REANALYSIS FOR STATIC AND STEADY STATE STRUCTURE RESPONSE

by

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Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

December 2010

ACKNOWLEDGEMENTS

First, I would like to thank my thesis advisor Dr. Bo Ping Wang without his help and continuous guidance I would not have been able to complete my thesis. He has always guided me towards the right approach and always had great ideas for my research.

Second, I would like to thank Dr. Kent Lawrence and Dr. Seiichi Nomura, for their time as serving as my committee members.

I would also like to thank all my friends here in University of Texas at Arlington for their continuous help and support throughout my studies here.

Last but not least, my special thanks to my parents, my grandparents and my brother Kaushal for their continuous encouragement and support in all my endeavors.

November 23, 2010

ABSTRACT

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STRUCUTURE RESPONSE

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The University of Texas at Arlington, 2010

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Reanalysis of a structure has become very important in structural analysis, as it reduces the cost and the computational time required to measure the response of the structure after it has been modified. As the importance of reanalysis has become clear, more and more research has been carried out in the field of reanalysis, which has lead to many different methods to carry out reanalysis. There are various methods which can be used for any single structural reanalysis problem and it would give us different results depending upon the method selected.

Due to lot of methods which are available to solve a single problem it sometimes becomes difficult to identify which method is better and which method gives results which are more accurate. In this thesis various methods

which are commonly used for static structure and steady state reanalysis have been compared to identify which one of them gives us more accurate solution without increasing the computational time and cost significantly. After comparing all the methods in this thesis, it has been found that Combined Approximation Method gives us better results than all the other methods.

All the methods are implemented in MATLAB R2007b to get all the results. All the results obtained by reanalysis methods are compared with the exact solution of the modified structure to check the validity of the results obtained by the reanalysis methods.

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CHAPTER 1

INTRODUCTION

1.1 Introduction to Structural Reanalysis

In today's world, where every company is looking for ways to reduce the production cost and time but at the same time are looking for ways to improve the quality of the product it becomes the responsibility of the engineers to find the ways to meet the company's objectives. When designing a structure it is subjected to various analyses to check if it meets the required design constraints. Sometime we perform static analysis and/or steady state response to check if the designed structure meets all the given design criteria. Many times it happens that the structure does not meet the required design constraints and the design has to be modified. It is very rare that the new design meets all the design constraints in the first attempt itself. So the design has to be modified numerous times before it meets all the design constraints. After each such modification the structure has to be analyzed again to check if it meets the required design constraints such as maximum allowable displacement, maximum stress which it can withstand, maximum allowable weight, etc. This repeated analysis for each such modification becomes very expensive and time consuming, especially if there are lots of degrees of freedoms.

The main purpose of reanalysis techniques is to analyze the modified structure without performing the complete analysis of the structure but give a reasonably accurate solution within the allowable tolerance limits. This helps in reducing both the computational time and cost. There are various methods to carry out static reanalysis and steady state response reanalysis. Some of the most commonly used methods are Direct Approximation method [1,11,12], Reciprocal Approximation method [1,14-16], Exponential Approximation method [1,19], Transformed Approximation Method [1,17,18] and Combined Approximation method [1,2,20,21]. In this thesis it has been shown that the Combined Approximation method gives us better results compared to all the other methods.

There are various ways in which the structural analysis can be carried out depending upon what the loading conditions. If the loading on the structure remains constant with respect to time and does not vary then static analysis of the structure can be carried out to find displacement, stress, strain and forces on the structure or the component of the structure. If the loading vary with time then dynamic analysis of the structure should be performed. If the loading varies very slowly the structure response may be carried out using static analysis but if the load varies quickly then dynamic analysis should be carried out on the structure. In this thesis we are only considering the static analysis and steady state analysis part and the numerical analysis are carried out considering that the structure is subjected to constant load, which does not vary with time.

Also, lately there has been lot of research being carried out in the field of structural reanalysis particularly for structure which is undergoing change in its shape or degree of freedom, it is known as topological modifications. Topological modification means that new members are added or the old members are deleted. Additionally, adding new joints or deleting old joints from the structure are also considered as cases of topological modification. Most of recently developed methods for structural reanalysis are for structure undergoing topological changes. These types of modifications are not considered for this thesis.

Numerical results from various examples shows that the Combined Approximation method gives better approximation results compared to other commonly used methods described in this thesis for the cases of static structure reanalysis and steady state forced vibration response reanalysis.

1.2 Outline of the Thesis

In Chapter 2, a brief introduction is given regarding structural analysis and structural reanalysis and different approaches.

In Chapter 3, various methods commonly used for the static response and the steady state response reanalysis are discussed briefly and the methods are explained briefly.

In Chapter 4, the results obtained from different examples are compared using the results obtained for various comparisons parameters and graphs are plotted for various results.

Chapter 5 contains conclusion and future research works.

CHAPTER 2

INTRODUCTION TO STRUCTURAL ANALYSIS AND STRUCTURAL REANALYSIS

2.1 Introduction to Structural Analysis

Structural analysis is only a support activity in the larger field of structural design. Structural analysis is a process of analyzing a model of structural system which helps us to predict the response of the real structure under the excitation of expected loading and external environment during its entire service life of the structure. The purpose of the structural analysis is to ensure the adequacy of the design from view point of safety, serviceability of the structure and the integrity of the structure under analysis, it is judged largely based upon its ability to withstand these loads. Structural analysis mainly incorporates the field of mechanics and dynamics as well as many failure theories. The primary goal of structural analysis from theoretical point of view is computation of deformation, internal forces and stresses. However in practice, structural analysis can also be used as a method to drive engineering design process or to prove the soundness of design without directly testing it. The accuracy of the analysis can be improved if a more redefined model is used. There is a tendency to employ more and more refined models to approximate as close to the actual structure as possible. This means that cost of an analysis and its practical feasibility depend to a considerable degree on the algorithms available for the solution of the resulting equations. The time required for solving the equilibriums equations can be a high percentage of the total solution time, particularly in non-linear analysis or in dynamic analysis, when solution must be repeated many times. Reanalysis methods are intended to analyze effectively structures modified due to changes in design. The object is to analyze the structural response for changes without solving the complete set of modified simultaneous equations.

2.2 Basics of Reanalysis

The design process of an engineering structure is an iterative modification process. In structural design, the procedure generally requires us to carry out repeated analysis of the structure as we continuously modify it to meet the required design constraints. Every modification requires us to carry out extensive calculations. Thus, it is necessary to seek a faster computational method for reanalysis. The main design problems which motivated the application of approximation concept in structural reanalysis are

1) The problems are usually complex and have large degrees of freedoms. So each element has at least one degree of freedom which needs to be considered for each change in design and loading condition
2) Every time we modify the design we need to perform extensive calculations. Design variables and constrains are usually related to each other, so for each change in the design variable, constraints have to calculated again.
3) Large number of modification in design has to be carried out. The solution for optimal design can only be obtained by repeated analysis and redesign.

Due to these difficulties a lot of research has been carried out in reanalysis methods. Also with improvements in computers, the speed at which the reanalysis can be performed has improved considerably. Now we can carry out reanalysis on structure with more degree of freedom and more modification in less time.

Reanalysis methods are used to analyze the structure that is modified due to various changes effectively. The objective of reanalysis is to evaluate structural response for successive modification in the design without repeatedly solving the complete set of modified analysis equations. The solution obtained by reanalysis is within the acceptable tolerance and it also reduces the computational time required to arrive at the solution. To avoid analysis after each modification many reanalysis methods have been developed, especially for the case of fixed layout modifications where the number of degree of freedom does not change. Reanalysis can be very useful for some problems such as:

1) In structural optimization problems, where the solution is iterative and consists of lot of analysis followed by redesigning the structure. The number of design variables is large in this type of problems and various failure modes under several load conditions are often considered also the constraints are implicit function of design variables. In these problems analysis takes up most of the computational time, so methods that are not time consuming and implicit are preferred.
2) In structural damage analysis problem, structure has to be analyzed for various changes due to deterioration, poor maintenance, damage or accidents. Various hypothetical scenarios have to be considered to check for the soundness of structure and numerous analyses are required to check it.
3) In design stage of complex structure, repeated analysis have to be performed for various modification during the design stage as modified structure are subjected to different loading conditions.

During design stage elements may be added or removed from the structure requiring us to performing more analysis.

4) In nonlinear analysis of structure, it is an iterative process and set of updated linear equation must be solved repeatedly. In vibration analysis of structure, to solve for mode shape, set of updated equations must be solved repeatedly.

5) In applications such as probabilistic analysis, controlled structures, smart structures, adaptive structures and conceptual design problems reanalysis methods might prove to be very useful and time saving.

The reanalysis methods are divided in two general categories [5,6,13]:

1) Direct Method or Exact Method

2) Iterative Method or Approximate Method

In exact solution basis vector is a linear combination of previous vectors. These methods give exact, closed-form solutions, which have the same effect of solving again the modified system of equations. Some of the exact methods are Initial Strain Technique, Parallel Element Technique, Modified Inverse of Matrices, Modified Displacement Vector, Modified Decomposed Matrices, etc. Exact methods are usually based on Sherman-Morrison and Woodbury formulae [8,9] for update of the inverse of matrix. It has been shown that various exact methods may be viewed as variant of these formulae.

There are also other various exact reanalysis methods but the drawback of these methods is that it is applicable only in cases when the structure has only changes in few of its members. In this thesis these types of methods are not reviewed, only the approximate reanalysis methods are considered in this thesis.

In the approximate reanalysis methods, the solutions are obtained based on the response of the modified structure using the information obtained during the full analysis of the initial structure. Approximate methods are generally derived from some form of a series expansion. Approximate methods of structural reanalysis reduce the number of steps required to obtain the solution which in turn reduces the computational cost and time. Approximate methods apply successive correction to the initial solution and converge to a more accurate solution for a modified structure.

In general, there is always a tradeoff between two conflicting factors which should be considered in choosing an approximate behavior model for specific problem. They are

1) The accuracy of the solution or the quality of the approximation.
2) The computational effort required or the efficiency of the method.

2.3 Different Approximation Approaches

There are various methods which fall under the category of Approximation method. These methods can be broadly divided into three main classes [3,4,6,7,10]. They are:

1) Local Approximation
2) Global Approximation
3) Combined Approximation

2.3.1. Local Approximation

This type of approximation is also called as single point approximation because these approximations are only valid within the close area of a point in the design space at which they were generated. First order Taylor series expansion is perhaps the most commonly used approximation method in the structural optimization, but other series expansion such as binomial series have also been used in local approximation. Although these methods are most efficient and reduce the computational time and cost considerably they are effective only in cases with small changes in design variable. For large changes in design, the accuracy of these types of approximation methods becomes meaningless. So the local approximations are most commonly used to generate approximate problem formulation that is to be solved for an optimum solution point. A new approximate problem is then generated at that point, and the process continues until convergence. Local approximations methods can be divided into two types:

1) Local function approximation
2) Local problem approximation

Local function approximations methods are variations on the Taylor series expansion; local problem approximations methods try to reduce the size of the active constraint set or the design variable set.

2.3.2. Global Approximation

This type of approximation is also called as multipoint approximation. These approximations are obtained by analyzing the structure at number of different design points, and they are valid for the whole design space or at least a very large region of design space. They are used to modify the formulation of the problem from the outset and generate an alternate formulation that is more tractable. Global approximations are particularly attractive when the problem has multiple local optima and one seeks to find the global optimum or when one is interested in finding several local optima. The effort associated with constructing the global approximation is typically independent of the number of local optima. A local approach, on other hand may become intractable for a problem with very large number of local optima. Global approximations methods can be divided into two types:

1) Global function approximation

2) Global problem approximation

Global function approximations methods include the generation of surface response; global problem approximations methods include the introduction of intermediate variable or response quantities as well methods to reduce the number of constraints or design variables in the problem.

