MODEL REFERENCE ADAPTIVE CONTROL USING STACKED IDENTIFIERS

by

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ABSTRACT

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Model reference adaptive control is a major design method for controlling plants with uncertain parameters. The primary objective of this dissertation is to develop a new design approach for the model reference adaptive control of a single-input single-output linear time-invariant plant. The proposed method, called the "Model reference adaptive control using stacked identifiers," uses a stacked identifier structure that is new to the field of adaptive control. The goal is to make the output of the plant asymptotically track the output of the first identifier, and then driving the output of the first identifier to track that of the second identifier, and so forth, up to the q-th identifier where q is the relative degree of the plant. Lastly, the output of the q-th identifier is forced to converge to that of the reference model. Simulation results show the

superiority of the proposed method over the traditional model reference adaptive control with augmented error in terms of the transient response. Since the resulting control systems are nonlinear and time-varying, the stability analysis of the overall system plays a central role in developing the theory.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

CHAPTER 1

INTRODUCTION

Model reference adaptive control (MRAC) was originally proposed to control plants with uncertain parameters. In traditional MRAC, the adaptive parameters are adjusted according to the output error, which is the difference between the output of the plant and that of the reference model. A basis of this method is that if the exact values of the plant parameters are known, then a controller is to be chosen such that the transfer function of the closed-loop system matches that of the reference model. The controller is designed to include adjustable parameters, which are updated through some parameter adaptation mechanism derived by Lyapunov's stability theory. Systems with a unity relative degree are straightforwardly designed using this approach. However, for the case of relative degree greater than one, where the transfer function is no longer strictly positive real [1], [6], [7], modification has to be introduced. This difficulty was first solved using the concept of augmented errors introduced by Monopoli [2]. After the stability problem was resolved in the early 1980's, most of the existing MRAC schemes use the same controller structure, which often suffers from poor transient responses during the initial adaptation stage. Attempts to remedy this situation include the variable structure control systems [3], [4], multiple models with switching and tuning [5], [28], [29], [39] and [43]. In this dissertation we present a method, called the

“Stacked Identifiers model reference adaptive control,” to improve the transient performance. The idea is to incorporate identifiers in the control scheme in order that the control structure comes closer to the structure of the plant. No state measurement of the plant is required. Simulations are given to show that the transient response of the proposed scheme is substantially better than that of the traditional augmented output error method.

The dissertation is organized as follows: Chapter 1 gives an introduction to the area of MRAC research. Chapter 2 begins with Identifier Tracking MRAC of plant with relative degree two, which is an essential milestone for extending the method to higher relative degrees. A considerable amount of stability proof is presented and the Identifier Tracking MRAC to the general case, i.e., plants of arbitrary relative degree q is extended. Chapter 3 deals with the design and analysis of another more systematic structure called the Stacked Identifiers MRAC. The performance of this scheme is much superior to the existing augmented output error and Identifier Tracking MRAC method as far as transient response is concerned. Chapter 4 gives a conclusion of this dissertation and possible areas for future research.

CHAPTER 2

IDENTIFIER TRACKING MODEL REFERENCE ADAPTIVE CONTROL

2.1 Introduction

For most adaptive schemes, the ability to deal with plants of relative degree two is an essential milestone for extending the method to higher relative degrees. We consider second order plants of relative degree $q = 2$, for which we will develop an adaptive scheme with a double-identifier structure using the following steps:

(i) Reparametrize the unknown plant into a form so that an appropriate identifier structure can be employed.

(ii) Derive the parameter update laws for Identifier #1, such that the identifier output y_{x1} asymptotically tracks the plant output y_p. Also derive parameter update laws for Identifier #2 so that the output of Identifier #2, y_{x2}, asymptotically tracks that of Identifier #1, y_{x1}.

(iii) Design a control law $u(t)$ to drive y_{x2} asymptotically towards the output of the reference model, y_m, and demonstrate that all variables in the feedback system are bounded.

The notation used in the adaptive control literature varies widely. In this paper, upper case letters are used to denote matrices, operators, or transfer functions and lower case letters are used for scalars or vectors. When $u(t)$ is a function of time, $u(s)$ denotes its Laplace transform; both u and $u(.)$ denote $u(t)$ or $u(s)$ according to the context. $P(s)$ is a plant transfer function or a plant transfer function operator with $s = \frac{d}{dt}$.

2.2 Identifier Tracking MRAC of plants with relative degree two

2.2.1. Reparameterization of the Unknown Plant

Consider a linear time-invariant plant $P(s)$ with an input-output pair $\{u(.), y_p(.)\}$ described by a transfer function

$$P(s) = \frac{k_p}{s^2 + a_{p1}s + a_{p0}} \qquad (2.2.1)$$

where k_p, a_{p0} and a_{p1} are constant but unknown parameters. The sign of the high frequency gain k_p and a lower bound for k_p are assumed to be known, i.e., for the case of a positive k_p, $k_p > k_{lower} > 0$; and for the case of a negative k_p, $k_p < k_{upper} < 0$. Throughout this paper, k_p is assumed to be positive.

We will reparametrize the plant into a form suitable for deriving the identifier and the parameter update laws.

Express (2.2.1) as

$$(s^2 + a_{p1}s + a_{p0})y_p(t) = k_p u(t), \quad k_p > k_{lower} > 0 \tag{2.2.2}$$

Dividing both sides of the above equation by $(s+\lambda_0)(s+\lambda_1)$, where λ_0 and λ_1 are positive constants, we obtain

$$\frac{s^2 + a_{p1}s + a_{p0}}{(s+\lambda_0)(s+\lambda_1)} y_p(t) = \frac{k_p}{(s+\lambda_0)(s+\lambda_1)} u(t) \tag{2.2.3}$$

Performing a long division on the L.H.S. gives

$$\text{L.H.S. of (2.2.3)} = y_p(t) + \frac{\left(a_{p1} - (\lambda_0 + \lambda_1)\right)s + \left(a_{p0} - \lambda_0\lambda_1\right)}{(s+\lambda_0)(s+\lambda_1)} y_p(t)$$

Conducting another long division on the second term yields

L.H.S. of (2.2.3) =

$$y_p(t) + \frac{1}{s+\lambda_1}\left[\left(a_{p1} - (\lambda_0 + \lambda_1)\right) + \frac{1}{(s+\lambda_0)}\left(a_{p0} - a_{p1}\lambda_0 + \lambda_0^2\right)\right] y_p(t)$$

Substituting this expression into (2.2.3) and moving all terms other than the term y_p to the R.H.S. gives

$$y_p(t) = \frac{1}{s+\lambda_1}\left[\left(\lambda_0 + \lambda_1 - a_{p1}\right)y_p(t) + \left(a_{p1}\lambda_0 - \lambda_0^2 - a_{p0}\right)\frac{1}{s+\lambda_0} y_p(t) + k_p \frac{1}{s+\lambda_0} u(t)\right] \tag{2.2.4a}$$

Define

$$c_0^* \triangleq k_p \tag{2.2.4b}$$

$$d_0^* \triangleq a_{p1}\lambda_0 - \lambda_0^2 - a_{p0} \tag{2.2.4c}$$

$$d_1^* \triangleq \lambda_0 + \lambda_1 - a_{p1} \tag{2.2.4d}$$

$$\tilde{u} \triangleq \frac{1}{s+\lambda_0} u(t) \tag{2.2.4e}$$

$$z_1 \triangleq \frac{1}{s+\lambda_0} y_p(t) \tag{2.2.4f}$$

Then, equation (2.2.4a) becomes

$$y_p(t) = \frac{1}{s+\lambda_1}\left(c_0^* \tilde{u}(t) + d_0^* z_1(t) + d_1^* y_p(t)\right) \tag{2.2.5}$$

In accordance with the form of (2.2.5), Identifier #1 is chosen as

$$y_{x1}(t) = \frac{1}{s+\lambda_1}\left((c_{10}\tilde{u}(t) + d_{10} z_1(t) + d_{11} y_p(t)\right) \tag{2.2.6}$$

Figure 2.1 shows a schematic diagram of Identifier #1.

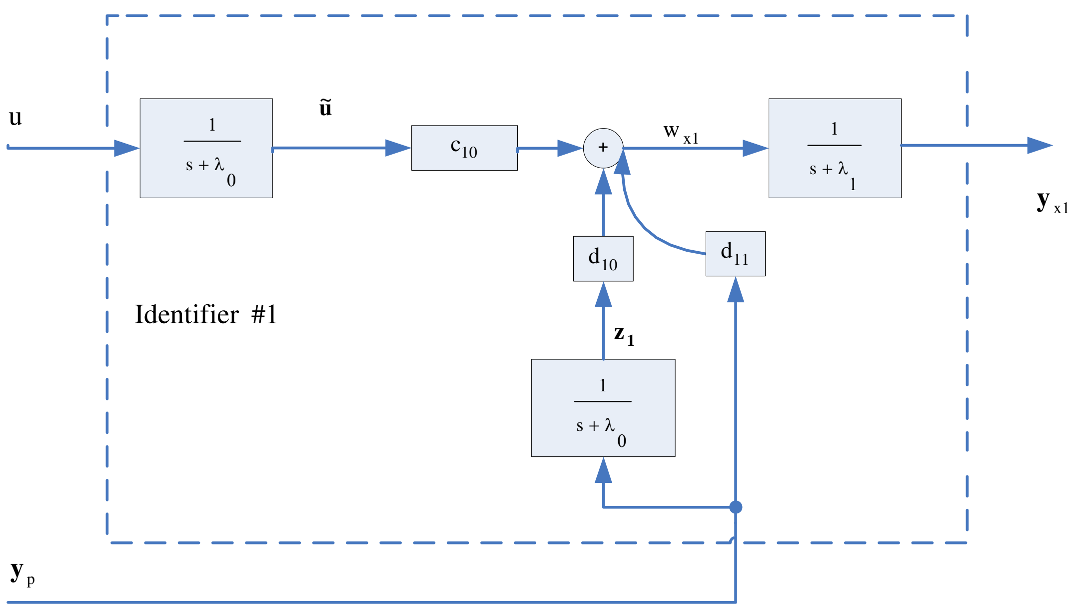

Figure 2.1: Identifier Tracking MRAC for Identifier #1.

Similar to the structure of Identifier #1, Identifier #2 is chosen as

$$y_{x2}(t) = \frac{1}{s+\lambda_1}\left(c_{20}\tilde{u}(t) + d_{20}z_2(t) + d_{21}y_{x1}(t)\right) \tag{2.2.7a}$$

where

$$z_2 \triangleq \frac{1}{s+\lambda_0} y_{x1}(t) \tag{2.2.7b}$$

The corresponding block diagram is shown in Figure 2.2.

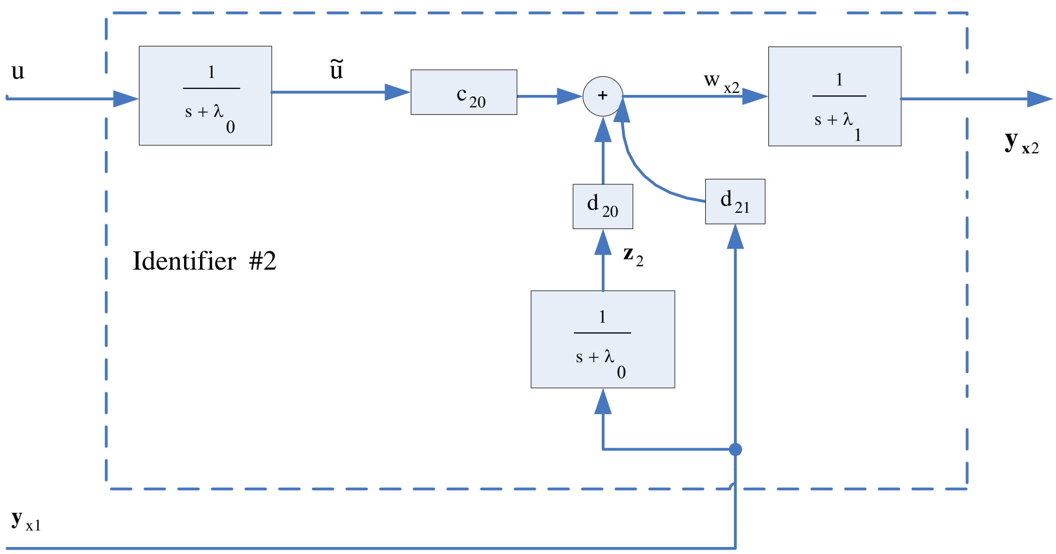

Figure 2.2: Identifier Tracking MRAC for Identifier #2.

2.2.2. Parameter Update Laws for the Identifiers

Identifier #1

Define

$$e_{x1}(t) \triangleq y_p(t) - y_{x1}(t) \tag{2.2.8a}$$

$$\tilde{c}_0 \triangleq c_0^* - c_{10},\ \tilde{d}_0 \triangleq d_0^* - d_{10},\ \tilde{d}_1 \triangleq d_1^* - d_{11} \tag{2.2.8b}$$

$$\varphi(t) \triangleq \begin{bmatrix} \tilde{c}_0 & \tilde{d}_0 & \tilde{d}_1 \end{bmatrix}^T \tag{2.2.8c}$$

$$w(t) \triangleq \begin{bmatrix} \tilde{u}\ (t) & z_1(t) & y_p(t) \end{bmatrix}^T \tag{2.2.8d}$$

From (2.2.8), (2.2.5) and (2.2.6), the error equation is given by

$$\begin{aligned} e_{x1}(t) &= y_p(t) - y_{x1}(t) \\ &= \frac{1}{s+\lambda_1}\left(\tilde{c}_0\tilde{u}(t) + \tilde{d}_0 z_1(t) + \tilde{d}_1 y_p(t)\right) \\ &= \frac{1}{s+\lambda_1}\varphi^T(t)w(t) \end{aligned} \tag{2.2.9}$$

Multiplying both sides of (2.2.9) by the polynomial operator $s+\lambda_1$ and moving all terms to the R.H.S. except for the $\dot{e}_{x1}$ term gives

$$\dot{e}_{x1}(t) = -\lambda_1 e_{x1}(t) + \tilde{c}_0\tilde{u}(t) + \tilde{d}_0 z_1(t) + \tilde{d}_1 y_p(t) \tag{2.2.10}$$

To go through a stability analysis, we choose a Lyapunov function candidate

$$V = \frac{1}{2}\left[e_{x1}^2 + \frac{1}{g}\left(\tilde{c}_0^2 + \tilde{d}_0^2 + \tilde{d}_1^2\right)\right], \quad g > 0 \tag{2.2.11}$$

The derivative of V is given by

$$\dot{V} = e_{x1}\dot{e}_{x1} + \frac{1}{g}\left(\tilde{c}_0\dot{\tilde{c}}_0 + \tilde{d}_0\dot{\tilde{d}}_0 + \tilde{d}_1\dot{\tilde{d}}_1\right)$$

Substituting $\dot{e}_{x1}$ from (2.2.10) yields

$$\dot{V} = -\lambda_1 e_{x1}^2 + \tilde{c}_0\left(e_{x1}\tilde{u} + \frac{1}{g}\dot{\tilde{c}}_0\right) + \tilde{d}_0\left(e_{x1}z_1 + \frac{1}{g}\dot{\tilde{d}}_0\right) + \tilde{d}_1\left(e_{x1}y_p + \frac{1}{g}\dot{\tilde{d}}_1\right) \quad (2.2.12)$$

Choose the parameter update laws as

$$\dot{c}_{10} = \begin{cases} 0, & \text{if } ge_{x1}\tilde{u} \le 0 \text{ and } c_{10} \le k_{lower} \quad (2.2.13a) \\ ge_{x1}\tilde{u}, & \text{otherwise} \quad (2.2.13b) \end{cases}$$

$$\dot{d}_{10} = ge_{x1}z_1 \quad (2.2.13c)$$

$$\dot{d}_{11} = ge_{x1}y_p \quad (2.2.13d)$$

Note that the adaptation for c_{10} is divided into two cases. The first case (2.2.13a) together with the rest of the adaptations (2.2.13c) and (2.2.13d) renders $\dot{V}$ in (2.2.12) as

$$\dot{V} = -\lambda_1 e_{x1}^2 + \tilde{c}_0 e_{x1}\tilde{u} \le 0 \quad (2.2.14a)$$

Similarly, the second case (2.2.13b) gives

$$\dot{V} = -\lambda_1 e_{x1}^2 \le 0 \quad (2.2.14b)$$

We see that in both cases, $\dot{V}$ is negative semi-definite. This implies that e_{x1}, $\tilde{c}_0$, $\tilde{d}_0$ and $\tilde{d}_1$ are bounded; and from (2.2.8b), c_{10}, d_{10} and d_{11} are also bounded.

The division of the adaptation of c_{10} in two cases as given in (2.2.13a) and (2.2.13b) is to ensure that

$$c_{10}(t) \ge k_{lower} > 0, \text{ for all } t \ge 0 \quad (2.2.14c)$$

This will be achieved by choosing an initial condition for the adaptive parameter $c_{10}(0) \ge k_{lower}$.

Identifier #2

Identifier #2 is chosen as

$$y_{x2}(t) = \frac{1}{s+\lambda_1}\left(c_{20}\tilde{u}(t) + d_{20}z_2(t) + d_{21}y_{x1}(t)\right) \tag{2.2.15a}$$

where

$$z_2 \triangleq \frac{1}{s+\lambda_0} y_{x1}(t) \tag{2.2.15b}$$

The purpose of the parameter update laws for Identifier #2 is to achieve $e_{x2} = (y_{x1} - y_{x2}) \to 0$ as $t \to \infty$.

They are chosen as

$$\dot{c}_{20} = \begin{cases} 0, & \text{if } ge_c\tilde{u} + \dot{c}_{10} \le 0 \text{ and } c_{20} \le c_{10} \quad (2.2.16a) \\ ge_c\tilde{u} + \dot{c}_{10}, & \text{otherwise } ge_c\tilde{u} + \dot{c}_{10} > 0 \quad (2.2.16b) \end{cases}$$

$$\dot{d}_{20} = \frac{(d_{10} - d_{20})\left(\alpha + \beta z_2^2 + z_2\dot{z}_2\right)}{1 + z_2^2} + \dot{d}_{10} \tag{2.2.16c}$$

$$\dot{d}_{21} = \frac{(d_{11} - d_{21})\left(\alpha + \beta y_{x1}^2 + y_{x1}\dot{y}_{x1}\right)}{1 + y_{x1}^2} + \dot{d}_{11} \tag{2.2.16d}$$

where the design parameters α, β and e_c are defined later in the Lyapunov analysis shown below.

Let

$$e_{x2} \triangleq y_{x1} - y_{x2} \tag{2.2.17}$$

From (2.2.6), (2.2.15a) and (2.2.17), we have

$$e_{x2} = \frac{1}{s+\lambda_1}\left(c_{10} - c_{20}\right)\tilde{u} + \frac{1}{s+\lambda_1}\left(d_{10}z_1 - d_{20}z_2\right) + \frac{1}{s+\lambda_1}\left(d_{11}y_p - d_{21}y_{x1}\right)$$

(2.2.18)

We first establish the boundedness of e_{x2}. This can be accomplished by requiring the same for the three R.H.S. terms, i.e., the boundedness of $\frac{1}{s+\lambda_1}\left(c_{10} - c_{20}\right)\tilde{u}$, $\frac{1}{s+\lambda_1}\left(d_{10}z_1 - d_{20}z_2\right)$ and $\frac{1}{s+\lambda_1}\left(d_{11}y_p - d_{21}y_{x1}\right)$, respectively, which can be achieved as follows.

(i). Boundedness of $\frac{1}{s+\lambda_1}\left(c_{10} - c_{20}\right)\tilde{u}$

Let

$$e_c \triangleq \frac{1}{s+\lambda_1}\left(c_{10} - c_{20}\right)\tilde{u} \tag{2.2.19}$$

Multiplying $s+\lambda_1$ to both sides of the equation yields

$$\dot{e}_c = -\lambda_1 e_c + \left(c_{10} - c_{20}\right)\tilde{u} \tag{2.2.20}$$

Choose a Lyapunov function candidate (to secure boundedness of e_c and $c_{10} - c_{20}$) as

$$V = \frac{1}{2}\left[g e_c^2 + \left(c_{10} - c_{20}\right)^2 \right] > 0 \tag{2.2.21}$$

The derivative of V is given by

$$\dot{V} = g e_c \dot{e}_c + \left(c_{10} - c_{20}\right)\left(\dot{c}_{10} - \dot{c}_{20}\right) \tag{2.2.22}$$

Substituting $\dot{e}_c$ from (2.2.20) yields

$$\dot{V} = -\lambda_1 g e_c^2 + (c_{10} - c_{20})[(g e_c \tilde{u} + \dot{c}_{10}) - \dot{c}_{20}] \qquad (2.2.23)$$

Use of the parameter update law (2.2.16a) and (2.2.16b) renders $\dot{V}$ as

$$\dot{V} = \begin{cases} -\lambda_1 g e_c^2 + (c_{10} - c_{20})(g e_c \tilde{u} + \dot{c}_{10}) \le 0, & \text{if } g e_c \tilde{u} + \dot{c}_{10} \le 0 \text{ and } c_{20} \le c_{10} \quad (2.2.24a) \\ -\lambda_1 g e_c^2 \le 0, & \text{otherwise} \quad (2.2.24b) \end{cases}$$

We can see that $\dot{V}$ is negative semi-definite. This implies from Lyapunov's theory that

$c_{10} - c_{20}$ and $e_c = \frac{1}{s+\lambda_1}(c_{10} - c_{20})\tilde{u}$ are bounded.

The division of the adaptation of c_{20} in two cases as given in (2.2.16a) and (2.2.16b) is to ensure that

$$c_{20}(t) \ge c_{10}(t), \text{ for all } t \ge 0 \qquad (2.2.24c)$$

This will be achieved by choosing an initial condition for the adaptive parameter $c_{20}(0) \ge c_{10}(0)$.

(ii). Boundedness of $\frac{1}{s+\lambda_1}(d_{10} z_1 - d_{20} z_2)$

Rearrange $d_{10} z_1 - d_{20} z_2$ as

$$d_{10} z_1 - d_{20} z_2 = d_{10}(z_1 - z_2) + (d_{10} - d_{20}) z_2 \qquad (2.2.25)$$

We shall treat the boundedness of the two terms on the R.H.S. separately.

In accordance with (2.2.4f), (2.2.15b) and (2.2.8a), $z_1 - z_2$ is given by

$$z_1 - z_2 = \frac{1}{s+\lambda_0}\left(y_p - y_{x1}\right) = \frac{1}{s+\lambda_0} e_{x1} \qquad (2.2.26)$$

$z_1 - z_2$ is bounded because the same is true for e_{x1}, (which is the input to the asymptotically stable system $\frac{1}{s+\lambda_0}$).

Thus, with a bounded d_{10}, $d_{10}(z_1 - z_2)$ is also bounded.

We now turn to the second term in (2.2.25) and let

$$e_d \triangleq \frac{1}{s+\lambda_1}(d_{10} - d_{20}) z_2 \qquad (2.2.27)$$

Multiplying $s+\lambda_1$ to both sides of the equation yields

$$\dot{e}_d = -\lambda_1 e_d + (d_{10} - d_{20}) z_2 \qquad (2.2.28)$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}\left[g e_d^2 + (d_{10} - d_{20})^2 + \left((d_{10} - d_{20}) z_2\right)^2 \right] > 0 \qquad (2.2.29)$$

The derivative of V is given by

$$\dot{V} = g e_d \dot{e}_d + (d_{10} - d_{20})(\dot{d}_{10} - \dot{d}_{20}) + (d_{10} - d_{20}) z_2 \left[(\dot{d}_{10} - \dot{d}_{20}) z_2 + (d_{10} - d_{20}) \dot{z}_2\right] \qquad (2.2.30)$$

Substituting $\dot{e}_d$ from (2.2.28) yields

$$\dot{V} = -\lambda_1 g e_d^2 + g e_d (d_{10} - d_{20}) z_2 + (d_{10} - d_{20})(\dot{d}_{10} - \dot{d}_{20}) + (d_{10} - d_{20}) z_2 \left[(\dot{d}_{10} - \dot{d}_{20}) z_2 + (d_{10} - d_{20}) \dot{z}_2\right] \qquad (2.2.31)$$

Let $\sigma_1 = e_d$ and $\sigma_2 = \sigma_3 z_2$. Our purpose is to select an adaptive law for d_{20} so that $\dot{V}$ will be in the form of

$$\dot{V} = -\alpha\sigma_3^2 - \left\{a\sigma_1^2 - b\sigma_1\sigma_2 + c\sigma_2^2\right\} \tag{2.2.32a}$$

where α, a, b and c are constant design parameters to be chosen so as to make $\dot{V} \le 0$. It is not difficult to see that (2.2.16c) would bring $\dot{V}$ to the required form (2.2.32a) with

$$a \triangleq \lambda_1 g,\ b \triangleq g,\ c \triangleq \beta,\ \sigma_3 \triangleq (d_{10} - d_{20}) \tag{2.2.32b}$$

With reference to (2.2.32a), by choosing $\alpha > 0$, the first term is non-positive. Any choice of a, b and c satisfying

$$ac > \left(\frac{b}{2}\right)^2 \tag{2.2.33}$$

will make the second (quadratic) term also non-positive. In other words, from (2.2.32b) and (2.2.33), the choice of the adaptive parameters

$$\lambda_1\beta > \frac{g}{4} \tag{2.2.34}$$

will render $\dot{V} \le 0$. This implies from Lyapunov's theory that $(d_{10} - d_{20})z_2$ is bounded. Hence, from (2.2.25), with both terms on the R.H.S. being bounded, $d_{10}z_1 - d_{20}z_2$ is bounded. Therefore, with bounded input $d_{10}z_1 - d_{20}z_2$, the output $\frac{1}{s+\lambda_1}(d_{10}z_1 - d_{20}z_2)$ is also bounded.

