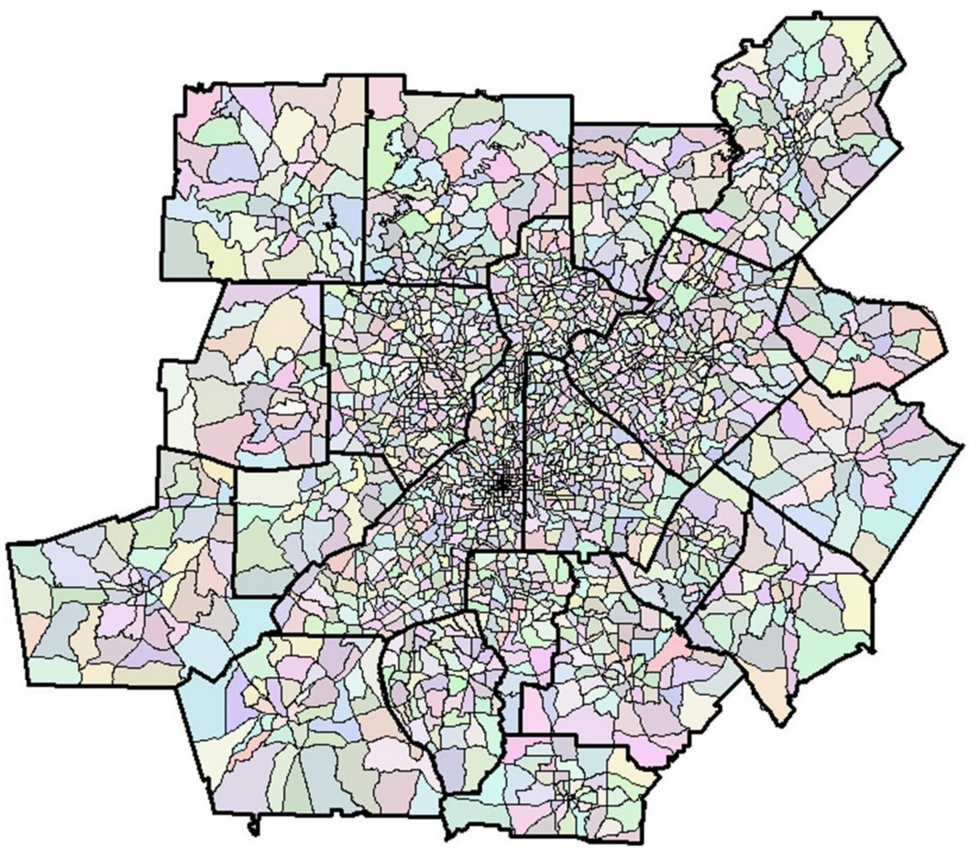


Figure 3.1 Atlanta 20-County Region with TAZs (Scale: 1Inch=20 Miles) [36]

In 2007, the Georgia Department of Transportation (GDOT) commissioned a traffic and revenue study to support its delivery of the solicited I-20 Managed Lanes (ML) project under GDOT’s Public Private Initiative (PPI) program. The existing I-20 is an interstate limited access freeway providing an essential east-west connection and running through downtown Atlanta. In addition to its interstate connectivity function, it is vital for commuters who live in the east suburbs of Atlanta and commute to the downtown area or to locations north and south of the downtown area. Within the project location, I-20 generally consists of 3 lanes per direction east of the I-285 interchange to Sigman Road and 4 lanes per direction, in general, from west of the I-285 interchange to Boulevard Rd. From Boulevard Rd to Hill St., it contains 5 lanes per direction. From Hill St to the I-75/I-85 ramps, it contains 4 lanes per direction. To the west of the I-75/I-85 ramps, it contains 3 lanes per direction. In addition, one lane of HOV lanes in each direction starts at a location just east of Boulevard Rd. and extends to a point just east of Colombia Drive [34].

The proposed project involves adding variable toll lanes in each direction, replacing the HOV lanes where they exist, and adding new lanes where there are no HOV lanes. The project limits are I-75/I-85 at its western terminus and Sigman Road at its eastern terminus, with a total length of approximately 20 miles [34].

The I-20 ML connects to two major interchanges, allowing access to two major freeways: I-75/I-85 and I-285 east of Atlanta. In addition, a number of interchanges have been proposed to access arterials and local roads. Figure 3.2 shows the proposed project overlaid on a regional map. Figure 3.3 shows the schematic and assumed connectivity as well as number of lanes on the Tolled Lane and General Purpose Lanes (GPL) [34].

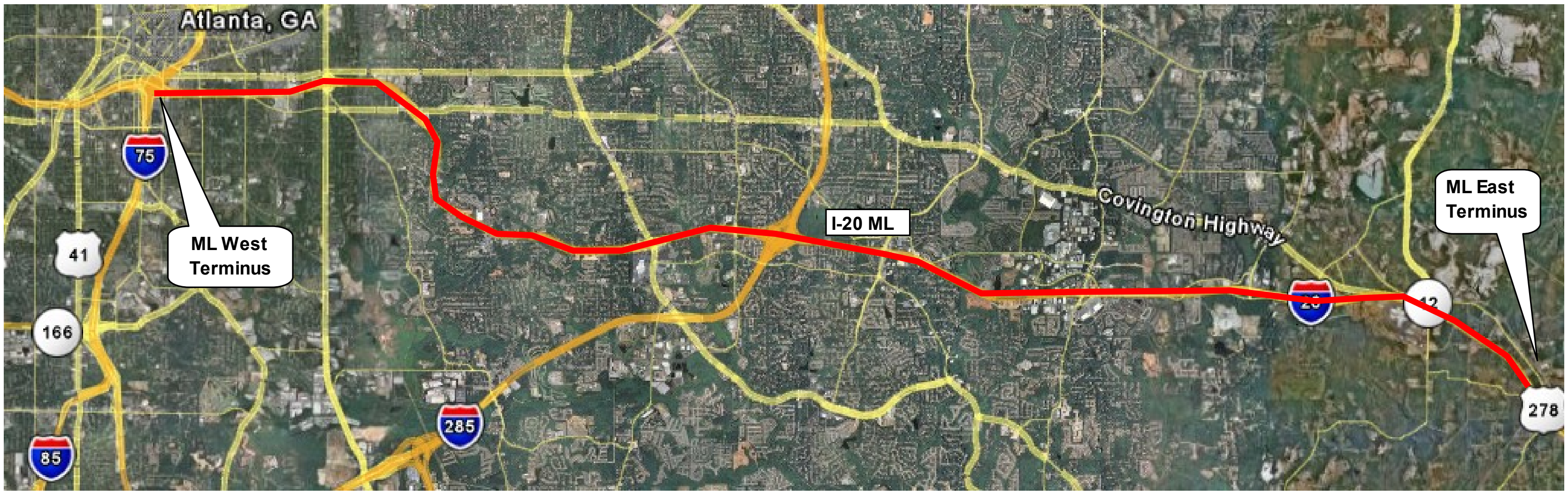


Figure 3.2 I-20 East Managed Lanes Project Location [34]

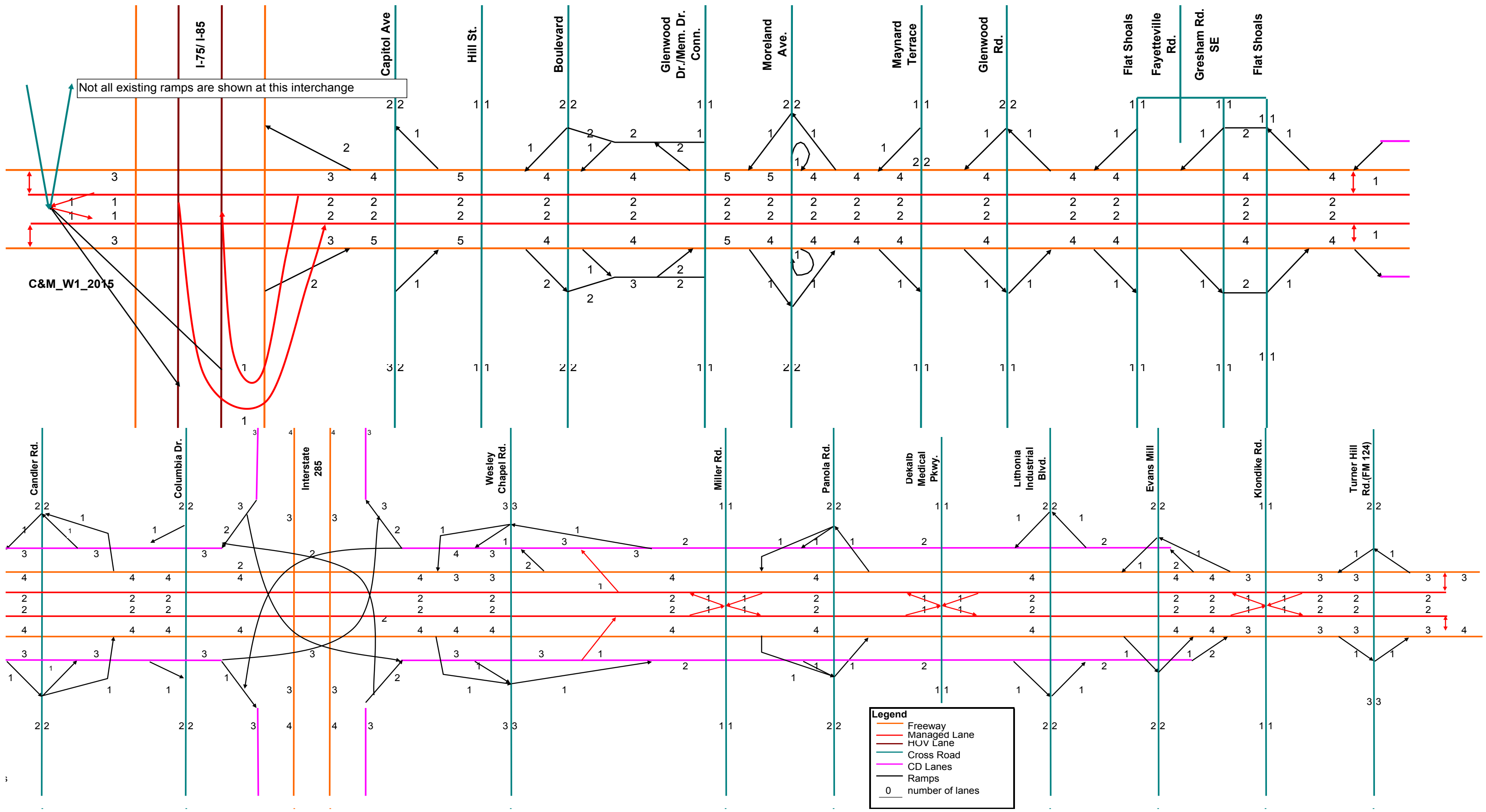


Figure 3.3 I-20 Managed Lanes Connectivity to the Roadway Network [34]

3.1.2 Data

Development of a trip assignment sub-model requires a set of information such as a geographical representation of the study area, corresponding descriptive information, and data related to traffic demand and travel cost. In this research, preliminary data inputs were adopted from available public information as well as studies that have been carried out by private practitioners relevant to this project. Throughout this research, adopted data from the private sector has been referred to as information from “final reports” or “available data from private resources.”

The initial modeling process to obtain data inputs for an assignment sub-model involved the use of the Atlanta Regional Commission (ARC) travel demand model as a base for the development of the preliminary travel demand model. The Atlanta Regional Commission travel demand model is a 4-step, trip-based model developed initially in 2003. Its current version, which includes some refinements applied in 2005-2006, includes 20 counties.

The model’s trip generation and distribution are based on an 8000-household survey conducted in 2001 and 2002. The trip distribution model is a gravity model using travel time on the highway and transit as impedance measures. The distribution model used the county-to-county census journey to work data of 2000. ARC’s mode choice model is a nested logit base utilizing both the household survey data and the transit on-board survey data. The current version of the model includes an enhancement in truck and commercial vehicle flow. Full documentation of the model, its development process, theoretical background, calibration, and individual components can be obtained from ARC.

Furthermore, this model was calibrated and validated by a private-sector organization focusing mainly on the project area of influence to replicate the year 2007 (base year) traffic conditions. The configuration of the managed facilities was determined and superimposed on the ARC future networks of 2015 (the project’s opening year) and 2030 (the design forecast year). However, the main assumption here is that the adopted data already is validated for reasonableness and is not subjected to an additional validation process throughout this research.

3.1.3 Network Coding and Preparation

Basically, a transportation network is the supply side of a transportation system which itself supplies a level of service that could be a function of time but also is associated with other monetary costs such as fuel and fares or tolls. A transportation network models the roadways as a directed graph, i.e. a system of nodes and links joining them. Most nodes are taken to represent junctions, and the links stand for homogeneous stretches of road between junctions; links are characterized by several attributes such as length, speed, number of lanes, and so on. A subset of the nodes is associated with zone centroids, and a subset of the links is associated with centroid connectors. Centroids are attached to the network through centroid connectors representing the average costs (time, distance) of joining the transport system for trips with origin or destination (zone) [33].

The network that has been used for this research is the ARC network, which has been calibrated to 2007 traffic conditions by applying updates to the 2005 ARC network within the project area. The network improvements included updating the speed and number of lanes, including recent projects related to roadway enhancement, checking network attributes for correctness, checking the network connectivity, and so on.

The network used as the basis of the assignment sub-model here is the “build network” with the tolled facilities coded inside it. The build network was developed based on the calibrated base year (2007) network with some extra modifications in order to bring it to corresponding opening year (2015) conditions. These modifications included regional future transportation projects and plans. Figure 3.4 depicts the opening year (2015) network (selected for this study) with the tolled facilities incorporated inside it.

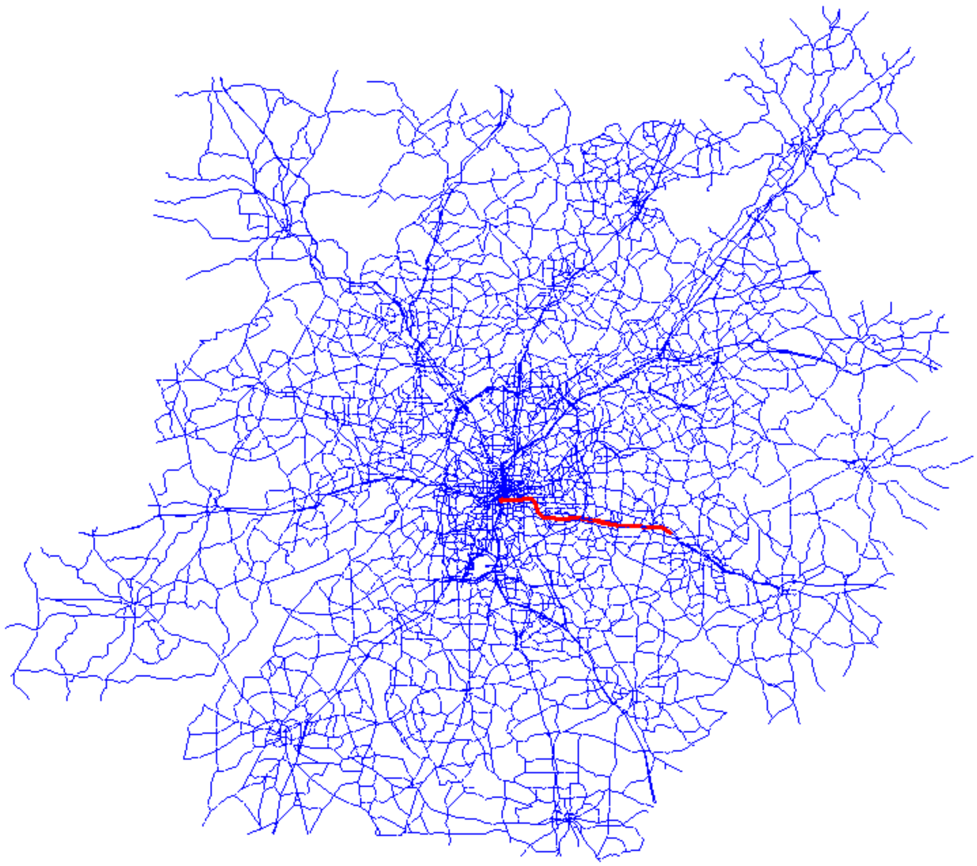


Figure 3.4 ARC Roadway System Network (Scale: 1Inch=20 Miles) [34]

3.1.4 Network Characteristics

A key decision in setting up a network is how many levels to include in the road hierarchy. What matters is to make route choices and flows as realistic as possible within the limitations of the study. Some investigations regarding the influence of network definition and details with respect to road assignment accuracy have suggested that the largest errors were obtained at the lower levels in the hierarchy of roads. Therefore, at least one level below the links of interest should be included in the network [33]. This link hierarchy presented within a network is managed by "roadway system functional classification" and "facility type." Facility type itself could be considered an identification code that changes the course of assignment procedure for a set of particular links. Facility type together with the “area type,” which is itself a regional (zonal) classification of links (such as urban, suburban, or rural) are the key elements to determine other attributes to the links such as speed and capacity.

A list of area type classification within the network is presented below. The ARC model uses 7 area types based on the density of the regions as listed below:

1—CBD/Very high density urban

2—High density urban

3—Medium density urban

4—Low density urban

5—Suburban

6—Exurban

7—Rural

Figure 3.5 illustrates the same selected roadway system network by functional roadway classification.

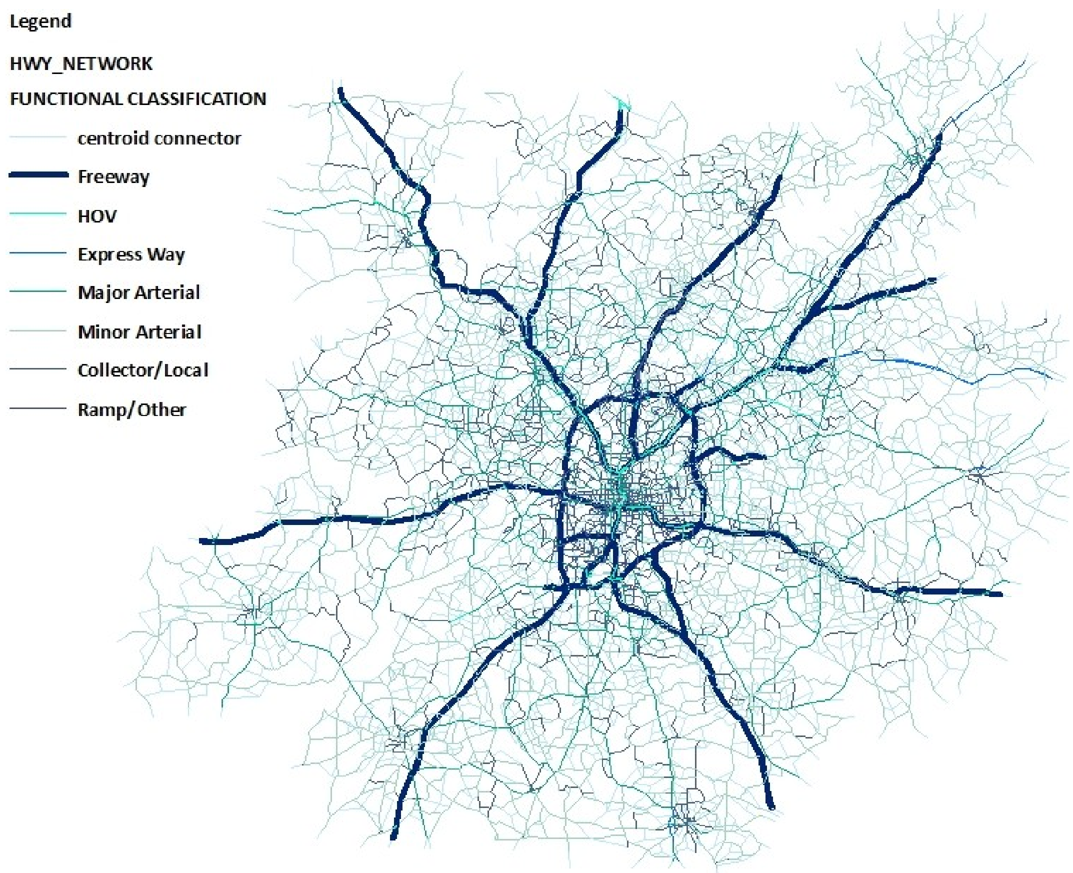


Figure 3.5 Roadway System Facility Type Classification (Scale: 1Inch=20 Miles)

The level of detail provided for the attributes of the links depends on the general resolution of the network and on the type of model used. At the very minimum, the data for each link should include length, travel speeds (either free-flow speeds or an observed value for a given flow level), and the capacity of the link, usually in passenger car equivalent units (PCU) per hour. Table 3.1 shows the required data fields for the initial highway network. Table 3.2 and Table 3.3 show speed and capacity classifications by facility type and area type pertaining to the selected network.

Table 3.1 Required Data Fields for Initial Highway Network

Variable	Definition
A	Beginning node
B	Ending node
Distance	Link distance
Prohibition	Link restriction parameters 0 = No restrictions 1 = Trucks prohibited 2 = HOV lanes 3 = Managed lanes 5 = Truck prohibition inside I-285
Lanes	Number of through lanes in one direction
Factype	0 Centroid connectors 1 Interstate/freeway 2 Parkway 3 HOV buffer separated 4 HOV barrier separated 5 High speed ramp/CD road 6 Medium speed ramp 7 Low speed ramp 8 Loop ramp 9 Off ramp w/ intersection 10 On ramp w/ intersection 11 Expressway 12 Principal arterial - Class I 13 Principal arterial - Class II 14 Minor arterial - Class I 15 Minor arterial - Class II 16 HOV - Arterial (all classes) 17 Major collector 18 Minor collector/Other local 19 Planned ramps w/ intersections 20 Planned directional ramps
Zone	Nearest traffic analysis zone
Area Type	Link area type 1—CBD/Very high density urban 2—High density urban 3—Medium density urban 4—Low density urban 5—Suburban 6—Exurban 7—Rural
Capacity	Total link capacity (1 Hr -LOS E)
Toll	Fixed toll cost
Speed	Free-flow speed

Table 3.2 Free-flow Speed by Area Type and Facility Type

	Area Type						
Facility Type	Urban Very High Density	Urban High Density	Urban Medium Density	Urban Low Density	Suburban	Exurban	Rural
Zone Centroid Connectors	7	11	11	11	11	14	14
Interstate / Freeway Free Flow	55	58	58	61	61	63	65
Parkway	50	50	55	55	57	60	60
HOV Buffer Separated	55	58	58	61	61	63	65
HOV Barrier Separated	55	58	58	61	61	63	65
High Speed Ramp / CD Road	50	50	55	55	57	60	60
Medium Speed Ramp	50	50	50	50	50	50	50
Low Speed Ramp	40	40	40	40	40	40	40
Loop Ramp	30	30	30	30	30	30	30
Off Ramp w/ Intersection	25	25	25	25	25	25	25
On Ramp w/ Intersection	40	40	40	40	40	40	40
Expressway	40	42	45	48	52	55	60
Principal Arterial - Class I	26	30	33	36	42	46	55
Principal Arterial - Class II	24	27	30	34	40	44	48
Minor Arterial - Class I	22	25	28	31	38	42	45
Minor Arterial - Class II	20	23	26	29	34	38	42
HOV - Arterial (all classes)	20	27	30	33	36	39	42
Major Collector	18	22	25	28	31	34	38
Minor Collector	15	18	21	24	27	30	35
Planned Ramps w/ Intersections	30	30	30	30	30	30	30
Planned Directional Ramps	45	45	45	45	45	45	45

Table 3.3 Free-flow Capacity by Area Type and Facility Type

	Area Type						
Facility Type	Urban Very High Density	Urban High Density	Urban Medium Density	Urban Low Density	Suburban	Exurban	Rural
Zone Centroid Connectors	10,000	10,000	10,000	10,000	10,000	10,000	10,000
Interstate / Freeway Free-flow	1,600	1,650	1,700	1,750	1,800	1,800	1,800
Parkway	1,600	1,600	1,600	1,700	1,700	1,800	1,800
HOV Buffer Separated	1,400	1,400	1,600	1,600	1,600	1,800	1,800
HOV Barrier Separated	1,600	1,650	1,700	1,750	1,800	1,800	1,800
High Speed Ramp / CD Road	1,600	1,600	1,600	1,700	1,700	1,800	1,800
Medium Speed Ramp	1,600	1,600	1,600	1,650	1,650	1,700	1,700
Low Speed Ramp	1,400	1,400	1,400	1,400	1,400	1,400	1,400
Loop Ramp	1,200	1,200	1,200	1,200	1,200	1,200	1,200
Off Ramp w/ Intersection	1,200	1,200	1,200	1,200	1,200	1,200	1,200
On Ramp w/ Intersection	1,200	1,200	1,200	1,200	1,200	1,200	1,200
Expressway	1,200	1,300	1,400	1,500	1,600	1,600	1,600
Principal Arterial - Class I	1,000	1,050	1,100	1,150	1,200	1,250	1,350
Principal Arterial - Class II	900	900	950	1,000	1,000	1,050	1,100
Minor Arterial - Class I	800	800	850	900	900	950	1,000
Minor Arterial - Class II	650	700	750	750	800	850	900
HOV - Arterial (all classes)	600	600	650	700	700	750	800
Major Collector	550	600	600	650	650	700	700
Minor Collector	400	400	450	450	500	550	600
Planned Ramps w/ Intersections	1,200	1,200	1,200	1,200	1,200	1,200	1,200
Planned Directional Ramps	1,600	1,600	1,600	1,700	1,700	1,800	1,800

In a network, “at-node” connectivity primarily is offered to each link, joining it at no cost. However, in practice, some turning movements at junctions may be much more difficult to perform than others; indeed, some turning movements may not be allowed at all. In order to represent these features of real road networks better, it is possible to penalize and/or ban some turning movements. This can be done manually by expanding the junction, providing separate (sometimes called dummy) links for each turning movement and associating a different cost with each. Alternatively, in some computer packages, user-defined movement restrictions can be entered through the model interface to introduce turn penalties [33]. The ARC model uses a turn penalty tabulation file to provide or ban movements at intersections or interchanges.

Generally elaborated models such as the one described in this chapter, relate delay to traffic flow using link information such as type of road, road width, number of lanes, or prohibitions of use by certain vehicles.

For practical reasons in traffic assignment, this type of relationship is handled in terms of travel time per unit distance versus flow, or more generally, as a cost-flow relationship, as shown in Figure 3.6. Note that the function allows the existence of an overload region i.e. it does not generate infinite travel time, even when flow is equal or greater than capacity.

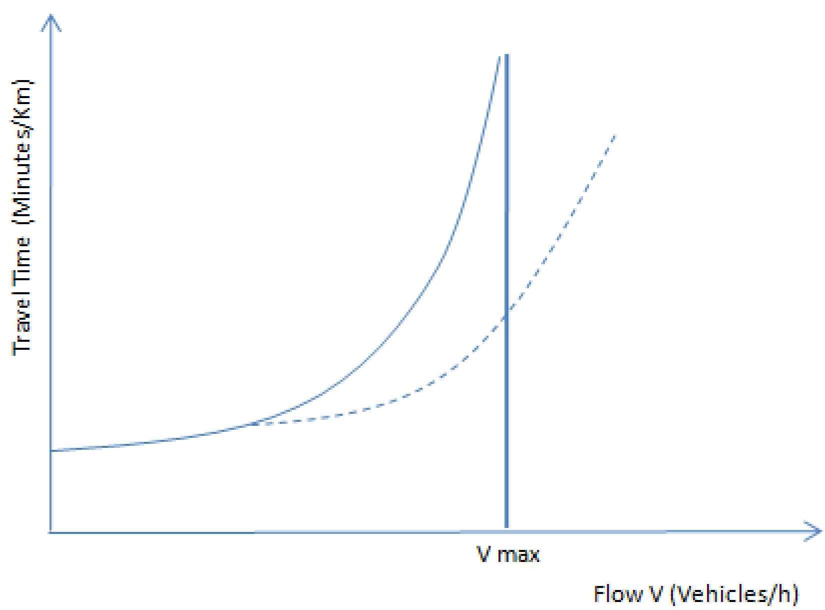


Figure 3.6 Cost-Flow Relationship

In traffic assignment methods that account for the congestion effects (in the form of suitable functions relating link attributes and flow on the network), the resulting speeds or costs can be written in general terms as [33]:

$$C_a = C_a(v_a)$$

That is, the cost on the link depends on its flow and the link characteristics. This assumption simplifies the estimation of these functions and the development and use of suitable trip assignment techniques [33].

In general, volume-delay functions (VDF) describe the rate at which delay is added to the travel time on a roadway segment as a function of the quantity of traffic being carried. Ratios of the assigned traffic volume versus the capacity (or the V/C ratio) are used to predict how travel times (and hence, delays) increase as roadway volumes build up to and beyond the capacity of the roadway [36].

Among all functions, the Bureau of Public Roads (1964) is probably the most common function of this type [33]:

$$t = t_0 [I + \alpha(V/C)^{\beta}]$$

where t_0 is travel time under free flow condition, α and β are parameters for calibration, and C is the capacity of the link. In some cases, a cut-off point in speed reductions is assumed; for example, the speed may be assumed to remain at F for V > F. All the above speed or cost-flow curves produce information about travel time on a link. However, it is recognized that most users might wish to minimize a combination of link attributes, including time and distance [33].

In the ARC model, for each time period hourly travel times were weighted by vehicle miles of travel during the hour to determine the overall average travel speed during that time period as a function of different volume loading levels (V/C ratios). The result was a unique set of volume-delay functions for each time period based on four general roadway classifications—freeways, urban expressways and rural streets and highways, urban arterial streets, and urban collector streets. Once the initial volume-delay functions were developed, the functions were modified to reflect local Atlanta travel conditions using data collected in two recent speed studies conducted in the fall of 2000 and the fall of 2001 using observed traffic counts [33].

The ARC model uses different functions (curves) in the form of tabulation tables for different roadway types and times of day. For each link, a lookup function assigns a new travel time (and speed) based on the corresponding flow on the link in each iteration of the assignment procedure. AM, MD, PM, and NT cost-V/C ratio relationships for a freeway roadway type are illustrated in Figure 3.7 as examples of cost-flow curves used in the ARC model [36].

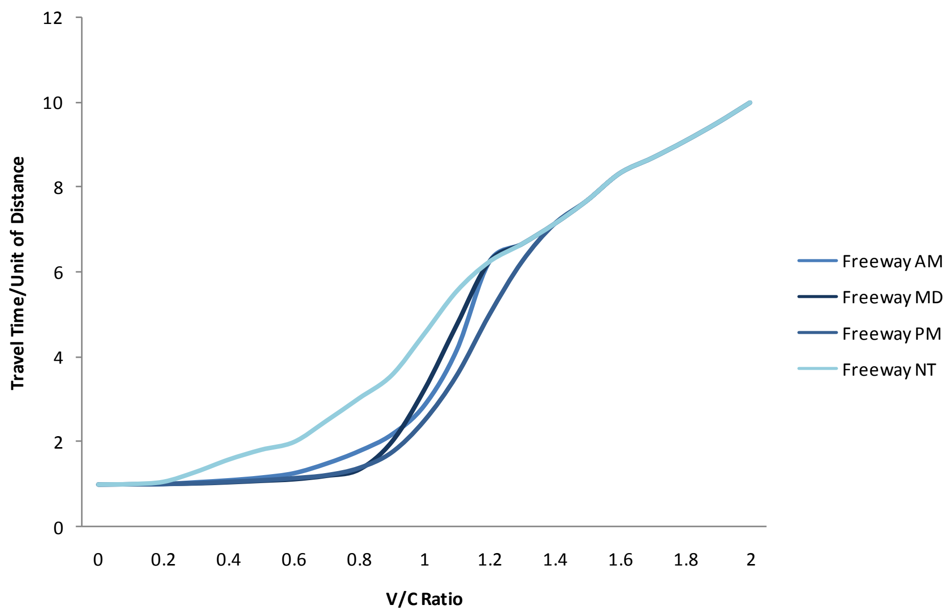


Figure 3.7 Cost-V/C Ratio Relationships [34]

The same relationship can be shown in terms of speed and v/c ratio. Figure 3.8 shows speed-v/c ratio for different facility types corresponding to the AM period.

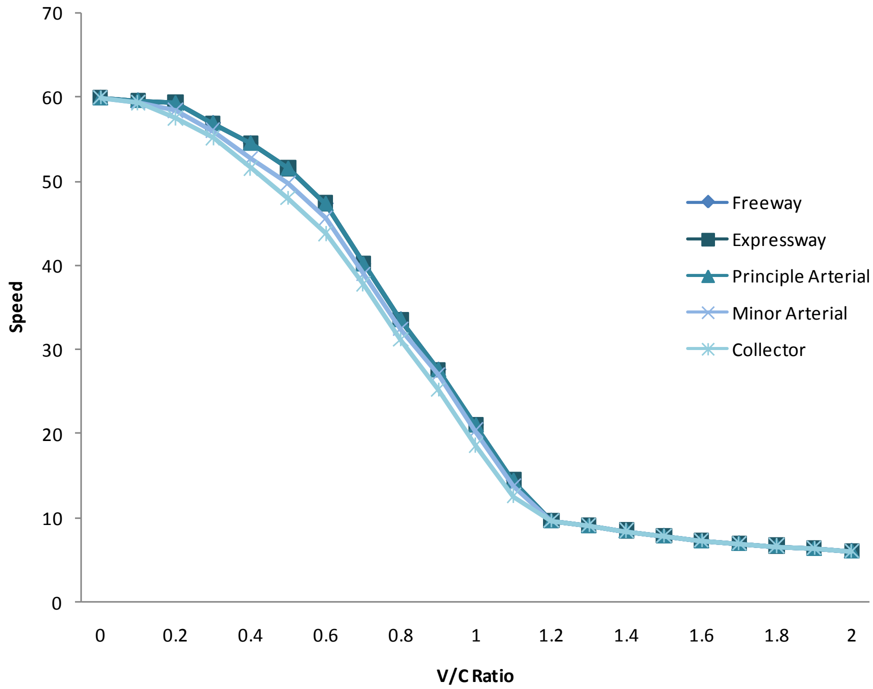


Figure 3.8 Speed-V/C Ratio Relationships [34]

3.1.5 Trip Purposes, Vehicle Class, and Time of Day Trip Tables

The trip pattern of a study area is represented by a trip matrix, which is essentially a two-dimensional array of cells in which rows and columns represent each of the zones (internal and external) in the study area. The cells in each row i contain the trips originating in that zone that have a destination zone j in the corresponding column. The main diagonal corresponds to intra-zonal trips [33]. A third dimension can be incorporated inside the trip matrix as well in order to define different trip purposes or vehicle classes.

The ARC model contains six trip purposes [36]:

1. Home based work: Trips made for the purpose of work that either begin or end at the traveler's home. This is a typical trip purpose that is related to the employment and the income of the traveler or the household.

2. Home based shopping: Trips made for any type of shopping that begin or end at the traveler's home. This trip purpose is related to socioeconomic and land-use characteristics, e.g. retail employment, income, and household size.
3. Home based school: Any trip between an elementary or high school and the home. Characteristics of these trips were determined from the Home Interview Survey. If the age of the traveler was under 19 years, the trip was classified as a school trip.
4. Home based university: All school travel made to a university with one end being at the traveler's home. Characteristics of these trips were determined from the Home Interview Survey. If the age of the traveler was 19 years or older, the trip was classified as a university trip.
5. Home based other: Any trip made with one end at the home except for the purpose of work, shopping, or school. This includes trips made for social visits, recreational trips, or personal-business.
6. Non-home based: Any trip that neither begins nor ends at home.

These six trip purposes were the core of the formation of the SOV and HOV trip tables. The commercial and truck trip tables, however, were shaped based on the recent ARC truck forecasting model developed in the mid-1990's from survey data and is updated to the most recent ARC model version, completed in April 2005. The ARC model has five vehicle classes as listed below [36]:

1. Single or Low Occupancy Vehicle(SOV)
2. High Occupancy Vehicle (HOV)
3. Commercial vehicle (COM)—any vehicle (passenger car, light duty truck) that is used for commercial purposes
4. Medium truck (MTK)— single-unit vehicles including class 4 (buses), class 5 (2 axle, 6 tires), class 6 (3 axles), and class 7 (4 axles)

5. Heavy truck (HTK)—classes 8-13 (either single- or multiple-trailer combination)

The ARC external/internal model is based on an origin-destination survey for passenger cars and trucks in external stations. This model initially was developed in 1994-1995 and recently updated and used for the formation of the external-external and external-internal trip tables. Figure 3.9 identifies the locations of ARC's 91 external stations.

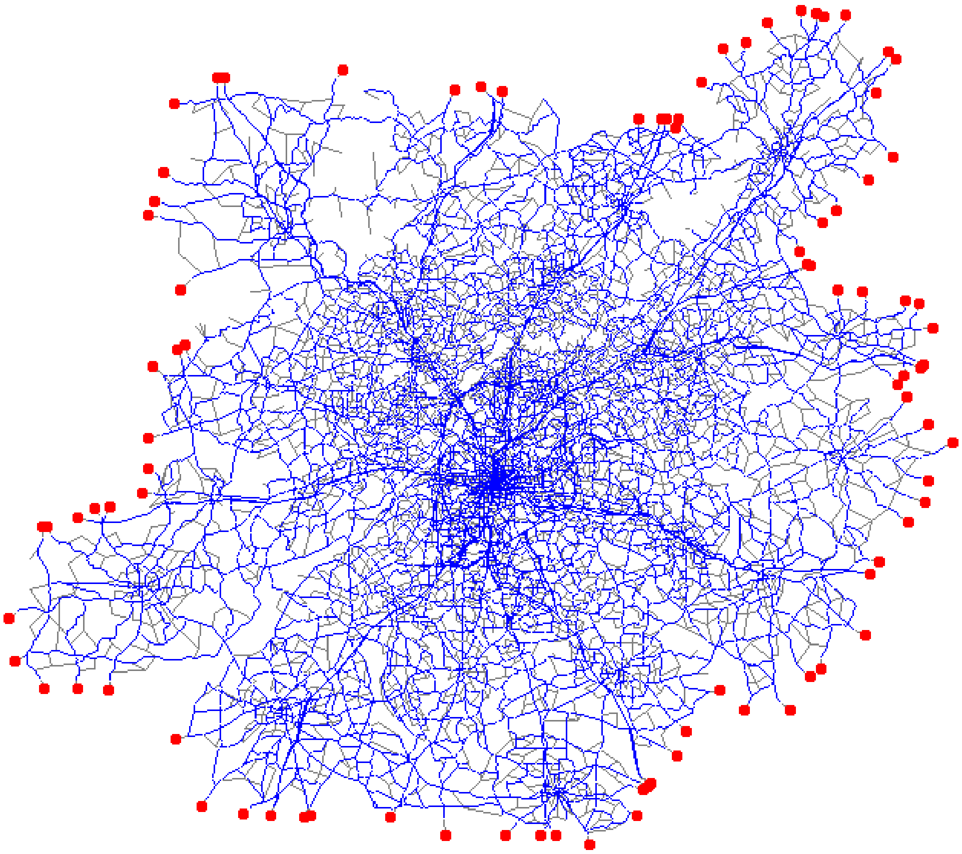


Figure 3.9 External Stations (Scale: 1Inch=20 Miles) [36]

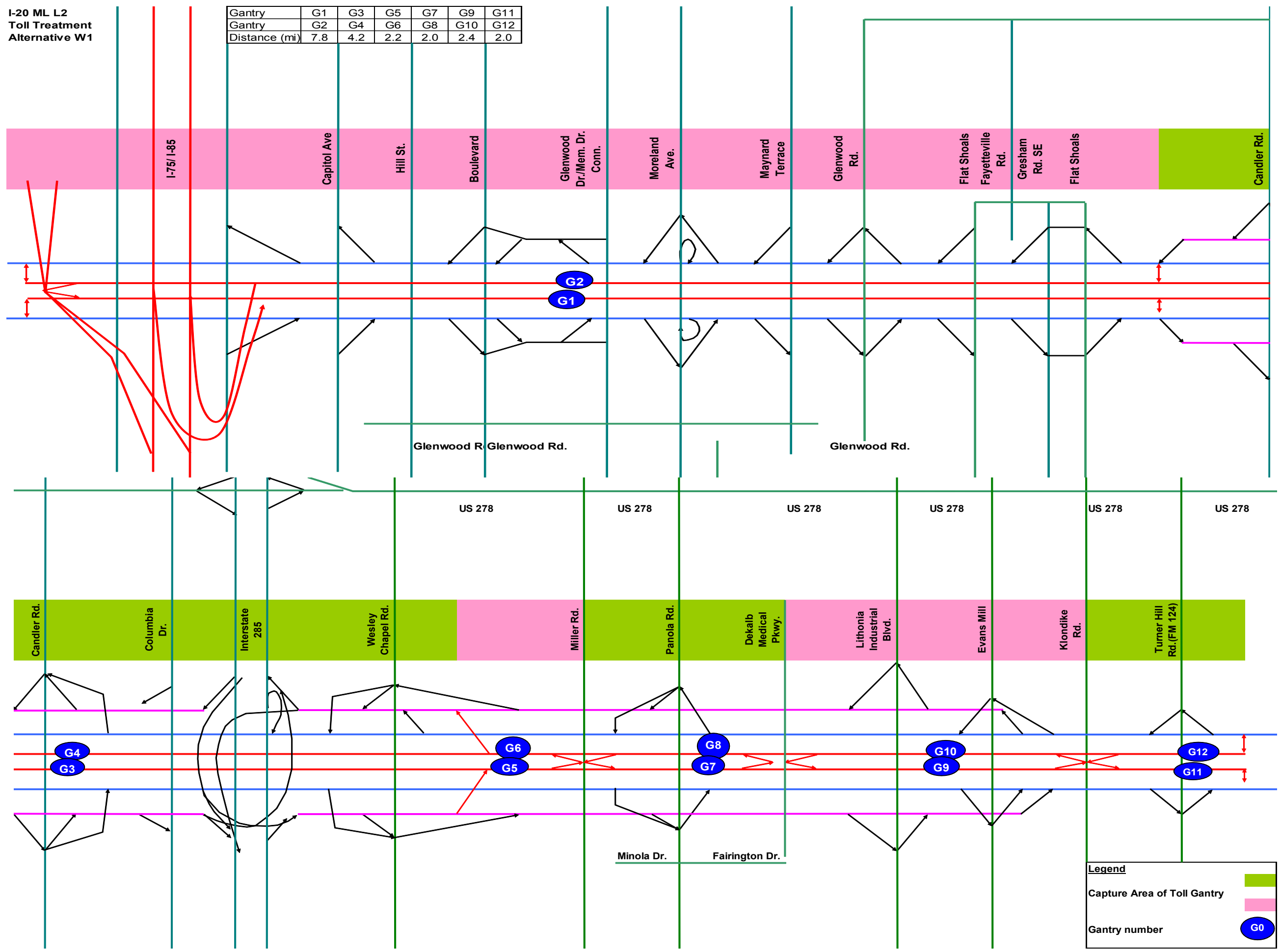
Five daily SOV, HOV, COM, MTK, and HTK trip tables were developed in the form of four separate trip matrices converted to time-of-day trip tables to be input into a traffic assignment sub-model. The ARC time-of-day model was calibrated using data from the 1990 Home Interview Survey. This data contained the beginning time of the trip and the ending time of the trip for each trip made by a traveler. This information was used to develop a series of factors showing the percentage of travel made in half-hour increments. These travel time factors were updated using 1999 and 2000 hourly traffic distribution counts for the entire study area. A two-year average of the 1999 and 2000 daily traffic counts was used to adjust the temporal trip distribution model for the four time-of-day (AM, PM, MD, and NT) period assignments [36]. The AM peak period consists of the hours between 6:00 AM and 10:00 AM. Midday (MD) was defined as 10:00 AM to 3:00 PM. The PM peak period consists of the hours 3:00 PM to 7:00 PM, while the night-time period (NT) consists of the remaining hours in the day: between 7:00 PM and 6:00 AM. Base year 2007, opening year 2015, and future year 2030 daily trip tables were generated using socioeconomic variables and utilizing the ARC trip generation and trip distribution models. These trip tables then were converted to time-of-day trip tables using the time of day factors [36].

