SIMULATION ANALYSIS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING SYSTEMS WITH DIVERSITY RECEPTION

by

SAI SRAVAN BHARADWAJ KARRI

Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

May 2008

ACKNOWLEDGEMENTS

I greatly thank my advisor Dr. Vasant Prabhu for constantly motivating me and for his positive criticism jelled with his most valuable advice. I would also like to thank Dr.Michael Vasilyev and Dr.Jonathan Bredow for their interest in my research and their valuable suggestions on my work. My special thanks to Dr.Iyad Alfalujah for his critical suggestions and constant care bestowed upon me in discussing the details of my work.

I would also like to extend thanks to my colleague and room mate, Richard Faramjee, Vamsi Guntuku for their support during the period of my work. Finally, my heart-felt regards to all the people who supported me directly or indirectly during my period of study.

February 12, 2008

ABSTRACT

SIMULATION ANALYSIS OF ORTHOGONAL FREQUENCY DIVISION
MULTIPLEXING SYSTEMS WITH
DIVERSITY RECEPTION

SAI SRAVAN BHARADWAJ KARRI, M.S

The University of Texas at Arlington, 2008

Supervising Professor: Vasant K Prabhu

With the rapid increase in the utility of voice and data services, there is a demand for higher data rates and reliability in wireless communications. A wireless communication channel is prone to degrading effects such as multi-path fading, inter symbol interference (ISI) etc. Orthogonal Frequency Division Multiplexing (OFDM), a candidate for the 4^{th} generation wireless systems, mitigates these effects and supports higher data rates. The performance analysis of a complex system like OFDM, is a challenging task where analytical methods become cumbersome to be used. Simulation based approaches provide a fair estimate of the performance in such cases.

In this thesis, the performance of an OFDM system on Additive White Gaussian(AWGN), slow-flat, slow, fast-frequency selective fading channels has been analyzed through Monte Carlo(MC) simulations. Some of the analytical techniques previously used to evaluate the performance of diversity combining schemes-Maximal Ratio Combining(MRC), Equal Gain Combining(EGC)on slow flat fading channels have been summarized. The effect of inter-carrier interference(ICI) and ISI on OFDM systems for different channel conditions has been studied and the performance of few techniques to mitigate the degradation due to ISI, ICI such as diversity-reception, equalization, coding and interleaving in OFDM have been investigated. System models for OFDM with post-FFT diversity-combining, equalization on dual antennas have been proposed and their performance is analyzed by MC simulation. Further, these results are compared with conventional detectors and it is found that OFDM systems with diversity are effective for lower signal to noise ratios in case of smaller Doppler spreads and for higher signal noise ratios in case of higher delay spreads.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 Wireless Channel Modeling

Increasing necessity for high data rates and portability in communication demands robust and effective techniques for wireless communications. Voice and data which form the two major sources of information are pre-processed to optimize for transmission rates and accuracy, providing robustness. Portability, on the other hand is achieved by implementing such pre-processing techniques through wireless communication systems (WCS).

In wireless transmission, there may or may not exist a line of sight depending on the distance between the mobile and the base station. Besides, absorption, destructive interference, shadowing etc are some phenomena that adversely effect the signal power in radio-wave propagation. All such phenomena cause fading in signal power which is classified into two types; large scale, small scale fading.

Large scale fading is due to loss of received signal power due to large scale phenomena such as absorption, shadowing etc. In addition to these, the power in an electromagnetic wave decreases with the square of distance as the wave propagates. Typically independent of frequency, large scale fading is most effective at the edge of the cell sites and hilly regions. Different models exist in literature for large scale fading such as free space propagation model[2], log-normal path loss model etc. Large scale models are more relevant to deployment of networks and are not a subject of interest in this thesis.

Small scale fading[3], [4] is due to the constructive and destructive interference caused by multi-path propagation. The electromagnetic wave undergoes different phenomena such as scattering, reflection, diffraction and refraction causing distinct delays in propagation of reflected, refracted waves. Due to the difference between these delays considerable difference in phases occur and when these waves superimpose, they can add destructively or constructively causing fades in the signal level. Small scale fading occurs in the milli and micro scale and can be modeled well enough to be considered in the design of WCS.

WCS involve modeling of complex phenomena such as fading, shadowing, multipath etc. The inherent randomness in a mobile channel due to fickle changes in propagation conditions is the main source of the uncertainties in the performance of a WCS. The probability of a channel under deep fade is a measure of the performance of such systems.

In the next section a physical model for a wireless channel for a typical wireless propagation scenario is considered for analysis based on the ray-tracing approach. Electro-magnetic wave equations are used to analyze the scenario briefly and the model so derived explains some basic characteristics of a wireless channel which are further used in developing a more general statistical model.

1.2 Physical Model

Consider a typical wireless scenario as shown in the fig1.1 in which a mobile user

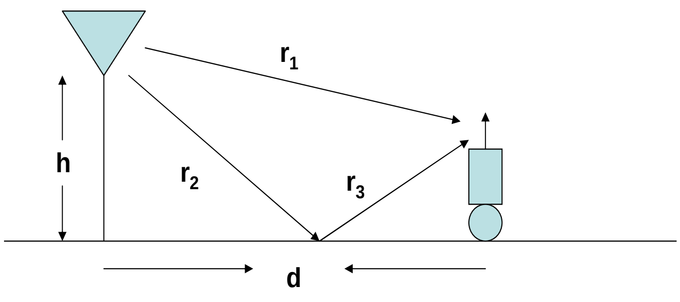

Figure 1.1. Typical wireless propagation model.

is moving with speed v away from a base station antenna of height h. Assume $d \gg h$. A line of sight path and a reflected path exist between the user and the base station.

The energy lost in reflection, absorption when the ray strikes the reflecting medium is considered to be negligible. The energy of the wave, $E \propto 1/d$, where d is the distance between the base station and the mobile unit. Two significant phenomena occur; the apparent motion of the user from the base station results in Doppler spread and a delay spread ($\triangle\theta$), owing to the difference between the delays in propagation.

Let the maximum amplitude of the wave at the antenna be E_{max} and the angular frequency of the carrier be ω. Let r_1, r_2, r_3 be the lengths of the direct path, incident and reflected wave components of the reflected path. Let $E_{max}\cos(\omega t)$ be the electric field intensity at the transmitting antenna. The intensity of the wave at the mobile is a function of the antenna pattern, beam width and the amplitude of the received wave can be written as

$$E(r,\theta,\Phi) = E_{max}(r,\theta,\Phi)\cdot\cos(\omega t)\cdot\Psi(r,\theta,\Phi) \tag{1.1}$$

Since, it was already assumed that the amplitude of the wave varies inversely with distance, the intensity of the wave after it travels a distance r_1, is

$$E = \frac{E_{max}}{r_1} \tag{1.2}$$

The phase of the direct path after it travels a distance r_1 is given by $\omega r_1/c$, where c is the velocity of light in the medium of propagation. Hence the equation for the direct wave at the mobile is given by

$$E_{dir} = (\frac{E_{max}}{r_1})\cos\omega(t - \frac{r_1}{c}) \tag{1.3}$$

Similarly, the reflected path after it travels a distance $r_2 + r_3$ undergoes a phase shift $\omega(r_2 + r_3)/c$. The equation for the reflected wave is given by

$$E_{ref} = \frac{-E_{max}}{r_2 + r_3}\cos\omega(t - \frac{r_2 + r_3}{c}) \tag{1.4}$$

The electromagnetic wave received at the antenna is a summation of the above two electromagnetic waves. After super imposing these two and making some approximations based on previous assumptions, the resultant equation reduces to,

$$E_{tot} = \frac{E}{r_1 r_2}\cos(\omega(t - \frac{r_1}{c}))\cos(\omega(\frac{r_1 - r_2}{c})) \tag{1.5}$$

Based on the time difference between direct and reflected waves, (the second term in the above equation) significant changes in the magnitude of the signal occur if the phase differences due to these are of the order $\lambda/4$. Similar changes occur in the magnitude of the wave as the wave propagates, if the frequency changes by

$$\Delta_f = \frac{1}{2\pi} \quad \frac{c}{4(r_1 - r_2)} \tag{1.6}$$

This difference is a measure of delay spread of the channel defined to be the difference between the propagation delays. Thus, if the frequency of the wave changes atleast by a quarter of the delay spread, there is a significant change in magnitude of the signal. Also called coherence bandwidth, this is the bandwidth over which the channel taps remain constant.

In addition to the above, there may be significant Doppler spread due to the motion of the mobile user from the base station resulting in frequencies given by,

$$\omega_{dopp} = -\omega(\frac{v}{c}) \tag{1.7}$$

leading to a time varying fading envelope within the received signal. Doppler spread describes the rate of traversal of the signal across this pattern and is inversely proportional to the coherence time of the channel.

The above physical model can be generalized to any wireless channel. Representing amplitudes and delays such that they are functions of frequency and time, eqn:1.7 describes the propagation effects for all frequencies and time scales. The attenuation in the signal, $a_i(f;t)$ is the product of attenuations from the antenna pattern, transmitter, receiver and a distance factor from the transmitting antenna, power loss due to reflection etc, along the i^{th} path. The attenuation is a function of frequency of the wave and the instantaneous time in which the wave propagates. Similarly, the delay across the i^{th} path, $\tau_i(f;t)$ is a function of frequency and the time of propagation. The generalized expression for the output of the channel is[3],[5]

$$y(t) = \sum_i a_i(f;t)x(t - \tau_i(f;t)) \tag{1.8}$$

where $x(t)$ is the input signal. Since the output for an input $x(t)$ to a linear time variant wireless channel $h(\tau, t)$is linear, it can be modeled as a response to a linear filter as,

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t)x(t - \tau)\mathrm{d}\tau. \tag{1.9}$$

The dependence of attenuation, delays on frequency can be ignored as systems that are dealt are limited to relatively narrow bands. The impulse response of the channel can be obtained by comparing $Eq1.7$ with $Eq1.8$ as below:

$$h(\tau, t) = \sum_i a_i(t)\delta(\tau - \tau_i(t)) \tag{1.10}$$

The expression above suggests that all the effects in a mobile channel such as reflection, absorption, etc accurately defined by Maxwell's equations can be modeled by a simple linear equation. If we assume further that the channel gains and delays vary slowly w.r.t time, the equation can be simplified to

$$h(\tau) = \sum_i a_i\delta(\tau - \tau_i) \tag{1.11}$$

Though communication is typically carried in passband, the signal processing, coding, etc are all done at the baseband. Hence the baseband equivalent of the channel model is more useful and can be obtained as

$$h_b(\tau, t) = \sum_i a_i(t)e^{-j2\pi f_c\tau_i(t)}(\delta(\tau - \tau_i(t)) \tag{1.12}$$

where f_c is the frequency of the carrier. The above representation can be related and easily modified for various phenomena in a wireless channel. The summation is over the replicas from multi-paths of a particular baseband input. The magnitude of i^{th} path can be represented as ${a_i}^b(t)$. It changes significantly with small changes in delay owing to the factor of $e^{-j2\pi f_c\tau_i(t)}$ in which f_c is large enough to magnify any small changes in $\tau_i(t)$. Significant changes in magnitude occur over path lengths of $\lambda/4$ and the corresponding change in the phase is $f_c\Delta\tau_i(t)$. Thus, it can be concluded that significant changes in phase occur if the delay along the i^{th} path τ_i changes by $c/4f_cv$, which is typically called the coherence time of the channel. The amplitude along each path is a function of time and each path has a delay that is a function time.

Further, the discrete baseband model can be obtained from the output of the channel for an input sampled at twice the highest frequency, W Hz and can be given by

$$h_l[m] = \sum_i {a_i}^b[m/W]sinc[l - \tau_i(m/W)W] \tag{1.13}$$

The output of the channel can be written in terms of the convolution sum,

$$Y[m] = \sum_l h_l[m]x[m-l] \tag{1.14}$$

1.2.1 Statistical Channel models

As the channel is random, to predict the performance of a WCS it is necessary to know the probability distributions of the channel tap gains. Though in real world the tap gains are not stationary, a simple channel model based on stationary assumption simplifies the performance analysis. Among the different stationary channel models existing in literature, two small scale fading channel models are used frequently for performance analysis; Rayleigh fading model in which the magnitude of the channel taps are modeled as Rayleigh and the phases as uniformly distributed random variables. Rayleigh fading model is used when no line of sight path component exists between the user and the base station. Rician fading, to model channel tap gains if a line of sight path exists between the user and the mobile station. Here the magnitude of the taps follow Rician distribution and the phase along the LOS path is uniformly distributed. The channel taps for the stationary models are characterized by their probability density functions.

1.2.1.1 Rayleigh fading

When there is no line of sight path and random multiple independent paths correspond to a single tap, such a tap can be regarded as a summation of independent complex random variables with zero mean where each random variable corresponds to the channel gain on a specific path. By the central limit theorem it can be concluded that the sum of these random variables form a complex symmetric Gaussian random variable with mean zero, whose magnitude distribution is Rayleigh and phase distribution is uniform. The density function for the Rayleigh random variable is given by,

$$f(x) = \frac{x}{\sigma_l}^2 e^{\frac{-x^2}{2\sigma_l^2}} \quad \forall \quad x \geq 0 \tag{1.15}$$

$$= 0 \quad elsewhere \tag{1.16}$$

1.2.1.2 Rician fading

When there is a specular path existing between the user and the base station along with random multiple independent paths due to reflections, the channel tap gains can be regarded as a summation of independent circular symmetric random variables and another random variable with constant magnitude and uniformly distributed phase. The distribution of the channel tap gains is given by

$$f(x) = \sqrt{\frac{K}{K+1}}\sigma_l e^{j\theta} + \sqrt{\frac{1}{K+1}} CN(0,1) \tag{1.17}$$

Here the channel taps are no longer circular since they have non-zero mean. The parameter K is the ratio of the energy in the specular path to the energy in the non-specular path. In all the above models the random process associated with the channel tap gains is assumed to be stationary. Hence the auto-correlation of the process is independent of time but is a function of the difference between the two distinct instants of time considered for correlation.

1.2.1.3 Additive White Gaussian Noise

White noise[5] is a wide sense stationary stochastic process with zero mean and a double-sided power spectral density $\frac{N_o}{2}$ over all the frequencies. Noise at the output of a bandpass filter defined over frequencies of interest is considered relevant. The noise power has to be N_o over a bandwidth of W hertz. Hence, the spectral density is defined to be $\frac{N_o}{2}$over both positive and negative frequencies. AWGN at the receiver accounts for the thermal noise at the receiver front end where $N_o = kT$ is the product of Boltzman's constant with the absolute temperature.

1.3 Parameters of a Wireless Channel

1.3.1 Diversity

Wireless channels provide multiple independent replicas of the signal at the receiver in one or more dimensions such as space, frequency, time, etc. Multiple copies of the signal are used to recover the transmitted signals efficiently. More about this is discussed in Chapter 3.

1.3.2 Doppler Spread

The parameter of the channel which indicates how quickly the channel taps vary with time. Equation 1.12 for the channel indicates that the taps can vary due to two factors; variation of the phase in the l^{th} tap due to difference in time varying delays of the paths contributing to the l^{th}tap i.e $f_c \tau_i(t)$, changes in the magnitude of the taps due to variation of the sinc function. The phase variations are significant as f_c is higher in magnitude and any small changes in $\tau_i(t)$ in i^{th} path are reflected on the phase of the complex gains provided by i^{th} tap. A measure of the magnitude of the change in the phase is given by

$$Max_{i \neq j} f_c |\tau_i(t) - \tau_j(t)| \tag{1.18}$$

Since the Doppler spread is given by $f_c v/c$ and $c/4f_c v$ is the path length over which the phase varies significantly, the coherence time, defined as the time over which a channel remains invariant is given in terms of Doppler spread[3] as

$$T_c = \frac{1}{4} Max(f_c|\tau_i(t) - \tau_j(t)|) \Rightarrow T_c = \frac{1}{4} D_s. \tag{1.19}$$

The factor of 4 is a result of the requirement on the path length to produce a significant change in $h[m]$. Typically coherence time is of the order of milli seconds and is usually greater than the delay spread.

The sinc varies slowly compared to change in phase of a tap. Each path is associated with a sinc function. The magnitude of the envelope of the sinc pulse for each path changes as a function of delay. However the factor $\tau_i(t)W$ is not significant compared to $\tau_i(t)f_c$ as the systems on which the channels are defined are sufficiently narrow band as

$$f_c \gg W.$$

Channels can be classified into two types based on coherence time. If the coherence time is greater than the delay requirement (typically the symbol duration), the channel is said to be slow fading. If the coherence time is smaller than the delay requirement, the channel is said to be fast fading. Fast fading channels are favorable for data transmission as they offer better time diversity.

1.3.3 Delay Spread

An indicator of how fast the channel is varying with frequency, it is referred to as the difference between propagation times of the longest and the shortest paths contributing significantly to each tap for the wireless channel.

$$T_d = Max_{i \neq j} |\tau_i(t) - \tau_j(t)| \tag{1.20}$$

Typically WCS are designed for delay spreads between 1 or 2 micro-secs. Delay spread of a channel determines the frequency coherence. This can be seen by taking the Fourier transform of the response of the channel.

$$H(f, t) = \sum_i a_i(t) exp(-j2\pi f \tau_i(t)) \tag{1.21}$$

In the above equation the product, $f(\tau_i(t) - \tau_j(t))$ determines frequency coherence of the channel for each frequency f. So the channel taps change significantly not only with changes in phase but also with a change in frequency. Hence coherence bandwidth can be defined as the frequency band over which a channel tap remains constant in frequency. If the delay spread is greater than the delay requirement (symbol time) then the channel exhibits frequency selective fading. If it is less than the delay requirement it is called flat fading. The coherence bandwidth of the channel is given in terms of delay spread and is defined to be

$$W_c = \frac{1}{2T_d} \tag{1.22}$$

Based on the delay spread, the channels can be classified into flat and frequency selective fading. Since in a flat fading channel, the delay spread is less than the symbol time, the multiple-paths in transmission arrive within one symbol time and the output of such a channel can be modeled as,

$$y[m] = h[m]x[m] + w[m]. \tag{1.23}$$

For a frequency selective fading channel, i.e the delay spread is greater than the symbol time, the multi-paths in transmission arrive at multiple symbol times leading to more than one tap spaced apart in time causing ISI in the system. The output on such channels can modeled as,

$$y[m] = \sum_{l} h[l]x[m-l] + w[m] \tag{1.24}$$

The next few sections provide a detailed explanation on modeling of a frequency selective fading channel, problem definition, previous work and organization of the thesis.

1.4 Frequency selective fading channels

Consider the output of a frequency selective selective fading channel $h[m]$ to the input $x[m]$ at $m = 3$, the output

$$y[m] = \sum_{l=1}^{L} h[l]x[m-l] + w[m] = h[3]x[0] + \Sigma_{l \neq 3} h[l]x[3-l] + w[m] \tag{1.25}$$

suffers from ISI due to the previous symbols. For a frequency selective fading channel with L independent taps, L-independent replicas of the transmitted symbol are available as it propagates through the channel [3]. The diversity offered by a frequency selective fading channel can be realized by modeling the channel into a multiple input single output channel with L transmit antennas and a single receive antenna with each tap experiencing flat fading.