(iii). Boundedness of $\frac{1}{s+\lambda_1}(d_{11}y_p - d_{21}y_{x1})$

The treatment of $\frac{1}{s+\lambda_1}(d_{11}y_p - d_{21}y_{x1})$ follows the same vein as that of

$$\frac{1}{s+\lambda_1}(d_{10}z_1 - d_{20}z_2)$$ above, and is therefore omitted.

Summarizing the results of (i)-(iii) in this section, it follows from (2.2.18) that e_{x2} is bounded. (The consequence of $e_{x2} \to 0$ will be shown later.) Figure 2.3 shows a schematic diagram of Identifier #1 and Identifier #2.

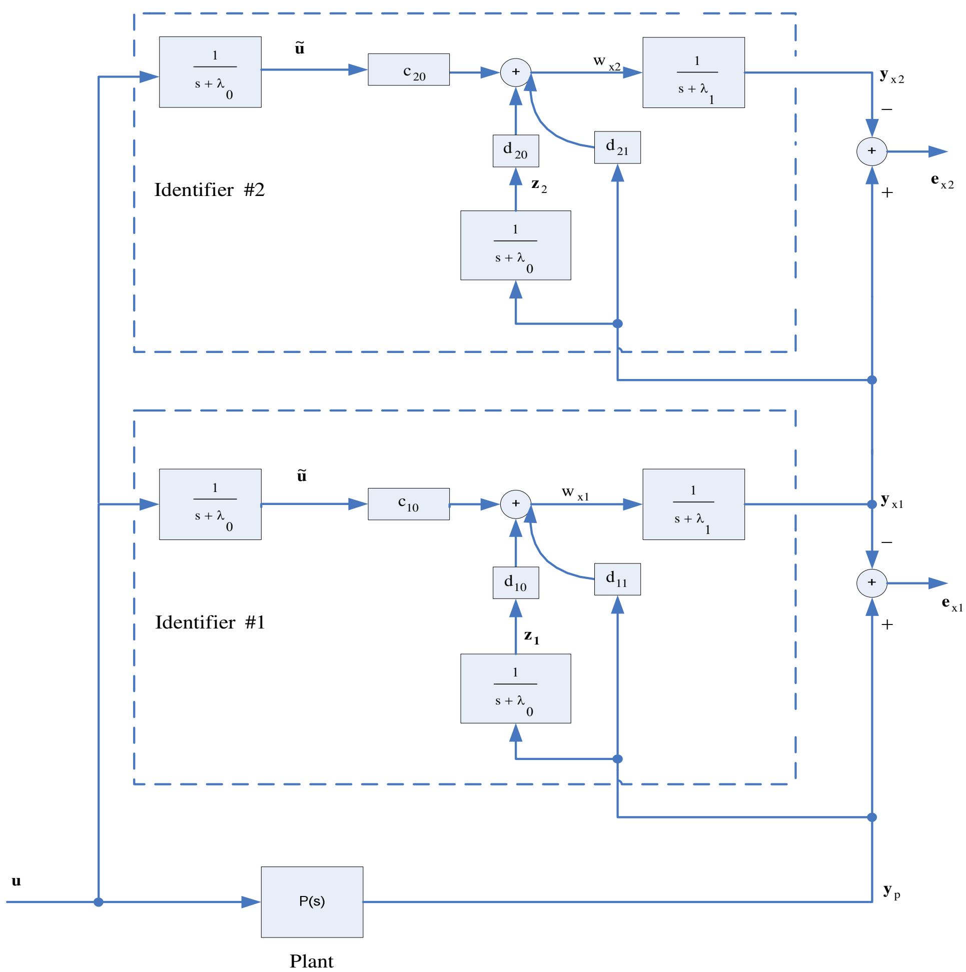

Figure 2.3: Identifier Tracking MRAC for Identifier #1 and Identifier #2.

2.2.3. Control Law u(t)

The reference model is given by an input-output pair $\{r(.), y_m(.)\}$ with a transfer function $M(s)$ given as

$$\frac{y_m(s)}{r(s)} = M(s) = \frac{k_m}{(s + a_{m1})(s + a_{m0})} \tag{2.2.35}$$

where k_m, a_{m1} and a_{m0} are positive design parameters, $r(t)$ is a bounded, piecewise continuous function of time for $t \geq 0$. The purpose here is to derive a control law such that y_{x2} asymptotically tracks y_m.

Define tracking error e as

$$e \triangleq (s + a_{m0})e_{m2} \tag{2.2.36a}$$

where

$$e_{m2} \triangleq y_{x2} - y_m \tag{2.2.36b}$$

(Note that if $e \to 0$, then $e_{m2} \to 0$ and $y_{x2} \to y_m$)

From (2.2.35) and (2.2.36), we have

$$e = (s + a_{m0})(y_{x2} - y_m) = (\dot{y}_{x2} + a_{m0}y_{x2}) - k_m r_x \tag{2.2.37a}$$

where

$$r_x \triangleq \frac{r}{s + a_{m1}} \tag{2.2.37b}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}e^2 > 0$$

A control law is now to be devised in order to make $\dot{V}$ negative definite.

From (2.2.37a), the derivative of e is given by

$$\dot{e} = (\ddot{y}_{x2} + a_{m0}\dot{y}_{x2}) - k_m \dot{r}_x \quad (2.2.38)$$

Multiplying $s + \lambda_1$ to both sides of (2.2.15a) yields

$$\dot{y}_{x2} = c_{20}\tilde{u} + d_{20}z_2 + d_{21}y_{x1} - \lambda_1 y_{x2} \quad (2.2.39)$$

The second derivative of y_{x2} reads

$$\ddot{y}_{x2} = c_{20}\dot{\tilde{u}} + m \quad (2.2.40a)$$

where

$$m \underline{\underline{\Delta}} \dot{c}_{20}\tilde{u} + \dot{d}_{20}z_2 + d_{20}\dot{z}_2 + \dot{d}_{21}y_{x1} + d_{21}\dot{y}_{x1} - \lambda_1 \dot{y}_{x2} \quad (2.2.40b)$$

Substituting (2.2.40a) into (2.2.38) yields

$$\dot{e} = c_{20}\dot{\tilde{u}} + m + a_{m0}\dot{y}_{x2} - k_m \dot{r}_x \quad (2.2.41)$$

Next we substitute $\dot{\tilde{u}}$ with $-\lambda_0\tilde{u} + u$ from (2.2.4e). The result is

$$\dot{e} = c_{20}u + \left(-c_{20}\,\lambda_0\tilde{u} + m + a_{m0}\dot{y}_{x2} - k_m \dot{r}_x\right) \quad (2.2.42)$$

Since our objective is to design a differentiator-free controller, setting $\dot{e} = -ke$ in (2.2.42) and replacing the derivative terms $\dot{y}_{x2}$ and $\dot{r}_x$ from (2.2.39) and (2.2.37b) gives the control law

$$u(t) = \left[\frac{1}{c_{20}}\left(c_{20}\lambda_0\tilde{u} - m - a_{m0}\left(c_{20}\tilde{u} + d_{20}z_2 + d_{21}y_{x1} - \lambda_1 y_{x2}\right)\right.\right.$$
$$\left.\left. + k_m\left(r - a_{m1}r_x\right)\right)\right] - ke, \quad c_{20} > 0,\ k > 0$$

(2.2.43a)

where

$$m = \dot{c}_{20}\tilde{u} + \dot{d}_{20}z_2 + d_{20}(y_{x1} - \lambda_0 z_2) + \dot{d}_{21}y_{x1} + d_{21}\left((c_{10}\tilde{u}(t) + d_{10}z_1(t) + d_{11}y_p(t) - \lambda_1 y_{x1}\right)$$
$$- \lambda_1(c_{20}\tilde{u} + d_{20}z_2 + d_{21}y_{x1} - \lambda_1 y_{x2})$$

(2.2.43b)

Note that the derivatives of the adaptive coefficients can be replaced by their respective adaptive laws in (2.2.16) and (2.2.13) in order to avoid actual differentiations. Also note from (2.2.14c) and (2.2.24c) that $c_{20} \geq c_{10} \geq k_{lower} > 0$ so division by zero in the control law would not occur.

Substituting the control law (2.2.43a) into (2.2.42) yields

$$\dot{e} = -ke \tag{2.2.44}$$

which makes

$$\dot{V} = e\,\dot{e} = -ke^2 \tag{2.2.45}$$

negative definite. This implies that the equilibrium state $e = 0$ is globally asymptotically stable, i.e. e is bounded and $e \to 0$ as $t \to \infty$.

Therefore, from (2.2.36),

$$e_{m2} \to 0 \text{ and } y_{x2} \to y_m \tag{2.2.46}$$

Figure 2.4 shows a schematic diagram of overall system.

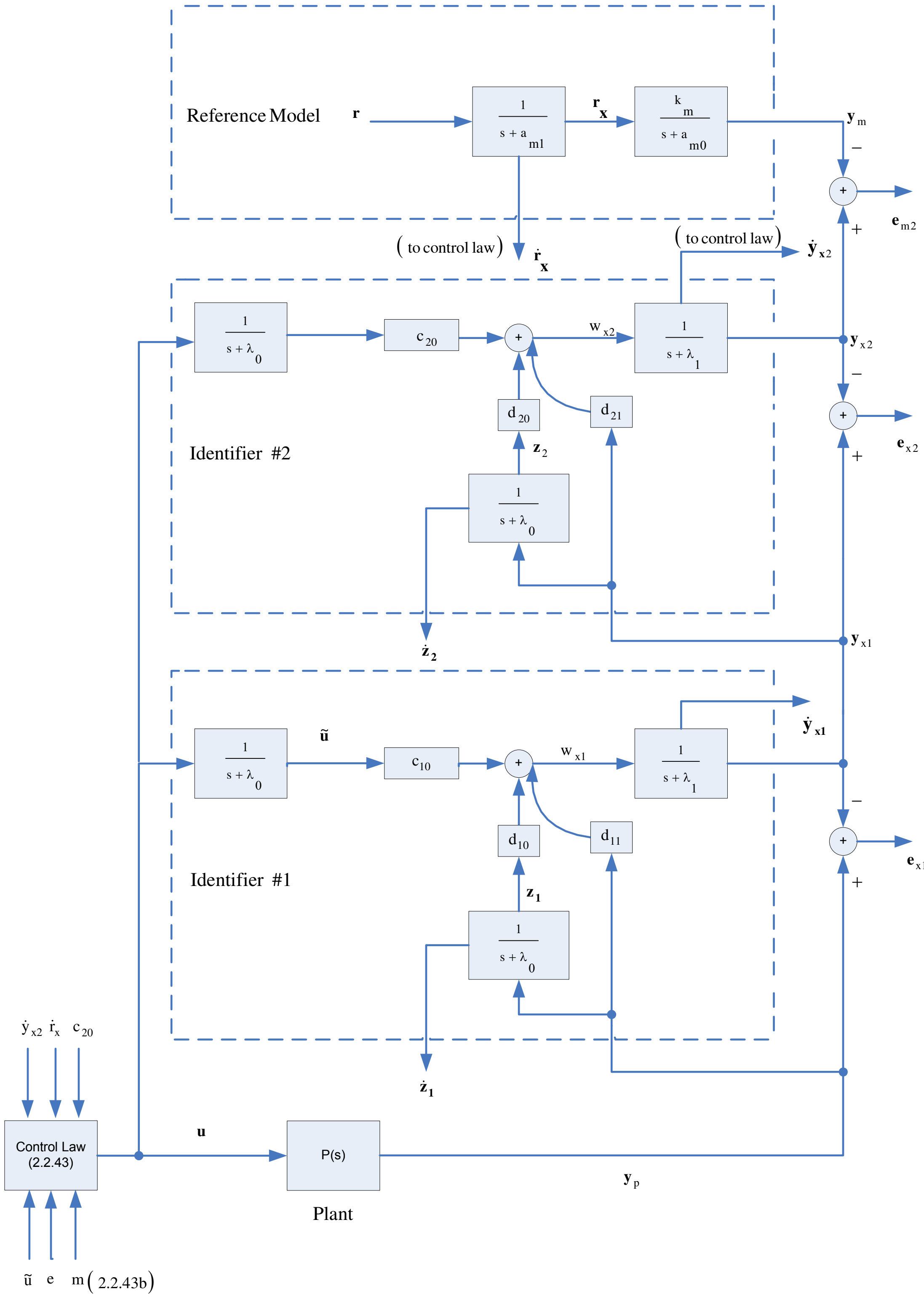

Figure 2.4: Identifier Tracking MRAC for 2nd order plant of relative degree two.

2.2.4. Boundedness of All Signals in the Entired Feedback System

With reference to the entire system in Figure 2.4, the following signals have been shown to be bounded:

From the analysis of Identifier #1: e_{x1}, c_{10}, d_{10} and d_{11}

From the analysis of Identifier #2: e_{x2}, c_{20}, d_{20}, d_{21}, $c_{10}-c_{20}$, $d_{10}z_1-d_{20}z_2$ and $d_{11}y_p-d_{21}y_{x1}$

From the analysis of control law: e and e_{m2}

From the reference model: r and y_m

The signals that remain to be shown bounded are:

y_p, y_{x1}, y_{x2}, z_1, z_2, $\dot{z}_1$, $\dot{z}_2$, $\dot{r}_x$, $\dot{y}_m$, $\dot{e}$, $\dot{e}_{m2}$, $\dot{y}_{x2}$, $\tilde{u}$, $\dot{e}_{x1}$, $\dot{e}_{x2}$, $\dot{y}_{x1}$, w_{x1}, w_{x2}, $\dot{c}_{10}$, $\dot{d}_{10}$, $\dot{d}_{11}$, $\dot{c}_{20}$, $\dot{d}_{20}$, $\dot{d}_{21}$, m and u

Boundedness of y_p, y_{x1} and y_{x2}:

Since y_m, e_{m2}, e_{x2} and e_{x1} are bounded, it follows from (2.2.17) and (2.2.8a) that y_{x2}, y_{x1} and y_p are bounded.

Boundedness of z_1, z_2, $\dot{z}_1$, $\dot{z}_2$, $\dot{r}_x$, $\dot{y}_m$, $\dot{e}$, $\dot{e}_{m2}$ and $\dot{y}_{x2}$:

The signals z_1, z_2, $\dot{z}_1$, $\dot{z}_2$, $\dot{r}_x$ and $\dot{y}_m$ are outputs of "proper" stable transfer functions with bounded inputs. Hence they are bounded. Also, from (2.2.44), we see that $\dot{e}$ is bounded. It follows from (2.2.36a) that the same is true of $\dot{e}_{m2}$. Consequently, from (2.2.36b), $\dot{y}_{x2}$ is bounded.

Boundedness of $\tilde{u}$, $\dot{e}_{x1}$, $\dot{e}_{x2}$, $\dot{y}_{x1}$, w_{x1} and w_{x2}:

With bounded $\dot{y}_{x2}$, the boundedness of $\tilde{u}$ is derived from (2.2.39). The signal $\dot{e}_{x1}$ in (2.2.10) is bounded because the signals ϕ and w in (2.2.9) are bounded. In a similar fashion, the boundedness of $\dot{e}_{x2}$ can be established. Finally, $\dot{y}_{x1}$ is also bounded due to the boundedness of $\dot{e}_{x2}$ and $\dot{y}_{x2}$. The signals w_{x1} and w_{x2} in Figure 2.4 are composed of a sum of bounded signals and are therefore bounded.

Boundedness of $\dot{c}_{10}$, $\dot{d}_{10}$, $\dot{d}_{11}$, $\dot{c}_{20}$, $\dot{d}_{20}$, $\dot{d}_{21}$, m and u:

The boundedness of $\dot{c}_{10}$, $\dot{d}_{10}$, $\dot{d}_{11}$, $\dot{c}_{20}$, $\dot{d}_{20}$, $\dot{d}_{21}$ and m follows from (2.2.13), (2.2.16) and (2.2.43b). Finally, the boundedness of u is established through (2.2.43a) because all signals appearing in the equations are bounded.

Thus, we have shown the boundedness of all signals in the entire control system. Next, we would like to demonstrate the convergence of the tracking errors.

2.2.5. Convergence of the Tracking Errors

With reference to the entire system in Figure 2.4, our purpose is to demonstrate that $y_p \to y_m$ as $t \to 0$. This is accomplished by showing the same for the signals e_{m2}, e_{x1}

and e_{x2}.

(i) *Convergence of* e_{m2}:

This has been shown in (2.2.46).

(ii) *Convergence of* e_{x1}:

We have shown in Section 2.2.4 that e_{x1}, $\dot{e}_{x1}$, $\dot{\tilde{c}}_0$ and $\dot{\tilde{u}}$ are bounded. Thus, from (2.2.14),

$$\ddot{V} = \begin{cases} -2\lambda_1 \dot{e}_{x1} e_{x1} + \dot{\tilde{c}}_0 e_{x1} \tilde{u} + \tilde{c}_0 \left(\dot{e}_{x1} \tilde{u} + e_{x1} \dot{\tilde{u}} \right), & \text{if } g e_{x1} \tilde{u} \le 0 \text{ and } c_{10} \le k_{lower} \\ -2\lambda_1 \dot{e}_{x1} e_{x1}, & \text{otherwise} \end{cases}$$

is bounded. According to Barbalat's Lemma, $\dot{V} \to 0$, which means $e_{x1} \to 0$ as $t \to \infty$.

(iii) *Convergence of* e_{x2}:

Consider e_{x2} in (2.2.18),

$$e_{x2} = \frac{1}{s+\lambda_1}(c_{10} - c_{20})\tilde{u} + \frac{1}{s+\lambda_1}(d_{10} z_1 - d_{20} z_2) + \frac{1}{s+\lambda_1}\left(d_{11} y_p - d_{21} y_{x1}\right)$$

Convergence of e_{x2} follows from the convergence of the signals $(c_{10} - c_{20})\tilde{u}$, $d_{10} z_1 - d_{20} z_2$ and $d_{11} y_p - d_{21} y_{x1}$. These are show as follows:

Convergence of $(c_{10} - c_{20})\tilde{u}$ **:**

Differentiating (2.2.24a) and (2.2.24b) gives

$$\ddot{V} = \begin{cases} -2\lambda_1 g\dot{e}_c e_c + (\dot{c}_{10} - \dot{c}_{20})(ge_c\tilde{u} + \dot{c}_{10}) + (c_{10} - c_{20})(g\dot{e}_c\tilde{u} + ge_c\dot{\tilde{u}} + \ddot{c}_{10}), \\ \qquad\qquad\qquad\qquad \text{if } ge_c\tilde{u} + \dot{c}_{10} \le 0 \text{ and } c_{20} \le c_{10} \\ -2\lambda_1 g\dot{e}_c e_c, \qquad\qquad \text{otherwise} \end{cases}$$

which can be seen to be bounded, (upon insertion of the parameter update laws from (2.2.13), (2.2.16), (2.2.20) and (2.2.4e)). With bounded $\ddot{V}$, $\dot{V} \to 0$ in accordance with Barbalat's Lemma. Since $\dot{V}$ in (2.2.23) consists of two non-positive terms, both terms must also converge to zero, in particular, $(c_{10} - c_{20})\tilde{u} \to 0$.

Convergence of $d_{10}z_1 - d_{20}z_2$:

Consider (2.2.25), the convergence of $d_{10}z_1 - d_{20}z_2$ will be demonstrated by the convergence of its two R.H.S. terms.

Differentiating (2.2.32a) gives

$$\ddot{V} = -2\alpha(d_{10} - d_{20})(\dot{d}_{10} - \dot{d}_{20}) - 2\lambda_1 ge_d\dot{e}_d + g\dot{e}_d\left((d_{10} - d_{20})z_2\right)$$
$$+ ge_d\left[\left((d_{10} - d_{20})\dot{z}_2\right) + \left((\dot{d}_{10} - \dot{d}_{20})z_2\right)\right] - 2\beta\left[\left((d_{10} - d_{20})\dot{z}_2\right) + \left((\dot{d}_{10} - \dot{d}_{20})z_2\right)\right]$$
$$\left((d_{10} - d_{20})z_2\right)$$

which can be shown to be bounded, if one inserts the parameter update laws (2.2.13), (2.2.16), (2.2.28) into the $\ddot{V}$ expression. With bounded $\ddot{V}$, $\dot{V} \to 0$ according to Barbalat's Lemma. Since $\dot{V}$ in (2.2.32a) consists of two non-positive terms, both terms must also converge to zero, in particular, $(d_{10} - d_{20})z_2 \to 0$. Therefore, the second term in (2.2.25) converges to zero. The convergence of the first term follows from the convergence of $z_1 - z_2$, which can be seen as follows. From (2.2.4f), (2.2.15b) and (2.2.8a), we have $z_1 - z_2 = \frac{1}{s+\lambda_0} e_{x1}$. Since e_{x1} is the input of an asymptotically stable system $\frac{1}{s+\lambda_0}$ and converges to zero, the output $z_1 - z_2$ also converges to zero. In conclusion, the convergence of $d_{10}z_1 - d_{20}z_2$ is established.

Convergence of $d_{11}y_p - d_{21}y_{x1}$:

The discussion of the convergence of $d_{11}y_p - d_{21}y_{x1}$ is similar to that of $d_{10}z_1 - d_{20}z_2$ and is omitted.

Summarizing, with the convergence of the three R.H.S. terms in (2.2.18), the convergence $e_{x2} \to 0$ as $t \to \infty$ is ensured.

2.2.6. Simulation Studies

The simulation studies presented in this section are to compare the effectiveness of the proposed adaptive scheme with the existing augmented output error method in [1]. This is done for the case of relative degree $q = 2$ (Simulations 2.2.1 and 2.2.2).