3.2 Modeling Assumption

This section will present the modeling assumptions as well as other terms, rates, and values selected to satisfy the entire process of traffic and revenue procedure. While this information will not all be used in the assignment procedure, listing them in this chapter will be worthwhile since these terms are used repeatedly in the related documents and literature, and it also will be important for the completeness of the data collection discussion. Table 3.4 lists the modeling assumptions and related terms, while Figure 3.10 illustrates the toll gantry configuration within the project area. Toll gantries are the designated locations for toll collection.

Table 3.4 Modeling Assumptions [34]

Period		Time of Day	
AM		6:00 to 10:00 AM	
PM		10:00 AM to 3:00 PM	
MD		3:00 to 7:00 PM	
NT		7:00 PM to the next day 6:00 AM	
Toll Collection System		Electronic Toll Collection System Video Tolling System	
Toll Gantry Configuration		As presented in Figure 3.10	
		Value Of Time ($/hr) in 2007 Dollars	
Year	Period	SOV	HOV & COM
2007	AM	11.96	17.55
	PM, MD, NT	11.32	17.55
2015	AM	12.80	18.87
	PM, MD, NT	12.11	19.87
2030	AM	14.53	21.32
	PM, MD, NT	13.75	22.32
SOV/HOV Toll Rate		$0.20 per mile	
COM Toll Rate		$0.30 per mile	
Prohibitions		Medium and Heavy trucks are prohibited from using the ML	
Violation		5% in 2015 declining to 2.5% in 2030	
ETC Ramp Up		80% in opening year (2015) to 100% in 5th year	



Gantry	G1	G3	G5	G7	G9	G11
Gantry	G2	G4	G6	G8	G10	G12
Distance (mi)	7.8	4.2	2.2	2.0	2.4	2.0

Figure 3.10 I-20 Managed Lanes Toll Gantry Location and Distance [34]

3.3 Modeling Platform

In order to meet the objective of this study, the Cube Voyager software developed by Citilabs, Inc. [37] was selected for the modeling environment. The Cube software suite is a comprehensive set of modules that support transportation planning, including transportation forecasting and system analysis. Cube integrates modeling methods with graphics technology for the study of transportation systems. Cube Base includes two additional tools, the Application Manager and the Scenario Manager. These tools simplify application and scenario management and include additional geographical information system (GIS) related features [37].

The application manager combines the functionality of a network editor, a transit line editor, a matrix editor, a database editor, a job script editor, and a model job launcher in one package [37]. The Application Manager also provides a flow-chart view of the programs and input/output that makes the entire modeling procedure easy to follow and comprehend.

Cube Voyager is designed to be an integrated modeling system for transportation planning applications. At the heart of the Cube Voyager system is a flexible control language referred to as a scripting language. This provides a flexible environment and grants control over all aspects of the modeling process [37].

The Cube Voyager system has four main assignment programs: Network, Matrix, Highway, and Public Transport. In addition, the system offers supplementary programs for common transportation planning tasks, such as the “Generate” program for trip generation and the “Distribution” program for trip distribution. These supplementary programs provide an easy-to-use interface to the basic four programs. Cube Voyager has no hard-coded mechanisms; users are free to change and modify runs as they progress. Cube Voyager is an excellent choice for model applications that require congestion feedback mechanisms [37].

Cube Voyager is a library of programs that employs a language allowing the user to write the script to provide instructions for performing all types of typical planning operations. The script is stored in a file and read when the system is executed. The individual programs are activated according to the instructions in the script. Each program is designed to perform certain operations, but only as specified by the user. A typical application could involve a very complicated set of instructions, or it can be as simple as computing and/or printing a number from a file [37]. Figure 3.11 shows the Cube Voyager's different management and graphic environments.

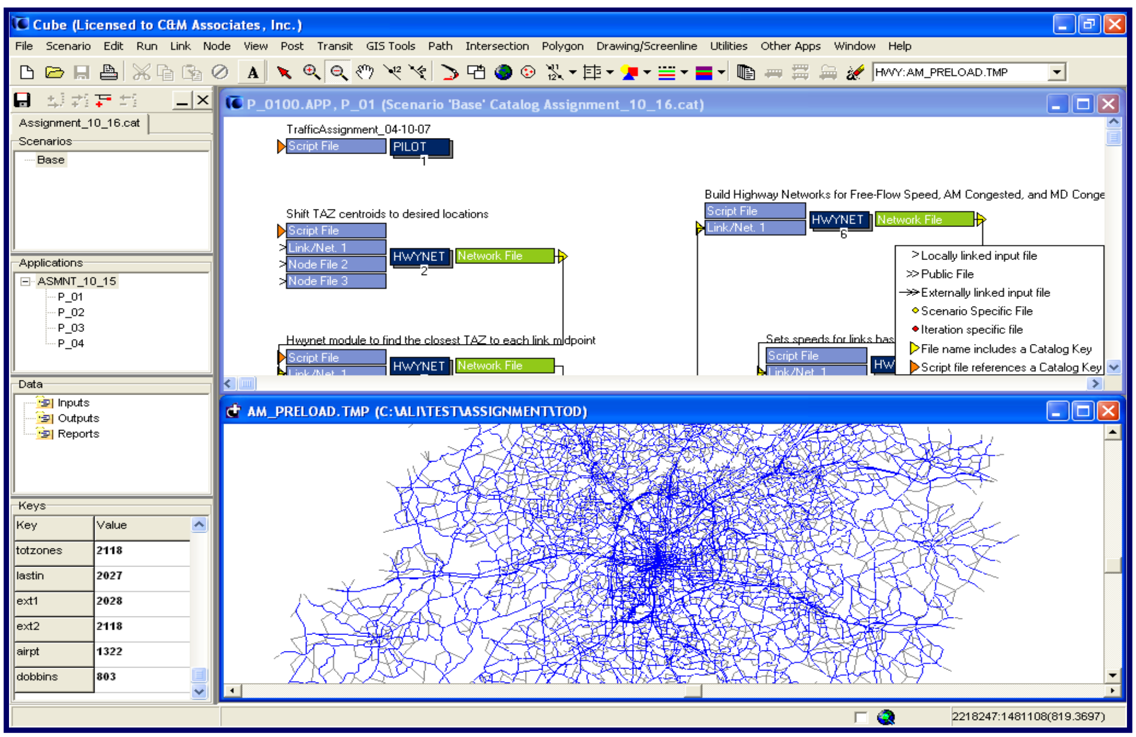


Figure 3.11 Cube Voyager Environment

3.4 Adopting a Discrete Choice Model

As utilized in this study, in general, discrete choice models assume that the probability of individuals choosing a given option is a function of their socioeconomic characteristics and the relative attractiveness of the option. To represent attractiveness, the concept of utility (U), a linear combination of variables, is used. Each variable in the utility function represents an attribute of a travel option.

The relative influence of each attribute to contribute to overall satisfaction of the alternative is represented by its coefficient. For example, if a coefficient is twice as much as another one, it has twice the influence. In order to predict whether an alternative will be chosen, the value of its utility should be translated into a probability value between 0 and 1. This is achieved, among others, through a Logit or probit model. Both models typically have an S-shaped plot [33].

A multinomial logit model has been set up for the purpose of this study to identify the share of toll road users among all I-20 corridor trip-makers. These types of models assume that alternatives are independent and that errors are randomly Gumbel distributed. In order to form the utility functions and derive the corresponding time and cost coefficients, a stated preference survey was conducted within the I-20 study corridor.

3.4.1 Stated Preference Survey

The stated preference survey is a survey that attempts to quantify how travelers will behave in a situation that is new to them. These surveys typically are used to estimate the value of time for proposed toll facilities, which generally cannot be captured in revealed preference surveys. Thus, a revealed preference survey provides a general quantification of the distribution, magnitude, and characteristics of a region’s or corridor’s travel activity, whereas a stated preference survey is used to estimate the impact of the imposition of pricing on the routes that the travelers who generate this activity would take. Stated preference surveys are designed to present different options to respondents, such as determining not only the value of time but also how their perceptions of that value would vary by time of day (i.e., by congestion level).

The stated preference (SP) survey for I-20 ML project was conducted in May and June, 2007. Two types of surveys were conducted: one designed for autos and the other for trucks. The auto version involved individuals over 16 years of age who had made at least one weekday trip of 15 minutes or more using the highways in the study area within the week preceding his/her interview. The truck survey included truck drivers, fleet managers, or dispatchers who drove, managed, or dispatched drivers making weekday trips on any of the highways in the study area. The surveys were conducted using a computer-assisted self-interview system, as well as an online survey [34]. Although it is not the objective of this study to review this SP survey in detail, a brief review of the outcomes and resulting coefficients will be presented below [32].

The total survey sample in all corridors consisted of 4,648 completed car drivers surveys. Of those, the total number of individuals who stated that they had traveled through a segment of I-20 was 1,011. Each questionnaire included 9 stated preference scenarios, providing a total of 9,099 answers. This survey sample was considered acceptable for the purposes of I-20 ML project [32].

Survey interviewees were asked about their trip purpose, time of day, trip travel time, delays, trip frequency, and occupancy. The results of this section of the interview are summarized below in Figure 3.12 to Figure 3.17 [32].

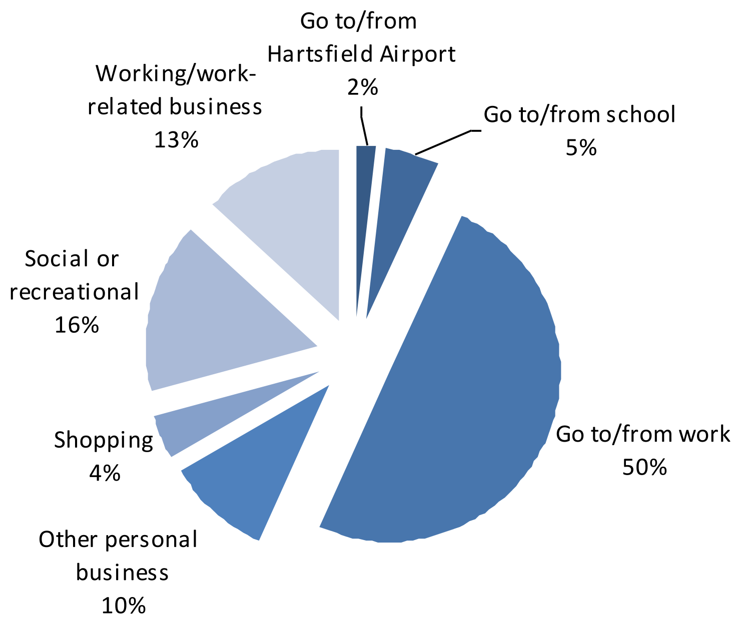


Figure 3.12 Trip Purpose Distribution

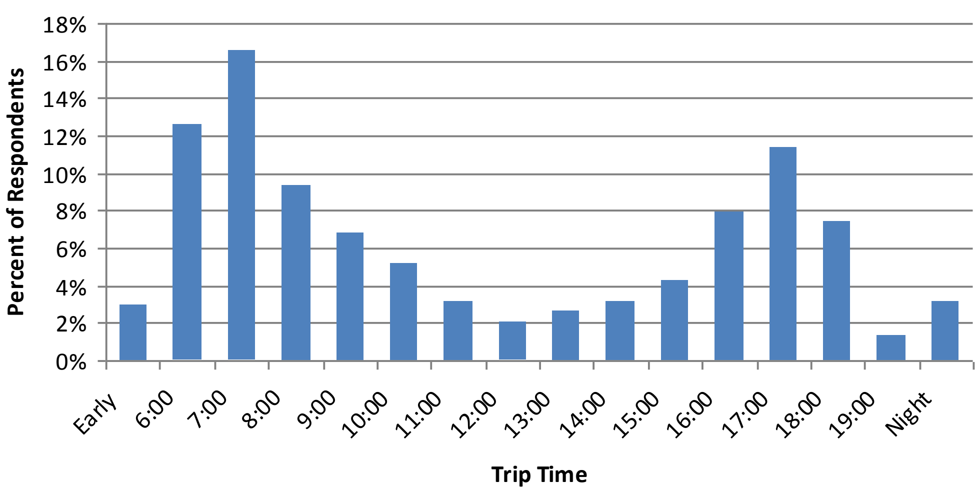


Figure 3.13 Time of Day at which Trip Started

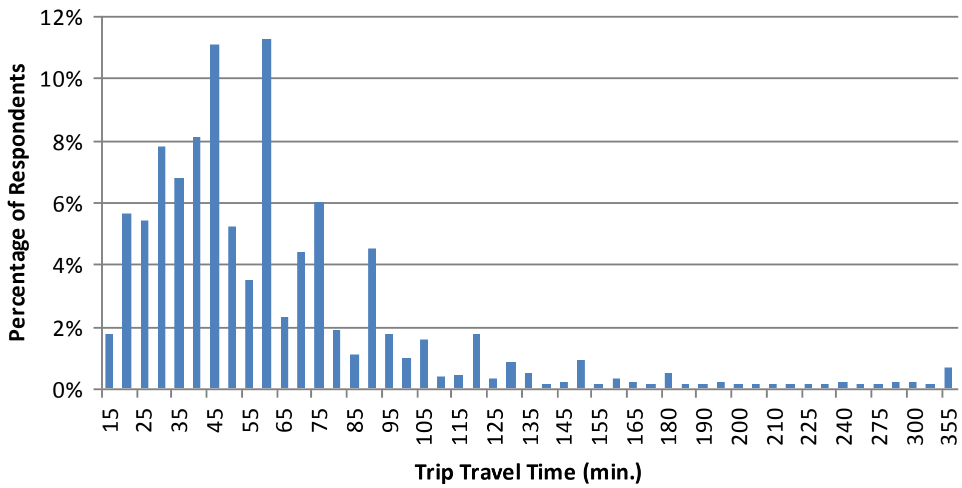


Figure 3.14 Trip Total Duration in Minutes

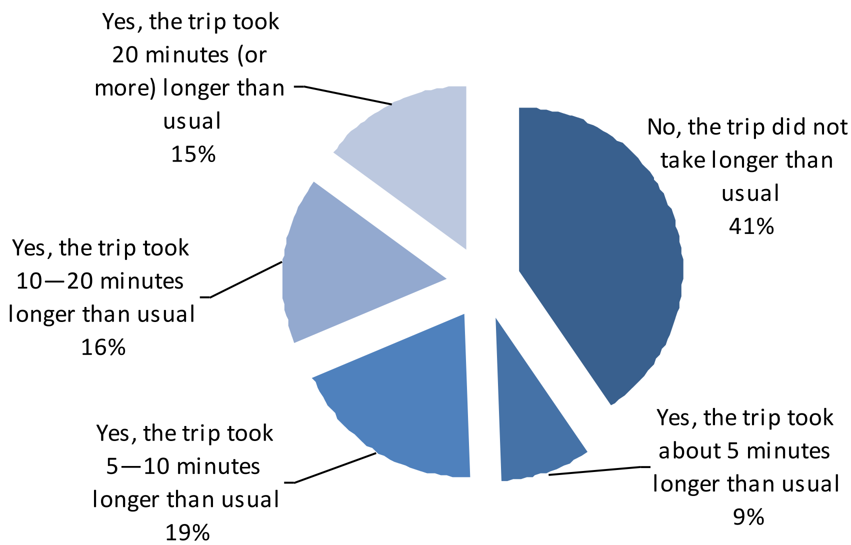


Figure 3.15 Perceived Delay on the last Trip of Interviewees that Included a Segment on I-20

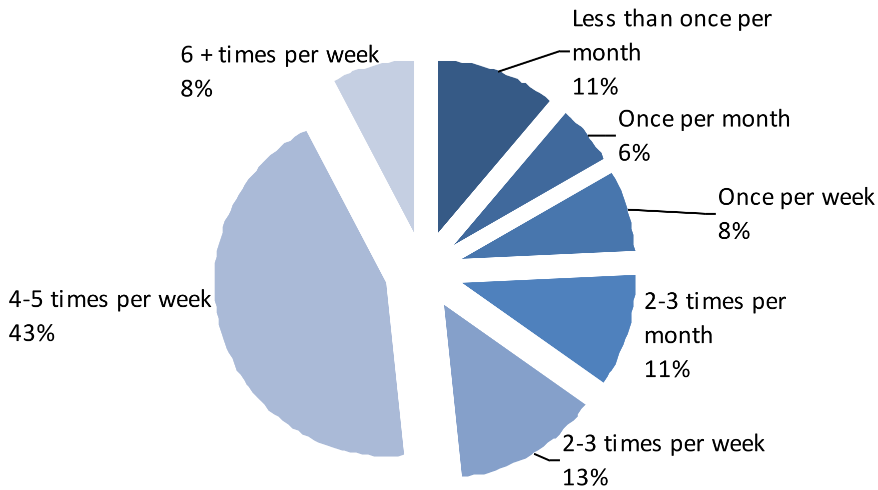


Figure 3.16 Trip Frequency

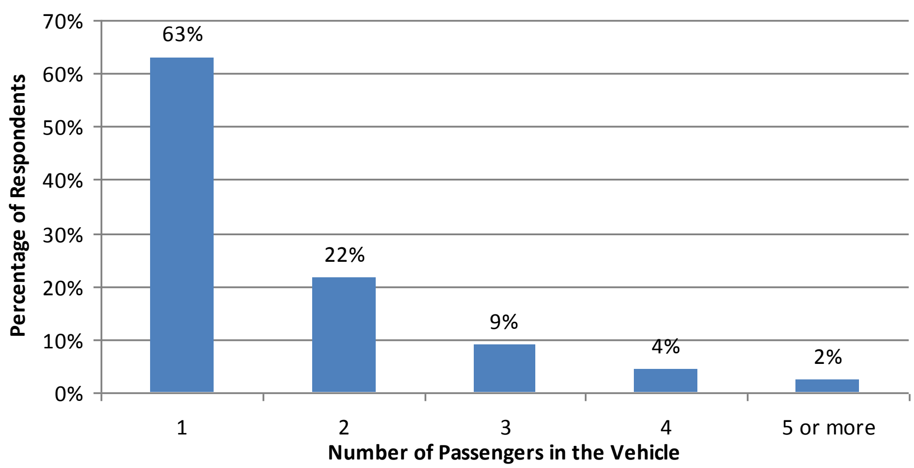


Figure 3.17 Vehicle Occupancy

I-20 users also were asked about their opinion on the project. 57% of respondents were either strongly or somewhat in favor. 25% of respondents declared themselves neutral to it, while the remaining 18% were opposed to it. Individuals who were in favor were asked their reasons for their support. Of those in favor of the project, 65% stated that the project would either alleviate congestion or make their travel time shorter. Similarly, 16% of individuals answered that the project could make their travel time more reliable. Individuals who were opposed to the project also were asked their reasons. 46% of individuals opposing the project declared that they opposed paying any type of toll, while 15% declared that tolls were normally set too high [32].

Individuals also were asked about several of their socioeconomic characteristics, such as household size, vehicle ownership, and income. Regarding the respondents' household size, 32% were from households of 2 people. 38% were from households of 3 or 4 people. Vehicle ownership on average for I-20 users was 1.44 vehicles per person, or 0.69 persons per vehicle. 42% of single occupancy drivers declared having 2 vehicles in their household, while 23% declared having 1 vehicle. The remaining 35% declared having more than 2 vehicles in their household. With regard to the question pertaining to annual income, most of the interviewees (51%) placed themselves in the $25,000 to $75,000 range. 17% of respondents declared having an income between $75,000 and $100,000, while 12% declared having an income less than $25,000 and 21% declared having an income higher than $100,000.

3.4.2 Model Results

The extracted data from the SP survey then resulted in the construction of a series of logit models using the logistic regression procedure. The alternatives included paying tolls on I-20 managed lanes without carpooling, sometimes paying tolls on I-20 managed lanes with carpooling, not paying tolls and staying in general purpose lanes. The logit models estimated for this study were calibrated using the principle of utility maximization. Utility functions representing the attractiveness of alternatives based on measured and unmeasured variables have a linear form and are estimated using data obtained through the aforementioned stated preference survey. Thus, for this application, the utility function could have the following form:

$$U_a = \alpha(Time) + \beta_1(\$)(Dummy_1) + \beta_2(\$)(Dummy_2) + \beta_3(\$)(Dummy_3) + \beta_4(Dummy_4) + \cdots$$

where:

U_a is the utility associated with alternative *a*,

Time is the travel time via alternative *a* in minutes,

$ is the out-of-pocket cost for traveling via alternative *a*,

Dummy are dummy or binary variables that indicate some socioeconomic characteristic (specific variables that were used in this study are listed below), and α, β1, β2, β3, β4, are estimated parameters.

The probability of choosing any given alternative will be given by the following formula for a case in which there are three alternatives: "a," "b," and "c":

$$P_a = \frac{e^{(U_a)}}{e^{(U_a)} + e^{(U_b)} + e^{(U_c)}}, \; P_b = \frac{e^{(U_b)}}{e^{(U_a)} + e^{(U_b)} + e^{(U_c)}} \text{ and } P_c = 1 - P_a - P_b$$

where:

P_a is the probability of choosing alternative a,

P_b is the probability of choosing alternative b,

P_c is the probability of choosing alternative c,

U_a is the utility associated with alternative a,

U_b is the utility associated with alternative b, and

U_c is the utility associated with alternative c.

Trip, system, and user attributes (listed below) then were tested for significance for insertion in the model. However, because of application matters, only travel time savings, vehicle occupancy, and toll costs were included in the resulting logit models and consequently in the toll assignment model.

The variables from the SP survey were as follows:

- Trip attributes such as:
    - Time savings (toll travel time-free travel time)
    - Time of travel
    - Day of travel
    - Trip frequency
- System attributes such as:
    - Travel costs (tolls)
- User attributes such as:
    - Income
    - Electronic toll collection availability
    - Household size
    - Vehicle ownership

The estimated probabilities then is used to partition the number of trips between a given origin-destination pair into toll road users (Toll) and non-tolled users (Free) based on cost (toll) and time-saving trade-off. The “Toll” portion is assigned to all the facilities, including managed facilities, and the “Free” portion will be assigned to non-tolled facilities.

Negative values are expected both for alternative specific constant and for all other parameters. The configuration for logit function that has been used in this project is illustrated below.

Prob.(toll-eligible trips)=1/(1+exp(time coefficient*($u_i^t - u_i^n$)+toll coefficient*(t_i^t)+ constant)

Where; $u_i^t - u_i^n$ is travel time saving and t_i^t is corresponding toll. This configuration uses travel time savings calculated by subtracting the values corresponding to a tolled network (A) from a non-tolled network (A-{Toll Links}) for each i-j pair as well as the costs obtained (tolls) from tolled-network. Therefore, a negative travel time saving matrix and a positive travel cost matrix would be built after skimming the networks within each assignment iteration or feedback. Then, negative toll and time difference parameters show, everything else being equal, increases in toll or decrease in time difference will result in decreases in the attractiveness of the toll alternative. For the alternative specific constant, a negative value shows that everything else being equal, the non-tolled alternative is preferred by auto respondents.

Figure 3.18 illustrates a series of typical diversion curves formed by the estimated logit models. The curves correspond to AM peak period, single-occupant passenger cars [32].

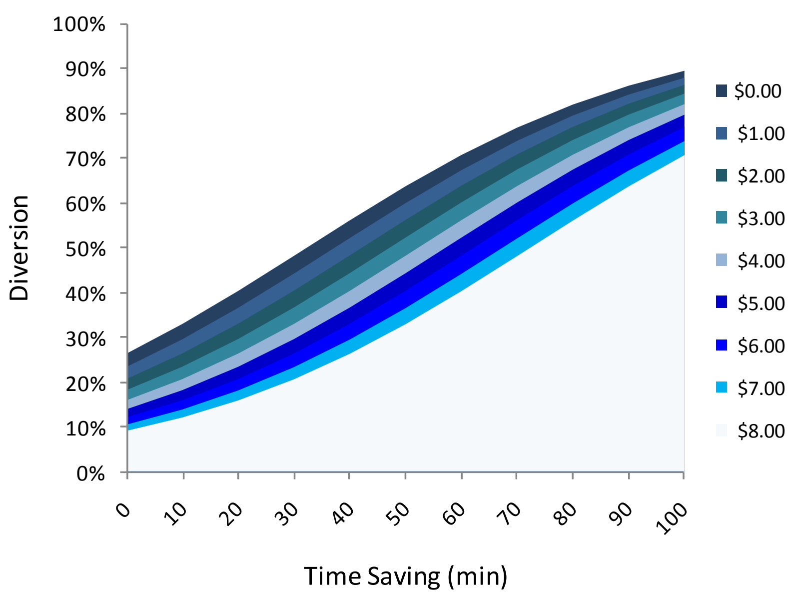


Figure 3.18 SOV Toll Diversion Curve—AM Peak Period

The adopted Logit models then are deployed within a user equilibrium assignment routine in order to estimate the percentage of the travelers that would potentially use the managed facilities and consequently those that would use the free alternatives to travel the same origin-destination pair. The next chapter will elaborate on deployment of these logit models as route choice functions within the formulation of a pseudo-probabilistic user equilibrium toll-assignment.

CHAPTER 4

DEVELOPMENT AND FORMULATION

Demand for new transportation facilities, a result of constantly growing congestion within existing roadway systems on the one hand and transportation budget shortfalls on the other, has caused the construction of new highways to be assigned to private companies, which operate these new facilities as concessions. Travelers on the proposed facility will then be charged tolls and provide revenues to finance the construction and operation of the highway for a certain period of time, after which the highway becomes property of the state government that awarded the concession. Estimating toll road usage and revenue then becomes very important, for both the mobility implications and the financial viability of a toll project. Basically, the new tolled facilities provide shorter travel times, and, given the value of time (VOT) of different user classes, one must determine the trade-off between increased travel costs and reduced travel time in order to predict usage of the toll road [40]. This chapter seeks, through literature review, to provide a sound algorithm and formulation for volume assignment problems associated with transportation networks involving managed facilities. This is, of course, to provide technical support for the main objective of the research, which is the implementation of such an assignment routine over a large-scale transportation network.

It should be noted that combined mode choice and assignment routines (Mode Choice models, followed by assignment programs), sometimes referred to as “toll mode choice” (also sometimes referred to as “mode choice toll assignment routine” in this research), are not of major interest within this study. These models, due to their common ground with a regular 4-step model, were found to be well-established and somewhat well-documented, and the only application of them in this study is to provide a measure for performance of the toll-assignment algorithm and formulation presented in this chapter. The results of analysis regarding the "mode choice toll assignment routine" for the purposes of this research are discussed in Chapter 6.

As mentioned in prior chapters, there are many assignment routines that deal with demand forecast and volume assignment related to "managed facilities," of which two were found to be more significant through a review of related literature. These are:

1. An assignment program including "Generalized Cost," based on VOT distribution among travelers, which arose from an SP survey [40]; and
2. A combined split (logit) model with a "deterministic users' equilibrium" loading approach [40].

The latter approach, through a multiclass volume assignment routine, attempts to solve a transportation network to determine the potential demand for toll roads and free alternatives. Logit functions that are usually acquired from a stated preference survey analysis are used to obtain the portion (or probability) of the demand that may use paths which include tolled facilities. These models utilize the value of time associated with the toll road users (in each class and over each given O-D pair) to determine the above-mentioned probabilities. A brief discussion of such logit models has already been provided in prior chapters, and the focus in this chapter is to provide a detailed discussion of the formulation and algorithm of the assignment models with embedded discrete route choice models. These explicit "choice tolled assignment" routines are further referred to as "pseudo-probabilistic assignment methods" in this research. Notations and formulations provided in this chapter are based mainly on the work of Michael Florian, which is presented in the article "Network Equilibrium Models for Analyzing Toll Highways [40]." This article has been used as the main technical reference for the algorithm of the "pseudo-probabilistic assignment method" as presented in this research.

This chapter is organized into two main sections. The first section is a quick review of standard volume assignment methods. It attempts to briefly introduce the general components and terminology of a users' equilibrium (UE) assignment routine. The second section presents the formulation and algorithm for the adopted probabilistic toll-assignment model and its underlying assumptions.

4.1 An Introduction to Equilibrium Traffic Assignment Programs

This section is intended to briefly introduce the general terminology of traffic assignment programs and the concepts within these models. The purpose is to provide a basis from which to commence the discussion regarding a pseudo-probabilistic users equilibrium toll-assignment (PT-Assignment) program.

A classic traffic assignment routine seeks, through a set of principles, to load a fixed trip matrix onto a network and produce a set of link flows. However, other outputs can be provided during a traffic assignment procedure, namely zone-to-zone travel cost (time), level of service, and level of congestion on each link. The basic principle in the assignment is the assumption that a rational traveler will choose the route which offers the lowest perceived individual cost. Among all factors that are thought to influence the choice of route, time and monetary cost are the factors most commonly employed within the assignment programs. Therefore, time and cost or in some assignment methods, time and distance are used to approximate a generalized cost for all factors involved in a transportation route choice [33].

Several steps within each assignment method must be followed in turn. The first step is "tree-building," which is to identify a set of routes which might be attractive to drivers. These routes are stored in a specific data structure called a "tree"--hence, the term "tree-building" [33].

The next step is "network loading," which is the assignment of suitable portions of the trip matrix to these routes of trees. This step would result in flows on the links within the network. The last step is "convergence." To search for convergence assignment routines, one follows an iterative pattern of successive approximations to an ideal solution [33]. One of the most common convergence methods is to search for Wardrop's equilibrium solution. The indicator δ defined below is often used to measure how close a solution is to Wardrop's equilibrium:

$$\delta = \frac{\sum_{ijr} T_{ijr} \left(C_{ijr} - C_{ij}^{*}\right)}{\sum_{ij} T_{ij} \, C_{ij}^{*}}$$

Where $\{T_{ijr}\}$ is a set of path flows and $C_{ijr} - C_{ij}^{*}$ is the excess cost of travel over a particular route relative to the minimum cost of travel for that $(i\,,j)$ pair.

These costs are calculated after the last assignment iteration is performed and total flows are obtained for each link. Therefore, δ is a measure of the total cost of excess travel through less than optimal routes [33]. Other methods regarding the test for convergence are provided in the "Convergence Test" subsection of this chapter.

4.1.1 Users' Equilibrium Traffic Assignment

Users' equilibrium traffic assignment methods, by utilizing a capacity restraint approach through cost-flow functions, seek to approximate the equilibrium condition as formally expressed by Wardrop [33]:

"Under equilibrium condition, traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes."

If all trip makers perceive costs in the same way, then:

"Under equilibrium condition, traffic arranges itself in congested networks such that all used routes between an O-D pair have equal and minimum costs while all unused routes have greater or equal costs" (Wardrop's first principle).

This expression could be mathematically formulated as follows [33]:

$$c_{ijr} \begin{cases} = c_{ij}^{*} & T_{ijr}^{*} > 0 \\ \geq c_{ij}^{*} & T_{ijr}^{*} = 0 \end{cases}$$

where $\{T^*_{ijr}\}$ is a set of path flows that satisfy Wardrop's first principle, and all the costs have been calculated after T^*_{ijr} has been loaded. In this case, link flows result from:

$$V_a = \sum_{ijr} T_{ijr} \delta^a_{ijr}$$

where δ^a_{ijr} is 1 if path r between i and j uses link a, and zero otherwise.

The cost along the path can then be calculated by:

$$C_{ijr} = \sum_a \delta^a_{ijr} c_a V^*_a$$

A few years after Wardrop presented his principle, a mathematical program was introduced that utilizes the properties of Wardrop's first principle by minimizing an objective function subject to constraints representing the properties of the flows, as illustrated below [33]:

$$Minimize\ Z\{T_{ijr}\} = \sum_a \int_0^{V_a} C_a(v)dv$$

Subject to

$$\sum_r T_{ijr} = T_{ij}$$

$$T_{ijr} \geq 0$$

The objective function corresponds to the sum of the areas under the cost-flow curves for all links in the network. This mathematical program can be solved using a number of methods; the most common algorithm is attributed to Frank and Wolfe (F-W). The Frank-Wolfe algorithm tends to converge rapidly over early assignment iterations, but less so as it approaches the optimum solution [33]. Frank-Wolfe is a well-known method that has also been used within the PT-Assignment algorithm, which is presented in following sections.

4.1.2 Multiclass Multimodal Users' Equilibrium Traffic Assignment

Within a volume assignment, routine different vehicle classes may need different treatment and may have to be assigned to separate sub-networks. The reason for this is that they may be subject to specific traffic management schemes and use different criteria for route choice. Detailed assignment models, such as the one used in this study, allow for several vehicle classes to be treated separately regarding the selective use of links and travel cost [35].

A multiclass multimodal traffic assignment program is a "generalized cost" assignment that allows simultaneous assignment of trips by individual modes or user classes (such as autos and trucks) to the same main network. This method allows for explicit modeling of the influences of different facilities with different functionalities, such as High-Occupancy Vehicle (HOV) links. In a multiclass traffic assignment program, each mode or class can have different link exclusions (which results in different sub-networks), congestion impacts (passenger car equivalent values), values of time, and toll costs. Multiclass traffic assignments are well-known and widely used methods within transportation problems involving several vehicle classes, and further discussion about them is not in the scope of this research.

4.2 Assignment Models with Explicit Choice of Managed Facilities

The formulation presented herein utilizes a multiclass users' equilibrium toll-assignment routine mainly due to tolled and non-tolled sub-networks that differ in the inclusion of the tolled links. Other classifications due to vehicle type and market segmentation, such as distribution of users by value of time, however, could also be implemented using the same formulation in this method.

There are different ways in which a probabilistic approach is attributed to the toll-assignment model. Through review of the related literature, it has been noted that pure probabilistic and stochastic volume assignments are those that employ a stochastic path building and/or network loading routine, whereas the assignment method utilized in this study uses a standard network loading method such as All-Or-Nothing routine. Therefore, the term "pseudo-probabilistic" assignment was selected to emphasize the use of choice models that are embedded within the assignment formulation presented in this research and to distinguish this technique from stochastic volume assignment methods.

4.2.1 Choice Models

As mentioned in prior chapters, logit functions of the form presented below are the results of a stated preference analysis to determine the choice that travelers make between “toll” and “non-toll” alternatives.

$$P(\text{using toll facility}) = \frac{1}{1+exp\ (\alpha^c \triangle cost + \beta^c \triangle time\ )}$$

where;

α^c and β^c : parameters

$\triangle\ cost$: difference in the cost of the trip

$\triangle\ time$: difference in the travel time

$\forall c \in C$: vehicle class [40].

4.2.2 Model Formulation

Assume a roadway network $R= (N, A)$ which consists of nodes n, $n \in N$, and links a, $a \in A$, that could carry vehicular traffic. The total demand for travel is subdivided into classes c, $c \in C$, that are segmented by different vehicle classes or socioeconomic characteristics. Let g_i^c denote the demand for travel of class c for each O-D (Origin-Destination) pair i, $i \in I \subset N$. Let g_i^{ct} denote the number of users in class *c* who are willing to use tolled facilities and g_i^{cn} denote the number of those who are not willing to pay the toll. That is, $g_i^c = g_i^{ct} + g_i^{cn}$, $i \in I$, $c \in C$. g_i^c is assumed to be fixed and known. In addition, let $k \in K_i^{ct}$ and $k \in K_i^{cn}$ denote the set of paths that contain tolled facilities and those that do not, respectively. The resulting multiclass UE model could then be stated as follows [44, 47]:

1. Multiclass-Network Users’ Equilibrium Model with explicit choice functions (Pseudo-Probabilistic Multiclass-Network Users’ Equilibrium), inequalities:

$$\left.\begin{aligned} S_k^{ct}(v) &= u_i^{ct}, if\ \ h_k > 0 \\ S_k^{ct}(v) &\geq u_i^{ct}, if\ \ h_k = 0 \end{aligned}\right\} k \in K_i^{ct},\ i \in I,\ c \in C$$

$$\left.\begin{aligned} S_k^{cn}(v) &= u_i^{cn}, if\ \ h_k > 0 \\ S_k^{cn}(v) &\geq u_i^{cn}, if\ \ h_k = 0 \end{aligned}\right\} k \in K_i^{cn},\ i \in I,\ c \in C$$

where $S_k^c(v)$ are travel costs (functions) on path k, h_k are path flows, and u_i^c are the shortest travel times for O-D pairs i, $i \in I$, and classes c, $c \in C$, subject to conservation of the flow and non-negativity constraints. This formulation assumes an additional term, p_k, to be discussed in following sections. The path flows h_k, then, may be written as:

$$\left.\begin{aligned} h_k &= p_k g_i^{ct}, k \in K_i^{ct} \\ h_k &= p_k g_i^{cn}, k \in K_i^{cn} \end{aligned}\right\} i \in I,\ c \in C$$

and the link flows could be expressed as:

$$v_a^{ct} = \sum_{k \in K_i^{ct}} \delta_{ak} p_k g_i^{ct} \ , c \in C, a \in A$$

$$v_a^{cn} = \sum_{k \in K_i^{cn}} \delta_{ak} p_k g_i^{cn} \ , c \in C, a \in A^n$$

$$A^n = A\text{-}\{Toll\ Links\}$$

$$v_a = \sum_{c \in C} (v_a^{cn} + v_a^{cn}) \ , a \in A$$

where $\delta_{ak} = 1$ if link a belongs to path k, and zero otherwise.

Then the cost of paths containing and not containing tolled facilities, respectively are:

$$S_k^{ct}(v) = \sum_{a \in A} \delta_{ak} s_a \ (\boldsymbol{v_a}), k \in K_i^{ct}, i \in I$$

$$S_k^{cn}(v) = \sum_{a \in A^n} \delta_{ak} s_a \ (\boldsymbol{v_a}), k \in K_i^{cn}, i \in I$$

2. The conservation of flow equations:

$$\sum_{k\ \epsilon\ K_i^{ct}} p_k g_i^{ct} - g_i^{ct} = 0 \Rightarrow g_i^{ct}\left(\sum_{k\ \epsilon\ k_i^{ct}} p_k - 1\right) = 0\ , i \epsilon I, c \epsilon C$$

$$\sum_{k\ \epsilon\ K_i^{cn}} p_k g_i^{cn} - g_i^{cn} = 0 \Rightarrow g_i^{cn}\left(\sum_{k\ \epsilon\ K_i^{cn}} p_k - 1\right) = 0\ , i \epsilon I, c \epsilon C$$

3. Toll-eligible and non-toll-eligible demand:

$$g_i^{ct} = g_i^c / \left\{1 + exp\left(\alpha^c\left[\sum_{k\ \epsilon\ K_i^{ct}} p_k t_k^{ct}\right] + \beta^c(u_i^{ct} - u_i^{cn})\right)\right\},$$

$$i \epsilon I,\ c \epsilon C;\ (g_i^{cn} = g_i^c - g_i^{ct})$$

where p_k are the path proportions and t_k^{ct} are the tolls for each class $c,\ c \epsilon C$ and O-D pair i :

$$t_k^{ct} = \sum_{k\ \epsilon\ K_i^{ct}} \delta_{ak} t_a^{ct}\ , c \epsilon C, a \epsilon A$$

where $\delta_{ak} = 1$ if link a belongs to path k, and zero otherwise. t_a^{ct} are link tolls ($a \epsilon A$) for class $c,\ c \epsilon C$.

4. Non-negativity constraints:

$$h_k \geq 0,\ k\ \epsilon\ K_i^{ct},\ k\ \epsilon\ K_i^{cn},\ i \epsilon I,\ c \epsilon C$$

$$g_i^{ct},\ g_i^{cn} \geq 0,\ i \epsilon I,\ c \epsilon C$$

As previously stated, g_i^{ct} and g_i^{cn} are toll-eligible and non-toll-eligible demands (that are not initially fixed but known at the equilibrium state). Since paths that are used are not known before computing the equilibrium flows, an equivalent formulation in terms of p_k, that is the proportion of the analyzed demand that uses path k, has been used in the presented formulation.

4.2.3 The PT-Assignment Algorithm

In theory, a PT-Assignment program performs a users' equilibrium assignment routine by measuring the users' perceived travel cost. Travel cost is measured not just in a generalized form of travel time, but in both time and monetary costs (toll) where routes involve managed facilities. In this program, trips associated with each origin-destination pair have the option of choosing managed facilities (if available) versus free alternatives along their routes. Therefore, through a set of embedded discrete route choice models and based on users' willingness to pay a toll, the demand associated with each (i, j) pair splits to:

- toll-eligible trips that potentially would use tolled links through their path from i to j;
- and non-toll-eligible trips that would use free alternative links through their path from i to j.

In practice, the assignment program presented in this study seeks_ without violating Wardrop's equilibrium_ to reach an equilibrium solution through the most commonly used optimization approaches, such as the Frank-Wolfe method, by utilizing a multiclass users' equilibrium assignment technique.

Convergence of this method is demonstrated through implementation of the algorithm over a large-scale network assignment by analyzing the results, as presented in the next chapter. Nevertheless, other convergence methods which are proven to reach Wardrop's equilibrium are also briefly discussed in this study.

Step-by-step algorithm of the PT-assignment method is as follows:

Step 0: Initialize and load the network information,

Set iteration counter to zero; $l = 0$

Choose a proper initial $g_i^{ct(0)}$ and $g_i^{cn(0)}$ or compute them by "skimming" free-flow networks A and $A^n = A\text{-}\{toll\ links\}$, obtain:

- shortest path based on free-flow travel times for each O-D pair:

$$S_k^{ct(0)}(v) = u_i^{ct(0)},\ k\ \epsilon\ K_i^{ct},\ i \epsilon I,\ c \epsilon C$$

$$S_k^{cn(0)}(v) = u_i^{cn(0)},\ k\ \epsilon\ K_i^{cn},\ i \epsilon I,\ c \epsilon C$$

and

- tolls paid for each class over each O-D pair (corresponding to the shortest paths that uses the toll roads)

$$t_i^{ct(0)} = \sum_{a\,\epsilon\,k_i^{ct}} t_k^{ct(0)}\ , k \epsilon K_i^{ct}, i \epsilon I, c \epsilon C$$

Compute:

$$g_i^{ct(0)} = g_i^c / \{1 + exp\ (\alpha^c t_i^{ct(0)} + \beta^c\ (u_i^{ct(0)},\ u_i^{cn(0)}))\}\ ,\ i \epsilon I\ ,\ c \epsilon C$$

$$g_i^{cn(0)} = g_i^c - g_i^{ct(0)}$$

Perform a multiclass All-Or-Nothing assignment based on free-flow travel times (shortest paths) and obtain:

$$v_a^{ct(0)},\ v_a^{cn(0)}\ ,\ a \epsilon A\ ,\ a \epsilon A^n$$

Set $l = l + 1$

Step 1: Update link travel times and solve a multiclass network UE problem.