Let the channel taps be $\{h_0 \, h_1 \, h_2 \, \ldots h_{L-1}\}$. Transmit an information symbol $x[m]$ on the first antenna while keeping the remaining antennas silent as shown in the fig:1.2 Turn on the second antenna and transmit the same symbol $x[m]$ one symbol time after

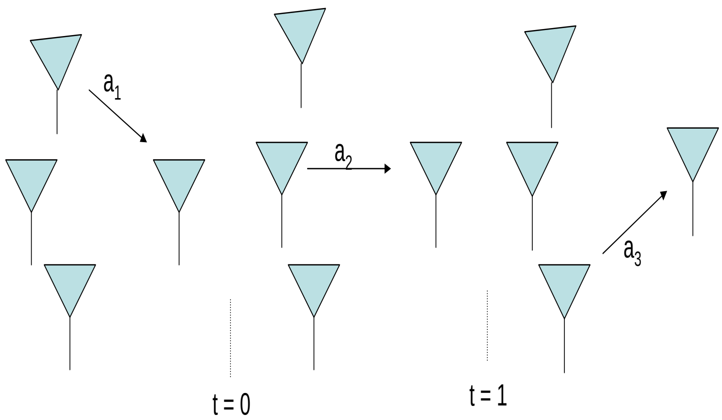

Figure 1.2. Frequency selective fading channel as MISO channel.

$x[m]$ is transmitted on the first, keeping the rest silent. Repeat the same till all the L antennas are covered. At the receive antenna, the output after L symbol times is same as that of an L-tap channel for an input symbol $x[m]$. In order to decode the data symbol from the L independent taps correctly, the decoder has to wait L symbol times i.e, the data rate has to be decreased by L.

To mitigate the effects of ISI, diversity techniques such as rake combining, equalization etc can be used. Though they mitigate ISI and better performance gains, they are not suitable for higher data rates due to an exponential increase in number of computations. A rake receiver may need to resolve higher number of multi-paths under higher data rates and collect multi-path energy in pulses of shorter durations which have less immunity to impulse noise etc. Time domain equalization, at higher data rates is not a cost effective solution. A trade-off exists between the performance of the equalizer and the number of taps and the cost of implementation of the equalizer increases with the resolution and the number of taps.

A new communication technique, Orthogonal Frequency Division Multiplexing (OFDM) supporting higher data rates in transmission while providing simpler equalization techniques is a better alternative and is considered in this thesis to reduce the impact of ISI and maximize the performance gain. OFDM involves transmitting N data symbols accumulated over N ($N > L$) symbol intervals at a time over N sub carriers and therefore minimizes ISI due to larger symbol duration than delay spread. In OFDM, equalization, usually a complex technique is highly simplified and the inherent frequency diversity in the system due to frequency selective fading is exploited to provide large performance gains[3]. OFDM ensures better performance with higher data rates per a user and has relaxed constraints on power control. The performance study of such systems is an interesting area to explore. Recently, communication systems with multiple antennas are receiving more attention as they provide diversity gains without any degradation in the supported data rates. Receive diversity systems employ multiple antennas placed sufficiently apart at the mobile receiver and exploit the independence of multiple replicas of the signal at the receiver. OFDM with multiple antennas was proven to be a lucrative choice to support higher data rates per channel per user with adequate quality of service. Motivated by the considerable improvements in performance gains, lower complexity in implementation and simpler equalization techniques, this thesis considers the analysis and performance of an OFDM system with multiple antennas at the receiver under Rayleigh fading conditions.

1.5 Previous work

OFDM also referred to as Discrete Multi-Tone modulation has been a topic of research from the late seventies. The advent of digital communications and advances in the design of digital filters provided feasible and cost effective architectures for its deployment and commercialization [6]. The basic idea of using orthogonal tones for transmission was first proposed by Chang and performance analysis of such systems was explored extensively through simulations in [7] and [8].

Performance analysis of OFDM with coding and interleaving was studied in [9] wherein the information theoretic capacity analysis shows that the probability of outage is the real performance indicator for any coded OFDM system with large OFDM block lengths. Closed form approximations for the outage probability have been derived based on averaging the instantaneous channel capacity over Rayleigh fading distribution.

Performance analysis of an alternative technique to achieve maximum frequency diversity is proposed in [10] where the analytical bounds for the probability of error for an OFDM block based on the union bound and pairwise error were calculated for a linear constellation pre-coded OFDM system.

Analytical evaluation of BER for OFDM on MQAM systems without diversity reception for imperfect channel estimation was discussed in [11]. The analysis is based on averaging the conditional symbol error probability (CSEP) over a particular distribution for channel fading. In [12] a systematic procedure for analyzing the performance of OFDM systems with equalization for imperfect linear channel estimates has been proposed. The systematic procedure is based on generalizing various linear and nonlinear estimates of the channel into the product of a weighting vector and the least squares estimate. The actual and the estimated channel taps are then treated as complex gaussian random variables and their joint distribution is computed and later used in evaluating the probability of symbol error for an equalized system. Some matched filter bounds for fast flat fading with OFDM systems have been provided in [13]. Analytical expressions for the probability of error for correlated fading in sub-carriers based on Karhunen-Loeve expansion theorem were also derived in [13]. In [14], the author analyzed the performance of various QAM constellations in frequency selective fading channels. In [15] the authors derive expressions for the BER performance of MPSK systems on frequency selective fading channels analytically. Later, these approaches are used for the performance analysis of OFDM with DPSK on frequency selective fading channels with perfect channel estimation in [16]. In [1], the performance of OFDM on slow time variant channels has been analyzed and analytical expressions for symbol error probability for a 16-QAM modulation scheme are obtained based on the Gram-Charliere expansion of probability of a linear combination of circular Gaussian random variables. The results indicate that there are considerably higher irreducible error floors at even high signal to noise ratios, lower Doppler spreads indicating that the performance of OFDM system degrades at higher data rates.

Simple techniques to reduce these error floors are of great interest as they support higher data rates with increased reliability. Error control coding with interleaving, diversity combining and equalization are some techniques that help to mitigate higher error floors. Of these, coding techniques are not suitable at higher data rates as coding itself reduces the spectral efficiency of the transmission scheme. The other alternatives, equalizers and diversity reception schemes are found to perform better and form a viable solution owing to their ease in implementation and the performance gains they provide.

Motivated by the curiosity to compare these alternatives, the thesis considers the problem of reducing higher error floors in OFDM systems through diversity reception, equalization techniques. Some background on the existing techniques for the performance analysis of flat fading channels with space-diversity has been provided for two important diversity reception schemes; equal gain combining, maximal ratio combining. These are further investigated as possible solutions to mitigate ICI, ISI in OFDM systems on frequency selective fading channels.

More specifically, this thesis investigates the problem of performance evaluation of BPSK/QPSK/16-QAM modulation in uncoded-OFDM systems for Rayleigh slow-flat and slow, fast-frequency selective fading channels by MC simulation. The work is later extended to performance analysis of diversity combining in time varying Rayleigh frequency selective fading channels with OFDM. A dual antenna case is considered for diversity in all cases as it is more practical and affordable to implement.

1.6 Organization of the Thesis

In Chapter 2, an introduction to OFDM as a technique to mitigate inter symbol interference has been provided. Some of the general principles and procedures involved in the implementation of OFDM were discussed. A brief overview on diversity reception schemes along with some analytic techniques for performance analysis of such schemes on flat fading channels have been summarized.

In Chapter 3, the performance of an OFDM system under different channel conditions has been investigated through MC simulations and some proposed remedies for performance improvement have been tested. Performance of an OFDM scheme on AWGN channel, slow-flat, frequency selective time invariant Rayleigh fading channels with and without the presence of ISI was simulated. The performance of diversity reception and equalization in the presence of ISI has been investigated. A new system model for post-detection equal gain combining in the absence of ISI was proposed and the performance of proposed system model was investigated.

The effects of inter carrier interference in OFDM system on slow time variant channels was later investigated. New system models incorporating diversity combining in already existing equalization schemes for the mitigation of inter-carrier interference have been proposed and the performance is evaluated through simulations. The results of the simulations were summarized in the 4^{th} chapter.

Chapter 5 provides conclusions on the work done and discusses the outcomes and the future developments of the thesis.

1.7 Conclusion

In this chapter, some of the back ground material required for the performance analysis of OFDM systems on frequency selective fading channels has been provided. A wireless channel model for frequency selective fading channels has been introduced and some of the previous research relevant to the work in this thesis has been provided. Finally an overview of the organization of the thesis has been provided.

CHAPTER 2

ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

A sinusoidal wave behaves as an eigen function for a linear time invariant (LTI) system and is less susceptible to interference. Hence a sine wave is suitable for transmission on a multi-tap channel. The sine waves used for transmission should be over sufficiently long intervals of time in order to preserve the eigen property. Therefore distinct multiple sinusoids of sufficient duration modulated by data symbols can be used to transmit data with a lesser degradation in performance. However, adjacent sub-carriers in frequency should be separated by guard bands to prevent overlap of information between them resulting in poor bandwidth efficiency.

2.1 What is OFDM

Orthogonal frequency division multiplexing is a wideband transmission technique in which the data symbols are modulated on N sub-carriers spread over a bandwidth of W hertz, with each sub-carrier centred at a frequency that is an integer multiple of $W/2N$ and occupying a bandwidth W/N Hz. The spectrum and the OFDM waveform are as shown in the figs:2.1 and 2.2. Certain sets of functions such as sine, cosine(separated by integer multiples of frequencies), sinc, etc possess a special property called 'Orthogonality' according to which their auto-correlation yields a maximum and the cross-correlation between distinct functions is minimum. This property of sinusoids aids in realizing an OFDM system. The frequency division in OFDM is made so that contribution from the spectra of the adjacent sub-channels becomes zero and the contribution from the required sub-channel is maximum ensuring the adjacent sub-carriers to be orthogonal.

The restriction on the separation between center frequencies in OFDM allows orthogonality between adjacent sub-bands facilitating easy recovery of transmitted symbols

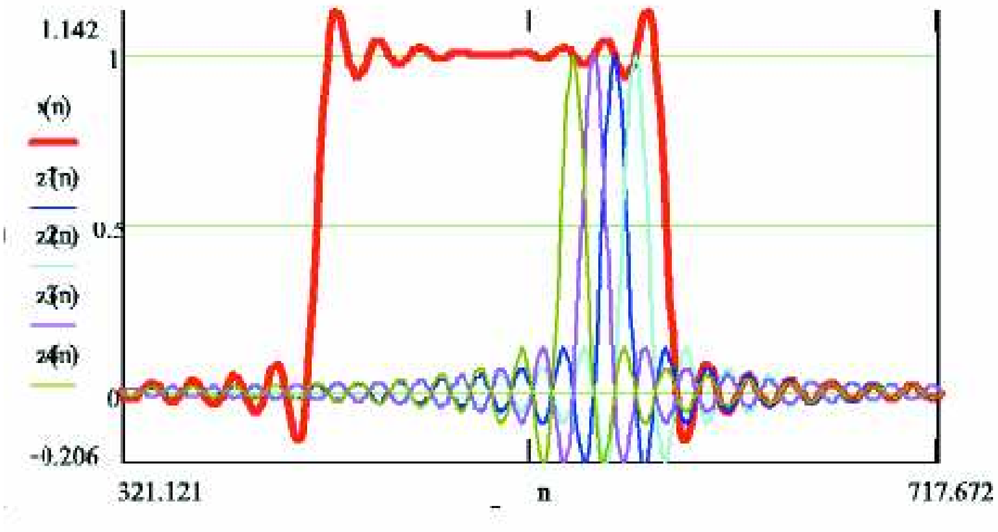

Figure 2.1. Spectrum of OFDM.

of a particular band though there is an overlap. OFDM modulates a serial data stream onto a set of parallel sub-carriers of sufficiently longer durations, keeping the data rate per a sub-carrier low. Thus, even in systems with higher data rates the symbol time is made larger than the delay spread in a wireless channel. For example in a typical 802.11(a) architecture which uses 48 sub-carriers for data, assuming the data rate at the binary source to be $24Mbps$, the data rate per sub-carrier is $(24)/(48) = 500Kbps$ and the duration of each OFDM symbol formed out of these 48 sub-carriers is atleast 2 micro-secs which is greater than 900 nano-secs, the delay spread in a typical indoor wireless channel.

In a frequency selective fading channel the signal on each sub-carrier in OFDM undergoes a fading characterized by the frequency response across its sub-band, which means that OFDM divides a frequency selective fading channel into a set of flat fading sub-channels. OFDM being a wideband system has larger bandwidth than the coherence bandwidth of the channel when used for transmission on wireless channels. Since, sub-carriers separated by more than the coherence bandwidth experience independent fading, there is an inherent frequency diversity which can be exploited further to improve the performance of the system in frequency selective fading channels.

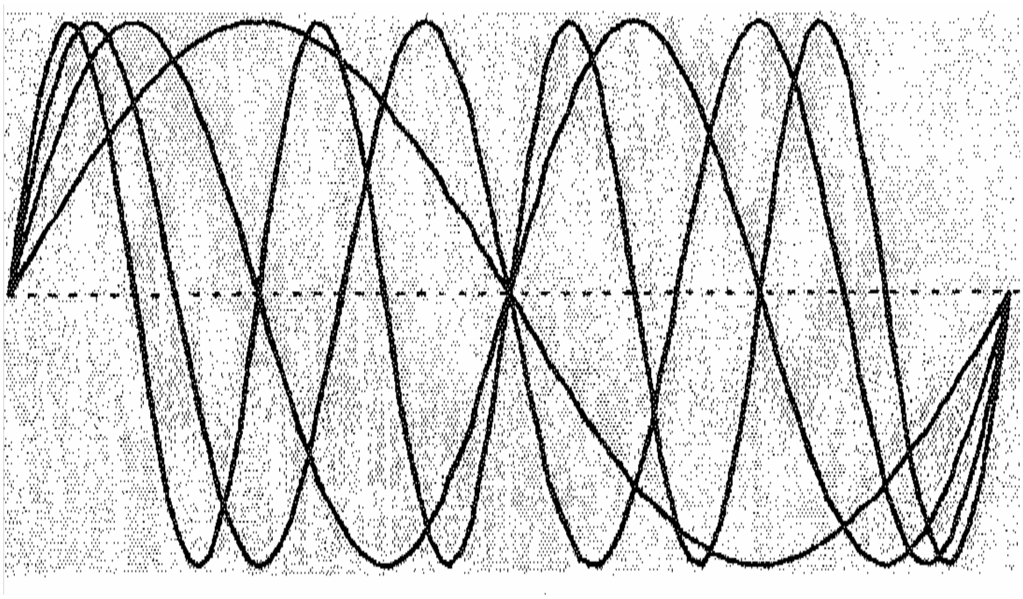

Figure 2.2. OFDM Transmitted signal.

2.2 Why OFDM ?

Of all the existing alternatives for mitigation of ISI in frequency selective fading channels OFDM is preferred for the following reasons:

- As discussed before, OFDM provides higher spectral efficiency by allowing overlap between sub-bands. This helps the designer to lift any stringent restrictions on guard bands in filter design.
- Modulation and demodulation in OFDM can be implemented using IFFT and FFT which are faster algorithms to implement.
- OFDM can be made completely immune to inter symbol interference by choosing required guard intervals and larger symbol lengths essentially greater than the delay spreads.
- Inter channel interference which is due to the loss of orthogonality caused by Doppler spreads can be adjusted as desired by a cyclic prefix and intentionally providing sufficient number of nulls with in an OFDM symbol.
- By dividing a frequency selective fading channel into a group of flat fading sub channels, OFDM utilizes frequency diversity among different sub-channels separated by coherence bandwidth which improves the performance of the system.

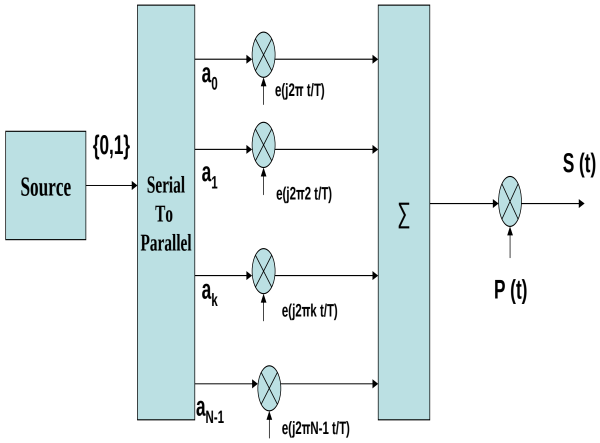

Figure 2.3. Analog OFDM Transmitter:Orthogonal tones modulate N symbols.

- OFDM offers higher data rates per user due to its parallel architecture at the transmitter and receiver mitigating ISI, an impending factor in systems with multi-path channels.
- OFDM simplifies the length and structure of the equalizer by transforming the symbols to frequency domain. For a channel with several taps, equalization is difficult in time domain due to large number of computations necessary for its implementation where as a single-tap frequency domain equalizer is used in OFDM systems.

2.3 Analog Model

2.3.1 System Description

Typical OFDM transmitter is as shown in the fig: 2.3. A binary source clocks out symbols every T_s secs. N or more than N bits are mapped onto complex M-QAM/M-PSK symbols, where 'N' is the number of sub-carriers used for transmission. The complex symbols are converted from serial to parallel and modulated onto N orthogonal sub-carriers. The modulators and demodulators are assumed to be analog and the N sub-carriers multiplexed are each over a duration of NT_s seconds. A cyclic prefix is added to the block of N sub-carriers. Such a block from now on will be referred to as an OFDM

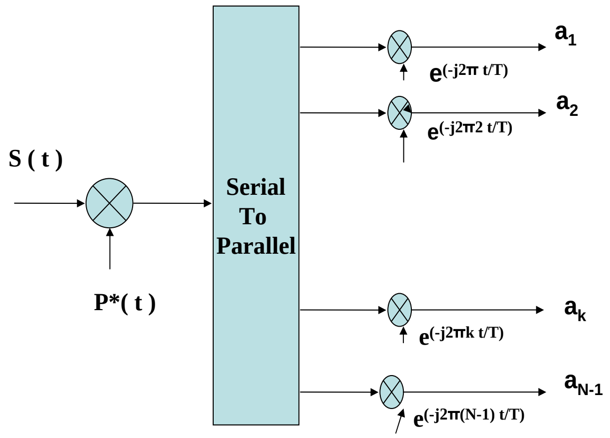

Figure 2.4. Analog OFDM Receiver:Received signal demodulated by N tones.

symbol. Each OFDM symbol is mixed with a desired high frequency R.F carrier and aired over a channel which is typically frequency selective fading. The output of the channel is the input signal convolved with impulse response of the channel. The OFDM receiver is as shown in the figure 2.4. At the receiver, the signal is down converted, matched, cyclic prefix removed and demodulated by the N sub-carriers. The N complex symbols obtained at the output of the demodulator are converted from serial to parallel and mapped back onto binary symbols. To obtain better performance gains, OFDM is implemented with forward error correction and interleaving (COFDM) protecting the data symbols from burst errors and exploiting frequency diversity. Adaptive modulation can also be implemented to optimize the transmitter power in the system. The continuous time modulator, demodulators can be replaced by their discrete equivalents which simplify the signal processing part. Though the analog architecture of OFDM was replaced by a more simpler discrete one, the analog model offers a more realistic insight into the OFDM principles.