Simulation 2.2.1: 2nd order Augmented Output Error Method [1]

The data for the simulation are as follows.

$$P(s) = \frac{1}{s^2 - s}, \ M(s) = \frac{1}{s^2 + 2s + 1}$$

$$\dot{c}_{00} = -ge_1 w_{11} / d, \ \dot{d}_{00} = -ge_1 w_{12} / d, \ \dot{d}_{01} = -ge_1 y_{x0} / d$$

$$w_{11} = M(s) w_1, \ w_{12} = M(s) w_2, \ y_{x0} = M(s) y_p$$

$$w_1 = \frac{1}{s+1} u, \ w_2 = \frac{1}{s+1} y_p$$

$$d = 1 + w_{11}^2 + w_{12}^2 + y_{x0}^2, \ r(t) = 1, \ e_p = y_p - y_m$$

$$c_{00}(0) = 1, \ d_{00}(0) = 0, \ d_{01}(0) = 0, \ c_{00}^* = -3, d_{00}^* = 6, d_{01}^* = -7$$

r.m.s parameter error $\triangleq p_1 = \left[\left(c_{00} - c_{00}^*\right)^2 + \left(d_{00} - d_{00}^*\right)^2 + \left(d_{01} - d_{01}^*\right)^2 \right]^{1/2}$

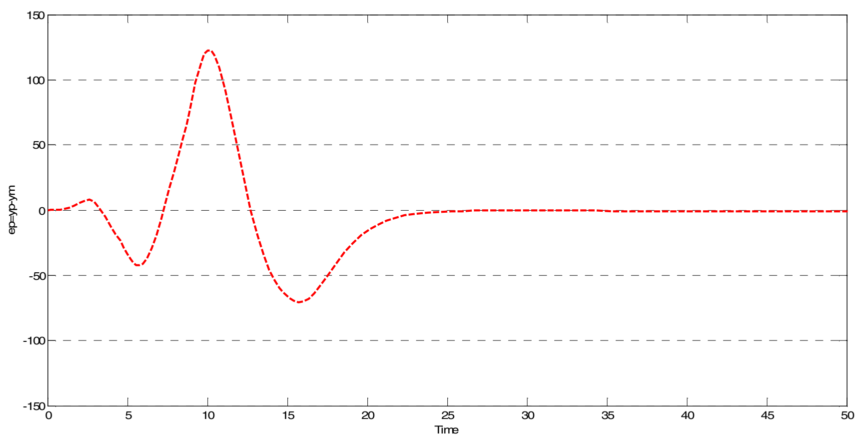

Figure 2.5: Augmented output error. (relative degree $q = 2$)

Simulation 2.2.2: 2nd order Identifier Tracking MRAC

The data for the simulation are as follows.

$$P(s) = \frac{1}{s^2 - s}, \; M(s) = \frac{1}{s^2 + 2s + 1}$$

$r(t) = 1, \; \lambda_0 = \lambda_1 = 1, \; \alpha = k = 1, \; \beta = 4, \; g = 10, \; e_p = y_p - y_m$

The initial conditions for the adaptive parameters are chosen in accordance with (2.2.14c) and (2.2.24c), in this case:

$c_{10}(0) = c_{20}(0) = 1 \geq k_{lower}$ (k_{lower} is taken to be 0.01),

$d_{10}(0) = d_{20}(0) = d_{11}(0) = d_{21}(0) = 0$,

$c_0^* = 1, \; d_0^* = -2, \; d_1^* = 3$

r.m.s parameter error $\underline{\underline{\Delta}} \, p_2 = \left[\left(c_{10} - c_0^*\right)^2 + \left(d_{10} - d_0^*\right)^2 + \left(d_{11} - d_1^*\right)^2 \right]^{1/2}$

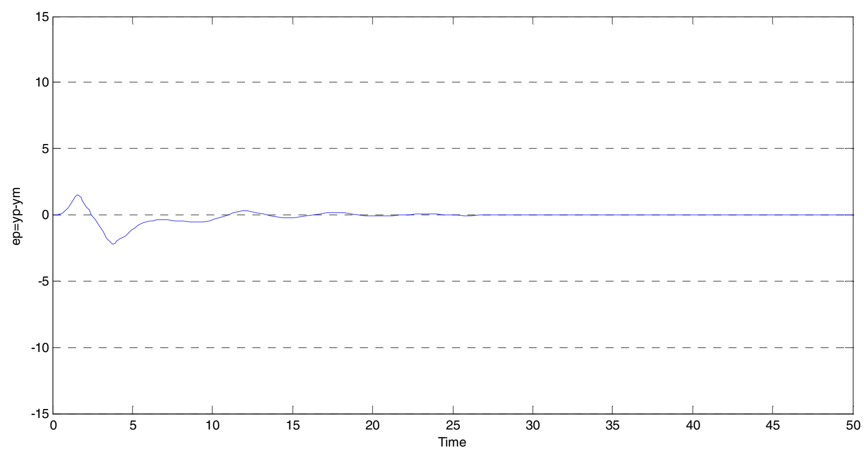

Figure 2.6: Identifier Tracking MRAC output error. (relative degree $q = 2$)

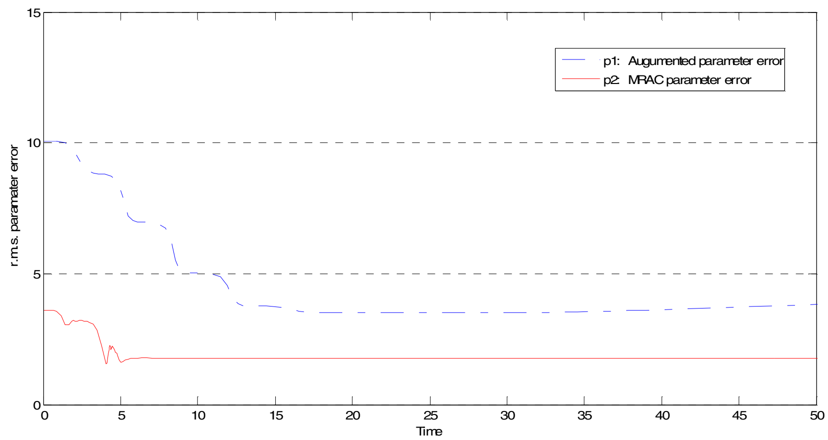

Figure 2.7: Augmented output parameter error p1 and Identifier Tracking MRAC output parameter error p2. (relative degree $q = 2$)

Discussion

Figure 2.5 gives the transient response of the tracking error e_p for the augmented output error method. The corresponding tracking error for the proposed Identifier-tracking MRAC scheme is shown in Figure 2.6. It can be seen that the tracking error for the proposed method is substantially smaller (about 60 times) than that of the augmented output error method (For the case of other initial conditions, the same superiority of the proposed method over the augmented output error method is observed). Figure 2.7 shows the r.m.s. parameter error for the augmented output error method and for the proposed Identifier Tracking MRAC. The reason that the above errors for the proposed method are smaller than those of the augmented output error method can be explained by the "closeness" of the proposed identifier structure to the plant structure.

2.3 Identifier Tracking MRAC of plants with relative degree greater than two

2.3.1. Reparameterization of the Unknown Plant

In this section, we extend the Identifier Tracking MRAC to the general case, i.e., plants of arbitrary relative degree q.

Consider a plant with an input-output pair $\{u(.), y_p(.)\}$ described by a transfer function

$$P(s) = \frac{N(s)}{D(s)} = \frac{b_{n-q}s^{n-q} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \quad n \geq q > 2 \qquad (2.3.1)$$

where $b_{n-q}s^{n-q} + \cdots + b_1 s + b_0$ is a Hurwitz polynomial in s. The sign of the high frequency gain b_{n-q} is assumed to be positive, with a known lower bound $b_{n-q} > k_{lower} > 0$.

We will reparametrize the plant into a form suitable for the derivation of the identifier structure and the parameter update laws

Express (2.3.1) as

$$(s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s \; + a_0)y_p = (b_{n-q}s^{n-q} + \cdots + b_1 s \; + b_0)u \qquad (2.3.2)$$

Let $\lambda\,(s)$, $\lambda_w(s)$ and $\lambda^*(s)$ be Hurwitz polynomials given as

$$\lambda\,(s) \triangleq s^{q-1} + \lambda_{n-2}s^{q-2} + \cdots + \lambda_{n-q+1}s \; + \lambda_{n-q} \qquad (2.3.3a)$$

$$\lambda_w(s) \triangleq s^{n-q} + \lambda_{n-q-1}s^{n-q-1} + \cdots + \lambda_1 s \; + \lambda_0 \qquad (2.3.3b)$$

$$\lambda_z(s) \triangleq \lambda\,(s)\lambda_w(s) = s^{n-1} + \tilde{\lambda}_{n-2}s^{n-2} + \ldots + \tilde{\lambda}_1 s + \tilde{\lambda}_0 \qquad (2.3.3c)$$

$$\lambda^*(s) \triangleq (s \; + \lambda_{n-1})\lambda_z(s) = s^n + \lambda^*_{n-1}s^{n-1} + \ldots + \lambda^*_1 s + \lambda^*_0 \qquad (2.3.3d)$$

First, divide both sides of (2.3.2) by $\lambda^*(s)$ using long division. Then, moving all terms to the R.H.S. except for the y_p-term and using the definitions in (2.3.3) yields

$$y_p = y_c + y_\lambda \tag{2.3.4a}$$

where

$$y_c = \frac{1}{s+\lambda_{n-1}}\left(\frac{b_{n-q}s^{n-q} + \cdots + b_1 s \; + b_0}{\lambda_w(s)} \cdot \frac{1}{\lambda(s)} u\right)$$

$$y_\lambda = \frac{1}{s+\lambda_{n-1}}\left(\frac{\left(\lambda^*_{n-1} - a_{n-1}\right)s^{n-1} + \left(\lambda^*_{n-2} - a_{n-2}\right)s^{n-2} + \cdots + \left(\lambda^*_1 - a_1\right)s \; + \left(\lambda^*_0 - a_0\right)}{\lambda_z(s)} y_p\right)$$

(2.3.4b)

Carrying out the long division by $\lambda_w(s)$ in the y_c expression gives

$$y_c = \frac{1}{s+\lambda_{n-1}}\left(c^*_{n-q} \frac{1}{\lambda(s)} u + \frac{c^*_{n-q-1}s^{n-q-1} + \cdots + c^*_1 s \; + c^*_0}{\lambda_w(s)} \cdot \frac{1}{\lambda(s)} u\right) \tag{2.3.4c}$$

In the same way, carrying out the long division by $\lambda_z(s)$ in the y_λ expression, we obtain

$$y_\lambda = \frac{1}{s+\lambda_{n-1}}\left(d^*_{n-1} y_p + \frac{d^*_{n-2}s^{n-2} + \cdots + d^*_1 s \; + d^*_0}{\lambda_z(s)} y_p\right) \tag{2.3.4d}$$

where c^*_i, $i = 0, 1, \cdots, n-q-1$, and d^*_j, $j = 0, 1, \cdots, n-2$, are the resulting coefficients after the long division.

Define

$$c^* \triangleq \left[c^*_{n-q-1} \dots \quad c^*_0\right]^T \tag{2.3.5a}$$

$$d^* \triangleq \left[d^*_{n-2} \dots \quad d^*_0\right]^T \tag{2.3.5b}$$

$$\tilde{u} \triangleq \frac{1}{\lambda(s)} u \tag{2.3.5c}$$

$$w \triangleq \begin{bmatrix} w_{n-q-1} \\ \vdots \\ w_1 \\ w_0 \end{bmatrix} \triangleq \begin{bmatrix} s^{n-q-1} \\ \vdots \\ s \\ 1 \end{bmatrix} \frac{1}{\lambda_w(s)} \tilde{u} \tag{2.3.5d}$$

$$z_1 \triangleq \begin{bmatrix} z_{1(n-2)} \\ \vdots \\ z_{11} \\ z_{10} \end{bmatrix} \triangleq \begin{bmatrix} s^{n-2} \\ \vdots \\ s \\ 1 \end{bmatrix} \frac{1}{\lambda_z(s)} y_p \tag{2.3.5e}$$

$$\overline{w}_1 \triangleq \left[\tilde{u} \quad w^T \quad y_p \quad z_1^T\right]^T \tag{2.3.5f}$$

$$\phi^* \triangleq \left[c^*_{n-q} \quad c^{*T} \quad d^*_{n-1} \quad d^{*T}\right]^T \tag{2.3.5g}$$

Then y_p in (2.3.4) is expressed as

$$y_p = \frac{1}{s+\lambda_{n-1}}\left(c^*_{n-q}\tilde{u} + c^{*T} w + d^*_{n-1} y_p + d^{*T} z_1\right)$$

$$= \frac{1}{s+\lambda_{n-1}}\left[\left(\phi^*\right)^T \overline{w}_1\right] \tag{2.3.6}$$

Equation (2.3.6) forms the basis of the identifier structures. Accordingly, Identifier #1 is constructed as

$$y_{x1} = \frac{1}{s+\lambda_{n-1}}\left[\phi_1^T \overline{w}_1\right] \tag{2.3.7a}$$

where

$$\phi_1 \triangleq \begin{bmatrix} c_{1(n-q)} & c_1^T & d_{1(n-1)} & d_1^T \end{bmatrix}^T \tag{2.3.7b}$$

$$c_1 \triangleq \begin{bmatrix} c_{1(n-q-1)} \cdots & c_{10} \end{bmatrix}^T \tag{2.3.7c}$$

$$d_1 \triangleq \begin{bmatrix} d_{1(n-2)} \cdots & d_{10} \end{bmatrix}^T \tag{2.3.7d}$$

are the adaptive coefficients.

Figure 2.8 shows a schematic diagram of Identifier #1.

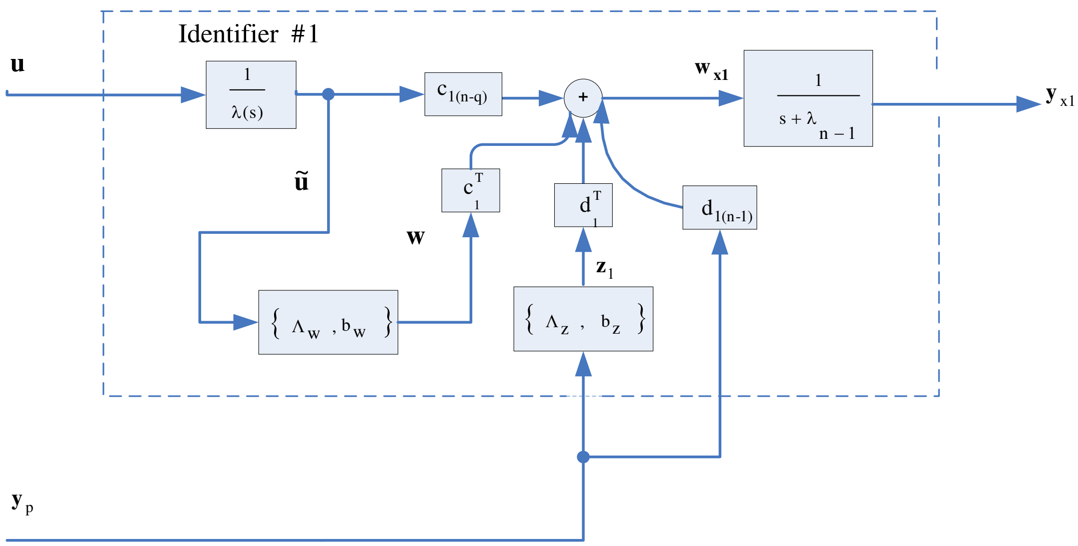

Figure 2.8: Identifier Tracking MRAC for Identifier #1.

For a relative degree of q, we need a total of q identifiers.

Identifier # γ ($\gamma = 2, 3, \cdots, q$) is constructed as

$$y_{x\gamma} = \frac{1}{s + \lambda_{n-1}} \left[\phi_\gamma^T \overline{w}_\gamma \right] \tag{2.3.8a}$$

where

$$\phi_\gamma \triangleq \left[c_{\gamma(n-q)} \quad c_\gamma^T \quad d_{\gamma(n-1)} \quad d_\gamma^T\right]^T \tag{2.3.8b}$$

$$c_\gamma \triangleq \left[c_{\gamma(n-q-1)} \cdots \quad c_{\gamma 0}\right]^T \tag{2.3.8c}$$

$$d_\gamma \triangleq \left[d_{\gamma(n-2)} \cdots \quad d_{\gamma 0}\right]^T \tag{2.3.8d}$$

$$\overline{w}_\gamma \triangleq \left[\tilde{u} \quad w^T \quad y_{x(\gamma-1)} \quad z_\gamma^T\right]^T \tag{2.3.8e}$$

$$z_\gamma \triangleq \begin{bmatrix} z_{\gamma(n-2)} \\ \vdots \\ z_{\gamma 1} \\ z_{\gamma 0} \end{bmatrix} \triangleq \begin{bmatrix} s^{n-2} \\ \vdots \\ s \\ 1 \end{bmatrix} \frac{1}{\lambda_z(s)} y_{x(\gamma-1)} \tag{2.3.8f}$$

The corresponding block diagram is shown in Figure 2.9.

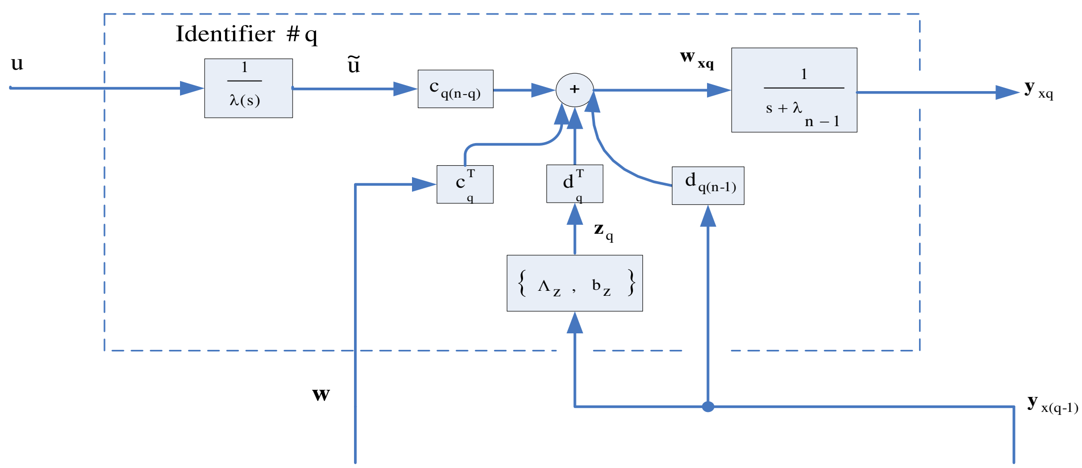

Figure 2.9: Identifier Tracking MRAC for Identifier #q.

In this diagram, the relation between $\tilde{u}$ and w as given in (2.3.5d) has been modeled in the controllable canonical form using (2.3.3b) as

$$\dot{w} = \Lambda_w w + b_w \tilde{u} \tag{2.3.9a}$$

where $\Lambda_w \in R^{(n-q)x(n-q)}$ and $b_w \in R^{n-q}$ are given by

$$\Lambda_w = \begin{bmatrix} -\lambda_{n-q-1} & \cdots & \cdot & \cdot & -\lambda_0 \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}$$

$$b_w = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{2.3.9b}$$

Similarly, the relation between y_p and z_1 in (2.3.5e) has been modeled in the controllable canonical form using (2.3.3c) as

$$\dot{z}_1 = \Lambda_z z_1 + b_z y_p \tag{2.3.10a}$$

where $\Lambda_z \in R^{(n-1)x(n-1)}$ and $b_z \in R^{n-1}$ are given by

$$\Lambda_z = \begin{bmatrix} -\tilde{\lambda}_{n-2} & \cdots & \cdot & \cdot & -\tilde{\lambda}_0 \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}$$

$$b_z = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{2.3.10b}$$

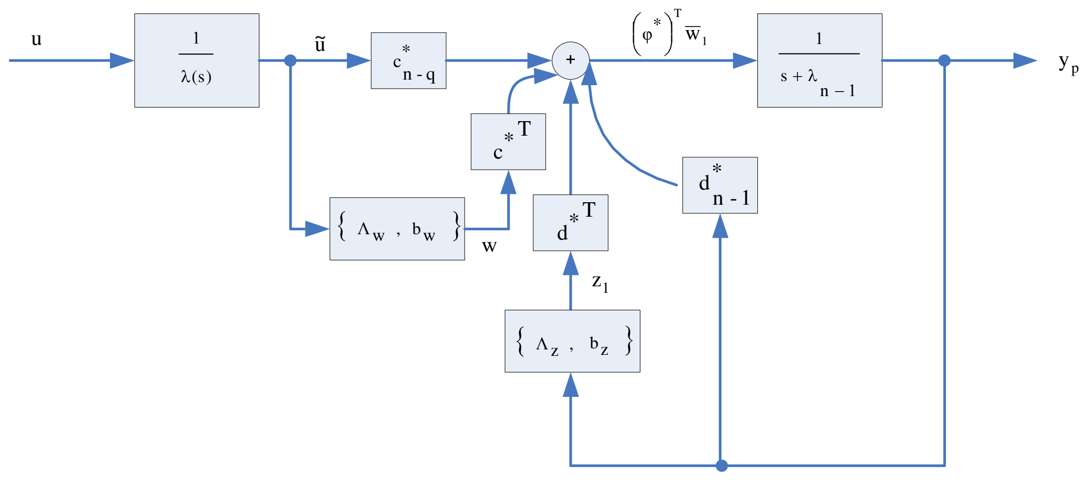

Figure 2.10: Plant parameterization -- nth order, relative degree $q > 2$

2.3.2. Parameter Update Laws for the Identifiers

Identifier #1

Define tracking error e_{x1} as

$$e_{x1} \triangleq y_p - y_{x1} = \frac{1}{s+\lambda_{n-1}}\left[\left(\tilde{\phi}\right)^T \overline{w}_1\right] \qquad (2.3.11a)$$

where

$$\tilde{\phi} \triangleq \phi^* - \phi_1$$

$$\phi_1 = \begin{bmatrix} c_{1(n-q)} & c_1^T & d_{1(n-1)} & d_1^T \end{bmatrix}^T$$

$$\overline{w}_1 = \begin{bmatrix} \tilde{u} & w^T & y_p & z_1^T \end{bmatrix}^T \qquad (2.3.11b)$$

Comparing (2.3.11a) with (2.2.9) and following the same Lyapunov analysis as in Section 2.2.2 leads to the following parameter update laws

$$\dot{c}_{1(n\text{-}q)} = \begin{cases} 0, & \text{if } ge_{x1}\tilde{u} \le 0 \text{ and } c_{1(n-q)} \le k_{lower} \quad (2.3.12a) \\ ge_{x1}\tilde{u}, & \text{otherwise} \quad (2.3.12b) \end{cases}$$

$$\dot{c}_1 = ge_{x1}w^T \tag{2.3.12c}$$

$$\dot{d}_{1(n-1)} = ge_{x1}y_p \tag{2.3.12d}$$

$$\dot{d}_1 = ge_{x1}z_1^T \tag{2.3.12e}$$

which is similar to (2.2.13). The division of the adaptation of $c_{1(n-q)}$ in two cases as given in (2.3.12a) and (2.3.12b) is to ensure that

$$c_{1(n-q)}(t) \ge k_{lower} > 0, \text{ for all } t \ge 0 \tag{2.3.13}$$

This will be achieved by choosing an initial condition for the adaptive parameter $c_{1(n-q)}(0) \ge k_{lower}$.

Identifier # $\boldsymbol{\gamma}$ ($\boldsymbol{\gamma = 2, 3, \cdots, q}$)

Define tracking error $e_{x\gamma}$ as

$$e_{x\gamma} \triangleq y_{x(\gamma-1)} - y_{x\gamma} = \frac{1}{s+\lambda_{n-1}}\left[\phi_{(\gamma-1)}^T \overline{w}_{(\gamma-1)} - \phi_\gamma^T \overline{w}_\gamma\right] \tag{2.3.14}$$

Parameter update laws are

$$\dot{c}_{\gamma(n\text{-}q)} = \begin{cases} 0, & \text{if } ge_{c\gamma}\tilde{u} + \dot{c}_{(\gamma-1)(n-q)} \le 0 \text{ and } c_{\gamma(n-q)} \le c_{(\gamma-1)(n-q)} \quad (2.3.15a) \\ ge_{c\gamma}\tilde{u} + \dot{c}_{(\gamma-1)(n-q)}, & \text{otherwise} \quad (2.3.15b) \end{cases}$$

where $e_{c\gamma} \triangleq \frac{1}{s+\lambda_{n\text{-}1}}\left(c_{(\gamma\text{-}1)(n\text{-}q)} - c_{\gamma(n-q)}\right)\tilde{u}$

$$\dot{c}_{\gamma i} = \left(c_{(\gamma-1)i} - c_{\gamma i}\right)\left(\alpha + \beta w_i^2\right) + g e_{x(\gamma-1)} w_i \quad , \; i = 0, 1, \cdots, n-q-1$$

(2.3.15c)

$$\dot{d}_{\gamma(n-1)} = \frac{\left(d_{(\gamma-1)(n-1)} - d_{\gamma(n-1)}\right)\left(\alpha + \beta y_{x(\gamma-1)}^2 + y_{x(\gamma-1)} \dot{y}_{x(\gamma-1)}\right)}{1 + y_{x(\gamma-1)}^2} + \dot{d}_{(\gamma-1)(n-1)}$$

(2.3.15d)

$$\dot{d}_{\gamma j} = \frac{\left(d_{(\gamma-1)j} - d_{\gamma j}\right)\left(\alpha + \beta z_{\gamma j}^2 + z_{\gamma j} \dot{z}_{\gamma j}\right)}{1 + z_{\gamma j}^2} + \dot{d}_{(\gamma-1)j}, \quad j = 0, 1, \cdots, n-2$$

(2.3.15e)

The division of the adaptation of $c_{\gamma(n-q)}$ in two cases as given in (2.3.15a) and (2.3.15b) is to ensure that

$$c_{\gamma(n-q)}(t) \geq c_{(\gamma-1)(n-q)}(t), \; \text{for all } t \geq 0 \tag{2.3.16}$$

This will be achieved by choosing an initial condition for the adaptive parameter $c_{\gamma(n-q)}(0) \geq c_{(\gamma-1)(n-q)}(0)$.

The update laws are derived in a similar way as the derivation of (2.2.16) in the simple case. The only difference is that we have one additional parameter update equation (2.3.15c) for the adaptive parameter $c_{\gamma i}$. This parameter is needed to take care of the numerator terms in the plant transfer function in this general case. The update laws are designed to produce a bounded tracking error $e_{x\gamma}$. This can be seen by substituting ϕ_γ^T, $\phi_{(\gamma-1)}^T$ from (2.3.8b) and $\overline{w}_\gamma$, $\overline{w}_{(\gamma-1)}$ from (2.3.8e), respectively, into (2.3.14), yielding

$$e_{x\gamma} = \frac{1}{s+\lambda_{n-1}}\left(c_{(\gamma-1)(n-q)} - (c_{\gamma(n-q)})\right)\tilde{u} + \frac{1}{s+\lambda_{n-1}}\left(d^T_{(\gamma-1)} z_{(\gamma-1)} - d^T_{\gamma} z_{\gamma}\right)$$

$$+ \frac{1}{s+\lambda_{n-1}}\left(d_{(\gamma-1)(n-1)} y_p - d_{\gamma(n-1)} y_{x(\gamma-1)}\right) + \frac{1}{s+\lambda_{n-1}}\left(c^T_{(\gamma-1)} - c^T_{\gamma}\right)w$$

(2.3.17)

The first three terms on the R.H.S. can be shown bounded as demonstrated for the simple case in Section 2.2.2. The boundedness of the last term is shown as follows.