Descent direction finding:

Perform a multiclass All-Or-Nothing assignment based on updated travel time and obtain auxiliary flows:

$$y_a^{c(l)},\ y_a^{cn(l)},\ \ c \epsilon C,\ a \epsilon A,\ a \epsilon A^n$$

Line search for optimal step size (an F-W method is intended):

Find λ^l that solves:

$$Min \sum_{a\,\epsilon\,A} \int_0^{v_a} S_a(x)dx + \sum_{c\,\epsilon\,C} \sum_{a\,\epsilon\,A} v_a^c\ \theta^c\ t_a^c$$

Subject to:

$$\sum_{a \,\epsilon\, k_i^{ct}} h_k = g_i^{ct(l)} , \sum_{a \,\epsilon\, k_i^{cn}} h_k = g_i^{cn(l)} ,$$

$$h_k \geq 0, \; k \;\epsilon\; K_i^{ct} , k \;\epsilon\; K_i^{cn} , i \,\epsilon\, I, c \,\epsilon\, C$$

$$0 \leq \lambda^{(l)} \leq 1$$

Note that the second component of this "objective function" is to account for the "generalized cost" of additional market segmentation by further disaggregating the total demand that could be eliminated if disaggregation of toll road users into smaller classes is no longer an objective. The O-D travel cost could then be written in the simpler form of:

$$u_i^{ct} = u_i^t \text{ and } u_i^{cn} = u_i^n , i \,\epsilon\, I, c \,\epsilon\, C$$

In other words, the toll is perceived at the demand function level, before the trip is made; once the decision to pay or not to pay the toll is made, the path choice is no longer governed by "generalized cost," but only by the time [40]. In the numerical example that is presented in the next chapter, this extra disaggregation due to "generalized cost" of different vehicle classes is not considered. It is assumed that the logit models maintain the element of the decision making process by splitting the demand accordingly, and vehicles associated with different classes are assumed to be homogenous. However, for the completeness of the presented method, this assumption is not made in the algorithm illustrated here.

Move:

Set:

$$v_a^{ct(l)} = (1- \lambda^{(l)}) * v_a^{ct(l-1)} + \lambda^{(l)} * y_a^{ct(l)} , c \,\epsilon\, C, a \,\epsilon\, A$$

$$v_a^{cn(l)} = (1- \lambda^{(l)}) * v_a^{cn(l-1)} + \lambda^{(l)} * y_a^{cn(l)} , c \,\epsilon\, C, a \,\epsilon\, A^n = A\text{-}\{Toll\ Links\}$$

Note that the tolled ($v_a^{ct(l)}$) and non-tolled ($v_a^{cn(l)}$) link flows are mentioned to emphasize the assignment of two classes of tolled and non-tolled trips among other vehicle classes. Nevertheless, this step is very similar to any other multiclass UE assignment.

Re-compute:

$$u_i^{ct(l)}, u_i^{cn(l)}, t_i^{ct(l)},\ k \in K_i^{ct}, k \in K_i^{cn}, i \in I, c \in C$$

where;

$$t_i^{ct(l)} = \sum_{a \in k_i^{ct}} p_k{}^{(l)} t_k^{ct(l)}$$

which are the (shortest) paths tolls for each class c, $c \in C$ and O-D pair i that include toll links.

Step 2: Modify demand

$\tilde{g}_i^{ct}$ $(and\ \tilde{g}_i^{cn})$, are (re)computed using logit function for each class c:

$$\tilde{g}_i^{ct} = g_i^c / \{1 + exp\ (\alpha^c t_i^{ct(l)} + \beta^c\ (u_i^{ct(l)},\ u_i^{cn(l)}))\}\ , i \in I, c \in C$$

and

$$g_i^{ct(l)} = (1 - \varphi^{(l)}) * g_i^{ct(l-1)} + \varphi^{(l)} * \tilde{g}_i^{ct}\ , i \in I, c \in C$$

$$g_i^{cn(l)} = g_i^c - g_i^{ct(l)}, i \in I, c \in C$$

$$0 \leq \varphi^{(l)} \leq 1$$

The step size $\varphi^{(l)}$ could be chosen to implement Method of Successive Averages (MSA), ($\varphi^{(l)} = \frac{1}{l}$), or any other reasonable sequence of step size [40].

Step 3: Convergence test

If the convergence criterion is met, stop. Otherwise, go to step 1.

The convergence criterion could be based on the similarity of two successive solutions or based on the reduction of the objective function values between two successive iterations. In terms of the convergence test, one notable difference in this method is to check the convergence for the toll-eligible (and consequently non-toll-eligible) demand(s). This is also presented below:

If max‖ $v_a^{(l)}$ - $v_a^{(l-1)}$‖ ≤ $\mathcal{E}_1$, stop. Otherwise, go to step 1.

If max‖ $g_i^{ct(l)}$ - $g_i^{ct(l-1)}$‖ ≤ $\mathcal{E}_2$, stop. Otherwise, go to step 1.

However, for an overall multiclass-network convergence, as illustrated in this section, estimated volume(s) and toll-eligible matrix should both converge to their corresponding solutions. However, if simulated volumes over a multiclass-network show a convergent pattern, this would indicate that toll-eligible and, consequently, non-toll-eligible matrices have also reached a stable condition.

According to the literature that was found related to this topic, the presented algorithm (volume assignment programs with embedded route choice models) has been used in some applications in Europe, North America and Asia. Also an application of this method was carried out on a network used for transportation planning in Southern California. The network included 2,450 zones and 46,000 links. The demand for travel was subdivided into High-Occupancy Vehicles (HOV) and Low-Occupancy Vehicles (LOV). Tolls were envisioned on some of the regional highways. The logit function illustrated below was used to determine the probability of a traveler using the toll facility [40].

$$Probability\ (using\ toll) = 1/(1 + exp(0.5647(u_i^t - u_i^n) + 0.4199(t_i^t)))$$

A two-class (HOV, LOV) network equilibrium model was used to find the initial travel times and toll costs. The logit function was used to obtain four matrices corresponding to the demand for HOV_{toll} , HOV_{notoll} , LOV_{toll} , and LOV_{notoll} , and a four-class network equilibrium assignment was carried out to determine the tolled and non-tolled link flows. This method was also applied in Mexico City for the evaluation of a 26-km section of an urban auto route (Chamapa Highway) [40]. Figure 4.1 illustrates a flowchart presentation of the PT-Assignment algorithm as presented in this chapter [40].

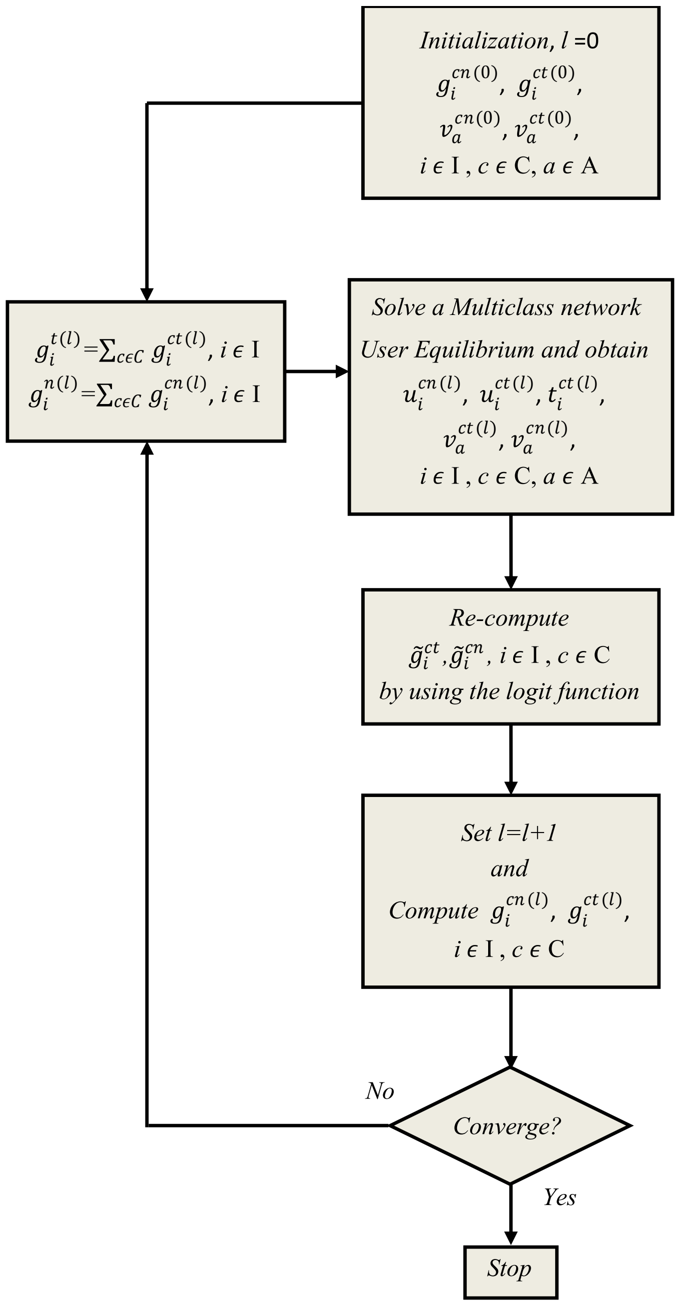


Figure 4.1 Flowchart of the PT-Assignment Model Formulation [40].

4.2.4 Commonly Used Methods for the Convergence Test

If an assignment algorithm is convergent, the objective function would have a minimum, and simulated volumes would converge to their equilibrium solutions. The proximity to the equilibrium solution and convergence pace depend on the convergence method used within the assignment algorithm. Some methods may converge very slowly, and some may converge rapidly. There are measures and criteria which are used to determine whether convergence exists or additional iterations are necessary.

The convergence test is performed in different ways with different criteria; however, all monitor and test the assignment outcomes after each iteration. Among these convergence criteria, *gap* and *relative gap* could be named as the most frequently-used criteria for the convergence test. Below is a description of *gap* and *relative gap,* as well as a number of other criteria that are sometimes used to test the convergence state of a transportation network:

- Gap between assignment outcomes (link volumes) for two successive iterations. Gap is calculated as defined below:

  $$Abs[\sum_{l}(v_n * c_n) - \sum_{l}(v_{n-1} * c_{n-1})]/\sum_{l}(v_{n-1} * c_{n-1})$$

  where:

  $\sum_l$ denotes summation over the links

  v_n is the equilibrium weighted volumes for iteration n

  c_n is the cost based on the equilibrium volumes v_n

  n is the current iteration

- Relative Gap, which is defined as:

  $$\sum_{l}(v_{n-1} * c_{n-1}) - \sum_{l}(VA_n * c_{n-1}))/\sum_{l}(v_{n-1} * c_{n-1})$$

  where:

VA_n is the link volume from an all-or-nothing assignment to the minimum cost paths based on c_{n-1}

- Average Absolute volume Difference (AAD) based upon successive iterations
- Relative AAD, which is calculated as: (diff v)/ v

  where:

  v is the equilibrium weighted volumes
- Percent of links whose change in v between iterations is less than a set value
- Root Mean Squared Error of the differences in v between iterations

Chapter 5 illustrates the implementation of the formulation presented in this chapter in the framework of the Cube Voyager software. Also, a description of the advantages of the PTA routine from a technical perspective is provided in Chapter 5.

CHAPTER 5

IMPLEMENTATION OF THE PTA ALGORITHM

Building on information from the previous chapters, Chapter 5 will describe the PT-assignment (PTA) algorithm and its implementation within the framework of the Cube Voyager software system. The outcome of this section of the study was development of an enhanced PTA-Cube module that resulted in substantial improvements in CPU time and convergence behavior in the toll-assignment procedure. These improvements are especially important when dealing with congested networks that show a non-convergent behavior which are the main concern of this study.

This chapter is based on real-size transportation projects utilizing state-of-the-practice toll demand forecasting. Because practitioners always are in search of new tools and technologies by which to minimize their operating costs, they typically are drawn to specific methods which are proven to save time, are user-friendly, and are easy-to-apply.

As such, this chapter will illustrate the many advantages that come from using the enhanced Cube-PTA tool, as opposed to other methods currently employed throughout the traffic and revenue forecast industry.

One of the most obvious advantages of the Cube-PTA module is its simple approach to toll-assignment problems. Generally speaking, this module is fully contained within a conventional assignment procedure, thereby making it more consistent with the framework of a 4-step travel demand forecast model. As a result, implementation of the Cube-PTA is relatively straightforward within most existing regional models used by MPOs.

Other advantages include:

- Faster convergence to the equilibrium solution.
- More consistent results and higher performance in highly-congested networks.
- Superior performance in terms of CPU time.
- Easier convergence configuration.

This chapter contains two sections. Section 5.1, illustrates implementation of the PTA algorithm within Cube Voyager and development of the “enhanced Cube-PTA” tool. Section 5.2 demonstrates the advantages of the “enhanced Cube-PTA” (Cube-PTA) module over other commonly used methods (applications) for toll-assignment.

5.1 Development of the PTA Tool in Cube

This section outlines the implementation of the PTA algorithm within the framework of Cube Voyager and illustrates the development of the Cube-PTA module. Cube Voyager allows the user to write scripts to perform a multitude of planning operations. This, in turn, provides a flexible environment and allows the user control over all aspects of the modeling process, two important reasons for selecting this software as the platform for a PTA application. It should be noted that altering and customizing Cube programs and consequently building the enhanced Cube-PTA application posed significant challenges. Some of these challenges came about as a result of adjustments that were not accounted for within the Cube software environment. However, some of these adjustments — such as enabling the assignment routine to run a “method of successive averages” over the toll-eligible matrices — became quite complex and presented additional challenges, as discussed later in this section.

Although the Cube-PTA module has been built based on the formulation presented in Chapter 4, the special conditions in this study dictated many deviations from the original formulation. This was mainly due to the congestion level within the project that was selected for this study. As described in Chapter 3, the selected project has a congestion level that makes it challenging for a conventional toll-assignment method to overcome. Therefore, many steps within the PTA formulation have been customized or “enhanced” to enable the Cube-PTA module to tackle such congestion level.

Cube Voyager uses the “Highway” program module to perform volume assignment procedures. The “Highway” program’s primary function is to assign trips to highway network links. There are basic default operations, but the user can control much of the process. The program operates by processing in various “phases,” each of which performs specific operations or stacks of operations provided by the user for that phase. In addition to phase operations, the user can enter “*FUNCTION*” statements that are to be invoked in lieu of default functions at appropriate times by the program.

During development of the Cube-PTA module in this study, the researcher modified a standard “Highway” program and enabled it to account for travelers’ route choice dilemma between toll and non-toll alternatives. This included customizing the Highway program by:

1- Implementation of required time and cost skims.

2- Embedding diversion curves.

3- Estimating the toll-eligible and non-toll matrices.

4- Utilizing a method of successive averages (MSA) over the estimated toll-eligible matrices.

All of these steps are performed in each assignment iteration.

Figure 5.1 shows a typical “Highway” program including the inputs and outputs to this program. Sections below describe the steps taken in this study to alter a standard assignment routine in order to develop the Cube-PTA module in a Cube Voyager environment.

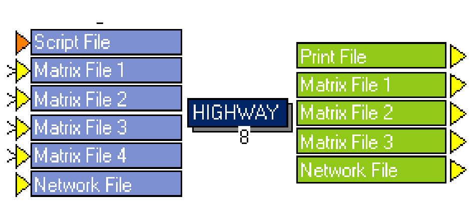


Figure 5.1 A Typical “Highway” Program Including Inputs and Outputs in Cube Voyager

5.1.1 Entering Demand and Network Information

In a “Highway” program, free-flow network, zonal matrices, and turn penalties can be inputted, and a loaded network and new matrices can be obtained. In this study SOV, HOV, TRK, MTK and HTK trip tables are the demand inputs. The ARC network, which includes the managed facilities, is the inputted free-flow network. Inputs enable the “Highway” program to completely access the demand and network information. However, not all the required parameters and link information can be included simply by introducing the inputs. Other required link attributes such as capacity, free-flow speeds and link times must be uploaded to the “Highway” program by means of additional procedures which are outlined below.

5.1.2 Assignment Parameters

A convergence approach such as the Frank-Wolfe (F-W) method and other parameters related to the convergence, are defined in this step of the Cube-PTA module. In addition, information such as *gap*, *relative gap*, *number of assignment iteration* and *zonal data* (number of traffic analysis zones) all of which is required for the PTA, will be introduced in this step.

5.1.3 Reading Network Information

Typically, link values are computed directly from variables within the input network, but because the network does not contain a fixed format, the required variables — including free-flow speed, time and capacity — may not be present. In such case, the “LINKREAD” phase can be used and formulated to provide these values.

“LINKREAD” phase is the first point of deviation from a standard volume assignment procedure toward implementation of the Cube-PTA module. Through user-defined functions, the Cube-PTA module obtains required link attributes, as well as other important information, in this phase of the “Highway” program. “Link prohibition” which forms the basis for shaping the toll and non-toll networks also are introduced in this phase. The “*LI.PROHIBITION*” function controls the exclusion of the link sets from a network based on vehicle class or in the case of the Cube-PTA a traveler's value of time.

Within this step and using the "*LI.PROHIBITION*" function, the network layer A, which contains all the links (including tolled links) and network layer $A^n = A\text{-}\{toll\ links\}$ which holds the free alternatives were produced. In this toll-assignment program, seven prohibition sets, which control the access of different vehicle classes to different network layers, are defined.

5.1.4 Assignment Functions

Volume-delay functions are introduced to the Cube-PTA procedure by means of "*FUNCTION*" block. This block allows users to include passenger car equivalency factors within the assignment procedure, and defines the congestion level caused by each vehicle class based on these factors. This is accomplished by the "*VOLUME*" function within the "*FUNCTION*" statement in this step of Cube-PTA module. When needed, these functions will be called by related sub-routines within the Cube-PTA routine.

At this point, the Cube-PTA module should have all the required information to begin the toll-assignment procedure. The following sections describe the iterative steps of the Cube-PTA procedure.

5.1.5 The Cube-PT Assignment Procedure

A typical assignment program builds paths based on link costs (impedances) and assigns trips to those paths for each zone. After all zones have been processed, link costs are updated based on the level of congestion on each link. The entire assignment process then is repeated, continuing on until some criteria for termination is reached. The volumes from each iteration are then combined to form a weighted assigned volume for each link.

The Cube-PTA method follows the same procedure. However, it differs from a typical assignment routine by utilizing diversion curves and updating toll and non-toll demands in each assignment iteration. A description of the steps forming the core of the Cube-PTA routine is provided below.

5.1.5.1 Cube-PTA Initialization

In this study, “Initialization” is the first added block to a standard volume assignment routine in order to initiate the Cube-PTA module. This step initializes the Cube-PTA routine by setting up the first set of demand splits, then, feeds the results to the “ILOOP” phase. In this step, the initial demand splits were shaped by means of utilizing diversion curves over free-flow travel time and cost information for each origin-destination pair.

A “*PATHLOAD*” statement builds the shortest paths for each I-J pair. Travel times and cost (toll) skims, based on constructed (shortest) paths are obtained by performing the “*PATHTRACE*” function over the toll and non-toll free-flow networks. Selected I-J path traces (travel time and tolls) are then written to the respective matrices to build the initial toll and non-toll demand split. These toll and non-toll demands, in turn, are used in the “ADJUST” phase of the Cube-PTA module for estimation of the toll and non-toll link volumes. Throughout this study, the process of developing the toll and non-toll demand matrices is called “matrix formation” or “matrix calculation.”

5.1.5.2 Multiclass Cube-PT Assignment

Similarly, based on the paths obtained by performing “*PATHLOAD*” statements, link volumes can be obtained by assigning toll and non-toll demand matrices along these paths. This step is performed by using “VOL” statements in conjunction with utilizing link exclusions (*EXCLUDEGRP*) based on the previously setup link prohibition sets. An All-Or-Nothing assignment method is used in order to assign the demands to related toll and non-toll paths (networks). Following the first set of PT-assignments and after acquiring tolled and non-tolled link volumes, travel times and demands are updated, as described in the next step.

5.1.6 Update Cube-PT Assignment Information

Equilibrium assignment is performed in the so-called "*ADJUST*" step. In addition, computations regarding "objective function," and finding the "weights" to apply to each iteration's link volume in the volume combining process, also are performed in the "*ADJUST*" phase. The researcher examined several methods for assignment convergence during the construction of the Cube-PTA module. MSA, MSA-D and Frank-Wolfe and Conjugate F-W methods were examined. The Conjugate F-W method is an improved link-based algorithm which is associated with faster convergence [44]. Figure 5.2 illustrates the results of this analysis over a selected tolled link within the selected project.

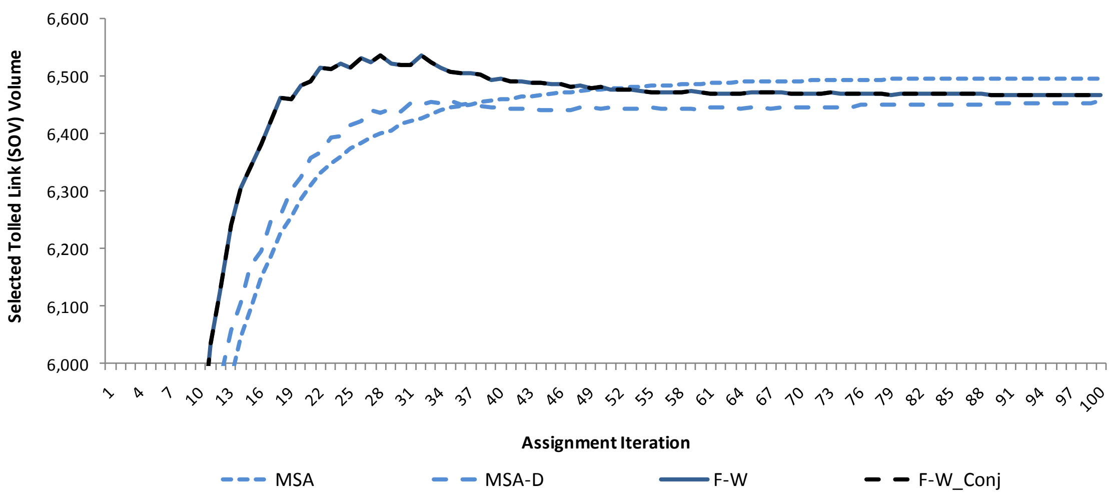


Figure 5.2 Analysis of Assignment Convergence Approach in Cube-PTA Module

As shown in the figure and concluded after further analysis of the overall results, the Conjugate F-W method does not show any advantages over the F-W regarding convergence (F-W Conj. exactly follows the F-W curve in the figure). More detailed information about MSA and MSA-D approach is provided in Chapter 7. As a result of this analysis, the F-W method is selected and utilized in this study based on its performance.

Other important estimations conducted in this step of Cube-PTA are estimations for link "congested time" and examinations for convergence of estimated link volumes. These analyses are further discussed in the following sections.

5.1.6.1 Update travel times

In this block within the "*ADJUST*" phase, "congested times" on each link are computed and "time" values for links are revised. In this step, volume-delay functions are utilized to obtain the "congested times" on each link to be used in the next assignment iteration.

5.1.6.2 Update Demand

This block is another step specific to Cube-PTA module. This block re-estimates demand splits similar to the initialization block. Also in this step, a method of successive averages over estimated toll-eligible demands is performed to obtain the toll and consequently, non-toll demands for iteration "l" of the Cube-PTA procedure. Demand splits of SOV, HOV and COM vehicle classes are estimated in this step in order to be used in the next iteration in the "*ILOOP*" phase. In this study, the researcher conducted many analyses to determine the best approach to the matrix formation problem. As described in Chapter 4, the PTA method uses a stopping criterion for matrix calculations as illustrated below:

If max$\| g_i^{ct(l)}$ - $g_i^{ct(l-1)} \| \leq \boldsymbol{\varepsilon}_2$, stop. Otherwise, go to step 1.

Due to the substantial influence of this step on the final results, careful examination of this step was important. One of the concerns in this study was to maintain a reasonable margin for the outcomes (estimated tolled volumes) in terms of closeness to the results of other methods utilized over the same project.

Selecting $\boldsymbol{\varepsilon}_2$ in line with the objectives of this step of the study itself became a complex task which warranted a series of analyses. This criterion was selected in line with achieving the following objectives:

- Reasonableness of the results (5% difference from the results of the MCTA method, which will be discussed in section 5.2)
- Minimizing the CPU time
- Reaching an acceptable state for convergence within estimated matrices

Several configurations such as, ($\mathcal{E}_2$ =1), ($\mathcal{E}_2$ =2) and ($\mathcal{E}_2$ =3) were examined. Due to level of congestion in the selected project in this study a small value for $\mathcal{E}_2$ would never be reached and a big number as well would have CPU time disadvantages. It was decided then, to use the number of iterations in matrix formation instead of finding a reasonable $\mathcal{E}_2$. In this study then, (mode 1) function, which is utilizing matrix formation in every other assignment iteration, (mode 5), (mode 10) and (mode 50) functions were tested. As illustrated in Figure 5.3, all approaches resulted in more deviations from acceptable range for reasonableness of the results. This means deviation from the results of the MCTA method by more than 5%. Detailed information about utilizing the MCTA method over the selected project and corresponding results is provided in Chapter 6.

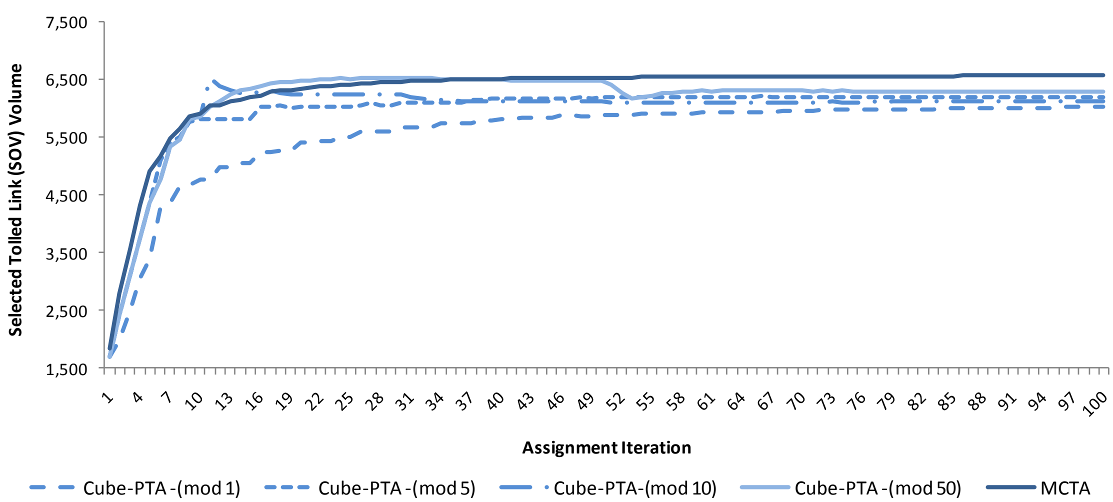


Figure 5.3 Examination of Different Approach to Matrix Formation

Overall, when the results (estimated tolled link volumes) of different approaches were compared to those of the MCTA method a range of 8% to 13% difference in %RMSE was found. All the results were above the acceptable 5% range and were therefore not accepted.

Another examination that was conducted, in line with the best approach in matrix formation, was utilizing a standard users' equilibrium assignment (UEA) procedure over the selected project. The UEA included all the aspects of the PTA except matrix formation and related calculations. Hence, a CPU time comparison between the UEA and the PTA became a good estimate of the (CPU) time penalty due to the matrix formation procedure within the PTA method. The results showed that an additional 65% CPU time over the entire assignment procedure can be attributed to the matrix formation process. Based on the sate-of-the-practice toll demand forecasting, the researcher determined that 65% additional CPU time is not unreasonable due to the congestion level within the project. Therefore, it was decided to continue with matrix formation within all toll-assignment iterations.

It was also determined that another advantage of utilizing the route choice functions in all assignment iterations would be the simplicity that it brings in model configuration by eliminating the analysis related to finding a proper $\mathcal{E}_2$.

Another major challenge in development of the Cube-PTA tool was setting up the method of successive averages (MSA) within the Cube platform. A portion of this challenge was finding the proper step size to be used within the MSA procedure. In this study, λ^l ($\lambda = step\ size\ due\ to\ F - W\ method$) as an additional value for a proper step size was tested. This approach resulted in some small advantages in CPU time as well as overall convergence of the assignment procedure. However, as illustrated in Figure 5.4 the MSA method ($\varphi^l = \frac{1}{l}$, $l = assignment\ iteration\ number$) showed a better convergence pattern and therefore was selected to be utilized within the Cube-PTA module.

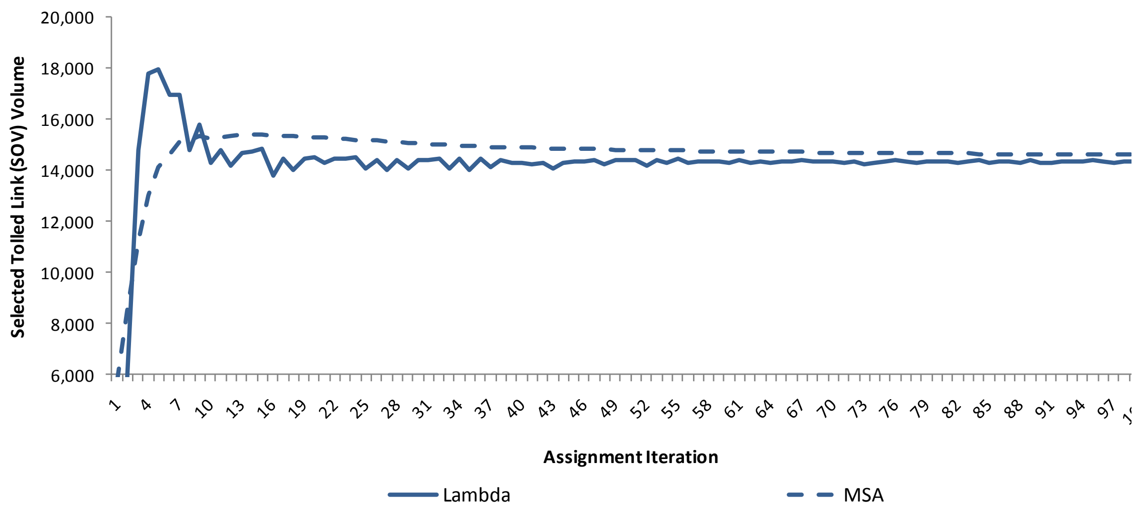


Figure 5.4 Examination of Different Step Size within MSA Procedure

Circulating the toll demand information within toll-assignment iterations, in order to utilize the MSA procedure, was also another challenge during the development of the Cube-PTA module. Normally, in each assignment iteration, all the previously estimated values are overwritten by newly estimated values in current assignment iteration. Therefore, preserving the toll-eligible demand information to be used in the next Cube-PTA assignment iteration became a complex task. After many trials, the researcher used the "ARRAY" function to overcome this challenge. Several ARRAYs then were set up in order to store and transfer toll-eligible demand information from one Cube-PTA iteration to the next. In this way, we managed to enable the Cube-PTA module to perform an MSA procedure over the toll-eligible demands, within toll-assignment iterations.

5.1.6.3 Convergence Test

The Cube-PTA program seeks an equilibrium state through its iterative procedure. The equilibrium state is reached when further adjustments in the link costs used for routing will not produce significant differences in the system as a whole. Various criteria are used to determine when sufficient iterations have been performed or whether more iterations are needed. Similar to a typical assignment routine, the Cube-PTA uses "*gap*" and "*relative gap*" parameters for examination of the convergence to the equilibrium solutions. If these criteria are reached within a Cube-PTA iteration, the program stops and writes the final estimated link volumes (tolled and non-tolled) on the respective link networks. However, if the convergence criteria are not reached, the program is redirected to "step 1" to start another assignment iteration ($set\ l = l + 1$). To better illustrate the preceding explanation, the Cube-PTA algorithm is summarized in Figure 5.5, which represents a flowchart of the algorithm within the Cube Voyager software framework.

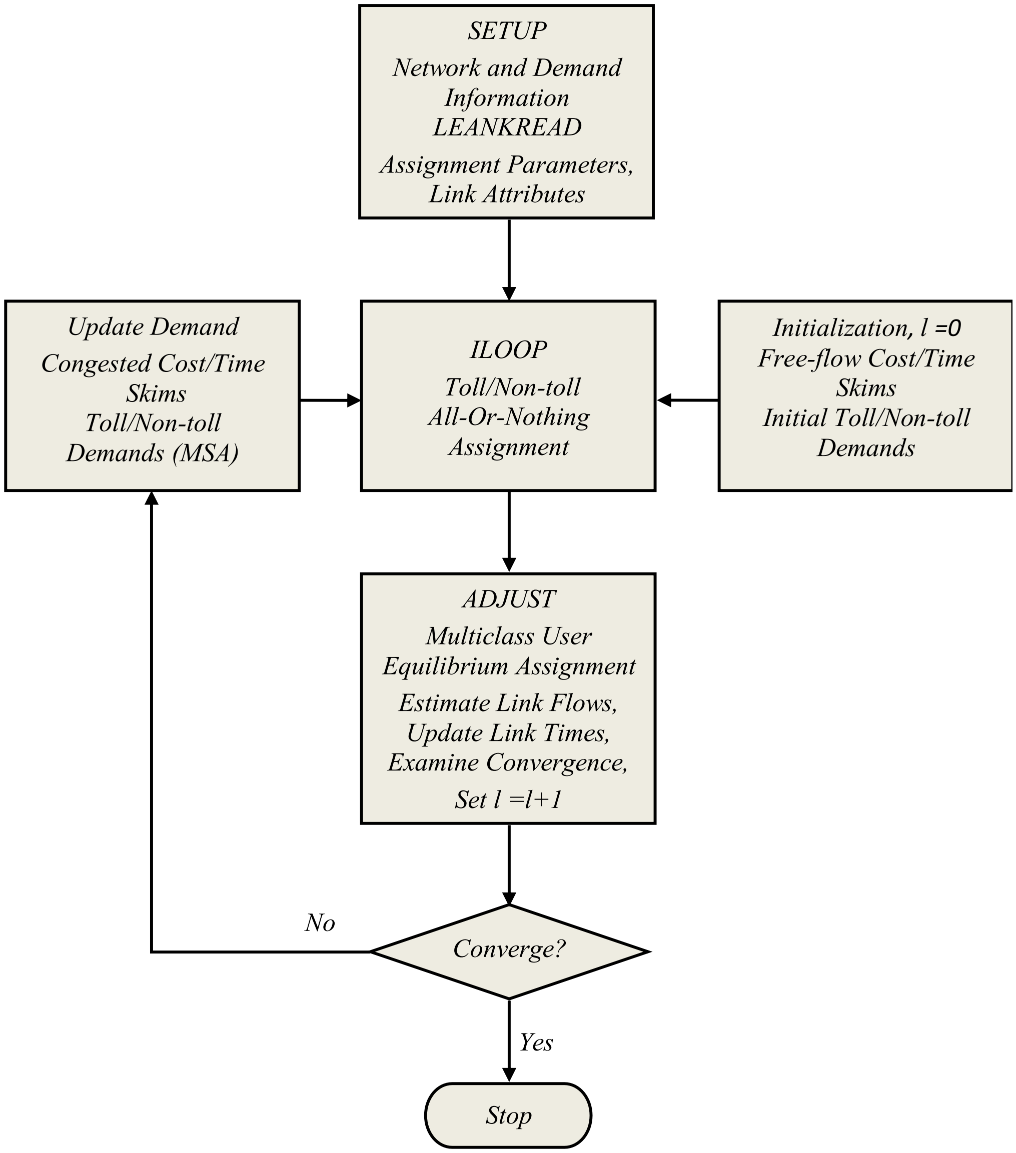


Figure 5.5 Flowchart of the Cube-PTA Module

5.2 Advantages of the Cube-PTA Module

While existing toll-assignment methods — even those with more sophisticated structures — are still subject to more development, there is little doubt that the Cube-PTA tool, with its superior approach to solving toll-assignment problems, has distinct advantages over other methods. The purpose of this section is to outline the benefits of implanting the enhanced Cube-PTA tool within a real-size transportation network, as opposed to another conventional tool known as the "toll mode choice" (MCTA). As discussed in Chapter 2, the MTCA method utilizes the mode choice component to solve for a traveler's choice of tolled versus non-tolled alternatives. The MCTA considers a set of toll and non-toll assignments for estimation of toll road demand in the assignment step, then uses a feedback procedure and seeks a stable state through an iterative process. This section will use the MCTA method as a basis for comparison with the Cube-PTA tool in an effort to demonstrate the advantages that have been gained through implementation of the Cube-PTA tool within a real-size-network.

As mentioned earlier in this chapter — within the context of state-of-the-practice traffic and revenue forecast projects — applications and methods that can lower the cost of a project, whether through time savings or other means, are constantly being sought. For example, because of the numerous (travel demand forecast) model runs required of traffic and revenue projects, "model run time" or CPU time, is always an important consideration. Such revenue forecasts constitute an essential step in securing private capital for toll projects, a step that is often conducted under time pressure for meeting various financial market requirements for issuance of bonds. Therefore, tools that offer CPU time savings can result in significant advantages with possible lowering of the ultimate cost of a project. This is especially true of projects with travel demand models that involve multiple time periods and numerous modeling years. In these cases, the CPU time savings can be considerable.

As found through literature review and mentioned in Chapter 4, there are some implications of earlier implementations of the PTA method within the real-size networks. However, none of the conclusions presented in this study could have been made based on such studies. The main reason is a general lack of information about specific study conditions such as information about the demand conditions (congestion levels) under which the studies were conducted.

The congestion level in the selected project in this study makes it almost a non-convergent network and a challenging case for conventional methods. The objective of this chapter has been to present the advantages of the Cube-PTA tool when utilized within the selected real-size-congested traffic and revenue forecast projects, specifically to illustrate substantial contribution of the Cube-PTA tool to the overall performance of the traffic and revenue forecast field.

The following sections will discuss the underlying theoretical reasons for the superior performance of the Cube-PTA module within the context of transportation problems involving managed facilities.

5.2.1 The Cube-PTA Approach

As discussed earlier in this chapter, the Cube-PTA application is fully contained within a conventional assignment procedure; an attribute which makes it simple to implement within a regional model. Some of the initial advantages of this tool when compared to the MCTA method, are highlighted as follows:

- Elimination of the feedback procedure.
- Less programming effort.
- Saving (computer) memory and space by eliminating many intermediate files.

The most notable characteristic of the Cube-PTA tool, identified in this study, is in its approach to solving toll-assignment problems. The Cube-PTA tool utilizes the route choice functions (toll-eligible matrix formation) and assignment procedure simultaneously, whereas the MCTA performs the matrix formation process (through feedbacks) after the assignment procedure is completed. In other words, under the MCTA method, matrix formation is done when the network is in the equilibrium state, while under the Cube-PTA approach, matrices are formed while solving for the equilibrium state. This suggests that route choice functions can process travel time savings and cost skims, while assignment is in progress. The results (immature demand matrices) can then be used in the next assignment iteration and an equilibrium condition can still be reached. The researcher determined that, theoretically, the same approach can be used in the MCTA method, which helps with non-convergent conditions by producing more consistent outputs. It should be noted that no numerical example for this analysis is provided in this study and further examinations can be performed in future studies.

5.2.2 Faster Convergence to the Equilibrium Solution

The basis for the converging pattern within the toll-eligible matrices in the Cube-PTA module is circulation of travel time and cost information (skims) by means of the iterative assignment procedure. Furthermore, to expedite the convergence process, the Cube-PTA tool uses a method of successive averages (MSA) over the estimated toll-eligible matrices. The MSA method reduces variations within the toll-eligible matrices estimated by route choice functions in each assignment iteration. Frequently updating the estimated demands is an important advantage of the Cube-PTA module, making it a superior tool within the context of toll-assignment problems. Typically within a toll-assignment problem, the Cube-PTA tool updates the estimated demands five to ten times more than a comparable MCTA method, which results in faster convergence.

5.2.3 More Consistent Results and Performance in Highly-Congested Networks

The same characteristic of the Cube-PTA tool that features frequently visiting route choice functions (in every assignment iteration) also results in more consistent outcomes (estimated link volumes) when compared to the MTCA method. The MCTA method predicts (estimates) the toll and non-toll demands in mode choice step using the last assignment procedure at the equilibrium state. Predicted demands then are assigned to a fresh (free-flow) network in the next assignment procedure. The difference in network condition at the time the information is collected (equilibrium state) and the time it is used (assignment over a free-flow state) is the source of some inconsistency in the MCTA outcomes. The Cube-PTA tool, however, collects the information (skims) for demand estimation in one assignment iteration and uses this information in the very next one, when the state of the network has not changed significantly. This approach leads to more consistent outcomes, making the Cube-PTA method a more effective planning tool in toll demand forecasting.

In the event of a highly-congested network condition, an MTCA method may never converge, and as will be shown in the next chapter, could actually fail to perform. The underlying reasons for this failure are issues with inter-connectivity within the two steps of the MCTA approach. Within the assignment step of the MCTA, in the presence of extreme congestion, volume-delay functions will override the functionality of the diversion curves by filling up all the links to their capacities (or even higher), thus resulting in estimation of low toll-eligible demand in the feedback process. This low toll-eligible demand would then leave the tolled links extremely underutilized in the next assignment procedure. Underutilized tolled links, in turn, could result in high CPU time savings and a very high toll-eligible demand matrix in the next feedback step. This means going back to the first step in the next assignment procedures and having all links congested again. This big oscillation can extend to a level that in a congested condition a non-convergent pattern occurs and MCTA fails to perform.

In the case of the Cube-PTA tool, however, even in a high level of congestion a converging pattern can be seen even in the early stages of the assignment procedure. This, of course, is due to frequent utilization of the embedded route choice functions that minimize the abovementioned effect. In practice, dealing with a non-convergent pattern can be a challenging task and may take much time and effort to overcome. Consistency of the Cube-PTA tool's results, coupled with its superior performance in a highly- congested network, gives this tool added advantages over the MCTA. These attributes can save time and extra effort that are usually required to turn a non-convergent network into a network with a nearly stable state when utilizing the MCTA method.

5.2.4 Higher Performance in Terms of CPU Time

While developing the Cube-PTA tool, combining several procedures into one uniform assignment procedure provided many advantages, including the CPU time advantage. This is particularly beneficial in cases where the congestion level is considerable and the network would only converge after a relatively high number of assignment iterations. The researcher determined that, in congested networks, under the MCTA method, waiting for an assignment procedure to finish before performing another round of demand estimation expends a significant amount of CPU time. Under the Cube-PTA module, however, simultaneous utilization of route choice functions and assignment procedure is associated with significant CPU time savings. Table 5.1 illustrates a side-by-side comparison of CPU time used by the MCTA and Cube-PTA in a similar and comparable toll-assignment problem. The table shows the distinct advantage of the Cube-PTA over the MCTA in CPU time usage. In practice, this time savings could translate into a notable benefit when dealing with real-size traffic and revenue forecast projects. The figures presented in Table 5.1 are derived from analysis conducted on a selected project, the details of which will be provided in Chapter 6.

Table 5.1 Side-by-Side Comparison of CPU Times

Method	**CPU Time[1]**
PTA	6:37:38
MCTA	31:43:43

[1]Hours:Minutes:Seconds

5.2.5 Following a Standard Convergence Configuration

When utilizing the MCTA method and dealing with congested networks, set-up and configuration of this method can itself be challenging for practitioners. Reaching a desired convergence criterion such as a "relative gap" of 0.01or 0.0001, as suggested in practice, or in related literature [44], can at times have a high cost in terms of CPU time. Therefore, finding a balance been accuracy of the assignment outputs (adjacency to real equilibrium solutions,) and CPU cost, becomes a concern in real-size projects. As will be shown through numerical examples in Chapter 6, the configuration of the MTCA method by selecting an appropriate number of feedbacks and assignment iterations could be a time consuming task, and one which could easily involve many trials and observations. The Cube-PTA module, on the other hand, behaves similarly to a standard assignment procedure in regard to selecting the convergence criteria. Under this method, achieving the desired criterion could be as easy as selecting a "gap" or a "relative gap," yet another characteristic that puts this tool at an advantage over the MTCA method.

Using numerical examples, a more in-depth examination of the advantages of the Cube-PTA tool will be presented in Chapter 6. Chapter 6 also will summarize the results of the implementation of the Cube-PTA tool, as presented in this chapter, when utilized within a large-scale transportation network such as the one presented in Chapter 3. In addition, Chapter 6 will demonstrate the convergent pattern of the Cube-PTA algorithm as briefly discussed in this chapter — as well as the reasonableness of the outcomes — by comparing the results to those of the MCTA method.