Approximate expressions of the structural behavior in terms of the independent variable can be introduced based on results of several exact analyses. The main challenge in generating such is to do so without performing an excessive number of analyses. Linear and quadratic polynomial approximations are the most common form employed, but other forms, such as polynomial in power of trigonometric function, polynomial fitting or reduced basis methods have been used as well. The approximation contains a number of unknown parameters that must be determined such that some preselected conditions must be satisfied. To do so, precise analyses are performed at number of design points. However, the number of analysis required to obtain adequate approximation might be large, particularly in problems with many design variables. In the reduced basis method, for example, the structural behavior is expressed as a linear combination of a reduced number of basis vectors, computed for some design points. One problem in using this approach is that multiple exact analyses at latter points must be carried out before introducing approximations. In addition, how to choose the design points effectively is not always clear.

2.3.3. Combined Approximation

This method tries to give local approximation global qualities. This can be done in number of ways. In this approximation the binomial series terms are used as basis vectors in reduced basis approximation. Similar to local approximation, the calculations are based on results of a single exact analysis. Each reanalysis involves a small computational effort, and calculation of derivatives is not required. The method is easy to implement and can be used with general finite element programs. Other approach is to introduce combined approximation is to scale the initial design so that the changes in the design variables are reduced. The advantage is that, it is similar to local approximations, as the solution is based in the results of a single point analysis. It has been shown that he scaling operation is useful for various types of design variables and behavior functions. In particular, simplified approximations of homogeneous displacement and stress functions can be achieved. Several criteria for selecting the scaling multiplier have been proposed, including geometrical considerations and mathematical criteria. The concept of scaling has been extended to include the approximate displacement, in addition to the initial stiffness matrix, thereby improving the results. This approach has been found to be most effective for various reanalysis problems.

CHAPTER 3

VARIOUS METHODS FOR STATIC REANALYSIS

3.1 Reanalysis Problem Formulation

The problem under consideration can be stated as follows:

Given an initial design variable vector X_o, the corresponding stiffness matrix is K_o and the displacement vector u_o can be calculated by the equilibrium equation

$$K_o u_o = R_o \tag{3.1}$$

where R_o is the load vector, whose elements are assumed to be independent of the design variable and K_o is symmetric. From the initial analysis, the stiffness matrix can be written in decomposed form as

$$K_o = U_o^T U_o \tag{3.2}$$

where U_o is an upper triangular matrix.

Assume a change ΔX in design variable, so that the modified design is

$$X = X_o + \Delta X \tag{3.3}$$

and the corresponding modified stiffness matrix is given as

$$K = K_o + \Delta K \tag{3.4}$$

where ΔK is the change in stiffness matrix due to change in the design ΔX.

The object of reanalysis is to find efficient and high quality approximations of the modified displacement vector y due to various changes in the design variables ΔX, without having to solve all the modified analysis equation

$$Ku = (K_o + \Delta K)u = R \tag{3.5}$$

where R is the load vector of the modified design, which may be also different from the load vector of the initial design R_o. The elements of the stiffness matrix are not restricted to certain forms and can be general function of design variables. That is, the design variable X may represent coordinates of joints, the structural shape, geometry, member's cross sections, etc. After displacements are evaluated, the stress can readily be determined explicitly by stress displacement relation.

$$\sigma = Su \tag{3.6}$$

where S is the system stress transformation matrix.

After finding the stresses σ, forces can be easily calculated using

$$N = W\sigma = WSu = Tu \tag{3.7}$$

where W is a diagonal matrix giving the force-stress ratio and matrix T is defined as

$$T = WS \tag{3.8}$$

The elements of matrices K and W are usually some explicit function of design variables, whereas the elements of R and S are often constant.

3.2 Various Methods for Static Reanalysis

In this section various commonly used methods for static structural reanalysis are described briefly

3.2.*1. Direct Approximation Method*

The Direct Approximation method is one of the first methods to be used for structural reanalysis, and it is perhaps the most commonly used approximation method in structural reanalysis. Direct Approximation method of first order is also known as the Taylor series expansion method of first order. Common approach is to consider the first terms of series expansion, to obtain the approximate displacement u_a

$$u_a = u_1 + u_2 + u_3 ... \tag{3.9}$$

The first three terms, obtained by Taylor series expansion about X_o, are given by

$$u_1 = u_o \tag{3.10}$$

$$u_2 = u_{ox} \Delta X \tag{3.11}$$

$$u_{3j} = \tfrac{1}{2} \Delta X^T H_{oj} \Delta X \tag{3.12}$$

where the displacement u_o, the gradient u_{ox}, and the Hessian matrix H_{oj}, are computed at X_o. u_{3j} is the j^{th} component of u_3.

The displacement vector for the modified structure can also be given as

$$u_{DA1} = u_o + \sum_{i=1}^{n} \frac{\partial u_o}{\partial X_i} (X_i - X_{oi}) \tag{3.13}$$

The simplest linear approximation method is based on the Taylor series. However for some application the linear approximation is inaccurate even for design points that are close to the original design points. The accuracy can be increased by retaining additional terms in the Taylor series expansion. This however requires costly calculations of higher-order derivatives.

One of the ways to improve the quality of the results obtained by this method without much increase in calculation is to take inverse or reciprocal of design variable. The procedure for this method is given in next sub section.

3.2.2. Reciprocal Approximation Method

The displacement vector for the modified structure can be given by the following formula

$$u_{RA1} = u_o + \sum_{i=1}^{n} y_i \frac{\partial u_o}{\partial X_i} \left(X_i - X_{oi} \right) \tag{3.14}$$

where $y_i = X_{oi} / X_i$ (3.15)

The design variables in these methods are usually cross-sectional areas of the truss elements and the thickness of the plane-stress elements. For statically determinate structures, stress and displacement constraints are linear functions of the reciprocals of these design variables. For statically redundant structures the constraint functions are thus expected to present some inherent linear characteristics, especially if the structure is only weakly redundant. One of the attractive features of the reciprocal approximation method is that it preserves the property of scaling, even in case of the statically indeterminate structures. If all the design variables are scaled by a factor, the displacement vector is scaled by a factor; the displacement vector is scaled by the reciprocal of that factor. The reciprocal approximation preserves this scaling property, and therefore it is exact for scaling the design.

The main problem with this method is that if X_i is zero then the corresponding y_i value approaches ∞.

The weaknesses of this approximation method are

i) It is a separable approximation and does not capture the coupling effect present in statically indeterminate structures.

ii) It cannot capture the higher order nonlinearity associated with stresses.

To overcome this difficulty, there are two other methods that can be used. They are similar to Reciprocal Approximation method but the modified multiplier y_i is selected by different method than that from Reciprocal Approximation method. Those two methods are Exponential Approximation method and Transformed Approximation method.

3.2.3. Exponential Approximation Method

One of the ways to improve the quality of results which we obtain from Reciprocal Approximation method is that we use a modified multiplier that contain an exponential term

$$u_{EA1} = u_o + \sum_{i=1}^{n} y_i \frac{\partial u_o}{\partial X_i} \left(X_i - X_{oi} \right) \tag{3.16}$$

where $y_i = \left(X_{oi} / X_i \right)^m$ (3.17)

where m is the parameter to be selected. The results obtained by this method are significantly better than the results obtained by Reciprocal Approximation method, but are highly dependent on the value of parameter m which we have selected. If we assume value of m to be one than the results obtained by the Exponential Approximation method and the Reciprocal Approximation method are the same.

3.2.4. Transformed Approximation Method

The Reciprocal Approximation method cannot be used when X_i is zero or become very small, to overcome this difficulty Transformed approximation method can be used. In this method we use a modified multiplier which contains extra term δX_i so that when X_i becomes zero the denominator term does not become zero.

$$u_{TA1} = u_o + \sum_{i=1}^{n} y_i \frac{\partial u_o}{\partial X_i} \left(X_i - X_{oi} \right) \tag{3.18}$$

where $y_i = \left(X_{oi} + \delta X_i \right) / \left(X_i + \delta X_i \right)$ (3.19)

In this method value of δX_i is typically very small compared to representative value of corresponding X_i. It is possible, however to take large values of δX_i and this results in an approximation which is closer to the linear approximation than the reciprocal approximation of these variables.

The main problem with this method is that the quality of results which we obtain from this method is highly dependent on what we have selected as our δX_i.

3.2.5. Combined Approximation Method

Let K_o and R_o be the stiffness matrix and load vector of initial design with n degree of freedom respectively. The corresponding displacement vector can be computed from the following equation

$$K_o u_o = R_o \tag{3.20}$$

If the modification in the stiffness matrix and load vector are ΔK and ΔR respectively then to calculate the displacement vector for modified system we have the following equation

$$(K_o + \Delta K)u = (R_o + \Delta R) \tag{3.21}$$

which can also be written as

$$Ku = R \tag{3.22}$$

where the modified values of K and R are given as

$$K = K_o + \Delta K \qquad R = R_o + \Delta R \tag{3.23}$$

Now the procedure to calculate the displacement vector for the Combined Approximation method is

The first of the basis vector is calculated using the terms of binomial series using following equation

$$u_1 = K_o^{-1} R \tag{3.24}$$

Rest of the terms of the basis vector are calculated using the following equation

$$u_i = -Bu_{i-1} \qquad i = 2,3,\ldots,s \tag{3.25}$$

where s is much smaller number the number of degree of freedom and matrix B is defined as

$$B = K_o^{-1}\Delta K \tag{3.26}$$

It can be observed that calculation of the basis vector involves only forward and backward substitutions. To perform reanalysis, we assume the response of the modified structure is a linear combination of the basis

$$u_a = y_1 u_1 + y_2 u_2 + ... + y_s u_s = u_B y \tag{3.27}$$

Substitute eq. (3.27) into eq. (3.22) leads to

$$K_u y = R_u \tag{3.28}$$

where $u_B = [u_1, u_2, ..., u_s]$ (3.29)

$$K_u = u_B^T K u_B \tag{3.30}$$

$$R_u = u_B^T R \tag{3.31}$$

$$y^T = \{y_1, y_2, ..., y_s\} \tag{3.32}$$

y is a set of generalized coordinates.

Once y is compute from eq. (3.30), we can compute the displacement vector for modified design by using the following equation

$$u_a = y_1 u_1 + y_2 u_2 + ... + y_s u_s = u_B y \tag{3.33}$$

Here number of basis vector s is much smaller than number of degree of freedom n. Thus, solution of eq. (2.28) is more efficient then solving eq. (3.22) directly.

3.3 Method Used for Steady State Forced Vibration Response Reanalysis

In steady state forced vibration the structure is subjected to external loading and it can be represented as

$$M\ddot{u} + Ku = Fe^{i\Omega t} \quad (3.34)$$

where $u = Ue^{i\Omega t}$ (3.35)

and $Fe^{i\Omega t}$ is the external force acting on the system

The equation can be modified and written as

$$ZU = F \quad (3.36)$$

where $Z = K - \Omega^2 M$ (3.37)

this equation is similar to that of static structure which is given by

$$Ku = F \quad (3.38)$$

As we can observe from eq. (3.36) and eq. (3.38) that the equation for steady state forced response is almost same as that of static response, we can apply almost all the methods which we can use for static reanalysis in steady state forced response reanalysis with almost no change in the procedure. We can substitute value of K with Z to use all the static reanalysis methods for steady state forced response reanalysis.

CHAPTER 4

RESULTS

Several numerical examples are considered in this chapter. The results show that the Combined Approximation Method gives us the best results.

4.1 Ten-Element Frame Model

In first case we will consider a 10 element frame structure as an example. In figure 4.1 the, original frame structure with its properties is given.