Let the last term in (2.3.17) be

$$e_w \underline{\underline{\Delta}} \frac{1}{s+\lambda_{n-1}}(c^T_{(\gamma-1)} - c^T_{\gamma})w \tag{2.3.18}$$

where $c_{(\gamma-1)}$, c_{γ} and w are defined as in (2.3.8c) and (2.3.5d).

The derivative of e_w is given by

$$\dot{e}_w = -\lambda_{n-1} e_w + (c^T_{(\gamma-1)} - c^T_{\gamma})w \tag{2.3.19}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}\left[g e_w^2 + \left(c^T_{(\gamma-1)} - c^T_{\gamma}\right)^2\right] > 0 \tag{2.3.20}$$

The derivative of V then becomes

$$\dot{V} = g e_w \dot{e}_w + \left(c^T_{(\gamma-1)} - c^T_{\gamma}\right)\left(\dot{c}^T_{(\gamma-1)} - \dot{c}^T_{\gamma}\right) \tag{2.3.21}$$

Substituting $\dot{e}_w$ from (2.3.19) and using the parameter update laws in (2.3.15c), we have

$$\dot{V} = -\alpha\zeta_3^2 - \left\{a'\zeta_1^2 - b'\zeta_1\zeta_2 + c'\zeta_2^2\right\} \tag{2.3.22a}$$

where

$$a' \triangleq \lambda_{n-1} g$$

$$b' \triangleq g$$

$$c' \triangleq \beta$$

$$\zeta_1 \triangleq e_w$$

$$\zeta_2 \triangleq \zeta_3 \; w$$

$$\zeta_3 \triangleq \left(c^T_{(\gamma-1)} - c^T_{\gamma}\right)$$

(2.3.22b)

Following the same argument as in (2.2.32), any choice of the parameter values satisfying $\lambda_{n-1}\beta > \frac{g}{4}$ will render $\dot{V} \le 0$.

This implies that $e_w = \frac{1}{s + \lambda_{n-1}}(c^T_{(\gamma-1)} - c^T_{\gamma})w$ is bounded. Consequently, from (2.3.17), $e_{x\gamma}$ is bounded.

Figure 2.11 shows a schematic diagram of Identifier #1 ,…, Identifier #q.

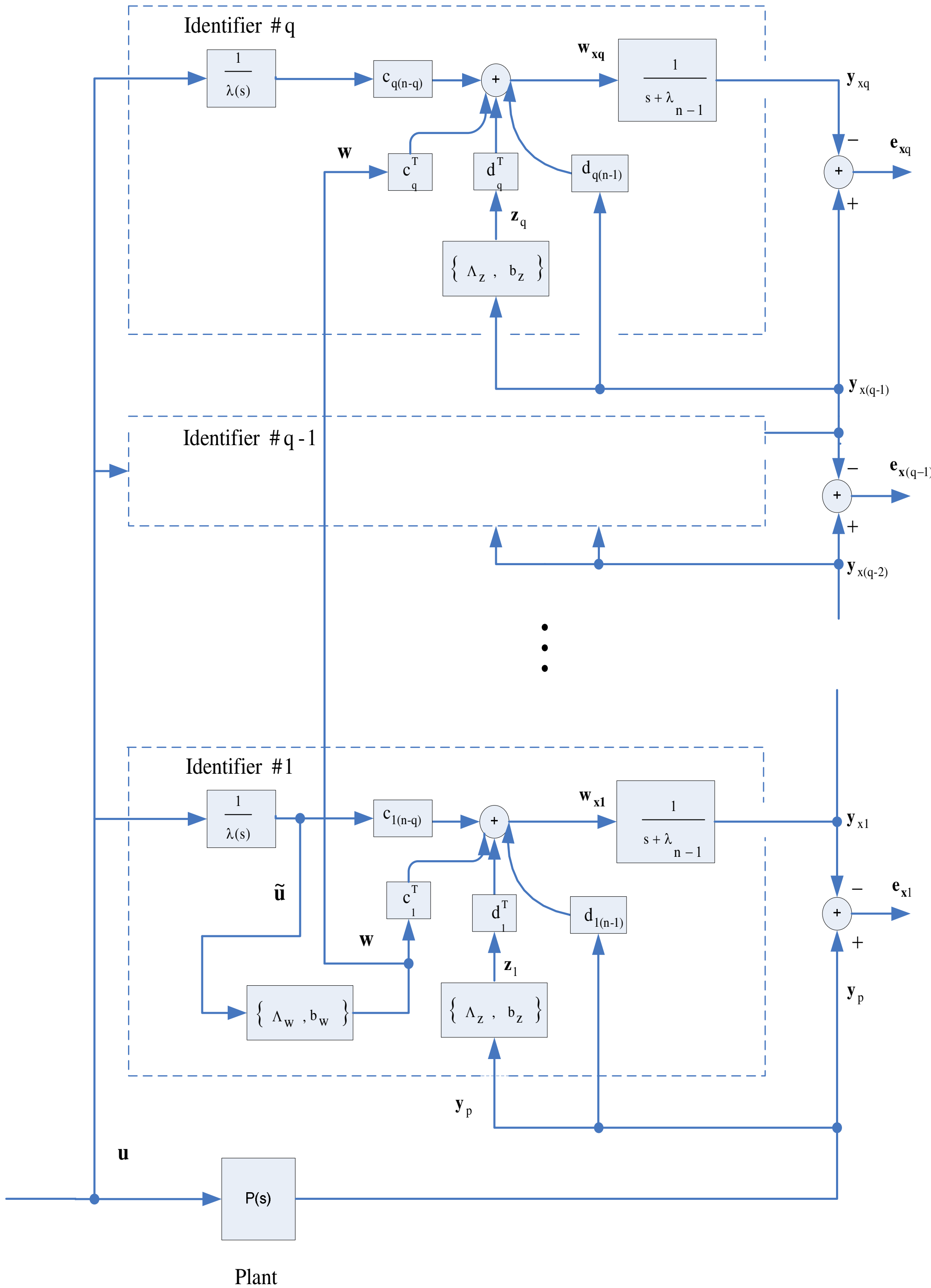

Figure 2.11: Identifier Tracking MRAC for Identifier #1 ,…, Identifier #q.

2.3.3. Control Law u(t)

The reference model is given by an input-output pair $\{r(.), y_m(.)\}$ with a transfer function $M(s)$ given as

$$\frac{y_m}{r} = M(s) = \frac{1}{s + a_{m(q-1)}} \cdot \frac{k_m}{s^{q-1} + a_{m(q-2)}s^{q-2} + \cdots a_{m1}s + a_{m0}} \tag{2.3.23}$$

The above transfer function consists of two blocks in series as shown in Figure 2.13.

Let the output of the first block be

$$r_x \triangleq \frac{r}{s + a_{m(q-1)}} \tag{2.3.24}$$

Then, (2.3.23) can be rewritten as

$$\frac{y_m}{r_x} = \frac{k_m}{s^{q-1} + a_{m(q-2)}s^{q-2} + \cdots a_{m1}s + a_{m0}} \tag{2.3.25a}$$

or, in time domain,

$$k_m r_x = y_m^{(q-1)} + a_{m(q-2)}y_m^{(q-2)} + \cdots a_{m1}\dot{y}_m + a_{m0}y_m \tag{2.3.25b}$$

Define tracking error e as

$$e \triangleq \left(s^{q-1} + a_{m(q-2)}s^{q-2} + \cdots a_{m1}s + a_{m0}\right)e_{mq} \tag{2.3.26a}$$

where

$$e_{mq} \triangleq y_{xq} - y_m \tag{2.3.26b}$$

(Note that if $e \to 0$, then $e_{mq} \to 0$ and $y_{xq} \to y_m$)

Substituting (2.3.26b) and (2.3.25b) into (2.3.26a) yields

$$e = \left[y_{xq}^{(q-1)} + a_{m(q-2)}y_{xq}^{(q-2)} + \cdots a_{m1}\dot{y}_{xq} + a_{m0}y_{xq}\right] - k_m r_x \tag{2.3.27}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}e^2 > 0 \tag{2.3.28}$$

A control law is now to be devised in order to make $\dot{V} = e\dot{e}$ negative definite. This can be achieved by setting $\dot{e} = -ke, \quad k > 0$.

From (2.3.27), the derivative of e is given by

$$\dot{e} = \left[y_{xq}^{(q)} + a_{m(q-2)}y_{xq}^{(q-1)} + \cdots a_{m1}\ddot{y}_{xq} + a_{m0}\dot{y}_{xq}\right] - k_m\dot{r}_x \tag{2.3.29}$$

The next step is to find an expression for $y_{xq}^{(q)}$.

Substituting (2.3.8b) and (2.3.8e) into (2.3.8a) and displaying the u-term explicitly, we have

$$\dot{y}_{xq} = c_{q(n-q)}\tilde{u} + r_{xq} \tag{2.3.30a}$$

where

$$r_{xq} \underline{\underline{\Delta}}\; c_q^T w \;\; + d_{q(n-1)}y_{x(q-1)} + d_q^T z_q - \lambda_{n-1}y_{xq} \tag{2.3.30b}$$

Successively differentiating (2.3.30a), gives the ν^{th} derivatives of y_{xq}

$$y_{xq}^{(\nu)} = \sum_{i=0}^{\nu-1} C_i^{\nu-1}\left(c_{q(n-q)}{}^{(\nu-i-1)}\tilde{u}^{(i)}\right) + r_{xq}^{(\nu-1)}, \quad \nu = 1,2,\cdots,q \tag{2.3.31}$$

(C_i^{q-1} is the combination symbol)

Letting $\nu = q$ and utilizing $\tilde{u}^{(q-1)} = -\lambda_{n-2}\tilde{u}^{(q-2)} - \cdots - \lambda_{n-q}\tilde{u} + u$ from (2.3.5c)

and (2.3.3a) yields

$$y_{xq}^{(q)} = c_{q(n-q)}u + \hat{y}_{xq}^{(q)} \tag{2.3.32a}$$

where

$$\hat{y}_{xq}^{(q)} \underline{\underline{\Delta}} \sum_{i=0}^{q-2} C_i^{q-1}\left(c_{q(n-q)}{}^{(q-i-1)}\tilde{u}^{(i)}\right) + r_{xq}^{(q-1)} - c_{q(n-q)}\left(\lambda_{n-2}\tilde{u}^{(q-2)} + \cdots + \lambda_{n-q}\tilde{u}\right)$$

(2.3.32b)

After $y_{xq}^{(q)}$ is found as in (2.3.32), the expression for $\dot{e}$ in (2.3.29) becomes

$$\dot{e} = \left[c_{q(n-q)}u + \hat{y}_{xq}^{(q)} + a_{m(q-2)}y_{xq}^{(q-1)} + \cdots a_{m1}\ddot{y}_{xq} + a_{m0}\dot{y}_{xq}\right] - k_m\dot{r}_x$$

(2.3.33)

Setting $\dot{e} = -ke$ and replacing the derivative terms $\hat{y}_{xq}^{(q)}$, $y_{xq}^{(q-1)}, \cdots, \dot{y}_{xq}$ and $\dot{r}_x$ in (2.3.33) with (2.3.32b), (2.3.31) and (2.3.24) gives the control law

$$u(t) = -\frac{m}{c_{q(n-q)}} - ke, \qquad c_{q(n-q)} > 0, \; k > 0$$

(2.3.34a)

where

$$m \underline{\underline{\Delta}} \sum_{i=0}^{q-2} C_i^{q-1}\left(c_{q(n-q)}{}^{(q-i-1)}\tilde{u}^{(i)}\right) + r_{xq}^{(q-1)} - c_{q(n-q)}\left(\lambda_{n-2}\tilde{u}^{(q-2)} + \cdots + \lambda_{n-q}\tilde{u}\right) +$$

$$\cdots + a_{m0}\left(c_{q(n-q)}\tilde{u} + r_{xq}\right) - k_m\left(r - a_{m(q-1)}r_x\right)$$

(2.3.34b)

Note that division by zero in (2.3.34a) will not occur because (2.3.13) and (2.3.16) guarantee that $c_{q(n-q)} \geq c_{(q-1)(n-q)} \geq \cdots \geq c_{1(n-q)} \geq k_{lower} > 0$. Also note that the signals $\tilde{u}^{(q-2)}, \cdots, \tilde{u}$, $r_{xq}^{(q-1)}$ can be obtained without actual differentiation because they are outputs of proper stable transfer functions with bounded inputs as shown in (2.3.5c). As for the derivatives of the adaptive parameters, they can be replaced by their respective adaptive laws, thus dispensing of the need of differentiations.

Finally, substituting the control law (2.3.34) into (2.3.33) yields

$$\dot{e} = -ke \tag{2.3.35}$$

which makes

$$\dot{V} = e\,\dot{e} = -ke^2 = -2kV \tag{2.3.36}$$

Solving (2.3.36) gives the solution

$$V(t) = V(0)\exp(-2kt) \tag{2.3.37}$$

In other words, from (2.3.36),

$$e(t) = \pm\sqrt{2V(0)}\exp(-kt) \tag{2.3.38}$$

This implies that the equilibrium state $e = 0$ is globally asymptotically stable and $e \to 0$ as $t \to \infty$. It follows from (2.3.26) that

$$y_{xq} \to y_m \text{ as } t \to \infty \tag{2.3.39}$$

Figure 2.12 shows a schematic diagram of overall system.

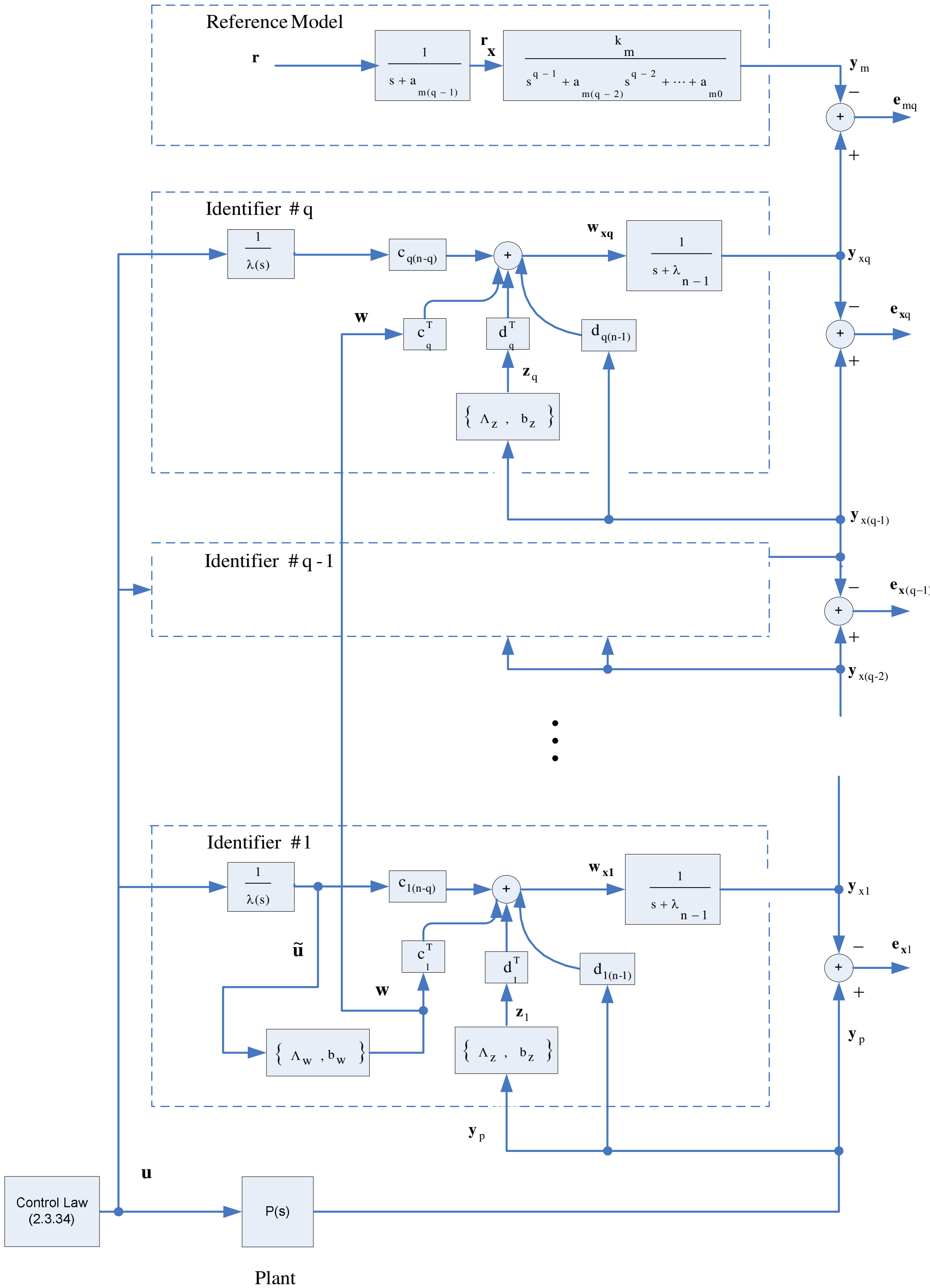

Figure 2.12: Identifier Tracking MRAC for an n[th] order plant of relative degree $q > 2$

2.3.4. Boundedness of All Signals in the Entired Feedback System

With reference to the entire adaptive control system in Figure 2.12, the following signals have been shown to be bounded:

From the analysis of Identifier#1, $\cdots$, Identifier#q: $e_{x1},\cdots,e_{xq}$, $c_{1(n-q)},\cdots,c_{q(n-q)}$, $c_1^T,\cdots,c_q^T$, $d_{1(n-1)},\cdots,d_{q(n-1)}$ and $d_1^T,\cdots,d_q^T$

From the analysis of the control law: e and e_{mq}

From the reference model: r and y_m

The signals that remain to be shown bounded are:

y_p, $y_{x1},\cdots,y_{x(q-1)}$, y_{xq}, $z_1,\cdots,z_q$, $\dot{z}_1,\cdots,\dot{z}_q$, $\dot{r}_x$, $\dot{y}_m,\cdots y_m^{(q)}$, $\dot{e}$, $\ddot{e}$, $\cdots,e^{(q)}$, $\dot{e}_{mq},\ddot{e}_{mq},\cdots,e_{mq}^{(q)}$, $\dot{y}_{xq},\ddot{y}_{xq},\cdots,y_{xq}^{(q)},\tilde{u}$, w, $\dot{e}_{x1},\cdots,\dot{e}_{xq}$, $\dot{y}_p$, $\dot{y}_{x1},\cdots\dot{y}_{x(q-1)}$, $w_{x1},\cdots,w_{xq}$, $\dot{c}_{1(n-q)},\cdots,\dot{c}_{q(n-q)},\dot{c}_1^T,\cdots,\dot{c}_q^T$, $\dot{d}_{1(n-1)},\cdots,\dot{d}_{q(n-1)}$, $\dot{d}_q^T,\cdots,\dot{d}_q^T$, $\dot{r}_{xq}$, $\ddot{y}_p,\cdots,y_p^{(q)}$, m and u.

They are shown to be bounded in accordance to the following grouping:

Boundedness of y_p, $y_{x1},\cdots,y_{x(q-1)}$ and y_{xq}:

Since y_m, e_{mq}, e_{xq} and $e_{x(q-1)},\cdots,e_{x1}$ are bounded, it follows from (2.3.14) and (2.3.11a) that $y_{xq},\cdots,y_{x1}$ and y_p are bounded.

Boundedness of $z_1,\cdots,z_q$, $\dot{z}_1,\cdots,\dot{z}_q$, $\dot{r}_x,\dot{y}_m,\cdots y_m^{(q)}$, $\dot{e}$,$\ddot{e}$,$\cdots,e^{(q)}$, $\dot{e}_{mq},\ddot{e}_{mq},\cdots,e_{mq}^{(q)}$ and $\dot{y}_{xq},\ddot{y}_{xq},\cdots,y_{xq}^{(q)}$:

The signals $z_1,\cdots,z_q$, $\dot{z}_1,\cdots,\dot{z}_q$, $\dot{r}_x$ and $\dot{y}_m,\cdots y_m^{(q)}$ are outputs of "proper" stable transfer functions with bounded inputs. Hence they are bounded. Also, from (2.3.38), we see that $\dot{e}$,$\ddot{e}$,$\cdots,e^{(q)}$ are bounded. It follows from (2.3.26a) that the same is true of $\dot{e}_{mq},\ddot{e}_{mq},\cdots,e_{mq}^{(q)}$. Consequently, from (2.3.26b), $\dot{y}_{xq},\ddot{y}_{xq},\cdots,y_{xq}^{(q)}$ are bounded.

Boundedness of $\tilde{u}$, w, $\dot{e}_{x1},\cdots,\dot{e}_{xq}$, $\dot{y}_p$, $\dot{y}_{x1},\cdots\dot{y}_{x(q-1)}$ and $w_{x1},\cdots,w_{xq}$:

With bounded $\dot{y}_{xq}$, eliminating w from (2.3.5d) and (2.3.30) gives the boundedness of $\tilde{u}$ because all other variables in the resulting equation are bounded. After we establish the boundedness of $\tilde{u}$, the boundedness of w follows. Next consider the signal $\dot{e}_{x1}$ in (2.3.11). It is bounded because φ_1 and $\overline{w}_1$ are bounded. In a similar fashion, the boundedness of $\dot{e}_{x2},\cdots,\dot{e}_{xq}$ can be established. Finally, $\dot{y}_{x(q-1)},\cdots\dot{y}_{x1}$, $\dot{y}_p$ are also bounded due to the boundedness of $\dot{y}_{xq}$ and $\dot{e}_{x2},\cdots,\dot{e}_{xq}$. The signals $w_{x1},\cdots,w_{xq}$ in Figure 2.12 are composed respectively of a sum of bounded signals, and are therefore bounded.

Boundedness of $\dot{c}_{1(n-q)},\cdots,\dot{c}_{q(n-q)}$, $\dot{c}_1^T,\cdots,\dot{c}_q^T$, $\dot{d}_{1(n-1)},\cdots,\dot{d}_{q(n-1)}$, $\dot{d}_q^T,\cdots,\dot{d}_q^T$ and $\dot{r}_{xq}$:

The boundedness of the variables $\dot{c}_{1(n-q)},\cdots,\dot{c}_{q(n-q)}$, $\dot{c}_1^T,\cdots,\dot{c}_q^T$, $\dot{d}_{1(n-1)},\cdots,\dot{d}_{q(n-1)}$ and

$\dot{d}_q^T, \cdots, \dot{d}_q^T$ as given in (2.3.12) and (2.3.15) can be seen through the substitution of all occurring derivative terms by their respective adaptive laws. For example, $\dot{c}_{q(n-q)}$ in (2.3.15b) has the derivative term $\dot{c}_{(q-1)(n-q)}$, which is the adaptive law in Identifier #(q-1). Consequently, $\dot{r}_{xq} = \dot{c}_q^T w + \dot{d}_{q(n-1)} y_{x(q-1)} + \dot{d}_q^T z_q + c_q^T \dot{w} + d_{q(n-1)} \dot{y}_{x(q-1)} + d_q^T \dot{z}_q$ $- \lambda_{n-1} \dot{y}_{xq}$ (as obtained from 2.3.30b) is also bounded. Note that by following the above procedure, we can also demonstrate the boundedness of any derivative term of an adaptive parameter *up to the q-th derivative*.

Boundedness of $\ddot{y}_p, \cdots, y_p^{(q)}$, u and m:

The boundedness of u is established from (2.3.1) if one can demonstrate the boundedness of $y_p, \dot{y}_p, \ddot{y}_p, \cdots, y_p^{(q)}$. This is demonstrated as follows. Substituting (2.3.5c) and (2.3.1) into (2.3.30a) gives

$$\dot{y}_{xq} = c_{q(n-q)} \frac{1}{\lambda(s)} \left(\frac{D(s)}{N(s)} y_p \right) + r_{xq}$$

Dividing $\lambda(s)N(s)$ into $D(s)$ yields

$$\dot{y}_{xq} = c_{q(n-q)} \left(\dot{y}_p + \xi_{n-1} y_p + \frac{\zeta_{n-2} s^{n-2} + \zeta_{n-3} s^{n-3} + \cdots \zeta_0}{\lambda(s)N(s)} y_p \right) + r_{xq} \tag{2.3.40}$$

Differentiating (2.3.40) once gives

$$\ddot{y}_{xq} = \dot{c}_{q(n-q)} \left(\dot{y}_p + \xi_{n-1} y_p + \frac{\zeta_{n-2} s^{n-2} + \zeta_{n-3} s^{q-3} + \cdots \zeta_0}{\lambda(s)N(s)} y_p \right)$$

$$+c_{q(n-q)}\left(\dddot{y}_p+\xi_{n-1}\dot{y}_p+\frac{\zeta_{n-2}s^{n-2}+\zeta_{n-3}s^{n-3}+\cdots\zeta_0}{\lambda(s)N(s)}\dot{y}_p\right)+\dot{r}_{xq}$$

(2.3.41)

Since $\ddot{y}_{xq}$, $\dot{c}_{q(n-q)}$, y_p, $\dot{y}_p$ and $\dot{r}_{xq}$ in (2.3.41) have been shown to be bounded, $\ddot{y}_p$ is bounded. Using the same approach, differentiating (2.3.40) twice will leads to the boundedness of $\dddot{y}_p$. Continuing on in this fashion would lead to the boundedness of $y_p^{(4)},\cdots,y_p^{(q)}$. With the boundedness of u, the boundedness of m is assured from (2.3.34a). In conclusion, all signals in the overall system are bounded.