2.3.2 Mathematical model

A mathematical model for the above system provides more freedom in further analysis and is as follows. The binary symbols emitted by the source every T_s seconds are mapped onto a complex symbols $\{a_k\}$. Each symbol a_k modulates a sinusoid whose frequency is an integer multiple of a fundamental tone to form $a_k e^{j\frac{2\pi kt}{T_o}}$ where $'T_o'$ represents the duration of a block of N modulated symbols. The output of the OFDM modulator for every T_o seconds is

$$y(t) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi kt}{T_o}} \tag{2.1}$$

The output of the modulator is shaped by filter $p(t)$ every T_o seconds and is up-converted by mixing the signal with a high frequency R.F carrier. Since, $T_o = NT_s$ one OFDM symbol containing N modulated symbols is clocked out every T seconds. A cyclic prefix is added to the block before transmission and is ignored in the model as it is removed right after a symbol is received. Therefore the output of the OFDM transmitter in the baseband is

$$s(t - nT_o) = \sum_{n=-\infty}^{\infty} p(t - nT_o)y(t - nT_o) \tag{2.2}$$

Including the time period of the cyclic prefix and the R.F carrier wave in the equation, the transmitted sub-carrier in the pass-band is obtained as

$$\begin{aligned} G_{RF,k}(t - kT_o) = Re\{p(t - kT_o) \sum_{m=0}^{N-1} a_m e^{j2\pi(f_c - \frac{k}{T_{FFT}})t - kT_o}\} \\ T_{s1} < T_o < T_{s2} \\ T_{s1} = kT_o - T_{window} - T_{cyclic} \\ T_{s2} = kT_o + T_{FFT} + T_{window} \end{aligned} \tag{2.3}$$

where:

T_o OFDM symbol duration i.e time between consecutive OFDM symbols

T_{fft} FFT or IFFT time window

T_{Win} Duration of the windowed prefix or post fix for pulse shaping

T_{cyc} Duration of the cyclic prefix

m Index of the sub-carrier

$P(t)$ Any pulse shaping filter,typically raised cosine filter

The output of the OFDM transmitter considering the OFDM symbols are transmitted every T_o secs is

$$S_{RF}(t) = \sum_{-\infty}^{\infty} G_{RF,k}(t - kT_0) \tag{2.4}$$

The above system can be represented by its discrete equivalent, by sampling the modulated wave every $T_s$seconds. Replacing t with $mT_s$and substituting $T_{FFT} = NT_s$ seconds, N samples of each OFDM symbol are obtained. The discrete model is explained further in detail later. The next section provides an overview of the general procedures and principles used in an OFDM system.

2.3.3 OFDM general Procedures

2.3.3.1 Serial to Parallel conversion

In an OFDM system, the source data emitted at a rate of R bits every T secs serially is converted into a group of symbols in parallel before modulation, every NT secs. The serial to parallel conversion involves storing complex symbols in a buffer which is flushed periodically every NT secs, and the modulated OFDM symbols are transmitted on N sub-carriers. A similar parallel to serial conversion also takes place at the OFDM receiver. This enables detection of each symbol independently at the receiver and eliminates the need for complex vector detection techniques.

2.3.3.2 Channel estimation

In practical OFDM systems the channel taps have to be properly estimated and tracked to ensure better performance. The probability of error depends on the error in the measurement of channel taps for coherent detection[3]. In pilot-based estimation[17]schemes, the receiver compares the estimate of a received sequence to a known sequence and computes the channel gains. The known sequence is called a pilot. The performance of such schemes depends on the coherence time of the channel. If the coherence time of the channel is too low, the frequency of the estimation of the taps increases causing the transmitter to send the known sequences more often than data, hampering data transfer. Decision feedback estimation is another technique in which the channel taps are updated based on previously detected symbols, but computing the channel taps for fast fading channel still remains a problem.

In [18] two different pilot based estimation techniques were analyzed. One technique uses a set of pilot symbols spaced uniformly in an OFDM symbol. The channel taps obtained after comparison are interpolated to obtain the complete channel response. Uniform distribution of pilot carriers is proved to be the optimum estimation technique. Another method is to use one full OFDM symbol to estimate the channel taps. The channel gains are assumed to remain constant for the next few OFDM symbols in such cases. This estimation is efficient only in the case of slow fading channel.

2.3.3.3 Equalization

Equalizers are used to nullify any degrading effects of channel on the transmitted signal, such as ISI. Since the channel is a multi-tap filter, the length of the equalizer has to be approximately the same as the length of the impulse response of the channel if the equalization is performed in time domain. Such equalizers are complex and expensive to implement if the impulse response of the channel is lengthy and higher accuracy of the taps is a requirement. OFDM over comes this problem by transforming problem of equalization to frequency domain. Simple single tap equalizers can be then used to recover the symbols.

Since the time domain output of a multi-tap channel is the convolution sum of the impulse response of the channel and the input, a fourier transform of the output would lead to a product of frequency response of the channel and the input leading to a simplified equalization of each tap of the channel. Two types of equalizers are typically used in practical OFDM systems. Zero Forcing, Minimum Mean Square Error equalizer(MMSE).

A Zero Forcing Equalizer(ZFE) divides the received signal by the estimated channel taps in frequency domain to recover the transmitted signal as shown here.

$$Y[k] = H[k]X[k] + W[k] \Rightarrow \frac{Y[k]}{H[k]} = X[k] + \frac{W[k]}{H[k]} \tag{2.5}$$

As seen from the above, there is an enhancement in the noise power when the channel is in deep fade. Though the noise power is enhanced, the signal to noise ratio remains the same owing to a corresponding increase in signal power after equalization. However in the presence of spectral nulls, $H[n]$ can be too low causing an irreducible error floor in the system. Since the channel taps have to be estimated for equalization, correlation between the data symbols and the pilots used to estimate the channel taps aids in designing efficient equalizers[7]. If the pilots are decorrelated w.r.t data, there is considerable degradation in the performance of the system.

For a flat fading case in OFDM, in the presence of a co-channel interferer, considerable degradation in the system performance[7]occurs due to the enhancement of the power of interferer with a ZFE. An optimum equalization technique reducing the effects of interferer induced noise was proposed in [7]. Some error bounds on the performance of the system were obtained in[16].

A MMSE provides improved BER performance compared to ZFE at the expense of complexity. The estimated channel taps are used to compute the weights of the equalizer designed based on the minimizing the mean square error between the received and transmitted symbols.

Let Y be the received signal and X be the transmitted signal, $H[n]$ the estimated channel response and $W[n]$is white noise with $N(0, N_o)$. The best linear estimate of X can be obtained by calculating the best $'a'$ that minimizes mean square error between X and aY.

$$min\|X - aY\|^2 \tag{2.6}$$

The best a can be calculated as[3]

$$a = \frac{H[n]^*}{\|H[n]\|^2 + \frac{N_0}{\sigma_x{}^2}} \tag{2.7}$$

$'\sigma_x'$ is the variance of the transmitted signal. All the equalizers mentioned above fall into a class that need channel estimates. The accuracy with which these estimates are calculated governs the performance of these schemes. The bit-error probability(BEP) performance of different equalizers with linear pilot based channel estimates was analyzed in[11] and estimation using maximum likelihood decoding in [19]. A general systematic approach for calculation of BEP using such equalizers has been proposed in [12] and the optimum linear channel estimates,(perfect channel knowledge) that minimize the BEP have also been analyzed. The procedure involves calculating a weighting vector for a defined channel estimation technique that is being used, and calculating some parameters associated with the derived expressions for BEP based on these estimates.

Some more general class of equalizers are feedback equalizers[13] which depend on the previously detected samples and Blind equalizers which do not need any channel state information. In OFDM systems, typically the MMSE is a preferred technique as it is considerably simple to implement and performs well for deep sub channel fades.

2.3.3.4 Receiver

For an OFDM system on AWGN and fading channels, optimum receiver principles are used for detection. Optimum receivers are designed in such a way that the probability of symbol error is minimized and consist of a demodulator and a detector. The demodulator converts the received waveform into a vector and the detector makes a decision as to which vector was transmitted. Demodulators use signal correlation or matched filtering to demodulate the received waveform.

A correlation demodulator is based on decomposing the received waveform into a set of orthonormal functions, using suitable filters, so that the coefficient of each orthonormal function acts as a component of the received vector. Whereas, a matched filter demodulator maximizes signal to noise ratio using a filter at the receiver whose impulse response matches the impulse response of the transmit filter. Matched filter receivers depend on the energy output from each filter while a correlation demodulator exploits the properties of the received signal.

In OFDM, in a way, both the types of demodulators are used, the orthogonal basis filters in the correlation demodulator are equivalent to the orthogonal sinusoids used in demodulation, i.e, the FFT at the receiver. A matched filter demodulator is used prior to FFT, right after the down-conversion of the carrier wave. The impulse response of the matched filter has the same shape as the pulse shaping filter used at the transmitter. The output of the matched filter is thus a linear combination of orthogonal sinusoids, which are later sampled and sent to the FFT.

2.3.3.5 Detection

Each symbol received is detected using any optimal detection techniques such as Maximum a-posteriori probability detection(MAP), Maximum likelihood decoding etc. A MAP decoder minimizes error probability by selecting the detector output that maximizes $p(m|r)$ where m is the message symbol r is the received vector and p is the probability. In other words

$$p(s_{i,sent}|r) > p(s_{j,sent}|r) \quad for \quad all \quad j \neq i. \tag{2.8}$$

A set of decision regions are defined for each received vector r such that the condition above is satisfied. If a received vector falls into one of these regions, the transmitted symbol corresponding to the region is assumed to be transmitted at the detector. In case of additive white Gaussian noise, the MAP decoding reduces to finding a transmitted data symbol that minimizes the Euclidean distance metric between the received and transmitted symbols

$$i.e \quad M(x_k, y_k) = |y_k - h_k x_k|^2 \tag{2.9}$$

2.3.3.6 Coding and Interleaving

An OFDM system employs conventional forward error correction codes and interleaving for protection against burst errors caused by deep fades in the channel. A code is an invertible mapping from a k-dimensional binary vector space to an n-dimensional binary vector space, whereas a digital modulation is a mapping from n-dimensional binary to n-dimensional complex space. This increases the minimum distance between the symbols improving the performance of the system. Earlier Shannon showed that the capacity of a channel can be achieved by using codes of infinite lengths which are not feasible in real world systems. Codes with finite length which nearly achieve the Shannon capacity also exist but are not being implemented due to the lack of fast decoding algorithms.

Convolution coding is often used for error protection often due to its burst error correction capabilities and simplicity in the decoding. Often it is concatenated with a block code for improving the performance. A Viterbi decoder for decoding a convolution code is easy to implement. Though coding improves the performance of the system,it decreases the spectral efficiency. Codes of different rates are used in conjunction with different modulation schemes to support different QOS.

While error correction codes provide coding gain in the system, interleaving provides diversity gain. For OFDM, when a deep fade occurs in the channel the bits within the deep fade are erased. Interleaving the bits across different frequency bins distributes the energy within a symbol among different sub-carriers. Since distinct sub-carriers undergo different fading conditions, the probability that all the bits corresponding to a symbol are lost,decreases significantly.

An uncoded OFDM system cannot exploit frequency diversity. Since in frequency selective fading channels, the sub-carriers separated by coherence bandwidth are independent of each other, the frequency diversity can be realized by spreading coded bits over different sub-carriers in such a way that adjacent bits are separated by coherence bandwidth. Coding and interleaving, diversity, equalization techniques decrease the irreducible error floors caused due to delay spread [20]. As will be discussed later, linear constellation complex field codes also provide diversity in the system with out coding and interleaving, but increase the complexity of the system as they are based on maximum likelihood decoding of a symbols on a group of sub-carriers.

2.3.3.7 Cyclic Prefix

The cyclic prefix in OFDM systems mitigates inter block interference due to last L data symbols in an OFDM symbol and inter channel interference from symbols in adjacent carriers within an OFDM block. If the channel impulse response is of length L, then the last L+1 symbols in an OFDM symbol are attached as prefix to the transmitted OFDM block. This is called the cyclic prefix and it serves two distinct purposes.

2.3.3.8 Reduction of Inter-block interference

Consider a multi-path channel whose impulse response is of length L, the output of the channel when a sequence of N symbols are transmitted through it is the convolution of the impulse response of the channel of length L with the sequence of N transmitted symbols. In the output, the last L-1 samples of the output overlap on the first L symbols of the next OFDM symbol causing significant interference. This is overcome by attaching a prefix of length L symbols before each OFDM symbol as a guard interval to account for the overlap and ignoring the the first L symbols of each block at the receiver as these are already corrupt by interference. Fig:2.5 depicts the inter block interference caused by a channel of length L symbols.

2.3.3.9 Reduction of Inter-channel Interference

Since a guard interval is being added already as a prefix, it can be used to make the OFDM symbol periodic over the transmission period. This can be done adding the last L symbols of the current OFDM block as a prefix to OFDM symbol. The OFDM block obtained is thus pseudo-periodic, if not periodic. Assuming the channel impulse

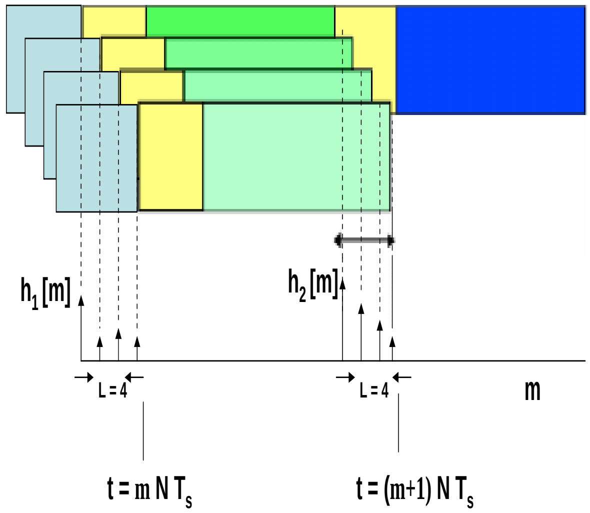

Figure 2.5. Left OFDM symbol overlaps on the right symbol in $m \leq L$ samples .

response of length L remains time invariant over the duration of the single OFDM block, the response of the channel is the convolution of the transmitted periodic sequence with that of the channel ;equivalent to the circular convolution of the two discrete periodic sequences of period N. In frequency domain this is equal to the product of the individual frequency responses of the channel and the transmitted sequence eliminating any inter-carrier interference.

The above assumption on the periodicity is valid only if the channel remains static over large durations which is practically not the case. When considerable Doppler spreads exist, there is always sufficient ICI in the system to cause a degradation in the performance of the system.

2.3.4 The 802.11 (a) architecture

The 802.11(a) architecture supports several coding rates enough to provide flexibility in data rates. The system parameters followed in this thesis are based on the IEEE 802.11 (a) WLAN standard. The architecture was designed for indoor wireless communications and where the delay spreads typically vary from $800 - 900ns$. A typical $802.11(a)$ architecture supports data rates between 54 Mbps to 6 Mbps with different qualities of service in the 5 GHz unlicensed spectrum and is based on OFDM. A 20 MHz wide band is divided into 64 sub carriers out of which 48 are used for data, 12 are zeroed out intentionally to reduce inter carrier interference and 4 are used as pilots for channel estimation. The length of the prefix is set to be 16*symbols* which has a higher duration than typical delay spreads in an indoor channel. The length of a single OFDM symbol is thus $80T_s$. The receiver provides a feedback to the transmitter about the packet error rate, and a modulation scheme is chosen based on the feedback for all sub carriers in a OFDM symbol at a time. Convolution coding is the FEC technique used with one of three possible coding rates $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ and the modulation scheme that can be used on the sub-channels are BPSK,QPSK, 16-QAM, or 64-QAM. The table below lists the parameters of the simulated model:

Table 2.1. System Parameters for simulations

System Parameters	values
System Bandwidth	20MHz
Modulation	2 or 4-PSK
Data Payload rate	6-12 Mbp/sec
No of data Sub-carriers	48
No of pilots	4
Sub-carrier frequency spacing	312.5 Khz
IFFT/FFT size	64
OFDM block length	320 μ secs
Preamble duration	16 symbols
Symbols per packet	80
Coding scheme	1/2,Convol
Antennas	2

2.3.4.1 Pre-Coded OFDM systems

A frequency diversity scheme offers multi-path diversity across the taps of the channel. The maximum diversity obtained is equal to the number of taps L. Frequency diversity in OFDM is obtained if the adjacent bits are encoded in sub-carriers separated by coherence bandwidth. Diversity in OFDM can be obtained in two ways one of which is already discussed in the section on coding and interleaving.

Another method of realizing diversity is to divide an OFDM symbol into K groups such that adjacent sub-carriers fall into different groups and pre-code each group using distinct linear constellation codes [10]. This procedure offers a diversity of channel tap length L, if $K > L + 1$ and the determinant of the coding difference matrix can be maximized. The disadvantage with this method is that the length of each group which determines the diversity gain of the system needs to be varied with the number of independent taps L. The number of taps are dependent on the channel conditions and hence not a fixed parameter. Maximum likelihood decoding is used to detect each coded block. The decoding complexity increases exponentially with the increase in the size of block and hence is a limiting factor on the diversity gain. Hence a trade-off exists between the length of the block and the diversity gain in the system.

The analysis of most digital communication systems is simplified when the analog models are discretized. In this section, the analog model for OFDM system has been shown to reduce to the discrete model.