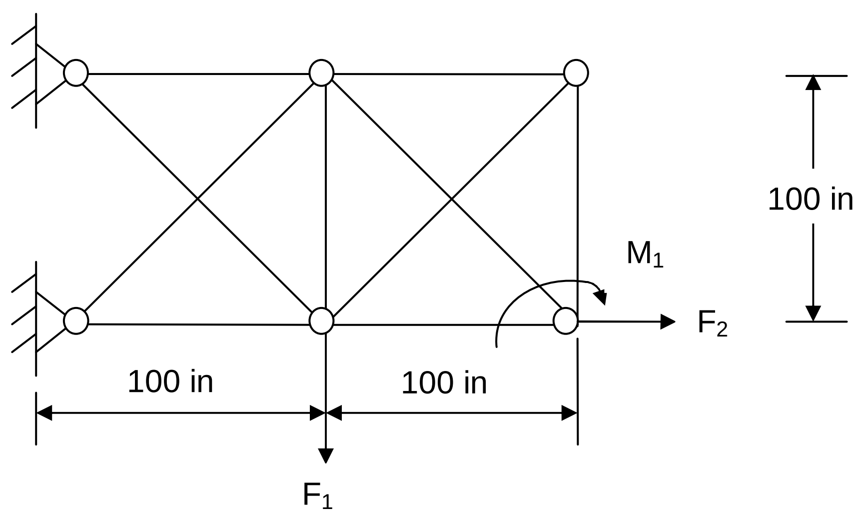

Figure 4.1 10 Element Frame Initial Design

Young's Modulus = 30e6 psi   Cross-sectional area = 1*1 in^2

F_1 = 100000 lb F_2 = 100000 lb M_1= 50000 lb-in

In the modified design the all the conditions are kept same except the cross-sectional area of two members has been changed. The members for which the cross-sectional area has been changed have been darkened for the purpose of identification. Figure 4.2 represents the modified frame design with its modified properties.

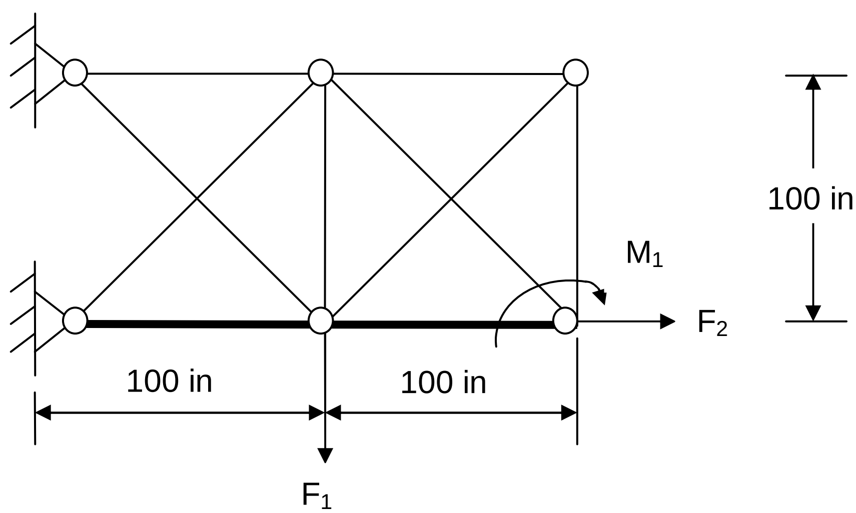

Figure 4.2 10 Element Frame Modified Design

Young's Modulus = 30e6 psi

Original Cross-sectional area = 1*1 in^2

F_1 = 100000 lb F_2 = 100000 lb M_1= 50000 lb-in

Modified Cross-sectional area = 1.2*1.2 in^2

In this example, only the first order basis vector is used in all the reanalysis methods to obtain the displacement and rotation of the modified structure. The computed displacement and rotation of the modified structure are tabulated below

Table 4.1 Displacement and rotation obtained by different methods after reanalysis of 10 element modified frame structure

Methods / Degree of Freedom	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method	Exact Solution
x_2	0.0693	0.1025	0.0881	0.0881	0.0829	0.0827
y_2	-0.5383	-0.5139	-0.5245	-0.5245	-0.5275	-0.5284
θ_2	-0.0293	-0.0193	-0.0184	-0.0184	-0.0266	-0.0232
x_3	0.2434	0.3577	0.3076	0.3076	0.2902	0.2893
y_3	-0.4193	-0.3158	-0.3611	-0.3611	-0.3761	-0.3777
θ_3	0.1222	0.1776	0.1543	0.1543	0.1446	0.1446
x_4	0.0821	0.0699	0.0753	0.0753	0.0770	0.0772
y_4	-0.4538	-0.3592	-0.4006	-0.4006	-0.4142	-0.4158
θ_4	-0.0165	-0.0259	-0.0225	-0.0225	-0.0201	-0.0205
x_5	0.1169	0.1138	0.1151	0.1151	0.1154	0.1156
y_5	-0.4526	-0.4406	-0.4458	-0.4458	-0.4470	-0.4478

Table 4.1 – *Continued*

θ_5	-0.0068	-0.0106	-0.0095	-0.0095	-0.0082	-0.0085

In figure 4.3 we have compared the displacement obtained in x-direction at free nodes for various reanalysis methods with that of exact solution for the given condition. The results show that Combined Approximation yields the best approximation.

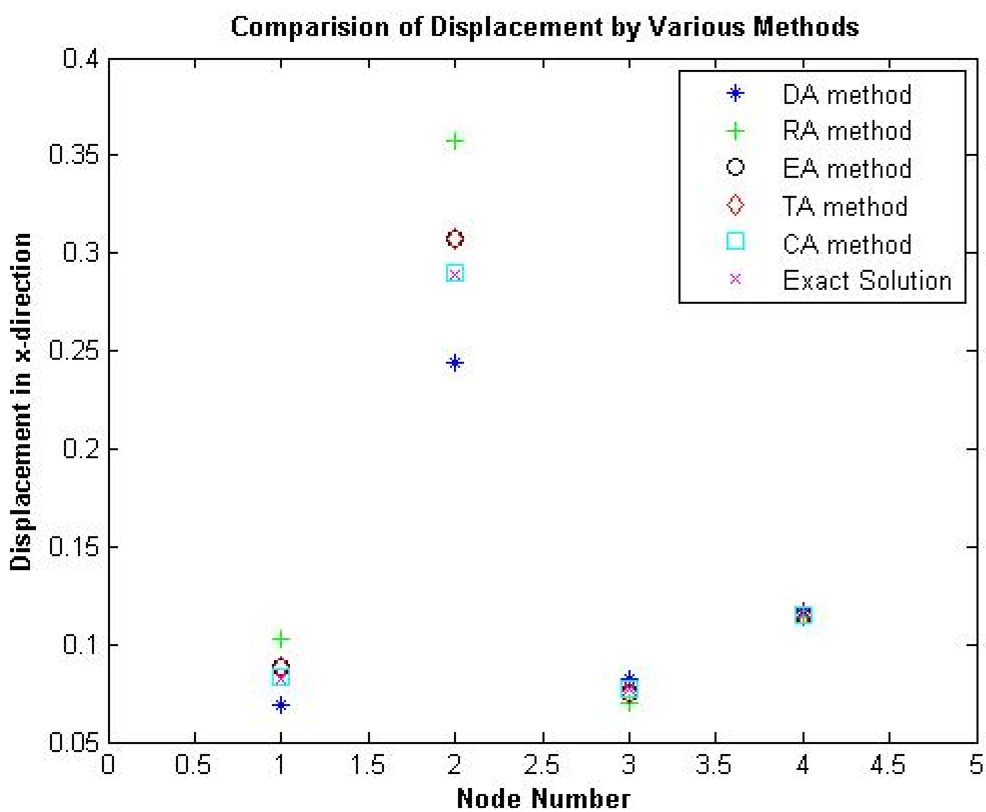

Figure 4.3 Comparison of displacement in x-direction for 10 element frame structure by various methods.

In figure 4.4 we have compared the displacement obtained in y-direction at free nodes for various reanalysis methods with that of exact solution for the given condition. The results show that Combined Approximation yields the best approximation.

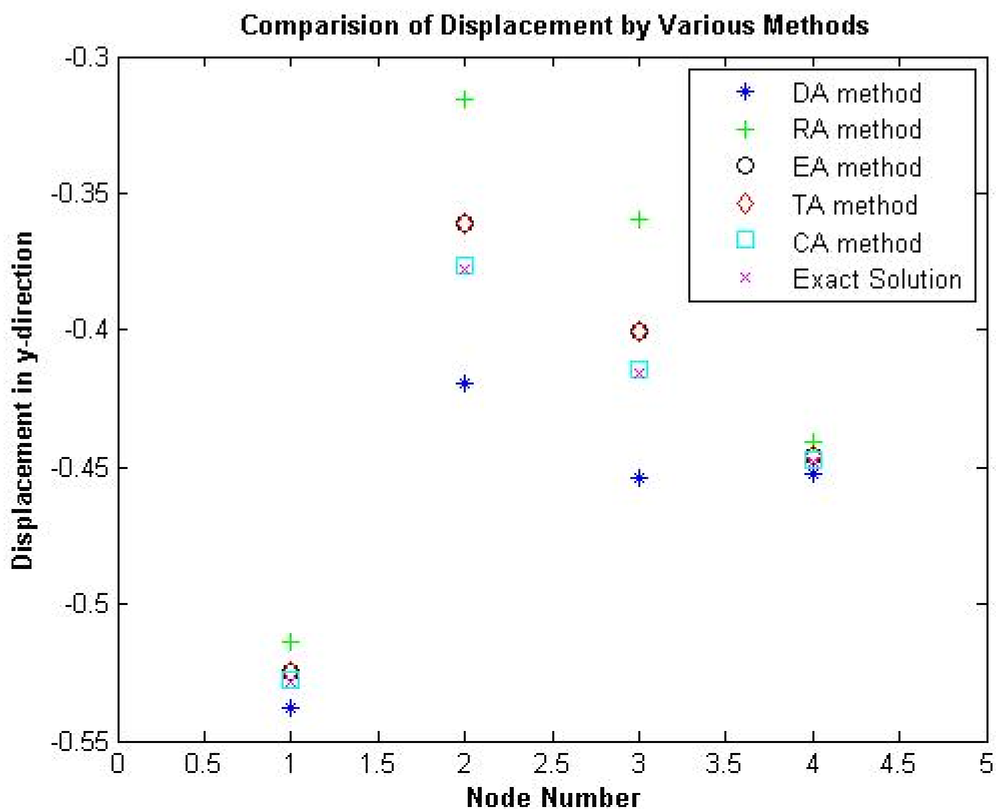

Figure 4.4 Comparison of displacement in y-direction for 10 element frame structure by various methods.

In figure 4.5 we have compared the rotation obtained at free nodes for various reanalysis methods with that of exact solution for the given condition. The results show that Combined Approximation yields the best approximation.

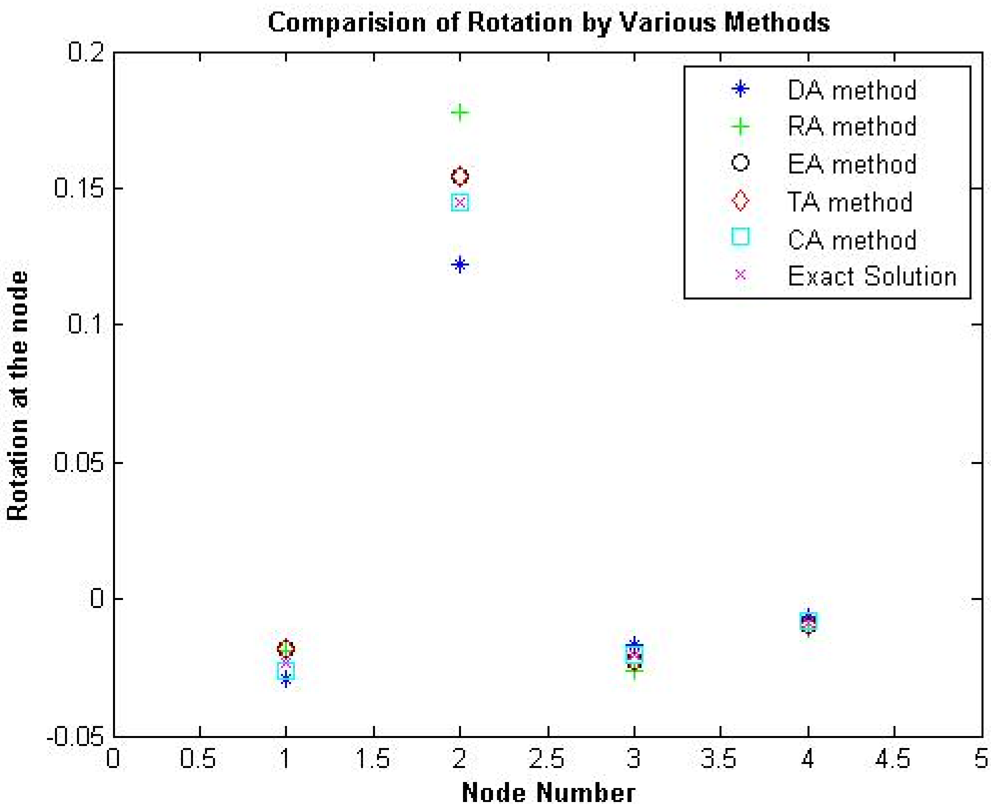

Figure 4.5 Comparison of rotation for 10 element frame structure by various methods.