CHAPTER 6

RESULTS

A revenue assessment of a proposed toll road is the ultimate objective of a traffic and revenue (T&R) study and solely depends on the accuracy of estimated traffic flow on the tolled facility. This describes the importance of estimated volumes on a selected set of links that would translate to the transactions on entrances (ramp-in), exits (ramp-off), main lanes of a managed facility, so called “gantries,” and finally would be used for revenue estimation.

Generally, for estimations of flow rates within road pricing applications in the literature, two types of path choice models are employed within network equilibrium assignment models [40]. The first type implements deterministic path choice models based on a generalized path cost (time) function in which path travel time (path cost) is weighted by a trip-maker’s value of time. Trip-makers’ values of time represent how much money the trip-makers are willing to trade off for a unit of time saving. The other type is characterized by a probabilistic discrete path choice (e.g., logit- or probit-based) model consisting mainly of travel time and out-of-pocket cost. Those discrete choice models can be constructed and calibrated from the analysis of revealed or stated preference surveys to determine the probability that a trip-maker will use paths that include tolled facilities [38]. Within the context of utilizing discrete choice models for route choice applications, two different methods have been mainly suggested [39]:

1. A conventional toll mode choice method for toll road assignment
2. A multiclass user equilibrium assignment program including discrete route choice models

The primary objective of this chapter is to illustrate the results of implementing an assignment program with embedded discrete path choice models within a large scale project. Throughout this chapter this method will be referred to as a “pseudo-probabilistic toll-assignment (PT-assignment) program.” The outcomes of the PT-assignment program discussed in this chapter are the estimated tolled volumes over tolled links. The scope of this research does not include providing a full set of traffic and revenue forecasts.

This chapter is organized into three sections. The first section presents the results of the implementation of a pseudo-probabilistic toll-assignment program utilized for the selected project previously introduced in Chapter 3. This section includes the results of the analysis regarding convergence test. Section two provides examinations over network components and model elements such as links, paths and matrices to illustrate the proper behavior of the utilized PT-assignment model. Section three compares the link volumes estimated by the PT-assignment method with those of other well-known methods.

6.1 Results of the Probabilistic Toll - Assignment Program

The analysis in this study was performed on the Atlanta Regional Commission (ARC) travel demand model. This model covers the Atlanta metropolitan area and consists of a detailed road network with 2118 traffic analysis zones. The model is a 4-step, trip-based model initially developed in 2003. Furthermore, this model was calibrated and validated focusing mainly on the area of influence of the I-20 Managed Lanes (I-20 ML) project, to replicate 2007 (base year) traffic conditions [34]. As described in Chapter 3, the proposed I-20 ML connects two major interchanges, allowing access to two major freeways: I-75/I-85 and I-285 east of Atlanta. The configuration of the managed facilities was determined and superimposed on the Atlanta Regional Commission (ARC) travel demand model network, corresponding to the project’s opening year of 2015 as well as the design forecast year of 2030. The ARC model was utilized for 4 times per day (AM, MD, PM, and NT) demand tables and for several vehicle classes, of which SOV, HOV, and Commercial vehicles were included in the analysis pertaining to the proposed I-20 ML project[34].

As described in Chapter 3 a series of available logit models derived from a stated preference survey calibrated to the I-20 study corridor was used to identify the share of toll road users among all I-20 corridor trip-makers.

To avoid cumbersome illustrations within the presented graphs throughout this chapter, model outcomes pertaining to the AM peak period of the model year 2015 (opening year) and single occupant vehicle (SOV) demand were selected. A complete set of PT-assignment model results including the estimated link volumes corresponding to all vehicle classes is provided in Appendix A. The SOV trip table corresponds to the largest share of the total trips.

As discussed in Chapter 5, in order to implement the PT-assignment method within the ARC model platform and to obtain the corresponding demand for the "I-20 ML" project the Cube Voyager transportation software package was used as the scripting/programming tool in this study. Next, in order to examine the results and the convergence to the equilibrium solution for this method, as well as to lay out a comparable platform for future comparisons, different checks in different steps were performed.

6.1.1 Convergence Test

A convergence test is to check if the assignment has reached an equilibrium solution or if additional iterations are necessary. The equilibrium state is reached when further adjustments in the link costs used for routing will not produce significant differences in the system as a whole. There are different philosophies as to what measures are best to determine if convergence has been reached or if further assignment iterations will improve the assignment depending on the assignment method used [37]. This is the most important check within the results of a PT-assignment routine as presented in this research. In this study, in absence of a mathematical proof for convergence, the overall performance of the presented method was gauged by analyzing the results. Therefore, before additional examinations, it was essential for the estimated toll volume outputs to show a reasonable converging pattern in order to support the overall consistency and workability of the method under study. This made the convergence test the most important check, as discussed in following section.

A combination of *gap*, *relative gap*, and *assignment iteration number* are the criteria used in this research to define and check the convergence state. As discussed in Chapter 4, the *gap* between assignment outcomes (link volumes) for two successive iterations is calculated as shown below:

$$\mathrm{Abs}(\textstyle\sum_l(v_n * c_n) - \sum_l(v_{n-1} * c_{n-1}))/\sum_l(v_{n-1} * c_{n-1})$$

Where:

$\sum_l$ denotes summation over the links

v_n is the equilibrium weighted volumes for iteration n

c_n is the cost based on the equilibrium volumes v_n

n is the current iteration

Similarly, the *relative gap* between assignment outcomes (link volumes) for two successive iterations is calculated as shown below:

$$\textstyle\sum_l(v_{n-1} * c_{n-1}) - \sum_l(VA_n * c_{n-1}))/\sum_l(v_{n-1} * c_{n-1})$$

Where:

VA_n is the link volume from an “all or nothing” assignment to the minimum cost paths based on c_{n-1}

Assignment iteration number is the selected number that stops the assignment procedure after it is reached. Although a convergence test is as easy as setting up some of criteria to stop the run at a desired stage, in practice, there is always a trade-off between computational costs in terms of model run time (CPU time) and accuracy in terms of proximity of the estimated link volumes to the equilibrium solutions. However, while overall smaller *gaps* are desired, an association of smaller *gaps* with larger CPU time sometimes makes the decision for convergence criteria somewhat difficult. This is especially true in most transportation projects where numerous model runs are required to achieve the project's objectives. Therefore, CPU time becomes an important factor within transportation projects. This is especially important within models with a congested condition such as the one under study in this research, which corresponds to the AM peak period and is expected to converge to an equilibrium solution after a rather high number of assignment iterations. Therefore, convergence criteria are usually defined in different ways within different modeling circumstances, and a standard set of assumptions may not be found that applies to all conditions.

Some studies suggest exhaustive measures to assure that the assignment is sufficiently converged to achieve stable link flows. For example some papers propose a *relative gap* of 0.0001 as a proper convergence criterion [45]. On the other hand in practice, many models default to a *relative gap* of 0.01 for convergence test [44].

In this study, however, several scenarios using above-mentioned criteria were examined to determine the proper convergence criterion for the purpose of this research. The necessity of an in-depth examination of a PT-assignment method's behavior and unavailability of similar comparable studies warranted a detailed analysis to define convergence criteria in this study. After a series of trials, 8 scenarios with different convergence criteria were selected to examine the existence and stability of the overall network convergence. Scenarios generally consisted of a set of selected *gaps* and *relative gaps* in addition to an unspecified *number of assignment iterations* or a set of unspecified *gaps* and *relative gaps* in conjunction with a selected *number of assignment iterations*. Referring to Table 6.1 below in scenarios 2, 6, and 7, a set of selected *number of assignment iterations* were defined and corresponding *gaps* and *relative gaps* were "recorded" after convergence criterion was met. Similarly, in scenarios 1,3,4,5, and 8, a set of *gaps* and *relative gaps* were selected to observe the convergence state and resulting number of assignment iterations were recorded after the convergence criteria were met. The results of different scenarios were then examined by calculating the percent root mean square error (%RMSE) between the estimated tolled volumes over all tolled links corresponding to each scenario. It should be noted that these results are specific to the selected project and are a function of the network size.

Table 6.1 presents the convergence assumptions and settings for all defined scenarios along with the corresponding CPU time (Intel Dual Core @ 2.53 GHz processor and 3 GB RAM) and %RMSE. The scenarios have been built sequentially by increasing the *gap*, *relative gap* and *number of assignment iterations*. Then %RMSE corresponding to each scenario was calculated successively with respect to the prior scenario. Note that the %RMSEs are not monotonically decreasing because the number of iteration in each row are different.

Table 6.1 Comparison of Different Convergence Scenarios for the PT-Assignment Method.

Scenario	Gap	Relative Gap	# of assignment iterations	CPU Time[1]	%RMSE
1	0.0005	0.005	28[2]	1:56:01	-
2	0.00013[2]	0.0015[2]	50	3:19:31	4.07%
3	0.0001	0.001	61[2]	4:19:38	0.58%
4	0.00005	0.0005	86[2]	5:53:49	0.98%
5	0.00001	0.0005	93[2]	6:22:52	0.19%
6	**0.000056[2]**	**0.00046[2]**	**100**	**6:37:38**	**0.11%**
7	0.000013[2]	0.00013[2]	200	13:06:06	0.62%
8	0.00001	0.0001	232[2]	15:39:18	0.10%

[1]Hours:Minutes:Seconds
[2]Recorded

The unavailability of similar detailed studies regarding PT-assignment prolonged the investigation of the convergence behavior of this method. However, this in-depth analysis allowed close examination to ensure the stability of the link flows and to check for any unreasonable and unexpected fluctuations after the convergence state is met.

Ultimately, since in this study time restriction was not a factor, convergence assumptions listed under “Scenario 8” (*gap* ≤ 0.00001 and *relative gap* ≤ 0.0001) were assumed to properly serve the objective of this section. The results of this scenario showed a proper and stable convergence for the estimated link flows to the equilibrium solutions utilizing the PT-assignment routine. Figure 6.1 and Figure 6.2 illustrate the overall network convergence pattern over the entire assignment iterations corresponding to “Scenario 8” by means of the *gap* and *relative gap,* respectively.

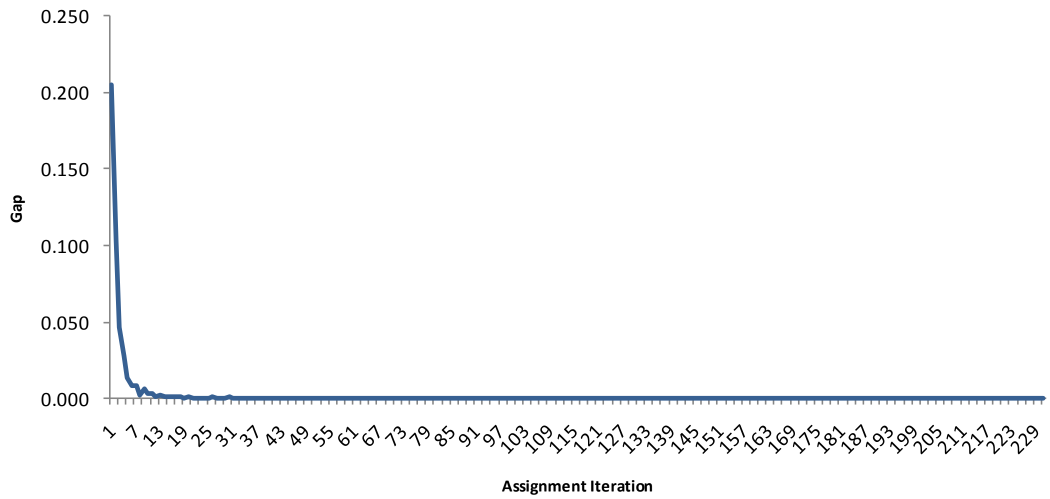


Figure 6.1 Illustration of the Gap over Assignment Iterations for the PT-Assignment Method.

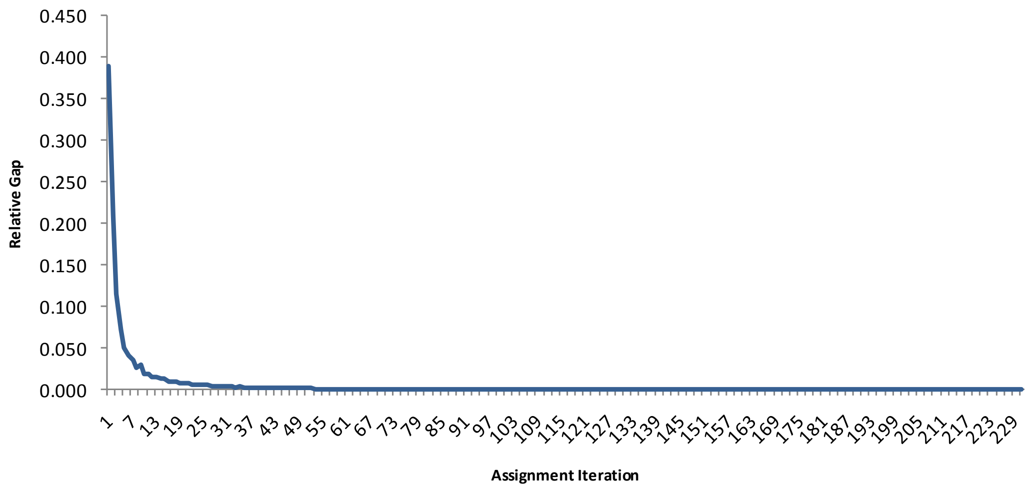


Figure 6.2 Illustration of the Relative Gap over Assignment Iterations for the PT-Assignment Method.

Similarly, Figure 6.3 and Figure 6.4 show a closer snap shot of the graphs presented above, respectively. These graphs provide a closer look and a better understanding of the overall converging pattern within the toll-assignment routine as presented in this study. This is also to check for reappearance of any unexpected and unreasonable fluctuations after the assumed convergence state is met.

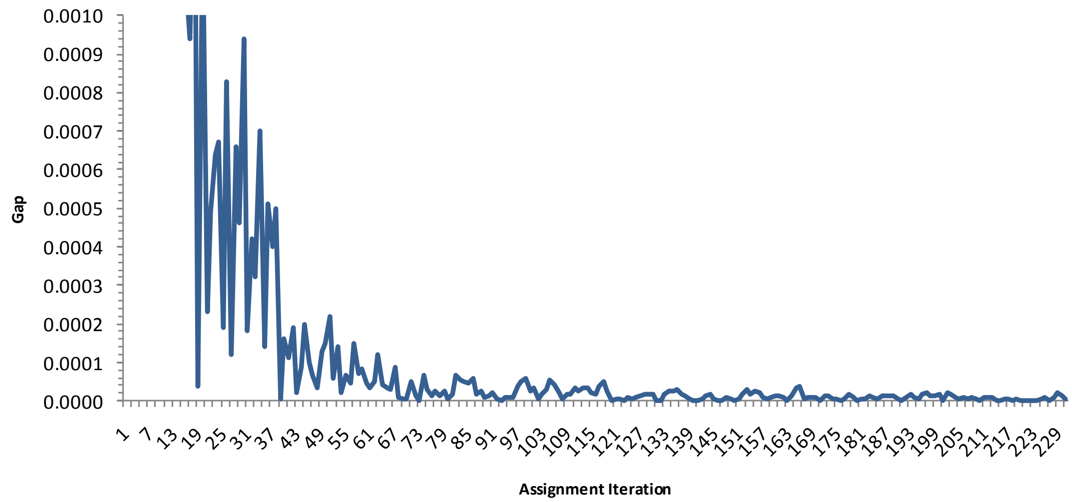


Figure 6.3 A Close-Up Illustration of the Gap over Assignment Iterations for the PT-Assignment Method.

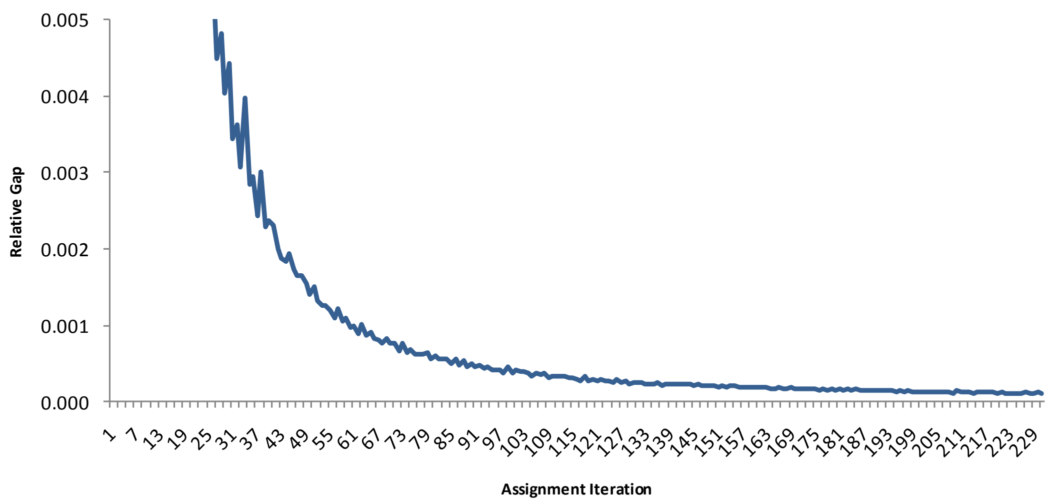


Figure 6.4 A Close-Up Illustration of the Relative Gap over Assignment Iterations for the PT-Assignment Method.

In all graphs, a descending pattern can be seen that stabilizes over assignment iterations and clearly indicates the overall convergence of the loaded network to the equilibrium state. This concludes the proper performance of the PT-assignment routine within a transportation network involving managed facilities and shows the converging pattern of the formulation provided for this method in Chapter 4.

However, later and through extensive model runs, additional factors to consider within a PT-assignment method with regards to convergence were identified. For example it is essential that the results of Logit route choice models which are the toll-eligible (and consequently non-toll-eligible) trip matrices to also reach and maintain a stable state. Therefore, in this method for setting the convergence criteria, care should be taken to check not only the estimated link volumes but also the toll-eligible matrices for stability and convergence. This became the basis for further examinations of the PT-assignment model components, as discussed in the following section. In order to provide further discussions in this regard, however, it seemed necessary to select a set of outcomes for presentation purposes.

Base on the numbers presented in Table 6.1, there are little (1.74 %RMSE) improvements (change in overall estimated link volumes) being gained by increasing assignment iterations between "Scenario 2" (50 assignment iterations) and "Scenario 6" (100 assignment iterations). However, in case of "Scenario 6" more assignment iterations (even 100 more), do not bring (almost) any improvements to the overall estimated link volumes. Therefore, "Scenario 6" was selected as a reasonable solution, regarding convergence, for further analysis on the model elements presented in following section. There were however, other reasons to select Scenario 6 for further analysis. These reasons are discussed later in this chapter.

6.2 Examination of the Probabilistic Toll - Assignment Model Components

The analysis presented in previous section alone can show the ample performance of a PT-assignment, as used in this study, for assignment problems involving managed facilities. However, in order to illustrate the sufficient performance of this method in its entirety, the following subsections are presented to independently review and show the results within the components of this model. These components include estimated tolled link volumes (link base analysis), probability calculations by logit route choice models (path base analysis), and estimation of the toll-eligible matrices. As previously mentioned, the results of the PT-assignment model with 100 assignment iterations (Scenario 6) are selected for the analysis presented in the following sections. It is essential to mention that this section does not intend to check or evaluate the performance of the PT-assignment model components. The main purpose of this section is to demonstrate the development of the results within each component or sub-model of the PT-assignment program. In addition, this section shapes a foundation for future comparisons presented in the next section.

For the purpose of illustrating the PT-assignment results within each model component, a path that passes through the tolled links and a tolled link on the selected path were chosen. The selected tolled path and tolled link were chosen from a big group of tolled paths and tolled links that have been reviewed through numerous model runs conducted for the purpose of this research. A complete review of the estimated tolled link volumes are presented in Appendix A.

Figure 6.5 is a snapshot of the network used in this study. This figure illustrates the selected path (in green) and link (in red). A small image of the selected path in relation with the entire network is also presented in the same figure. The selected path for the presentation of the route choice analysis is the longest (toll) path and encompasses almost the entire managed facility in the westbound direction. The origin (zone number 1213) and destination (zone number 2) of this path are also shown in Figure 6.5. As noted before the selected link (A=15715 and B=15752) used for the presentation of the link-based analysis, is also part of the same path. This link is a “main lane gantry”. Hereafter in this chapter, “selected tolled path” or “tolled path” is used interchangeably to refer to the same path. This is also applicable to “selected tolled link” or “tolled link” terms.

“Link-based”, “path-based” and “trip matrix calculation” presented in this section are mainly performed in line with a common practice that is usually conducted by practitioners in similar studies.

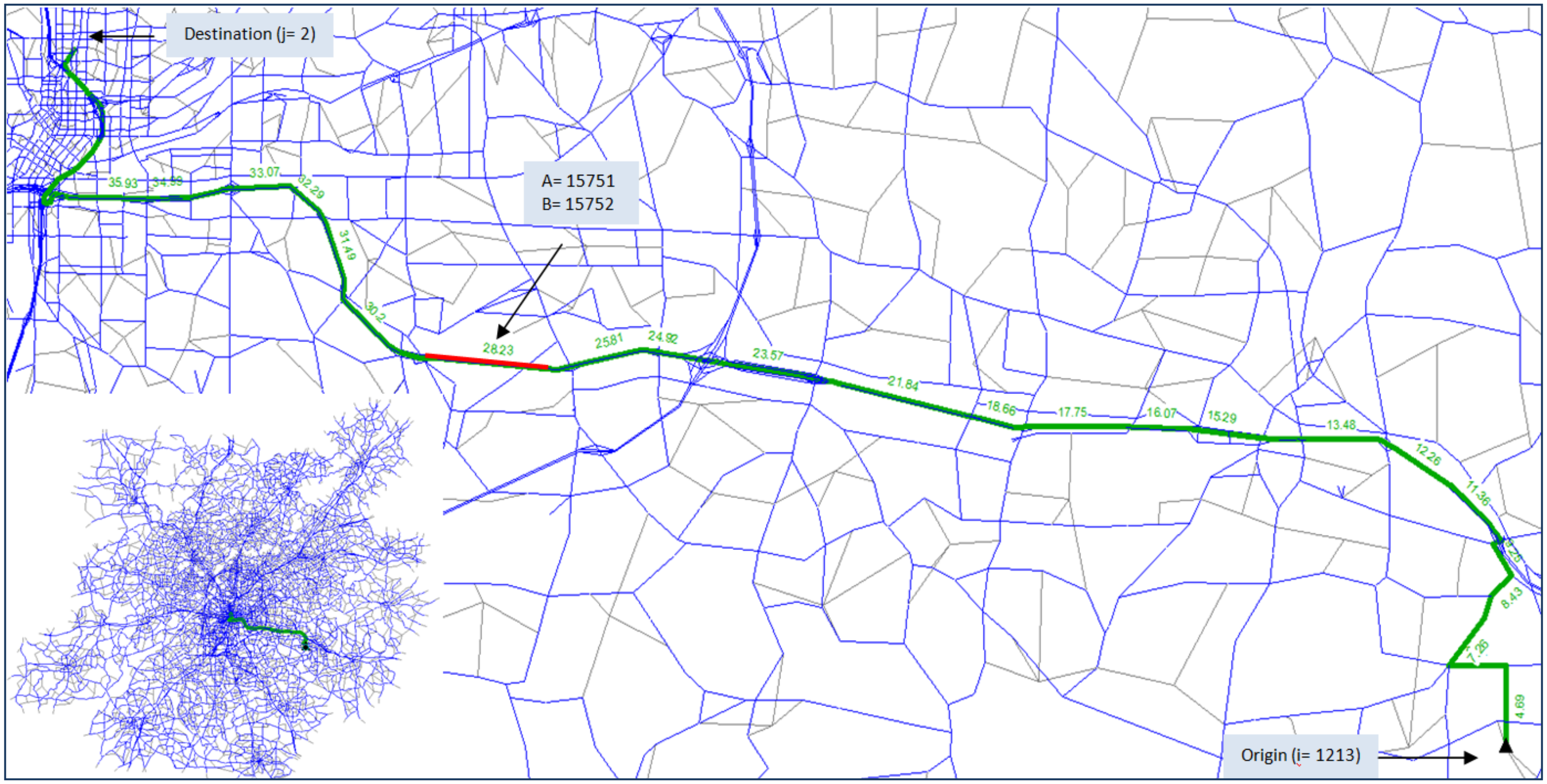


Figure 6.5 Model Roadway Network with the Selected Path and Link along the Managed Facility.

6.2.1 Link-Based Analysis

The link-based analysis demonstrates the development of the link flow estimated for the selected tolled link over PT-assignment iterations. The selected tolled link (as discussed earlier in this chapter) was used as an example of the numerous tolled links that were checked for performance and consistency in the PT-assignment method regarding volume estimation.

As previously mentioned, in traffic and revenue studies, some links, or so-called "gantries," are usually used to represent the tolled traffic or tolled transactions within the roadway system networks involving managed facilities. Therefore, examination of the tolled-link flows, in terms of reasonable development toward equilibrium solutions, was considered as one of the checks regarding the sound functionality of the PT-assignment method in this study.

Figure 6.6 presents the assigned tolled volumes on the selected tolled link over the assignment iterations. This graph also illustrates model run time over the entire 100 assignment iterations corresponding to Scenario 6. Similar to previous sections, for the analysis in this section, the SOV demand matrix was selected for presentation purposes. Similar behavior and patterns also were observed for toll-eligible trip matrices associated with other vehicle classes that are omitted throughout this chapter due to redundancy in concept and functionality.

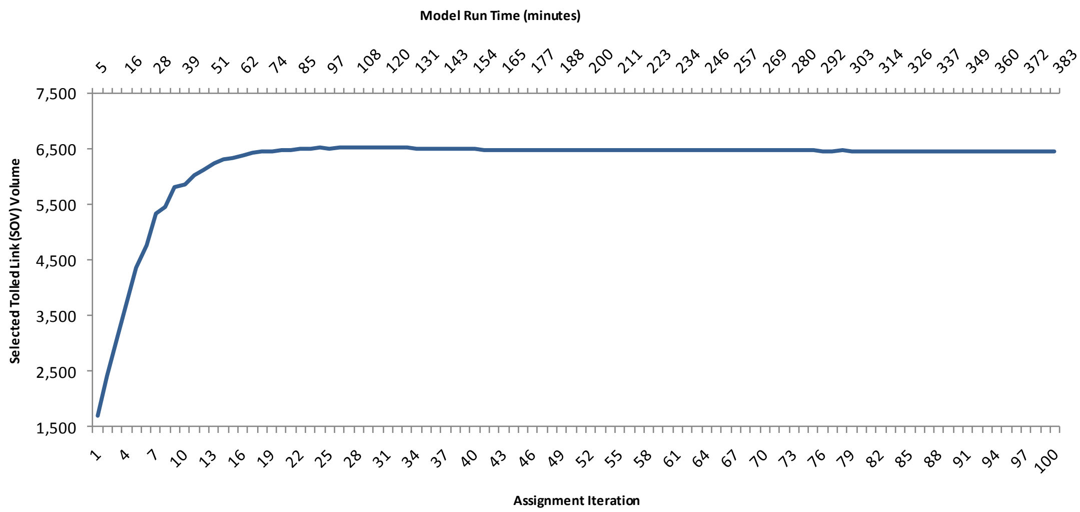


Figure 6.6 Development of the Selected Tolled Link Volume to the Equilibrium Solution for the PT- Assignment Method.

Figure 6.6 illustrates the development of the tolled volumes toward the equilibrium solution associated with the selected tolled link. This figure shows that the estimated volume reached a stable state and this state is maintained through the entire assignment procedure. In the PT-assignment, although total demand for each origin-destination is fixed, toll-eligible and non-toll-eligible demands can change from one iteration to the next. This is due to the effects of the discrete route choice models that are embedded within the PT-assignment routine. These route choice functions, within each assignment iteration, partition the total demand between toll-eligible and non-toll-eligible trips. Some extra oscillations within this method could be partially due to its unique approach to estimating the link volumes while utilizing the route choice models. As the iterative assignment procedure continues, toll-eligible trips that indicate potential toll road users as well as non-toll-eligible trips, which are non-toll road users, are built. Then, these trip matrices are iteratively assigned to the corresponding networks (facilities) similar to a standard multiclass UEA method.

In this study, a free-flow network was used in the PT-assignment program as the starting point for toll-eligible and non-toll-eligible trip table calculations. This may have resulted in some extra oscillations within the first few assignment iterations. Then, as illustrated in Figure 6.6 flow rates were gradually built up to the equilibrium solution by constantly skimming the networks (A and A^n = A- {Toll Links}) for time savings and costs (toll) information.

Initial oscillations shown in Figure 6.6 however, may be alleviated by taking a different approach to introduce initial toll/non-toll-eligible trip matrices to the PT-assignment program. This approach could be selecting a different starting point for calculation of the toll-eligible and non-toll-eligible trip matrices. This can be easily achieved by disabling the matrix calculations for the first few assignment iterations. These adjustments however, are not in the scope of this study and can be performed as potential improvements to this method through future developments.

To take a closer look at the convergence process over the selected tolled link, the results of the toll-assignment have been summarized over each assignment iteration in Figure 6.7. This figure has been configured in terms of percentage difference between successive estimated volumes.

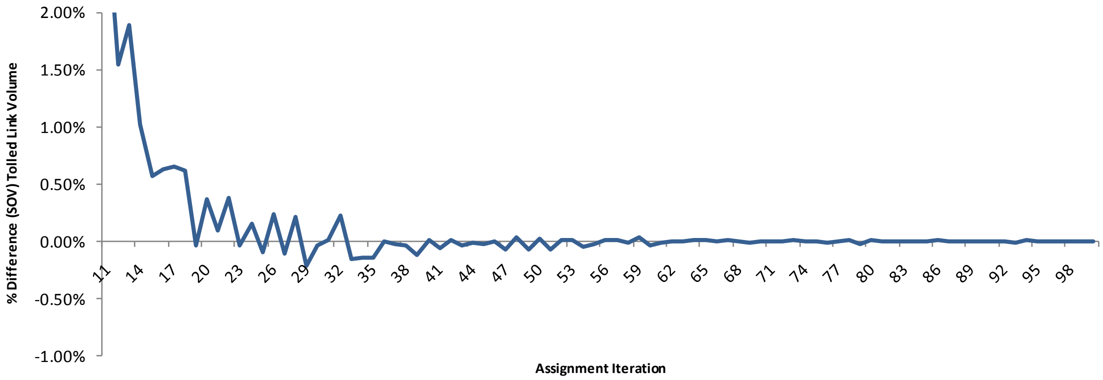


Figure 6.7 Percent Difference between Successive Estimation of the Volumes for the Selected Tolled Link.

6.2.2 Path-Based Analysis

The path-based analysis in this study monitors the calculations of the toll-eligible and non-toll-eligible trip probabilities by origin/destination pairs. This section provides an illustration of the probabilities calculated by logit models within each assignment iteration for the selected path. In order to guarantee the convergence of the assigned tolled and non-tolled volumes detecting a converging pattern within calculated probabilities within PT-assignment routine is important. There is, of course, a direct relationship between assigned tolled volumes and toll-eligible trip matrices, which are in turn a products of probabilities calculated by logit models for each i-j pair. This section summarizes the effort undertaken to carefully assess the pattern of the calculated probabilities associated with toll-eligible trips within the PT-assignment routine. In this study, many paths have been examined. In this section, the previously introduced selected tolled path was chosen for illustration purposes.

As discussed in Chapter 3, the logit route choice models are designed to directly calculate the probability associated with potential toll road users. In this setup, functions generally show a form of attractiveness for travel time savings, which means more travel time saving makes the toll alternative more attractive. In terms of travel cost though, the functions show a reverse form that associates more travel cost with less attractiveness for tolled facilities.

The logit models in this project were calibrated to calculate the probability of toll-eligible trips for each i-j pair using corresponding travel time savings and costs in a general form, as follows:

$$\text{Prob. (toll} - \text{eligible trips)} = \frac{1}{1 + e^{(\text{time coefficient}*(\,u_i^t - u_i^n\,) + \text{toll coefficient}*(t_i^t) + \text{ constant})}}$$

The probability of using free alternatives then would be calculated by subtracting this value from “1.” As illustrated above and mentioned in prior chapters, this configuration uses travel time savings calculated by subtracting travel times corresponding to the tolled network (A) from the non-tolled network (A^n = A-{Toll Links}). This generates a negative value for travel time saving for each i-j pair. Travel cost (toll) for each origin-destination also is obtained by directly skimming the tolled-network. Any increase in travel time saving and any decrease in travel cost will result in higher attractiveness for the toll alternative.

In the PT-assignment method, time saving/cost skims are performed simultaneously within the equilibrium assignment procedure. Figure 6.8 illustrates the calculated probabilities for all 100 assignment iterations within the PT-assignment method as well as corresponding time savings (top axis) and tolls (bottom axis) for the selected path (corresponding to SOV trips). Figure 6.8 shows a converging pattern for toll-eligible probability calculations over volume assignment iterations.

As previously mentioned, a free-flow network condition was used for the starting point of the toll-eligible trip probability calculations. This is the underlining reason for low probability calculations for potential toll road users in the beginning of the toll-assignment procedure (first point in Figure 6.8). With this configuration, as congestion within the network increases, toll-eligible probability calculations using logit route choice models are gradually formed in each assignment iteration. The PT-assignment routine continues iteratively by skimming the current loaded networks (A and A^n = A-{Toll Links}) for time savings/costs information. As a result, probabilities associated to toll-alternative trip makers are constantly updated by means of the logit route choice models. This routine continues until a convergence state is reached as illustrated in Figure 6.8. The output of this phase of the PT-assignment routine is a series of probabilities by each (i, j) pair. These probabilities are saved in a matrix format and are used in the calculation of the toll-eligible and consequently non-toll-eligible trips as is discussed in the next section.

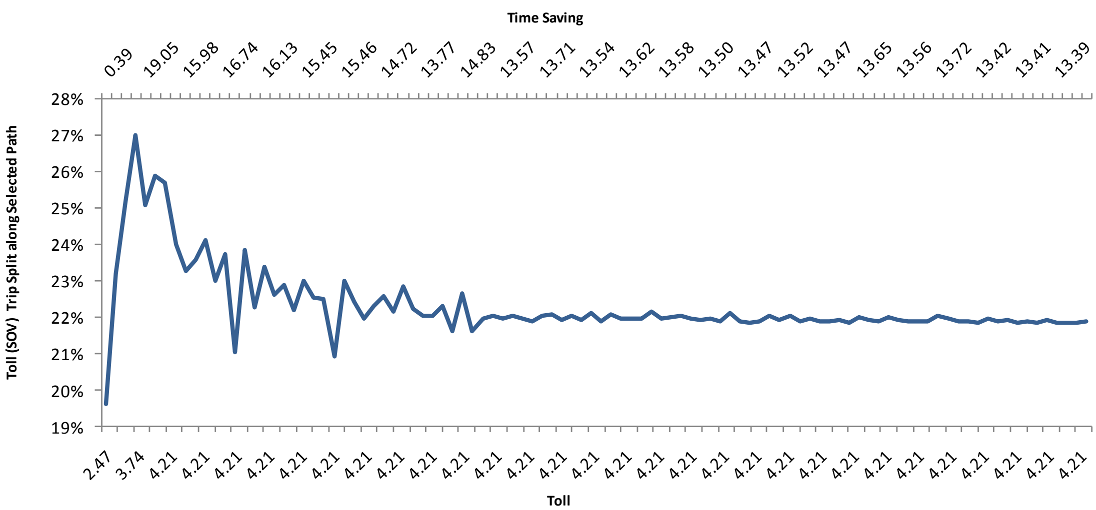


Figure 6.8 Development of the Toll-Eligible Trips for the Selected Path Pertaining to the PT-Assignment Method.

6.2.3 Toll-Eligible Trip Matrix Calculation

As described in the previous section, toll-eligible matrices are direct productions of toll-eligible probability calculations resulting from logit models. A direct cell to cell multiplication of the calculated toll-eligible probabilities and total trip tables in each origin/destination pair is the foundation of the toll-eligible trip matrix formation. In a PT-assignment, this is done before each volume assignment iteration to first determine toll-eligible and non-toll-eligible trip matrices.

Therefore two matrices of toll-eligible and non-toll-eligible trips for each vehicle class are calculated based on the estimated probabilities that reflect the route choice behavior of toll and non-toll route users and their perceived values of time. These two trip matrices in turn act similarly to demands for two different vehicle classes within the assignment procedure. Therefore, a multi-class assignment for three SOV, HOV, and Commercial vehicle classes would be performed over six matrices (2×3 original vehicle classes) of toll-eligible and non-toll-eligible SOV, HOV, and Commercial vehicle trips.

The following discussion presents the results of the toll-eligible trip matrix estimation. The toll-eligible matrices would not be necessarily assigned entirely to the tolled facilities. Rather, the assignment procedure would still search for the shortest travel time for each origin-destination pair in both free and tolled alternatives in order to assign these matrices to the network. Therefore, resulting tolled volumes may not follow the same exact pattern of toll-eligible matrices. However, both toll-eligible matrices and simulated tolled volumes should show a converged condition at the network equilibrium state.

As discussed in the previous chapter, the PT-assignment program uses a method of successive averages (MSA) over the calculated matrices in each assignment iteration. After each assignment iteration, toll probability calculations and consequently toll/non toll-eligible matrix formations are performed and the results are fed to the next assignment iteration.

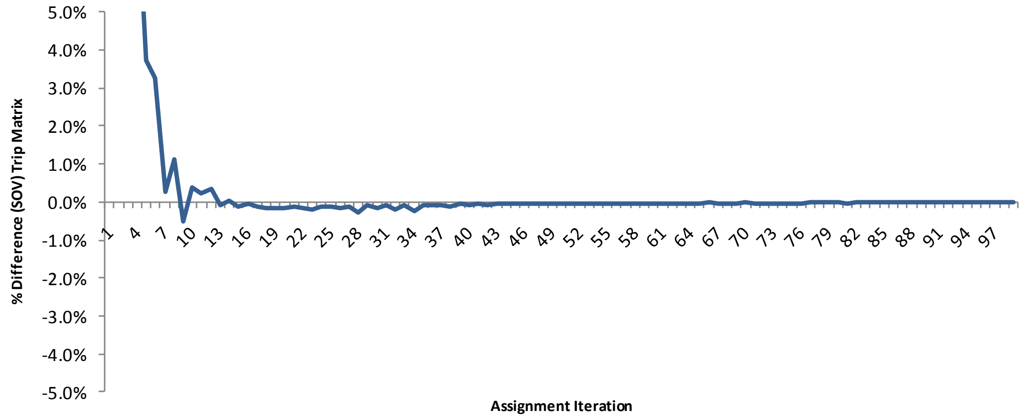


Figure 6.9 Percent Difference between Successive Estimated Toll-Eligible (SOV) Trip Matrices for the PT-Assignment Method.

Figure 6.9 illustrates the results of the toll-eligible matrix calculation over assignment iterations associated with the SOV demand matrix. In order to illustrate the converging pattern of the calculated matrices, the figure is configured to show the percent differences between the successive results of the matrix calculation in each assignment iteration. It is clearly shown that after some oscillations in toll-eligible matrices the calculated toll-eligible matrices reach a stable state and remain in this condition. The oscillations are decreased over the assignment iterations rather fast. This is largely due to performing the MSA method over calculated matrices. The variability within calculated matrices could be carried over to the estimated volumes (as discussed before in the link-based analysis) and could explain part of the extra oscillations within the estimated volumes in a PT- assignment routine over the (initial) assignment iterations.

Convergence in the estimated matrices over assignment iterations, illustrated in Figure 6.9, shows the convergence pattern consistent with the formulation discussed in Chapter 4.

$$g_i^{ct(l)} = (1-\varphi^{(l)}) * g_i^{ct(l-1)} + \varphi^{(l)} * \tilde{g}_i^{ct}\ ,\ i \in \mathrm{I},\ c \in \mathrm{C}$$

$$g_i^{cn(l)} = \mathrm{g}_\mathrm{i}^\mathrm{c} - g_i^{ct(l)},\ i \in \mathrm{I},\ c \in \mathrm{C}$$

$$0 \le \varphi^{(l)} \le 1$$

where g_i^{ct} and g_i^{cn} are toll-eligible and non-toll-eligible demands and $\varphi^{(l)}$ is estimated based on a MSA approach, $\varphi^{(l)} = \frac{1}{l}$, $l = iteration\,\#$.

In conclusion, the analyses presented above showed that a PT-assignment method such as the one utilized and discussed in this study can perform well within transportation assignment problems involving managed facilities. Despite of the ample performance of the PT-assignment in simulating link volumes in a desired fashion, it was necessary to confirm the credibility of the results by employing other well-known methods. The objective of the following portion of the research is to confirm and validate the final outcomes of the implemented pseudo-probabilistic toll-assignment program. The following sections also try to identify possible advantages of the PT-assignment method over other comparable methods that are known to perform well within the environment under study in this research. Therefore in addition to the PT-assignment method two other techniques were utilized to validate the results of the PT-assignment program. Below is a list of the added programs:

1. A conventional toll mode choice method for toll road assignment (referred to as MCTA in this text for the simplicity)
2. A standard user equilibrium assignment (UEA) program

6.3 Examination of the Probabilistic Toll - Assignment Model Results

As mentioned above, in this study a second technique for the toll-assignment was also implemented on the same project (I-20 ML project). This method which is, in some literatures, known as toll mode choice model [31] (referred to as MCTA within this text) was utilized to present a measure for the reasonableness of the results of the utilized PT-assignment program.

In the MCTA model setup, similar modeling assumptions and configurations were presumed and used. The goal of this comparison, however, was not to exactly match the outcomes within both methods. Instead, it was to observe comparable results within an acceptable range. The MCTA method is a set of discrete choice models coupled with a multiclass user equilibrium assignment (UEA) procedure that has been wildly used to solve the network assignment program involving toll roads. As discussed in Chapter 2, this approach is a well-known method that has been used by practitioners in many traffic and revenue projects. It uses an outer-loop mode choice (feedback) step to split the toll and non-toll trip tables and an inner-loop UEA step to assign these trip tables to the tolled and non-tolled links accordingly. The MCTA method uses this iterative mechanism to circulate the obtained information from one assignment procedure to another until a stable state is maintained.

The configuration and setup associated with a MCTA method itself could be a challenging task. Different setups in assignment step and feedbacks could result in the production of different outcomes with varying accuracy. On the other hand, using an extensive configuration would have costly consequences in terms of CPU time. Besides, as it will be shown, sometimes the MCTA method is inconsistent in producing results, especially in a congested model environment. Therefore, model accuracy/CPU time trade-off and lack of a criterion for overall convergence for this method are usually major concerns and typically cause a dilemma in the toll projects handled by a MCTA method. In this study, setting up a proper configuration for this method in order to layout a comparable platform became a cumbersome task as well. In practice, however, different techniques are used to expedite or manage the convergence for a MCTA method of which successive averaging over simulated tolled volumes as well as using a high number of assignment iterations or feedbacks could be mentioned.

While utilizing a MCTA method for the I-20 ML project, numerous trials were performed, and it was realized that the number of assignment iterations would be the best approach to expedite the convergence within the outcomes as well as to keep the consistency between both methods in terms of convergence criteria. Below is a list and brief description of selected cases for the MCTA method with different convergence criteria that have been considered in this study:

- The case with 20 assignment iterations and 20 feedbacks—

  The assumed convergence in this case occurred after the 8th feedback with a 0.77% (less than 1%) percent root mean square error compared to its next immediate feedback. A CPU time of 6 hours and 7 minutes was recorded for this convergence setup up to the point of assumed convergence.

- The case with 30 assignment iterations and 30 feedbacks—

  This case was assumed to converge after the 7th feedback with a 0.81% percent root mean square error compared to its next immediate feedback. A CPU time of 8 hours and 33 minutes was recorded for this convergence setup up to the point of assumed equilibrium.