2.4 Discrete System Model

In the actual implementation of OFDM system, the OFDM modulators and demodulators are realized using digital signal processors which reduces the complexity of the system by a fair extent. Degradation in the performance of the system due to the presence of mixers and local oscillators in the analog counter parts is eliminated. The discrete model of the system is as in fig:2.6 Consider the output of the analog modulator given by:

$$s_n(t) = \sum_n p(t - nT_o)y(t - nT_o) \tag{2.10}$$

$$y(t) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi kt}{T_o}} \tag{2.11}$$

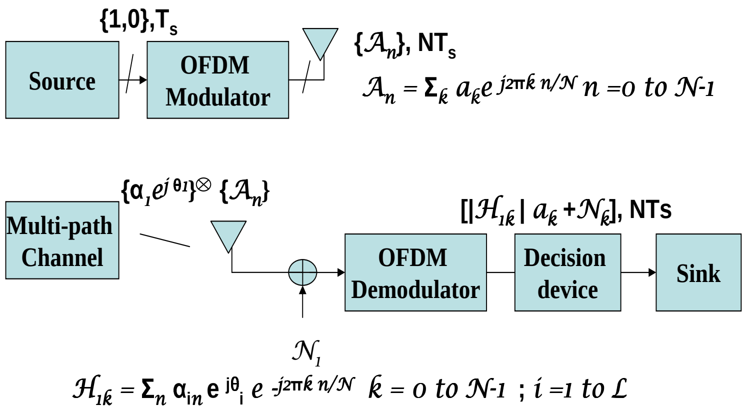

Figure 2.6. The Discrete architecture of an OFDM system .

where

$$p(t) = A \quad rect(t/T_o) \tag{2.12}$$

The sinusoids used for modulation are with a duration NT_s, can be sampled every T_s seconds and the output of the modulator is a sequence of N samples. Sampling $y(t)$ at the rate of $1/T_s$ samples, the equation becomes

$$y(mT_s) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi k(mT_S)}{T_o}} \quad 0 \leq m \leq N-1 \tag{2.13}$$

replacing T_o with NT_s

$$y[m] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi km}{N}} \tag{2.14}$$

The discrete signal at the output of the modulator, $y[m]$ is a sequence of N symbols which are later converted from digital to analog. A cyclic prefix atleast of length equal to the number of taps is added and the resulting waveform is up-converted and transmitted over the air. Thus a block of atleast $N+L$ symbols are are transmitted at a time. For the sake of simplicity, the D/A and up-conversion are ignored in this model. The channel model considered in the analysis is frequency selective fading whose discrete impulse response is $h[l]$ and the output of the channel is

$$z[m] = \sum_{l=L+1}^{N+L-1} h_m(l)_N y(m-l)_N + w[m]. \tag{2.15}$$

Though the channel can be time-variant causing significant Doppler spreads, it is assumed to be quasi-static for few OFDM symbols and the output of the channel is a convolution sum of the sequence of the transmitted symbols with that of the channel. At the receiver, the cyclic prefix is removed prior to the matched filtering. At this point the received signal is equivalent to the circular convolution of the zero padded impulse response of the channel with the input sequence of length N. After matched filtering, down, A/D conversions, the output of A/D converter a discrete sequence, is demodulated by taking the FFT, AWGN is added to account for the noise at the receiver front end.

$$A[k] = \sum_{m=0}^{N-1} z[m]e^{-j\frac{2\pi km}{N}} + w[m] \qquad where \quad 0 \leq k \leq N-1; \tag{2.16}$$

$A[k]$ obtained after taking the FFT is compensated for the magnitude, phase effects of the channel by dividing the demodulated symbols with the channel taps assumed to be perfectly known at the receiver. From the properties of the Fourier transform, convolution in time domain is equivalent to the product of the fourier transforms of the sequences given by,

$$A[k] = H[k]Y[k] + W[k] \qquad where \quad 0 \leq k \leq N-1. \tag{2.17}$$

where $H[k]$ is the Fourier transform of the channel impulse response, $h[m]$ and $Y[k]$ is the FFT of the output of the modulator,$y[m]$. Since $y[m]$ itself is an IFFT of the complex data symbols a_k,

$$Y[k] = \sum_{m=0}^{N-1} y[m]e^{-j\frac{2\pi km}{N}} \qquad where \quad 0 \leq k \leq N-1 \tag{2.18}$$

$$y[m] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi k(m)}{N}} \quad where \quad 0 \leq k \leq N-1. \tag{2.19}$$

Substituting in the above equation, the output of the demodulator is obtained as

$$Y[k] = H[k]a_k + W[k]. \tag{2.20}$$

After zero forcing equalization assuming perfect channel knowledge, the output of the demodulator is

$$\frac{Y[k]}{H[k]} = a_k + \frac{W[k]}{H[k]} \tag{2.21}$$

The variance of the noise is scaled by $\|H[k]\|^2$. The above model can be used to analyze the performance of the system. The assumptions made on the OFDM system in the above model are listed here.

- Coherent detection is assumed, i.e the receiver reconstructs the carrier with perfect phase and frequency.
- Ideal synchronization is assumed between the timing circuits of the transmitter and receiver.
- Perfect channel knowledge is assumed at the receiver.
- No co-channel interferer exists on a given sub-carrier and noise is uncorrelated and independent of channel fading.
- The taps in the multi-path channel are assumed to be independent and quasi-static.

Coherent detection is practically impossible to realize and receivers are often designed with circuitry that lock with the phase of the carrier additionally transmitted. However in this case also a random phase error may occur due to phase imbalances. In such cases optimum detectors are designed to minimize the probability of phase error, assuming a probability distribution for the random phase such as a Tikhonov distribution. Alternatively, pilot channel estimation techniques provide an estimate of the phase on each sub-carrier.

2.4.1 Performance Analysis: Analytical

Analytical methods in the performance analysis of an OFDM system are same as that on a flat fading channel except that here a block of sub-carriers need to be detected rather than a single sub-carrier. The analysis is based on the derived discrete model. Matrix notation is convenient for analysis and is considered here. The assumptions made in the discrete model are maintained here. The system model is as shown in the fig:2.7

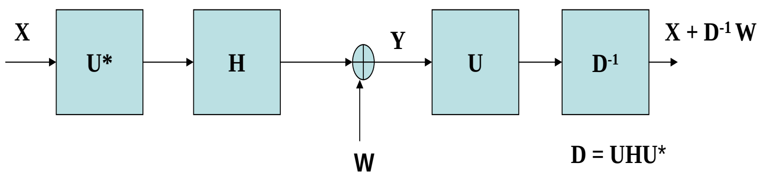

Figure 2.7. A simple OFDM system model for performance analysis.

2.4.1.1 System model

The source sends a set of complex symbols X in parallel, to the OFDM modulator. The symbols are the result of mapping a sequence of binary bits. The mapped complex symbols are modulated onto orthogonal tones, equivalent to taking an IDFT, U^* on the complex symbols. The output of the IDFT is fed to the Rayleigh frequency selective fading channel with L taps. The remaining taps of the channel are assumed to be zeros and the channel matrix,H has each column to be a cyclic shifted version of the impulse response of the channel. Noise, $W(0, N_o)$ is added to account for random voltages at the receiver front-end. The received signal is demodulated by taking the DFT of the received signal.

U^* : $N \times N$ IFFT matrix whose $(m,k)^{th}$ entry is given by $\frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi k(m)}{N}}$.

$\vec{H}$: $N \times N$ circulant channel matrix obtained after padding zeroes to L channel taps.

U : $N \times N$ FFT matrix whose $(m,k)^{th}$ entry is given by $\frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k e^{j\frac{-2\pi k(m)}{N}}$.

X : $N \times 1$ transmitted vector containing modulated symbols in frequency domain.

$\vec{X}$: $N \times 1$ vector in time domain after IFFT.

$\vec{W}$: $N \times 1$ noise vector in time domain.

$\vec{Y}$: $N \times 1$ received output vector in frequency domain.

Assume the channel has 'L' taps which are Rayleigh faded, the output at the receiver before taking the FFT is

$$\vec{Y} = \vec{H}U^*X + W \tag{2.22}$$

Taking FFT on both sides,

$$U\vec{Y} = U\vec{H}U^*X + UW \tag{2.23}$$

U and U^* being orthogonal matrices diagonalize $\vec{H}$, the output of the receiver detected on a per symbol basis. A parallel to serial conversion is made on the parallel stream to obtain

$$Y = HX + W \Rightarrow y_n = h_n x_n + w_n \quad \forall \quad n = 0 \quad to \quad N - 1. \tag{2.24}$$

After demodulating the signal,the channel effects are removed by a zero forcing equalizer and the output after equalization is,

$$\frac{y_n}{h_n} = x_n + \frac{w_n}{h_n} \tag{2.25}$$

The variance of the noise now is $No/\|h\|^2$ after equalization, however the SNR remains unaffected. The probability of error for frequency selective fading channels can thus be calculated from a group of independent flat fading channels. Since there are a group of N sub-channels, the probability of error is dominated by the P_e of the sub-carrier in deepest fade. Assuming h is such a sub channel, if the mapped symbols are BPSK modulated, the conditional symbol error probability(CSEP) can be calculated as

$$P(e/h) = Q(\sqrt{2}\|h\|^2 SNR). \tag{2.26}$$

$SNR = \frac{E_s}{N_o}$. The average symbol error (SEP) is obtained by averaging the (CSEP) over the distribution of the fading channel, which is assumed to be Rayleigh. The probability of symbol error for imperfect channel estimation on a QPSK modulated system obtained in [11]. An outline of this procedure is given here.

With an imperfect channel phase estimation, a phase error can occur in the equalized symbols. Let α be the estimated channel gain and θ be the estimated phase of the signal and ϕ be the actual phase of the channel, the output of the equalizer y_e is

$$y_e = \frac{\|h_n\| e^{j\phi-\theta} x_n + w_n}{\|\alpha\|} \tag{2.27}$$

Let the power of the modulated symbol to be ρ^2, the modulation to be QPSK, assuming $\o = \phi - \Theta$ a random variable uniformly distributed between $0 \quad to \quad 2\pi$,

$$y_e = \frac{\rho \|h_n\| e^{j\o-\pi/4} + w_n}{\|\alpha\|} \tag{2.28}$$

Splitting the received signal into its real and imaginary parts, the conditional error probability that a symbol is in error is detected correctly is given by

$$P(c/\|\alpha\|) = P(Re\{y_e\}) > 0 \wedge P(Imag\{y_e\}) > 0 \tag{2.29}$$

The variance of the noise in each branch is ρ_N. The instantaneous probability that a symbol is correct can be obtained from

$$P_{c,inst} = Q(\frac{-\rho_x h_n(\cos\o - \sin\o)}{\rho_n}) \times Q(\frac{-\rho_x h_n(\cos\o + \sin\o)}{\rho_n}). \tag{2.30}$$

The instantaneous SEP $P_{e,inst}$ can be obtained from $(1 - P_c)(1 - P_c)$, neglecting the square terms,

$$\begin{aligned} P_{e,inst} = & Q(\frac{\rho_x h_n(\cos\o - \sin\o)}{\rho_n}) + Q(\frac{\rho_x h_n(\cos\o - \sin\o)}{\rho_n}) \\ & - Q(\frac{\rho_x h_n(\cos\o - \sin\o)}{\rho_n}) \times Q(\frac{\rho_x h_n(\cos\o + \sin\o)}{\rho_n}) \end{aligned} \tag{2.31}$$

Assuming the distribution of h_n to be Rayleigh and averaging the CEP over the channel gains one can obtain the expression for $P_{e,Avg}$as

$$\begin{aligned} P_{e,Avg} = & 1 - 0.5[\sqrt{\frac{SNR_{rec}}{2}} \int \frac{(\cos\o - \sin\o)}{1 + \frac{SNR_{rec}(1-\sin(2\o))}{2}} \times p(\o)d\o] \\ & - 0.5[\sqrt{\frac{SNR_{rec}}{2}} \int \frac{(\cos\o + \sin\o)}{1 + \frac{SNR_{rec}(1+\sin(2\o))}{2}} \times p(\o)d\o] \end{aligned} \tag{2.32}$$

The above expression originally proposed in [11], can be used to extend the analysis to any modulation scheme.

2.4.2 OFDM on flat fading channels

OFDM on flat fading behaves slightly differently from frequency selective fading. Under flat fading conditions the resultant OFDM signal after it passes through channel is the signal attenuated by a random single tap, typically Rayleigh faded. After demodulation by FFT this transforms to a convolution causing ISI in the system. However this happens, only if the channel taps vary with in one symbol time. In slow fading cases, the received signal can be corrected pre or post FFT using the channel estimates. If the channel is fast fading, pre-correction of the received signal is necessary to facilitate easy recovery of symbols. In such cases there shall be noise enhancement in the signal even after equalization and some correction is required to minimize this. An optimum limited gain correction technique was proved to be providing higher performance gains compared to a single carrier flat fading system in[7]. A discrete model for the flat fading case is as below

$$y[n] = h[n]x[n] + w[n] \tag{2.33}$$

After FFT

$$Y[k] = \sum_{l=0}^{N-1} H[k]x[k-l] + N[k] \tag{2.34}$$

If $H[k] = \delta(k)$ where $'\delta' =$ Kronecker delta, then $Y[k] = ax[n]$. Gain correction techniques are employed at the receiver to make $H[k] = \delta$. For slow flat fading, $H[k] = \alpha$, a constant and hence can be corrected. For fast flat fading channels, multi-tap and frequency domain equalizers are necessary.

2.4.3 Performance Analysis: Simulation

In this section the basic system model used to simulate an un-coded and coded OFDM system has been discussed. The channel model used models an outdoor environment. The system model discussed is implemented on MATLAB and is shown in fig:2.8

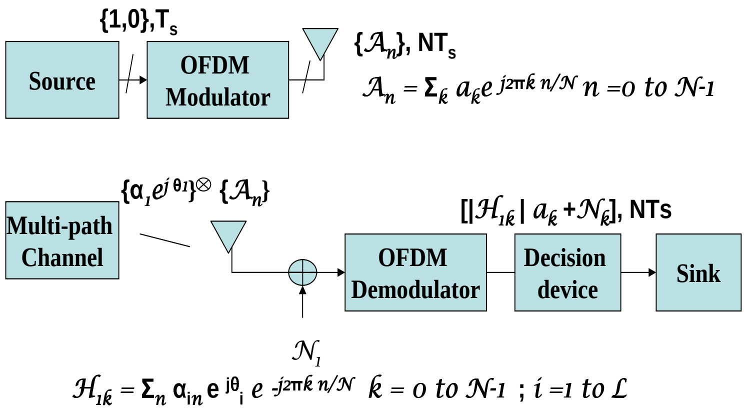

Figure 2.8. System model for performance analysis.

Differences w.r.t an 802.11(a) architecture: The 802.11 (a) supports several coding rates such as $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ offering flexibility to support different data rates and multiple qualities of service. Since the purpose of the thesis is to study the performance of OFDM with multiple antennas, the choice is restricted to a single coding rate$\frac{1}{2}$. Support for several modulation schemes is still provided. The steps in the simulation are covered here and the results are summarized in the next chapter.

2.4.4 Simulation Description

2.4.4.1 Data source

A random generator of binary data is used as a source. The binary data from the generator must be uniformly distributed. The function `randint` in MATLAB generates uniformly distributed binary bits 0 or 1. Though the output is expected to be ideally random, negligible correlation exists between the generated data sequences as the random generators used in computers are periodic. This correlation is assumed to negligible here but a more accurate simulation shall be based on an ideal random generator. Data blocks sufficient for 48 sub-carriers, 12 zero symbols and 4 pilots are sent to the channel encoder for coding.

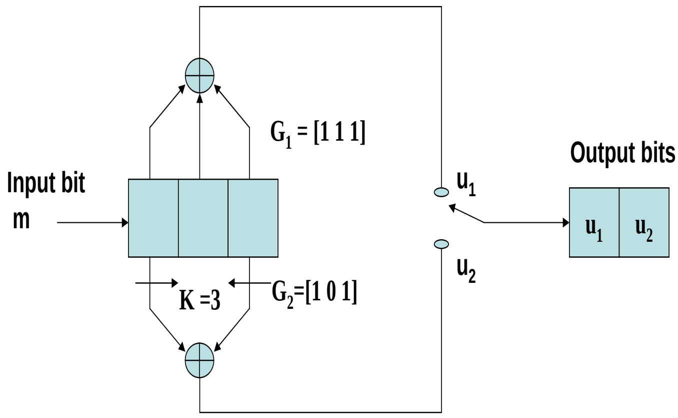

Figure 2.9. A rate $\frac{1}{2}$ convolution coder,K = 3.

2.4.4.2 Channel coding

A simple convolution coder with rate $\frac{1}{2}$ is implemented. The encoder takes 1 bit at a time and codes it using generators [111,101]. Every data bit is provided with an extra coded bit. This causes a decrease in spectral efficiency of the system by 2 and a decrease in the energy per a coded bit. The encoder is as shown in the fig:2.8 The convolutional encoder consists of three single bit registers. 2 adders are connected across these single bit registers each producing an output bit. The generators for each adder are specified by using the generator functions. In MATLAB an octal number is used to indicate the generator functions. The generators used should be in such a way that the code is non-catastrophic. The input bit is considered to occupy one of three single-bit registers. The constraint length of the code is given by number of shift registers plus one which in this case is 2 registers + 1 (input), K = 3. The generators chosen for the rate $\frac{1}{2}$ code are 5 and 7 i.e. [111], [101] which are assured to be non-catastrophic. The generating functions can be represented as

$$Y_1 = D^2 + D + 1 \tag{2.35}$$

$$Y_2 = D^2 + 1 \tag{2.36}$$

Thus the output is a combination of a previous state of the encoder with the current input bit. The parameter trace back length is kept to five times the constraint length. The initial state of the encoder determines the mapping between the input bits and the output code word.

2.4.4.3 Interleaving

The coded data bits are interleaved to ensure that the adjacent bits in a codeword after interleaving are separated sufficiently far apart. When these code words are later put on sub-carriers in the form of complex symbols, adjacent bits in the actual symbols (before interleaving) are separated by sub-carriers differing by coherence bandwidth. Since, the sub carriers separated by coherence band width experience independent fades frequency diversity can be exploited. Thus at least one of the bits can be prevented from erasure which can be detected later on.

In the simulation, the depth of interleaver is kept to be equal to 64 which is the OFDM symbol length so that the interleaving gain is maximized. The bits are arranged in a 100 by 64 matrix in the form of rows and are read out column-wise.

2.4.4.4 Complex symbol Mapping

The binary bits are mapped onto complex symbols by using standard modulation schemes such as BPSK, QPSK. The mapping here follows gray coding, i.e adjacent symbols differ by a single bit. Gray coding ensures that if there is an error in one of the bits, the symbol detected is one of the adjacent symbols of the transmitted symbol. The 802.11 (a) architecture supports several modulation schemes such as QPSK, 16-QAM, 64 QAM and utilizes them based on the required rates for transmission, in the increasing order of their spectral efficiencies. However, with the increasing spectral efficiencies we expect degradation in the performance of the system.

In MATLAB, the modulations are defined by specifying the mapping between all the possible sequences from n-bits to a complex number in 2 dimensions. A switch-case statement is written allowing the choice of different modulation schemes. Alternatively MATLAB also provides functions such as `qammod`,`pskmod` in the tool-box which require integers as inputs for symbols. Binary sequences produced can be converted to integers which in turn will be mapped onto complex numbers.

2.4.4.5 Serial to parallel conversion

The mapped symbols are converted from serial to parallel streams. As stated in section serial to parallel conversion helps in delaying each symbol by NT_s where T_s is the signaling time with out losing data rates. Here serial to parallel conversion is done by storing N data symbols produced every T_s seconds in buffer and flushing the buffer every NT_s secs. In MATLAB this is equivalent to constructing a matrix in which each column is an OFDM symbol.

2.4.4.6 Zero symbol insertion

In order to mitigate the effects of inter channel interference in the OFDM symbol due to variable fading conditions and reduce the effects of phase noise across a single OFDM symbol zeros are placed in the OFDM symbol uniformly. These zero symbols prevent interference accumulated over a group of sub carriers from affecting the adjacent channels. The loss of orthogonality between the sub-carriers can also be reduced in this way at the expense of data rate.

12 zeros are placed between sub groups of 4 sub carriers. Loss in spectral efficiency can be overcome by employing a modulation scheme with higher spectral efficiency.

2.4.4.7 Pilot symbol Insertion

The stream obtained after mapping is adjusted to carry 4 known data symbols which are also called pilot carriers. They are used to estimate the channel at the receiver. Typically the receiver compares the pilots received to the known data symbols and estimates the magnitude and phase of each channel tap. The channel response is obtained by interpolating the channel gains estimated. This pilot insertion also causes a loss in data rate. The 4 pilots are also placed uniformly in the OFDM symbol.