In table 4.2, we have tabulated the difference between the displacements and rotation which we obtained from various reanalysis methods to that of exact solution for the modified system for 10 element frame structure.

Table 4.2 Difference between the results obtained by exact solution and results obtained by reanalysis methods for 10 element frame structure

Methods / Degree of Freedom	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
x_2	0.0134	-0.0198	-0.0054	-0.0054	-0.0002
y_2	0.0098	-0.0146	-0.0039	-0.0039	0.0010
θ_2	0.0061	-0.0040	-0.0048	-0.0048	0.0033
x_3	0.0459	-0.0685	-0.0183	-0.0183	-0.0009
y_3	0.0416	-0.0619	-0.0166	-0.0166	-0.0016
θ_3	0.0224	-0.0331	-0.0098	-0.0098	-0.0000
x_4	-0.0049	0.0073	0.0019	0.0019	0.0002
y_4	0.0380	-0.0566	-0.0152	-0.0152	-0.0016
θ_4	-0.0040	0.0054	0.0020	0.0020	-0.0004
x_5	-0.0013	0.0018	0.0005	0.0005	0.0002
y_5	0.0048	-0.0072	-0.0019	-0.0019	-0.0007
θ_5	-0.0017	0.0021	0.0010	0.0010	-0.0003

In table 4.3 we have written the norm of values for the difference between the exact displacement and the reanalysis method displacement solution.

Table 4.3 The norm of value of difference between exact method and reanalysis method for 10 element frame model

Methods	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
	0.0785	0.1165	0.0319	0.0319	0.0044

In table 4.4 we have show the percentage of error for each of the free degree of freedom. It has been calculated using

$$err = \left(\frac{extu - ru}{extu}\right) * 100$$

where extu = exact displacement of the modified structure

ru = displacement obtained by reanalysis method.

Table 4.4 Percentage of Error at each Degree of freedom for 10 element frame model

Methods / Degree of Freedom	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
x_2	16.1644	-23.9485	-6.5250	-6.5178	-0.1992
y_2	-1.8565	2.7571	0.7447	0.7439	0.1850
θ_2	-26.2354	17.0747	20.7073	20.7087	-14.3838
x_3	15.8551	-23.6611	-6.3372	-6.3301	-0.3064
y_3	-11.0023	16.3916	4.4032	4.3983	0.4343
θ_3	15.4634	-22.8729	-6.7643	-6.7577	-0.0210
x_4	-6.3018	9.4052	2.5194	2.5165	0.3191
y_4	-9.1398	13.6072	3.6611	3.6570	0.3784
θ_4	19.7389	-26.4106	-9.8989	-9.8921	1.8538
x_5	-1.0841	1.5994	0.4393	0.4388	0.1595
y_5	-1.0821	1.6004	0.4354	0.4349	0.1625
θ_5	20.3289	-24.1013	-11.5157	-11.5106	4.0166

It can be observed from all the results obtained that the Combined Approximation method gives us superior results compared to all the other commonly used reanalysis methods which we have considered.

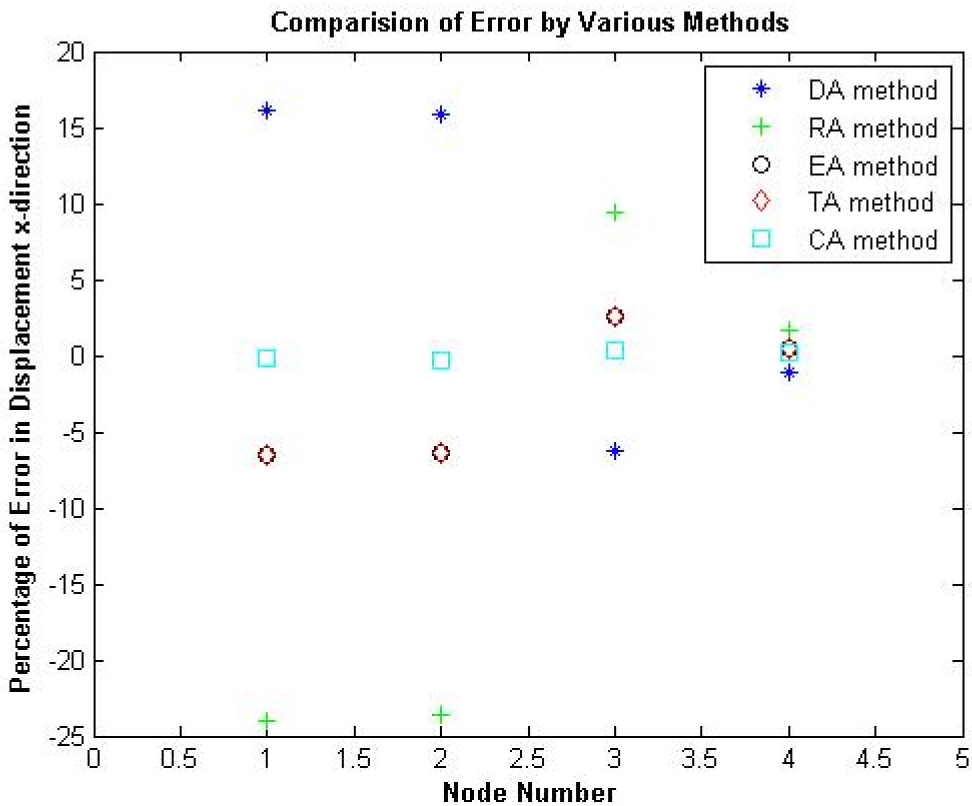

Figure 4.6 Comparison of error in x-direction of 10 element frame structure by various methods.

In figure 4.6 we compare the percentage of error in displacement in x-direction at the free nodes for results obtained from various methods against that of exact solution for the modified structure.

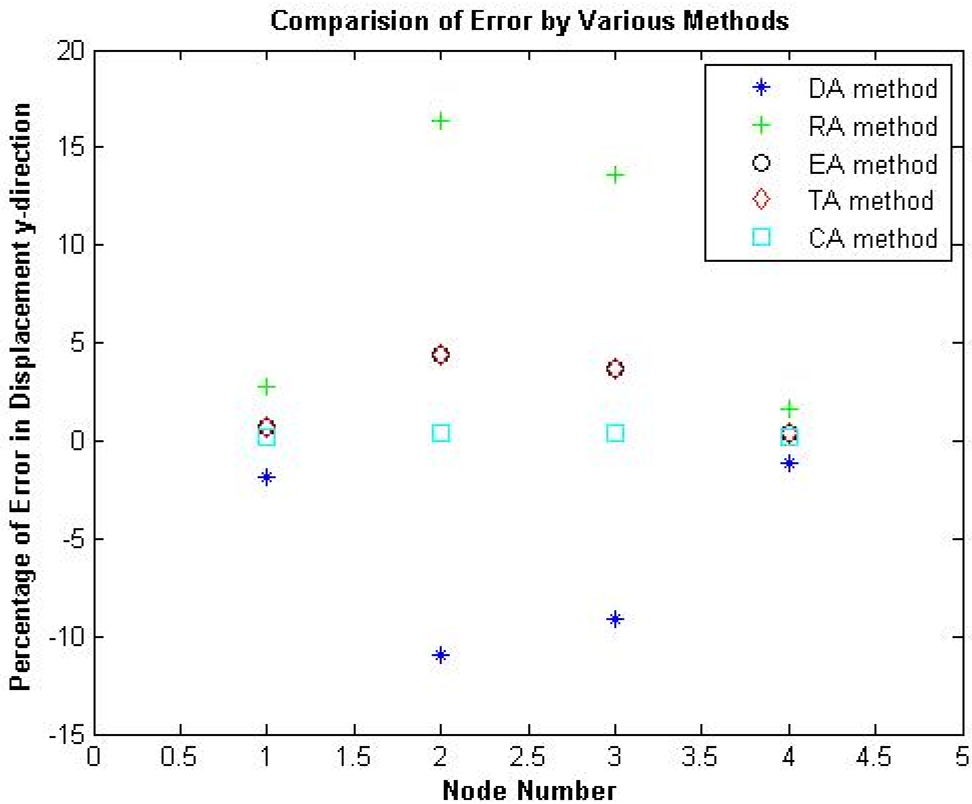

Figure 4.7 Comparison of error in y-direction of 10 element frame structure by various methods.

In figure 4.7 we compare the percentage of error in displacement in y-direction `at the free nodes for results obtained from various methods against that of exact solution for the modified structure.

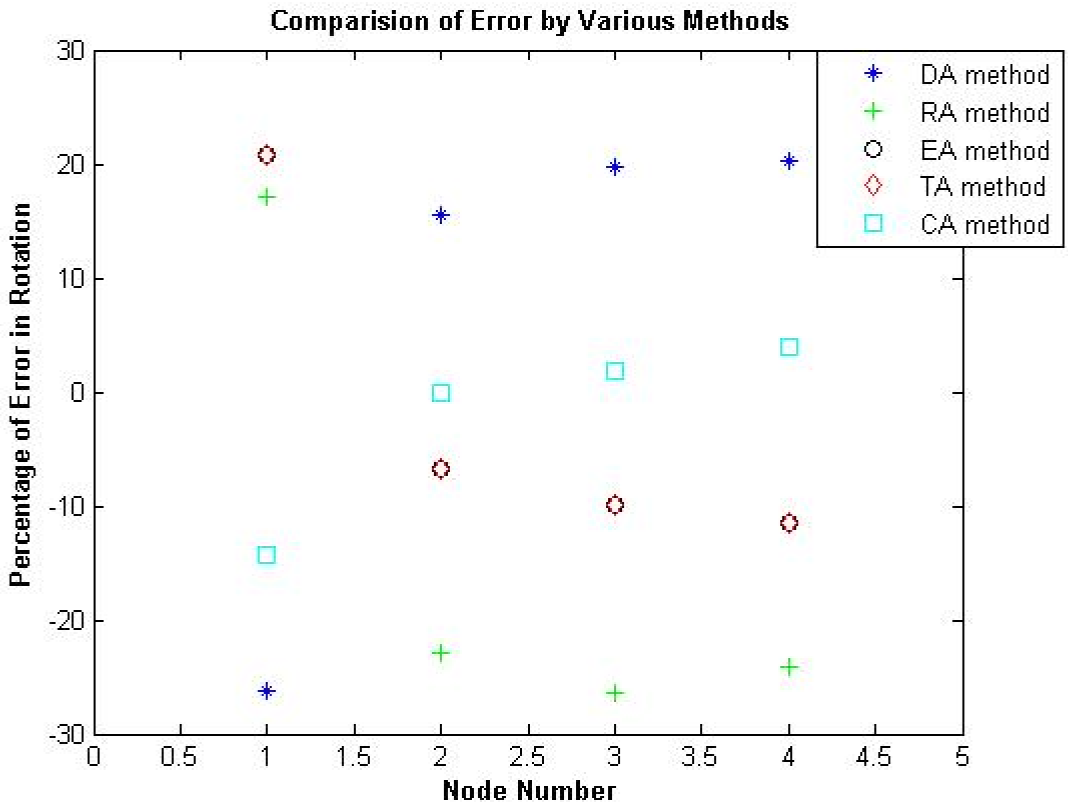

Figure 4.8 Comparison of error for rotation of 10 element frame structure by various methods.