2.3.5. Convergence of the Tracking Errors

The discussion of the convergence is the same as that in Section 2.2.5 and is omitted.

2.3.6. Simulation Studies

Simulation 2.3.1: 3^{rd} order Identifier Tracking MRAC

The data for the simulation are as follows.

$$P(s)=\frac{1}{s^3+2s^2+s-1},\ M(s)=\frac{1}{(s^2+2s+1)(s+1)}$$

$$r(t)=1$$

$$\lambda^*=s^3+3s^2+3s+1$$

$$\alpha=k=1,\ \beta=4,\ g=2,\ e_p=y_p-y_m$$

The initial conditions for the adaptive parameters are chosen in accordance with

(2.3.13) and (2.3.16), in this case:

$c_{i0}(0) = 1 \geq k_{lower}$ (k_{lower} is taken to be 0.01)

$d_{ij}(0) = 0$, $i = 1 \ldots 3$, $j = 0 \ldots 2$

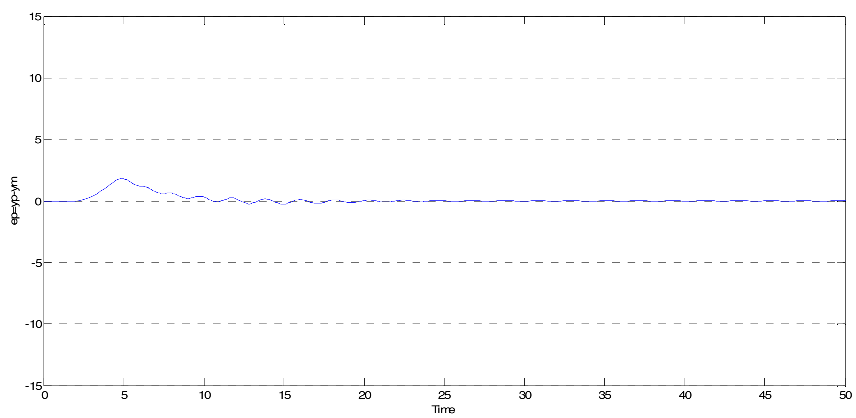

Figure 2.13: Identifier Tracking MRAC output error. (relative degree $q = 3$)

Discussion

Figure 2.13 shows the simulation for the case of relative degree $q = 3$, which has not been reported in the current literature. It is seen that the transient response of the tracking error is also small.

CHAPTER 3

STACKED IDENTIFIERS MODEL REFERENCE ADAPTIVE CONTROL

3.1 Introduction

The primary objective of this chapter is to develop a new design approach for the model reference adaptive control of a single-input single-output linear time-invariant plant. The proposed method, called the "Model reference adaptive control using stacked identifiers," uses a stacked identifier structure that is new to the field of adaptive control. The goal is to make the output of the plant asymptotically track the output of the first identifier, and then driving the output of the first identifier to track that of the second identifier, and so forth, up to the q-th identifier where q is the relative degree of the plant. Lastly, the output of the q-th identifier is forced to converge to that of the reference model. Simulation results show the superiority of the proposed method over the traditional model reference adaptive control with augmented error in terms of the transient response. Since the resulting control systems are nonlinear and time-varying, the stability analysis of the overall system plays a central role in developing the theory.

3.2 Stacked Identifiers MRAC of plants with relative degree two

3.2.1. Reparameterization of the Unknown Plant

Consider a linear time-invariant plant $P(s)$ with an input-output pair $\{u(.), y_p(.)\}$

described by a transfer function

$$P(s) = \frac{k_p}{s^2 + a_{p1}s + a_{p0}} \tag{3.2.1}$$

where k_p, a_{p0} and a_{p1} are constant but unknown parameters. The sign of the high frequency gain k_p and a lower bound for k_p are assumed to be known, i.e., for the case of a positive k_p, $k_p > k_{lower} > 0$; and for the case of a negative k_p, $k_p < k_{upper} < 0$. Throughout this paper, k_p is assumed to be positive.

We will reparametrize the plant into a form suitable for deriving the identifier and the parameter update laws.

Express (2.1) as

$$(s^2 + a_{p1}s + a_{p0})y_p(t) = k_p u(t), \quad k_p > k_{lower} > 0 \tag{3.2.2}$$

Dividing both sides of the above equation by $(s+\lambda)^2$, where λ is a positive constant, we obtain

$$\frac{s^2 + a_{p1}s + a_{p0}}{(s+\lambda)^2} y_p(t) = \frac{k_p}{(s+\lambda)^2} u(t) \tag{3.2.3}$$

Performing long division on the L.H.S. gives

$$\text{L.H.S. of (3.2.3)} = y_p(t) + \frac{(a_{p1} - 2\lambda)s + (a_{p0} - \lambda^2)}{(s+\lambda)^2} y_p(t)$$

Conducting another long division on $\frac{(a_{p1} - 2\lambda)s + (a_{p0} - \lambda^2)}{s+\lambda}$ yields

L.H.S. of (3.2.3) =

$$y_p(t) + \frac{1}{s+\lambda}\left[\left(a_{p1} - 2\lambda\right) + \frac{1}{s+\lambda}\left(a_{p0} - a_{p1}\lambda + \lambda^2\right)\right] y_p(t)$$

Substituting this expression into (3.2.3) and moving all terms other than the term y_p to the R.H.S. gives

$$y_p(t) = \frac{1}{s+\lambda}\left[\left(2\lambda - a_{p1}\right) y_p(t) + \left(a_{p1}\lambda - \lambda^2 - a_{p0}\right)\frac{1}{s+\lambda} y_p(t) + k_p \frac{1}{s+\lambda} u(t)\right]$$

(3.2.4)

Let

$$\alpha_1^* \triangleq k_p \tag{3.2.5a}$$

$$\beta_1^* \triangleq a_{p1}\lambda - \lambda^2 - a_{p0} \tag{3.2.5b}$$

$$\beta_0^* \triangleq 2\lambda - a_{p1} \tag{3.2.5c}$$

$$\tilde{u} \triangleq \frac{1}{s+\lambda} u(t) \tag{3.2.5d}$$

$$\tilde{y}_p \triangleq \frac{1}{s+\lambda} y_p(t) \tag{3.2.5e}$$

Equation (3.2.4) becomes

$$y_p(t) = \frac{1}{s+\lambda}\left(\alpha_1^* \tilde{u}(t) + \beta_0^* y_p(t) + \beta_1^* \tilde{y}_p(t)\right) \tag{3.2.6}$$

In accordance with the form of (3.2.6), Identifier #1 is chosen as

$$y_{x1}(t) = \frac{1}{s+\lambda}\left(\alpha_{11} \tilde{u}(t) + \beta_{10} y_p(t) + \beta_{11} \tilde{y}_p(t)\right) \tag{3.2.7}$$

Figure 3.1 shows a schematic diagram of Identifier #1.

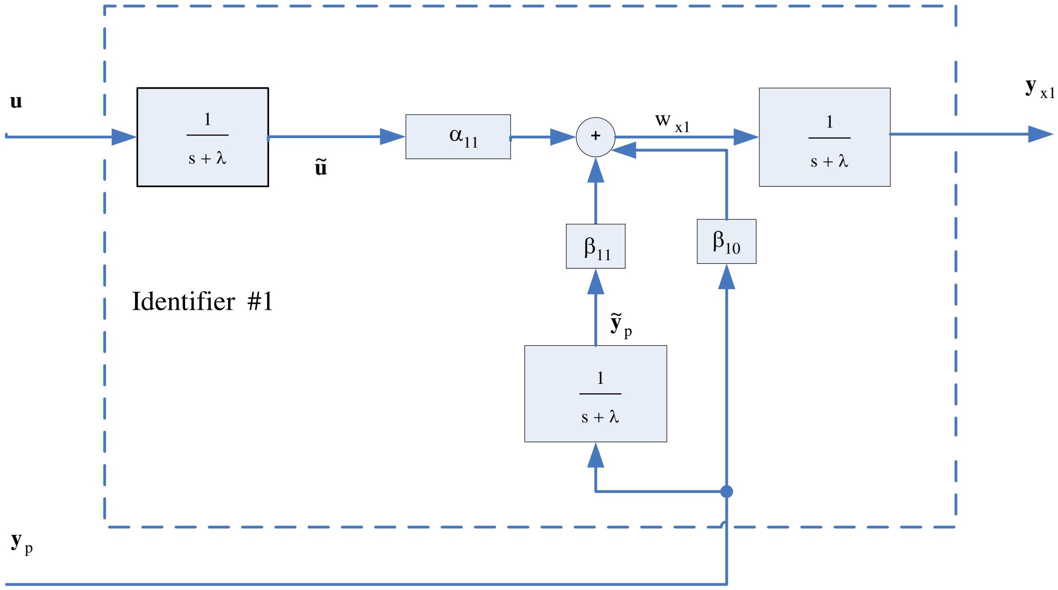

Figure 3.1: Stacked Identifiers MRAC for Identifier #1.

Similar to the structure of Identifier #1, Identifier #2 is chosen as

$$y_{x2}(t) = \frac{1}{s+\lambda}\left(\alpha_{21}\tilde{u}(t) + \beta_{20} y_{x1}(t) + \beta_{21}\tilde{y}_{x1}(t)\right) \tag{3.2.8a}$$

where

$$\tilde{y}_{x1} \triangleq \frac{1}{s+\lambda} y_{x1}(t) \tag{3.2.8b}$$

The corresponding block diagram is shown in Figure 3.2.

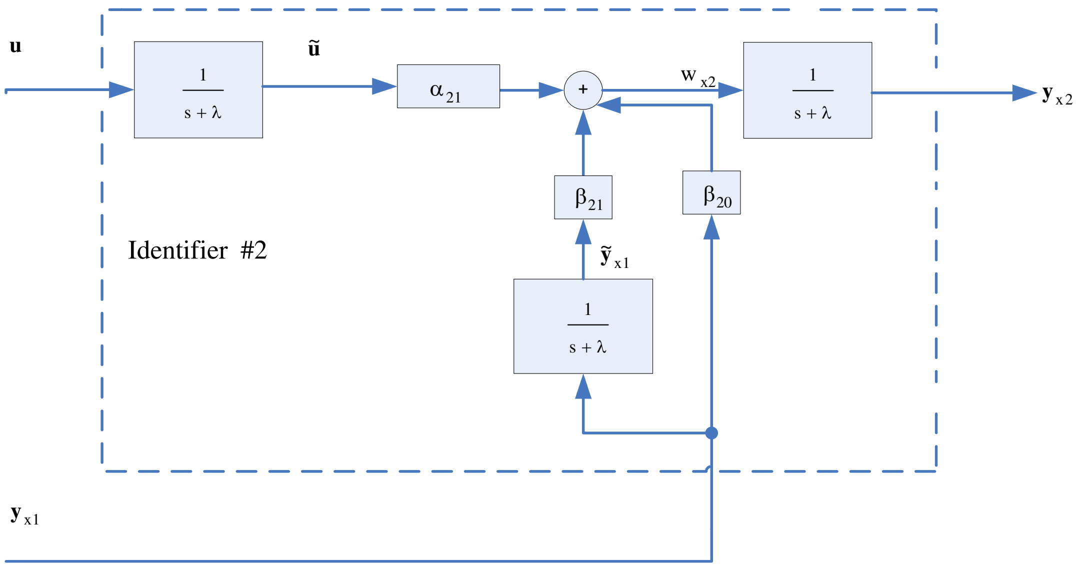

Figure 3.2: Stacked Identifiers MRAC for Identifier #2.

3.2.2. Parameter Update Laws for the Identifiers

Identifier #1

Define

$$e_{x1}(t) \triangleq y_p(t) - y_{x1}(t) \tag{3.2.9a}$$

$$\tilde{\alpha}_1 \triangleq \alpha_1^* - \alpha_{11},\ \ \tilde{\beta}_0 \triangleq \beta_0^* - \beta_{10},\ \ \tilde{\beta}_1 \triangleq \beta_1^* - \beta_{11} \tag{3.2.9b}$$

From (3.2.9), (3.2.6) and (3.2.7), the error equation is given by

$$e_{x1}(t) = y_p(t) - y_{x1}(t)$$

$$= \frac{1}{s+\lambda}\left(\tilde{\alpha}_1 \tilde{u} + \tilde{\beta}_0 y_p + \tilde{\beta}_1 \tilde{y}_p\right) \tag{3.2.10}$$

Multiplying both sides of (3.2.10) by the polynomial operator $s+\lambda$ and moving all terms to the R.H.S. except for the $\dot{e}_{x1}$ term gives

$$\dot{e}_{x1}(t) = -\lambda e_{x1}(t) + \tilde{\alpha}_1\tilde{u} + \tilde{\beta}_0 y_p + \tilde{\beta}_1\tilde{y}_p \tag{3.2.11}$$

To go through a stability analysis, we choose a Lyapunov function candidate

$$V = \frac{1}{2}\left[e_{x1}^2 + \frac{1}{g}\left(\tilde{\alpha}_1^2 + \tilde{\beta}_0^2 + \tilde{\beta}_1^2\right)\right], \quad g > 0 \tag{3.2.12}$$

The derivative of V is given by

$$\dot{V} = e_{x1}\dot{e}_{x1} + \frac{1}{g}\left(\tilde{\alpha}_1\dot{\tilde{\alpha}}_1 + \tilde{\beta}_0\dot{\tilde{\beta}}_0 + \tilde{\beta}_1\dot{\tilde{\beta}}_1\right)$$

Substituting $\dot{e}_{x1}$ from (3.2.11) yields

$$\dot{V} = -\lambda e_{x1}^2 + \tilde{\alpha}_1\left(e_{x1}\tilde{u} + \frac{1}{g}\dot{\tilde{\alpha}}_1\right) + \tilde{\beta}_0\left(e_{x1}y_p + \frac{1}{g}\dot{\tilde{\beta}}_0\right) + \tilde{\beta}_1\left(e_{x1}\tilde{y}_p + \frac{1}{g}\dot{\tilde{\beta}}_1\right)$$

(3.2.13)

Choosing the parameter update laws as

$$\dot{\alpha}_{11} = -\dot{\tilde{\alpha}}_1 = \begin{cases} 0, & \text{if } ge_{x1}\tilde{u} \le 0 \text{ and } \alpha_{11} \le k_{lower} \quad (3.2.14a) \\ ge_{x1}\tilde{u}, & \text{otherwise} \quad (3.2.14b) \end{cases}$$

$$\dot{\beta}_{10} = -\dot{\tilde{\beta}}_0 = ge_{x1}y_p \tag{3.2.14c}$$

$$\dot{\beta}_{11} = -\dot{\tilde{\beta}}_1 = ge_{x1}\tilde{y}_p \tag{3.2.14d}$$

renders

$$\dot{V} = \begin{cases} -\lambda g e_{x1}^2 + \tilde{\alpha}_1 e_{x1}\tilde{u} \le 0, & \text{if } ge_{x1}\tilde{u} \le 0 \text{ and } \alpha_{11} \le k_{lower} \quad (3.2.15a) \\ -\lambda g e_{x1}^2 \le 0, & \text{otherwise} \quad (3.2.15b) \end{cases}$$

Remark:

(a) We see that in both cases, $\dot{V}$ is negative semi-definite. This implies that e_{x1}, $\tilde{\alpha}_1$, $\tilde{\beta}_0$ and $\tilde{\beta}_1$ are bounded; and from (3.2.9b), α_{11}, β_{10} and β_{11} are also bounded. The convergence to zero of e_{x1} will be shown later using Barbalat's Lemma after establishing the boundedness of all signals in the entire system.

(b) The division of the adaptation of α_{11} into two cases as given by (3.2.14a) and (3.2.14b) is to ensure that, with the choice of initial condition $\alpha_{11}(0) \geq k_{lower}$,

$$\alpha_{11}(t) \geq k_{lower} > 0, \text{ for all } t \geq 0 \tag{3.2.16}$$

This is needed in order to avoid division by zero later.

Identifier #2

Identifier #2 is chosen as

$$y_{x2}(t) = \frac{1}{s+\lambda}\left(\alpha_{21}\tilde{u}(t) + \beta_{20}y_{x1}(t) + \beta_{21}\tilde{y}_{x1}(t)\right) \tag{3.2.17a}$$

where

$$\tilde{y}_{x1} \underline{\underline{\Delta}} \frac{1}{s+\lambda} y_{x1}(t) \tag{3.2.17b}$$

Let

$$e_{x2}(t) \underline{\underline{\Delta}}\ y_{x1}(t) - y_{x2}(t) \tag{3.2.18}$$

The purpose of the parameter update laws for Identifier #2 is to achieve $e_{x2} = (y_{x1} - y_{x2}) \to 0$ as $t \to \infty$.

They are chosen as

$$\dot{\alpha}_{21} = \begin{cases} 0, & \text{if } d(\alpha_{11} - \alpha_{21}) + g e_\alpha \tilde{u} + \dot{\alpha}_{11} \le 0 \text{ and } \\ & \alpha_{21} \le \alpha_{11}, \ d > 0 \qquad (3.2.19a) \\ d(\alpha_{11} - \alpha_{21}) + g e_\alpha \tilde{u} + \dot{\alpha}_{11}, & \text{otherwise} \qquad (3.2.19b) \end{cases}$$

$$\dot{\beta}_{20} = d(\beta_{10} - \beta_{20}) + g e_{\beta 0} y_{x1} + \dot{\beta}_{10} \qquad (3.2.19c)$$

$$\dot{\beta}_{21} = d(\beta_{11} - \beta_{21}) + g e_{\beta 1} \tilde{y}_{x1} + \dot{\beta}_{11} \qquad (3.2.19d)$$

where $$e_\alpha \triangleq \frac{1}{s+\lambda}(\alpha_{11} - \alpha_{21})\tilde{u} \qquad (3.2.19e)$$

$$e_{\beta 0} \triangleq \frac{1}{s+\lambda}(\beta_{10} - \beta_{20}) y_{x1} \qquad (3.2.19f)$$

$$e_{\beta 1} \triangleq \frac{1}{s+\lambda}(\beta_{11} - \beta_{21}) \tilde{y}_{x1} \qquad (3.2.19g)$$

(The update laws are derived as a natural consequence of observing the $\dot{V}$ expression in the Lyapunov analysis, which will be shown later.)

From (3.2.18), (3.2.7) and (3.2.17a), the error equation is given by

$$e_{x2}(t) = \frac{1}{s+\lambda}(\alpha_{11} - \alpha_{21})\tilde{u} + \frac{1}{s+\lambda}(\beta_{10} y_p - \beta_{20} y_{x1}) + \frac{1}{s+\lambda}(\beta_{11} \tilde{y}_p - \beta_{21} \tilde{y}_{x1})$$

(3.2.20)

Based on this equation, it is noted that though a traditional treatment of Lyapunov's analysis is possible for the first term, it is not possible for the second and third term. The reason is due to the occurrence of a product of a parameter with a signal, such as $\beta_{10} y_p$, instead of the product of a "*parameter deviation*" with a signal as in the first term. So, we resort to establishing the boundedness of "*all*" signals in the overall system first, and then assuring the convergence to zero of e_{x2} through the use of Barbalat's Lemma later.

We would like to demonstrate the boundedness of e_{x2} when the update laws (3.2.19) are used. This can be accomplished by requiring the same for the three R.H.S. terms of (3.2.20).

(i). Boundedness of $\frac{1}{s+\lambda}(\alpha_{11}-\alpha_{21})\tilde{u}$

Consider

$$e_{\alpha} = \frac{1}{s+\lambda}(\alpha_{11}-\alpha_{21})\tilde{u} \qquad (3.2.19e)$$

Multiplying $s+\lambda$ to both sides of the equation yields

$$\dot{e}_{\alpha} = -\lambda e_{\alpha} + (\alpha_{11}-\alpha_{21})\tilde{u} \qquad (3.2.21)$$

Choose a Lyapunov function candidate (to secure boundedness of e_{α} and $\alpha_{11}-\alpha_{21}$) as

$$V = \frac{1}{2}\left[ge_{\alpha}^{2} + (\alpha_{11}-\alpha_{21})^{2}\right] > 0, \quad g > 0 \qquad (3.2.22)$$

The derivative of V is given by

$$\dot{V} = ge_{\alpha}\dot{e}_{\alpha} + (\alpha_{11}-\alpha_{21})(\dot{\alpha}_{11}-\dot{\alpha}_{21}) \qquad (3.2.23)$$

Substituting $\dot{e}_{\alpha}$ from (3.2.21) yields

$$\dot{V} = -\lambda ge_{\alpha}^{2} + (\alpha_{11}-\alpha_{21})\left[(ge_{\alpha}\tilde{u} + \dot{\alpha}_{11}) - \dot{\alpha}_{21}\right] \qquad (3.2.24)$$

It is seen that the application of the parameter update laws as given in (3.2.19a) and (3.2.19b) will render

$$\dot{V} = \begin{cases} -\lambda g e_\alpha^2 + (\alpha_{11} - \alpha_{21})(g e_\alpha \tilde{u} + \dot{\alpha}_{11}) \le -\lambda g e_\alpha^2 - d(\alpha_{11} - \alpha_{21})^2, & (3.2.25a) \\ \qquad \text{if } d(\alpha_{11} - \alpha_{21}) + g e_\alpha \tilde{u} + \dot{\alpha}_{11} \le 0 \text{ and } \alpha_{21} \le \alpha_{11} & \\ -\lambda_1 g e_\alpha^2 - d(\alpha_{11} - \alpha_{21})^2, \quad \text{otherwise} & (3.2.25b) \end{cases}$$

(The inequality in the first case follows as a result of the condition imposed for this case.)

Thus, $\dot{V}$ is negative definite, implying that the equilibrium state is globally asymptotically stable. Hence, both e_α and $\alpha_{11} - \alpha_{21}$ are bounded, and

$$e_\alpha \to 0, \ \alpha_{11} - \alpha_{21} \to 0 \qquad (3.2.26)$$

The division of the adaptation of α_{21} into two cases as given in (3.2.19a) and (3.2.19b) is to ensure that

$$\alpha_{21}(t) \ge \alpha_{11}(t), \ \text{for all } t \ge 0 \qquad (3.2.27)$$

This will be achieved as long as the initial conditions are chosen such that $\alpha_{21}(0) \ge \alpha_{11}(0)$. Condition (3.2.27) is needed in order to avoid division by zero later.

(ii). Boundedness of $\frac{1}{s+\lambda}(\beta_{10} y_p - \beta_{20} y_{x1})$

Rearrange $\frac{1}{s+\lambda}(\beta_{10} y_p - \beta_{20} y_{x1})$ as

$$\frac{1}{s+\lambda}\left(\beta_{10}y_p - \beta_{20}y_{x1}\right) = \frac{1}{s+\lambda}\beta_{10}\left(y_p - y_{x1}\right) + \frac{1}{s+\lambda}\left(\beta_{10} - \beta_{20}\right)y_{x1} \qquad (3.2.28)$$

We shall treat the boundedness of the two terms on the R.H.S. separately.