2.4.4.8 Inverse Fourier Transform

An Inverse Fast Fourier transform is taken on the parallel data stream to simultaneously modulate the complex symbols with orthogonal carriers and convert the symbols in frequency domain to time domain. The carriers here are discrete signals. The numbers of sub-carriers used are 64. The IFFT is a faster algorithm for Inverse Discrete Fourier Transform. Taking the IDFT involves multiplication of an OFDM symbol with a matrix whose $(n,k)^{th}$ element is $\frac{\sqrt{(N)}}{e^{j2\pi\frac{kn}{N}}}$. A 2N point IFFT is used to handle complex data samples in the real world to account for the imaginary part of the transmitted symbol.

2.4.4.9 Cyclic prefix

Just before the modulated symbol is ready to be converted to an analog waveform a cyclic prefix is added to the data stream. The cyclic prefix is formed from the last 16 symbols of the data. It is inserted before the OFDM symbol block so that the sequence is long enough (greater than the length of multi-path channel) and to make the OFDM symbol sequence periodic. As explained in the section before, a periodic sequence circularly convolved with another sequence is equivalent to the product of the Fourier transforms preventing inter channel interference.

2.4.4.10 AWGN

Receiver experiences thermal noise due to random motion of electrons at room temperatures. The power spectral density of the thermal noise at the receiver is given by $N_o Watts/Hertz$ where $N_o = kT$. This noise power distributed uniformly over bandwidth can be modeled as a noise source. The ratio of the transmitted signal power to noise power can be varied in the simulation by properly calibrating noise power. The variance of the transmitted signal be P and the noise power is given by $N_o = P \quad 10^{-SNR/10}$.

Noise power for the required SNR can be obtained by properly scaling the complex noise generated by `randn` function with the variance N_o. Typically the signal power is kept to unity. The above calibration is valid only for channels with AWGN.

In case of fading channels, the variance of the fading envelope is included in the calibration and variance can be calculated from,

$$10log_{10}\frac{E[\alpha^2]E_s}{No} = SNR \quad in \quad db \tag{2.37}$$

A similar equation to an AWGN channel can be obtained after reordering and calculating the variance of noise. If coding is employed, the SNR has to be scaled based on the coding rate. This is done in order to equate the energy transmitted per bit the same for un-coded and coded systems. If $E_b$is the energy per bit and E_c the energy per coded bit k/n the coding rate then, $kE_b = nE_c$ which yields $E_c = nE_b/k$. The BER curves are plotted w.r.t E_b/N_o. The obtained curve is in terms of E_c/N_o if no change is made in the scaling process for an un-coded system and the curve has to be shifted towards the right,upwards depending on the magnitude of coding rate to convert the scale from E_c/N_o to E_b/N_o.

2.4.4.11 Rician /Rayliegh fading channel

The channel model considered here is a 2-tap uncorrelated Rayleigh frequency selective fading and is simulated in MATLAB. The maximum Doppler spread $f_D NT_s$ is set to 0.0005 and the maximum delay spread is set to be less than the length of the cyclic prefix. Hence the effects of ICI,ISI are eliminated from the system. Two zero mean, uncorrelated real Gaussian random variables are used to simulate a complex Gaussian random variable, used to simulate a Rayleigh flat faded signal. The LOS component, i.e the modulated signal, is properly scaled and added to the Rayleigh flat faded signal component. The tap delays are set to be more than T_s, the symbol duration. Detailed modeling and simulation of the channel considered is explained in the appendix.

2.4.4.12 FFT Demodulation

The demodulation of the signal is implemented by taking the fourier transform of the received signal. The cyclic prefix is removed before any detection takes place. The fourier transform is implemented using the fast fourier transform algorithm. The size of the transform is 64 sub carriers.

2.4.4.13 Equalizer

A Zero Forcing Equalizer is considered here. The output of the FFT is divided by the channel gain H. Due to he possibility of having channel nulls, the zero forcing equalizer is not considered to be the optimum equalizer. MMSE single tap equalizer can be used in such cases.

2.4.4.14 Detector

The detection technique used in recovering the modulated symbols is minimum distance detection. In this, the Euclidean distance between all the modulated symbols and the received signal is calculated and the modulated symbol yielding minimum magnitude is assumed to be transmitted.

2.4.4.15 De-Interleaver

If the data symbols in the form of bits are interleaved across different sub-carriers, the reverse process is employed at the receiver to reconstruct the coded symbols. De-interleaver used here has a depth of 64, i.e the adjacent bits are separated by 64 bits. Increasing the depth of interleaver increases the delays in recovery of data symbols. A depth of 64 requires one OFDM symbol time to recover the full codeword for a data symbol. This delay is practically tolerable.

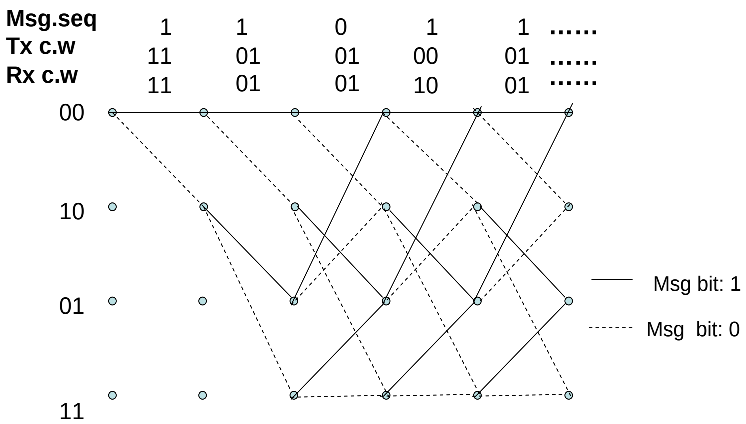

Figure 2.10. Trellis used for decoding .

2.4.4.16 Viterbi Decoder

A trellis constructed from the encoder states is used to recover the data symbols. Viterbis algorithm for maximum likelihood decoding is used to decode the coded symbols. Although soft decoding yields better performance, hard decoding is considered. The algorithm selects the sequence of bits which has a minimum hamming distance with the transmitted one. The algorithm is optimized in complexity and performance gains. The trellis for the decoder is as shown in the fig:2.9

The Viterbi algorithm for hard decoding is explained as follows:

- Consider an initial state t = m, all the partial metrics for the single path entering each state from that state are calculated. Store the path and its metric for each state.
- Move to the next state,i.e increment t by 1. The partial metric for all the 2^k paths from that state, entering the next state are calculated by adding the branch metric of the current state to the path metric calculated for the previous time unit. After comparing the metrics of all 2^k paths entering that state,the path with the largest metric(the survivor) is stored along with the metric and the remaining paths are deleted.

- Repeat step 2 until all $h + m$ states are traversed after the initial state, where h is the length of the sequence to be coded and m is the number of time units required for the encoder to return to its initial state.

2.4.4.17 BER estimator

The decoded data finally obtained is compared with the transmitted data and the number of errors are calculated. Data of size 6400 bits are processed into 80 OFDM symbols of length 80 each(including CP etc). Each OFDM symbol is transmitted over a Rayleigh /Rician flat and frequency selective fading channels with AWGN added. The difference between the transmitted and received sequence is estimated bit wise and the number of errors occurred for the total number of transmitted bits is estimated for different signal to noise ratios at the receiver. To obtain the BER at a given signal to noise ratio,the simulation is repeated 10^7 times for flat fading and 10^5 times for frequency selective fading and BER values are averaged. In case of frequency selective fading channels though the fading conditions change, the large scale characteristics of the channel such as the delay spread, power-delay profile are kept constant for 80 blocks.

2.4.5 OFDM: Drawbacks

2.4.5.1 Frequency offset

To maintain the orthogonality of sub carriers in OFDM the sub carriers are to be demodulated at exactly the same frequency. In a time variant channel there is significant Doppler spread in the system causing the carriers to spread in to the neighboring channels leading to a loss of orthogonality within the sub-carriers. Device imperfections can cause the non-zero sub-carriers to leak out of their bands even if the receiver estimates and compensates the Doppler spread. Thus the FFT peaks at the receiver do not line up close with the received signals leading to significant ICI to adjacent sub carriers. If the interference is from a large number of carriers it acts as additive white gaussian noise. A correction factor can reduce the ICI in the system.

2.4.5.2 LO phase offset

A phase offset also exists between the transmitter and the receiver. This occurs due to the propagation delay between transmitter and the receiver. The affect of such an offset is to cause rotation of each symbol by a certain amount. A frequency domain equalizer can be used to correct the offset if the offset is sufficiently low. If the offset is high an equalizer cannot compensate for phase error and as a result the constellation points may rotate beyond the symbol decision regions.

2.4.5.3 Carrier interference

OFDM system suffers from the single-carrier interference from other sources with in the same frequency range of interest. Interference is due to nearby circuits or other transmission sources. In order not to hamper the performance of the system the noise from such sources should be made random. By turning off the sub carriers within these frequencies, an OFDM system can avoid such interference.

2.4.5.4 Phase noise

Local oscillators at the transmitter and receiver may experience phase noise which can add to the desired signal during transmission. Phase noise typically exists around the carrier of a transmitted signal. For a narrow band signal , the effect of phase noise is not significant as it forms a smaller percentage of the total frequency band width. Since OFDM systems are wide band the the affect of phase noise is significant compared to single carrier systems. Two kinds of phase noise may exist in an OFDM system; phase noise in narrow band single sub carriers, phase noise across multiple sub carriers. The later is significant and may lead to demodulation errors while the former causes an arcing effect on the demodulated signal s. OFDM can over come the problem at the cost of data rates by using zero pilot sub carriers for reference between different sub- carriers.

2.4.5.5 Peak to Average Power Ratio

An OFDM signal has higher peak to average power ratios(PAPR) due to the symbols being a coherent combination of sub-carriers. On an addition of N sub-carriers the peak power is increased by N times w.r.t the average power. The peak power is defined as the power of a sine wave with an amplitude equal to the maximum envelope value. A large PAPR causes non linearities in amplifiers, reducing their efficiency and causing non-linear distortion. The complexity in implementation of A/D, D/A converters increases with increase in PAPR. Certain remedies such as clipping, coding and scrambling have been suggested in literature to mitigate the effects of higher PAPR.

2.5 OFDM:Deviations from Ideality

In the last few sections the system model and the general principles of OFDM system have been described and simulation steps in the performance analysis of an OFDM system on flat, frequency selective fading channels has been provided. Till now all the models described here assume zero ISI,ICI which is far from being practical. This results in slight modifications to the existing channel and received signal models. There is considerable degradation in the performance of the system leading to high irreducible error floors even at large signal to noise ratios. Both effect the performance of the system in the same manner but the cause for each of them is different.

2.5.1 Inter Symbol Interference

Although a guard interval or the cyclic prefix attached to an OFDM symbol block eliminates the effects of ISI, the technique is effective only when the maximum delay spread of the channel is less than the length of the guard interval. Since the channel is random, there may exist situations where the above condition is not satisfied. In such cases ISI is not entirely eliminated and results in degradation of the performance of the system causing high error floors. In order to mitigate the effects of fading techniques such as forward error correction, space diversity and equalization techniques can be used. A combination of diversity and equalization techniques provides significant reduction in the error floors caused otherwise. Diversity and single tap equalizers can be implemented easily and hence are an interesting area to study.

2.5.2 Inter Carrier Interference

The sub-carriers remain orthogonal to each other only when the carriers are separated by $1/T_o$ or there is no overlap between the sub-carriers. Though orthogonality is maintained between the sub-carriers at the transmitter, the relative motion between transmitter and receiver results in significant Doppler spread of each sub-carrier causing overlap between adjacent sub-carriers which is otherwise called intra-symbol interference or inter carrier interference. This has a similar effect as that of ISI leading to irreducible error floors in the performance. To mitigate these effects again, diversity, equalization techniques can be used. The next chapter discusses in detail the system models with diversity and equalization for the mitigation of ICI.

Simple techniques such as diversity reception, coding and interleaving, equalization etc are investigated in[4],[21] to improve the performance of these systems. Considering complexity and the performance gains that each alternative provides, diversity reception is expected to be the optimum one. Rigorous analysis of the performance of an OFDM scheme using DMPSK modulation for fast fading channels with different path profiles was provided in [22]. Error rate floors for both slow and fast fading channels with maximal ratio combining diversity receiver are obtained as a function of the normalized RMS delay spread, τ/T_s. These results indicate higher performance gains with 2 antennas, supporting the stand taken on diversity.

In systems where ISI is the major impending factor, though MRC provides maximum diversity gains[23], the receiver needs an accurate estimate of both magnitude and phase of each sub-channel of the OFDM system which might turn out to be complicated if more sub-channels are used. Though interpolation can be used in such cases, it can adversely affect the performance of the system. In such cases the complexity can be reduced by limiting the estimation to the phases of the channel taps only by employing equal gain combining at the receiver. The performance of OFDM systems in the absence of ISI with equal gain combining has not received much attention in the available literature.

In the next few sections, the performance analysis of various diversity combining schemes is considered. After providing an overview of diversity reception, two combining schemes yielding optimum, sub-optimum diversity gains, maximal ratio combining(MRC), equal gain combining (EGC) are introduced for flat fading channels and few analytical techniques used previously for the analysis of such schemes on flat fading channels are summarized. Later on a receiver model for a post-detection, post-FFT diversity based OFDM system is proposed and its performance is analyzed through simulations.

2.6 Introduction to Diversity

Communication systems designed for simple AWGN channels do not ensure similar performance for wireless channels due to random fluctuations in the channel. There is a significant probability that a wireless channel is in deep fade due to which the probability of symbol error decreases slowly with signal-to-noise ratio(SNR). Also, critical information about the channel, such as magnitude of the channel gains, phase are needed at the receiver to recover the transmitted signal.

Consider a typical WCS in which nothing about the propagation channel is known to the receiver. Let channel $h_o[m]$ be Rayleigh flat fading, i.e single tap whose magnitude varies according to Rayleigh distribution and phase varies uniformly. If the modulation used is assumed to be BPSK, then the received signal $Y[m]$ at the output of the demodulator for a transmitted symbol $x[m]$ can be modeled as

$$Y[m] = h_o[m]x[m] + w[m] \tag{2.38}$$

Where $x[m] = +1$ or -1 with h_o a $CN(0,1)$ and the noise at the receiver front-end $w[m]$ a $CN(0, N_o)$. Since there is no prior information about the channel taps at the receiver, the detection of $x[m]$ is impossible even with out noise because $h[m]x[m]$ has uniform phase distribution and hence can take any value. Though orthogonal symbols can be employed for non-coherent detection, it results in loss of spectral efficiency. Hence for efficient use of degrees of freedom in a channel and reliable detection of data symbols prior information on channel taps is necessary. Even when the channel taps are known at the receiver, there is considerable degradation in performance due to significant probability that the channel is in deep fade. When binary symbols are used in transmission on an AWGN channel, the symbol error probability is an exponential function of SNR where as for a Rayleigh flat fading channel, the conditional symbol error probability(CSEP) depends on the instantaneous SNR expressed as a function of the rayleigh distributed random variable. If a correlation demodulator is used, the SEP in case of AWGN and the CSEP in case of Rayleigh flat fading can be obtained in terms of the SNR as,

$$P_{b2} = Q(\sqrt{2SNR}) \tag{2.39}$$

where

$$SNR = \frac{(Average \quad received \quad signal \quad energy/cmplx \quad symbol \quad time)}{(Noise \quad energy \quad /cmplx \quad symbol \quad time)}.$$

SNR in case of Rayleigh fading channel is called the instantaneous as it is randomly varying w.r.t the symbol time. Averaging over a Rayleigh distributed fading channel, the CSEP is obtained as,[5]

$$P_b = \frac{K}{1 + SNR} \tag{2.40}$$

where K is any real constant. In the above equation the probability of error decreases exponentially with SNR where as for the fading channel it decreases as the inverse of

SNR. BPSK modulation over an AWGN channel gains approximately 17dB compared to a Rayleigh fading channel at high SNR. The probability of a channel in deep fade is given by

$$P(||h^2||SNR) < 1 = \frac{1}{SNR} \tag{2.41}$$

For modulation schemes with higher spectral efficiencies, the probability of error decreases even slower. Thus fading is an undesirable aspect in WCS. To cause a faster decay of probability of error with SNR, diversity techniques are adopted. Diversity enables independent replicas of a transmitted symbol to be available at the receiver so that the receiver can make a choice between these replicas as to what is the transmitted symbol.

Diversity techniques spread the information over the available-time frequency dimensions which helps in reconstructing the signal, if a deep fade occurs across a single unit of time or frequency bin resulting in the loss of information. Diversity thus allows efficient use of one or more available degrees of freedom. Based on the nature of the channel available, diversity can be realized across three degrees of freedom for a channel; time, frequency, space. There is also polarization diversity which is ignored in the present discussion.

2.6.1 Time diversity

Time diversity is obtained when the coherence time of a wireless channel is too low, i.e the taps in the channel vary rapidly. Due to this, a symbol when spread over multiple symbol intervals, can be easily detected though a deep fade occurs for a shorter time interval. Coding and interleaving are used to achieve time diversity over a slow fading channel. Coding ensures protection against noise and burst errors for shorter durations while interleaving achieves diversity gain. If coded symbols are interleaved over different coherence periods so that different parts of a code word experience independent fades, only few information symbols from a given code word are lost. A codeword undergoing

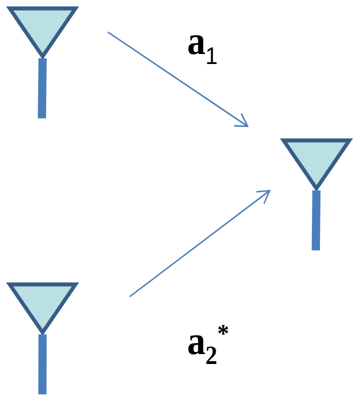

Figure 2.11. Transmit diversity:2Tx 1Rx.

independent fades has lesser probability of deep fade and hence lower probability of error. The probability of deep fade for L independent channels is proportional to $\frac{1}{SNR^L}$.

2.6.2 Space Diversity

Diversity can be induced in the channel by using multiple antennas spaced far enough, at the transmitter and the receiver. If multiple antennas are spaced sufficiently far apart, the channel gains experienced by each antenna vary independently creating multiple independent paths for each transmitted symbol. Typically antennas separated by thirty to forty carrier wavelengths ensure independency between the fades experienced on each antenna. Based on the number of antennas at the transmitter and the receiver, three different kinds of space diversity schemes exist; transmit, receive diversity and both transmit and receive diversities called MIMO.

2.6.3 Transmit Diversity

A typical transmit diversity system for L antennas is as shown in the fig:3.1. In this case the diversity is obtained by incorporating L antennas at the base station and a single antenna at the receiver. A simple case of transmit diversity is to transmit

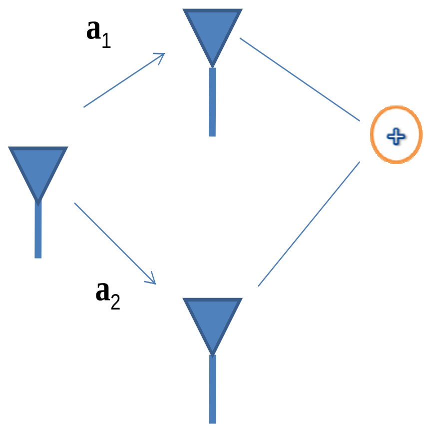

Figure 2.12. 1Tx 2Rx;Receive Diversity .

a data symbol L times each time on a different antenna. However this decreases the spectral efficiency. An efficient way of transmission is to code L symbols over L intervals of time. One such coding technique is the Alamouti's space time code which transmits two complex data symbols over two symbol intervals. The symbols are coded in a way that the pair of symbols transmitted during the first interval is orthogonal to the pair of symbols transmitted in the second interval. This converts a 2×1 channel over 2 symbol intervals into a 2×2 channel with orthogonal columns. Alamouti codes offer full diversity which is 2 and decompose the detection into two separate orthogonal scalar problems. Such codes ensure maximum diversity. Similar orthogonal codes are shown to exist for $4 \times 4, 8 \times 8$ cases etc.