In figure 4.8 we compare the percentage of error for rotation at the free nodes for results obtained from various methods against that of exact solution for the modified structure.

4.2 Fifty-Element Frame Model

In second case we will be considering a fifty element frame model to prove that Combined Approximation method give good results for even large degree of freedom. In figure 4.9 50 elements frame structure is given along with it loading conditions and its properties.

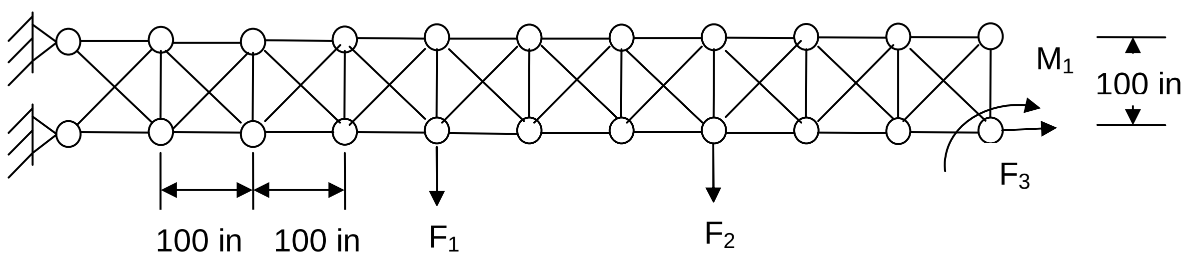

Figure 4.9 50 Element Frame Original Structure

Young's Modulus = 30e6 psi   Cross-sectional area = 2.5*2.5 in^2

F_1 = 10000 lb F_2 = 5000 lb F_3 = 10000 lb M_1= 5000 lb-in

In the modified design the all the conditions are kept same except the cross-sectional area of four members has been changed. The members for which the cross-sectional area has been changed have been darkened for the purpose of identification. In figure 4.10 modified structure is given with its properties.

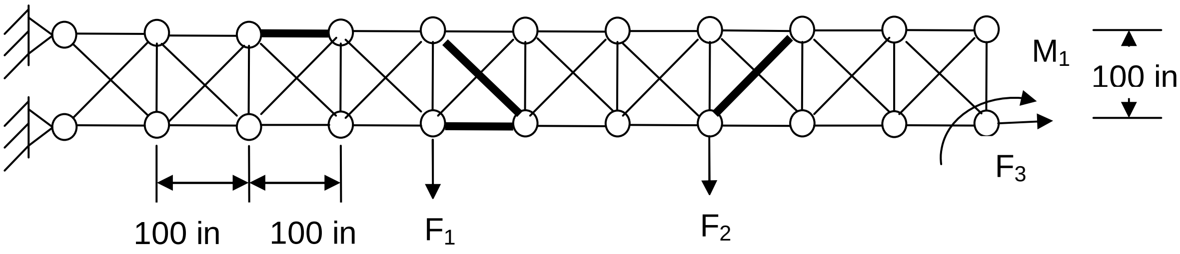

Figure 4.10   50 Element Frame Modified Structure

Young's Modulus = 30e6 psi

Original Cross-sectional area = 2.5*2.5 in^2

F_1 = 10000 lb F_2 = 5000 lb F_3 = 10000 lb M_1= 5000 lb-in

Modified Cross-sectional area = 3*3 in^2

In the above example, only the first order basis vector is used in all the reanalysis methods to obtain the displacement and rotation of the modified structure. In table 4.5 we have tabulated the computed values of displacement and rotation for the modified 50 element frame structure.

Table 4.5 Displacement and rotation obtained by different methods after reanalysis of 50 element modified frame structure

Methods / Degree of Freedom	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method	Exact Solution
x_2	-0.0312	-0.0312	-0.0312	-0.0312	-0.0312	-0.0312
y_2	-0.0440	-0.0441	-0.0441	-0.0441	-0.0441	-0.0441
θ_2	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007
x_3	-0.0544	-0.0543	-0.0543	-0.0543	-0.0543	-0.0543
y_3	-0.1477	-0.1475	-0.1476	-0.1476	-0.1476	-0.1476
θ_3	-0.0012	-0.0012	-0.0012	-0.0012	-0.0012	-0.0012
x_4	-0.0685	-0.0691	-0.0688	-0.0688	-0.0687	-0.0687
y_4	-0.2891	-0.2917	-0.2903	-0.2903	-0.2902	-0.2902
θ_4	-0.0015	-0.0016	-0.0015	-0.0015	-0.0015	-0.0015
x_5	-0.0759	-0.0764	-0.0761	-0.0761	-0.0761	-0.0761
y_5	-0.4535	-0.4622	-0.4574	-0.4574	-0.4571	-0.4572
θ_5	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017
x_6	-0.0774	-0.0782	-0.0777	-0.0777	-0.0778	-0.0777
y_6	-0.6202	-0.6356	-0.6270	-0.6270	-0.6267	-0.6266

Table 4.5 – *Continued*

θ_6	-0.0017	-0.0018	-0.0017	-0.0017	-0.0017	-0.0017
x_7	-0.0764	-0.0773	-0.0768	-0.0768	-0.0768	-0.0768
y_7	-0.7943	-0.8159	-0.8039	-0.8039	-0.8035	-0.8034
θ_7	-0.0017	-0.0018	-0.0018	-0.0018	-0.0018	-0.0018
x_8	-0.0730	-0.0738	-0.0733	-0.0733	-0.0734	-0.0733
y_8	-0.9687	-0.9965	-0.9811	-0.9811	-0.9805	-0.9804
θ_8	-0.0017	-0.0018	-0.0017	-0.0017	-0.0017	-0.0017
x_9	-0.0683	-0.0691	-0.0686	-0.0686	-0.0686	-0.0686
y_9	-1.1346	-1.1684	-1.1496	-1.1496	-1.1489	-1.1487
θ_9	-0.0016	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017
x_{10}	-0.0634	-0.0642	-0.0637	-0.0637	-0.0637	-0.0637
y_{10}	-1.2953	-1.3353	-1.3131	-1.3131	-1.3123	-1.3121
θ_{10}	-0.0016	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017
x_{11}	-0.0585	-0.0593	-0.0588	-0.0588	-0.0589	-0.0588
y_{11}	-1.4508	-1.4971	-1.4714	-1.4714	-1.4704	-1.4702
θ_{11}	-0.0010	-0.0011	-0.0011	-0.0011	-0.0011	-0.0011
x_{12}	0.0941	0.0995	0.0965	0.0965	0.0964	0.0964
y_{12}	-1.4513	-1.4976	-1.4719	-1.4719	-1.4710	-1.4707
θ_{12}	-0.0016	-0.0017	-0.0016	-0.0016	-0.0016	-0.0016

Table 4.5 – *Continued*

x_{13}	0.0946	0.1000	0.0971	0.0971	0.0969	0.0969
y_{13}	-1.2962	-1.3363	-1.3141	-1.3140	-1.3132	-1.3130
θ_{13}	-0.0016	-0.0017	-0.0016	-0.0016	-0.0016	-0.0016
x_{14}	0.0951	0.1005	0.0975	0.0975	0.0974	0.0974
y_{14}	-1.1356	-1.1694	-1.1507	-1.1507	-1.1500	-1.1498
θ_{14}	-0.0016	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017
x_{15}	0.0958	0.1011	0.0982	0.0982	0.0980	0.0980
y_{15}	-0.9686	-0.9964	-0.9810	-0.9810	-0.9804	-0.9803
θ_{15}	-0.0017	-0.0018	-0.0017	-0.0017	-0.0017	-0.0017
x_{16}	0.0950	0.1004	0.0975	0.0975	0.0973	0.0973
y_{16}	-0.7953	-0.8168	-0.8049	-0.8049	-0.8044	-0.8043
θ_{16}	-0.0017	-0.0018	-0.0018	-0.0018	-0.0018	-0.0018
x_{17}	0.0914	0.0969	0.0939	0.0939	0.0937	0.0937
y_{17}	-0.6216	-0.6368	-0.6284	-0.6284	-0.6280	-0.6280
θ_{17}	-0.0017	-0.0018	-0.0017	-0.0017	-0.0017	-0.0017
x_{18}	0.0859	0.0911	0.0882	0.0882	0.0881	0.0881
y_{18}	-0.4526	-0.4611	-0.4565	-0.4565	-0.4562	-0.4562
θ_{18}	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017
x_{19}	0.0747	0.0798	0.0770	0.0770	0.0768	0.0769

Table 4.5 – *Continued*

y_{19}	-0.2894	-0.2925	-0.2908	-0.2908	-0.2907	-0.2907
θ_{19}	-0.0015	-0.0015	-0.0015	-0.0015	-0.0015	-0.0015
x_{20}	0.0628	0.0629	0.0629	0.0629	0.0628	0.0629
y_{20}	-0.1478	-0.1481	-0.1479	-0.1479	-0.1479	-0.1479
θ_{20}	-0.0013	-0.0012	-0.0013	-0.0013	-0.0013	-0.0013
x_{21}	0.0354	0.0354	0.0354	0.0354	0.0354	0.0354
y_{21}	-0.0451	-0.0451	-0.0451	-0.0451	-0.0451	-0.0451
θ_{21}	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007

In figure 4.11, displacement in x-direction obtained at free nodes for various reanalysis methods is compared with that of exact solution for the given condition. It can be observed from the results obtained that the Combined Approximation yields the best approximation.

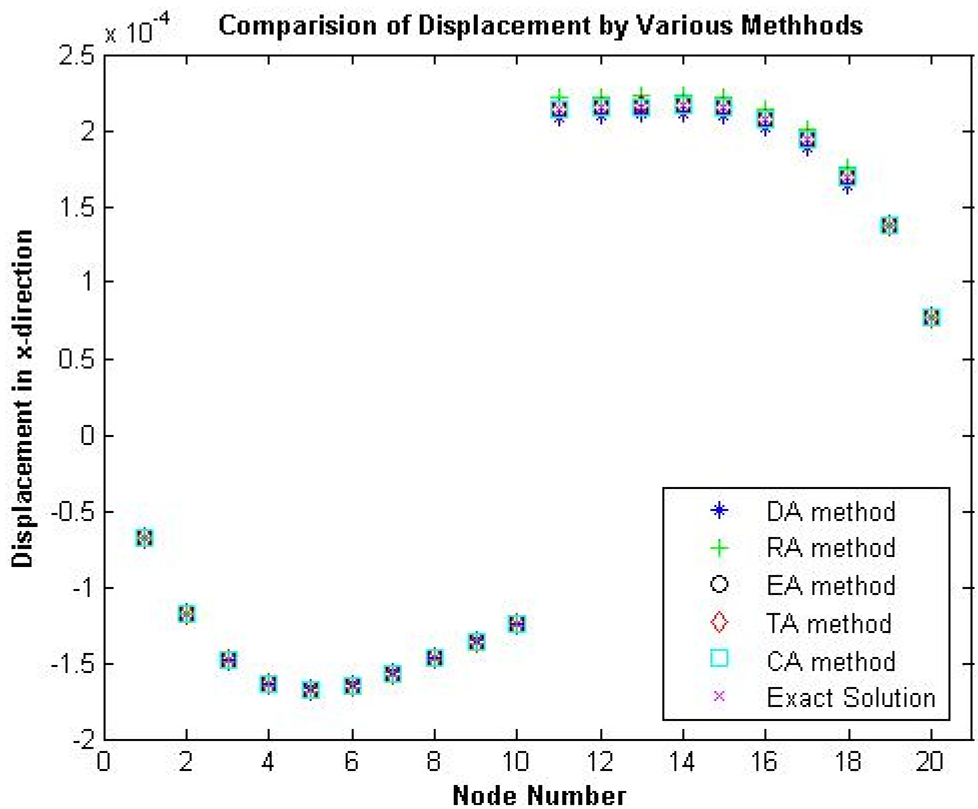

Figure 4.11 Comparison of displacement in x-direction for 50 element frame structure by various methods.