- The case with 100 assignment iterations and 15 feedbacks—

  This case was assumed to converge after the 6th feedback with a 0.89% percent root mean square error compared to its next immediate feedback. A CPU time of 31 hours and 43 minutes was recorded for this convergence setup up.

As depicted in Figure 6.10, different cases regarding convergence assumptions resulted in different outcomes. An overall comparison of the final results between above-mentioned cases showed a difference of 2.18% (between case 20×20 and case 30×30), 3.30% (between case 30×30 and case 100×15) and 4.58% (between case 20×20 and case 100×15) in terms of %RMSE. Figure 6.10 illustrates the variations within the estimated tolled volume over the same selected tolled link using a MCTA method with described different convergence criteria. As shown in the figure, the case with 100 assignment iterations and 15 feedbacks showed the fewest oscillations within the outcomes among others and therefore, was selected for the purposes of the comparison and further analysis.

In practice, such a configuration could not be maintained due to a very high CPU time associated with it, and perhaps, other scenarios shown in Figure 6.10 could be used if a real traffic and revenue project is under study. In this study, however, this setup was selected to compare the results of both MTCA and PT-assignment methods, while both methods are extended to their highest performances.

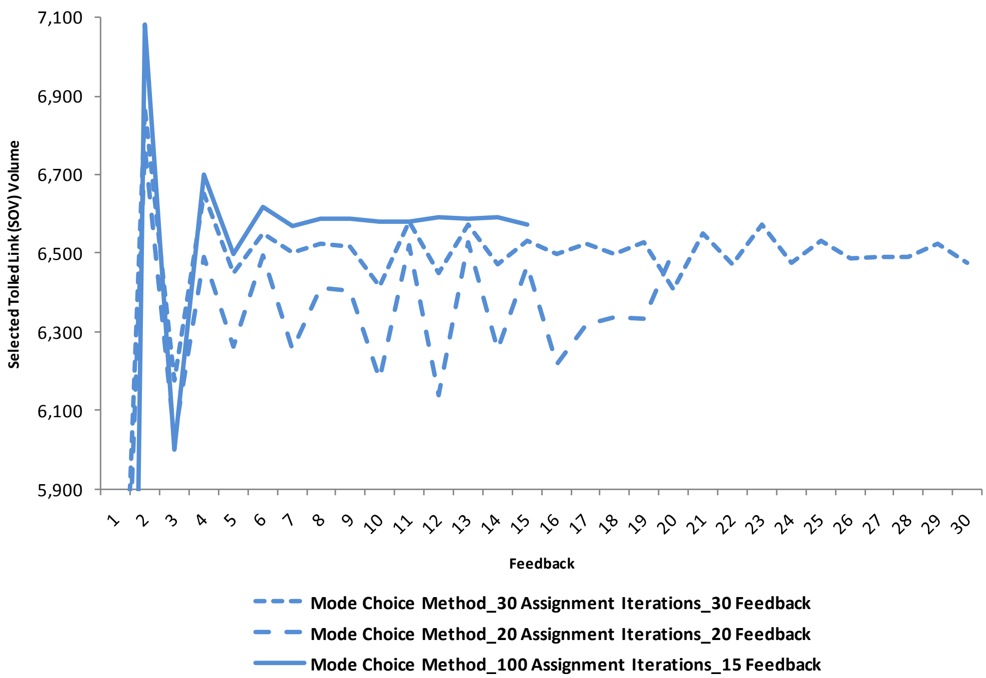


Figure 6.10 Comparison of Different Convergence Configurations for MCTA Method.

The same method of %RMSE explained in the prior chapter was selected to assess the outcomes of the utilized MCTA method in terms of overall convergence between successive feedbacks. Table 6.2 presents the %RMSE calculated for estimated toll volumes over all tolled links, between successive feedbacks. This table also shows the CPU time associated with the “100 assignment iterations and 15 feedbacks” scenario.

Table 6.2 Convergence Pattern over Feedbacks for the MCTA Method with 100 Assignment Iterations.

Feedback	CPU Time[1]	%RMSE
1	11:05:15 [2]	-
2	15:03:23	475.86%
3	19:13:28	21.45%
4	23:23:33	14.43%
5	27:33:38	3.85%
6	**31:43:43**	**2.16%**
7	**35:53:48**	**0.89%**
8	**40:03:53**	**0.32%**
9	44:13:58	0.34%
10	48:24:03	0.41%
11	52:34:08	0.35%
12	56:44:13	0.28%
13	60:54:18	0.33%
14	65:04:23	0.35%
15	69:14:28	0.39%

[1]Hours:Minutes:Seconds

[2] "Feedback 1" includes Warm-up Time

The table shows that after six feedbacks, the estimated tolled volumes became stable. This state of convergence is defined based on a 1% RMSE mark. This (optional) criterion was selected in this study to determine the acceptable convergence state for the results of the MTCA method in each feedback, when compared to those of previous feedback. Therefore, solutions associated with feedbacks 6, 7, and 8 were selected as candidates to be compared against the corresponding outcomes of the PT-assignment method.

Table 6.3 summarizes the comparison between results of selected feedbacks associated with the MCTA method and the same for Scenario 6 of the PT-assignment program. The estimated tolled volumes in both methods over all tolled links were compared using %RMSE, and as presented in Table 6.3, results of the 7th feedback of the MCTA method showed a better fit, among others, with respect to the results of “Scenario 6” of the PT-assignment program.

Table 6.3 Comparison of the Results of the MCTA and PT-Assignment Methods.

Mode Choice Feedback	PT-Assignment Scenario	%RMSE
6	6	2.94%
7	**6**	**2.34%**
8	6	2.47%

Furthermore, a linear regression method was used to closely compare the results of the two aforementioned scenarios (the 7th feedback of the MCTA method and the Scenario 6 of the PT-assignment program). The outcome of this comparison, as presented in Figure 6.11, showed a very tight correlation between the results produced by the two methods along the 45-degree line illustrated in the figure.

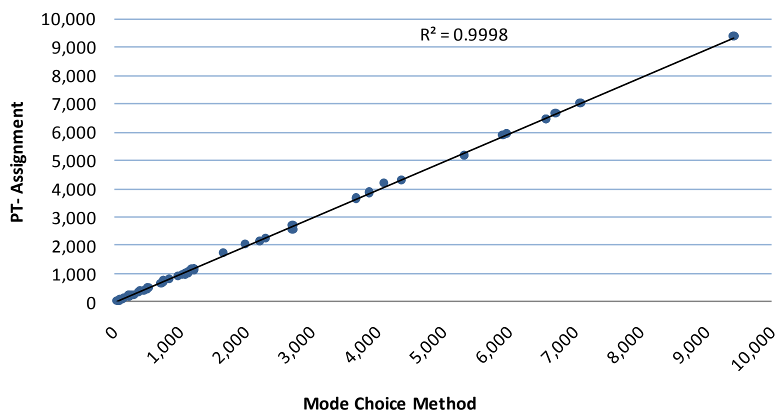


Figure 6.11 Comparison of the final Results of MCTA and PT-Assignment Methods.

In conclusion, based on the results of this study it is shown that a PT-assignment program can properly perform within the transportation models and projects involving tolled facilities. This model shows proper behavior in terms of convergence to the equilibrium solution and provides outcomes that are comparable to other leading methods. The objective of this study in using a MCTA method was to present a measure for the performance and accuracy of the PT-assignment program in all its aspects. A subsequent objective, however, led to more findings about the overall performance of the presented PT-assignment program in this research. Based on the results of this study when compared to a conventional MCTA method, the PT-assignment program showed better interconnectivity within the model components, which resulted in more consistency in the final outcomes. Moreover, the PT-assignment program offered more advantages in terms of CPU time, programming effort, and efficiency in a congested transportation network environment. Within the context of this study, the following sections attempt to elaborate on the aforementioned advantages of a PT-assignment method over a comparable MCTA technique. Following sections also address some additional capabilities of a PT-assignment method such as possible compatibilities with a "dynamic assignment" environment.

6.3.1 Performance, Efficiency and Consistency

In addition to the comparisons presented above and in order to further investigate the behavior of a PT-assignment routine, the volume assignment patterns of both methods were compared side-by-side. A conventional MCTA method uses a standard user equilibrium assignment routine for volume estimation utilizing a feedback process that results in producing outputs in an oscillating pattern as depicted in Figure 6.10. These oscillations, of course, have a minimum and maximum with a range that differs by using different convergence criteria (please see Figure 6.10). The final volume outcome for each link then desirably falls in the center of these maximum and minimum estimations. In order to observe the development of the results of a PT-assignment program (estimated tolled volumes) in relation to those of a MCTA method, a complete set of estimated tolled volumes over all assignment iterations associated with the two models were recorded. Figure 6.12 illustrates these recorded estimated volumes over the selected tolled link for all 100 assignment iterations. Figure 6.12 shows that throughout the procedure, over feedbacks and within each volume assignment iteration, the assumed final outcomes of the MCTA method (7^{th} feedback) falls between a maximum (second feedback) and minimum (third feedback) set of the estimated volumes (the starting point in this method was a free UEA and was not included in this comparison). The estimated tolled volumes resulting from the PT-assignment method started close to the lower limit (second feedback) and after a few iterations tended to grow to the higher limit (third feedback) and finally converged to the last assignment results in the 7th feedback of the MCTA method. This means the PT-assignment method converges faster to the optimum solution while the MCTA method bounces back and forth within maximum and minimum limits and slowly tries to converge to the optimum solution.

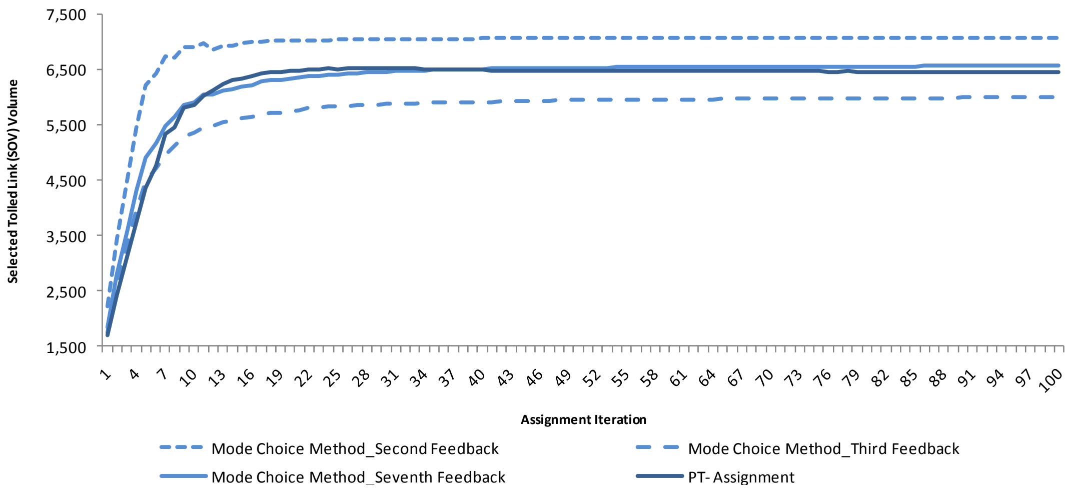


Figure 6.12 Development of the Selected Tolled Link Volumes for the PT-Assignment Program Compared to those of the MCTA Method.

Basically, the MCTA method tries to predict the potential toll road users in an outer-loop (feedback—toll-eligible matrix calculation by means of logit route choice models) step and assigns these predicted toll-eligible trips in a separate inner-loop step (volume assignment routine). In order to maintain consistency between the predicted and assigned trips, this method deals with the inter-connectivity of the two steps by providing previously mentioned remedies such as successive averaging over results of assignment (estimated volumes) or feedback (calculated toll-eligible trip tables) steps. In practice, on some occasions, the issues with interconnectivity between the steps of a MCTA model could be very difficult to deal with and may result in producing very inconsistent results. This issue will be further addressed in the following sections.

A PT-assignment method, on the other hand, improves efficiency by incorporating the route choice models within the assignment process and eliminating the need for the feedback step. This method also resolves any inconsistencies between predicted and assigned toll trips by utilizing a direct link between a toll-eligible matrix calculation (toll-trip prediction) and volume assignment routine (trip assignment). In addition, a successive averaging system over toll-eligible matrices within each assignment iteration eliminates the need for any further efforts to maintain the overall consistency and convergent behavior of this method.

In line with the evaluation of the performance of a PT-assignment program against a MCTA method, a third model which is, in this study, referred to as the "UE-free-assignment," was utilized by means of a standard UEA routine. This model helped to better understand the special characteristics of a PT-assignment routine when compared to a standard UEA procedure. This model also was the closest attempt to replicate the PT-assignment results by means of a MCTA method given the fact that (as described above) a MCTA method uses a standard UEA procedure in its inner-loop assignment step. The "UE-free-assignment" model used the resulting toll-eligible and non-toll-eligible trip matrices from the PT-assignment program as its inputs and performed a multiclass volume assignment over six vehicle classes of SOVs (toll and non-toll), HOVs (toll and non-toll), and TRKs (toll and non-toll). Similar modeling assumptions and configurations including the same convergence setup as Scenario 6 of the PT-assignment method were used in this model as well.

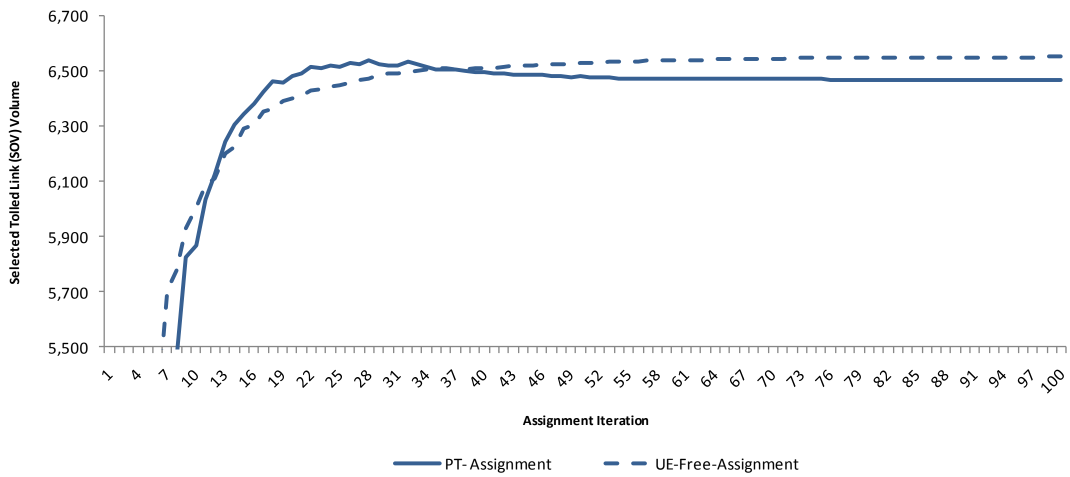


Figure 6.13 Development of the Selected Tolled Link Volume for the PT-Assignment Program Compared to the UE-Free-Assignment.

Figure 6.13 summarizes the estimated link flows over assignment iterations against those of the PT-assignment, associated with the SOV trips over the selected toll link. As expected though, there were differences between the results of the two model runs. An overall comparison of all tolled links showed a 9.7% difference in %RMSE. This result, although expected, could be partially due to the special method that PT-assignment employs to deal with the demand for tolled facilities. A PT-assignment constantly controls the assignment of the volumes over the tolled links by the travel time advantage over all alternatives (through volume delay functions). At the same time, it constantly filters the demand by users' value of time (logit route choice models). A standard UEA routine, such as the one employed in the UE-free-assignment model however, just assigns the demand according to the travel time advantage of the roads through volume delay functions.

Of course, working logit functions within the PT-assignment routine brought some variation in estimated volumes (as also depicted in Figure 6.13) which was due to the oscillations within resultant toll-eligible and non-toll eligible demands. This extra oscillation may have disturbed the development of the estimated volumes to the equilibrium solutions as illustrated in Figure 6.13. However, the embedded logit models did not convert the optimized assignment routine (utilized with a F-W method) to a non-convergent procedure. Instead, the variations ultimately stabilized and the iterative nature of the assignment procedure made the results converge to the equilibrium solutions.

A further detailed comparison between the aforementioned scenarios is not in the scope of this study and can be considered as a topic for future studies. This brief analysis, however, might suggest the insufficient performance of a standard UEA to develop solutions for tolled links since the expectation of limited use of tolled links is masked by the (high) demand in the project corridor. Therefore, given that a MCTA method uses a similar approach to assign the predicted toll trips to a (tolled) network, this deficiency of performance especially within a highly congested network may overshadow capabilities of this method in a toll-assignment problem. Therefore, the following section is developed and tailored to better investigate the performances of MCTA and PT-assignment methods within a highly congested environment.

6.3.2 Performance within a Highly Congested Transportation Network

This section shows both the incapability of a MCTA method to produce consistent results and measures the performance of a PT-assignment program in an extremely congested transportation model environment. Although, as discussed in previous sections, the MCTA method showed acceptable performance in a well-behaved model environment, the test described in this section shows that it performs poorly in a highly congested model. Figure 6.14 compares the estimated tolled volumes using both the PT-assignment and the MCTA methods over the same selected toll link (and SOV trips) in an artificially congested network that was tailored for this scenario. This figure was configured to show the percent differences between successive outcomes of the assignment iterations or feedbacks. In this scenario, trip tables for the design forecast year of 2030 were assigned to the base year (2005) network within the AM peak period. The result has become a highly congested network due to the absence of the additional capacity associated with future year (2030) transportation project plans as well as the growth in demand. A 20-iteration PT-assignment routine and a 20-assignment iteration $\times$ 20-assignment feedback MCTA program were used to generate the results for this scenario. This configuration for the MCTA was mainly used to expand the oscillations in this artificially congested network to better show the incapability of a MCTA method to perform reasonably under such congestion level.

The never ending oscillations in Figure 6.14 clearly show the incapability of the MCTA method to converge toward the equilibrium solution in this congested network environment, whereas the PT-assignment method showed a converging pattern. Basically, when extreme congestion exists, the lack of interconnectivity between the outer-loop and the inner-loop steps in a MCTA impairs the functionality of the logit models. Then, (successive) high and low toll-matrices within the matrix calculation procedure (outer-loop step) are built. This effect in turn makes the (inner-loop) volume assignment step keep producing oscillating results over feedbacks.

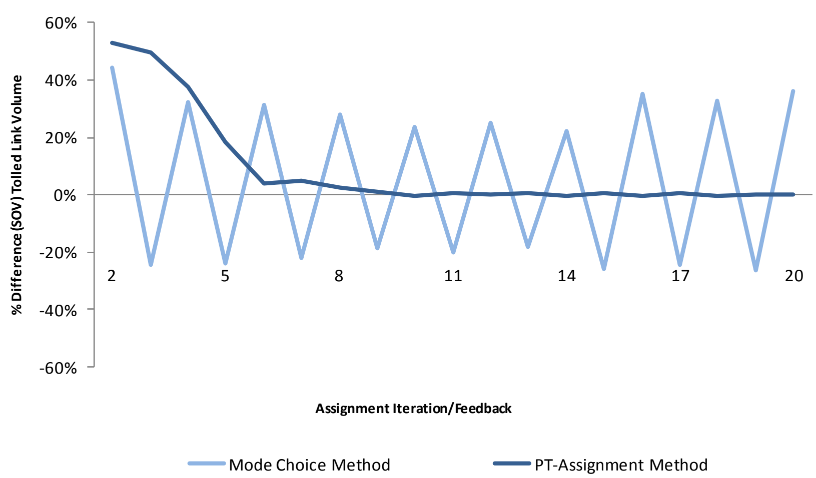


Figure 6.14 Volume Pattern over the Selected Tolled Link for the PT-Assignment Program Compared to the MCTA Method in the Case of Extreme Congestion.

However, if a system has a significant degree of congestion, it may be difficult (practically impossible) to reach a true state of equilibrium for any volume assignment routine. This could even be the case for the example used in this section. The low number of assignment iterations and consequently immature assignment procedures in the MCTA method may add to the non-convergent behavior of the model in this example. Similarly, it would be expected that at some degree of elevated congestion, similar to any other standard routine, a PT-assignment method might also fail to converge. Nevertheless, the results of this model setup in this illustration did not show such a non-convergent state for the PT-assignment model.

6.3.3 Model Run Time and Programming Effort

Embedded logit models (matrix calculation process) within the assignment routine in a PT-assignment program have advantages and disadvantages in CPU time when compared to the two-step (inner-loop/outer-loop) configuration of a MCTA method. A PT-assignment program takes advantage of using just one assignment routine in its entirety. The disadvantage of the PT-assignment method in terms of CPU time is in its excessive matrix calculations, which are as many as the assignment iterations. This may work to the benefit of this method by creating a more convergent pattern along the volume assignment procedure. Nevertheless, it definitely costs more CPU time. The previously utilized UE-free-assignment model is the best example of a rough measure of the CPU cost associated with the matrix calculations within a PT-assignment method. As discussed previously, the UE-free-assignment model performed a standard UEA routine while adopting similar assumptions and configurations of the PT-assignment. This model included all the inputs and excluded the embedded logit models and related matrix calculation block. Therefore, the difference in CPU time between these two models was a good estimate of the CPU time associated with the (toll-eligible and non-toll-eligible) matrix calculations within a PT-assignment method. After comparing the CPU time for both models, it was realized that with the same computational performance (Intel Dual Core @

2.53 GHz processor and 3 GB RAM) the PT-assignment (as configured in this study) performed 1.65 times (65%) slower than a standard UEA. This figure was considered very promising within the context of a toll-assignment project and when compared to the same for the MCTA method. In fact, the model utilized with the MCTA method (7^{th} feedback) performed 5.6 slower than the PT-assignment method (scenario 6), as shown in Table 6.1 and Table 6.2.

As previously mentioned, the MCTA case used 100 assignment iterations and 7 feedbacks, which translates to 7 matrix-calculation procedures and 8 volume assignment routines (the MCTA method used an additional warm-up volume assignment procedure). Considering that an assignment procedure is much costlier than a corresponding matrix-calculation program, a PT-assignment program is much more efficient in terms of CPU time. To sum up, based on the analysis presented in this study, it is evident that a PT-assignment program performs much faster than a corresponding MCTA method.

The programming effort was also explicitly lower in a PT-assignment approach. Because this approach combines several programs in one assignment routine, the coding of the entire model involved fewer programming tasks. In addition, the PT-assignment method largely saved memory space by eliminating a large number of middle-part temporary files that are usually needed to transfer information between different steps within a MCTA method. Figure 6.15 compares two modeling environments and clearly illustrates the differences between the numbers of programs used within each method. As shown in the figure, there are noticeably more programs within a MCTA method (on the right) compare to a PT-assignment program (on the left). This, of course, could be translated into more programming time and effort associated with a MCTA method as well as extensive memory space requirements to save temporary files to transfer data between outer/inner loops.

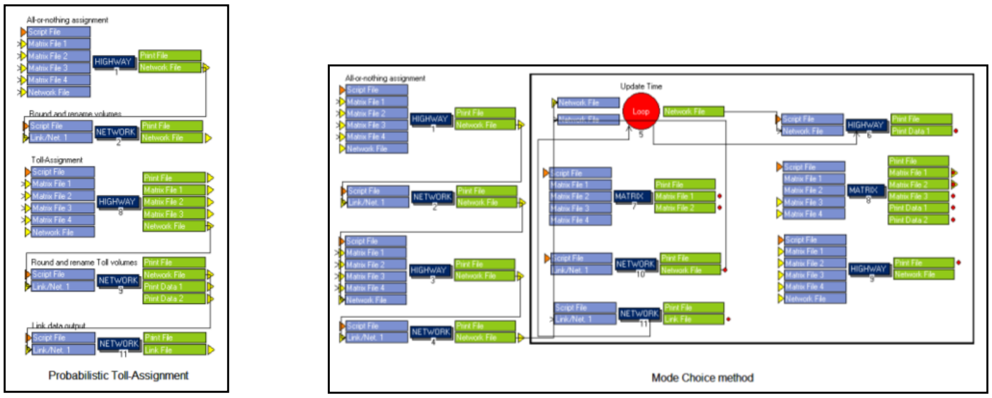


Figure 6.15 Comparison of the Modeling Environment between the PT-Assignment (left) and the MCTA Methods (right).

It should be again noted that it is not the intention of this study to discard the abilities and functionalities of a conventional MCTA method, as this method has been used by many public and private transportation entities, as discussed in Chapter 2, and is known to be well-suited within toll road assignment problems. The main purpose of utilizing a MCTA method in this study was to provide a measure for the accuracy and performance of a PT-assignment program.

The analyses presented in this chapter showed that components and elements of the PT-assignment method functioned desirably. Also, the outcomes and results illustrated a reasonable converging pattern to the equilibrium solutions. Moreover simulated tolled link volumes were found similar to those of other methods. A summary of the findings of this study along with directions for further studies in regard with potential improvements to a PT-assignment program is presented in the next chapter. Chapter 7 also includes discussions regarding possible applications of other probabilistic toll-assignment methods for transportation problems involving managed facilities.

CHAPTER 7

CONCLUSIONS AND FUTURE DIRECTIONS

Throughout previous chapters, this study examined the implementation of a pseudo-probabilistic toll-assignment program for estimation of the toll traffic for a selected large scale toll project. The results of the study showed the performance of this method in producing the tolled volumes associated with the large scale I-20 toll project located in Atlanta, GA. Development of the link volumes to the equilibrium solutions illustrated the converging pattern of the algorithm and formulations presented in Chapter 4. This algorithm has been used in numerous applications in Europe, North America, and Asia. Most of these applications are confidential, and the results may not be reported in academic papers [40]. There is also a brief description of this approach that was carried out on the large-scale network used for transportation planning in Southern California [40].

This chapter has two sections. The first section summarizes the findings of the research including the advantages of the PT-Assignment over a MTAC method. The second section suggests a number of topics for further related researches.

7.1 Conclusions

The presented discussion in this section summarizes the overall results of this research. It includes the findings of this study related to utilizing a PT-assignment method for a large-scale transportation network involving managed facilities. This section also lists the advantages of a PT-assignment program over other well-known methods that are widely used in similar traffic and revenue studies. Moreover, this section suggests some improvements to the

PT-assignment program utilized in this study. Based on the results of this study, the findings and conclusions can be listed as follow:

- The utilized PT-assignment program was found to be capable of properly performing within large-scale transportation projects involving tolled facilities. The proper performance of this method was concluded based on the demonstration of a suitable converging pattern to the equilibrium solution by this method, within the selected large-scale transportation network toll-volume assignment.
- The estimated volumes by the PT-assignment method were similar to those of other methods. In this study, a toll mode choice model (referred to as MCTA method within this text, for simplicity) was used to measure the reasonableness of the results of the utilized PT-assignment program. The simulated volumes utilizing the two methods, assuming the same modeling inputs and assumptions, showed a difference of 2.34% RMSE over all estimated tolled volumes.
- Convergence to the equilibrium state within a PT-assignment program can be defined and determined by commonly used convergence criteria similar to a standard user equilibrium assignment routine. The determination of an overall convergence state in a MTCA method however, can sometimes be complicated and subject to extra analysis.
- There are inconsistencies within a MTCA method. These inconsistencies are created by interconnectivity issues between outer-loop/feedback step and inner-loop/assignment step. This interconnectivity issues cause extra variations within a MTCA routine and make the overall convergence more difficult. A PT-assignment method, by incorporating the route choice models within the assignment process, eliminates the need for the feedback step and therefore, resolves the problem with extra variations caused by the feedback step.

- The limited number of calculations for toll-eligible demand (feedbacks) within a MTCA method can lead to overestimation of the tolled link flows. Low involvement of the route choice functions within a MTCA method makes the final results deviate from the latest estimation of the toll-eligible demand. Within this study, all comparable scenarios utilized by means of the MTCA method, showed a higher estimated flows for almost all tolled links when compared to those of the PT-assignment method. This comparison suggests that in a MTCA method, the performance of the route choice functions in the feedback step can be suppressed within the assignment step. The PT-assignment method, on the other hand, by revisiting the route choice functions in each assignment iteration, creates the outcomes in line with the latest estimation of the toll-eligible demand.
- In the presence of extreme congestion within a transportation network, the lack of interconnectivity between the outer-loop and the inner-loop steps in a MCTA method impairs the functionality of the logit models. In this study, the utilized MCTA method failed to function in a very congested network whereas the PT-assignment method showed a proper converging pattern within the same congested condition. Therefore, the PT-assignment method is a better method for transportation projects involving congested corridors when compared to a MTCA method.
- The PT-assignment program offered more advantages in terms of CPU time when compared to a MTCA method. The PT-assignment (as configured in this study) performed 5.6 times faster than the equally configured MTCA method. The PT-assignment model performed just 1.65 times slower than a similar standard (free) user equilibrium assignment (UEA). This extra run time within a PT-assignment routine (compared to a free UEA) can be associated to the extra calculations related to the embedded route choice functions.

- Compared to a MTCA method, the programming effort was also explicitly lower in a PT-assignment approach. Given that this approach combines several programs in one assignment routine, the coding of the entire model involves fewer programming tasks. In addition, this method largely saves memory space by eliminating a large number of middle-part temporary files that are needed to transfer information between different steps in a MCTA method.
- Embedded logit models within the assignment routine in a PT-assignment program also have a disadvantage in terms of CPU time. Excessive matrix calculations, which are as many as the assignment iterations, work to the benefit of this method by creating a more convergent pattern along the volume assignment procedure. Nevertheless, it definitely costs more CPU time. However, within this method, a simple revised model setup can be considered to optimize the use of the route choice functions. This revised setup can utilize the route choice functions less frequently by employing a simple function. For example, this simple function could identify odd (or even) iteration numbers in order to enable the route choice functions. Thus, the number of calculations for toll-eligible and non-toll-eligible demands can reduced to half and therefore, a considerable amount of CPU time can be saved.

Toll-assignment procedures are mostly developed by practitioners for implementation within actual large-scale projects. As such, due to confidentiality, there is little information in the literature about the core of these large-scale toll diversion models and almost no algorithms or any clear explanations of the procedure. The PT-Assignment program presented in this study, although proven to perform properly within the context of transportation assignment problems involving managed facilities, is still subject to many improvements. The following sections discuss some possible improvements to the pseudo-probabilistic toll-assignment program in the form of future studies. Some suggestions for future directions regarding a PT-assignment program are also included in the sections below.

7.2 Proposals for Future Studies

Formulations and algorithms that have been found throughout this research needed further developments and elaborations. The topics listed below are provided to point out some issues which require improvements within the context of a “pseudo-probabilistic toll assignment routines”.

7.2.1 Altering Convergence Approach

Termination of the assignment procedure, by reaching the desired convergence criteria, proved that the estimated flows approximately satisfy the model formulation [40] and provide the equilibrium solutions. However, other methods that can replace the current convergence method within the presented PT-assignment program could also be explored. The formulation presented in this study for a PT-assignment program employs an optimization method for assignment convergence utilizing a F-W approach. This section initiates the discussion of utilizing other convergence approaches for a PT-assignment program.

An iterative approach instead of optimization methods was the first technique examined in this study. In an iterative assignment algorithm approach, the “current” flow on a link is calculated by a linear combination of the current flow of the previous assignment iteration and an auxiliary flow resulting from an all-or-nothing assignment in the present iteration, as illustrated below:

$$v_a^{(l)} = (1- \varphi^{(l)}) * v_a^{(l-1)} + \varphi^{(l)} * y_a^{(l)}$$

Where $y_a^{(l)}$ is the auxiliary flow, l is the assignment iteration number, and φ is the fixed weight given to auxiliary flows within each assignment iteration [33].

Iterative assignment algorithms differ in the method used to give a value to φ. A well-known approach is to make $\varphi^l = 1/l$. In this case, equal weight is given to auxiliary flow, and for this reason, the formulation is also known as a method of successive averages (MSA) [33]. Making $\varphi^l = 1/l$ has been shown to produce solutions convergent to Wardrop's equilibrium [43]. This method is of course not an optimization approach and therefore is not very efficient.

The result of the MSA method employed over the same I-20 Atlanta, GA project with the same modeling assumptions is summarized in the figure below. As expected, the MSA results converged to the equilibrium solution more slowly. Therefore, a higher number of assignment iterations were needed to replicate the same outcomes of the F-W approach. However, utilizing a MSA approach with a different value for φ can alter the performance of this method by expediting the convergence pattern. For example, a different fixed weight of $\varphi^l = 1/(1 + l)$, referred to as MSA-D [41], was employed, within another model run. The results of the MSA-D toll assignment are demonstrated over the same selected tolled link in Figure 7.1.

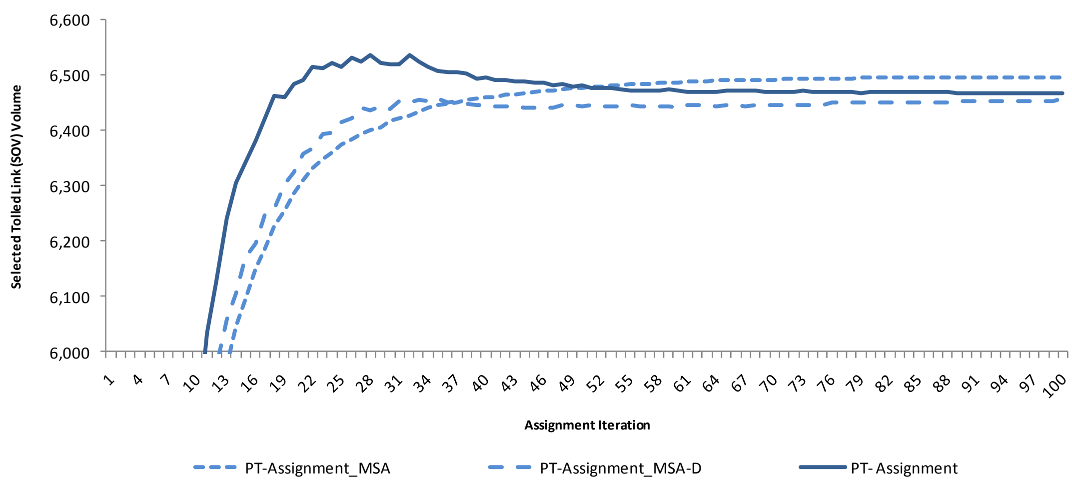


Figure 7.1 Comparison of Link Flows for MSA, MSA-D and PT-Assignment methods over Selected Tolled Link

Figure 7.1 illustrates the results of all three convergence methods that were utilized in this section. As shown in the figure, all methods approximately resulted in similar outcomes. The MSA-D method showed closer results after 100 fixed assignment iterations which were assumed for all three methods. Comparisons of the %RMSE also showed a tight correlation between the results produced by MSA and MSA-D with the outcomes of the PT-assignment method over all tolled links. Table 7.1 illustrates dilates of the assignment assumptions and convergence results for MSA and MSA-D methods when compared to those of the PT-assignment. The table shows that with the same number of assignment iterations, the MSA-D method produced better results and converged faster (smaller *relative gaps*) when compared to the MSA method, although both methods showed acceptable proximity to the PT-assignment results in an overall comparison. MSA and MSA-D methods also demonstrated acceptable proximity to the MCTA method by %RMSEs of 2.30% and 2.56%, respectively.

Table 7.1 Comparison of the Overall Results for MSA, MSA-D and PT-Assignment Methods

Convergence Method	Gap	Relative Gap	# of assignment iterations	CPU Time[1]	%RMSE
PT-assignment	0.000056	0.00046	100	6:37:38	-
MSA	0.000037	0.00186	100	6:21:56	0.66%
MSA-D	0.000021	0.00056	100	6:17:43	0.37%

[1]Hours:Minutes:Seconds

As shown in the Table 7.1, all three toll-assignment methods used almost similar CPU time. However it is possible to reduce the CPU time in all these methods. One way is to save in the matrix calculation process, meaning less CPU time usage could be achieved by using less matrix calculations. Currently, a free-flow network has been used for the starting point of the matrix calculations. However, calculation of toll-eligible and non-toll-eligible matrices could be started after a few first (free) assignment iterations, resulting in a lower number of matrix calculations and consequently less CPU time.

The brief analysis presented above suggested the use of other methods for PT-assignment convergence approach. Further investigation in this regard could be a subject for future studies. However, the limited examinations presented above were sufficient to trigger the thoughts of further investigations about functionality of a toll-assignment routine with embedded route choice models with more sophisticated assignment routines such as “dynamic” and “stochastic user equilibrium” assignment routines. The section below briefly explores the possible application of a PT-Assignment program within a dynamic assignment routine as a future research direction.

7.2.2 Dynamic Assignment and Congestion Pricing

Basically, solving transportation problems involving toll facilities with the utilization of one uniform assignment routine enables us (when compared to a MCTA method) to take advantage of more advanced assignment routines such as the “dynamic” assignment. The “dynamic” assignment routine deals with the assignment problem by dividing the modeling period into shorter time intervals, typically a few minutes. Each time interval is then treated as a static (steady-state) assignment problem. This captures some of the effects of the congestion. However, it assumes that all vehicles in the same time interval are faced with the same set of costs. This model assigns vehicles individually or in small groups (or packets) and releases them sequentially throughout each time slice. This approach provides a better representation of the delays facing vehicles at each stage in their passage through the network [33]. Capacity restrain are strictly enforced using “flow gates” within a “dynamic” assignment procedure.

A static assignment deals with the demand and path choices associated with each i-j pair, and through assignment procedure and within network paths, highly congested links might be produced. However, routes and flow rates change during each time slice within a “dynamic” assignment routine. The simulation of queues affects preceding link volume and stands to generate more realistic results about network operation issues such as queue formations and link blockages.

Shorter time periods, and therefore much higher number of path building, associated with this method enables it to better perform in utilizing the travel time saving advantages that are provided by a link (or set of links). This is expected to better present the effects of a managed lane facility within a network, especially when combined with variable tolling (congestion pricing). Overall, a “dynamic” assignment utilized with embedded route choice models can make a perfect tool to approach the dynamic nature of projects dealing with managed lanes and “dynamic pricing.” A detailed investigation of “dynamic” assignments and their possible applications to a toll-assignment problem, however, is not in the scope of this study and could be further examined in future studies.

7.2.3 Stochastic User Equilibrium

In stochastic user equilibrium (SUE) assignment case, a spread of routes between two points is produced because of variability in the perceived routes costs and the capacity-restraint effects. The SUE models seek an equilibrium condition where [33]:

> *“Each user chooses the route with the minimum perceived travel cost; in other words, under SUE, no user has a route with lower perceived costs and therefore all stay with their current routes.”*

The difference between stochastic and Wardrop's user equilibrium is that in SUE models, each driver is meant to define “travel costs” individually instead of using a single definition of cost applicable to all drivers. This different perception in travel cost by travelers is the basis of a stochastic approach to transportation assignment problems. This concept could also fit very well within the notion of this study which is a separation of the roadway users based on the cost (and time savings) of alternative facilities (tolled or non-tolled) to traverse the same origin-destination, due to differences in users’ VOT. Therefore, SUE models could potentially be applicable to the transportation problems involving managed facilities.

In this study, these problems were dealt with utilizing a UE toll-assignment routine with common techniques such as link prohibitions and a multiclass assignment approach. In contrast, SUE assignment routines emphasize the variability in drivers’ perception of costs and seek the second-best route between each i-j pair. These models use a stochastic assignment technique, such as Burrel’s, which utilizes a Monte Carlo simulation to represent the variability in drivers’ perceptions of link costs. It is shown that these models converge to a SUE solution, albeit slowly, using a MSA method [33]. However, detailed information about SUE models is not provided in this study. The possible application of these models within transportation problems including managed facilities is suggested as a topic for future studies due to conceptual similarities. A brief literature review on this subject suggested that there is some existing research regarding this topic. These studies try to fit a diversion curve (% use of toll road vs. alternative route as a function of the toll free) resulting from a stated preference survey within a SUE assignment routine by adjusting a “spread parameter” accordingly [42].