2.6.4 Receive diversity

A simple receive diversity system is shown as in fig:3.2. The diversity is obtained by incorporating a single antenna at the transmitter and L antennas at the receiver. If the channel fades experienced by each antenna are uncorrelated, a diversity gain proportional to the number of antennas is obtained. Two different gains, namely diversity gain and power gain are obtained in a receive diversity system. Diversity gain is due to the presence of uncorrelated replicas of the signal at the receiver and power gain is due to coherent combining. An increment in L results in a power gain of 3 db. However with the increase in L, the channel taps for each antenna become correlated and resulting in a decrease in diversity gain. If $y_l[m]$ is the received signal on antenna $'l'$ with tap gain $h_l[m]$ and $x[m]$ be the transmitted signal, the received signal can be modelled as

$$y_l[m] = h_l[m]x[m] + w_l[m] \quad \forall l = 1 \dots L \tag{2.42}$$

where $w_l[m]$ is the additive noise at the receiver front end. The L replicas at on L antennas are combined together yield a single copy of the transmitted signal. Various combining techniques are used to obtain this single copy based on the constraints imposed by complexity, performance etc.

2.6.5 Frequency Diversity

Diversity is achieved by making the bandwidth of the transmitted signal greater than the coherence bandwidth of the channel. Since coherence bandwidth, W_c is the bandwidth over which the the frequency response of the channel remains constant, if the bandwidth of the transmitted signal is greater than W_c, the signal experiences frequency selective fading. Hence wideband signals are used for transmission. The behavior of the channel in time domain, in such cases, is modeled as a linear time varying filter with taps over finite lengths, with same frequency response as that of the channel. The delay spread, a measure of coherence bandwidth of the channel is around $1 - 10\mu$ sec in such channels. The output $y[m]$ of such a channel is the impulse response of the channel $h[l]$ convolved with the input sequence and is obtained as,

$$y[m] = \sum_l h[l]x[m-l] + w[m]. \tag{2.43}$$

where $w[m]$ is the additive white gaussian noise at the receiver front end.

2.7 Diversity Reception: An Overview

Space diversity involves the usage of multiple antennas at the transmitter or receiver to mitigate the fading effects in transmission, providing multiple copies of the signal. Uncorrelated replicas of the signal can be obtained by placing the antennas such that they are separated by atleast thirty to forty wavelengths[23] of the transmitted signal. A power gain is obtained due to the presence of multiple antennas if the replicas are correlated. In typical implementations of receive diversity, the antennas are separated by $10-15$ wavelengths resulting in a correlation of about 0.6 between the antennas. Two types of spatial diversity exist between the base station and the mobile, they are

- Macroscopic Diversity
- Microscopic Diversity

Macroscopic diversity refers to the diversity obtained when a mobile takes advantage of multiple antennas at the base station whereas microscopic diversity is obtained due to multiple replicas at the mobile receiver. The available replicas of the signal are combined or used for selection, at the receiver depending on convenience resulting in diversity gains. The presence of multiple antennas at the mobile increases the complexity of implementation and consumes higher power for the their maintenance and additional processing. The resulting performance gains from these systems can be utilized by the networks to increase the scheduling rates when needed and in turn decrease the power consumption by decreasing the time over which the mobile is in a call. In short, there are higher gains at lower costs both to the network and the user.

Depending on the number of degrees of freedom exploited at the receiver, diversity techniques can be classified into pure and hybrid combining. Pure combining exploits single degree of freedom i.e one of space, time, frequency etc. Based on the weights used for combining, pure combining can be classified into maximal ratio combining (MRC), equal gain combining(EGC) etc. Though a few other combining techniques exist, they are not considered here owing to their less than sub-optimal performance. Hybrid combining exploits diversity in more than one degree of freedom and hence is a potential candidate for improving the performance. It is further divided into two categories; generalized, multi dimensional diversity combining.

Generalized combining schemes[24][4]perform adaptive pure combining, i.e SC is used with EGC or MRC. They are designed to bridge the gap in performance between SC and MRC, EGC and enjoy reduced complexity with the performance approaching that of EGC, MRC asymptotically. SC-EGC is shown to out perform conventional post detection EGC in certain cases as it is less affected by the 'combining noise' of the very noisy low-SNR paths. Multi dimensional diversity, which is considered in this thesis, involves the combination of diversity from two or more degrees of freedom in a communication system such as time-space, space-frequency etc.

In addition to the above, yet another classification of combining techniques exists depending on where weighting and combining[4] are performed. They are; pre-detection, post-detection combining. In pre-detection combining the signals are weighted prior to detection. The received signals from the coherent demodulator (at the output of the low pass filter) are weighted and combined prior to sending them to the integrator. The weights chosen determine the type of combining considered.

In post-detection combining, multiple replicas of the signal are weighted and combined after the detection i.e post-integrator, to yield a decision variable. Pre-detection and post-detection have the same form of the decision variable and the difference between them [23] depends on the relation between SNR_{RF} and SNR_o for a given modulation scheme. Pre-detection is associated with combining of received signals in R.F and the later is associated with combining at the output of the receiver. Typically the difference between them is significant in FM systems, where the relations between SNR_{RF} and SNR_o vary with in few dB.

In the next few sections, the receiver structure for the two different diversity techniques MRC, EGC will be introduced and some techniques used in literature for the performance analysis of these schemes have been summarized.

2.8 General Receiver structure-MRC,EGC in Flat fading channels

From now on until mentioned explicitly, coherent demodulation with post-detection is assumed. The receiver is assumed to estimate the phase of the channel accurately. Practically, the carrier phase in a phase modulated signal can be obtained from an additional pilot carrier transmitted with the received signal. A phase locked loop is then used to lock with the phase of carrier, provided the phase is within its locking range. In phase modulation systems on fading channels, where the phase of the received signal includes the random phase due to the channel and the modulated signal, the channel phase is estimated by taking a non-linear transformation on the modulated signal to remove the phase effects due to modulated symbols leading to an ambiguity in the estimated phase. To eliminate such ambiguities, a pre-defined symbol sequence which acts as a phase reference for the transmitted signal or a differential modulation schemes is employed in the transmitted signal. Even if the phase of the channel is accurately estimated there can still be an additional phase error due to phase imbalances in the local oscillator causing random phase shifts in the received signal. In OFDM systems, where a set of sub-channels exist, the phase of each sub channel is estimated by interpolating the phases of the pilots incorporated with in each OFDM symbol specifically for the purpose of channel estimation. Typical receiver architecture for post detection combining is shown in the fig:3.3. The assumptions involved in the model are listed here:

- The channel taps are perfectly known at the receiver.
- Noise is zero mean, complex Gaussian with variance N_o, independent of channel taps.
- The channel is slow, flat fading and Rayleigh distributed with average power $2\sigma^2$.
- The transmitted symbols are MPSK modulated i.e have equal energies but differ in phase.
- The channel taps on each antenna are independent and identically distributed.
- Noise is uncorrelated and has equal variance on each path.

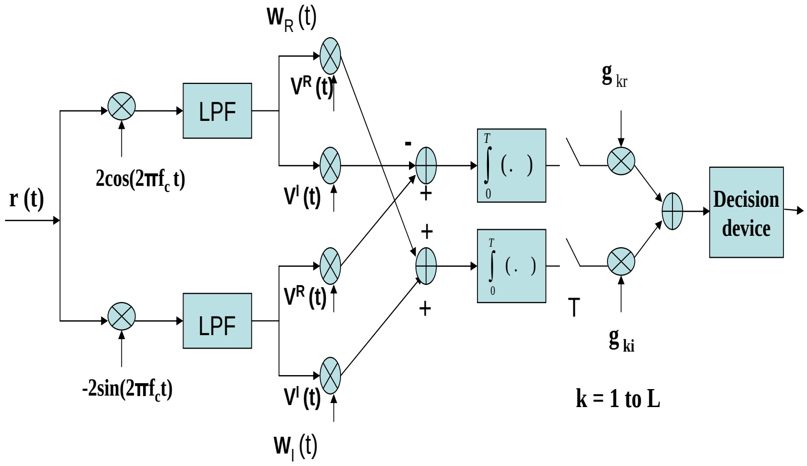

Figure 2.13. Receiver model for a Post-detection EGC system.

The receiver structure is similar to that for a single antenna on a flat fading channel, except that there are multiple replicas from different receiver branches available for combining. The received signal is down converted, low pass filtered, noise added and the inphase, quadrature components of the signal obtained are weighted by the real and imaginary components of the complex baseband message signal. The resulting signal is integrated over one symbol period and the output of the integrator is weighted by suitable complex constants which depend on the channel taps. The weighted replicas are later combined to yield the diversity signal. The weights determine the type of combining at the receiver.

On a flat fading channel with L antennas the optimum MAP receiver used for detection requires additional information about the fading characteristics of the channel i.e phase, magnitude. Let w_k be the received vector on the k^{th} antenna and v_m be the transmitted vector and g_k be the complex channel gain on the k^{th} antenna. The MAP receiver makes a decision on the received vector w_k, given the transmitted vector is v_m and the channel gain g_k, by searching for that transmitted vector from the set of all transmitted vectors V_m which maximizes $P(w_k/g_k v_m)$, where P stands for probability. In the presence of AWGN, the decision rule is equivalent to choosing a $v_i \epsilon V_m$ which minimizes the Euclidean distance between the received vector w_k and v_i [4]. Hence the receiver chooses a vector v_i that maximizes the metric[5]

$$\mu(v_i) = -\|w_k - g_k v_i\|^2 = -\|w_k\|^2 + 2Re[w_k {g_k}^* {v_i}^*] - \|g_i\|^2 {v_i}^2 \tag{2.44}$$

The $\|w_k\|^2$ term can be dropped since it is the same for all vectors and $\|v_i\|^2$, being constant for MPSK signals due to equal energies of the symbols, can be ignored. Letting $\|v_i\|^2$ to be E_i, the only factor that changes with the choice of v_i is the product term. The metric can further be written in its functional form as

$$\mu(v_i(t)) = \sum_{k=1}^{Ł} Re[{g_k}^* \int_0^T w_k(t) v_i(t)^*] \tag{2.45}$$

The vectors can be replaced by their equivalent signals and the projection of the received vector on the transmitted vector can be replaced with integration over one symbol,interval. The integration operation can be shown to satisfy the properties of an inner product. The above receiver structure is derived from this decision metric. Finally, combining the outputs of the integrators on L receive antennas after weighing them the decision metric is,

$$\mu(v_i(t)) = \sum_{k=1}^{Ł} Re[g_k^* \int_0^T w_k(t) v_i(t)^*] \tag{2.46}$$

An MRC technique has the complex weights g_k set to the conjugates of the channel gains, assumed to be ideally known to the receiver. Let the complex gains of the channel be $\alpha e^{j\Theta_c}$. Assuming the MPSK symbol has energy E_s, the received signal at the output of the combiner on each of the n antennas after weighting, is obtained as

$$W = \|\alpha_n\|^2 e^{-j(\theta_m)} \sqrt{E_s} + \|\alpha_n\| N \quad for \quad n = 1 \quad to \quad L \tag{2.47}$$

where θ_m is the MPSK symbol with the phase $\frac{m\pi}{M} \quad for \quad m = 1 \quad to \quad M$. Note that the phase of the complex channel tap has no effect on noise power and N is still the complex noise at the front end of the receiver with noise power N_o. The distribution of the fading envelope is Rayleigh with the average power $2\sigma^2$ and the instantaneous signal to noise ratio for the MRC case is,

$$\gamma_{mrc} = \frac{\sum_{k=1}^{L} \alpha^2 E_s}{\sum_{k=1}^{L} N_o} \tag{2.48}$$

where L is the number of antennas at the receiver. Since, the channel noise in uncorrelated and independent of channel gains with equal variance, the average SNR can be calculated as

$$\gamma_{mrc}{}^{avg} = L\gamma_k = L\gamma_c \tag{2.49}$$

Where γ_k is the bit energy to noise ratio per diversity branch and γ_c is the transmitted bit energy to noise ratio. The L fold increase in the signal to noise ratio indicates a power gain and the diversity gain obtained is L only if the L replicas of the channel are independent. If the channel taps are fully correlated only a power gain is obtained. Similarly, for the EGC scheme, the weights are conjugates of the phases of the channel response and the signal obtained at the receiver after egc is,

$$W = \sum_{k=1}^{L} \alpha_k + \sum_{k=1}^{L} N_o \tag{2.50}$$

The instantaneous signal to noise ratio at the output of the receiver can be obtained as,

$$\gamma_{egc} = \frac{(\sum_{k=1}^{L} \alpha_k)^2}{LN_0}. \tag{2.51}$$

The average signal to noise ratio is obtained as,

$$\gamma_{egc}{}^{avg} = \gamma_c(1 + \frac{(L-1)\pi}{4}). \tag{2.52}$$

which indicates that MRC provides higher gains than EGC. This is valid only when the noise across different branches is uncorrelated and independent of the channel fading.

2.8.1 Deviations from ideality

2.8.1.1 Effect of Correlated Noise

The noise across L branches of the diversity receiver can be correlated. If such a correlation exists, the noise power which is calculated based on the assumption of independence of the noise components across these branches, LN_0 is invalid. In such cases the basic equation for calculating the noise power should be used to evaluate the noise power and is calculated as,

$$P_N = E[\sum_{k=1}^{L} N_k]^2 = E[\sum_{k=1}^{L} {N_k}^2] + E[\sum_{k \neq j} N_i N_k] = [LN_o + L(L-1)\rho] \tag{2.53}$$

Hence the increase in noise power causes the decrement of SNR only when the correlation between the noise components is positive. A negative correlation would in effect lead to better performance in both the cases.

2.8.1.2 Effects of variable noise powers

Though it was assumed that the noise powers across different branches to be constant from the assumption of wide-sense stationarity, practically noise powers change rapidly and some times noise can itself act as a rayleigh random variable causing a degradation in the performance. Also sudden bursts of noise have an adverse effect on EGC than MRC as there is no choice for selectively weighing a path when needed as it is for an MRC case.

2.8.1.3 Correlated fading

Correlated fading causes an increment in power gain at the cost of diversity. Equal gain combining performs a little better compared to MRC under correlated fading but the performance of both the systems degrades in the absolute sense. The performance of both the schemes gradually shift towards a selection diversity scheme.

2.8.2 Complexity and performance tradeoffs for EGC and MRC

Each combining technique is optimal in its own sense if both complexity and performance are considered. If only performance is the criterion, MRC, EGC and Selection combining (SC) are optimal in the decreasing order. MRC requires an accurate estimate of the magnitude and phases of the channel gains making it relatively complex while

EGC requires the accurate estimates of only the phases of the channel gains. The later technique is favorable for implementation as it is sufficiently simple and provides considerable gains in the performance compared to a single antenna system. For a practical dual antenna case EGC differs from performance of the MRC by few dB which can be considered manageable considering the complexity of channel estimation for MRC. In implementation EGC is employed with modulation schemes having same power for all the symbols in the constellation such as PSK, DPSK etc and hence requires only the phase of the channel gain for detection. Any other modulation scheme with symbols having distinct powers such as M-QAM or an M-PAM will still require an estimate of the magnitude of the channel gains in EGC as symbol detection is based on the energy of the symbols. If the magnitudes of the tap gains need to be estimated for an EGC scheme, it is better to use MRC since it provides maximum performance gain.

The performance analysis of various modulation schemes under flat fading, in diversity combining receivers has been a topic of interest for many researchers and certain analytical techniques have been proposed for the performance analysis. In the next section some of the previous research in the analysis of these systems has been provided.

2.9 Performance of Diversity Receivers :Previous research

In [23] Brennan develops accurate and ideal models for performance analysis of EGC, MRC, then considers the deviations of these models from ideality and analyzes them rigorously. Diversity improvement curves for each combining scheme on independent Rayleigh slow flat fading channels have also been provided. Closed form expressions for exact symbol error probability(SEP) for MPSK signals even on AWGN channels proved to be a difficult task due to the presence of error functions. A convenient series expansion based on the fourier series is obtained by [25]. Later analytic expressions for the SEP of MPSK signals for L-Rayleigh flat, slow fading with MRC receivers was obtained by Chennakeshu [26]. EGC received less attention in the beginning, as the distribution for a sum of L-Rayleigh random variables is yet to be obtained. Altman and Sichak obtained the density function for a sum of two independent Rayleigh random variables.

Beaulieu [27] obtained an approximate series for a summation of L Rayleigh random variables which was later used for performance analysis of EGC on Nakagami-m flat fading channels. Zhang [28] obtained a closed form expression for the SEP of the performance of coherent and non-coherent receivers with equal gain combining for $L = 3$ case. He used the characteristic function of the Rayleigh fading distribution to evaluate the density function for the summation of 3 Rayleigh random variables. D.Zogas [29] evaluates the moments for Nakagami-m(Rice) and Nakagami-Q(Hoyt) random variables and uses these moments in evaluating the SEP for various modulation schemes based on Pade's approximants. Annamalai [30] evaluated the SEP for EGC on L independent and correlated fading channels based on fourier transform techniques.

Infinite series approximations for symbol error in PSK schemes in Nakagami fading channels were evaluated by Iyad [31]. The series approaches based on an expansion for P_{CSEP} developed by [25] for MPSK systems converge considerably faster. Najib [32] evaluated the probability of symbol error due to random phase error in partially coherent Rayleigh faded signals based on a Gram Charliere expansion. Though the series has slow convergence, it provides SEP s for BPSK and QPSK modulation upto L =4 EGC systems. Alouini [24] provides an approach based on moment generating function approach that unifies the performance analysis of different modulation schemes but the method is cumbersome to evaluate the probability of error for EGC with Rayleigh fading for L antennas.

2.9.1 Analytical techniques in performance analysis of EGC, MRC

The techniques used in the past for the performance analysis in flat fading channels provide a good insight into the performance of OFDM systems on frequency selective fading channels with diversity reception. On sufficiently slow fading channels OFDM in fact reduces the problem of performance analysis of frequency selective fading channels to that of flat fading due to its inherent orthogonality of sub-carriers. However, in case of fast fading channels significant ICI adds up to the noise causing error floors in the BER. Here an overview of the existing techniques has been provided and it is hoped that one of these techniques can be used in evaluating the performance of the OFDM system on diversity receivers.