In accordance with (3.2.9a), the first term $\frac{1}{s+\lambda}\beta_{10}\left(y_p - y_{x1}\right)$ is bounded because β_{10} and $e_{x1} = y_p - y_{x1}$ are bounded. We now turn to the second term and let

$$e_{\beta 0} = \frac{1}{s+\lambda}\left(\beta_{10} - \beta_{20}\right)y_{x1} \qquad (3.2.19f)$$

Multiplying $s + \lambda_1$ to both sides of the equation yields

$$\dot{e}_{\beta 0} = -\lambda e_{\beta 0} + \left(\beta_{10} - \beta_{20}\right)y_{x1} \qquad (3.2.29)$$

Choose a Lyapunov function candidate (to secure boundedness of $e_{\beta 0}$ and $\beta_{10} - \beta_{20}$)

$$V = \frac{1}{2}\left[ge_{\beta 0}^2 + \left(\beta_{10} - \beta_{20}\right)^2\right] > 0 \qquad (3.2.30)$$

The derivative of V is given by

$$\dot{V} = ge_{\beta 0}\dot{e}_{\beta 0} + \left(\beta_{10} - \beta_{20}\right)\left[\dot{\beta}_{10} - \dot{\beta}_{20}\right] \qquad (3.2.31)$$

Substituting $\dot{e}_{\beta 0}$ from (3.2.29), we have

$$\dot{V} = -\lambda ge_{\beta 0}^2 + \left(\beta_{10} - \beta_{20}\right)\left[\left(ge_{\beta 0}y_{x1} + \dot{\beta}_{10}\right) - \dot{\beta}_{20}\right] \qquad (3.2.32)$$

Applying the parameter update law in (3.2.19c), renders

$$\dot{V} = -\lambda e_{\beta 0}^2 - d\left(\beta_{10} - \beta_{20}\right)^2 \qquad (3.2.33)$$

which is negative definite. This implies that the equilibrium state is globally asymptotically stable. Hence, $e_{\beta 0}$ and $\beta_{10} - \beta_{20}$ are bounded, and

$$e_{\beta 0} = \frac{1}{s+\lambda}(\beta_{10} - \beta_{20})y_{x1} \to 0 \text{ as } t \to \infty \tag{3.2.34}$$

Consequently, from (3.2.28), with both terms on the R.H.S. being bounded, $\frac{1}{s+\lambda}(\beta_{10} y_p - \beta_{20} y_{x1})$ is also bounded.

(iii). Boundedness of $\frac{1}{s+\lambda}(\beta_{11} \tilde{y}_p - \beta_{21} \tilde{y}_{x1})$

The treatment of $\frac{1}{s+\lambda}(\beta_{11} \tilde{y}_p - \beta_{21} \tilde{y}_{x1})$ follows the same pattern as that of $\frac{1}{s+\lambda}(\beta_{10} y_p - \beta_{20} y_{x1})$ above, and is therefore omitted.

Summarizing the results of (i)-(iii) in this section, it follows from (3.2.20) that e_{x2} is bounded. (The convergence $e_{x2} \to 0$ will be shown later.)

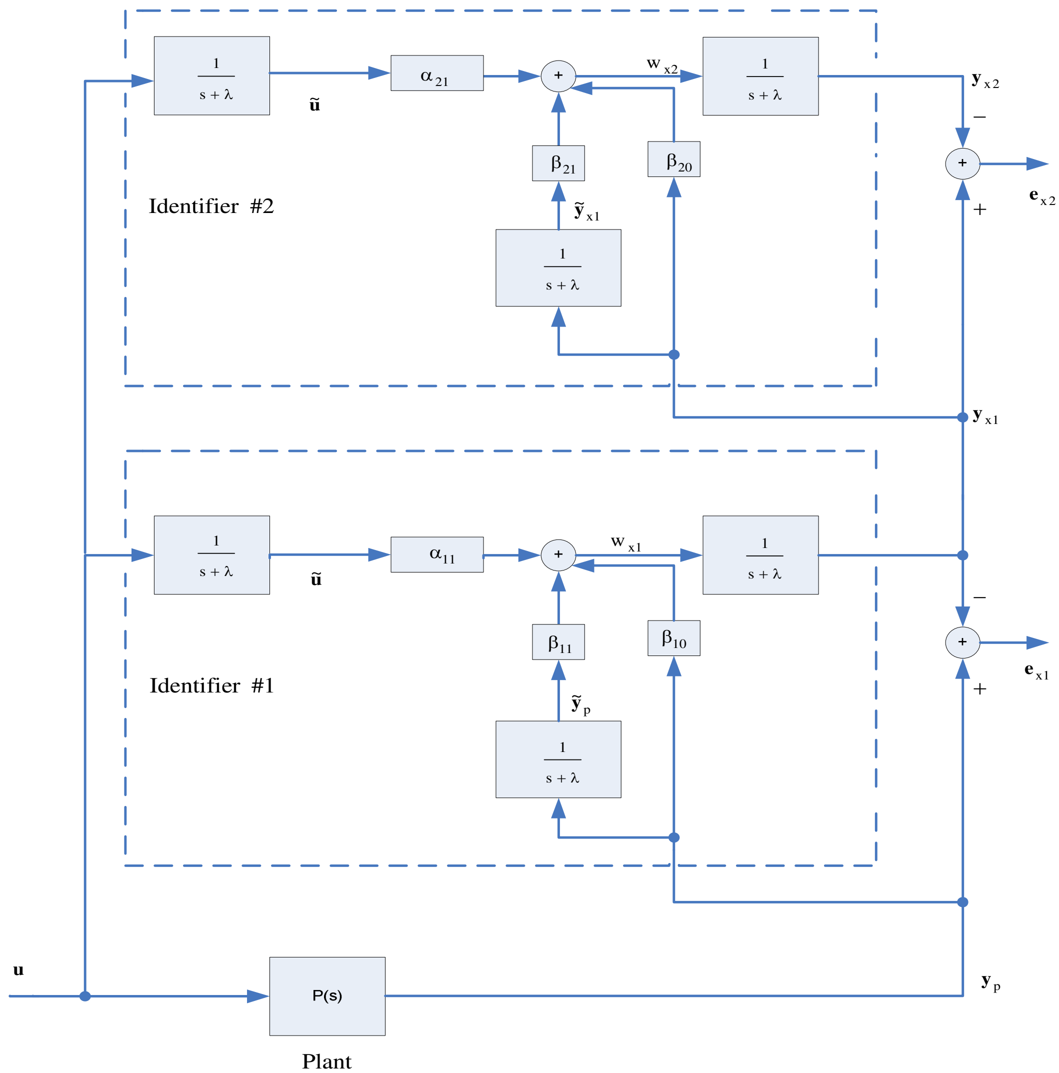

Figure 3.3: Stacked Identifiers MRAC for Identifier #1 and Identifier #2.

3.2.3. Control Law u(t)

Let the reference model with the input-output pair $\{r(.), y_m(.)\}$ be

$$\frac{y_m(s)}{r(s)} = M(s) = \frac{k_m}{(s + a_{m1})(s + a_{m0})} \tag{3.2.35}$$

where k_m, a_{m1} and a_{m0} are positive design parameters, $r(t)$ is a bounded, piecewise continuous function of time for $t \geq 0$. The purpose here is to derive a control law such that y_{x2} asymptotically tracks y_m.

Define tracking error e as

$$e \triangleq (s + a_{m0}) e_{m2} \tag{3.2.36a}$$

where

$$e_{m2} \triangleq y_{x2} - y_m \tag{3.2.36b}$$

(Note that if $e \to 0$, then $e_{m2} \to 0$ and $y_{x2} \to y_m$.)

From (3.2.35) and (3.2.36), we have

$$e = (s + a_{m0})(y_{x2} - y_m) = (\dot{y}_{x2} + a_{m0} y_{x2}) - k_m r_x \tag{3.2.37a}$$

where

$$r_x \triangleq \frac{r}{s + a_{m1}} \tag{3.2.37b}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2} e^2 > 0 \tag{3.2.38}$$

A control law is now to be devised in order to make

$$\dot{V} = e\dot{e} = -ke^2, \quad k > 0 \tag{3.2.39}$$

negative definite. This will be done by making

$$\dot{e} = -ke \tag{3.2.40}$$

through an appropriate control law to be derived as follows:

From (3.2.37a), the derivative of e is given by

$$\dot{e} = (\ddot{y}_{x2} + a_{m0}\dot{y}_{x2}) - k_m\dot{r}_x \tag{3.2.41}$$

Multiplying $s+\lambda$ to both sides of (3.2.17a) yields

$$\dot{y}_{x2} = \alpha_{21}\tilde{u}(t) + \beta_{20}y_{x1}(t) + \beta_{21}\tilde{y}_{x1}(t) - \lambda y_{x2} \tag{3.2.42}$$

The second derivative is given by

$$\ddot{y}_{x2} = \alpha_{21}\dot{\tilde{u}}(t) + m \tag{3.2.43a}$$

where

$$m \triangleq \dot{\alpha}_{21}\tilde{u}(t) + \dot{\beta}_{20}y_{x1}(t) + \dot{\beta}_{21}\tilde{y}_{x1}(t) + \beta_{20}\dot{y}_{x1}(t) + \beta_{21}\dot{\tilde{y}}_{x1}(t) - \lambda\dot{y}_{x2} \tag{3.2.43b}$$

Substituting (3.2.43a) into (3.2.41) yields

$$\dot{e} = \alpha_{21}\dot{\tilde{u}}(t) + m + a_{m0}\dot{y}_{x2} - k_m\dot{r}_x \tag{3.2.44}$$

Next we substitute $\dot{\tilde{u}}$ with $-\lambda\tilde{u} + u$ from (3.2.5d) and $\dot{e} = -ke$ from (3.2.40). The result is

$$-ke = \alpha_{21}u + \left(-\alpha_{21}\lambda\ \tilde{u} + m + a_{m0}\dot{y}_{x2} - k_m\dot{r}_x\right) \tag{3.2.45}$$

Since our objective is to design a differentiator-free controller, replacing $\dot{y}_{x2}$ and $\dot{r}_x$ in (3.2.45) with (3.2.42) and (3.2.37b), respectively, gives the control law

$$u(t) = \frac{1}{\alpha_{21}}\Big(\alpha_{21}\lambda\ \tilde{u} - m - a_{m0}\left(\alpha_{21}\tilde{u}(t) + \beta_{20}y_{x1}(t) + \beta_{21}\tilde{y}_{x1}(t) - \lambda y_{x2}\right)$$

$$+ k_m\left(r - a_{m1}r_x\right)\Big) - ke, \qquad \alpha_{21} > 0,\ k > 0 \tag{3.2.46}$$

Remarks:

(a). Note that the term m above as defined by (3.2.43b) still contains the derivatives of some adaptive coefficients. They can be replaced by their respective adaptive laws in (3.2.19). Also the other derivative terms $\dot{y}_{x1}$, $\dot{\tilde{y}}_{x1}$ and $\dot{y}_{x2}$ can be substituted by their expressions in (3.2.7), (3.2.17b) and (3.2.42), respectively, so as to dispense with the need of differentiations.

(b). Note from (3.2.16) and (3.2.27) that $\alpha_{21} \geq \alpha_{11} \geq k_{lower} > 0$ so that division by zero in the control law would not occur.

Thus, with $\dot{V}$ in (3.2.39) being negative definite, the equilibrium state $e = 0$ is globally asymptotically stable, i.e. e is bounded and $e \to 0$ as $t \to \infty$.

Consequently, from (3.2.36a) and (3.2.36b),

$$e_{m2} \to 0 \text{ and } y_{x2} \to y_m \tag{3.2.47}$$

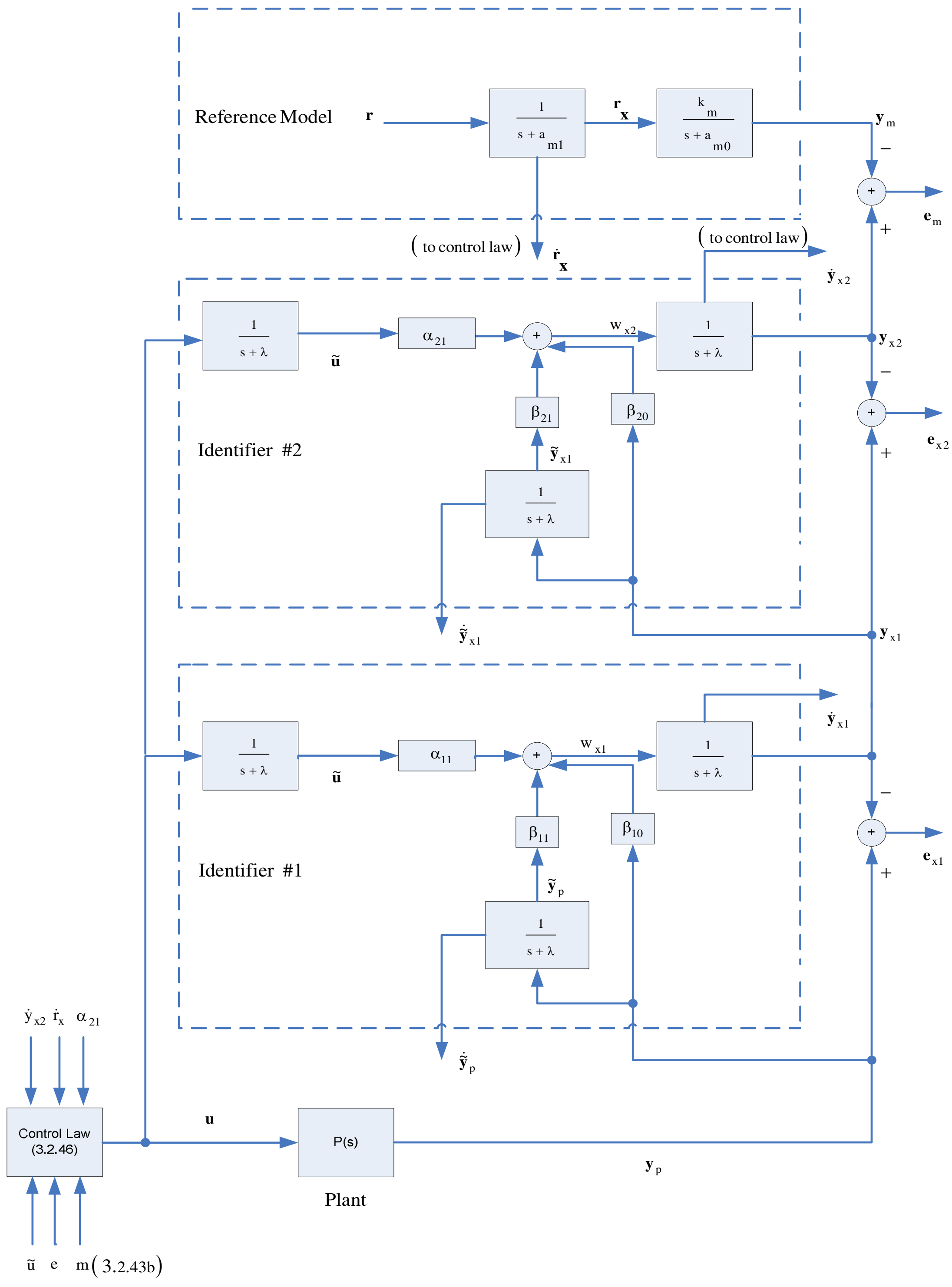

Figure 3.4: Stacked Identifiers MRAC for 2nd order plant of relative degree two.

3.2.4. Boundedness of All Signals in the Entired Feedback System

With reference to the entire system in Figure 3.4, the following signals have been shown to be bounded:

From the analysis of Identifier #1: e_{x1}, α_{11}, β_{10} and β_{11}

From the analysis of Identifier #2: e_{x2}, α_{21}, β_{20} and β_{21}

From the analysis of the control law: e and e_{m2}

From the reference model: r and y_m

The signals that remain to be shown bounded are as follows:

Boundedness of y_p, y_{x1} and y_{x2}:

Since y_m, e_{m2}, e_{x2} and e_{x1} are bounded, it follows from (3.2.18) and (3.2.9a) that y_{x2}, y_{x1} and y_p are bounded.

Boundedness of $\tilde{y}_p$, $\tilde{y}_{x1}$, $\dot{\tilde{y}}_p$, $\dot{\tilde{y}}_{x1}$, $\dot{r}_x$, $\dot{y}_m$, $\ddot{y}_m$, $\dot{e}$, $\ddot{e}$, $\dot{e}_{m2}$, $\ddot{e}_{m2}$, $\dot{y}_{x2}$ and $\dot{y}_{x2}$:

The signals $\tilde{y}_p$, $\tilde{y}_{x1}$, $\dot{r}_x$, $\dot{y}_m$, $\ddot{y}_m$, $\dot{\tilde{y}}_p$ and $\dot{\tilde{y}}_{x1}$ are outputs of "proper" stable transfer functions with bounded inputs. Hence they are bounded. Also, from (3.2.40), we see that $\dot{e}$ and $\ddot{e}$ are bounded. It follows from (3.2.36a) that the same is true of $\dot{e}_{m2}$ and $\ddot{e}_{m2}$. Consequently, from (3.2.36b), $\dot{y}_{x2}$ and $\ddot{y}_{x2}$ are bounded.

Boundedness of $\tilde{u}$, $\dot{e}_{x1}$, $\dot{e}_{x2}$, $\dot{y}_{x1}$, $\dot{y}_p$, w_{x1} and w_{x2}:

With bounded $\dot{y}_{x2}$, the boundedness of $\tilde{u}$ is derived from (3.2.42). The signal $\dot{e}_{x1}$ in (3.2.11) is bounded because the signals $\tilde{u}$, y_p and $\tilde{y}_p$ are bounded. In a similar fashion, the boundedness of $\dot{e}_{x2}$ can be established. Finally, $\dot{y}_{x1}$ and $\dot{y}_p$ are also bounded due to the boundedness of $\dot{y}_{x2}$, $\dot{e}_{x2}$ and $\dot{e}_{x1}$. The signals w_{x1} and w_{x2} in Figure 3.4 are composed of a sum of bounded signals and are therefore bounded.

Boundedness of $\dot{\alpha}_{11}$, $\dot{\beta}_{10}$, $\dot{\beta}_{11}$, $\dot{\alpha}_{21}$, $\dot{\beta}_{20}$ and $\dot{\beta}_{21}$:

The boundedness of the variables $\dot{\alpha}_{11}$, $\dot{\beta}_{10}$, $\dot{\beta}_{11}$, $\dot{\alpha}_{21}$, $\dot{\beta}_{20}$ and $\dot{\beta}_{21}$ as given in (3.2.14) and (3.2.19) can be seen through the substitution of all occurring derivative terms by their respective adaptive laws. For example, $\dot{\alpha}_{21}$ in (3.2.19b) has the derivative term $\dot{\alpha}_{11}$. It can be substituted with $\dot{\alpha}_{11}$ from (3.2.14), which is the adaptive law in Identifier #1.

Boundedness of $\ddot{y}_p$, u and m:

The boundedness of u is established from (3.2.2) if one can show the boundedness of y_p, $\dot{y}_p$ and $\ddot{y}_p$. This is demonstrated as follows. Substituting (3.2.5d) and (3.2.1) into (3.2.42) gives

$$\dot{y}_{x2} = \alpha_{21}\frac{1}{s+\lambda}\left(\frac{s^2 + a_{p1}s + a_{p0}}{k_p}y_p\right) + \beta_{20}y_{x1}(t) + \beta_{21}\tilde{y}_{x1}(t) - \lambda y_{x2}$$

Dividing $s+\lambda$ into $s^2 + a_{p1}s + a_{p0}$ yields

$$\dot{y}_{x2} = k_p^{-1}\alpha_{21}\left(\dot{y}_p + (a_{p1} - \lambda)y_p + \frac{a_{p0} - a_{p1}\lambda + \lambda^2}{s+\lambda} y_p\right) + \beta_{20} y_{x1}(t) + \beta_{21}\tilde{y}_{x1}(t) - \lambda y_{x2} \tag{3.2.48}$$

Differentiating (3.2.48) once gives

$$\ddot{y}_{x2} = k_p^{-1}\left[\dot{\alpha}_{21}\left(\dot{y}_p + (a_{p1} - \lambda)y_p + \frac{a_{p0} - a_{p1}\lambda + \lambda^2}{s+\lambda} y_p\right) + \alpha_{21}\left(\ddot{y}_p + (a_{p1} - \lambda)\dot{y}_p + \frac{a_{p0} - a_{p1}\lambda + \lambda^2}{s+\lambda}\dot{y}_p\right)\right] + \dot{\beta}_{20} y_{x1}(t) + \beta_{20}\dot{y}_{x1}(t) + \dot{\beta}_{21}\tilde{y}_{x1}(t) + \beta_{21}\dot{\tilde{y}}_{x1}(t) - \lambda\dot{y}_{x2} \tag{3.2.49}$$

Since $\dot{y}_{x2}$, $\ddot{y}_{x2}$, $\tilde{y}_p$, $\dot{\tilde{y}}_p$, $\tilde{y}_{x1}$, $\dot{\tilde{y}}_{x1}$, $\dot{\alpha}_{21}$, $\dot{\beta}_{20}$, $\dot{\beta}_{21}$, y_p and $\dot{y}_p$ have been shown to be bounded, $\ddot{y}_p$ is bounded. With the boundedness of u, the boundedness of m is assured from (3.2.46). Thus, we have shown the boundedness of all signals in the entire control system. Next, we would like to demonstrate the convergence of the tracking errors.

3.2.5. Convergence of the Tracking Errors

With reference to the entire system in Figure 3.4, our purpose is to demonstrate that $y_p \to y_m$ as $t \to 0$. This is accomplished by showing the same for the signals e_{m2}, e_{x1} and e_{x2}, which is given as follows:

(i) *Convergence of* e_{m2}:

This has been shown in (3.2.47).

(ii) *Convergence of* e_{x1} :

We have shown in Section 3.2.4 that e_{x1}, $\dot{e}_{x1}$, $\dot{\tilde{\alpha}}_1$ and $\dot{\tilde{u}}$ are bounded. Thus, from (3.2.15),

$$\ddot{V} = \begin{cases} -2\lambda\dot{e}_{x1}e_{x1} + \dot{\tilde{\alpha}}_1 e_{x1}\tilde{u} + \tilde{\alpha}_1\left(\dot{e}_{x1}\tilde{u} + e_{x1}\dot{\tilde{u}}\right), & \text{if } \; ge_{x1}\tilde{u} \le 0 \text{ and } \alpha_{11} \le k_{lower} \\ -2\lambda\dot{e}_{x1}e_{x1}, & \text{otherwise} \end{cases}$$

is bounded. According to Barbalat's Lemma, $\dot{V} \to 0$, which means from (3.2.15) that $e_{x1} \to 0$ as $t \to \infty$.

(iii) *Convergence of* e_{x2} :

Consider e_{x2} in (3.2.20),

$$e_{x2}(t) = \frac{1}{s+\lambda}\left(\alpha_{11} - \alpha_{21}\right)\tilde{u} + \frac{1}{s+\lambda}\left(\beta_{10}y_p - \beta_{20}y_{x1}\right) + \frac{1}{s+\lambda}\left(\beta_{11}\tilde{y}_p - \beta_{21}\tilde{y}_{x1}\right)$$

Convergence of e_{x2} will follows from the convergence of each individual term.

Convergence of $\frac{1}{s+\lambda}\left(\alpha_{11} - \alpha_{21}\right)\tilde{u}$ **:**

This has been shown in (3.2.26).

Convergence of $\frac{1}{s+\lambda}\left(\beta_{10}y_p - \beta_{20}y_{x1}\right)$:

Consider $\frac{1}{s+\lambda}(\beta_{10}y_p - \beta_{20}y_{x1}) = \frac{1}{s+\lambda}\beta_{10}(y_p - y_{x1}) + \frac{1}{s+\lambda}(\beta_{10} - \beta_{20})y_{x1}$ from (3.2.28), the convergence of the first term on the R.H.S. $\frac{1}{s+\lambda}\beta_{10}(y_p - y_{x1})$ follows from the convergence of $e_{x1} = y_p - y_{x1}$. The convergence of the second term $\frac{1}{s+\lambda}(\beta_{10} - \beta_{20})y_{x1}$ is assured by (3.2.34). Therefore, the convergence of $\frac{1}{s+\lambda}(\beta_{10}y_p - \beta_{20}y_{x1})$ is established.

Convergence of $\frac{1}{s+\lambda}(\beta_{11}\tilde{y}_p - \beta_{21}\tilde{y}_{x1})$:

The discussion of the convergence of $\frac{1}{s+\lambda}(\beta_{11}\tilde{y}_p - \beta_{21}\tilde{y}_{x1})$ is similar to that of $\frac{1}{s+\lambda}(\beta_{10}y_p - \beta_{20}y_{x1})$ and is here omitted.

Summarizing, with the convergence of all three R.H.S.-terms in (3.2.20), the convergence $e_{x2} \to 0$ as $t \to \infty$ is assured.

3.2.6. Simulation Studies

The simulation studies presented in this section are to compare the effectiveness of the proposed adaptive scheme with the existing augmented output error method in [1]. This is done for the case of relative degree $q = 2$ (Simulations 2.2.1 and 3.2.1).