APPENDIX A

CUBE-PTA_ ESTIMATED TOLLED LINK VOLUMES

A	B	DISTANCE	V_SOVAM	V_HOVAM	V_COMAM	V_TotalAM
2933	12428	0.007	420	64	92	576
4007	15751	0.007	2043	219	80	2342
4028	15790	0.007	139	30	46	215
4521	10738	0.206	5180	507	275	5962
5897	16477	0.012	3870	355	270	4495
9677	15776	0.110	735	144	166	1045
9678	15766	0.440	6466	615	268	7349
10384	12004	1.725	9383	908	575	10866
10385	12001	0.225	904	132	214	1250
10385	12002	0.105	242	39	34	315
10720	59040	0.980	481	81	116	678
10722	12003	0.110	1146	171	248	1565
10725	59049	1.071	999	152	231	1382
10727	16440	0.834	1043	158	234	1435
10729	59046	0.847	5939	514	382	6835
10732	12000	1.071	7035	646	445	8126
10733	10734	0.036	9383	908	575	10866
10734	10735	0.352	5180	507	275	5962
10734	59036	0.094	4204	400	300	4904
10738	15747	0.982	5180	507	275	5962
12000	10384	0.223	6672	585	402	7659
12000	12002	0.121	363	61	43	467
12001	10723	0.477	999	152	231	1382
12002	10384	0.103	2711	322	173	3206
12002	12001	0.121	95	20	17	132
12003	10385	1.725	1146	171	248	1565
12004	10733	0.103	9383	908	575	10866
12189	12428	0.024	80	27	19	126
12430	59042	0.216	500	91	111	702
12431	15771	0.040	735	144	166	1045
12432	59043	0.153	6466	615	268	7349
12436	12437	0.217	4305	359	174	4838
12437	2728	0.007	1727	169	125	2021
12437	12189	0.024	2578	190	49	2817
12773	12774	0.133	806	115	173	1094
12773	59050	0.069	237	43	62	342
12774	16442	0.961	1178	179	206	1563

12787	12788	0.133	3670	316	244	4230
12787	59050	0.065	200	40	26	266
12788	16482	0.272	5939	514	382	6835
13539	15753	0.190	6466	615	268	7349
13540	13552	0.310	735	144	166	1045
13551	13539	0.300	6466	615	268	7349
13552	15789	0.220	735	144	166	1045
15548	59053	0.099	235	53	55	343
15750	15751	0.700	5180	507	275	5962
15751	4007	0.007	757	111	88	956
15770	15770	1.600	6466	615	268	7349
15789	15790	1.600	735	144	166	1045
15790	4028	0.007	394	93	96	583
15794	15794	0.700	481	81	116	678
16441	16441	0.268	1043	158	234	1435
16446	16446	0.887	1178	179	206	1563
16481	16481	0.968	3870	355	270	4495
16483	16483	0.303	5939	514	382	6835
59039	59040	0.061	665	90	132	887
59040	10721	0.096	1146	171	248	1565
59042	12431	0.150	735	144	166	1045
59043	12435	0.054	4305	359	174	4838
59043	59060	0.015	2161	256	94	2511
59045	59047	0.070	87	17	20	124
59045	59048	0.063	1152	148	83	1383
59046	59045	0.071	56	15	19	90
59046	59048	0.133	5884	499	362	6745
59047	10726	0.709	1043	158	234	1435
59048	10730	0.177	7035	646	445	8126
59049	59045	0.063	44	11	17	72
59049	59047	0.132	956	141	214	1311
59050	12774	0.065	372	64	33	469
59050	12788	0.068	2269	198	137	2604
59058	59065	0.071	235	53	55	343
59064	59067	0.201	2161	256	94	2511
59065	59068	0.021	235	53	55	343
59067	59066	0.055	2161	256	94	2511
59069	59069	0.025	235	53	55	343

A	B	V_SOV_it01	V_SOV_it02	V_SOV_it03	V_SOV_it04	V_SOV_it05
2933	12428	392	315	304	325	338
4007	15751	1262	1257	1414	1546	1662
4028	15790	62	119	121	120	123
4521	10738	1293	1952	2484	2948	3425
5897	16477	2842	2251	2272	2405	2597
9677	15776	443	496	512	533	555
9678	15766	1699	2403	3129	3741	4367
10384	12004	2994	3299	3796	4362	5140
10385	12001	67	162	256	331	418
10385	12002	243	241	256	255	268
10720	59040	102	227	267	301	342
10722	12003	309	403	512	586	686
10725	59049	119	240	350	437	540
10727	16440	171	289	396	482	584
10729	59046	2728	2832	3042	3344	3758
10732	12000	2862	3061	3323	3711	4250
10733	10734	2994	3299	3796	4362	5140
10734	10735	1293	1952	2484	2948	3425
10734	59036	1701	1347	1312	1415	1715
10738	15747	1293	1952	2484	2948	3425
12000	10384	2815	2945	3179	3578	4120
12000	12002	47	116	144	133	130
12001	10723	119	240	350	437	540
12002	10384	179	354	617	785	1020
12002	12001	52	78	94	106	122
12003	10385	309	403	512	586	686
12004	10733	2994	3299	3796	4362	5140
12189	12428	36	152	165	150	133
12430	59042	428	467	469	474	470
12431	15771	443	496	512	533	555
12432	59043	1699	2403	3129	3741	4367
12436	12437	1633	2025	2405	3069	3214
12437	2728	801	844	969	1076	1212
12437	12189	832	1181	1436	1993	2002
12773	12774	89	179	264	331	412
12773	59050	83	110	133	151	172
12774	16442	401	478	562	631	717

12787	12788	2595	2056	2085	2218	2417
12787	59050	247	196	187	187	180
12788	16482	2728	2832	3042	3344	3758
13539	15753	1699	2403	3129	3741	4367
13540	13552	443	496	512	533	555
13551	13539	1699	2403	3129	3741	4367
13552	15789	443	496	512	533	555
15548	59053	15	29	42	59	85
15750	15751	1293	1952	2484	2948	3425
15751	4007	856	806	769	752	720
15770	15770	1699	2403	3129	3741	4367
15789	15790	443	496	512	533	555
15790	4028	404	388	366	352	337
15794	15794	102	227	267	301	342
16441	16441	171	289	396	482	584
16446	16446	401	478	562	631	717
16481	16481	2842	2251	2272	2405	2597
16483	16483	2728	2832	3042	3344	3758
59039	59040	208	175	245	285	344
59040	10721	309	403	512	586	686
59042	12431	443	496	512	533	555
59043	12435	1633	2025	2405	3069	3214
59043	59060	66	378	723	672	1152
59045	59047	82	84	83	83	83
59045	59048	172	267	325	428	572
59046	59045	38	38	44	62	80
59046	59048	2690	2795	2998	3282	3678
59047	10726	171	289	396	482	584
59048	10730	2862	3061	3323	3711	4250
59049	59045	29	34	36	38	39
59049	59047	90	205	313	400	501
59050	12774	312	298	298	300	306
59050	12788	133	777	958	1126	1341
59058	59065	15	29	42	59	85
59064	59067	66	378	723	672	1152
59065	59068	15	29	42	59	85
59067	59066	66	378	723	672	1152
59069	59069	15	29	42	59	85

A	B	V_SOV_it06	V_SOV_it07	V_SOV_it08	V_SOV_it09	V_SOV_it10
2933	12428	348	357	368	375	377
4007	15751	1711	1833	1829	1869	1859
4028	15790	123	125	125	129	129
4521	10738	3775	4205	4340	4660	4714
5897	16477	2742	2949	3017	3185	3222
9677	15776	576	600	610	627	633
9678	15766	4777	5343	5457	5824	5868
10384	12004	5701	6478	6692	7314	7419
10385	12001	479	559	585	651	665
10385	12002	272	261	259	252	250
10720	59040	363	390	399	418	422
10722	12003	751	820	843	903	915
10725	59049	611	700	724	789	801
10727	16440	655	748	772	836	847
10729	59046	4062	4456	4570	4889	4955
10732	12000	4621	5104	5245	5638	5719
10733	10734	5701	6478	6692	7314	7419
10734	10735	3775	4205	4340	4660	4714
10734	59036	1926	2273	2352	2653	2706
10738	15747	3775	4205	4340	4660	4714
12000	10384	4474	4924	5054	5416	5492
12000	12002	148	181	190	222	226
12001	10723	611	700	724	789	801
12002	10384	1227	1554	1638	1898	1927
12002	12001	132	141	139	138	136
12003	10385	751	820	843	903	915
12004	10733	5701	6478	6692	7314	7419
12189	12428	122	108	105	97	96
12430	59042	470	465	473	472	473
12431	15771	576	600	610	627	633
12432	59043	4777	5343	5457	5824	5868
12436	12437	3469	3606	3783	3905	3869
12437	2728	1313	1427	1472	1549	1567
12437	12189	2156	2180	2311	2355	2303
12773	12774	468	541	561	611	621
12773	59050	187	207	211	225	226
12774	16442	779	860	881	937	949

12787	12788	2560	2767	2828	3000	3034
12787	59050	182	182	189	184	188
12788	16482	4062	4456	4570	4889	4955
13539	15753	4777	5343	5457	5824	5868
13540	13552	576	600	610	627	633
13551	13539	4777	5343	5457	5824	5868
13552	15789	576	600	610	627	633
15548	59053	106	135	138	155	159
15750	15751	3775	4205	4340	4660	4714
15751	4007	710	695	712	706	705
15770	15770	4777	5343	5457	5824	5868
15789	15790	576	600	610	627	633
15790	4028	335	335	337	338	340
15794	15794	363	390	399	418	422
16441	16441	655	748	772	836	847
16446	16446	779	860	881	937	949
16481	16481	2742	2949	3017	3185	3222
16483	16483	4062	4456	4570	4889	4955
59039	59040	388	430	445	484	493
59040	10721	751	820	843	903	915
59042	12431	576	600	610	627	633
59043	12435	3469	3606	3783	3905	3869
59043	59060	1308	1737	1674	1919	1999
59045	59047	83	89	88	88	88
59045	59048	635	719	744	816	829
59046	59045	76	71	70	66	66
59046	59048	3986	4385	4501	4823	4889
59047	10726	655	748	772	836	847
59048	10730	4621	5104	5245	5638	5719
59049	59045	40	41	41	41	41
59049	59047	572	659	683	748	759
59050	12774	311	318	321	326	328
59050	12788	1502	1689	1743	1889	1921
59058	59065	106	135	138	155	159
59064	59067	1308	1737	1674	1919	1999
59065	59068	106	135	138	155	159
59067	59066	1308	1737	1674	1919	1999
59069	59069	106	135	138	155	159

A	B	V_SOV_it11	V_SOV_it12	V_SOV_it13	V_SOV_it14	V_SOV_it15
2933	12428	381	384	390	390	395
4007	15751	1890	1896	1914	1943	1952
4028	15790	130	131	130	131	131
4521	10738	4850	4935	5038	5072	5095
5897	16477	3325	3373	3439	3476	3531
9677	15776	644	649	660	664	671
9678	15766	6033	6127	6242	6306	6342
10384	12004	7758	7920	8159	8257	8424
10385	12001	699	716	739	751	773
10385	12002	248	246	244	243	243
10720	59040	430	434	439	442	447
10722	12003	947	962	983	994	1015
10725	59049	830	845	867	881	900
10727	16440	876	891	912	926	945
10729	59046	5147	5238	5355	5419	5510
10732	12000	5955	6068	6215	6296	6411
10733	10734	7758	7920	8159	8257	8424
10734	10735	4850	4935	5038	5072	5095
10734	59036	2908	2986	3121	3184	3328
10738	15747	4850	4935	5038	5072	5095
12000	10384	5714	5814	5955	6027	6130
12000	12002	241	254	259	268	280
12001	10723	830	845	867	881	900
12002	10384	2043	2106	2204	2229	2293
12002	12001	131	129	128	130	127
12003	10385	947	962	983	994	1015
12004	10733	7758	7920	8159	8257	8424
12189	12428	93	92	89	87	86
12430	59042	475	475	479	478	481
12431	15771	644	649	660	664	671
12432	59043	6033	6127	6242	6306	6342
12436	12437	3860	3881	3857	3831	3852
12437	2728	1616	1626	1643	1657	1667
12437	12189	2243	2255	2214	2174	2184
12773	12774	646	659	677	689	705
12773	59050	230	232	235	237	240
12774	16442	978	993	1014	1028	1046

12787	12788	3135	3185	3249	3285	3338
12787	59050	190	189	191	191	194
12788	16482	5147	5238	5355	5419	5510
13539	15753	6033	6127	6242	6306	6342
13540	13552	644	649	660	664	671
13551	13539	6033	6127	6242	6306	6342
13552	15789	644	649	660	664	671
15548	59053	169	174	182	186	191
15750	15751	4850	4935	5038	5072	5095
15751	4007	706	704	711	710	706
15770	15770	6033	6127	6242	6306	6342
15789	15790	644	649	660	664	671
15790	4028	344	347	351	353	356
15794	15794	430	434	439	442	447
16441	16441	876	891	912	926	945
16446	16446	978	993	1014	1028	1046
16481	16481	3325	3373	3439	3476	3531
16483	16483	5147	5238	5355	5419	5510
59039	59040	517	528	544	552	568
59040	10721	947	962	983	994	1015
59042	12431	644	649	660	664	671
59043	12435	3860	3881	3857	3831	3852
59043	59060	2174	2246	2385	2475	2490
59045	59047	87	87	87	87	87
59045	59048	872	893	922	937	961
59046	59045	64	63	62	61	60
59046	59048	5083	5175	5293	5358	5450
59047	10726	876	891	912	926	945
59048	10730	5955	6068	6215	6296	6411
59049	59045	41	41	42	42	42
59049	59047	789	804	825	839	858
59050	12774	332	334	337	339	341
59050	12788	2012	2053	2106	2134	2172
59058	59065	169	174	182	186	191
59064	59067	2174	2246	2385	2475	2490
59065	59068	169	174	182	186	191
59067	59066	2174	2246	2385	2475	2490
59069	59069	169	174	182	186	191

A	B	V_SOV_it16	V_SOV_it17	V_SOV_it18	V_SOV_it19	V_SOV_it20
2933	12428	395	399	399	401	400
4007	15751	1959	1970	1971	1972	1966
4028	15790	133	134	135	134	136
4521	10738	5130	5149	5185	5182	5218
5897	16477	3558	3594	3627	3648	3675
9677	15776	673	678	681	686	688
9678	15766	6382	6423	6463	6461	6484
10384	12004	8502	8598	8696	8753	8841
10385	12001	781	795	806	815	826
10385	12002	242	242	242	241	240
10720	59040	450	453	456	458	460
10722	12003	1024	1037	1048	1056	1066
10725	59049	910	923	932	940	951
10727	16440	954	967	977	984	995
10729	59046	5554	5610	5663	5685	5722
10732	12000	6465	6537	6603	6635	6683
10733	10734	8502	8598	8696	8753	8841
10734	10735	5130	5149	5185	5182	5218
10734	59036	3372	3449	3511	3571	3623
10738	15747	5130	5149	5185	5182	5218
12000	10384	6179	6244	6303	6334	6377
12000	12002	286	294	300	301	307
12001	10723	910	923	932	940	951
12002	10384	2323	2354	2393	2419	2465
12002	12001	128	128	126	125	125
12003	10385	1024	1037	1048	1056	1066
12004	10733	8502	8598	8696	8753	8841
12189	12428	85	83	83	83	83
12430	59042	480	482	481	484	482
12431	15771	673	678	681	686	688
12432	59043	6382	6423	6463	6461	6484
12436	12437	3850	3876	3868	3873	3880
12437	2728	1677	1683	1693	1696	1707
12437	12189	2173	2193	2175	2177	2172
12773	12774	714	725	734	740	750
12773	59050	241	242	243	244	246
12774	16442	1056	1070	1080	1088	1099

12787	12788	3365	3399	3433	3450	3477
12787	59050	194	195	194	199	199
12788	16482	5554	5610	5663	5685	5722
13539	15753	6382	6423	6463	6461	6484
13540	13552	673	678	681	686	688
13551	13539	6382	6423	6463	6461	6484
13552	15789	673	678	681	686	688
15548	59053	194	196	200	202	205
15750	15751	5130	5149	5185	5182	5218
15751	4007	707	696	693	693	700
15770	15770	6382	6423	6463	6461	6484
15789	15790	673	678	681	686	688
15790	4028	357	359	360	363	363
15794	15794	450	453	456	458	460
16441	16441	954	967	977	984	995
16446	16446	1056	1070	1080	1088	1099
16481	16481	3558	3594	3627	3648	3675
16483	16483	5554	5610	5663	5685	5722
59039	59040	574	584	593	598	606
59040	10721	1024	1037	1048	1056	1066
59042	12431	673	678	681	686	688
59043	12435	3850	3876	3868	3873	3880
59043	59060	2532	2547	2595	2588	2604
59045	59047	87	87	87	87	87
59045	59048	971	986	999	1008	1020
59046	59045	60	59	59	58	58
59046	59048	5494	5551	5604	5627	5664
59047	10726	954	967	977	984	995
59048	10730	6465	6537	6603	6635	6683
59049	59045	42	42	42	42	42
59049	59047	868	881	890	898	909
59050	12774	343	345	347	348	350
59050	12788	2190	2211	2230	2236	2245
59058	59065	194	196	200	202	205
59064	59067	2532	2547	2595	2588	2604
59065	59068	194	196	200	202	205
59067	59066	2532	2547	2595	2588	2604
59069	59069	194	196	200	202	205

A	B	V_SOV_it21	V_SOV_it22	V_SOV_it23	V_SOV_it24	V_SOV_it25
2933	12428	402	402	404	404	406
4007	15751	1970	1977	1975	1993	2013
4028	15790	135	135	135	136	135
4521	10738	5220	5239	5241	5230	5195
5897	16477	3688	3703	3719	3736	3745
9677	15776	691	693	696	698	701
9678	15766	6490	6515	6512	6522	6515
10384	12004	8875	8923	8968	8987	8986
10385	12001	832	837	841	844	849
10385	12002	240	239	238	238	238
10720	59040	461	463	464	465	466
10722	12003	1072	1076	1080	1082	1087
10725	59049	956	961	965	968	972
10727	16440	1000	1005	1008	1012	1015
10729	59046	5739	5763	5781	5801	5808
10732	12000	6707	6738	6762	6788	6798
10733	10734	8875	8923	8968	8987	8986
10734	10735	5220	5239	5241	5230	5195
10734	59036	3656	3685	3728	3757	3790
10738	15747	5220	5239	5241	5230	5195
12000	10384	6399	6427	6450	6471	6482
12000	12002	307	311	312	316	316
12001	10723	956	961	965	968	972
12002	10384	2476	2497	2518	2515	2504
12002	12001	124	125	123	124	123
12003	10385	1072	1076	1080	1082	1087
12004	10733	8875	8923	8968	8987	8986
12189	12428	83	83	82	82	81
12430	59042	484	485	486	486	487
12431	15771	691	693	696	698	701
12432	59043	6490	6515	6512	6522	6515
12436	12437	3874	3857	3894	3897	3909
12437	2728	1709	1709	1711	1712	1712
12437	12189	2166	2148	2183	2186	2197
12773	12774	754	760	765	769	772
12773	59050	246	245	244	243	243
12774	16442	1105	1112	1118	1123	1127

12787	12788	3487	3504	3520	3536	3542
12787	59050	201	199	200	200	203
12788	16482	5739	5763	5781	5801	5808
13539	15753	6490	6515	6512	6522	6515
13540	13552	691	693	696	698	701
13551	13539	6490	6515	6512	6522	6515
13552	15789	691	693	696	698	701
15548	59053	207	209	211	212	214
15750	15751	5220	5239	5241	5230	5195
15751	4007	700	701	704	701	693
15770	15770	6490	6515	6512	6522	6515
15789	15790	691	693	696	698	701
15790	4028	365	366	367	368	370
15794	15794	461	463	464	465	466
16441	16441	1000	1005	1008	1012	1015
16446	16446	1105	1112	1118	1123	1127
16481	16481	3688	3703	3719	3736	3745
16483	16483	5739	5763	5781	5801	5808
59039	59040	611	613	616	617	621
59040	10721	1072	1076	1080	1082	1087
59042	12431	691	693	696	698	701
59043	12435	3874	3857	3894	3897	3909
59043	59060	2616	2657	2618	2624	2606
59045	59047	87	87	87	87	87
59045	59048	1026	1033	1039	1044	1047
59046	59045	58	58	57	57	57
59046	59048	5681	5705	5724	5744	5751
59047	10726	1000	1005	1008	1012	1015
59048	10730	6707	6738	6762	6788	6798
59049	59045	43	43	43	43	43
59049	59047	914	918	922	925	929
59050	12774	351	352	353	354	355
59050	12788	2251	2259	2261	2265	2266
59058	59065	207	209	211	212	214
59064	59067	2616	2657	2618	2624	2606
59065	59068	207	209	211	212	214
59067	59066	2616	2657	2618	2624	2606
59069	59069	207	209	211	212	214

A	B	V_SOV_it26	V_SOV_it27	V_SOV_it28	V_SOV_it29	V_SOV_it30
2933	12428	407	407	407	408	409
4007	15751	2014	2018	2028	2022	2020
4028	15790	136	136	136	136	136
4521	10738	5215	5207	5213	5206	5209
5897	16477	3758	3763	3777	3784	3795
9677	15776	703	704	706	707	709
9678	15766	6531	6523	6537	6523	6520
10384	12004	9034	9044	9067	9063	9093
10385	12001	853	857	860	861	865
10385	12002	239	239	240	240	241
10720	59040	468	468	469	470	471
10722	12003	1092	1096	1100	1102	1105
10725	59049	975	978	981	982	986
10727	16440	1019	1022	1024	1026	1029
10729	59046	5828	5836	5856	5862	5874
10732	12000	6824	6836	6860	6868	6884
10733	10734	9034	9044	9067	9063	9093
10734	10735	5215	5207	5213	5206	5209
10734	59036	3819	3837	3855	3857	3884
10738	15747	5215	5207	5213	5206	5209
12000	10384	6503	6513	6535	6542	6554
12000	12002	321	322	325	327	330
12001	10723	975	978	981	982	986
12002	10384	2531	2530	2532	2521	2539
12002	12001	122	121	120	121	121
12003	10385	1092	1096	1100	1102	1105
12004	10733	9034	9044	9067	9063	9093
12189	12428	81	81	81	81	81
12430	59042	488	488	488	489	490
12431	15771	703	704	706	707	709
12432	59043	6531	6523	6537	6523	6520
12436	12437	3907	3906	3943	3932	3971
12437	2728	1713	1713	1713	1713	1714
12437	12189	2193	2193	2229	2220	2257
12773	12774	776	779	782	784	789
12773	59050	242	243	242	241	240
12774	16442	1133	1136	1140	1143	1148

12787	12788	3555	3561	3573	3580	3590
12787	59050	203	203	204	204	205
12788	16482	5828	5836	5856	5862	5874
13539	15753	6531	6523	6537	6523	6520
13540	13552	703	704	706	707	709
13551	13539	6531	6523	6537	6523	6520
13552	15789	703	704	706	707	709
15548	59053	215	216	218	218	220
15750	15751	5215	5207	5213	5206	5209
15751	4007	698	701	703	705	709
15770	15770	6531	6523	6537	6523	6520
15789	15790	703	704	706	707	709
15790	4028	371	372	373	374	375
15794	15794	468	468	469	470	471
16441	16441	1019	1022	1024	1026	1029
16446	16446	1133	1136	1140	1143	1148
16481	16481	3758	3763	3777	3784	3795
16483	16483	5828	5836	5856	5862	5874
59039	59040	625	628	631	632	635
59040	10721	1092	1096	1100	1102	1105
59042	12431	703	704	706	707	709
59043	12435	3907	3906	3943	3932	3971
59043	59060	2624	2617	2595	2591	2550
59045	59047	87	87	87	87	87
59045	59048	1053	1056	1060	1063	1065
59046	59045	57	57	56	56	56
59046	59048	5771	5780	5800	5806	5818
59047	10726	1019	1022	1024	1026	1029
59048	10730	6824	6836	6860	6868	6884
59049	59045	43	43	43	43	43
59049	59047	932	935	938	939	943
59050	12774	356	357	358	359	360
59050	12788	2273	2276	2283	2282	2284
59058	59065	215	216	218	218	220
59064	59067	2624	2617	2595	2591	2550
59065	59068	215	216	218	218	220
59067	59066	2624	2617	2595	2591	2550
59069	59069	215	216	218	218	220

A	B	V_SOV_it31	V_SOV_it32	V_SOV_it33	V_SOV_it34	V_SOV_it35
2933	12428	410	410	411	412	412
4007	15751	2020	2021	2015	2013	2007
4028	15790	136	136	136	136	136
4521	10738	5210	5225	5223	5222	5220
5897	16477	3798	3803	3806	3813	3815
9677	15776	711	712	713	715	716
9678	15766	6521	6535	6525	6516	6507
10384	12004	9095	9120	9120	9146	9143
10385	12001	865	868	868	871	872
10385	12002	241	242	241	241	241
10720	59040	471	472	472	473	473
10722	12003	1106	1110	1110	1112	1113
10725	59049	987	989	990	994	995
10727	16440	1030	1032	1034	1037	1038
10729	59046	5879	5880	5884	5894	5896
10732	12000	6890	6904	6909	6920	6923
10733	10734	9095	9120	9120	9146	9143
10734	10735	5210	5225	5223	5222	5220
10734	59036	3886	3895	3897	3924	3923
10738	15747	5210	5225	5223	5222	5220
12000	10384	6559	6571	6575	6584	6586
12000	12002	331	333	334	337	337
12001	10723	987	989	990	994	995
12002	10384	2536	2550	2545	2562	2558
12002	12001	122	121	122	123	123
12003	10385	1106	1110	1110	1112	1113
12004	10733	9095	9120	9120	9146	9143
12189	12428	81	81	81	80	80
12430	59042	491	491	492	493	493
12431	15771	711	712	713	715	716
12432	59043	6521	6535	6525	6516	6507
12436	12437	3950	3936	3976	4024	4015
12437	2728	1714	1714	1715	1717	1716
12437	12189	2236	2222	2262	2307	2299
12773	12774	790	793	794	798	799
12773	59050	240	239	239	239	239
12774	16442	1150	1153	1155	1160	1161

12787	12788	3595	3601	3605	3611	3613
12787	59050	203	202	201	203	202
12788	16482	5879	5880	5884	5894	5896
13539	15753	6521	6535	6525	6516	6507
13540	13552	711	712	713	715	716
13551	13539	6521	6535	6525	6516	6507
13552	15789	711	712	713	715	716
15548	59053	220	221	222	223	223
15750	15751	5210	5225	5223	5222	5220
15751	4007	709	711	712	720	721
15770	15770	6521	6535	6525	6516	6507
15789	15790	711	712	713	715	716
15790	4028	376	377	377	378	379
15794	15794	471	472	472	473	473
16441	16441	1030	1032	1034	1037	1038
16446	16446	1150	1153	1155	1160	1161
16481	16481	3798	3803	3806	3813	3815
16483	16483	5879	5880	5884	5894	5896
59039	59040	635	638	638	639	639
59040	10721	1106	1110	1110	1112	1113
59042	12431	711	712	713	715	716
59043	12435	3950	3936	3976	4024	4015
59043	59060	2571	2599	2549	2492	2491
59045	59047	87	87	87	87	87
59045	59048	1067	1080	1080	1082	1082
59046	59045	56	56	56	56	56
59046	59048	5823	5824	5828	5838	5840
59047	10726	1030	1032	1034	1037	1038
59048	10730	6890	6904	6909	6920	6923
59049	59045	43	43	43	43	43
59049	59047	944	946	947	950	951
59050	12774	360	361	361	362	362
59050	12788	2284	2280	2280	2284	2283
59058	59065	220	221	222	223	223
59064	59067	2571	2599	2549	2492	2491
59065	59068	220	221	222	223	223
59067	59066	2571	2599	2549	2492	2491
59069	59069	220	221	222	223	223

A	B	V_SOV_it36	V_SOV_it37	V_SOV_it38	V_SOV_it39	V_SOV_it40
2933	12428	413	413	414	414	414
4007	15751	2014	2017	2023	2022	2022
4028	15790	136	137	137	137	137
4521	10738	5213	5209	5202	5197	5199
5897	16477	3820	3822	3827	3828	3833
9677	15776	717	718	719	719	720
9678	15766	6506	6504	6502	6495	6495
10384	12004	9155	9160	9171	9172	9187
10385	12001	874	875	877	878	880
10385	12002	241	241	241	242	242
10720	59040	474	474	475	475	475
10722	12003	1115	1116	1119	1120	1122
10725	59049	996	997	998	999	1000
10727	16440	1039	1040	1041	1042	1043
10729	59046	5903	5900	5897	5898	5904
10732	12000	6932	6935	6942	6943	6950
10733	10734	9155	9160	9171	9172	9187
10734	10735	5213	5209	5202	5197	5199
10734	59036	3942	3951	3969	3975	3988
10738	15747	5213	5209	5202	5197	5199
12000	10384	6593	6597	6602	6605	6610
12000	12002	338	338	340	339	340
12001	10723	996	997	998	999	1000
12002	10384	2561	2563	2569	2567	2577
12002	12001	122	122	121	121	120
12003	10385	1115	1116	1119	1120	1122
12004	10733	9155	9160	9171	9172	9187
12189	12428	80	80	80	80	80
12430	59042	493	493	494	494	494
12431	15771	717	718	719	719	720
12432	59043	6506	6504	6502	6495	6495
12436	12437	4020	4015	4084	4119	4106
12437	2728	1717	1718	1719	1719	1719
12437	12189	2303	2297	2366	2400	2387
12773	12774	801	802	804	804	806
12773	59050	238	238	238	238	237
12774	16442	1164	1165	1167	1169	1170

12787	12788	3619	3620	3626	3626	3630
12787	59050	201	201	201	202	203
12788	16482	5903	5900	5897	5898	5904
13539	15753	6506	6504	6502	6495	6495
13540	13552	717	718	719	719	720
13551	13539	6506	6504	6502	6495	6495
13552	15789	717	718	719	719	720
15548	59053	224	224	225	225	226
15750	15751	5213	5209	5202	5197	5199
15751	4007	721	721	723	724	725
15770	15770	6506	6504	6502	6495	6495
15789	15790	717	718	719	719	720
15790	4028	380	380	381	381	382
15794	15794	474	474	475	475	475
16441	16441	1039	1040	1041	1042	1043
16446	16446	1164	1165	1167	1169	1170
16481	16481	3820	3822	3827	3828	3833
16483	16483	5903	5900	5897	5898	5904
59039	59040	641	642	644	645	647
59040	10721	1115	1116	1119	1120	1122
59042	12431	717	718	719	719	720
59043	12435	4020	4015	4084	4119	4106
59043	59060	2486	2490	2418	2376	2389
59045	59047	87	87	87	87	87
59045	59048	1085	1091	1100	1101	1101
59046	59045	56	56	56	56	56
59046	59048	5847	5844	5842	5843	5849
59047	10726	1039	1040	1041	1042	1043
59048	10730	6932	6935	6942	6943	6950
59049	59045	43	43	44	44	44
59049	59047	952	953	955	955	957
59050	12774	363	363	364	364	365
59050	12788	2284	2279	2271	2272	2274
59058	59065	224	224	225	225	226
59064	59067	2486	2490	2418	2376	2389
59065	59068	224	224	225	225	226
59067	59066	2486	2490	2418	2376	2389
59069	59069	224	224	225	225	226

A	B	V_SOV_it41	V_SOV_it42	V_SOV_it43	V_SOV_it44	V_SOV_it45
2933	12428	415	415	416	416	416
4007	15751	2025	2025	2024	2023	2022
4028	15790	137	137	137	137	137
4521	10738	5193	5194	5194	5195	5196
5897	16477	3834	3836	3837	3839	3840
9677	15776	721	722	723	723	724
9678	15766	6491	6491	6489	6488	6487
10384	12004	9192	9204	9212	9221	9231
10385	12001	881	882	883	884	885
10385	12002	242	242	242	242	242
10720	59040	476	476	476	476	477
10722	12003	1123	1124	1125	1126	1127
10725	59049	1001	1002	1003	1003	1004
10727	16440	1044	1045	1046	1046	1047
10729	59046	5909	5914	5911	5909	5913
10732	12000	6955	6960	6962	6965	6970
10733	10734	9192	9204	9212	9221	9231
10734	10735	5193	5194	5194	5195	5196
10734	59036	4000	4010	4018	4027	4036
10738	15747	5193	5194	5194	5195	5196
12000	10384	6613	6617	6619	6621	6625
12000	12002	342	343	343	344	345
12001	10723	1001	1002	1003	1003	1004
12002	10384	2579	2587	2593	2600	2607
12002	12001	120	120	119	119	119
12003	10385	1123	1124	1125	1126	1127
12004	10733	9192	9204	9212	9221	9231
12189	12428	80	80	80	80	80
12430	59042	495	495	496	496	496
12431	15771	721	722	723	723	724
12432	59043	6491	6491	6489	6488	6487
12436	12437	4158	4144	4177	4165	4170
12437	2728	1720	1720	1720	1721	1721
12437	12189	2438	2424	2457	2444	2449
12773	12774	807	808	808	809	810
12773	59050	237	237	237	237	237
12774	16442	1172	1173	1174	1175	1176

12787	12788	3632	3635	3636	3639	3641
12787	59050	202	201	200	200	200
12788	16482	5909	5914	5911	5909	5913
13539	15753	6491	6491	6489	6488	6487
13540	13552	721	722	723	723	724
13551	13539	6491	6491	6489	6488	6487
13552	15789	721	722	723	723	724
15548	59053	226	227	227	227	228
15750	15751	5193	5194	5194	5195	5196
15751	4007	727	728	729	730	731
15770	15770	6491	6491	6489	6488	6487
15789	15790	721	722	723	723	724
15790	4028	382	383	384	384	385
15794	15794	476	476	476	476	477
16441	16441	1044	1045	1046	1046	1047
16446	16446	1172	1173	1174	1175	1176
16481	16481	3834	3836	3837	3839	3840
16483	16483	5909	5914	5911	5909	5913
59039	59040	647	648	649	650	650
59040	10721	1123	1124	1125	1126	1127
59042	12431	721	722	723	723	724
59043	12435	4158	4144	4177	4165	4170
59043	59060	2333	2348	2312	2323	2317
59045	59047	87	87	87	87	87
59045	59048	1101	1102	1107	1112	1112
59046	59045	56	56	56	56	56
59046	59048	5854	5858	5855	5853	5858
59047	10726	1044	1045	1046	1046	1047
59048	10730	6955	6960	6962	6965	6970
59049	59045	44	44	44	44	44
59049	59047	957	958	959	959	960
59050	12774	365	365	366	366	366
59050	12788	2277	2278	2274	2270	2273
59058	59065	226	227	227	227	228
59064	59067	2333	2348	2312	2323	2317
59065	59068	226	227	227	227	228
59067	59066	2333	2348	2312	2323	2317
59069	59069	226	227	227	227	228

A	B	V_SOV_it46	V_SOV_it47	V_SOV_it48	V_SOV_it49	V_SOV_it50
2933	12428	417	417	417	417	418
4007	15751	2023	2021	2022	2020	2021
4028	15790	137	137	137	137	137
4521	10738	5196	5196	5197	5196	5197
5897	16477	3843	3844	3846	3846	3848
9677	15776	724	725	726	726	727
9678	15766	6487	6482	6484	6479	6481
10384	12004	9240	9249	9257	9263	9269
10385	12001	886	887	888	889	889
10385	12002	242	242	242	242	242
10720	59040	477	477	477	477	477
10722	12003	1128	1129	1130	1131	1131
10725	59049	1004	1005	1006	1005	1005
10727	16440	1048	1048	1049	1048	1048
10729	59046	5911	5915	5919	5914	5917
10732	12000	6973	6976	6980	6979	6982
10733	10734	9240	9249	9257	9263	9269
10734	10735	5196	5196	5197	5196	5197
10734	59036	4044	4053	4060	4067	4072
10738	15747	5196	5196	5197	5196	5197
12000	10384	6627	6630	6633	6634	6635
12000	12002	346	346	347	345	346
12001	10723	1004	1005	1006	1005	1005
12002	10384	2613	2618	2624	2629	2633
12002	12001	119	118	118	116	116
12003	10385	1128	1129	1130	1131	1131
12004	10733	9240	9249	9257	9263	9269
12189	12428	80	80	80	80	80
12430	59042	496	496	497	497	497
12431	15771	724	725	726	726	727
12432	59043	6487	6482	6484	6479	6481
12436	12437	4164	4201	4186	4218	4204
12437	2728	1721	1722	1722	1722	1722
12437	12189	2443	2480	2465	2496	2482
12773	12774	811	812	812	811	812
12773	59050	237	237	237	237	236
12774	16442	1177	1178	1179	1178	1179

12787	12788	3643	3644	3647	3646	3648
12787	59050	200	200	199	201	201
12788	16482	5911	5915	5919	5914	5917
13539	15753	6487	6482	6484	6479	6481
13540	13552	724	725	726	726	727
13551	13539	6487	6482	6484	6479	6481
13552	15789	724	725	726	726	727
15548	59053	228	228	229	229	229
15750	15751	5196	5196	5197	5196	5197
15751	4007	732	734	735	737	738
15770	15770	6487	6482	6484	6479	6481
15789	15790	724	725	726	726	727
15790	4028	385	385	386	386	386
15794	15794	477	477	477	477	477
16441	16441	1048	1048	1049	1048	1048
16446	16446	1177	1178	1179	1178	1179
16481	16481	3843	3844	3846	3846	3848
16483	16483	5911	5915	5919	5914	5917
59039	59040	651	652	653	653	654
59040	10721	1128	1129	1130	1131	1131
59042	12431	724	725	726	726	727
59043	12435	4164	4201	4186	4218	4204
59043	59060	2323	2281	2298	2261	2276
59045	59047	87	87	87	87	87
59045	59048	1117	1117	1117	1121	1120
59046	59045	56	56	56	56	56
59046	59048	5855	5859	5863	5859	5862
59047	10726	1048	1048	1049	1048	1048
59048	10730	6973	6976	6980	6979	6982
59049	59045	44	44	44	44	44
59049	59047	961	961	962	961	961
59050	12774	367	367	367	367	367
59050	12788	2268	2270	2272	2269	2269
59058	59065	228	228	229	229	229
59064	59067	2323	2281	2298	2261	2276
59065	59068	228	228	229	229	229
59067	59066	2323	2281	2298	2261	2276
59069	59069	228	228	229	229	229

A	B	V_SOV_it51	V_SOV_it52	V_SOV_it53	V_SOV_it54	V_SOV_it55
2933	12428	418	418	418	418	418
4007	15751	2019	2020	2020	2019	2022
4028	15790	138	138	138	138	138
4521	10738	5197	5197	5197	5197	5193
5897	16477	3851	3851	3852	3853	3854
9677	15776	727	727	727	728	728
9678	15766	6476	6477	6477	6474	6472
10384	12004	9277	9282	9288	9293	9293
10385	12001	890	891	891	892	892
10385	12002	242	242	242	242	241
10720	59040	478	478	478	478	478
10722	12003	1132	1132	1133	1133	1134
10725	59049	1006	1005	1004	1003	1002
10727	16440	1049	1048	1047	1046	1045
10729	59046	5921	5919	5922	5920	5923
10732	12000	6985	6987	6989	6992	6993
10733	10734	9277	9282	9288	9293	9293
10734	10735	5197	5197	5197	5197	5193
10734	59036	4080	4085	4090	4096	4100
10738	15747	5197	5197	5197	5197	5193
12000	10384	6638	6639	6641	6643	6644
12000	12002	347	348	348	349	349
12001	10723	1006	1005	1004	1003	1002
12002	10384	2639	2643	2646	2650	2650
12002	12001	116	114	113	111	110
12003	10385	1132	1132	1133	1133	1134
12004	10733	9277	9282	9288	9293	9293
12189	12428	80	80	80	80	80
12430	59042	497	497	497	498	498
12431	15771	727	727	727	728	728
12432	59043	6476	6477	6477	6474	6472
12436	12437	4215	4204	4189	4199	4207
12437	2728	1722	1723	1722	1723	1723
12437	12189	2493	2482	2466	2476	2484
12773	12774	813	812	811	810	809
12773	59050	236	236	236	236	236
12774	16442	1180	1180	1179	1178	1178

12787	12788	3651	3651	3652	3654	3654
12787	59050	200	200	200	199	199
12788	16482	5921	5919	5922	5920	5923
13539	15753	6476	6477	6477	6474	6472
13540	13552	727	727	727	728	728
13551	13539	6476	6477	6477	6474	6472
13552	15789	727	727	727	728	728
15548	59053	230	230	230	230	230
15750	15751	5197	5197	5197	5197	5193
15751	4007	740	740	740	742	744
15770	15770	6476	6477	6477	6474	6472
15789	15790	727	727	727	728	728
15790	4028	387	387	387	387	388
15794	15794	478	478	478	478	478
16441	16441	1049	1048	1047	1046	1045
16446	16446	1180	1180	1179	1178	1178
16481	16481	3851	3851	3852	3853	3854
16483	16483	5921	5919	5922	5920	5923
59039	59040	654	654	655	655	655
59040	10721	1132	1132	1133	1133	1134
59042	12431	727	727	727	728	728
59043	12435	4215	4204	4189	4199	4207
59043	59060	2261	2272	2288	2275	2265
59045	59047	87	87	87	87	87
59045	59048	1119	1123	1123	1127	1126
59046	59045	56	56	56	56	56
59046	59048	5866	5864	5867	5865	5867
59047	10726	1049	1048	1047	1046	1045
59048	10730	6985	6987	6989	6992	6993
59049	59045	44	44	44	44	44
59049	59047	962	961	960	959	958
59050	12774	368	368	368	368	368
59050	12788	2271	2268	2270	2266	2268
59058	59065	230	230	230	230	230
59064	59067	2261	2272	2288	2275	2265
59065	59068	230	230	230	230	230
59067	59066	2261	2272	2288	2275	2265
59069	59069	230	230	230	230	230

A	B	V_SOV_it56	V_SOV_it57	V_SOV_it58	V_SOV_it59	V_SOV_it60
2933	12428	418	418	418	419	419
4007	15751	2022	2023	2023	2024	2024
4028	15790	138	138	138	138	138
4521	10738	5194	5194	5193	5194	5194
5897	16477	3854	3854	3855	3855	3856
9677	15776	728	729	729	729	730
9678	15766	6472	6473	6472	6474	6471
10384	12004	9297	9302	9306	9311	9315
10385	12001	893	893	894	894	895
10385	12002	241	241	241	241	241
10720	59040	478	478	479	479	479
10722	12003	1134	1135	1135	1136	1136
10725	59049	1003	1003	1004	1003	1003
10727	16440	1046	1046	1047	1046	1047
10729	59046	5925	5928	5926	5928	5927
10732	12000	6995	6997	6998	7001	7002
10733	10734	9297	9302	9306	9311	9315
10734	10735	5194	5194	5193	5194	5194
10734	59036	4104	4108	4113	4117	4120
10738	15747	5194	5194	5193	5194	5194
12000	10384	6644	6646	6647	6649	6650
12000	12002	350	351	351	352	352
12001	10723	1003	1003	1004	1003	1003
12002	10384	2653	2656	2659	2662	2665
12002	12001	110	110	110	109	109
12003	10385	1134	1135	1135	1136	1136
12004	10733	9297	9302	9306	9311	9315
12189	12428	80	80	80	80	80
12430	59042	498	498	498	498	498
12431	15771	728	729	729	729	730
12432	59043	6472	6473	6472	6474	6471
12436	12437	4193	4204	4211	4200	4206
12437	2728	1723	1723	1724	1724	1724
12437	12189	2470	2480	2487	2476	2483
12773	12774	810	810	811	810	810
12773	59050	236	236	236	236	236
12774	16442	1178	1179	1180	1179	1180

12787	12788	3654	3655	3656	3656	3658
12787	59050	199	199	199	199	199
12788	16482	5925	5928	5926	5928	5927
13539	15753	6472	6473	6472	6474	6471
13540	13552	728	729	729	729	730
13551	13539	6472	6473	6472	6474	6471
13552	15789	728	729	729	729	730
15548	59053	231	231	231	231	231
15750	15751	5194	5194	5193	5194	5194
15751	4007	744	744	745	745	747
15770	15770	6472	6473	6472	6474	6471
15789	15790	728	729	729	729	730
15790	4028	388	388	388	389	389
15794	15794	478	478	479	479	479
16441	16441	1046	1046	1047	1046	1047
16446	16446	1178	1179	1180	1179	1180
16481	16481	3854	3854	3855	3855	3856
16483	16483	5925	5928	5926	5928	5927
59039	59040	656	656	657	657	657
59040	10721	1134	1135	1135	1136	1136
59042	12431	728	729	729	729	730
59043	12435	4193	4204	4211	4200	4206
59043	59060	2279	2269	2261	2274	2265
59045	59047	87	87	87	87	87
59045	59048	1125	1125	1128	1128	1131
59046	59045	56	56	56	56	56
59046	59048	5869	5872	5870	5873	5871
59047	10726	1046	1046	1047	1046	1047
59048	10730	6995	6997	6998	7001	7002
59049	59045	44	44	44	44	44
59049	59047	959	960	960	959	960
59050	12774	369	369	369	369	369
59050	12788	2270	2273	2270	2272	2269
59058	59065	231	231	231	231	231
59064	59067	2279	2269	2261	2274	2265
59065	59068	231	231	231	231	231
59067	59066	2279	2269	2261	2274	2265
59069	59069	231	231	231	231	231

A	B	V_SOV_it61	V_SOV_it62	V_SOV_it63	V_SOV_it64	V_SOV_it65
2933	12428	419	419	419	419	419
4007	15751	2024	2024	2025	2025	2026
4028	15790	138	138	138	138	138
4521	10738	5195	5195	5194	5195	5195
5897	16477	3856	3856	3856	3857	3857
9677	15776	730	730	730	730	731
9678	15766	6470	6470	6470	6471	6471
10384	12004	9319	9321	9325	9327	9331
10385	12001	895	895	896	896	897
10385	12002	241	241	241	241	241
10720	59040	479	479	479	479	479
10722	12003	1136	1137	1137	1138	1138
10725	59049	1002	1002	1001	1001	1000
10727	16440	1046	1045	1045	1044	1044
10729	59046	5929	5927	5930	5929	5927
10732	12000	7004	7004	7006	7006	7008
10733	10734	9319	9321	9325	9327	9331
10734	10735	5195	5195	5194	5195	5195
10734	59036	4124	4126	4131	4133	4136
10738	15747	5195	5195	5194	5195	5195
12000	10384	6651	6651	6652	6653	6654
12000	12002	353	353	354	354	354
12001	10723	1002	1002	1001	1001	1000
12002	10384	2668	2670	2673	2675	2678
12002	12001	107	107	105	104	104
12003	10385	1136	1137	1137	1138	1138
12004	10733	9319	9321	9325	9327	9331
12189	12428	80	80	80	80	80
12430	59042	499	499	499	499	499
12431	15771	730	730	730	730	731
12432	59043	6470	6470	6470	6471	6471
12436	12437	4215	4216	4246	4234	4228
12437	2728	1724	1724	1725	1724	1724
12437	12189	2491	2492	2521	2509	2503
12773	12774	810	809	808	808	807
12773	59050	236	236	236	236	236
12774	16442	1179	1179	1178	1178	1177