- Averaging the PDF.
- Moment Generating Function.
- Parseval's theorem.
- Infinite series approximations.
- Method of Moments

2.9.1.1 Averaging the PDF

In this method the conditional symbol error probability (CSEP) of a modulation scheme for a specific channel condition is derived and the CSEP is averaged over the fading distribution. This method requires the pdf of the fading distribution to be known, which might not be readily available as it is, for the case of EGC on Rayleigh fading channels. The computation of CSEP is similar to evaluation of the error probability of any modulation scheme on AWGN channel except that it is conditioned on a given channel gain,

$$CSEP = P(\epsilon/\alpha). \tag{2.54}$$

Where $'\epsilon'$ represents the symbol error probability conditioned on the fading random variable α. Let $P(\alpha)$ be the probability density function of the random variable. The probability of error can be calculated by averaging over the distribution of α and is obtained as

$$\int_{-\infty}^{\infty} P(\epsilon/\alpha)P(\alpha)d\alpha \tag{2.55}$$

This approach is adopted for evaluating the error probabilities for fading distributions with a convenient pdf such as Nakagami, Chy-square etc.

2.9.1.2 Moment Generating function approach

In certain cases, the moment generating function has a simpler form than the probability density function itself. In this approach the moment generating function of the fading distribution is used to evaluate the average symbol error probability. By transforming the CSEP for a given modulation scheme, which is usually in terms of an error function or a Q function, into a standard integral referred to as the Craig's representation an integral with finite limits in terms of moment generating functions can be obtained for the average error probability. The complementary error function $Q(x)$ can be expressed as [24]

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{\frac{-x^2}{2sin^2(\Theta)}} d\Theta \tag{2.56}$$

This form has an advantage of having finite limits of integration, independent of x. The exponential function in terms of x simplifies the CSEP by representing it in terms of the moment generating function. For example, assume γ to be the instantaneous SNR on a fading channel and let $'a'$ be a constant which depends on the modulation scheme used, the CSEP for a modulation scheme is given by $Q(a\sqrt{\gamma})$ and let the density function be $p(\gamma)$, then the average symbol error probability is

$$I = \int_0^{\infty} Q(a\sqrt{\gamma})p(\gamma)d\gamma \tag{2.57}$$

Substituting the expression for $Q(x)$ in the above equation

$$I = \int_0^{\infty} \frac{1}{\pi}\int_0^{\pi/2} e^{\frac{-a^2\gamma}{2\sin^2\Theta}} d\Theta p(\gamma)d\gamma. \tag{2.58}$$

Changing the order of integration one can write I as,

$$I = \frac{1}{\pi}\int_0^{\pi/2} M_{\gamma}(\frac{-a^2}{2sin^2\Theta})d\Theta \tag{2.59}$$

where M_{γ} is the moment generating function for the pdf $p(\gamma)$.

2.9.1.3 Parseval's theorem

In this approach the fourier transform of the pdf of the fading distribution and the CSEP are taken transforming the problem to frequency domain. A general form of CSEP is obtained for most of the existing modulation schemes. The CSEP can be expressed as

$$P_s(\epsilon/x) = \sum \int_0^{\eta_u} a_u(\theta) e^{-x^2 b_u(\theta)} d\theta \tag{2.60}$$

where u corresponds to the number of distinct exponential integrals in the CSEP of the modulation scheme and both $a_u(\theta), b_u(\theta)$ are the coefficients independent of γ and the SEP is obtained by applying the Parseval's theorem to the CSEP as

$$SEP = \frac{1}{2\pi} \int_{-\infty}^{\infty} FT[P_s(\epsilon/x)] \phi_x{}^*(\omega) d\omega. \tag{2.61}$$

Where $'FT'$ stands for the Fourier transform of the CSEP and $'\phi_x{}^*(\omega)'$ is the characteristic function of fading distribution.

2.9.1.4 Infinite Series approach

The technique used here involves finding an expansion for the the CSEP in terms of a converging infinite series. Each term in the series is then averaged over the fading distribution which again depends on the availability of the PDF for a particular fading. This is useful when a particular CEP is periodic or if it can be expressed as an infinite series. The advantage with such a series is that there is a choice to select the number of terms that should be used to evaluate the error probability based on the degree of accuracy needed thus decreasing the number of computations required. Certain infinite series expansions are not convergent at all which requires obtaining functions that approximate the resultant of the series over a well defined interval. In fact such asymptotic expansions when evaluated are more advantageous than a slowly convergent series which needs more than around 50 terms to converge. There are also some specialized series expansions to express or approximate the probability density function of an unknown random variable in terms of a normal distribution. Listed below are some series expansions encountered frequently in literature.

Gram-Charliere and Edgeworth series are based on expressing an unknown pdf in terms of weighted functions of a normal distribution. The weights being functions of moments of the distribution function. Gram-Charliere series-A (GCA) is mostly used to approximate functions that are similar to Gaussian distribution functions. However GCA might not provide a convergent series in certain cases. This is due to the error between the pdf and the expansion not being convergent. A series based on the asymptotic expansion of a function can be used to define the density functions over various disjoint intervals in such cases. Here instead of the non-central moments of a pdf, centralized moments are used in determining the expansion.

The Gram-Charliere series -A is given by:

$$f(x) = \sum_{j=0}^{\infty} c_j w(x) \alpha(x) \tag{2.62}$$

where $\alpha(x)$ is the pivot function that the pdf is to be expressed in and $w(x)$ is the weighting function for the pivot and c_j is a constant which depends on successive moments of the pdf to be obtained. To simplify the series, orthogonal hermite polynomials are used as weighting functions and the coefficients c_j can be calculated based on the moments of the unknown pdf. Let $H_n(x)$ be the hermite polynomial, then the c_js are calculated as below

$$c_j = \frac{1}{j!} \int_{-\infty}^{\infty} f(x) H_j(x) dx \tag{2.63}$$

$$c_j = \frac{1}{j!} (\hat{\mu}_j - \frac{j^{(2)}}{2 \times 1!} \hat{\mu}_{j-2} + \frac{j^{(4)}}{2^1 \times 2!} \hat{\mu}_{j-4} \ldots) \tag{2.64}$$

$$j^{(k)} = \frac{j!}{(j-k)!} \qquad \mu_j \quad is \quad the \quad j^{th} \quad moment \tag{2.65}$$

A recursive algorithm for the computation of c_j s may be used in this case. In [32] the Gram-Charliere expansion is used to obtain a series approximation for the probability of symbol error due to random phase error for BPSK, QPSK on Rayleigh flat fading with EGC for L =4 channels. With the increase in L the convergence of the series slows down. Edge worth series is similar to Gram-Charliere series and is the actual asymptotic expansion for a given function, the convergence of which is more faster. This series requires cummulants to evaluate the coefficients.

In [27], the author obtained a different series approximation for the sum of L EGC random variables. The approach followed here is similar to Chernoff bounds where the tail of the distribution is upper bounded by defining a step function of the random variable over the region of interest. Since the Rayleigh distribution extends till infinity a periodic step function of the sum of random variables is considered for upper bounding. The series was expanded using the Fourier series and an expectation of the random variable yields the characteristic function.

2.9.1.5 Method of Moments

In [29], the authors evaluate the analytical expression for n^{th} moment of the signal to noise ratio and use this moments to obtain an approximate rational function for the moment generating function using Pade's approximants. The Pade's approximants involves approximating the moment generating function with a rational function whose coefficients are determined by the moments of the distribution function. Later an inverse laplace transform of this function gives an approximate pdf of the distribution function which can atleast be used to provide some error bounds. This is a unified technique and used to evaluate the error probabilities various modulation schemes approximately.

2.10 Conclusion

An overview of OFDM systems and performance analysis through simulation in the absence of ISI,ICI is provided. An introduction to diversity receivers and some techniques for performance analysis on flat fading channels have been summarized.

CHAPTER 3

OFDM WITH DIVERSITY RECEPTION

The performance evaluation of a complex system like OFDM on a frequency selective slow and considerably fast fading channel analytically is a cumbersome task. Diversity reception and equalization schemes add to the complexity of the process. In such cases model based simulations provide a better alternative.

A simulation involves a set of random experiments based on the frequency of whose outcomes a person or entity judges a designed system and is equivalent to estimating the performance by means of an experiment. Simulation can address two types of characteristics of a system; transient, steady state. A transient state is a localized state of a specific component in the system, these characteristics are estimated and dealt within the sub-system level and are not taken into account while considering the steady state characteristics of the system. Performance measures such as bit-error rates, mean square error, signal to noise ratios are considered to be steady state as these are judged over longer periods of time. Sufficient care should be taken during the simulation so that the output remains unbiased. Though simulations are performed to estimate the performance they should never be viewed as a substitute for mathematical analysis, they guide a system designer as to what parameters have a significant influence on the performance and what parameters are to be neglected.

In the next section the performance analysis of a diversity reception scheme for an OFDM system on slow frequency selective fading channels with and without ISI has been considered using Monte Carlo simulations.

3.1 OFDM with no Cyclic Prefix

As discussed in the previous chapter, OFDM suffers from three major impairments depending on the nature of the channel used for transmission; ISI, ICI and AWGN. ISI and ICI cause irreducible error floors resulting in degraded performance of the system at higher data rates, even at larger signal to noise ratios. Diversity systems are a natural solution for such impairments as they sufficiently average out the deep fades by providing multiple replicas of the signal at the receiver. In case of a slow fading channel with delay spreads larger or smaller than a single OFDM symbol interval(ISI,no ISI), the MRC scheme provides maximum diversity gains. Channel estimation which involves the estimation of both magnitude and phase of the complex filter taps for a large set of sub-carriers is a complex process. Limiting the estimation to phase of the channel taps saves both time and resources. Thus an equal gain combining scheme is a better alternative.

Diversity combining schemes with post-FFT architecture are analyzed in performance by simulation. These schemes are considered to yield better gains than the existing pre-FFT schemes and hence are favorable to improve the performance of the system.

The OFDM system considered here is similar to the one considered in the previous chapter, however some additional components to account for diversity reception are added at the receiver. Since different degrading factors such as ISI, ICI arise in the system based on the channel characteristics, each of them is dealt independently. In the first case, the cyclic prefix which is used in any conventional implementation of OFDM to prevent ISI, is not included to consider the performance of the system on slow frequency selective fading channels with higher delay spreads. A coherent detection receiver with equal gain combining has been proposed and the performance is estimated by simulations.

The system model is as in the fig:3.4. The transmitter architecture remains the same as for a typical OFDM system with a binary source clocking out bits every T_s seconds and the bits mapped onto real/complex symbols depending on the modulation scheme selected. These symbols are later modulated onto orthogonal tones by taking an IFFT on the modulated symbols to obtain N samples of each tone every NT_s seconds.

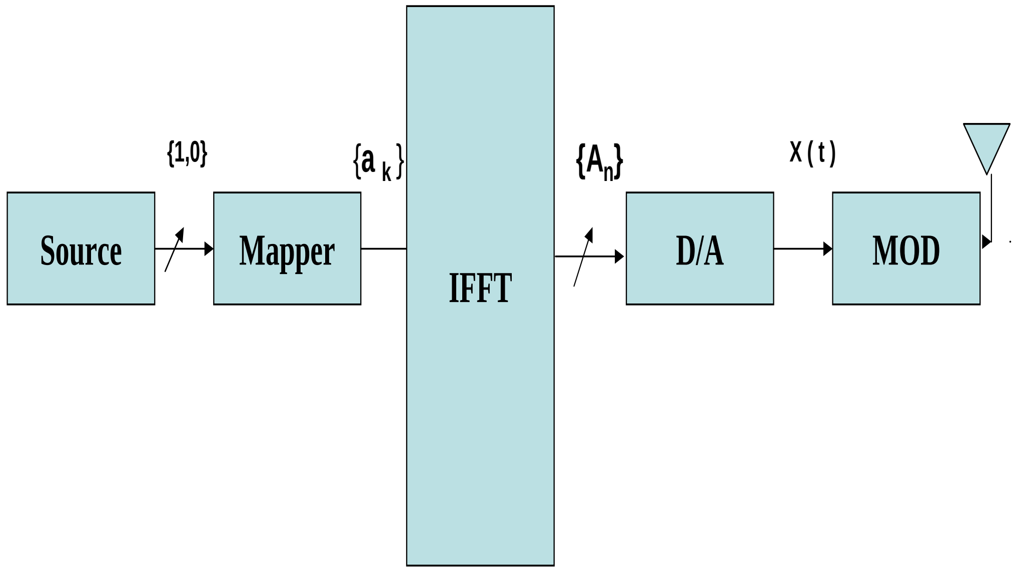

Figure 3.1. Transmitter model remains the same for Receive diversity.

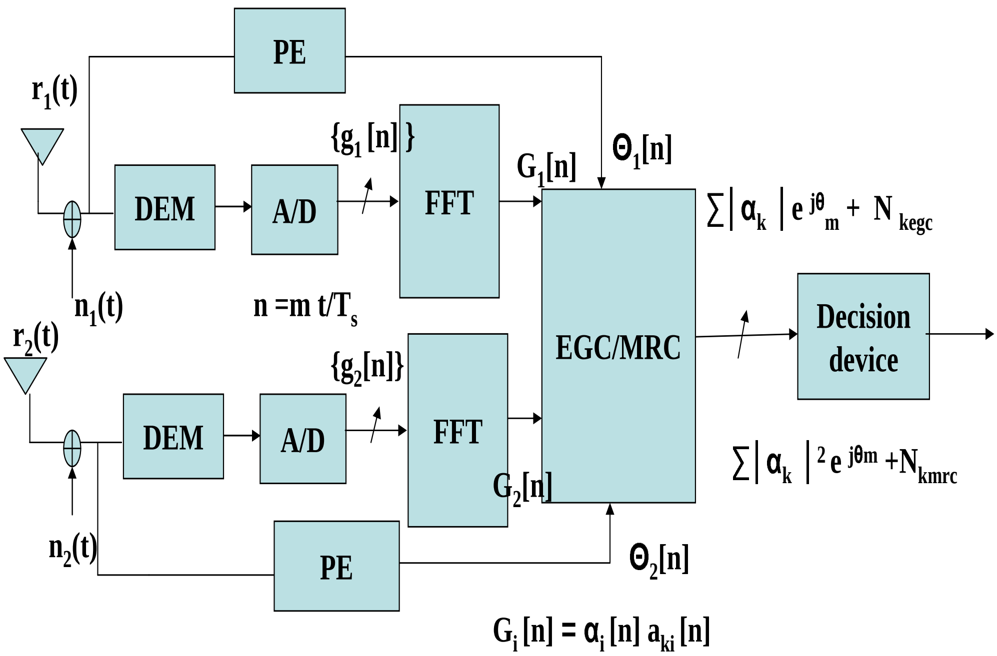

Figure 3.2. Receiver model for an OFDM-EGC system.

The samples are further converted from digital to analog waveforms after modulation and a cyclic prefix is added based upon the necessity. The resulting signal is up-converted and aired over a time invariant fading channel considered to be frequency selective.

At the EGC enabled receiver, the signals on both the antennas are down converted and low pass filtered. A digital to analog (D/A) converter converts the signals on both the antennas to discrete symbols which are demodulated later on by taking the FFT at each receiver. Thus an OFDM scheme with L antennas requires L receive chains making it expensive. The phase on each subcarrier within an OFDM symbol is estimated from the modulated signal through any standard pilot channel estimation techniques. In the system model considered, perfect knowledge of phase is assumed at the receiver. While a parallel OFDM symbol is being converted to serial, the phase estimator provides the phase estimates to the combiner which further co-phases the signal on each sub-channel to combine with the symbols on the corresponding sub-carriers to yield a copy of the transmitted signal. This will be further used to make a decision on the transmitted signal. The detection is made symbol wise rather than block wise.

3.1.1 Mathematical model

Let the sequence of complex symbols obtained after mapping a stream of N or more than N bits be grouped into a block of N symbols. Let the n^{th} block be $a[n]$. Each block of N symbols are then modulated onto N orthogonal carriers through an IFFT. The IFFT is equivalent to multiplication of the matrix ${F_N}^H$ where F_N is a matrix whose $(i,k)^{th}$ element is given by $e^{\frac{j2\pi ik}{N}}$ and $'H'$ stands for the transpose conjugate of a matrix. The output of the transmitter is obtained as

$$x[n] = {F_N}^H a[n] \tag{3.1}$$

The transmitted signal on the i^{th} sub-carrier can thus be obtained as

$$x_i[n] = \sum_{k=0}^{N-1} a_k[n] e^{\frac{j2\pi ik}{N}} \quad i = 0 \quad to \quad N-1 \tag{3.2}$$

The D/A conversion and the up-conversion are ignored for now. The output of the transmitter is aired over a slow frequency selective fading channel. Note that there is no cyclic prefix attached to the transmitted OFDM symbol. Let the discrete time invariant taps of the slow frequency selective fading channel on the s^{th} antenna be $\{h[l]^s\}$. The resolvable taps are placed in time separated by integer multiples of symbol times mT_s. Since there is no cyclic prefix, the output of the channel at the receiver input is the linear convolution of the transmitted signal $x_i[n]$ with the impulse response of the channel $\{h[l]^s\}$ given by,

$$y_m{}^s[n] = \sum_{l=0}^{L-1} h[l]^s x_{m-l}[n] \qquad 0 \leq m \leq L+N-1 \tag{3.3}$$

Where the number of taps in the channel is $'L'$ and $'N'$ is the number of sub-carriers or transmitted symbols. The output $y[m]^s$ which is the convolved output of the two discrete sequences of length L, N is of length $L+N-1$. Hence the last L symbols of the output of the channel y^s overlap on the successive L symbols of the next OFDM block causing Inter symbol interference. The above steps can be modeled in terms of matrices as,

$$Y_s = H_s \cdot X \tag{3.4}$$

where

$$H_s = \begin{bmatrix} h_{L-1}{}^s & h_{L-2}{}^s & \dots & h_0{}^s & 0 & 0 & \dots & 0 \\ 0 & h_{L-1}{}^s & h_{L-2}{}^s & \dots & h_0{}^s & 0 & \dots & 0 \\ 0 & 0 & h_{L-1}{}^s & h_{L-2}{}^s & \dots & h_0{}^s & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & h_{L-1}{}^s & h_{L-2}{}^s & \dots & h_0{}^s \end{bmatrix} ; \quad X = \begin{bmatrix} x_{N-(L-1)}[n-1] \\ x_{N-(L-2)}[n-1] \\ \dots \\ x_{N-1}[n-1] \\ x_0[n] \\ x_1[n] \\ \dots \\ x_{N-1}[n] \end{bmatrix} \tag{3.5}$$

From eqn : 3.1, each block $x[n]$ is the IFFT of the n^{th} block of data. Noise at the front-end of the receiver in the form of AWGN is added to the received signal and the resulting signal is demodulated by taking taking the Fast fourier transform. The above equation hence can be further decomposed into the required signal and interference from the previous block and inter-carrier interference. Let $a[n-1], a[n]$ be the $(N, 1)$ modulated symbol blocks successively transmitted. Let the IFFT matrix for the last $'L-1'$ symbols of $a[n-1]$ be ${F_{L-1,N}}^H$ and K be channel coefficient matrix of size,$(N, L-1)$ corresponding to the overlapped portion from the previous block. Let Z be the channel matrix of size (N, N) corresponding to the intra-symbol interference within the transmitted block. This intra-symbol interference results due to the interference from symbols within an OFDM symbol,$a[n]$ and not due to the previous OFDM symbol$a[n-1]$. Let W be the AWGN added at the receiver front end, the output of the receiver after down, A/D conversion can be modelled as,

$$y_s[n] = h_0 {F_N}^H a[n] + K {F_{L-1,N}}^H a[n-1] + Z {F_N}^H a[n] + W \tag{3.6}$$

where

$$K_s[n] = \begin{bmatrix} {h_{L-1}}^s & {h_{L-2}}^s & {h_{L-3}}^s & \dots & \dots & \dots & {h_1}^s \\ 0 & {h_{L-1}}^s & {h_{L-2}}^s & {h_{L-3}}^s & \dots & \dots & {h_2}^s \\ 0 & 0 & {h_{L-1}}^s & {h_{L-2}}^s & {h_{L-3}}^s & \dots & {h_3}^s \\ 0 & 0 & 0 & {h_{L-1}}^s & {h_{L-2}}^s & \dots & {h_4}^s \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & {h_{L-1}}^s \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \tag{3.7}$$

and

$$Z_s[n] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ {h_1}^s & 0 & 0 & 0 & 0 & \dots & 0 \\ {h_2}^s & {h_1}^s & 0 & 0 & 0 & \dots & 0 \\ {h_3}^s & {h_2}^s & {h_1}^s & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ {h_{L-2}}^s & \dots & {h_1}^s & 0 & 0 & \dots & 0 \\ {h_{L-1}}^s & {h_{L-2}}^s & \dots & {h_1}^s & 0 & \dots & 0 \\ 0 & \dots & {h_{L-1}}^s & \dots & {h_1}^s & \dots & 0 \\ \vdots & & & & & & \\ 0 & \dots & 0 & {h_{L-1}}^s & \dots & {h_1}^s & 0 \end{bmatrix} \tag{3.8}$$

After demodulation, the received signal is

$$F_N y[n]^s = F_N {h_0}^s I {F_N}^H a[n] + F_N K_s {F_{L-1,N}}^H a[n-1] + F_N Z_s {F_N}^H a[n] + F_N W \tag{3.9}$$

I corresponds to an (N, N) identity matrix. As can be seen from the above equation, the first term corresponds to the purely transmitted signal whereas, the second and third terms contribute to inter block, intra-block interference. After combining first and the third terms, it can be seen that intra-symbol interference is negligible due to the fact that $F_N({h_0}^s I + Z_s){F_N}^H$ can be approximated by the diagonal matrix obtained by eigen decomposition of the (N,N) circulant matrix formed from the channel taps for the OFDM symbol $a[n]$. This is due to the toeplitz structure of $({h_0}^s I + Z_s)$ which is asymptotically equivalent to the circulant matrix defined by the zero padded N-point channel matrix. A sequence of circulant matrix is asymptotically equivalent to a Toeplitz matrix with sufficient sparsity as stated in[33]. Asymptotic equivalence guarantees the eigen values, inverse, products etc to behave the same way for both the matrices.