In figure 4.12, displacement in y-direction obtained at free nodes for various reanalysis methods is compared with that of exact solution for the given condition. It can be observed from the results obtained that the Combined Approximation yields the best approximation.

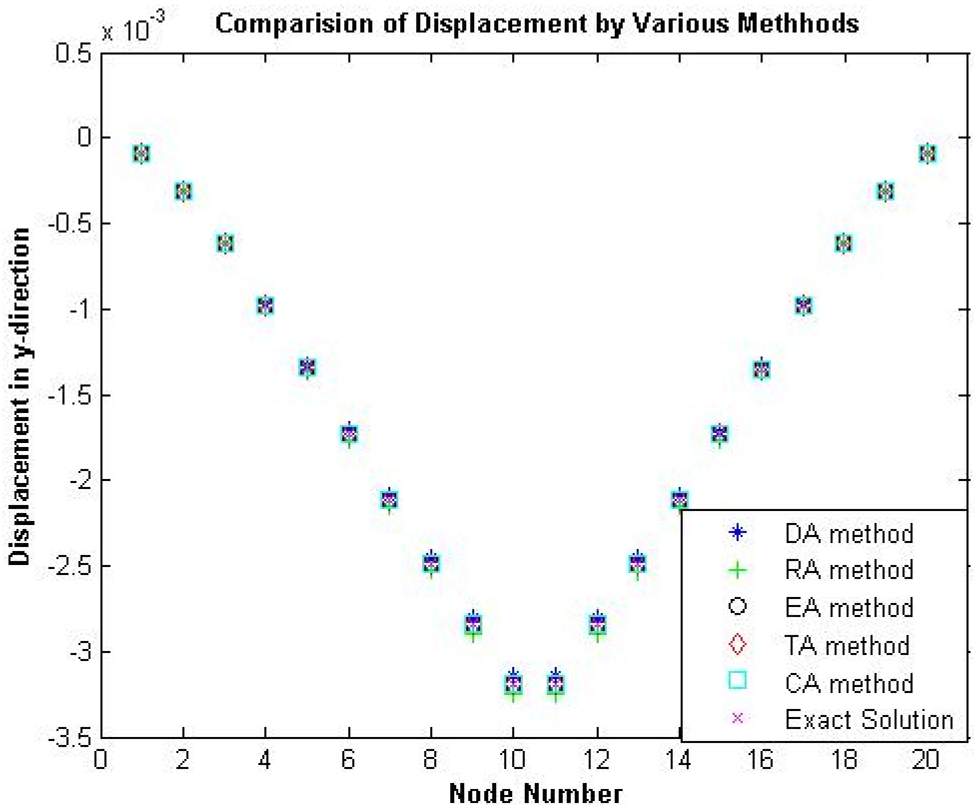

Figure 4.12 Comparison of displacement in y-direction for 50 element frame structure by various methods.

In figure 4.13, rotation obtained at free nodes for various reanalysis methods is compared with that of exact solution for the given condition. It can be observed from the results obtained that the Combined Approximation yields the best approximation.

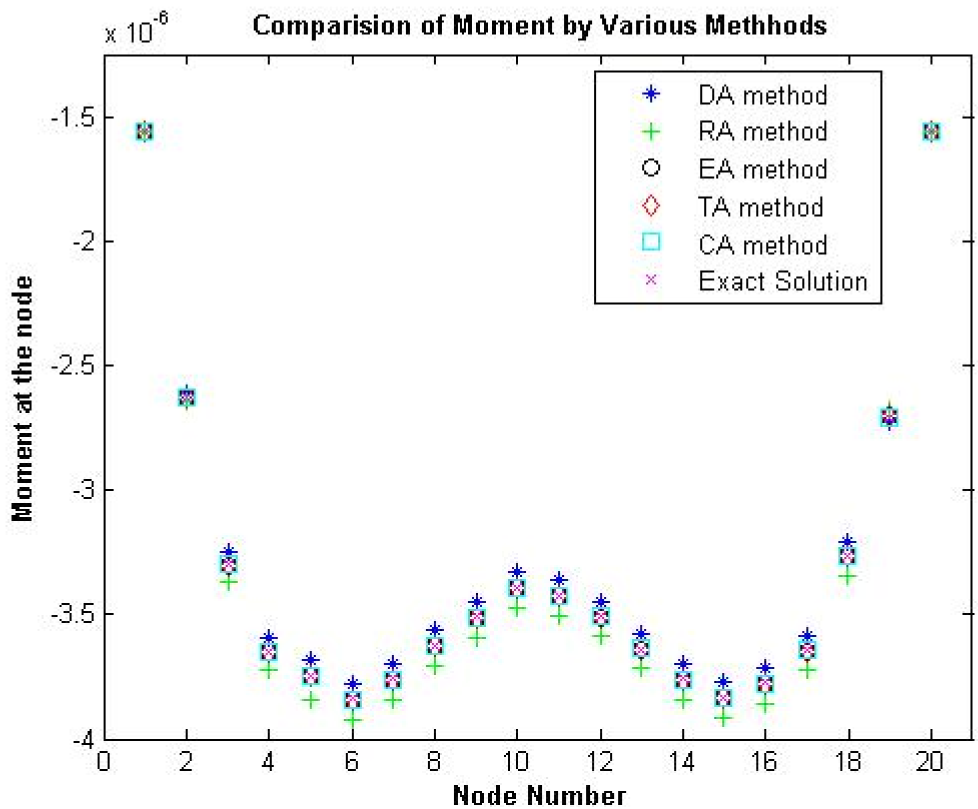

Figure 4.13 Comparison of rotation for 50 element frame structure by various methods.

In table 4.6, difference between the exact solution and the displacement and rotation obtained by reanalysis methods for modified 50 element frame problem is tabulated.

Table 4.6 Difference between the results obtained by exact solution and results obtained by reanalysis methods for 50 element frame structure

Methods / Degree of Freedom	Direct Approx. Method (10^{-3})	Reciprocal Approx. Method (10^{-3})	Exponential Approx. Method (10^{-3})	Transformed Approx. Method (10^{-3})	Combined Approx. Method (10^{-3})
x_2	-0.0031	0.0041	0.0002	0.0002	-0.0015
y_2	-0.0118	0.0159	0.0007	0.0007	-0.0022
θ_2	0.0001	0.0000	-0.0000	-0.0000	0.0000
x_3	0.0236	-0.0317	-0.0015	-0.0015	-0.0024
y_3	0.1145	-0.1538	-0.0072	-0.0071	-0.0061
θ_3	-0.0046	0.0063	0.0003	0.0003	0.0000
x_4	- 0.2327	0.3125	0.0146	0.0145	-0.0049
y_4	- 1.0961	1.4718	0.0689	0.0683	-0.0218
θ_4	-0.0214	0.0286	0.0013	0.0013	-0.0003
x_5	-0.2137	0.2919	0.0131	0.0130	-0.0024
y_5	-3.6887	4.9813	0.2301	0.2281	-0.0332
θ_5	-0.0242	0.0345	0.0015	0.0015	0.0008
x_6	-0.3260	0.5115	0.0156	0.0154	0.0352
y_6	-6.4638	8.9330	0.3898	0.3863	0.0642

Table 4.6 – *Continued*

θ_6	-0.0282	0.0422	0.0014	0.0014	0.0022
x_7	-0.3336	0.5264	0.0158	0.0156	0.0379
y_7	-9.0405	12.4901	0.5455	0.5406	0.0910
θ_7	-0.0259	0.0355	0.0016	0.0016	0.0001
x_8	-0.3347	0.5281	0.0158	0.0156	0.0381
y_8	-11.6671	16.1429	0.7022	0.6959	0.1335
θ_8	-0.0258	0.0364	0.0016	0.0015	0.0007
x_9	-0.3166	0.4974	0.0153	0.0151	0.0348
y_9	-14.1950	19.6276	0.8560	0.8483	0.1570
θ_9	-0.0258	0.0359	0.0016	0.0015	0.0004
x_{10}	-0.3184	0.5005	0.0153	0.0151	0.0354
y_{10}	-16.7999	23.2423	1.0121	1.0030	0.1954
θ_{10}	-0.0262	0.0366	0.0016	0.0016	0.0005
x_{11}	-0.3182	0.5002	0.0153	0.0151	0.0356
y_{11}	-19.4147	26.8739	1.1686	1.1580	0.2359
θ_{11}	-0.0261	0.0362	0.0016	0.0015	0.0004
x_{12}	2.2956	-3.1298	-0.1411	-0.1399	-0.0048
y_{12}	-19.4145	26.8736	1.1685	1.1580	0.2358
θ_{12}	-0.0261	0.0362	0.0016	0.0015	0.0004

Table 4.6 – *Continued*

x_{13}	2.2954	-3.1294	-0.1411	-0.1399	-0.0048
y_{13}	-16.8016	23.2452	1.0122	1.0030	0.1956
θ_{13}	-0.0263	0.0367	0.0016	0.0016	0.0005
x_{14}	2.2973	-3.1327	-0.1412	-0.1399	-0.0051
y_{14}	-14.1789	19.6003	0.8554	0.8478	0.1539
θ_{14}	-0.0252	0.0331	0.0016	0.0015	-0.0006
x_{15}	2.2792	-3.1022	-0.1406	-0.1394	-0.0017
y_{15}	-11.6502	16.1141	0.7017	0.6954	0.1302
θ_{15}	-0.0259	0.0360	0.0016	0.0015	0.0004
x_{16}	2.2803	-3.1038	-0.1406	-0.1394	-0.0018
y_{16}	-9.0491	12.5066	0.5457	0.5408	0.0936
θ_{16}	-0.0260	0.0359	0.0016	0.0016	0.0003
x_{17}	2.2878	-3.1186	-0.1408	-0.1396	-0.0046
y_{17}	-6.3978	8.8026	0.3884	0.3850	0.0412
θ_{17}	-0.0270	0.0375	0.0016	0.0016	0.0004
x_{18}	2.2142	-2.9729	-0.1392	-0.1381	0.0208
y_{18}	-3.5964	4.8155	0.2270	0.2251	-0.0560
θ_{18}	-0.0254	0.0356	0.0015	0.0015	0.0006
x_{19}	2.1953	-2.9524	-0.1377	-0.1366	0.0174

Table 4.6 – *Continued*

y_{19}	-1.3334	1.7953	0.0835	0.0828	-0.0209
θ_{19}	-0.0237	0.0322	0.0016	0.0016	-0.0001
x_{20}	0.0237	-0.0318	-0.0015	-0.0015	0.0031
y_{20}	-0.1142	0.1532	0.0072	0.0071	-0.0079
θ_{20}	0.0095	-0.0147	-0.0005	-0.0005	-0.0010
x_{21}	-0.0031	0.0041	0.0002	0.0002	0.0016
y_{21}	0.0117	0.0157	0.0007	0.0007	0.0020
θ_{21}	-0.0009	0.0014	0.0000	0.0000	0.0001

In table 4.7 the norm of values for the difference between the exact solution and the reanalysis method displacement solution for 50 element frame member is tabulated.

Table 4.7 The norm of value of difference between exact displacement and reanalysis method displacement for 50 element frame model

Methods	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
	0.1113	0.1541	0.0068	0.0068	0.0009

In table 4.8, we have show the percentage of error for each of the free degree of freedom. It has been calculated using

$$err = \left(\frac{extu - ru}{extu}\right) * 100$$

where extu = exact displacement of the modified structure

ru = displacement obtained by reanalysis method.