12787	12788	3658	3658	3658	3659	3659
12787	59050	199	199	199	198	199
12788	16482	5929	5927	5930	5929	5927
13539	15753	6470	6470	6470	6471	6471
13540	13552	730	730	730	730	731
13551	13539	6470	6470	6470	6471	6471
13552	15789	730	730	730	730	731
15548	59053	231	231	232	232	232
15750	15751	5195	5195	5194	5195	5195
15751	4007	748	748	749	749	749
15770	15770	6470	6470	6470	6471	6471
15789	15790	730	730	730	730	731
15790	4028	389	389	389	390	390
15794	15794	479	479	479	479	479
16441	16441	1046	1045	1045	1044	1044
16446	16446	1179	1179	1178	1178	1177
16481	16481	3856	3856	3856	3857	3857
16483	16483	5929	5927	5930	5929	5927
59039	59040	657	658	658	658	659
59040	10721	1136	1137	1137	1138	1138
59042	12431	730	730	730	730	731
59043	12435	4215	4216	4246	4234	4228
59043	59060	2255	2254	2224	2237	2244
59045	59047	87	87	87	87	87
59045	59048	1130	1132	1131	1133	1137
59046	59045	56	56	56	56	56
59046	59048	5874	5872	5874	5873	5871
59047	10726	1046	1045	1045	1044	1044
59048	10730	7004	7004	7006	7006	7008
59049	59045	44	44	44	44	44
59049	59047	959	958	958	957	957
59050	12774	369	370	370	370	370
59050	12788	2271	2270	2272	2270	2268
59058	59065	231	231	232	232	232
59064	59067	2255	2254	2224	2237	2244
59065	59068	231	231	232	232	232
59067	59066	2255	2254	2224	2237	2244
59069	59069	231	231	232	232	232

A	B	V_SOV_it66	V_SOV_it67	V_SOV_it68	V_SOV_it69	V_SOV_it70
2933	12428	419	419	419	419	419
4007	15751	2026	2026	2026	2027	2027
4028	15790	138	138	138	139	139
4521	10738	5195	5195	5195	5195	5195
5897	16477	3858	3859	3859	3860	3861
9677	15776	731	731	731	731	732
9678	15766	6471	6471	6471	6470	6471
10384	12004	9334	9337	9340	9342	9345
10385	12001	897	897	898	898	898
10385	12002	241	241	241	241	241
10720	59040	479	479	479	480	480
10722	12003	1138	1138	1139	1139	1139
10725	59049	1000	999	999	998	998
10727	16440	1043	1043	1042	1042	1041
10729	59046	5929	5928	5929	5931	5930
10732	12000	7009	7010	7012	7012	7013
10733	10734	9334	9337	9340	9342	9345
10734	10735	5195	5195	5195	5195	5195
10734	59036	4139	4142	4145	4147	4150
10738	15747	5195	5195	5195	5195	5195
12000	10384	6654	6655	6656	6656	6657
12000	12002	355	355	356	356	356
12001	10723	1000	999	999	998	998
12002	10384	2680	2682	2684	2686	2688
12002	12001	103	102	101	100	100
12003	10385	1138	1138	1139	1139	1139
12004	10733	9334	9337	9340	9342	9345
12189	12428	80	80	80	80	80
12430	59042	499	499	499	499	499
12431	15771	731	731	731	731	732
12432	59043	6471	6471	6471	6470	6471
12436	12437	4233	4239	4233	4254	4247
12437	2728	1724	1725	1725	1725	1725
12437	12189	2509	2514	2509	2529	2522
12773	12774	807	807	806	806	805
12773	59050	236	236	236	236	236
12774	16442	1177	1177	1176	1176	1176

12787	12788	3659	3660	3660	3661	3662
12787	59050	199	199	199	199	199
12788	16482	5929	5928	5929	5931	5930
13539	15753	6471	6471	6471	6470	6471
13540	13552	731	731	731	731	732
13551	13539	6471	6471	6471	6470	6471
13552	15789	731	731	731	731	732
15548	59053	232	232	232	232	232
15750	15751	5195	5195	5195	5195	5195
15751	4007	749	750	750	751	751
15770	15770	6471	6471	6471	6470	6471
15789	15790	731	731	731	731	732
15790	4028	390	390	390	390	390
15794	15794	479	479	479	480	480
16441	16441	1043	1043	1042	1042	1041
16446	16446	1177	1177	1176	1176	1176
16481	16481	3858	3859	3859	3860	3861
16483	16483	5929	5928	5929	5931	5930
59039	59040	659	659	659	660	660
59040	10721	1138	1138	1139	1139	1139
59042	12431	731	731	731	731	732
59043	12435	4233	4239	4233	4254	4247
59043	59060	2238	2232	2238	2217	2223
59045	59047	87	87	87	87	87
59045	59048	1136	1138	1138	1136	1139
59046	59045	56	56	56	56	56
59046	59048	5873	5872	5874	5876	5874
59047	10726	1043	1043	1042	1042	1041
59048	10730	7009	7010	7012	7012	7013
59049	59045	44	44	44	44	44
59049	59047	956	956	955	955	954
59050	12774	370	370	370	370	370
59050	12788	2270	2268	2270	2270	2268
59058	59065	232	232	232	232	232
59064	59067	2238	2232	2238	2217	2223
59065	59068	232	232	232	232	232
59067	59066	2238	2232	2238	2217	2223
59069	59069	232	232	232	232	232

A	B	V_SOV_it71	V_SOV_it72	V_SOV_it73	V_SOV_it74	V_SOV_it75
2933	12428	419	419	419	420	420
4007	15751	2028	2028	2029	2029	2029
4028	15790	139	139	139	139	139
4521	10738	5195	5194	5194	5194	5194
5897	16477	3863	3862	3863	3863	3864
9677	15776	732	732	732	732	732
9678	15766	6471	6470	6471	6471	6470
10384	12004	9348	9350	9353	9354	9356
10385	12001	899	899	899	899	900
10385	12002	241	241	241	241	241
10720	59040	480	480	480	480	480
10722	12003	1140	1140	1140	1140	1141
10725	59049	998	999	998	999	999
10727	16440	1042	1042	1042	1042	1043
10729	59046	5932	5930	5933	5931	5935
10732	12000	7014	7015	7016	7017	7019
10733	10734	9348	9350	9353	9354	9356
10734	10735	5195	5194	5194	5194	5194
10734	59036	4153	4155	4158	4160	4162
10738	15747	5195	5194	5194	5194	5194
12000	10384	6658	6658	6659	6659	6661
12000	12002	357	357	357	358	358
12001	10723	998	999	998	999	999
12002	10384	2690	2691	2694	2695	2694
12002	12001	100	100	99	99	100
12003	10385	1140	1140	1140	1140	1141
12004	10733	9348	9350	9353	9354	9356
12189	12428	80	80	80	80	80
12430	59042	499	499	499	499	499
12431	15771	732	732	732	732	732
12432	59043	6471	6470	6471	6471	6470
12436	12437	4269	4264	4257	4271	4275
12437	2728	1725	1725	1726	1726	1726
12437	12189	2544	2538	2532	2545	2549
12773	12774	806	806	806	806	807
12773	59050	236	236	236	236	236
12774	16442	1176	1177	1176	1177	1177

12787	12788	3663	3663	3664	3664	3664
12787	59050	200	200	200	200	200
12788	16482	5932	5930	5933	5931	5935
13539	15753	6471	6470	6471	6471	6470
13540	13552	732	732	732	732	732
13551	13539	6471	6470	6471	6471	6470
13552	15789	732	732	732	732	732
15548	59053	233	233	233	233	233
15750	15751	5195	5194	5194	5194	5194
15751	4007	752	752	752	752	752
15770	15770	6471	6470	6471	6471	6470
15789	15790	732	732	732	732	732
15790	4028	391	391	391	391	391
15794	15794	480	480	480	480	480
16441	16441	1042	1042	1042	1042	1043
16446	16446	1176	1177	1176	1177	1177
16481	16481	3863	3862	3863	3863	3864
16483	16483	5932	5930	5933	5931	5935
59039	59040	660	660	660	661	661
59040	10721	1140	1140	1140	1140	1141
59042	12431	732	732	732	732	732
59043	12435	4269	4264	4257	4271	4275
59043	59060	2201	2207	2214	2200	2196
59045	59047	87	87	87	87	87
59045	59048	1138	1140	1139	1141	1140
59046	59045	56	56	56	56	56
59046	59048	5876	5875	5877	5876	5879
59047	10726	1042	1042	1042	1042	1043
59048	10730	7014	7015	7016	7017	7019
59049	59045	44	44	44	44	44
59049	59047	955	955	955	955	956
59050	12774	370	370	371	371	371
59050	12788	2269	2268	2269	2268	2271
59058	59065	233	233	233	233	233
59064	59067	2201	2207	2214	2200	2196
59065	59068	233	233	233	233	233
59067	59066	2201	2207	2214	2200	2196
59069	59069	233	233	233	233	233

A	B	V_SOV_it76	V_SOV_it77	V_SOV_it78	V_SOV_it79	V_SOV_it80
2933	12428	420	420	420	420	420
4007	15751	2029	2029	2032	2032	2033
4028	15790	139	139	139	139	139
4521	10738	5193	5193	5191	5191	5191
5897	16477	3864	3865	3865	3865	3865
9677	15776	732	733	733	733	733
9678	15766	6469	6469	6470	6468	6469
10384	12004	9357	9359	9359	9361	9363
10385	12001	900	900	900	901	901
10385	12002	241	241	241	241	241
10720	59040	480	480	480	480	480
10722	12003	1141	1141	1142	1142	1142
10725	59049	999	999	999	999	999
10727	16440	1043	1042	1043	1043	1043
10729	59046	5934	5932	5934	5936	5934
10732	12000	7020	7020	7021	7022	7023
10733	10734	9357	9359	9359	9361	9363
10734	10735	5193	5193	5191	5191	5191
10734	59036	4164	4166	4168	4170	4172
10738	15747	5193	5193	5191	5191	5191
12000	10384	6662	6662	6662	6663	6663
12000	12002	358	358	359	359	359
12001	10723	999	999	999	999	999
12002	10384	2695	2697	2696	2698	2699
12002	12001	99	99	99	99	98
12003	10385	1141	1141	1142	1142	1142
12004	10733	9357	9359	9359	9361	9363
12189	12428	80	80	80	80	80
12430	59042	499	499	500	500	500
12431	15771	732	733	733	733	733
12432	59043	6469	6469	6470	6468	6469
12436	12437	4284	4275	4270	4287	4278
12437	2728	1726	1726	1726	1726	1726
12437	12189	2558	2550	2544	2561	2552
12773	12774	806	806	806	807	807
12773	59050	236	236	236	236	236
12774	16442	1177	1177	1177	1178	1177

12787	12788	3665	3665	3665	3666	3666
12787	59050	200	200	200	200	200
12788	16482	5934	5932	5934	5936	5934
13539	15753	6469	6469	6470	6468	6469
13540	13552	732	733	733	733	733
13551	13539	6469	6469	6470	6468	6469
13552	15789	732	733	733	733	733
15548	59053	233	233	233	233	233
15750	15751	5193	5193	5191	5191	5191
15751	4007	753	753	753	754	755
15770	15770	6469	6469	6470	6468	6469
15789	15790	732	733	733	733	733
15790	4028	391	391	392	392	392
15794	15794	480	480	480	480	480
16441	16441	1043	1042	1043	1043	1043
16446	16446	1177	1177	1177	1178	1177
16481	16481	3864	3865	3865	3865	3865
16483	16483	5934	5932	5934	5936	5934
59039	59040	661	661	661	662	662
59040	10721	1141	1141	1142	1142	1142
59042	12431	732	733	733	733	733
59043	12435	4284	4275	4270	4287	4278
59043	59060	2185	2194	2200	2182	2191
59045	59047	87	87	87	87	87
59045	59048	1142	1144	1143	1142	1144
59046	59045	56	56	56	56	56
59046	59048	5878	5877	5878	5880	5879
59047	10726	1043	1042	1043	1043	1043
59048	10730	7020	7020	7021	7022	7023
59049	59045	44	44	44	44	44
59049	59047	955	955	955	956	956
59050	12774	371	371	371	371	371
59050	12788	2269	2267	2268	2270	2269
59058	59065	233	233	233	233	233
59064	59067	2185	2194	2200	2182	2191
59065	59068	233	233	233	233	233
59067	59066	2185	2194	2200	2182	2191
59069	59069	233	233	233	233	233

A	B	V_SOV_it81	V_SOV_it82	V_SOV_it83	V_SOV_it84	V_SOV_it85
2933	12428	420	420	420	420	420
4007	15751	2033	2033	2033	2036	2037
4028	15790	139	139	139	139	139
4521	10738	5191	5190	5190	5188	5186
5897	16477	3866	3866	3866	3867	3868
9677	15776	733	733	733	734	734
9678	15766	6469	6468	6469	6469	6469
10384	12004	9365	9367	9368	9368	9368
10385	12001	901	901	901	902	902
10385	12002	241	241	241	241	241
10720	59040	480	480	480	480	480
10722	12003	1142	1143	1143	1143	1143
10725	59049	999	999	999	999	998
10727	16440	1042	1042	1043	1042	1042
10729	59046	5936	5935	5936	5935	5936
10732	12000	7024	7024	7024	7025	7025
10733	10734	9365	9367	9368	9368	9368
10734	10735	5191	5190	5190	5188	5186
10734	59036	4175	4176	4178	4180	4181
10738	15747	5191	5190	5190	5188	5186
12000	10384	6664	6664	6664	6665	6665
12000	12002	360	360	360	360	360
12001	10723	999	999	999	999	998
12002	10384	2701	2702	2704	2703	2703
12002	12001	98	97	98	97	97
12003	10385	1142	1143	1143	1143	1143
12004	10733	9365	9367	9368	9368	9368
12189	12428	80	80	80	80	80
12430	59042	500	500	500	500	500
12431	15771	733	733	733	734	734
12432	59043	6469	6468	6469	6469	6469
12436	12437	4281	4284	4288	4302	4293
12437	2728	1726	1726	1726	1726	1726
12437	12189	2555	2558	2562	2576	2567
12773	12774	806	806	806	806	806
12773	59050	236	236	236	236	236
12774	16442	1177	1177	1177	1177	1177

12787	12788	3666	3666	3667	3668	3668
12787	59050	200	200	200	200	200
12788	16482	5936	5935	5936	5935	5936
13539	15753	6469	6468	6469	6469	6469
13540	13552	733	733	733	734	734
13551	13539	6469	6468	6469	6469	6469
13552	15789	733	733	733	734	734
15548	59053	233	234	234	234	234
15750	15751	5191	5190	5190	5188	5186
15751	4007	755	755	755	755	755
15770	15770	6469	6468	6469	6469	6469
15789	15790	733	733	733	734	734
15790	4028	392	392	392	392	392
15794	15794	480	480	480	480	480
16441	16441	1042	1042	1043	1042	1042
16446	16446	1177	1177	1177	1177	1177
16481	16481	3866	3866	3866	3867	3868
16483	16483	5936	5935	5936	5935	5936
59039	59040	662	662	663	663	663
59040	10721	1142	1143	1143	1143	1143
59042	12431	733	733	733	734	734
59043	12435	4281	4284	4288	4302	4293
59043	59060	2188	2184	2181	2167	2176
59045	59047	87	87	87	87	87
59045	59048	1143	1145	1144	1146	1145
59046	59045	56	56	56	56	56
59046	59048	5881	5879	5881	5879	5880
59047	10726	1042	1042	1043	1042	1042
59048	10730	7024	7024	7024	7025	7025
59049	59045	44	44	44	44	44
59049	59047	955	955	955	955	955
59050	12774	371	371	371	371	371
59050	12788	2270	2269	2269	2267	2268
59058	59065	233	234	234	234	234
59064	59067	2188	2184	2181	2167	2176
59065	59068	233	234	234	234	234
59067	59066	2188	2184	2181	2167	2176
59069	59069	233	234	234	234	234

A	B	V_SOV_it86	V_SOV_it87	V_SOV_it88	V_SOV_it89	V_SOV_it90
2933	12428	420	420	420	420	420
4007	15751	2038	2039	2039	2041	2041
4028	15790	139	139	139	139	139
4521	10738	5186	5185	5185	5183	5183
5897	16477	3867	3868	3868	3868	3869
9677	15776	734	734	734	734	734
9678	15766	6469	6469	6468	6468	6468
10384	12004	9370	9370	9372	9371	9373
10385	12001	902	902	902	903	903
10385	12002	241	241	241	241	241
10720	59040	481	481	481	481	481
10722	12003	1143	1144	1144	1144	1144
10725	59049	998	998	998	999	999
10727	16440	1042	1042	1042	1042	1043
10729	59046	5934	5935	5937	5936	5939
10732	12000	7028	7028	7029	7030	7032
10733	10734	9370	9370	9372	9371	9373
10734	10735	5186	5185	5185	5183	5183
10734	59036	4184	4185	4187	4188	4190
10738	15747	5186	5185	5185	5183	5183
12000	10384	6667	6667	6668	6669	6671
12000	12002	361	361	361	361	361
12001	10723	998	998	998	999	999
12002	10384	2703	2702	2704	2702	2702
12002	12001	96	96	96	96	96
12003	10385	1143	1144	1144	1144	1144
12004	10733	9370	9370	9372	9371	9373
12189	12428	80	80	80	80	80
12430	59042	500	500	500	500	500
12431	15771	734	734	734	734	734
12432	59043	6469	6469	6468	6468	6468
12436	12437	4286	4296	4289	4300	4294
12437	2728	1726	1726	1726	1726	1726
12437	12189	2560	2569	2563	2573	2567
12773	12774	805	805	806	806	806
12773	59050	236	236	236	236	237
12774	16442	1177	1177	1177	1177	1178

12787	12788	3668	3668	3668	3669	3669
12787	59050	200	200	200	200	200
12788	16482	5934	5935	5937	5936	5939
13539	15753	6469	6469	6468	6468	6468
13540	13552	734	734	734	734	734
13551	13539	6469	6469	6468	6468	6468
13552	15789	734	734	734	734	734
15548	59053	234	234	234	234	234
15750	15751	5186	5185	5185	5183	5183
15751	4007	755	755	756	756	756
15770	15770	6469	6469	6468	6468	6468
15789	15790	734	734	734	734	734
15790	4028	392	393	393	393	393
15794	15794	481	481	481	481	481
16441	16441	1042	1042	1042	1042	1043
16446	16446	1177	1177	1177	1177	1178
16481	16481	3867	3868	3868	3868	3869
16483	16483	5934	5935	5937	5936	5939
59039	59040	663	663	663	663	664
59040	10721	1143	1144	1144	1144	1144
59042	12431	734	734	734	734	734
59043	12435	4286	4296	4289	4300	4294
59043	59060	2183	2173	2180	2169	2175
59045	59047	87	87	87	87	87
59045	59048	1149	1148	1148	1150	1149
59046	59045	56	56	56	56	56
59046	59048	5879	5880	5881	5880	5883
59047	10726	1042	1042	1042	1042	1043
59048	10730	7028	7028	7029	7030	7032
59049	59045	44	44	44	44	44
59049	59047	955	954	955	955	955
59050	12774	371	371	371	372	372
59050	12788	2267	2268	2269	2267	2270
59058	59065	234	234	234	234	234
59064	59067	2183	2173	2180	2169	2175
59065	59068	234	234	234	234	234
59067	59066	2183	2173	2180	2169	2175
59069	59069	234	234	234	234	234

A	B	V_SOV_it91	V_SOV_it92	V_SOV_it93	V_SOV_it94	V_SOV_it95
2933	12428	420	420	420	420	420
4007	15751	2043	2042	2042	2043	2042
4028	15790	139	139	139	139	139
4521	10738	5181	5182	5181	5181	5181
5897	16477	3869	3869	3869	3870	3869
9677	15776	734	734	734	735	735
9678	15766	6468	6468	6467	6468	6468
10384	12004	9372	9374	9375	9377	9378
10385	12001	903	903	903	903	903
10385	12002	241	241	241	241	242
10720	59040	481	481	481	481	481
10722	12003	1144	1144	1145	1145	1145
10725	59049	999	999	999	999	999
10727	16440	1043	1042	1043	1043	1043
10729	59046	5938	5939	5938	5937	5938
10732	12000	7032	7033	7033	7033	7034
10733	10734	9372	9374	9375	9377	9378
10734	10735	5181	5182	5181	5181	5181
10734	59036	4191	4193	4194	4196	4197
10738	15747	5181	5182	5181	5181	5181
12000	10384	6671	6671	6671	6671	6671
12000	12002	362	362	362	362	362
12001	10723	999	999	999	999	999
12002	10384	2702	2703	2704	2706	2707
12002	12001	96	95	96	96	95
12003	10385	1144	1144	1145	1145	1145
12004	10733	9372	9374	9375	9377	9378
12189	12428	80	80	80	80	80
12430	59042	500	500	500	500	500
12431	15771	734	734	734	735	735
12432	59043	6468	6468	6467	6468	6468
12436	12437	4296	4290	4300	4302	4304
12437	2728	1727	1727	1727	1727	1727
12437	12189	2570	2563	2573	2576	2577
12773	12774	806	806	806	806	806
12773	59050	237	237	237	237	237
12774	16442	1178	1177	1178	1178	1178

12787	12788	3669	3669	3669	3670	3669
12787	59050	200	200	200	200	200
12788	16482	5938	5939	5938	5937	5938
13539	15753	6468	6468	6467	6468	6468
13540	13552	734	734	734	735	735
13551	13539	6468	6468	6467	6468	6468
13552	15789	734	734	734	735	735
15548	59053	234	234	234	234	234
15750	15751	5181	5182	5181	5181	5181
15751	4007	756	756	756	756	756
15770	15770	6468	6468	6467	6468	6468
15789	15790	734	734	734	735	735
15790	4028	393	393	393	393	393
15794	15794	481	481	481	481	481
16441	16441	1043	1042	1043	1043	1043
16446	16446	1178	1177	1178	1178	1178
16481	16481	3869	3869	3869	3870	3869
16483	16483	5938	5939	5938	5937	5938
59039	59040	664	664	664	664	664
59040	10721	1144	1144	1145	1145	1145
59042	12431	734	734	734	735	735
59043	12435	4296	4290	4300	4302	4304
59043	59060	2172	2178	2168	2165	2163
59045	59047	87	87	87	87	87
59045	59048	1150	1149	1151	1152	1152
59046	59045	56	56	56	56	56
59046	59048	5882	5883	5882	5881	5882
59047	10726	1043	1042	1043	1043	1043
59048	10730	7032	7033	7033	7033	7034
59049	59045	44	44	44	44	44
59049	59047	955	955	955	956	955
59050	12774	372	372	372	372	372
59050	12788	2268	2270	2269	2267	2268
59058	59065	234	234	234	234	234
59064	59067	2172	2178	2168	2165	2163
59065	59068	234	234	234	234	234
59067	59066	2172	2178	2168	2165	2163
59069	59069	234	234	234	234	234

A	B	V_SOV_it96	V_SOV_it97	V_SOV_it98	V_SOV_it99	V_SOV_it100
2933	12428	420	420	420	420	420
4007	15751	2042	2042	2042	2042	2043
4028	15790	139	139	139	139	139
4521	10738	5181	5181	5181	5181	5180
5897	16477	3869	3869	3870	3870	3870
9677	15776	735	735	735	735	735
9678	15766	6468	6467	6467	6466	6466
10384	12004	9380	9381	9382	9384	9383
10385	12001	904	904	904	904	904
10385	12002	242	242	242	242	242
10720	59040	481	481	481	481	481
10722	12003	1145	1145	1145	1146	1146
10725	59049	999	999	999	999	999
10727	16440	1042	1043	1043	1043	1043
10729	59046	5939	5938	5939	5940	5939
10732	12000	7034	7034	7035	7035	7035
10733	10734	9380	9381	9382	9384	9383
10734	10735	5181	5181	5181	5181	5180
10734	59036	4199	4200	4201	4203	4204
10738	15747	5181	5181	5181	5181	5180
12000	10384	6671	6672	6672	6672	6672
12000	12002	363	363	363	363	363
12001	10723	999	999	999	999	999
12002	10384	2709	2709	2710	2712	2711
12002	12001	95	95	95	95	95
12003	10385	1145	1145	1145	1146	1146
12004	10733	9380	9381	9382	9384	9383
12189	12428	80	80	80	80	80
12430	59042	500	500	500	500	500
12431	15771	735	735	735	735	735
12432	59043	6468	6467	6467	6466	6466
12436	12437	4298	4300	4302	4304	4305
12437	2728	1727	1727	1727	1727	1727
12437	12189	2571	2573	2575	2577	2578
12773	12774	806	806	806	806	806
12773	59050	237	237	237	237	237
12774	16442	1178	1178	1178	1178	1178

12787	12788	3669	3669	3669	3670	3670
12787	59050	200	200	200	200	200
12788	16482	5939	5938	5939	5940	5939
13539	15753	6468	6467	6467	6466	6466
13540	13552	735	735	735	735	735
13551	13539	6468	6467	6467	6466	6466
13552	15789	735	735	735	735	735
15548	59053	234	234	235	235	235
15750	15751	5181	5181	5181	5181	5180
15751	4007	756	756	757	757	757
15770	15770	6468	6467	6467	6466	6466
15789	15790	735	735	735	735	735
15790	4028	393	393	393	394	394
15794	15794	481	481	481	481	481
16441	16441	1042	1043	1043	1043	1043
16446	16446	1178	1178	1178	1178	1178
16481	16481	3869	3869	3870	3870	3870
16483	16483	5939	5938	5939	5940	5939
59039	59040	664	664	665	665	665
59040	10721	1145	1145	1145	1146	1146
59042	12431	735	735	735	735	735
59043	12435	4298	4300	4302	4304	4305
59043	59060	2169	2167	2165	2162	2161
59045	59047	87	87	87	87	87
59045	59048	1151	1152	1152	1151	1152
59046	59045	56	56	56	56	56
59046	59048	5883	5882	5883	5884	5884
59047	10726	1042	1043	1043	1043	1043
59048	10730	7034	7034	7035	7035	7035
59049	59045	44	44	44	44	44
59049	59047	955	955	956	955	956
59050	12774	372	372	372	372	372
59050	12788	2270	2269	2269	2270	2269
59058	59065	234	234	235	235	235
59064	59067	2169	2167	2165	2162	2161
59065	59068	234	234	235	235	235
59067	59066	2169	2167	2165	2162	2161
59069	59069	234	234	235	235	235

APPENDIX B

MCTA _ESTIMATED TOLLED LINK VOLUMES

A	B	DISTANCE	V_SOVAM	V_HOVAM	V_COMAM	V_TOTAM
2933	12428	0.007	443	90	84	617
4007	15751	0.007	2030	260	74	2364
4028	15790	0.007	146	40	43	229
4521	10738	0.206	5343	588	233	6164
5897	16477	0.012	3919	411	243	4572
9677	15776	0.110	748	163	144	1054
9678	15766	0.440	6617	725	232	7574
10384	12004	1.725	9462	1040	517	11019
10385	12001	0.225	965	167	198	1330
10385	12002	0.105	255	46	34	335
10720	59040	0.980	505	102	101	708
10722	12003	0.110	1220	213	232	1665
10725	59049	1.071	1080	193	220	1493
10727	16440	0.834	1125	199	223	1548
10729	59046	0.847	5997	601	340	6938
10732	12000	1.071	7136	755	402	8293
10733	10734	0.036	9462	1040	517	11019
10734	10735	0.352	5343	588	233	6164
10734	59036	0.094	4120	452	283	4855
10738	15747	0.982	5343	588	233	6164
12000	10384	0.223	6759	687	360	7805
12000	12002	0.121	378	68	42	487
12001	10723	0.477	1080	193	220	1493
12002	10384	0.103	2704	353	157	3213
12002	12001	0.121	116	26	21	163
12003	10385	1.725	1220	213	232	1665
12004	10733	0.103	9462	1040	517	11019
12189	12428	0.024	80	23	16	118
12430	59042	0.216	522	113	99	734
12431	15771	0.040	748	163	144	1054
12432	59043	0.153	6617	725	232	7574
12436	12437	0.217	4403	460	154	5016
12437	2728	0.007	1671	224	99	1994
12437	12189	0.024	2732	236	55	3023
12773	12774	0.133	835	137	158	1130
12773	59050	0.069	290	62	65	417
12774	16442	0.961	1209	204	191	1604

12787	12788	0.133	3688	364	215	4267
12787	59050	0.065	230	47	28	305
12788	16482	0.272	5997	601	340	6938
13539	15753	0.190	6617	725	232	7574
13540	13552	0.310	748	163	144	1054
13551	13539	0.300	6617	725	232	7574
15548	59053	0.099	226	50	44	320
15750	15751	0.700	5343	588	233	6164
15751	4007	0.007	756	123	75	954
15770	15770	1.600	6617	725	232	7574
15789	15790	1.600	748	163	144	1054
15790	4028	0.007	390	101	85	575
15794	15794	0.700	505	102	101	708
16441	16441	0.268	1125	199	223	1548
16446	16446	0.887	1209	204	191	1604
16481	16481	0.968	3919	411	243	4572
16483	16483	0.303	5997	601	340	6938
59039	59040	0.061	715	111	131	957
59040	10721	0.096	1220	213	232	1665
59042	12431	0.150	748	163	144	1054
59043	12435	0.054	4403	460	154	5016
59043	59060	0.015	2214	265	78	2557
59045	59047	0.070	88	18	20	126
59045	59048	0.063	1197	170	81	1447
59046	59045	0.071	57	16	19	92
59046	59048	0.133	5940	585	321	6846
59047	10726	0.709	1125	199	223	1548
59048	10730	0.177	7136	755	402	8293
59049	59045	0.063	43	11	16	71
59049	59047	0.132	1037	181	204	1422
59050	12774	0.065	374	67	33	473
59050	12788	0.068	2309	237	125	2671
59058	59065	0.071	226	50	44	320
59064	59067	0.201	2214	265	78	2557
59065	59068	0.021	226	50	44	320
59067	59066	0.055	2214	265	78	2557
59069	59069	0.025	226	50	44	320

APPENDIX C

CUBE-PTA_CONVERGENCE

Iter	AAD	RAAD	RMSE	Gap	RelGap	Lambda	Factor
1	--	--	--	0	0	1	0.00048
2	449	1.478	1,185	0.20501	0.38866	0.20776	0.00013
3	223	0.258	643	0.10336	0.21045	0.16604	0.00012
4	135	0.139	432	0.04605	0.11494	0.1406	0.00012
5	119	0.105	373	0.02754	0.07211	0.18054	0.00019
6	84	0.049	244	0.01349	0.05005	0.14479	0.00018
7	96	0.071	271	0.00823	0.04081	0.21184	0.00033
8	53	0.026	170	0.00854	0.03607	0.08581	0.00014
9	81	0.045	253	0.00192	0.02626	0.2131	0.00046
10	41	0.018	144	0.00669	0.03048	0.06427	0.00015
11	57	0.031	193	0.00322	0.01911	0.16924	0.00047
12	40	0.019	133	0.00306	0.01957	0.09208	0.00028
13	45	0.023	155	0.00142	0.01578	0.13602	0.00048
14	37	0.018	138	0.00185	0.01586	0.09014	0.00035
15	45	0.022	147	0.00143	0.01304	0.14101	0.00063
16	33	0.015	120	0.00137	0.01361	0.08011	0.00039
17	32	0.016	124	0.00106	0.01032	0.10969	0.0006
18	35	0.017	128	0.00094	0.01003	0.10965	0.00068
19	30	0.014	104	0.00148	0.01012	0.08825	0.0006
20	31	0.016	120	3.81E-05	0.00794	0.11627	0.00089
21	24	0.012	94	0.00121	0.00849	0.07464	0.00062
22	22	0.011	85	0.00023	0.007	0.08394	0.00076
23	23	0.012	83	0.00049	0.00654	0.09256	0.00092
24	25	0.013	88	0.00064	0.00665	0.09243	0.00101
25	20	0.01	76	0.00067	0.00614	0.0658	0.00077
26	23	0.013	88	0.00019	0.00515	0.10802	0.00142
27	17	0.008	68	0.00083	0.00589	0.05673	0.00079
28	21	0.012	71	0.00012	0.0045	0.09896	0.00153
29	17	0.008	61	0.00066	0.00483	0.05923	0.00097
30	19	0.011	69	0.00046	0.00403	0.09464	0.00172
31	14	0.007	57	0.00094	0.00442	0.05186	0.00099
32	17	0.009	74	0.00018	0.00345	0.08211	0.00171
33	11	0.006	39	0.00042	0.00363	0.04421	0.00097
34	18	0.01	57	0.00032	0.00308	0.10354	0.00252
35	10	0.005	39	0.0007	0.00397	0.03582	0.0009
36	14	0.008	51	0.00014	0.00284	0.07742	0.00212

37	10	0.006	35	0.00051	0.00295	0.04544	0.0013
38	15	0.008	53	0.0004	0.00244	0.08252	0.00258
39	11	0.006	42	0.0005	0.00301	0.04474	0.00146
40	12	0.007	38	3.84E-06	0.00229	0.06786	0.00238
41	13	0.007	43	0.00016	0.00238	0.06109	0.00228
42	11	0.006	41	0.00011	0.00231	0.05626	0.00223
43	9	0.005	31	0.00019	0.00201	0.04697	0.00195
44	9	0.005	39	2.11E-05	0.00187	0.05159	0.00226
45	10	0.006	34	8.53E-05	0.00184	0.05953	0.00277
46	9	0.005	31	0.0002	0.00193	0.04972	0.00244
47	10	0.005	35	0.0001	0.00173	0.05329	0.00276
48	9	0.005	36	6.71E-05	0.00166	0.05247	0.00287
49	8	0.005	26	3.48E-05	0.00165	0.04658	0.00267
50	8	0.004	31	0.00013	0.00155	0.04421	0.00265
51	9	0.005	31	0.00015	0.00141	0.05602	0.00356
52	8	0.004	27	0.00022	0.0015	0.03907	0.00258
53	7	0.004	26	5.86E-05	0.00132	0.03857	0.00265
54	7	0.004	28	0.00014	0.00126	0.04536	0.00327
55	7	0.004	23	2.27E-05	0.00125	0.04009	0.00301
56	6	0.004	24	6.73E-05	0.0012	0.03581	0.00279
57	7	0.004	27	4.72E-05	0.0011	0.0445	0.00363
58	7	0.004	22	0.00015	0.00121	0.03699	0.00313
59	7	0.004	25	7.16E-05	0.00106	0.04141	0.00366
60	6	0.003	25	8.14E-05	0.0011	0.03452	0.00316
61	6	0.004	20	4.44E-05	0.00098	0.03962	0.00377
62	5	0.003	19	3.50E-05	0.001	0.02779	0.00272
63	6	0.004	24	5.19E-05	0.00088	0.04392	0.0045
64	5	0.003	17	0.00012	0.00101	0.02671	0.00281
65	5	0.003	20	4.20E-05	0.00086	0.03491	0.00381
66	5	0.003	19	3.30E-05	0.00091	0.03174	0.00358
67	5	0.003	18	2.86E-05	0.00083	0.02979	0.00346
68	5	0.003	19	8.70E-05	0.0008	0.02865	0.00342
69	5	0.003	21	7.45E-06	0.00077	0.03314	0.0041
70	5	0.003	15	3.75E-06	0.00083	0.02995	0.00382
71	5	0.003	16	5.17E-06	0.00076	0.03343	0.00441
72	4	0.002	15	4.97E-05	0.00077	0.02365	0.00319
73	5	0.003	17	1.12E-05	0.00066	0.0328	0.00458
74	4	0.002	18	1.34E-06	0.00077	0.02373	0.00339

75	5	0.003	17	6.68E-05	0.00064	0.03004	0.00443
76	4	0.002	16	3.16E-05	0.00069	0.02428	0.00367
77	4	0.002	16	1.26E-05	0.00063	0.02673	0.00415
78	4	0.002	16	2.55E-05	0.00063	0.02773	0.00443
79	4	0.003	16	1.11E-05	0.00063	0.02842	0.00467
80	3	0.002	15	2.51E-05	0.00064	0.02068	0.00347
81	4	0.002	14	3.23E-06	0.00055	0.02856	0.00493
82	4	0.002	12	1.73E-05	0.00061	0.02513	0.00445
83	4	0.002	15	6.51E-05	0.00056	0.02516	0.00457
84	4	0.002	13	5.47E-05	0.00055	0.02534	0.00472
85	3	0.002	13	4.99E-05	0.00055	0.01974	0.00376
86	4	0.002	13	4.74E-05	0.0005	0.02726	0.00533
87	3	0.002	11	5.69E-05	0.00055	0.01723	0.00343
88	4	0.002	12	1.74E-05	0.00047	0.02655	0.00543
89	3	0.002	13	2.67E-05	0.00053	0.0196	0.00409
90	3	0.002	13	1.07E-05	0.00046	0.02368	0.00506
91	3	0.002	12	1.12E-05	0.00049	0.01882	0.0041
92	3	0.002	13	2.32E-05	0.00045	0.02492	0.00556
93	3	0.002	12	5.23E-06	0.00047	0.01824	0.00415
94	3	0.002	11	9.95E-07	0.00044	0.02285	0.00532
95	3	0.002	10	9.26E-06	0.00046	0.01762	0.00417
96	3	0.002	12	1.04E-05	0.00042	0.02331	0.00565
97	3	0.002	11	8.87E-06	0.00042	0.01777	0.00439
98	2	0.001	12	3.83E-05	0.00042	0.01457	0.00365
99	3	0.002	12	4.89E-05	0.00038	0.02462	0.00632
100	2	0.001	8	5.66E-05	0.00046	0.01379	0.00359
101	3	0.002	11	2.43E-05	0.00037	0.02167	0.00577
102	3	0.002	11	3.53E-05	0.00041	0.01756	0.00476
103	3	0.002	9	3.11E-06	0.00039	0.02097	0.0058
104	2	0.001	9	1.71E-05	0.0004	0.01598	0.0045
105	2	0.001	9	3.12E-05	0.00037	0.0159	0.00454
106	3	0.002	11	5.55E-05	0.00034	0.0184	0.00536
107	3	0.002	10	4.20E-05	0.00037	0.01665	0.00493
108	3	0.002	9	2.12E-05	0.00035	0.01904	0.00575
109	2	0.001	10	5.37E-06	0.00037	0.0151	0.00463
110	2	0.001	9	1.63E-05	0.00032	0.01648	0.00514
111	2	0.001	8	1.80E-05	0.00033	0.01491	0.00472
112	2	0.001	10	3.22E-05	0.00033	0.01598	0.00514

113	2	0.001	7	2.57E-05	0.00033	0.01475	0.00481
114	2	0.001	8	3.43E-05	0.00033	0.01608	0.00533
115	2	0.001	9	3.50E-05	0.00031	0.0171	0.00577
116	2	0.001	8	2.32E-05	0.00032	0.01451	0.00497
117	2	0.001	9	1.77E-05	0.0003	0.01095	0.00379
118	2	0.002	9	3.61E-05	0.00027	0.01959	0.00692
119	2	0.001	7	5.12E-05	0.00034	0.01154	0.00412
120	2	0.001	7	2.54E-05	0.00027	0.01609	0.00584
121	2	0.001	9	3.02E-07	0.0003	0.0132	0.00486
122	2	0.001	8	6.32E-06	0.00028	0.01604	0.006
123	2	0.001	8	2.95E-06	0.00029	0.01285	0.00487
124	2	0.001	6	1.24E-06	0.00027	0.01464	0.00563
125	2	0.001	8	7.10E-06	0.00028	0.01146	0.00446
126	2	0.001	7	4.46E-06	0.00025	0.0162	0.0064
127	2	0.001	6	9.33E-06	0.00029	0.01206	0.00483
128	2	0.001	7	1.13E-05	0.00026	0.01493	0.00607
129	2	0.001	8	1.63E-05	0.00027	0.01057	0.00434
130	2	0.001	8	1.56E-05	0.00024	0.01288	0.00535
131	2	0.001	6	1.80E-05	0.00025	0.0143	0.00603
132	2	0.001	7	1.60E-06	0.00026	0.01393	0.00596
133	2	0.001	6	1.44E-06	0.00026	0.01253	0.00543
134	2	0.001	7	1.52E-05	0.00024	0.01172	0.00514
135	1	0.001	7	2.70E-05	0.00024	0.00966	0.00427
136	2	0.001	7	2.73E-05	0.00022	0.0147	0.00661
137	1	0.001	5	3.07E-05	0.00026	0.00934	0.00423
138	2	0.001	6	1.58E-05	0.00021	0.01212	0.00556
139	2	0.001	5	1.23E-05	0.00024	0.01127	0.00523
140	2	0.001	7	6.23E-06	0.00022	0.0108	0.00507
141	2	0.001	7	1.31E-06	0.00023	0.01128	0.00536
142	2	0.001	7	1.55E-06	0.00022	0.01283	0.00617
143	2	0.001	5	6.04E-06	0.00023	0.01075	0.00523
144	2	0.001	7	1.37E-05	0.00022	0.01248	0.00615
145	2	0.001	6	1.52E-05	0.00022	0.01076	0.00535
146	2	0.001	6	4.17E-06	0.00021	0.01164	0.00586
147	1	0.001	5	1.44E-06	0.00022	0.00872	0.00443
148	1	0.001	5	1.84E-06	0.0002	0.0107	0.0055
149	1	0.001	5	8.23E-06	0.0002	0.01037	0.00538
150	2	0.001	6	5.83E-06	0.00021	0.01054	0.00553

151	1	0.001	6	1.58E-06	0.0002	0.00986	0.00522
152	2	0.001	6	3.78E-06	0.00019	0.01111	0.00595
153	1	0.001	5	1.89E-05	0.0002	0.00766	0.00413
154	2	0.001	6	2.84E-05	0.00019	0.01247	0.00682
155	2	0.001	6	1.57E-05	0.00021	0.01072	0.00592
156	1	0.001	6	2.36E-05	0.0002	0.01026	0.00573
157	1	0.001	5	2.14E-05	0.00019	0.00889	0.00501
158	1	0.001	5	7.56E-06	0.00019	0.00904	0.00514
159	1	0.001	5	4.28E-06	0.00018	0.011	0.00632
160	1	0.001	5	8.61E-06	0.00019	0.00981	0.00569
161	1	0.001	6	1.39E-05	0.00019	0.00982	0.00575
162	1	0.001	5	1.23E-05	0.00019	0.00933	0.00552
163	1	0.001	5	7.38E-06	0.00018	0.01085	0.00649
164	1	0.001	5	1.12E-06	0.00018	0.00875	0.00528
165	1	0.001	5	1.42E-05	0.00017	0.00832	0.00506
166	1	0.001	6	3.21E-05	0.00017	0.01122	0.0069
167	1	0.001	5	3.78E-05	0.00018	0.00787	0.00488
168	1	0.001	4	2.94E-06	0.00017	0.0096	0.00601
169	1	0.001	6	1.08E-05	0.00017	0.01043	0.0066
170	1	0.001	5	9.00E-06	0.00018	0.00916	0.00585
171	1	0.001	5	8.45E-06	0.00017	0.00842	0.00542
172	1	0.001	5	4.54E-07	0.00016	0.00906	0.00589
173	1	0.001	5	1.11E-05	0.00017	0.00977	0.00641
174	1	0.001	4	1.23E-05	0.00017	0.00837	0.00554
175	1	0.001	5	3.18E-06	0.00016	0.0096	0.00641
176	1	0.001	5	4.88E-06	0.00016	0.00738	0.00497
177	1	0.001	5	1.40E-06	0.00015	0.00873	0.00593
178	1	0.001	5	6.56E-06	0.00017	0.00672	0.00459
179	1	0.001	4	1.66E-05	0.00014	0.00973	0.00672
180	1	0.001	4	1.02E-05	0.00016	0.00675	0.00469
181	1	0.001	4	2.63E-06	0.00015	0.00826	0.00579
182	1	0.001	5	4.90E-06	0.00016	0.00751	0.0053
183	1	0.001	4	5.81E-06	0.00014	0.00998	0.00712
184	1	0.001	4	1.36E-05	0.00017	0.00638	0.00458
185	1	0.001	5	9.76E-06	0.00014	0.01084	0.00787
186	1	0.001	5	3.55E-06	0.00016	0.00699	0.00511
187	1	0.001	5	1.38E-05	0.00014	0.00946	0.00698
188	1	0.001	4	1.26E-05	0.00015	0.00653	0.00485