Simulation 3.2.1: 2nd order Stacked Identifiers MRAC

The data for the simulation are as follows.

$$P(s) = \frac{1}{s^2 - s}, \; M(s) = \frac{1}{s^2 + 2s + 1}$$

$r(t) = 1, \; \lambda = 1, \; g = 10, \; e_p = y_p - y_m$

The initial conditions for the adaptive parameters are chosen in accordance with (3.2.16) and (3.2.27), in this case:

$\alpha_{11}(0) = \alpha_{21}(0) = 1 \geq k_{lower}$ (k_{lower} is taken to be 0.01)

$\beta_{10}(0) = \beta_{20}(0) = \beta_{11}(0) = \beta_{21}(0) = 0, \; \alpha_1^* = 1, \; \beta_0^* = 3, \; \beta_1^* = -2$

r.m.s parameter error $\underline{\underline{\Delta}}\, p_2 = \left[(\alpha_{11} - \alpha_1^*)^2 + (\beta_{10} - \beta_0^*)^2 + (\beta_{11} - \beta_1^*)^2 \right]^{1/2}$

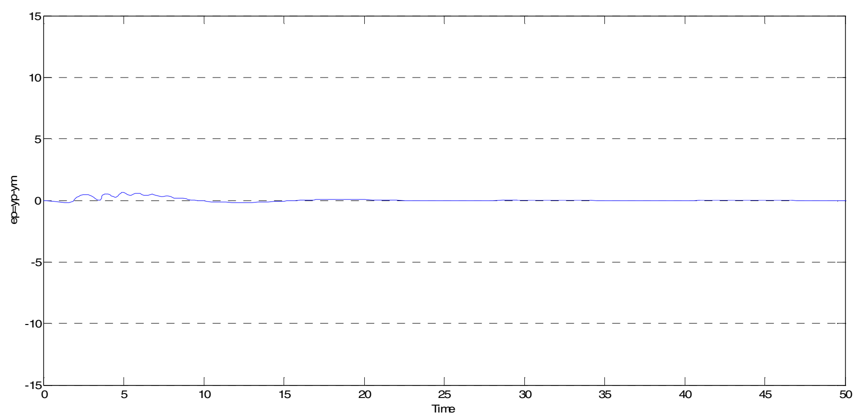

Figure 3.5: Stacked Identifiers MRAC output error. (relative degree $q = 2$)

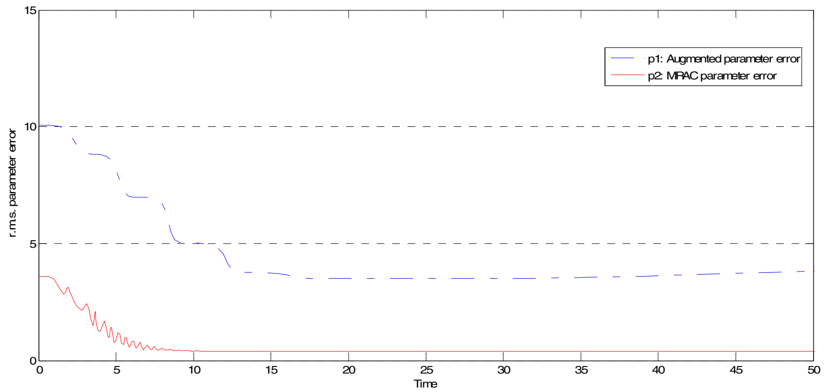

Figure 3.6: Augmented output parameter error p1 and Stacked Identifiers MRAC output parameter error p2. (relative degree $q = 2$)

3.3 Stacked Identifiers MRAC of plants with relative degree greater than two

3.3.1. Reparameterization of the Unknown Plant

In this section, we extend the Stacked Identifiers MRAC to the general case, i.e., plants of arbitrary relative degree q. Consider a plant with an input-output pair $\{u(.), y_p(.)\}$ described by a transfer function

$$P(s) = \frac{N(s)}{D(s)} = \frac{b_{n-q}s^{n-q} + \cdots + b_1 s \ + b_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s \ + a_0}, \quad n \geq q > 2 \tag{3.3.1}$$

where $b_{n-q}s^{n-q} + \cdots + b_1 s + b_0$ is a Hurwitz polynomial in s. The sign of the high frequency gain b_{n-q} is assumed to be positive, with a known lower bound $b_{n-q} > k_{lower} > 0$.

We will reparametrize the plant into a form suitable for the derivation of the identifier and the parameter update laws

Express (3.3.1) as

$$(s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s \ + a_0)y_p = (b_{n-q}s^{n-q} + \cdots + b_1 s \ + b_0)u$$

(3.3.2)

Let the plant be parametrized by

$$y_p = \frac{1}{s+\lambda}\left(\alpha^*_{q-1}\tilde{u}_{(q-1)} + \alpha^*_q\tilde{u}_q + \cdots + \alpha^*_{n-1}\tilde{u}_{(n-1)} + \beta^*_0 y_p + \beta^*_1\tilde{y}_{p1} + \cdots + \beta^*_{n-1}\tilde{y}_{p(n-1)}\right)$$

(3.3.3a)

where λ is a positive constant and

$$\tilde{u}_\mu \triangleq \frac{1}{(s+\lambda)^\mu}u, \quad \mu = q-1, \cdots, n-1 \tag{3.3.3b}$$

$$\tilde{y}_{p\sigma} \triangleq \frac{1}{(s+\lambda)^\sigma}y_p, \quad \sigma = 1, \cdots, n-1 \tag{3.3.3c}$$

Coefficient matching of terms of like powers in s in (3.3.2) and (3.3.3a) gives the relationship between the parametrized coefficients w and the original plant coefficients z.

$$\Lambda\, w = z\,, \ \Lambda = \left[\begin{array}{c|c} \Lambda_1 & 0 \\ \hline 0 & \Lambda_2 \end{array}\right] \tag{3.3.4a}$$

where $w, z \in R^{2n-q+1}$, $\Lambda \in R^{(2n-q+1)x(2n-q+1)}$, $\Lambda_1 \in R^{(n-q+1)x(n-q+1)}$ and $\Lambda_2 \in R^{nxn}$ are given by

$$w = \left[\begin{array}{cccc|cccc} \alpha^*_{q-1} & \alpha^*_q & \cdots \ \alpha^*_{n-2} & \alpha^*_{n-1} & \beta^*_0 & \beta^*_1 & \cdots \ \beta^*_{n-2} & \beta^*_{n-1} \end{array}\right]^T$$

(3.3.4b)

$$z = \left[b_{n-q} \quad b_{n-q-1} \quad \cdots \quad b_1 \quad b_0 \;\vdots\; a_{n-1} - \binom{n}{1}\lambda \quad a_{n-2} - \binom{n}{2}\lambda^2 \quad \cdots \quad a_1 - \binom{n}{n-1}\lambda^{n-1} \quad a_0 - \binom{n}{n}\lambda^n \right]^T$$

(3.3.4c)

$$\Lambda_1 = \begin{bmatrix} 0 & \cdots & 0 & 0 & 1 \\ 0 & \cdots & 0 & 1 & \binom{n-q}{1}\lambda \\ \vdots & \iddots & \iddots & \vdots & \vdots \\ 0 & 1 & \binom{2}{1}\lambda^1 & \cdots & \binom{n-q}{n-q-1}\lambda^{n-q-1} \\ 1 & \binom{1}{1}\lambda & \binom{2}{2}\lambda^2 & \cdots & \binom{n-q}{n-q}\lambda^{n-q} \end{bmatrix} \qquad (3.3.4d)$$

$$\Lambda_2 = \begin{bmatrix} 0 & \cdots & 0 & 0 & -1 \\ 0 & \cdots & 0 & -1 & \binom{n-1}{1}\lambda \\ \vdots & \iddots & \iddots & \vdots & \vdots \\ 0 & -1 & \binom{2}{1}\lambda & \cdots & \binom{n-1}{n-2}\lambda^{n-2} \\ -1 & \binom{1}{1}\lambda & \binom{2}{2}\lambda^2 & \cdots & \binom{n-1}{n-1}\lambda^{n-1} \end{bmatrix} \qquad (3.3.4e)$$

$\binom{\mu}{\nu}$ are combination symbols employed in the expansion of

$$(s+\lambda)^{\mu} = s^{\mu} + \sum_{\nu=1}^{\mu}\left(\binom{\mu}{\nu}\lambda^{\nu}\right)s^{\mu-\nu}, \qquad \nu = 1, 2, \cdots, n \qquad (3.3.4f)$$

which is used in the derivation of (3.3.4).

To re-write (3.3.3a) in a more compact form, let

$$\varphi^* \triangleq \left[\alpha^*_{q-1} \quad \alpha^{*T} \quad \beta^*_0 \quad \beta^{*T}\right]^T \tag{3.3.5a}$$

$$\overline{w} \triangleq \left[\tilde{u}_{(q-1)} \quad \tilde{u} \quad y_p \quad \tilde{y}_p\right]^T \tag{3.3.5b}$$

$$\alpha^* \triangleq \left[\alpha^*_q \dots \quad \alpha^*_{(n-1)}\right]^T \tag{3.3.5c}$$

$$\beta^* \triangleq \left[\beta^*_1 \dots \quad \beta^*_{(n-1)}\right]^T \tag{3.3.5d}$$

$$\tilde{u} \triangleq \left[\tilde{u}_q \dots \quad \tilde{u}_{(n-1)}\right]^T \tag{3.3.5e}$$

$$\tilde{y}_p \triangleq \left[\tilde{y}_{p1} \dots \quad \tilde{y}_{p(n-1)}\right]^T \tag{3.3.5f}$$

Then y_p in (3.3.3a) can be expressed as

$$y_p = \frac{1}{s+\lambda}\left(\alpha^*_{q-1}\tilde{u}_{(q-1)} + \alpha^{*T}\tilde{u} + \beta^*_0 y_p + \beta^{*T}\tilde{y}_p\right)$$

$$= \frac{1}{s+\lambda}\varphi^{*T}\overline{w} \tag{3.3.6}$$

For a relative degree of q, we need a total of q identifiers.

In accordance with the form of (3.3.6), Identifier #1 is constructed as

$$y_{x1} = \frac{1}{s+\lambda}\left(\alpha_{1(q-1)}\tilde{u}_{(q-1)} + \alpha_1^T\tilde{u} + \beta_{10} y_p + \beta_1^T\tilde{y}_p\right)$$

$$= \frac{1}{s+\lambda}\varphi_1^T\overline{w} \tag{3.3.7a}$$

where

$$\varphi_1 \triangleq \left[\alpha_{1(q-1)} \quad \alpha_1^T \quad \beta_{10} \quad \beta_1^T\right]^T \tag{3.3.7b}$$

$$\alpha_1 \triangleq \left[\alpha_{1q} \dots \quad \alpha_{1(n-1)}\right]^T \tag{3.3.7c}$$

$$\beta_1 \triangleq \left[\beta_{11} \cdots \quad \beta_{1(n-1)}\right]^T \tag{3.3.7d}$$

Figure 3.7 shows a schematic diagram of Identifier #1.

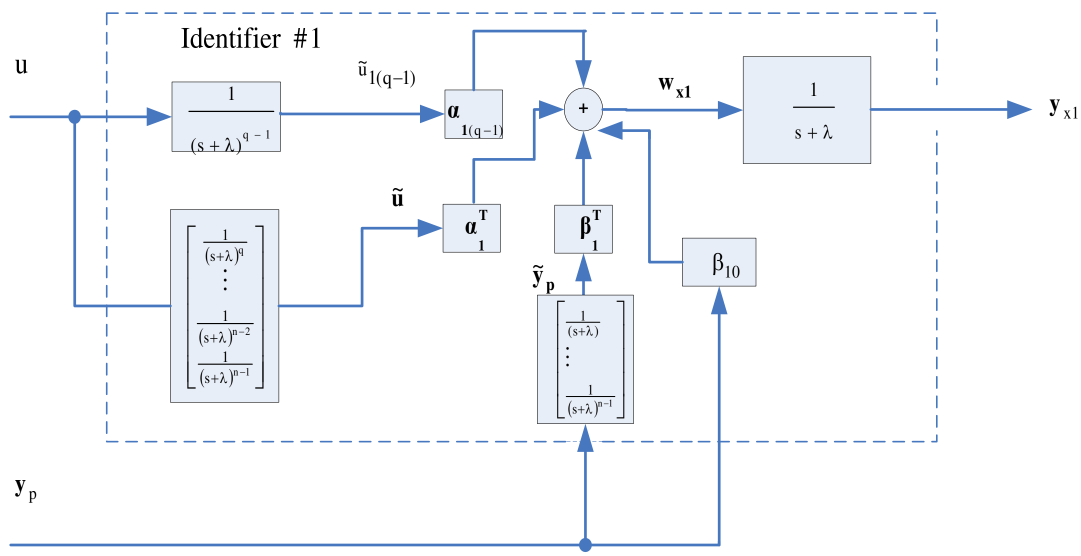

Figure 3.7: Stacked Identifiers MRAC for Identifier #1.

As for the rest of the identifiers, Identifier # γ ($\gamma = 2, 3, \cdots, q$) is constructed as

$$y_{x\gamma} = \frac{1}{s+\lambda}\left(\alpha_{\gamma(\gamma-1)}\tilde{u}_{(\gamma-1)} + \alpha_\gamma^T \tilde{u} + \beta_{\gamma 0} y_{x(\gamma-1)} + \beta_\gamma^T \tilde{y}_{x(\gamma-1)}\right)$$

$$= \frac{1}{s+\lambda}\varphi_\gamma^T \overline{w}_\gamma \tag{3.3.8a}$$

where

$$\varphi_\gamma \triangleq \left[\alpha_{\gamma(\gamma-1)} \quad \alpha_\gamma^T \quad \beta_{\gamma 0} \quad \beta_\gamma^T\right]^T \tag{3.3.8b}$$

$$\overline{w}_{\gamma} \triangleq \left[\tilde{u}_{(\gamma-1)} \quad \tilde{u} \quad y_{x(\gamma-1)} \quad \tilde{y}_{x(\gamma-1)}\right]^{T} \tag{3.3.8c}$$

$$\alpha_{\gamma} \triangleq \left[\alpha_{\gamma\gamma} \cdots \quad \alpha_{\gamma(n-1)}\right]^{T} \tag{3.3.8d}$$

$$\beta_{\gamma} \triangleq \left[\beta_{\gamma 1} \cdots \quad \beta_{\gamma(n-1)}\right]^{T} \tag{3.3.8e}$$

$$\tilde{y}_{x(\gamma-1)} \triangleq \left[\tilde{y}_{x(\gamma-1)1} \cdots \quad \tilde{y}_{x(\gamma-1)(n-1)}\right]^{T} \tag{3.3.8f}$$

$$\tilde{y}_{x(\gamma-1)\sigma} \triangleq \frac{1}{(s+\lambda)^{\sigma}} y_{x(\gamma-1)}, \quad \sigma = 1, \cdots, n-1 \tag{3.3.8g}$$

The corresponding block diagram is shown in Figure 3.8.

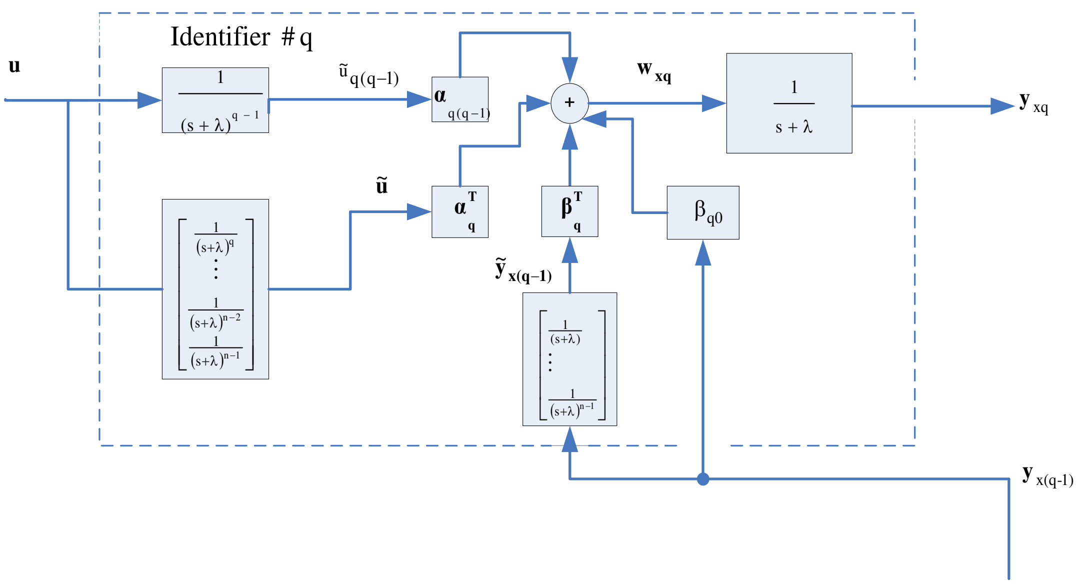

Figure 3.8: Stacked Identifiers MRAC for Identifier #q.

3.3.2. Parameter Update Laws for the Identifiers

Identifier #1

Define tracking error e_{x1} as

$$e_{x1} \triangleq y_p - y_{x1}$$

$$= \frac{1}{s+\lambda}\left[\left(\alpha^*_{q-1} - \alpha_{1(q-1)}\right)\tilde{u}_{(q-1)} + \left(\alpha^{*T} - \alpha_1^T\right)\tilde{u} + \left(\beta^*_0 - \beta_{10}\right)y_p + \left(\beta^{*T} - \beta_1^T\right)\tilde{y}_p\right]$$

(3.3.9)

Comparing (3.3.9) with (3.2.10) and following the same Lyapunov analysis as in Section 3.2.2 leads to the following parameter update laws

$$\dot{\alpha}_{1(q-1)} = \begin{cases} 0, & \text{if } ge_{x1}\tilde{u} \le 0 \text{ and } \alpha_{1(q-1)} \le k_{lower} \quad (3.3.10a) \\ ge_{x1}\tilde{u}, & \text{otherwise} \quad (3.3.10b) \end{cases}$$

$$\dot{\alpha}_1 = ge_{x1}\tilde{u} \quad (3.3.10c)$$

$$\dot{\beta}_{10} = ge_{x1}y_p \quad (3.3.10d)$$

$$\dot{\beta}_1 = ge_{x1}\tilde{y}_p \quad (3.3.10e)$$

which are similar to (3.2.14), with the exception of an additional update law for α_1 in (3.3.10c). Choosing an initial condition for the adaptive parameter $\alpha_{1(q-1)}(0) \ge k_{lower}$ will ensure

$$\alpha_{1(q-1)}(t) \ge k_{lower} > 0, \quad t \ge 0 \quad (3.3.10f)$$

Identifier # $\boldsymbol{\gamma}$ ($\boldsymbol{\gamma = 2, 3, \cdots, q}$)

Define tracking error $e_{x\gamma}$ as

$$e_{x\gamma} \triangleq y_{x(\gamma-1)} - y_{x\gamma}$$

$$= \frac{1}{s+\lambda}\Big[\big(\alpha_{(\gamma-1)(\gamma-1)} - \alpha_{\gamma(\gamma-1)}\big)\tilde{u}_{(\gamma-1)} + \big(\alpha_{(\gamma-1)}^{T} - \alpha_{\gamma}^{T}\big)\tilde{u} + \big(\beta_{(\gamma-1)0}\, y_{x(\gamma-1)} - \beta_{\gamma 0}\, y_{x\gamma}\big)$$

$$+ \big(\beta_{(\gamma-1)}^{T}\tilde{y}_{x(\gamma-1)} - \beta_{\gamma}^{T}\tilde{y}_{x\gamma}\big)\Big] \tag{3.3.11}$$

Comparing (3.3.11) with (3.2.19) and following the same Lyapunov analysis as in Section 3.2.2 leads to the following parameter update laws

$$\dot{\alpha}_{\gamma(\gamma\text{-}1)} = \begin{cases} 0, \\ \text{if } \; d\big(\alpha_{(\gamma\text{-}1)(\gamma\text{-}1)} - \alpha_{\gamma(\gamma\text{-}1)}\big) + g e_{\alpha\gamma}\tilde{u} + \dot{\alpha}_{(\gamma-1)(\gamma-1)} \le 0 \;\text{ and }\; \alpha_{\gamma(\gamma-1)} \le \alpha_{(\gamma-1)(\gamma-1)} & \text{(3.3.12a)} \\ d\big(\alpha_{(\gamma\text{-}1)(\gamma\text{-}1)} - \alpha_{\gamma(\gamma\text{-}1)}\big) + g e_{\alpha\gamma}\tilde{u} + \dot{\alpha}_{(\gamma-1)(\gamma-1)}, \\ \text{otherwise} & \text{(3.3.12b)} \end{cases}$$

$$\dot{\alpha}_{\gamma} = d\big(\alpha_{(\gamma\text{-}1)} - \alpha_{\gamma}\big) + g e_{\alpha\gamma}\tilde{u} + \dot{\alpha}_{(\gamma-1)} \tag{3.3.12c}$$

$$\dot{\beta}_{\gamma 0} = d\big(\beta_{(\gamma\text{-}1)0} - \beta_{\gamma 0}\big) + g e_{\beta\gamma 0}\, y_{x(\gamma-1)} + \dot{\beta}_{(\gamma-1)0} \tag{3.3.12d}$$

$$\dot{\beta}_{\gamma} = d\big(\beta_{(\gamma\text{-}1)} - \beta_{\gamma}\big) + g e_{\beta\gamma}\tilde{y}_{x(\gamma-1)} + \dot{\beta}_{(\gamma-1)} \tag{3.3.12e}$$

$$e_{\alpha\gamma} \triangleq \frac{1}{s+\lambda}\big(\alpha_{(\gamma-1)}^{T} - \alpha_{\gamma}^{T}\big)\tilde{u} \tag{3.3.12f}$$

$$e_{\beta\gamma 0} \underset{=}{\Delta} \frac{1}{s+\lambda}\left(\beta_{(\gamma-1)0} - \beta_{\gamma 0}\right) y_{x(\gamma-1)} \tag{3.3.12g}$$

$$e_{\beta\gamma} \underset{=}{\Delta} \frac{1}{s+\lambda}\left(\beta^T_{(\gamma-1)} - \beta^T_{\gamma}\right) \tilde{y}_{x(\gamma-1)} \tag{3.3.12h}$$

which are similar to (3.2.19), with the exception of an additional update law for α_γ in (3.3.12c).

The choice of an initial condition for the adaptive parameter $\alpha_{\gamma(\gamma-1)}(0) \geq \alpha_{(\gamma-1)(\gamma-1)}(0)$ will ensure

$$\alpha_{\gamma(\gamma-1)}(t) \geq \alpha_{(\gamma-1)(\gamma-1)}(t) > 0, \quad t \geq 0 \tag{3.3.12i}$$

Figure 3.9 shows a schematic diagram of Identifier #1 ,…, Identifier #q.

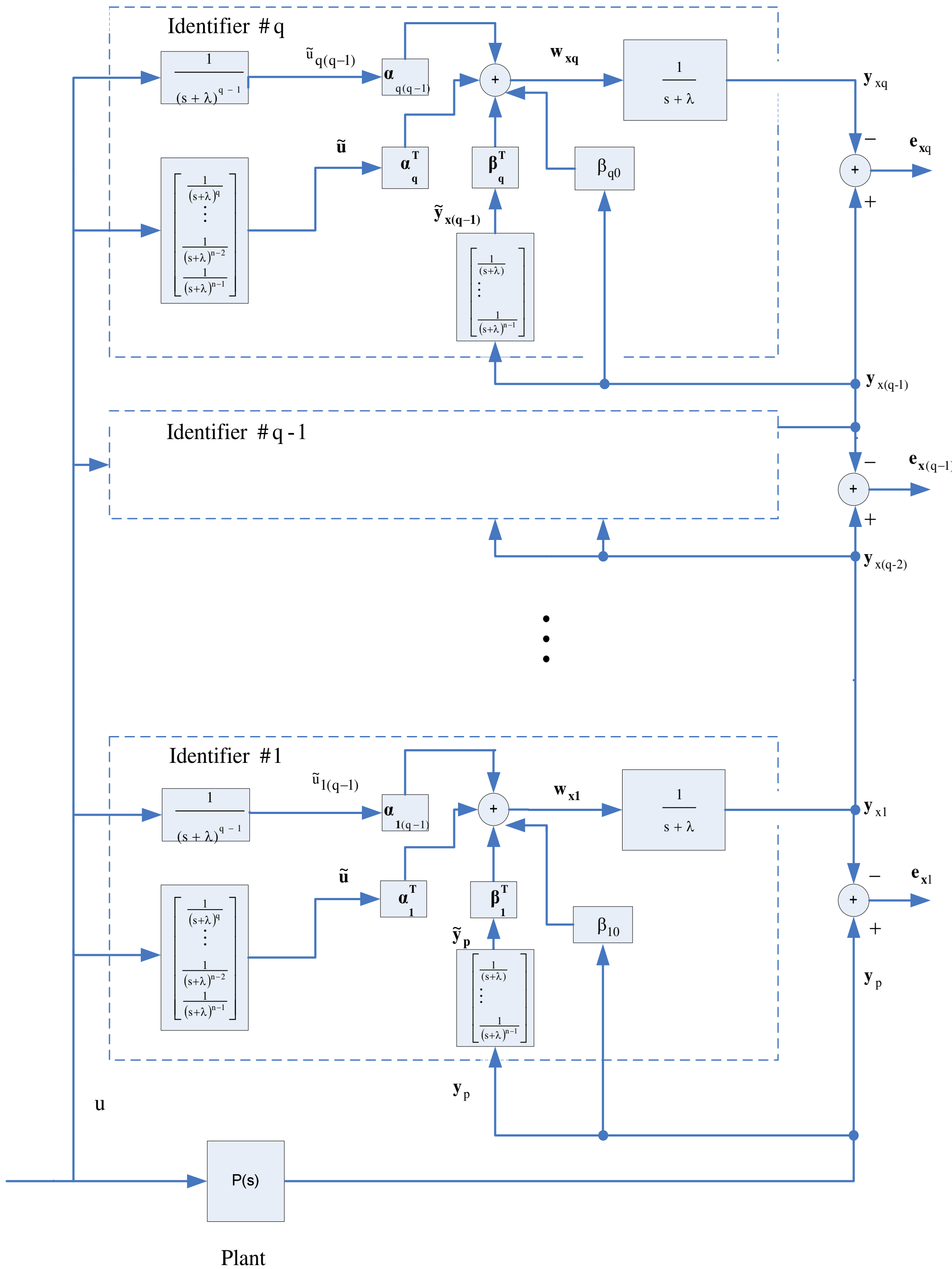

Figure 3.9: Stacked Identifiers MRAC for Identifier #1 ,…, Identifier #q.