Table 4.8 Percentage of Error at each Degree of freedom for 50 element frame model

Methods / Degree of Freedom	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
x_2	0.0098	-0.0132	-0.0006	-0.0006	0.0048
y_2	0.0268	-0.0360	-0.0017	-0.0017	0.0049
θ_2	-0.0104	-0.0008	0.0015	0.0015	-0.0036
x_3	-0.0435	0.0583	0.0027	0.0027	0.0043
y_3	-0.0776	0.1041	0.0049	0.0048	0.0041
θ_3	0.3760	-0.5201	-0.0221	-0.0219	-0.0012
x_4	0.3385	-0.4546	-0.0213	-0.0211	0.0072
y_4	0.3777	-0.5071	-0.0237	-0.0235	0.0075

Table 4.8 – *Continued*

θ_4	1.3989	-1.8684	-0.0873	-0.0866	0.0204
x_5	0.2807	-0.3833	-0.0172	-0.0171	0.0032
y_5	0.8068	-1.0896	-0.0503	-0.0499	0.0073
θ_5	1.4449	-2.0533	-0.0877	-0.0869	-0.0467
x_6	0.4195	-0.6581	-0.0201	-0.0199	-0.0452
y_6	1.0315	-1.4256	-0.0622	-0.0617	-0.0102
θ_6	1.6410	-2.4613	-0.0799	-0.0789	-0.1290
x_7	0.4346	-0.6858	-0.0206	-0.0203	-0.0493
y_7	1.1253	-1.5547	-0.0679	-0.0673	-0.0113
θ_7	1.4648	-2.0100	-0.0896	-0.0888	-0.0084
x_8	0.4565	-0.7202	-0.0216	-0.0213	-0.0520
y_8	1.1900	-1.6466	-0.0716	-0.0710	-0.0136
θ_8	1.4989	-2.1139	-0.0904	-0.0895	-0.0400
x_9	0.4615	-0.7250	-0.0223	-0.0220	-0.0508
y_9	1.2357	-1.7086	-0.0745	-0.0738	-0.0137
θ_9	1.5655	-2.1727	-0.0941	-0.0932	-0.0230
x_{10}	0.5001	-0.7860	-0.0241	-0.0238	-0.0556
y_{10}	1.2804	-1.7714	-0.0771	-0.0764	-0.0149
θ_{10}	1.5859	-2.2104	-0.0947	-0.0938	-0.0286
x_{11}	0.5410	-0.8503	-0.0261	-0.0257	-0.0605

Table 4.8 – *Continued*

y_{11}	1.3206	-1.8279	-0.0795	-0.0788	-0.0160
θ_{11}	2.4781	-3.4376	-0.1484	-0.1471	-0.0384
x_{12}	2.3816	-3.2471	-0.1464	-0.1451	-0.0050
y_{12}	1.3201	-1.8272	-0.0795	-0.0787	-0.0160
θ_{12}	1.5854	-2.1988	-0.0950	-0.0941	-0.0226
x_{13}	2.3685	-3.2291	-0.1456	-0.1443	-0.0049
y_{13}	1.2796	-1.7703	-0.0771	-0.0764	-0.0149
θ_{13}	1.6126	-2.2524	-0.0961	-0.0952	-0.0318
x_{14}	2.3592	-3.2171	-0.1450	-0.1437	-0.0053
y_{14}	1.2331	-1.7046	-0.0744	-0.0737	-0.0134
θ_{14}	1.5162	-1.9940	-0.0938	-0.0930	0.0383
x_{15}	2.3247	-3.1641	-0.1434	-0.1422	-0.0017
y_{15}	1.1884	-1.6438	-0.0716	-0.0709	-0.0133
θ_{15}	1.5014	-2.0894	-0.0899	-0.0891	-0.0251
x_{16}	2.3431	-3.1893	-0.1445	-0.1433	-0.0019
y_{16}	1.1250	-1.5549	-0.0678	-0.0672	-0.0116
θ_{16}	1.4762	-2.0375	-0.0894	-0.0886	-0.0151
x_{17}	2.4407	-3.3270	-0.1502	-0.1489	-0.0049
y_{17}	1.0188	-1.4017	-0.0619	-0.0613	-0.0066
θ_{17}	1.5582	-2.1655	-0.0935	-0.0927	-0.0245

Table 4.8 – *Continued*

x_{18}	2.5136	-3.3749	-0.1581	-0.1567	0.0236
y_{18}	0.7883	-1.0555	-0.0498	-0.0493	0.0123
θ_{18}	1.5151	-2.1255	-0.0895	-0.0887	-0.0349
x_{19}	2.8560	-3.8409	-0.1792	-0.1777	0.0227
y_{19}	0.4586	-0.6175	-0.0287	-0.0285	0.0072
θ_{19}	1.5596	-2.1186	-0.1039	-0.1030	0.0046
x_{20}	0.0377	-0.0506	-0.0024	-0.0024	0.0050
y_{20}	0.0772	-0.1036	-0.0049	-0.0048	0.0053
θ_{20}	-0.7586	1.1688	0.0384	0.0379	0.0829
x_{21}	-0.0087	0.0117	0.0005	0.0005	0.0046
y_{21}	-0.0260	0.0348	0.0016	0.0016	0.0045
θ_{21}	0.1277	-0.1968	-0.0065	-0.0064	-0.0085

It can be observed from all the results obtained that the Combined Approximation method gives us superior results compared to all the other commonly used reanalysis methods which we have considered.

In figure 4.14, percentage of error in results obtained from various methods is compared against that of exact solution for displacement in x-direction at free nodes of the modified structure.

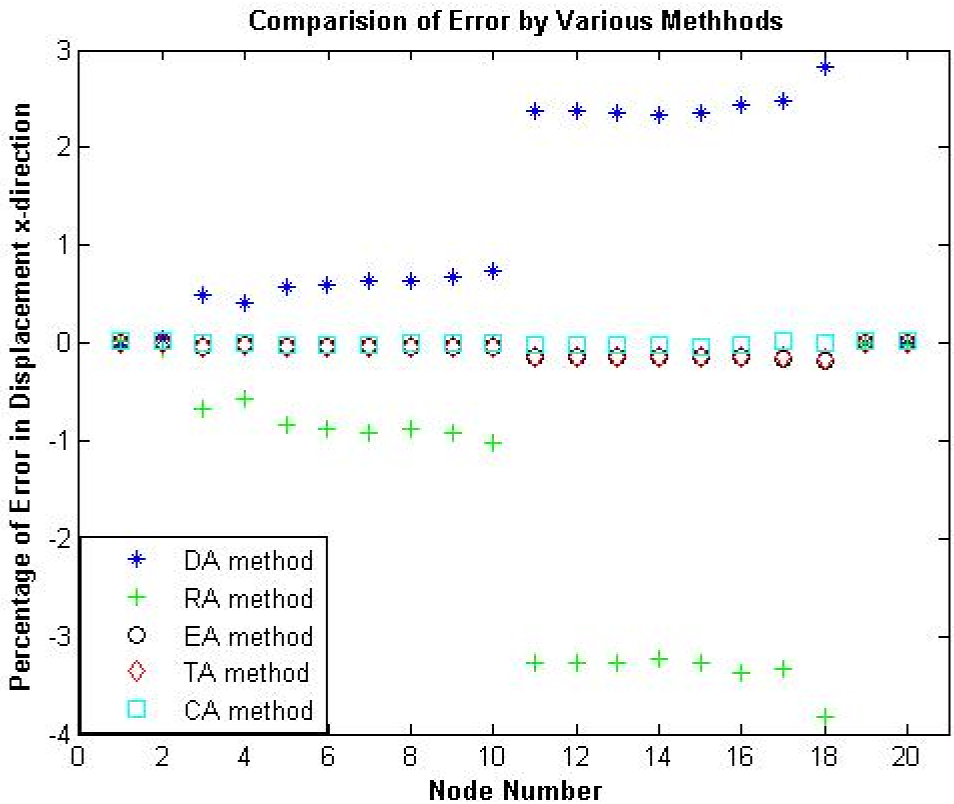

Figure 4.14 Comparison of error in x-direction of 50 element frame structure by various methods.

In figure 4.15, percentage of error in results obtained from various methods is compared against that of exact solution for displacement in y-direction at free nodes of the modified structure.

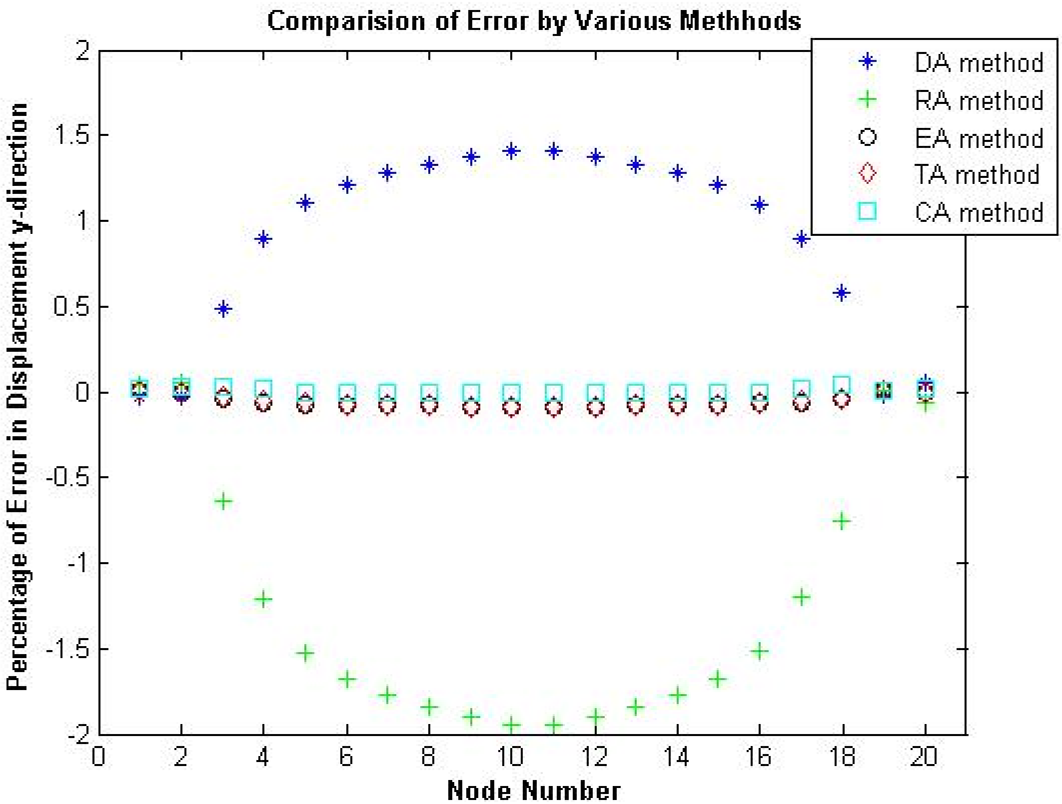

Figure 4.15 Comparison of error in y-direction of 50 element frame structure by various methods.

In figure 4.16, percentage of error in results obtained from various methods is compared against that of exact solution for rotation at free nodes of the modified structure.