189	1	0.001	4	1.23E-05	0.00014	0.0067	0.00501
190	1	0.001	4	1.24E-05	0.00014	0.00808	0.00609
191	1	0.001	4	5.18E-06	0.00015	0.00717	0.00545
192	1	0.001	4	9.08E-08	0.00014	0.00868	0.00665
193	1	0.001	4	1.08E-05	0.00014	0.00782	0.00603
194	1	0.001	4	1.67E-05	0.00014	0.00741	0.00576
195	1	0.001	4	1.08E-05	0.00014	0.0068	0.00533
196	1	0.001	4	3.84E-06	0.00013	0.00695	0.00548
197	1	0.001	3	1.80E-05	0.00014	0.00664	0.00527
198	1	0.001	4	1.94E-05	0.00013	0.00839	0.00672
199	1	0.001	4	1.24E-05	0.00014	0.00726	0.00586
200	1	0.001	4	1.29E-05	0.00013	0.00821	0.00668
201	1	0.001	4	1.82E-05	0.00013	0.0062	0.00507
202	1	0.001	4	1.41E-06	0.00013	0.0069	0.00569
203	1	0.001	4	2.21E-05	0.00013	0.0073	0.00606
204	1	0.001	4	1.79E-05	0.00013	0.00644	0.00538
205	1	0.001	4	7.72E-06	0.00012	0.00776	0.00654
206	1	0.001	4	3.67E-06	0.00013	0.00682	0.00578
207	1	0.001	3	8.56E-06	0.00013	0.0067	0.00572
208	1	0.001	4	4.99E-06	0.00012	0.00769	0.00662
209	1	0	4	7.44E-06	0.00013	0.0049	0.00423
210	1	0.001	4	3.67E-06	0.00011	0.009	0.00785
211	1	0.001	4	1.65E-06	0.00014	0.00633	0.00556
212	1	0.001	4	1.10E-05	0.00012	0.0079	0.00699
213	1	0.001	3	7.53E-06	0.00013	0.00592	0.00527
214	1	0	3	8.32E-06	0.00012	0.00561	0.00502
215	1	0	4	2.20E-06	0.00011	0.00597	0.00537
216	1	0	3	6.98E-07	0.00012	0.00574	0.0052
217	1	0.001	3	5.05E-06	0.00012	0.00732	0.00668
218	1	0.001	4	6.12E-06	0.00012	0.00627	0.00575
219	1	0.001	3	8.78E-07	0.00012	0.00693	0.00641
220	1	0	3	3.39E-06	0.00012	0.00501	0.00465
221	1	0.001	3	2.57E-06	0.00011	0.00818	0.00766
222	1	0.001	3	2.84E-07	0.00013	0.00612	0.00577
223	1	0.001	4	2.79E-06	0.00011	0.0067	0.00635
224	1	0	4	1.21E-06	0.00011	0.00599	0.00572
225	1	0	3	4.05E-07	0.00011	0.00599	0.00575
226	1	0	4	3.65E-06	0.00011	0.0056	0.0054

227	1	0.001	3	9.61E-06	0.00011	0.00657	0.00638
228	1	0	3	2.77E-06	0.00012	0.00596	0.00582
229	1	0	3	7.87E-06	0.00011	0.0054	0.00531
230	1	0.001	3	2.00E-05	0.0001	0.00752	0.00745
231	1	0	3	1.35E-05	0.00012	0.00428	0.00426
232	1	0	3	3.20E-06	9.76E-05	0.00513	0.00513

APPENDIX D

CUBE-PTA_SOURCE CODE

```
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```

SET-UP INPUTS/OUTPUTS

```
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ARRAY SOV1=4485924 SOV2=4485924 HOV1=4485924 HOV2=4485924 TRK1=4485924
TRK2=4485924



zones={totzones}
  parameters maxiters = {iteration},                    ;Max numbers of
iterations allowed
  gap=0.0005,RELATIVEGAP=0.005,aad=0, raad=0, combine=Equi

  phase=linkread              ;read in link variables from input file
    C  = li.capacity*{capacity}
    T0 = li.distance*60/li.speed

    lw.v_sovee = li.preld_sov
    lw.v_comee = li.preld_com
    lw.v_mtkee = li.preld_mtk
    lw.v_htkee = li.preld_htk

    linkclass=li.factype+1      ;define linkclass by assignment group

   ; Set Prohibitions
    IF (li.prohibition==1) ADDTOGROUP=2  ;no-truck LINKS
    IF (li.prohibition==2) ADDTOGROUP=3  ;HOV 2 LINKS
    IF (LI.prohibition==3) ADDTOGROUP=6  ; Managed lanes
    IF (LI.prohibition==4) ADDTOGROUP=7  ; Truck only lanes
    if (li.factype> 49)    ADDTOGROUP=4  ;transit only
    IF (li.prohibition==5) ADDTOGROUP=5  ;links within I-285 (for HTK)
IF (li.prohibition==10) ADDTOGROUP=10 ;; CM_Tolled Facilities

   ; Set Fixed Toll Penalties
   ; Truck toll = 3 * auto toll, VOT = $60/hr (1.0 min/$)
    lw.trkaddtime=(li.toll*3)*1
   ; HOV toll = 0.4 * auto toll (avg 2.5 persons/veh), VOT = $15/hr
(4.0 min/$)
    lw.hovaddtime=(li.toll*0.4)*4
```

```
   ; SOV toll, VOT = $15/hr (4.0 min/$)
    lw.sovaddtime=li.toll*4


   ; Add Time Penalty for Trucks at the Financial Center Tunnel
(GA400)
    if ((a=8650 & b=8679) | (a=8678 & b=8651))
lw.trkaddtime=lw.trkaddtime+5   ;PKS Corrected 06/08/04

   ; Set Time
    lw.trktime= T0 + lw.trkaddtime
    lw.hovtime= T0 + lw.hovaddtime
    lw.sovtime= T0 + lw.sovaddtime

   lw.sovtoll = li.distance*li.SOV_TOLL
   lw.comtoll = li.distance*li.COM_TOLL
   lw.hovtoll = li.distance*li.HOV_TOLL

  endphase

;setup phase
  FUNCTION {                                            ;change TP+
default functions
    TC[1]=   T0/CURVE{period}(1,V/C)
    TC[2]=   T0/CURVE{period}(2,V/C)
    TC[3]=   T0/CURVE{period}(3,V/C)
    TC[4]=   T0/CURVE{period}(4,V/C)
    TC[5]=   T0/CURVE{period}(5,V/C)
    TC[6]=   T0/CURVE{period}(6,V/C)
    TC[7]=   T0/CURVE{period}(7,V/C)
    TC[8]=   T0/CURVE{period}(8,V/C)
    TC[9]=   T0/CURVE{period}(9,V/C)
    TC[10]=  T0/CURVE{period}(10,V/C)
    TC[11]=  T0/CURVE{period}(11,V/C)
    TC[12]=  T0/CURVE{period}(12,V/C)
    TC[13]=  T0/CURVE{period}(13,V/C)
    TC[14]=  T0/CURVE{period}(14,V/C)
    TC[15]=  T0/CURVE{period}(15,V/C)
    TC[16]=  T0/CURVE{period}(16,V/C)
    TC[17]=  T0/CURVE{period}(17,V/C)
    TC[18]=  T0/CURVE{period}(18,V/C)
    TC[19]=  T0/CURVE{period}(19,V/C)
    TC[20]=  T0/CURVE{period}(20,V/C)
    TC[21]=  T0/CURVE{period}(21,V/C)


;Re-label volumes:
;1 = SOV
 ;2 = HOV
; 3 = COM
; 4 = MTK
```

```
; 5 = HTK

;PCE factors are applied to medium duty trucks (1.5) and heavy duty
trucks (2.0) in the vdf lookup.
;The output VC ratios include the PCE factors, however, the volumes do
not!!!

;total volume function (with preloaded EE)
    V = vol[1] + vol[2] + vol[3] +vol[4] + vol[5] + vol[6]+ lw.v_sovee
+ 1.5 * (vol[8]+lw.v_mtkee) +
        2.0 * (vol[7] + lw.v_htkee) + lw.v_comee

    COST[1] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[2] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[3] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[4] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[5] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[6] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[7] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[8] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[9] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[10] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[11] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[12] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[13] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[14] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[15] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[16] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[17] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[18] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
    COST[19] = min(time+lw.sovaddtime, 163)              ; Use same
"cost" for paths and equilibrium
```

```
    COST[20] = min(time+lw.sovaddtime, 163)               ; Use same
"cost" for paths and equilibrium
    COST[21] = min(time+lw.sovaddtime, 163)               ; Use same
"cost" for paths and equilibrium
    }

  phase=iloop
   ; Congested Time with Toll Penalties

IF(iteration>1)
JLOOP
k=(I-1)*_zones+J
SOV2[k]= SOV1[k]
MW[806]= SOV2[k]
HOV2[k]= HOV1[k]
MW[906]= HOV2[k]
TRK2[k]= TRK1[k]
MW[916]= TRK2[k]
ENDJLOOP
ENDIF

MW[810]= SOV2[k]
MW[811]= SOV1[k]


    mw[1]=mi.1.1  ; SOV
    mw[2]=mi.1.2  ; HOV
    mw[3]=mi.2.{TT period}         ; COM
    mw[4]=mi.3.{TT period}         ; MTK
    mw[5]=mi.4.{TT period}         ; HTK



jloop
      if(i>{lastin} && j>{lastin})
        mw[1]=0
        mw[2]=0
        mw[3]=0
        mw[4]=0
        mw[5]=0
      endif
    endjloop


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;Toll_time and cost
pathload path=cost, excludegroup=3,4,6,7,
             MW[300] = PATHTRACE(lw.sovtime), noaccess=0,

             MW[400] = PATHTRACE(lw.sovtoll), noaccess=0,
```

```
            MW[401] = PATHTRACE(lw.comtoll), noaccess=0,

            MW[402] = PATHTRACE(lw.hovtoll), noaccess=0

COMP MW[300][I] = rowmin(300) * 0.5 ; Intrazonal time

pathload path=cost, excludegroup=3,4,6,7,
            MW[301] = PATHTRACE(lw.hovtime), noaccess=0
                    COMP MW[301][I] = rowmin(301) * 0.5 ; Intrazonal
time


;if (I < {From_Node})    _skiptoI={From_Node}
IF (I={From_Node})
pathload path=cost, excludegroup=3,4,6,7 MW[111] =
PATHTRACE(lw.sovtime),TRACE=(I={From_Node}&J={TO_Node})

pathload path=cost, excludegroup=3,4,6,7 MW[222] =
PATHTRACE(lw.sovtoll),TRACE=(I={From_Node}&J={TO_Node})

jloop
IF (J={TO_Node})
PRINT,
LIST='FromI','_','ToJ','_','Time-Toll','_','Cost-Toll'
;;;;;;;;;;;;;;;;from i to j time and cost tolled links included
LIST={From_Node}(6.0),{TO_Node}(6.0),mw[111](10.5),mw[222](10.5)
ENDIF
Endjloop
ENDIF
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;; Free_time and cost


pathload path=cost, Excludegrp=3,4,6,7,10,
                    MW[200] = PATHTRACE(lw.sovtime), noaccess=0
                            COMP MW[200][I] = rowmin(200) * 0.5 ;
Intrazonal time


pathload path=cost, Excludegrp=3,4,6,7,10,
                    MW[201] = PATHTRACE(lw.hovtime), noaccess=0
                            COMP MW[201][I] = rowmin(201) * 0.5 ;
Intrazonal time



;if (I < {From_Node})     _skiptoI={From_Node}
IF (I={From_Node})

pathload path=cost, Excludegrp=3,4,6,7,10 MW[333] =
PATHTRACE(lw.sovtime),TRACE=(I={From_Node}&J={TO_Node})
```

```
pathload path=cost, Excludegrp=3,4,6,7,10 MW[444] =
PATHTRACE(lw.sovtoll),TRACE=(I={From_Node}&J={TO_Node})

jloop
IF (J={TO_Node})

PRINT,
LIST='FromI','_','ToJ','_','Time_Free','_','Cost_Free'
;;;;;;;;;;;;;;;;;from i to j time and cost tolled links not included
LIST={From_Node}(6.0),{TO_Node}(6.0),mw[333](10.5),mw[444](10.5)

ENDIF
Endjloop

ENDIF

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;Matrix calculations_Probs.Free-Toll
jloop
MW[500][J] = (MW[200][j]- MW[300][j])  ;TD   SOV_Time Difference
(Free-Toll)
MW[501][J] = (MW[201][j]- MW[301][j])  ;TD   HOV_Time Difference

MW[600][J] = MW[400][j]  ;TollSOV   Cost Difference
MW[601][J] = MW[401][j]  ;TollHOV   Cost Difference
MW[602][J] = MW[402][j]  ;TollCOM   Cost Difference


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;SOV
if (MW[500]<=0 || MW[600]<=0)  ;threshold
MW[800][j]=0
else
MW[800][j]= 1-(1/(1+EXP(({SOV_TCOEFF}*MW[500][j])+
({SOV_CCOEFF}*MW[600][j])+ ({SOV_CONST}))))  ;SOV_tollprob.

Endif
MW[799][j]= mw[1][j]* MW[800][j] ;SOV_TOLL_TRIPS_gct
MW[801][j]= (1-(1/iteration))*MW[806]+(1/iteration)*MW[799][j]
;SOV_TOLL_TRIPS_gctlMW[802][j]= mw[1][j]- MW[801][j] ;SOV_FREE_Trips
MW[803][j]= MW[801][j]+ MW[802][j] ;SOV_ All_Trips
if (MW[501]<=0 | MW[601]<=0) ;threshold<=5 for time could work as well
MW[900][j]=0
else
MW[900][j]= 1-(1/(1+EXP(({HOV_TCOEFF}*MW[501][j])+
({HOV_CCOEFF}*MW[601][j])+ ({HOV_CONST}))))  ;HOV_tollpro.
Endif
MW[899][j]= mw[2][j]*MW[900][j]
MW[901][j]= (1-(1/iteration))*MW[906]+(1/iteration)*MW[899][j]
;;HOV_TOLL_TRIPS_gctl
MW[902][j]= mw[2][j]-MW[901][j] ;HOV_FREE_Trips
```

```
MW[903][j]=MW[901][j]+MW[902][j] ;HOV_ All_Trips
if (MW[500]<=0 | MW[602]<=0) ;threshold<=5
MW[910][j]=0
else
MW[910][j]= 1-(1/(1+EXP(({COM_TCOEFF}*MW[500][j])+
({COM_CCOEFF}*MW[602][j])+ ({COM_CONST}))))    ;COM_tollpro.
Endif
MW[999][j]= mw[3][j]*MW[910][j] ;COM_TOLL_Trips
MW[911][j]= (1-(1/iteration))*MW[916]+(1/iteration)*MW[999][j] ;
MW[912][j]= mw[3][j]-mw[911][j] ;COM_FREE_Trips
MW[913][j]=MW[911][j]+MW[912][j] ;COM ALL_Trips
endjloop


JLOOP
k=(I-1)*_zones+J
SOV1[k]= MW[801]
MW[805]= SOV1[k]
HOV1[k]= MW[901]
MW[905]= HOV1[k]
TRK1[k]= MW[911]
MW[915]= TRK1[k]
ENDJLOOP


Jloop

MW[123][j]=abs(MW[801][j]-MW[799][j])
MW[321][j]=abs(MW[901][j]-MW[899][j])
MW[132][j]=abs(MW[911][j]-MW[999][j])

Endjloop


jloop

IF (I={From_Node}&& J={TO_Node})
PRINT,
LIST='FromI','_','ToJ','_','SOV-Prob','_','SOV-Trp','_','SOV-
TOLL','_','SOV-Free'
LIST={From_Node}(6.0),{TO_Node}(6.0),mw[800](10.5),mw[1](10.5),mw[801]
(10.5),mw[802](10.5)

LIST='FromI','_','ToJ','_','HOV-Prob','_','HOV-Trp','_','HOV-
TOLL','_','HOV-Free'
LIST={From_Node}(6.0),{TO_Node}(6.0),mw[900](10.5),mw[2](10.5),mw[901]
(10.5),mw[902](10.5)

LIST='FromI','_','ToJ','_','COM-Prob','_','COM-Trp','_','COM-
TOLL','_','COM-Free'
```

```
LIST={From_Node}(6.0),{TO_Node}(6.0),mw[910](10.5),mw[3](10.5),mw[911
](10.5),mw[912](10.5)

LIST='FromI','_','ToJ','_','SOV-TD','_','HOV-TD','_','SOV-
TOLL','_','HOV-TOLL','_','COM-TOLL'
LIST={From_Node}(6.0),{TO_Node}(6.0),mw[500](10.5),mw[501](10.5),mw[60
0](10.5),mw[601](10.5),mw[602](10.5)

ENDIF
endjloop

;Separate heavy trucks for I-285 Bypass
    jloop
      if(i=109,112-113,116-123,125,131-133,182-216,226-446,462-
463,480-483,
        488-494,499-503,526-533,535-547,549-556,580,582,584-608,611-
628,
        633,641-748,753-838,842-843,847-848,852-1321,1324-1326,1328-
1645,
        1669-1671,1679,1683-2118 &&
        j=109,112-113,116-123,125,131-133,182-216,226-446,462-463,480-
483,
        488-494,499-503,526-533,535-547,549-556,580,582,584-608,611-
628,
        633,641-748,753-838,842-843,847-848,852-1321,1324-1326,1328-
1645,
        1669-1671,1679,1683-2118)

        mw[10]=mw[5] ;Outside I-285 to outside I-285
      else
        mw[11]=mw[5] ;Origin or destination inside I-285
      endif
    endjloop

;Assign SOV,HOV, COM_toll links included
    pathload path=lw.sovtime,vol[1]=mw[801],vol[3]=mw[911],
excludegrp=3,4,6,7  ;SOV,COM
    pathload path=lw.hovtime,vol[2]=mw[901],excludegrp=4,7
;HOV

;Assign SOV,HOV, COM_toll links NOT included
    pathload path=lw.sovtime,
vol[4]=mw[802],vol[6]=mw[912],excludegrp=3,4,6,7,10  ;SOV,COM
    pathload path=lw.hovtime, vol[5]=mw[902],excludegrp=4,7,10
;HOV

;Assign some Heavy Trucks to a path that does not go inside I-285
    pathload path=lw.trktime, vol[7]=mw[10], excludegrp=2,3,4,5,6,10

; Assign other Heavy Trucks and all Medium Trucks to "normal" path
```

```
    pathload path=lw.trktime, vol[8]=mw[4],vol[7]=mw[11],
excludegrp=2,3,4,6,10

; Volume/delay functions, by period
    LOOKUP INTERPOLATE=T, NAME=CURVEAM,      ;eqv. to speed curves in
TRANPLAN setup
    LOOKUP[1] =1, RESULT=7,                  ;centroid connector
    LOOKUP[2] =1, RESULT=2,                  ;freeway
    LOOKUP[3] =1, RESULT=2,                  ;parkway
    LOOKUP[4] =1, RESULT=2,                  ;HOV buffer seperated
    LOOKUP[5] =1, RESULT=2,                  ;hov barrier seperated
    LOOKUP[6] =1, RESULT=2,                  ;High speed ramp
    LOOKUP[7] =1, RESULT=2,                  ;Medium speed ramp
    LOOKUP[8] =1, RESULT=2,                  ;low speed ramp
    LOOKUP[9] =1, RESULT=2,                  ;Loop Ramp
    LOOKUP[10]=1, RESULT=2,                  ;Off Ramp/with
intersection
    LOOKUP[11]=1, RESULT=2,                  ;On Ramp/with intersection
    LOOKUP[12]=1, RESULT=3,                  ;Express Way
    LOOKUP[13]=1, RESULT=4,                  ;Principle Arterial -
Class I
    LOOKUP[14]=1, RESULT=4,                  ;Principle Arterial -
Class II
    LOOKUP[15]=1, RESULT=5,                  ;Minor Arterial - Class 1
    LOOKUP[16]=1, RESULT=5,                  ;Minor Arterial - Class 2
    LOOKUP[17]=1, RESULT=5,                  ;HOV-arterial
    LOOKUP[18]=1, RESULT=6,                  ;Collector
    LOOKUP[19]=1, RESULT=6,                  ;Other Local
    LOOKUP[20]=1, RESULT=2,                  ;Planned Ramp/with
intersections
    LOOKUP[21]=1, RESULT=2,                  ;Planned directional ramp
with intersections

;   V/C  Freeway Exrswy Prin Art Min Art Collector Cent.
 R='0.00   1.000  1.000   1.000   1.000   1.000   1.000 ',
   '0.10   0.995  0.995   0.995   0.992   0.990   0.960 ',
   '0.20   0.990  0.990   0.990   0.975   0.960   0.920 ',
   '0.30   0.950  0.950   0.950   0.935   0.920   0.880 ',
   '0.40   0.910  0.910   0.910   0.880   0.860   0.800 ',
   '0.50   0.860  0.860   0.860   0.830   0.800   0.720 ',
   '0.60   0.790  0.790   0.790   0.760   0.730   0.640 ',
   '0.70   0.670  0.670   0.670   0.650   0.630   0.560 ',
   '0.80   0.560  0.560   0.560   0.540   0.520   0.480 ',
   '0.90   0.460  0.460   0.460   0.450   0.420   0.400 ',
   '1.00   0.350  0.350   0.350   0.340   0.310   0.360 ',

   '1.10   0.240  0.240   0.240   0.230   0.210   0.320 ',
   '1.20   0.160  0.160   0.160   0.160   0.160   0.280 ',
   '1.30   0.150  0.150   0.150   0.150   0.150   0.240 ',
   '1.40   0.140  0.140   0.140   0.140   0.140   0.200 ',
   '1.50   0.130  0.130   0.130   0.130   0.130   0.160 ',
```

```
   '1.60   0.120  0.120   0.120   0.120   0.120   0.120 ',
   '1.70   0.115  0.115   0.115   0.115   0.115   0.080 ',
   '1.80   0.110  0.110   0.110   0.110   0.110   0.080 ',
   '1.90   0.105  0.105   0.105   0.105   0.105   0.080 ',
   '2.00   0.100  0.100   0.100   0.100   0.100   0.080 ',
   '99.00   0.010  0.010   0.010   0.010   0.010   0.010 '

    LOOKUP INTERPOLATE=T, NAME=CURVEMD,      ;eqv. to speed curves in
TRANPLAN setup
    LOOKUP[1] =1, RESULT=7,                  ;centroid connector
    LOOKUP[2] =1, RESULT=2,                  ;freeway
    LOOKUP[3] =1, RESULT=2,                  ;parkway
    LOOKUP[4] =1, RESULT=2,                  ;HOV buffer seperated
    LOOKUP[5] =1, RESULT=2,                  ;hov barrier seperated
    LOOKUP[6] =1, RESULT=2,                  ;High speed ramp
    LOOKUP[7] =1, RESULT=2,                  ;Medium speed ramp
    LOOKUP[8] =1, RESULT=2,                  ;low speed ramp
    LOOKUP[9] =1, RESULT=2,                  ;Loop Ramp
    LOOKUP[10]=1, RESULT=2,                  ;Off Ramp/with
intersection
    LOOKUP[11]=1, RESULT=2,                  ;On Ramp/with intersection
    LOOKUP[12]=1, RESULT=3,                  ;Express Way
    LOOKUP[13]=1, RESULT=4,                  ;Principle Arterial -
Class I
    LOOKUP[14]=1, RESULT=4,                  ;Principle Arterial -
Class II
    LOOKUP[15]=1, RESULT=5,                  ;Minor Arterial - Class 1
    LOOKUP[16]=1, RESULT=5,                  ;Minor Arterial - Class 2
    LOOKUP[17]=1, RESULT=5,                  ;HOV-arterial
    LOOKUP[18]=1, RESULT=6,                  ;Collector
    LOOKUP[19]=1, RESULT=6,                  ;Other Local
    LOOKUP[20]=1, RESULT=2,                  ;Planned Ramp/with
intersections
    LOOKUP[21]=1, RESULT=2,                  ;Planned directional ramp
with intersections

;  V/C  Freeway  Exrswy Prin Art Min Art Collector Cent.
   R= ' 0.00   1.000   1.000   1.000   1.000   1.000   1.000 ' ,
  ' 0.10   0.995   0.995   0.995   0.992   0.990   0.960 ' ,
  ' 0.20   0.985   0.985   0.985   0.975   0.970   0.920 ' ,
  ' 0.30   0.975   0.975   0.975   0.960   0.950   0.880 ' ,
  ' 0.40   0.950   0.950   0.950   0.930   0.915   0.800 ' ,
  ' 0.50   0.920   0.920   0.920   0.900   0.870   0.720 ' ,
  ' 0.60   0.890   0.890   0.890   0.865   0.830   0.640 ' ,
  ' 0.70   0.830   0.830   0.830   0.800   0.770   0.560 ' ,
  ' 0.80   0.740   0.740   0.740   0.710   0.660   0.480 ' ,
  ' 0.90   0.500   0.500   0.500   0.500   0.500   0.400 ' ,
  ' 1.00   0.310   0.310   0.310   0.310   0.310   0.360 ' ,
  ' 1.10   0.210   0.210   0.210   0.210   0.210   0.320 ' ,
  ' 1.20   0.160   0.160   0.160   0.160   0.160   0.280 ' ,
  ' 1.30   0.150   0.150   0.150   0.150   0.150   0.240 ' ,
```

```
 ' 1.40   0.140   0.140   0.140   0.140   0.140   0.200 ' ,
 ' 1.50   0.130   0.130   0.130   0.130   0.130   0.160 ' ,
 ' 1.60   0.120   0.120   0.120   0.120   0.120   0.120 ' ,
 ' 1.70   0.115   0.115   0.115   0.115   0.115   0.080 ' ,
 ' 1.80   0.110   0.110   0.110   0.110   0.110   0.080 ' ,
 ' 1.90   0.105   0.105   0.105   0.105   0.105   0.080 ' ,
 ' 2.00   0.100   0.100   0.100   0.100   0.100   0.080 ' ,
 '99.00   0.010   0.010   0.010   0.010   0.010   0.010 '

    LOOKUP INTERPOLATE=T, NAME=CURVEPM,      ;eqv. to speed curves in
TRANPLAN setup
    LOOKUP[1] =1, RESULT=7,                  ;centroid connector
    LOOKUP[2] =1, RESULT=2,                  ;freeway
    LOOKUP[3] =1, RESULT=2,                  ;parkway
    LOOKUP[4] =1, RESULT=2,                  ;HOV buffer seperated
    LOOKUP[5] =1, RESULT=2,                  ;hov barrier seperated
    LOOKUP[6] =1, RESULT=2,                  ;High speed ramp
    LOOKUP[7] =1, RESULT=2,                  ;Medium speed ramp
    LOOKUP[8] =1, RESULT=2,                  ;low speed ramp
    LOOKUP[9] =1, RESULT=2,                  ;Loop Ramp
    LOOKUP[10]=1, RESULT=2,                  ;Off Ramp/with
intersection
    LOOKUP[11]=1, RESULT=2,                  ;On Ramp/with intersection
    LOOKUP[12]=1, RESULT=3,                  ;Express Way
    LOOKUP[13]=1, RESULT=4,                  ;Principle Arterial -
Class I
    LOOKUP[14]=1, RESULT=4,                  ;Principle Arterial -
Class II
    LOOKUP[15]=1, RESULT=5,                  ;Minor Arterial - Class 1
    LOOKUP[16]=1, RESULT=5,                  ;Minor Arterial - Class 2
    LOOKUP[17]=1, RESULT=5,                  ;HOV-arterial
    LOOKUP[18]=1, RESULT=6,                  ;Collector
    LOOKUP[19]=1, RESULT=6,                  ;Other Local
    LOOKUP[20]=1, RESULT=2,                  ;Planned Ramp/with
intersections
    LOOKUP[21]=1, RESULT=2,                  ;Planned directional ramp
with intersections

;        V/C   Freeway  Exrsway Prin Art Min Art Collector Cent.
    R=   '0.00   1.000   1.000   1.000   1.000   1.000   1.000' ,

  '0.10   0.995   0.995   0.995   0.995   0.990   0.960' ,
  '0.20   0.990   0.990   0.990   0.990   0.980   0.920' ,
  '0.30   0.970   0.970   0.970   0.960   0.950   0.880' ,
  '0.40   0.940   0.940   0.940   0.930   0.915   0.800' ,
  '0.50   0.910   0.910   0.910   0.900   0.870   0.720' ,
  '0.60   0.870   0.870   0.870   0.865   0.830   0.640' ,
  '0.70   0.820   0.820   0.820   0.800   0.770   0.560' ,
  '0.80   0.720   0.720   0.720   0.710   0.680   0.480' ,
  '0.90   0.570   0.570   0.570   0.560   0.560   0.400' ,
  '1.00   0.400   0.400   0.400   0.390   0.390   0.360' ,
```

```
  '1.10   0.280   0.280   0.280   0.280   0.280   0.320' ,
  '1.20   0.200   0.200   0.200   0.200   0.200   0.280' ,
  '1.30   0.160   0.160   0.160   0.160   0.160   0.240' ,
  '1.40   0.140   0.140   0.140   0.140   0.140   0.200' ,
  '1.50   0.130   0.130   0.130   0.130   0.130   0.160' ,
  '1.60   0.120   0.120   0.120   0.120   0.120   0.120' ,
  '1.70   0.115   0.115   0.115   0.115   0.115   0.080' ,
  '1.80   0.110   0.110   0.110   0.110   0.110   0.080' ,
  '1.90   0.105   0.105   0.105   0.105   0.105   0.080' ,
  '2.00   0.100   0.100   0.100   0.100   0.100   0.080' ,
 '99.00   0.010   0.010   0.010   0.010   0.010   0.010 '

    LOOKUP INTERPOLATE=T, NAME=CURVENT,      ;eqv. to speed curves in
TRANPLAN setup
    LOOKUP[1] =1, RESULT=7,                  ;centroid connector
    LOOKUP[2] =1, RESULT=2,                  ;freeway
    LOOKUP[3] =1, RESULT=2,                  ;parkway
    LOOKUP[4] =1, RESULT=2,                  ;HOV buffer seperated
    LOOKUP[5] =1, RESULT=2,                  ;hov barrier seperated
    LOOKUP[6] =1, RESULT=2,                  ;High speed ramp
    LOOKUP[7] =1, RESULT=2,                  ;Medium speed ramp
    LOOKUP[8] =1, RESULT=2,                  ;low speed ramp
    LOOKUP[9] =1, RESULT=2,                  ;Loop Ramp
    LOOKUP[10]=1, RESULT=2,                  ;Off Ramp/with
intersection
    LOOKUP[11]=1, RESULT=2,                  ;On Ramp/with intersection
    LOOKUP[12]=1, RESULT=3,                  ;Express Way
    LOOKUP[13]=1, RESULT=4,                  ;Principle Arterial -
Class I
    LOOKUP[14]=1, RESULT=4,                  ;Principle Arterial -
Class II
    LOOKUP[15]=1, RESULT=5,                  ;Minor Arterial - Class 1
    LOOKUP[16]=1, RESULT=5,                  ;Minor Arterial - Class 2
    LOOKUP[17]=1, RESULT=5,                  ;HOV-arterial
    LOOKUP[18]=1, RESULT=6,                  ;Collector
    LOOKUP[19]=1, RESULT=6,                  ;Other Local
    LOOKUP[20]=1, RESULT=2,                  ;Planned Ramp/with
intersections
    LOOKUP[21]=1, RESULT=2,                  ;Planned directional ramp
with intersections

;         V/C  Freeway  Exrswy Prin Art Min Art Collector Cent.
   R= ' 0.00   1.000   1.000   1.000   1.000   1.000   1.000 ' ,
 ' 0.10   0.990   0.990   0.990   0.990   0.980   0.960 ' ,
 ' 0.20   0.940   0.940   0.940   0.930   0.920   0.920 ' ,
 ' 0.30   0.770   0.770   0.770   0.775   0.780   0.880 ' ,
 ' 0.40   0.630   0.630   0.630   0.660   0.700   0.800 ' ,
 ' 0.50   0.550   0.550   0.550   0.590   0.630   0.720 ' ,
 ' 0.60   0.500   0.500   0.500   0.550   0.600   0.640 ' ,
 ' 0.70   0.400   0.400   0.400   0.450   0.500   0.560 ' ,
 ' 0.80   0.330   0.330   0.330   0.350   0.400   0.480 ' ,
```

```
 ' 0.90   0.280   0.280   0.280   0.280   0.280   0.400 ' ,
 ' 1.00   0.220   0.220   0.220   0.220   0.220   0.360 ' ,
 ' 1.10   0.180   0.180   0.180   0.180   0.180   0.320 ' ,
 ' 1.20   0.160   0.160   0.160   0.160   0.160   0.280 ' ,
 ' 1.30   0.150   0.150   0.150   0.150   0.150   0.240 ' ,
 ' 1.40   0.140   0.140   0.140   0.140   0.140   0.200 ' ,
 ' 1.50   0.130   0.130   0.130   0.130   0.130   0.160 ' ,
 ' 1.60   0.120   0.120   0.120   0.120   0.120   0.120 ' ,
 ' 1.70   0.115   0.115   0.115   0.115   0.115   0.080 ' ,
 ' 1.80   0.110   0.110   0.110   0.110   0.110   0.080 ' ,
 ' 1.90   0.105   0.105   0.105   0.105   0.105   0.080 ' ,
 ' 2.00   0.100   0.100   0.100   0.100   0.100   0.080 ' ,
 '99.00   0.010   0.010   0.010   0.010   0.010   0.010 '

  endphase

PHASE=ADJUST


            lw.trktime= time + lw.trkaddtime
            lw.hovtime= time + lw.hovaddtime
            lw.sovtime= time + lw.sovaddtime

            lw.sovtoll = li.distance*li.SOV_TOLL
            lw.comtoll = li.distance*li.COM_TOLL
            lw.hovtoll = li.distance*li.HOV_TOLL


ENDPHASE

PROCESS  PHASE=CONVERGE

IF (GAP <= GAPCUTOFF && RGAP<= RGAPCUTOFF && iteration> {iteration})
BALANCE=1

ENDPROCESS

ENDRUN
```

REFERENCES

[1] National Research Council. *Estimating Toll Road Demand and Revenue*. NCHRP 364. Washington D.C.: Transportation Research Board, 2007. Print.

[2] Urban Analytics Inc., and URS Corporation. *Estimating Demand for Value Pricing Projects—State of the Practice*. Arlington, TX: North Central Texas Council of Governments, 2004.

[3] Regan, E. “Moving Off the Gas Tax Implications for the Toll Industry.” *Tollways* 1.1 (2004): 2–5.

[4] Gustavo, B.A. *Investment-Quality Surveys*. Austin, TX: Texas Turnpike Authority, 2004.

[5] Muller, R., and K. Buono. *Start-up Toll Roads: Separating Winners from Losers.* New York: Municipal Credit Monitor, J.P. Morgan Securities Inc., 2002.

[6] General Accounting Office. *Private Sector Sponsorship of and Investment in Major Projects Has Been Limited.* GAO-04-419. Washington D.C.: n.p. 2004.

[7] George, C., W. Streeter, and S. Trommer. *Bliss, Heartburn, and Toll Road Forecasts*. New York: Public Finance, Fitch Ratings, 2003. Print.

[8] Muller, R. *Examining Toll Road Feasibility Studies*. New York: Municipal Credit Monitor, J.P. Morgan Securities Inc., 1996. Print.

[9] Bain, R., and M. Wilkins. *Credit Implications of Traffic Risk in Start-up Toll Facilities*. New York: Standard & Poor’s, 2002. Print.

[10] Meyer, M., and E. Miller. *Urban Transportation Planning: A Decision-Oriented Approach*. 2nd ed. New York: McGraw– Hill, 2001. Print.

[11] Delcan Corporation, ADEC Consultants, and iTRANS Consulting. *Costs of Congestion in Canada's Transportation Sector, Final Report*. Ottawa, ON, Canada: Transport Canada, 2005.

[12] Dehghani, Y., and W. Olsen. *Potential Pitfalls in Forecasting Travel Demand for Toll Roads: Experience in the U.S. and Overseas*. Paper 99051. Washington D.C.: Transportation Research Board, 1998.

[13] Spears, B. "A Summary of the Current State of the Practice in Modeling Road Pricing." *Expert Forum on Road Pricing and Travel Demand Modeling*. Washington D.C.: U.S. Department of Transportation, 2005.

[14] Vollmer Associates LLP. *183A Project Traffic and Revenue Study.* Final Report, Austin, TX: Central Texas Regional Mobility Authority. 2004.

[15] Flyvbjerg, B., M.K.S. Holm, and S.L. Buhl. "How (in) Accurate Are Demand Forecasts in Public Works Projects? The Case of Transportation". *Journal of the American Planning Association* 71.2 (2005): 131–146.

[16] URS Consultants Inc. *Mid-Bay Bridge Actual Revenue Growth.* Annual Report. Niceville: State of Florida Mid-Bay Bridge Authority, 1995.

[17] URS Consultants Inc., and Coverdale & Colpitts. *Traffic and Earnings Report for the Proposed Choctawhatchee Bay Bridge.* Niceville, FL: State of Florida Mid-Bay Bridge Authority Revenue Bonds, Series 1991B, 1991.

[18] URS Consultants Inc. *Garcon Point Bridge, Emergency Transportation Operations.* Annual Report. Tallahassee, FL: Florida Department of Transportation, 2004.

[19] URS Consultants Inc., and Coverdale & Colpitts. *Traffic and Earnings Report for the Proposed Garcon Point–Redfish Point Toll Bridge.* Santa Rosa, FL: Santa Rosa Bay Bridge Authority Revenue Bonds. Series 1996, 1996.

[20] Wilbur Smith Associates. *Route 895 Connector Comprehensive Traffic and Revenue Study*. Richmond, VA: Pocahontas Parkway Association Route 895 Connector Toll Road Revenue Bonds, 1998.

[21] Vollmer Associates LLP, PBConsult Inc., Cambridge Systematics Inc., and Carter & Burgess. *Northwest Parkway Traffic and Revenue Study.* Broomfield, CO: Northwest Parkway Public Highway Authority Revenue Bonds, 2001.

[22] Bain, R., and L. Polakovic. *Traffic Forecasting Risk: Study Update 2005: Through Ramp-up and Beyond*. New York: Standard & Poor's, 2005. Print.

[23] Vollmer Associates LLP. *Final Traffic and Revenue Report—Transportation Corridor System—Including the San Joaquin Hills, Foothill and Eastern Transportation Corridors*. Orange County, CA: Transportation Corridor Agencies, 2003.

[24] U.S. Department of Transportation. *The Value of Saving Travel Time: Department Guidance for the Valuation of Travel Time in Economic Analysis.* Washington, D.C.: U.S. Department of Transportation, 1997.

[25] Bain, R. and J. Plantagie. *Traffic Forecasting Risk: Study Update 2003*. New York, NY: Reason Public Policy Institute, Standard & Poor's, 2003.

[26] Bruce, D. *Comments Related to Recommended Best Practices in Disclosure for Toll Road Financings*. Unpublished memorandum. Ottawa, ON: HLB Decision Economics, 2005.

[27] Portland Metro and Oregon Department of Transportation. *Traffic Relief Options Study* Technical Appendix: Recommendations, Preliminary Findings, and Working Papers. Portland, Oregon: Portland Metro and Oregon Department of Transportation, 2000.

[28] Travel Model Improvement Program. *A System of Activity-Based Models for Portland, Oregon.* Portland, Oregon: U.S. Department of Transportation and U.S. Environmental Protection Agency, 1998.

[29] DeCorla-Souza, P. "Evaluation of Toll Options Using Quick-Response Analysis Tools: A Case Study of Capital Beltway." *Transportation Research Record* 1839 (2003): 65-73.

[30] DeCorla-Souza, P. "An Evaluation of High Occupancy Tolls and Fast and Intertwined Regular Networks." Paper No.04-4000. Washington D.C.: Transportation Research Board Annual Meeting, 2004.

[31] Youssef Dehghani. *Deployments of a New Toll Mode Choice Modeling System. Ocoee*, FL: Florida's Turnpike Enterprise, 2002.

[32] HNTB Corporation. *Discrete Choice Model Estimation for I-20 Managed Lanes. Documentation*. Atlanta, Georgia: Georgia Department of Transportation, 2007. Electronic Resource.

[33] Ortúzar, Juan de Dios, and Luis G. Willumsen. *Modelling Transport*. 3rd ed. Chichester, UK: John Wiley & Sons, 2001. Print.

[34] C&M Associates, Inc. *Preliminary Traffic and Revenue Study for I-20 East Managed Lanes. Final Report*. Atlanta, Georgia: Georgia Department of Transportation, 2008. Electronic Resource.

[35] Caliper Corporation. *TransCAD Software Package User's Guide.* Newton, MA: Caliper Corporation, 2010. Electronic Resource.

[36] Atlanta Regional Commission. *The Travel Forecasting Model Set For the Atlanta Region. Documentation.* Atlanta, Georgia: Atlanta Regional Commission, 2007. Electronic Resource.

[37] Citilabs, Inc. *Cube Voyager Software Package User's Guide*. Tallahassee, FL: Citilabs, Inc., 2010.

[38] Cheng Lu, Chung, Hani S Mahmassani and Xuesong. "A bi-criterion dynamic user equilibrium traffic assignment model and solution algorithm for evaluating dynamic road pricing strategies". *Transportation Research Part C: Emerging Technologies* 16.4 (2008): 371-389. Web. 21 Nov. 2010.

[39] Nielsen, A., Daly, A., and Frederiksen, R. "A Stochastic Route Choice Model for Car Travelers in the Copenhagen Region." *Networks and Spatial Economics 2.4 (*2002): 327-346. Web. 21 Nov. 2010.

[40] Florian, M. *Network Equilibrium Models for Analyzing Toll Highways*. Springer eBook. Springer, 5 Jun. 2006. Web. 21 Nov. 2010.

[41] Leurent, F. "Curbing the computational difficulty of logit equilibrium assignment model". Transportation Research Part B: Methodological 31.4 (1997): 315-326. Web. 21 Nov. 2010.

[42] Van Zyl, NJW, Oberholzer, JdeV and Chen, YS. "South African experience with the estimation of values of time from stated preference studies and their use in toll road models." *Southern African Transport Conference (20th : 2001 : Pretoria, South Africa).* n.Ed. Pretoria, South Africa: Southern African Transport Conference, 2001. Web. 21 Nov. 2010.

[43] Sheffi, Y. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Techniques*. Englewood Cliffs, New Jersey**:** Prentice Hall, 1984. Print.

[44] Martimo, M., Z. Zhou and C. Brown. *New Assignment Methods in Cube voyager.* Citilabs Webinars 11 Mar. 2010. Web. 21 Nov. 2010.

[45] Unnikrishnan, A., S. Boyles, and S. T. Waller. *User Equilibrium under Operational, Demand-Side Uncertainty: Formulation, Properties, and Solution Methods*. In review, *Transportation Research Part B*. 15 May 2008. Web. 21 Nov. 2010.

BIOGRAPHICAL INFORMATION

Alireza Khaleghi-Soroush received a Bachelor of Science degree in civil engineering from the Azad University, Tehran, Iran in 1999. He earned a Master of Science degree in transportation planning and engineering from the University of Texas at Arlington in May 2005. In fall 2005, he continued his studies in the transportation engineering field at the Civil and Environmental Engineering Department of the University of Texas at Arlington. This dissertation is the result of the research conducted by Khaleghi-Soroush during his Ph.D. program at UTA.

Soroush's interests include steady-state and dynamic "pseudo-probabilistic" volume assignment methods, specifically, how those methods might be applied to transportation networks involving tolled facilities. Throughout his career as a transportation system modeler, he has been, and currently is, responsible for transportation modeling tasks, as well as for providing assistance to project managers in various aspects of travel demand modeling, traffic and revenue analysis.