3.3.3. Control Law u(t)

The reference model has an input-output pair $\{r(.), y_m(.)\}$ and a transfer function $M(s)$ given by

$$\frac{y_m}{r} = M(s) = \frac{1}{s + a_{m(q-1)}} \cdot \frac{k_m}{s^{q-1} + a_{m(q-2)} s^{q-2} + \cdots a_{m1} s + a_{m0}} \tag{3.3.13}$$

The above transfer function consists of two blocks in series as shown in Fig. 3.10.

Let the output of the first block be

$$r_x \triangleq \frac{r}{s + a_{m(q-1)}} \tag{3.3.14}$$

Then, (3.3.13) can be rewritten as

$$\frac{y_m}{r_x} = \frac{k_m}{s^{q-1} + a_{m(q-2)} s^{q-2} + \cdots a_{m1} s + a_{m0}} \tag{3.3.15a}$$

or, in time domain,

$$k_m r_x = y_m^{(q-1)} + a_{m(q-2)} y_m^{(q-2)} + \cdots a_{m1} \dot{y}_m + a_{m0} y_m \tag{3.3.15b}$$

Define tracking error e as

$$e \triangleq \left(s^{q-1} + a_{m(q-2)} s^{q-2} + \cdots a_{m1} s + a_{m0}\right) e_{mq} \tag{3.3.16a}$$

where

$$e_{mq} \triangleq y_{xq} - y_m \tag{3.3.16b}$$

(Note that if $e \to 0$, then $e_{mq} \to 0$ and $y_{xq} \to y_m$)

Substituting (3.3.16b) and (3.3.15b) into (3.3.16a) yields

$$e = \left[y_{xq}^{(q-1)} + a_{m(q-2)} y_{xq}^{(q-2)} + \cdots a_{m1} \dot{y}_{xq} + a_{m0} y_{xq}\right] - k_m r_x \tag{3.3.17}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}e^2 > 0 \tag{3.3.18}$$

A control law is now to be devised in order to make

$$\dot{V} = e\dot{e} = -ke^2 \tag{3.3.19}$$

negative definite. This will be achieved by making

$$\dot{e} = -ke\,, \quad k > 0 \tag{3.3.20}$$

through the use of an appropriate control law to be derived as follows:

From (3.3.17), the derivative of e is given by

$$\dot{e} = \left[y_{xq}^{(q)} + a_{m(q-2)}y_{xq}^{(q-1)} + \cdots a_{m1}\ddot{y}_{xq} + a_{m0}\dot{y}_{xq}\right] - k_m\dot{r}_x \tag{3.3.21}$$

The next step is to find an expression for $y_{xq}^{(q)}$.

From (3.3.8a), displaying the $\tilde{u}_{(q-1)}$ term explicitly, we have

$$\dot{y}_{xq} = \alpha_{q(q-1)}\tilde{u}_{(q-1)} + r_{xq} \tag{3.3.22a}$$

where

$$r_{xq} \triangleq \alpha_{q(q)}\tilde{u}_{(q)} + \cdots + \alpha_{q(n-1)}\tilde{u}_{(n-1)} + \beta_{q0}y_{x(q-1)} + \cdots + \beta_{q(n-1)}\tilde{y}_{x(q-1)(n-1)} - \lambda y_{xq}$$

(3.3.22b)

Successively differentiating (3.3.22a), gives the ν^{th} derivatives of y_{xq}

$$y_{xq}^{(\nu)} = \sum_{i=0}^{\nu-1}\binom{\nu-1}{i}\left(\alpha_{q(q-1)}{}^{(\nu-i-1)}\tilde{u}_{(q-1)}^{(i)}\right) + r_{xq}^{(\nu-1)}, \quad \nu = 1,2,\cdots,q \tag{3.3.23}$$

Letting $\nu = q$ and utilizing $\tilde{u}_{(q-1)}^{(q-1)} = -\left(\left(\sum_{\nu=1}^{q-1}\left(\binom{q-1}{\nu}\lambda^\nu\right)s^{(q-1)-\nu}\right)\tilde{u}_{(q-1)}\right) + u$ from (3.3.3b) and (3.3.4f) yields

$$y_{xq}^{(q)} = \alpha_{q(q-1)} u + \hat{y}_{xq}^{(q)} \qquad (3.3.24a)$$

where

$$\hat{y}_{xq}^{(q)} \underline{\underline{\Delta}} \sum_{i=0}^{q-2} \binom{q-1}{i} \left(\alpha_{q(q-1)}{}^{(q-i-1)} \tilde{u}_{(q-1)}^{(i)} \right) + r_{xq}^{(q-1)} - \alpha_{q(q-1)} \left(\left(\sum_{\nu=1}^{q-1} \left(\binom{q-1}{\nu} \lambda^{\nu} \right) s^{(q-1)-\nu} \right) \tilde{u}_{(q-1)} \right)$$

(3.3.24b)

After $y_{xq}^{(q)}$ is found as in (3.3.24), the expression for $\dot{e}$ in (3.3.21) becomes

$$\dot{e} = \left[\alpha_{q(q-1)} u + \hat{y}_{xq}^{(q)} + a_{m(q-2)} y_{xq}^{(q-1)} + \cdots a_{m1} \ddot{y}_{xq} + a_{m0} \dot{y}_{xq} \right] - k_m \dot{r}_x \qquad (3.3.25)$$

Next we substitute $\dot{e} = -ke$ from (3.3.20). The result is

$$-ke = \left[\alpha_{q(q-1)} u + \hat{y}_{xq}^{(q)} + a_{m(q-2)} y_{xq}^{(q-1)} + \cdots a_{m1} \ddot{y}_{xq} + a_{m0} \dot{y}_{xq} \right] - k_m \dot{r}_x \qquad (3.3.26)$$

Since our objective is to design a differentiator-free controller, replacing the derivative terms $\hat{y}_{xq}^{(q)}$, $y_{xq}^{(q-1)}, \cdots, \dot{y}_{xq}$ and $\dot{r}_x$ in (3.3.26) with (3.3.24b), (3.3.24a) and (3.3.14), respectively, gives the control law

$$u(t) = -\frac{m}{\alpha_{q(q-1)}} - ke, \qquad \alpha_{q(q-1)} > 0,\ k > 0$$

(3.3.27a)

where

$$m \underline{\underline{\Delta}} \sum_{i=0}^{q-2} \binom{q-1}{i} \left(\alpha_{q(q-1)}{}^{(q-i-1)} \tilde{u}^{(i)} \right) + r_{xq}^{(q-1)} - \alpha_{q(q-1)} \left(\left(\sum_{\nu=1}^{q-1} \left(\binom{q-1}{\nu} \lambda^{\nu} \right) s^{(q-1)-\nu} \right) \tilde{u}_{(q-1)} \right) +$$

$$\cdots + a_{m0}\left(\alpha_{q(q-1)}\tilde{u}_{(q-1)} + r_{xq}\right) - k_m\left(r - a_{m(q-1)}r_x\right) \qquad (3.3.27b)$$

Note that division by zero in (3.3.27a) will not occur because (3.3.10f) and (3.3.12i) guarantee that $\alpha_{q(q-1)} \geq \alpha_{(q-1)(q-1)} \geq \cdots \geq \alpha_{1(q-1)} \geq k_{lower} > 0$. Also note that the signals $\tilde{u}^{(q-2)}, \cdots, \tilde{u}$ and $r_{xq}^{(q-1)}$ can be obtained without actual differentiation because they are outputs of proper stable transfer functions with bounded inputs as shown in (3.3.3b). As for the derivatives of the adaptive parameters, they can be replaced by their respective adaptive laws, thus dispensing of the need of differentiations.

Thus, with $\dot{V}$ in (3.3.19) being negative definite, the equilibrium state $e = 0$ is globally asymptotically stable, i.e. e is bounded and $e \to 0$ as $t \to \infty$.

Consequently, from (3.3.16a) and (3.3.16b),

$$e_{mq} \to 0 \text{ and } y_{xq} \to y_m \qquad (3.3.28)$$

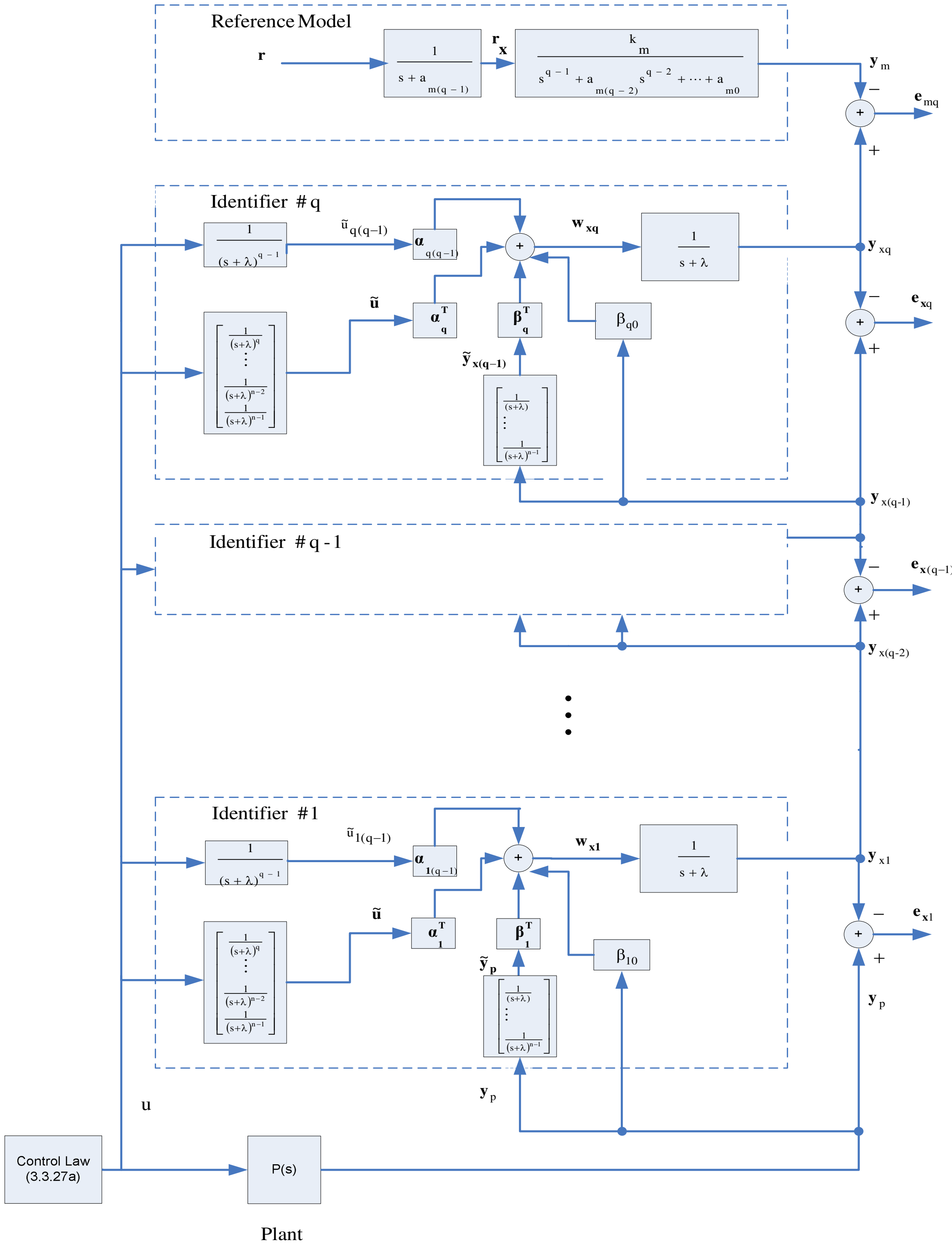

Figure 3.10: Stacked Identifiers MRAC for an n^{th} order plant of relative degree $q > 2$

3.3.4. Boundedness of All Signals in the Entired Feedback System

With reference to the entire adaptive control system in Figure 3.10, the following signals have been shown to be bounded:

From the analysis of Identifier#1, $\cdots$, Identifier#q: $e_{x1},\cdots,e_{xq}$, $\alpha_{1(q-1)},\cdots,\alpha_{q(q-1)}$, $\alpha_1^T,\cdots,\alpha_q^T$, $\beta_{10},\cdots,\beta_{q0}$ and $\beta_1^T,\cdots,\beta_q^T$

From the analysis of the control law: e and e_{mq}

From the reference model: r and y_m

The signals that remain to be shown bounded are, in appropriate groups:

Boundedness of y_p, $y_{x1},\cdots,y_{x(q-1)}$ and y_{xq}:

Since y_m, e_{mq}, e_{xq} and $e_{x(q-1)},\cdots,e_{x1}$ are bounded, it follows from (3.3.11) and (3.3.9) that $y_{xq},\cdots,y_{x1}$ and y_p are bounded.

Boundedness of $\tilde{y}_p$, $\tilde{y}_{x1},\cdots,\tilde{y}_{x(q-1)}$, $\dot{\tilde{y}}_{x1},\cdots,\dot{\tilde{y}}_{x(q-1)}$, $\dot{r}_x$, $\dot{y}_m,\cdots y_m^{(q)}$, $\dot{e}$, $\ddot{e}$, $\cdots$, $e^{(q)}$, $\dot{e}_{mq},\ddot{e}_{mq},\cdots,e_{mq}^{(q)}$ and $\dot{y}_{xq},\ddot{y}_{xq},\cdots,y_{xq}^{(q)}$:

The signals $\tilde{y}_p$, $\tilde{y}_{x1},\cdots,\tilde{y}_{x(q-1)}$, $\dot{\tilde{y}}_{x1},\cdots,\dot{\tilde{y}}_{x(q-1)}$, $\dot{r}_x$ and $\dot{y}_m,\cdots y_m^{(q)}$ are outputs of "proper" stable transfer functions with bounded inputs. Hence they are bounded. Also, from (3.3.20), we see that $\dot{e}$, $\ddot{e}$, $\cdots$, $e^{(q)}$ are bounded. It follows from (3.3.16a) that the same is true of $\dot{e}_{mq},\ddot{e}_{mq},\cdots,e_{mq}^{(q)}$. Consequently, from (3.3.16b), $\dot{y}_{xq},\ddot{y}_{xq},\cdots,y_{xq}^{(q)}$ are bounded.

Boundedness of $\tilde{u}_{(q-1)}, \tilde{u}_{(q)}, \cdots, \tilde{u}_{(n-1)}$, $\tilde{u}$, $\dot{\tilde{u}}_{(q-1)}, \cdots, \dot{\tilde{u}}_{(n-1)}$, $\dot{e}_{x1}, \cdots, \dot{e}_{xq}$, $\dot{y}_p$, $\dot{y}_{x1}, \cdots \dot{y}_{x(q-1)}$ and $w_{x1}, \cdots, w_{xq}$:

With bounded $\dot{y}_{xq}$, eliminating $\tilde{u}_q, \cdots, \tilde{u}_{(n-1)}$ from (3.3.3b), and (3.3.22) gives the boundedness of $\tilde{u}_{(q-1)}$ because all other variables in the resulting equation are bounded. After we establish the boundedness of $\tilde{u}_{(q-1)}$, the boundedness of $\tilde{u}_{(q)}, \cdots, \tilde{u}_{(n-1)}$ and $\tilde{u}$ follows. Furthermore, the signals $\dot{\tilde{u}}_{(q-1)}, \cdots, \dot{\tilde{u}}_{(n-1)}$ are outputs of "proper" stable transfer functions with bounded inputs $\tilde{u}_{(q-1)}$. Hence they are bounded. Next consider the signal $\dot{e}_{x1}$ in (3.3.9). It is bounded because $\tilde{u}$, y_p and $\tilde{y}_p$ are bounded. In a similar fashion, the boundedness of $\dot{e}_{x2}, \cdots, \dot{e}_{xq}$ can be established. Finally, $\dot{y}_{x(q-1)}, \cdots \dot{y}_{x1}$, $\dot{y}_p$ are also bounded due to the boundedness of $\dot{y}_{xq}$ and $\dot{e}_{x2}, \cdots, \dot{e}_{xq}$. The signals $w_{x1}, \cdots, w_{xq}$ in Figure 3.10 are composed respectively of a sum of bounded signals, and are therefore bounded.

Boundedness of $\dot{\alpha}_{1(q-1)}, \cdots, \dot{\alpha}_{q(q-1)}$, $\dot{\alpha}_1^T, \cdots, \dot{\alpha}_q^T$, $\dot{\beta}_{10}, \cdots, \dot{\beta}_{q0}$, $\dot{\beta}_1^T, \cdots, \dot{\beta}_q^T$ and $\dot{r}_{xq}$:

The boundedness of the variables $\dot{\alpha}_{1(q-1)}, \cdots, \dot{\alpha}_{q(q-1)}$, $\dot{\alpha}_1^T, \cdots, \dot{\alpha}_q^T$, $\dot{\beta}_{10}, \cdots, \dot{\beta}_{q0}$ and $\dot{\beta}_1^T, \cdots, \dot{\beta}_q^T$ as given in (3.3.10) and (3.3.12) can be seen through the substitution of all occurring derivative terms by their respective adaptive laws. For example, $\dot{\alpha}_{q(q-1)}$ in

(3.3.12a) has the derivative term $\dot{\alpha}_{(q-1)(q-1)}$. It can be substituted with $\dot{\alpha}_{(q-1)(q-1)} = ge_{\alpha(q-1)}\tilde{u} + \dot{\alpha}_{(q-2)(q-1)}$, which is the adaptive law in Identifier #(q-1). Consequently, $\dot{r}_{xq}$ (as obtained from 3.3.22b) is also bounded. Note that by following the above procedure, we can also demonstrate the boundedness of any derivative term of an adaptive parameter *up to the q-th derivative*.

Boundedness of $\ddot{y}_p, \cdots, y_p^{(q)}$ u and m:

The boundedness of u is established from (3.3.1) if one can show the boundedness of $y_p, \dot{y}_p, \ddot{y}_p, \cdots, y_p^{(q)}$. This is demonstrated as follows. Substituting (3.3.3b) and (3.3.1) into (3.3.22a) to give

$$\dot{y}_{xq} = \alpha_{q(q-1)} \frac{1}{\lambda(s)} \left(\frac{D(s)}{N(s)} y_p \right) + r_{xq}$$

Dividing $\lambda(s)N(s)$ into $D(s)$ yields

$$\dot{y}_{xq} = \alpha_{q(q-1)} \left(\dot{y}_p + \xi_{n-1} y_p + \frac{\zeta_{n-2}s^{n-2} + \zeta_{n-3}s^{n-3} + \cdots \zeta_0}{\lambda(s)N(s)} y_p \right) + r_{xq} \qquad (3.3.29)$$

Differentiating (3.3.29) once gives

$$\ddot{y}_{xq} = \dot{\alpha}_{q(q-1)} \left(\dot{y}_p + \xi_{n-1} y_p + \frac{\zeta_{n-2}s^{n-2} + \zeta_{n-3}s^{q-3} + \cdots \zeta_0}{\lambda(s)N(s)} y_p \right)$$

$$+ \alpha_{q(q-1)} \left(\ddot{y}_p + \xi_{n-1} \dot{y}_p + \frac{\zeta_{n-2}s^{n-2} + \zeta_{n-3}s^{n-3} + \cdots \zeta_0}{\lambda(s)N(s)} \dot{y}_p \right) + \dot{r}_{xq} \qquad (3.3.30)$$

Since $\ddot{y}_{xq}$, $\dot{\alpha}_{q(q-1)}$, y_p, $\dot{y}_p$ and $\dot{r}_{xq}$ have been shown to be bounded, $\ddot{y}_p$ is bounded.

Using the same approach, differentiating (3.3.29) twice will leads to the boundedness of $\ddot{y}_p$. Continuing on in this fashion would lead to the boundedness of $y_p^{(4)}, \cdots, y_p^{(q)}$. With the boundedness of u, the boundedness of m is assured from (3.3.27a). In conclusion, all systems in the overall system are bounded.

3.3.5. Convergence of the Tracking Errors

The discussion of the convergence is exactly the same as that in Section 3.2.5 and is omitted.

3.3.6. Simulation Studies

We include a simulation for the case of $q = 3$ (Simulation 3) which has not been done in the literature.

Simulation 3.3.1: 3rd order Stacked Identifiers MRAC

The data for the simulation are as follows.

$$P(s) = \frac{s+1}{s^4 + 3s^3 - s^2}, \; M(s) = \frac{1}{(s^2 + 2s + 1)(s+1)}$$

$$r(t) = 1$$

$$\lambda = 1, \; g = 2, \; e_p = y_p - y_m$$

The initial conditions for the adaptive parameters are chosen in accordance with (3.3.10f) and (3.3.12i), in this case:

$\alpha_{\gamma(\gamma-1)}(0) = \cdots = \alpha_{\gamma(n-1)}(0) = 1 \geq k_{lower}$ (k_{lower} is taken to be 0.01)

$\beta_{\gamma 0}(0) = \cdots = \beta_{\gamma(n-1)}(0) = 0$

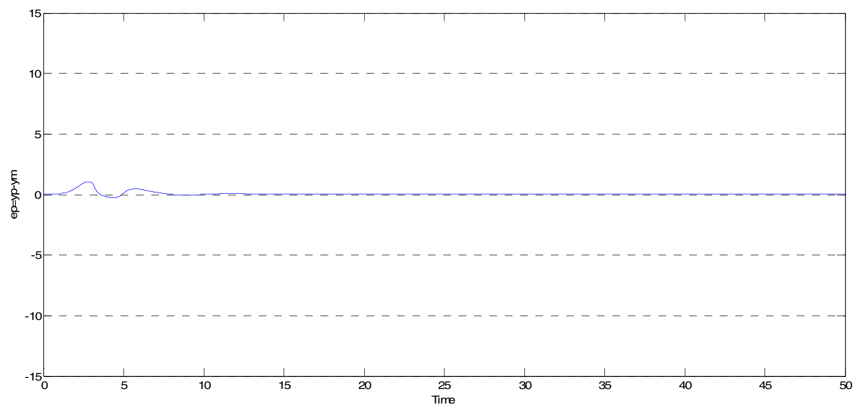

Figure 3.11: Stacked Identifiers MRAC output error. (relative degree $q = 3$)

Discussion

Figure 3.11 shows the simulation for the case of relative degree $q = 3$, which has not been reported in the current literature. It is seen that the transient response of the tracking error is also small.

CHAPTER 4

CONCLUSIONS

4.1 General Conclusions

In this dissertation, a new adaptive control scheme (referred to as the Stacked Identifiers model reference adaptive control) is proposed for controlling a single-input single-output, linear time-invariant plant containing uncertain parameters. The scheme incorporates a total of q (q being the plant relative degree) layers of identifiers in the control. Each identifier mimics the structure the plant *directly*, so that the control adaptations deviates less from the true plant values than other conventional methods (which adapt in such a way that the transfer function of the *entire control loop* matches that of the reference model). To achieve this, we adopt the following steps:

1. Reparametrize the unknown plant into a form so that an identifier can be constructed.
2. Choose an identifier and a parameter update algorithm such that the plant output asymptotically tracks the identifier output.
3. Design a control law to make the identifier output asymptotically track the reference model output. That means output of plant will track reference model asymptotically.
4. Give proof that all states generated are bounded.
5. Give proof that all tracking errors are converged.

The Stacked Identifiers MRAC design method is much superior than the existing augmented output error method as far as transient response is concerned. Simulations for the cases of $q = 2$ and $q = 3$ are given to demonstrate the effectiveness of the method. In conclusion, this work introduces an adaptive framework, which is completely different from existing ones and which produces much smaller transient excursions from the desired output response.

4.2 Future Research

We have developed only a fundamental theory for identifier-tracking MRAC. There remains much to do. On the basis of its structure, some future works are as follows:

- Extension of the continuous time schemes to the discrete time case.
- Extension of the single-input single-output plants to multi-input multi-output plants.
- Design a real-time system parameter identification algorithm.
- Robustness in the presence of unmodeled dynamics, time-varying parameters, and other perturbations.
- Relaxing assumptions.
- Implementations and applications.

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