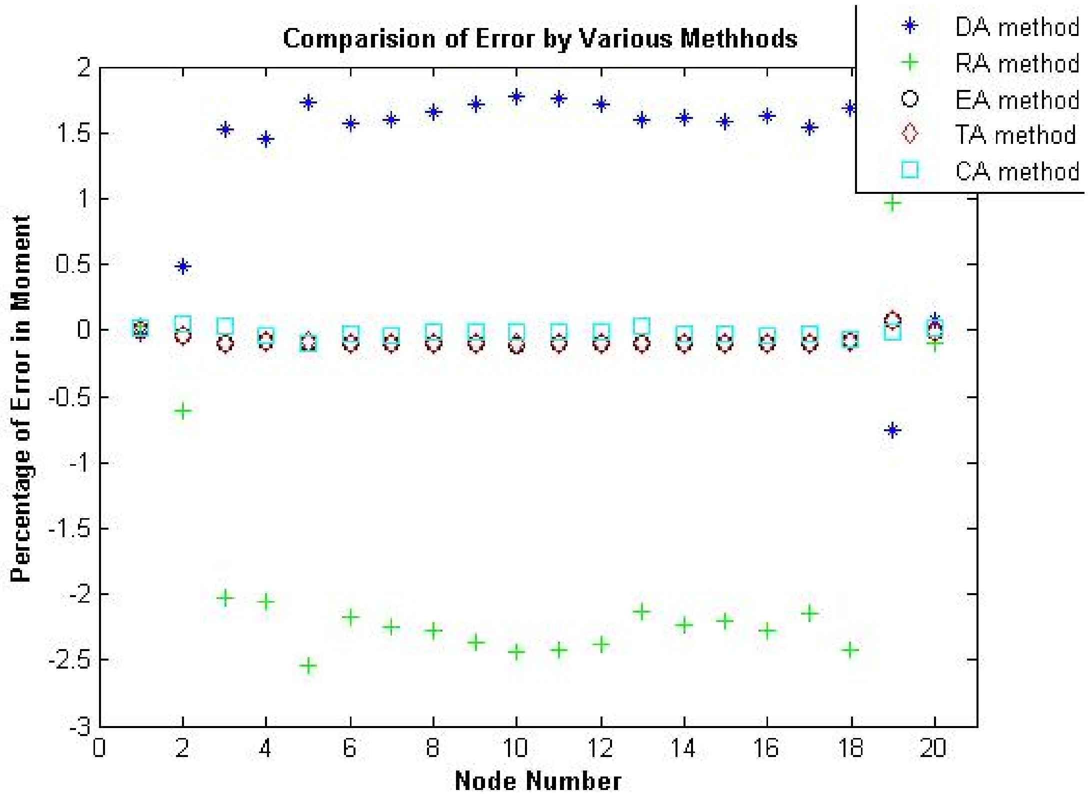

Figure 4.16 Comparison of error in rotation of 50 element frame structure by various methods.

4.3 Steady State Forced Response Spring Mass Example

Consider a 3-dof spring mass system shown in figure 4.17 for steady state forced response example. Spring stiffness and mass of system is given below.

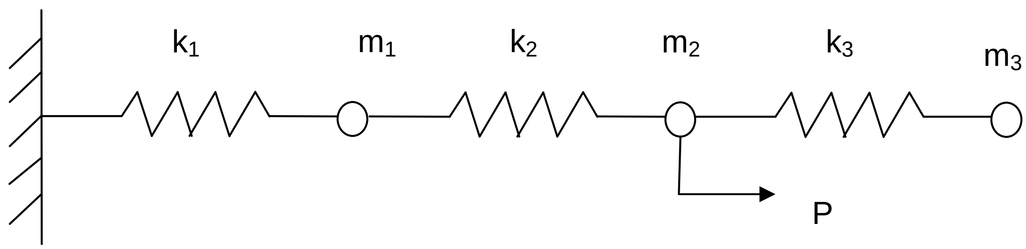

Figure 4.17 Original Steady State Forced Response Spring Mass Example

k_1 = 100 lb/in k_2 = 200 lb/in k_3 = 100 lb/in

m_1 = 1 lb m_2 = 2 lb m_3 = 1 lb

P = 50 lb

The forcing frequency Ω = 16.2462 rad/sec

In the modified design the all the conditions are kept same except the stiffness of all the springs will be changed. The springs darkened for the purpose of identification. The modified system has

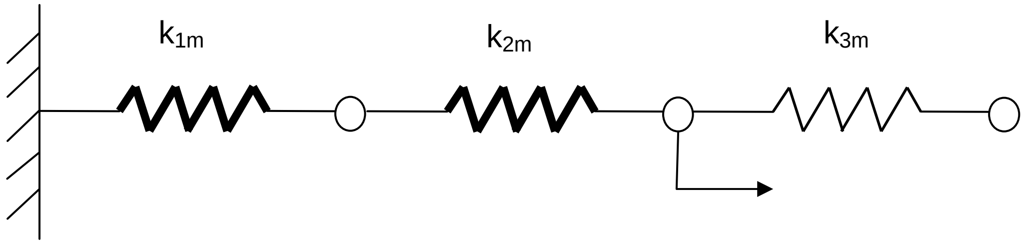

Figure 4.18 Modified Steady State Forced Response Spring Mass Example

k_{1m} = 120 lb/in k_{2m} = 180 lb/in k_{3m} = 100 lb/in

We will using only the first order basis vector in all the reanalysis methods to obtain the displacement. The amplitude of steady state responses are tabulated in the following table 4.9.

Table 4.9 Amplitude of displacement obtained by different methods after reanalysis of modified structure

Methods / Nodes	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method	Exact Solution
2	0.3890	0.4954	0.5214	0.5214	0.5825	0.5806
3	-0.2508	-0.3115	-0.3120	-0.3120	-0.3427	-0.3365
4	0.1091	0.1502	0.1424	0.1424	0.1367	0.1542

These amplitudes of displacements are plotted in figure 4.19, with the exact solution for the given conditions.

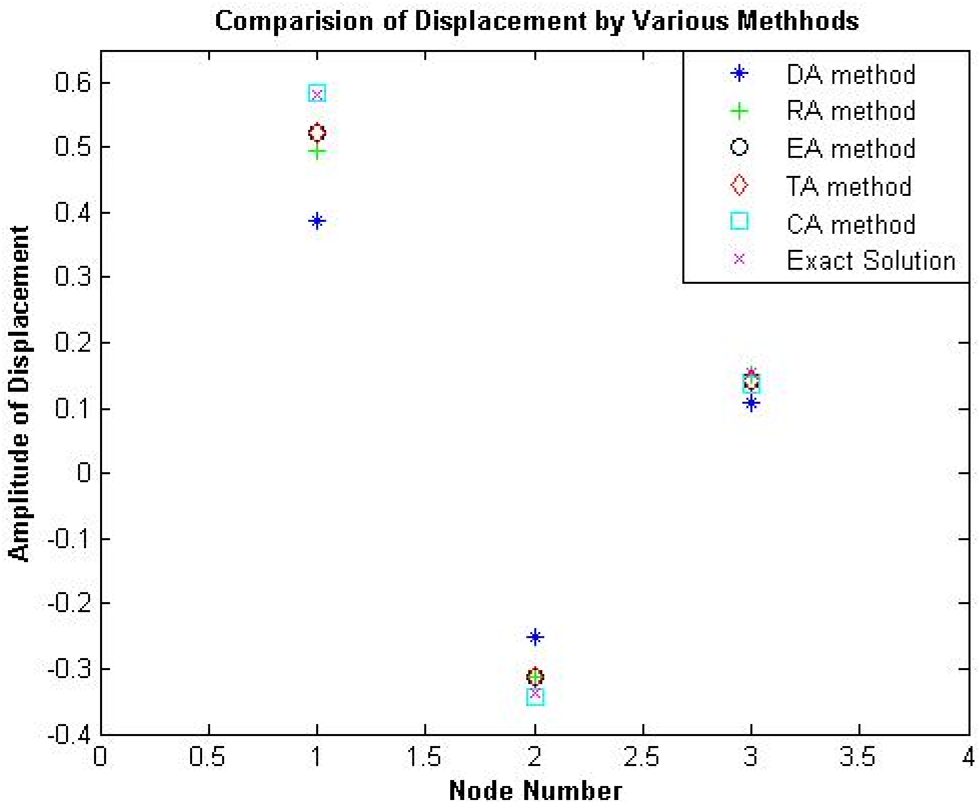

Figure 4.19 Comparison of amplitude of displacement in by various methods.

The difference between the displacement which we obtained from various reanalysis methods and that of exact solution for the modified system is tabulated in table 4.10.

Table 4.10 Difference between the exact solution and results obtained by reanalysis methods

Methods / Nodes	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
2	0.1916	0.0852	0.0591	0.0591	-0.0020
3	-0.0857	-0.0250	-0.0245	-0.0245	0.0062
4	0.0451	0.0040	0.0118	0.0118	0.0176

The norm for the difference between the exact displacement and the reanalysis method displacement solution are tabulated in table 4.11.

Table 4.11 Normalized value of difference between exact displacement and reanalysis method displacement

Methods	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
	0.2147	0.0889	0.0651	0.0651	0.0187

In table 4.12, we have show the percentage of error for each of the free degree of freedom. It has been calculated using

$$err = \left(\frac{extu - ru}{extu}\right) * 100$$

where extu = exact displacement of the modified structure

ru = displacement obtained by reanalysis method.

Table 4.12 Percentage of Error at each Degree of freedom

Methods / Nodes	Direct Approx. Method	Reciprocal Approx. Method	Exponential Approx. Method	Transformed Approx. Method	Combined Approx. Method
2	33.0003	14.6710	10.1822	10.1815	-0.3399
3	25.4661	7.4275	7.2838	7.2836	-1.8548
4	29.2476	2.5747	7.6788	7.6789	11.3835

It can be observed from all the results obtained that the Combined Approximation method gives us superior results compared to all the other commonly used reanalysis methods which we have considered.

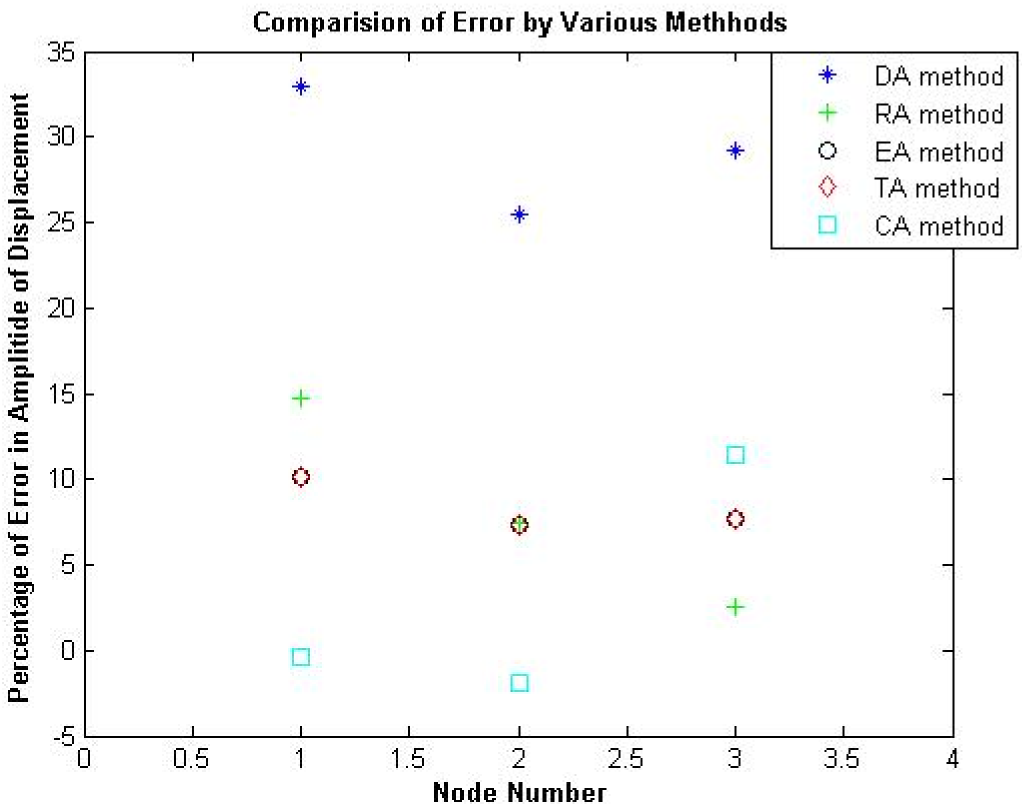

Figure 4.20 Comparison of error in amplitude of displacement by various methods.

In figure 4.20, we compare the percentage of error in amplitude of displacement at the free nodes for results obtained from various methods.

CHAPTER 5

CONCLUSION AND FUTURE WORKS

5.1 Conclusion

The results obtained for various problem shows that Combined Approximation method is better compared to other commonly used reanalysis methods.

5.2 Future Works

In future more research could be carried out to the quality of results which we can obtain for using Combined Approximation method in other type of modification in structure such as topological changes, etc.

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BIOGRAPHICAL INFORMATION

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