Inter symbol interference in the form of IBI in the system causes significant degradation in the performance. To improve the performance, diversity combining is adapted. In EGC, the replicas of the obtained signal are co-phased and combined on a sub-carrier basis to yield the diversity signal. The received signals on the s^{th} antenna are compensated with the phase of the channel which is assumed to be perfectly available at the receiver. Typically the phase is estimated with the help of pilot symbols assumed to be transmitted along with the OFDM symbols. Representing the diagonalized form of the circulant to be D_s, the signal obtained post-compensation and equal gain combining is as shown below,

$$\sum_{s=1}^{2} F_N y[n]^s = \sum_{s=1}^{2} \|D_s\| a[n] + F_N K_s F_{L-1,N}{}^H a[n-1] + F_N W \tag{3.10}$$

where $\|D_s\|$ stands for the channel matrix whose elements are the magnitudes of the frequency responses of the adjacent channel gains. Phase compensation does not effect the power of the interferer and diversity combining reduces the probability of the channel in deep fade. The performance analysis of this system for a 2-tap multi-path slow channel model has been estimated by simulation. The channel taps remain constant over the duration of a single OFDM block. Since an 80 symbol packet is already being aired over the channel, it is assumed that there is no guard interval for it and all the 80 symbols are regarded as data symbols. The results of the simulation are further explained in the next chapter.

The inter symbol interference due to larger delay spreads in the channel can be mitigated by employing a guard interval with each transmitted OFDM symbol provided, the length of the guard interval is greater than the maximum number of taps in the channel and is removed at the receiver prior to demodulation. Thus the corrupted portion of the transmitted signal is removed without any loss of information. Guard intervals can be either zero or non-zero modulated symbols where, in the former case, power required to transmit a non-zero /cyclic prefix can be saved.

3.2 EGC enabled OFDM with cyclic prefix

In this section, the performance of an OFDM system with cyclic prefix included is considered on slow frequency selective fading channels with considerably higher delay spreads but lower than the length of the cyclic prefix. The cyclic prefix here also mitigates the inter carrier interference while protecting the orthogonality of sub-carriers. The channel here again, is assumed to remain static over atleast a single OFDM block. The performance of equal gain combining in such cases is similar to that of equal gain combining on flat fading channels. Even in the presence of ISI, OFDM reduces the frequency selective fading channel to a set of flat fading sub-channels and diversity combining is used on each sub channel providing high improvement in the performance of the system.

The system model is same as the above as shown in the fig:3.1 The system description is along the same lines of model used for OFDM with ISI except for a small change in the transmitted signal. A prefix which includes the last L symbols of the OFDM symbol to be transmitted is padded before each OFDM symbol. Thus the prefix is a cyclic.The length of the OFDM symbol is now $N + G - 1$ where G stands for the length of the prefix. Since the cyclic prefix is fixed in length and the impulse response of channel keeps varying based on propagation conditions, it is difficult to optimize the length of prefix. Typically the length of the cyclic prefix is more than the maximum delay spread LT_s of the channel, i.e atleast equal to length of the channel. The output of the channel is again a linear convolution of the input with the channel

$$y_m{}^s[n] = \sum_{l=0}^{L-1} h[l]^s x_{m-l}[n] \qquad 0 \leq m \leq L + N - 1 \tag{3.11}$$

Note here that the linear convolution of two sequences of lengths $L, N + L - 1$ should yield a sequence of length $N + 2L - 1$. In this case the first L samples involve an overlap from the preceding transmitted block and the last $L - 1$ samples overlap on the first L samples of the succeeding block. Since the first L samples of each block are removed, the last N samples in the output provide the response of channel. Hence the output is taken from the L^{th} sample to $N + L - 1^{th}$ sample. The output is equivalent to

$$y_m{}^s[n] = \sum_{l=0}^{L-1} h[l]^s d_{(m-L-l)mod(N)}[n] \qquad 0 \leq m \leq L + N - 1 \tag{3.12}$$

where

$$x = [d[N - L + 1)], d[N - L + 2], \ldots, d[N - 1], d[0], d[1], \ldots, d[N]]. \tag{3.13}$$

Hence the linearly convolved output from L to $N+L-1$ replaced by a cyclic convolution of two N-point discrete sequences with one of them being, with one of them being N-points of the data portion of the OFDM symbol, and another being a sequence with L channel taps padded with $N-L$ zeros.

A zero padded sequence such as the N-point impulse response of the channel above renders the sequence periodic with period N, since an $N-L$ zero padded version of an L point sequence when circularly shifted N times produces the same signal. On the other hand the cyclic prefix attached to the data portion makes it periodic as well. The DFT of the convolution of two periodic sequences of period N is the product of the N-point DFTs of each sequence. This property is important in the context of removing intra-channel interference in a slow frequency selective fading channel and providing single tap equalization techniques for OFDM. The output of the channel thus obtained is added with noise and demodulated using the FFT to yield the product of frequency responses of each sub-channel with the transmitted signal. The phase response of the channel on each sub carrier is estimated and used to co-phase each sub-carrier.

3.3 Mathematical model

The mathematical model used is a simplified form of the eqn: and can be written as

$$F_N y[n]^s = \|D_s\| a[n] + F_N W \tag{3.14}$$

Note here that the term corresponding to the IBI is vanished owing to the removal of cyclic prefix. Also it can be seen that the output has a simple diagonalized form in which each transmitted symbol is associated with the frequency response at that sub-carrier.After equal gain combining, the output is

$$\sum_{s=1}^{2} F_N y[n]^s = \sum_{s=1}^{2} \|D_s\| a[n] + F_N W \tag{3.15}$$

The performance of the above system is simulated on 64 sub-carriers with a cyclic prefix of 16. It should be noted that the channel taps remain invariant atleast over a single block of OFDM symbol. Any time variations in the system lead to additional degradation as will be discussed in the net section. Since the OFDM system divides a frequency selective fading channels into a set of flat fading channels, equal combining scheme yields the same results as that on a flat fading sub channels providing huge gains. The simulation results for the slow frequency selective fading channel case are summarized further in the next chapter.

3.4 OFDM with Cyclic Prefix on Fast fading channels

In this section, the performance of an OFDM system with cyclic prefix included is considered on a fast frequency selective fading channel with considerable delay spreads. Though the cyclic prefix eliminates ISI, the orthogonality of the sub-carriers is lost, due to significant doppler spread in each sub-carrier owing to rapid motion of the mobile from base station, causing high inter- carrier interference. To minimize such effects, diversity-equalization receivers can be used and they perform better than most other complex existing equalization techniques. A measure of the degradation in the performance is the ratio of the doppler spread to the amount of sub-carrier spacing, other wise called the normalized doppler spread is also used to calculate error floors due to inter- carrier interference.

The system model is a shown in the figure:3.3 The transmitter functions similar to that as explained before w.r.t modulation, mapping etc but since the channel is fast frequency selective, to account for additional degradation due to ICI, the receiver is modified to support both diversity combining and equalization. At the receiver, the received signal on both the antennas, added with noise is demodulated by taking the FFT, weighted by the conjugates of the channel gains on each sub-carrier and combined (MRC)prior to equalization. Later a single tap frequency domain equalizer whose weights are determined by the channel gains is used to account for the effects of the channel.

The channel used in the the system model is assumed to be wide sense stationary and uncorrelated scattering(WSSUS), characterized by a time-frequency correlation

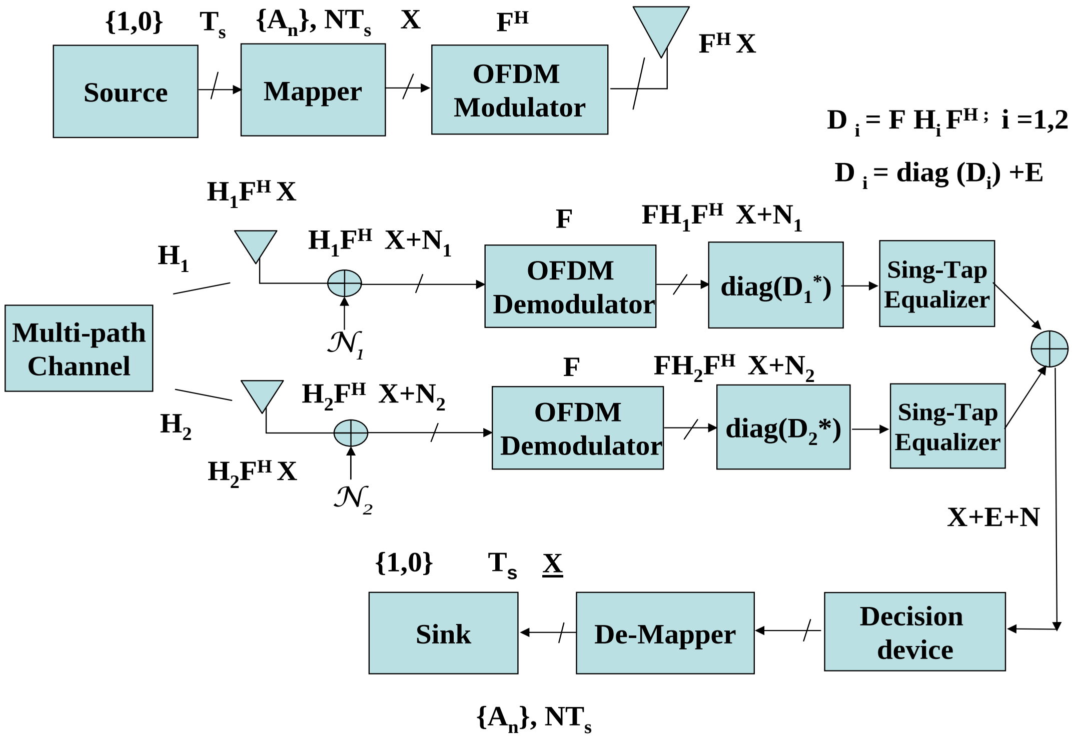

Figure 3.3. Receiver model for an OFDM-MRC system.

function in refs. The model being WSSUS, allows the separation of the auto-correlation function into time and frequency functions. Hence the time variations of the channel can be modelled independent of the frequency characteristics of the channel. On each sub-carrier the channel is assumed to be frequency flat-faded and the coherence bandwidth is determined by the frequency correlation function which in turn is specified by the multi-path power intensity profile. Exponential multi-path profile, with as many as whose time constant defines the coherence bandwidth of the channel is employed.

The modulated signal after transmission over such channels over the air is cyclic prefix removed and added with AWGN. Later the signals on each branch of the antenna are demodulated and weighted by the conjugates of the channel gains on each sub-carrier and combined to yield a diversity signal. A single-tap zero forcing equalizer whose coefficients are determined by the channel gains assumed to be perfectly known or estimated is used to equalize the transmitted signal. The equalized symbols are detected and mapped back onto complex symbols and then onto bits.

3.5 Mathematical model

The transmitted signal remains the same but the received signal after the signal passes through a time variant fading channel is

$$y_m{}^s[n] = \sum_{l=0}^{L-1} h[m,l]^s x_{m-l}[n] \qquad 0 \le m \le L+N-1 \tag{3.16}$$

where the variables used in the above equation have their usual meaning as before except that the channel is time variant with an impulse response $h[m,l]^s$ at the m^{th} instant on s^{th} antenna. In matrix form, the received signal can be written as,

$$y[n]^s = H_s F_N{}^H a[n] + K_s F_{L-1,N}{}^H a[n-1] + W_s \tag{3.17}$$

where H_s is a matrix consisting of taps which are time variant. H_s can be written as

$$H_s = \begin{bmatrix} h_{1,0}{}^s & 0 & \dots & 0 & h_{1,L-1}{}^s & h_{1,L-2}{}^s & \dots & h_{1,1}{}^s \\ h_{2,1}{}^s & h_{2,0}{}^s & 0 & \dots & 0 & h_{2,L-1}{}^s & \dots & h_{2,2}{}^s \\ h_{3,2}{}^s & h_{3,1}{}^s & h_{3,0}{}^s & 0 & \dots & \dots & \dots & h_{3,3}{}^s \\ \multicolumn{8}{c}{\dots\dots\dots\dots\dots\dots\dots\dots} \\ \multicolumn{8}{c}{\dots\dots\dots\dots\dots\dots\dots\dots} \\ \multicolumn{8}{c}{\dots\dots\dots\dots\dots\dots\dots\dots} \\ \multicolumn{8}{c}{\dots\dots\dots\dots\dots\dots\dots\dots} \\ \multicolumn{8}{c}{\dots\dots\dots\dots\dots\dots\dots\dots} \\ 0 & \dots & 0 & h_{N,1}{}^s & \dots & h_{N,L-2}{}^s & h_{N,L-1}{}^s & h_{N,0}{}^s \end{bmatrix} \tag{3.18}$$

and

$$K_s = O_{N\times N} \tag{3.19}$$

The second term on the R.H.S which is due to inter block interference form the previous OFDM symbol is zeroed out. This is because of the cyclic prefix used which is longer than the length of the impulse response of the channel. Hence, IBI is eliminated. Unlike a slow frequency selective fading channel, the matrix $F_N H_s F_N{}^H$ is not diagonal due to time variant taps. H_s can be further decomposed into a sum of a slow fading channel matrix and an error matrix. The error matrix consists of difference between H_s and the slow fading channel matrix which can be diagonalized. Let the slow fading, error magnitude be G_s,$H_{s,ICI}$ on the s^{th} antenna, the output at the receiver is

$$y[n]^s = G_s {F_N}^H a[n] + H_{s,ICI} {F_N}^H a[n] + W_s \tag{3.20}$$

The second term on the R.H.S acts as inter-carrier interference causing degradation in the performance. The first term yields the diagonal matrix corresponding to circulant structure of the block channel matrix. After demodulation by FFT the received signal on each antenna is,

$$F_N y[n]^s = F_N G_s {F_N}^H a[n] + F_N H_{s,ICI} {F_N}^H a[n] + F_N W_s \tag{3.21}$$

Let the diagonal matrix $F_N G_s {F_N}^H$ be D_s in the first term on the RHS. The received signal is further weighted by the conjugate of the elements on the diagonal matrix ${D_s}^*$. These coefficients in matrix D_s need to be estimated by means of pilot carriers. Here the channel gains are assumed to be known perfectly. The resultant signal obtained after weighting is,

$$\sum_{s=1}^{2} y[n]^s = \sum_{s=1}^{2} {D_s}^* D_s a[n] + {D_s}^* F_N H_{s,ICI} {F_N}^H a[n] + {D_s}^* W \tag{3.22}$$

where the diversity combined signal is later on equalized by a single tap equalizer whose channel gains are the sum of the squares of the absolute values of channel gains. The signal on sub-carrier m after diversity combining and equalization is obtained as

$$\sum_{s=1}^{2} \omega_m y_m[n]^s = \omega_m \sum_{s=1}^{2} {d_{m,s}}^* d_{m,s} a_m[n] + \omega_m {d_{m,s}}^* {h_m}^{ICI} a_m[n] + \omega_m {d_{m,s}}^* w_m \tag{3.23}$$

where

$$\omega_m = \frac{1}{\sum_{s=1}^{2} |d_{m,s}|^2} \tag{3.24}$$

The performance of the above diversity equalization techniques are simulated for 64 sub carriers under different doppler spreads. The above techniques yield significant performance gains w.r.t to a single antenna OFDM system at even higher doppler spreads. The performance gains are even better than a single antenna OFDM scheme with singular value decomposition technique for the mitigation of Inter- carrier interference for lower signal to noise ratios.

The above system models introduced before and simulated later on were based on some more assumptions. There always exist some deviations in few assumptions when the model is modified to include some more practical aspects of communication on wireless channels. Listed here are some of the assumptions in the analysis of performance of diversity based systems

3.5.1 Assumptions on the system model

- The channel noise is un-correlated on both the branches of the receiver.
- The noise variance remains constant with time; noise is assumed to be wide sense stationary.
- Noise on each channel is independent of the channel taps.
- The multi-path channels seen on different antennas are assumed to be independent of each other.
- The channel is frequency selective fading with independent taps

3.6 Conclusion

A system model for an OFDM system with equal gain combining has been proposed for slow frequency selective fading channels and its performance is studied by simulations. Required modifications for the above model to mitigate the effect of Inter carrier interference on linear time variant multi-path channels, have been suggested after a simulations are performed with these modifications. Some previous research on performance analysis of diversity combining schemes on flat fading channels have been summarized. Finally the assumptions on the performance of the system have been mentioned.

CHAPTER 4

RESULTS

4.1 Performance analysis through Simulation

As no analytical techniques are available for the performance analysis of OFDM with diversity combining on time, frequency selective fading channels, the performance analysis is done through Monte-Carlo simulations. The simulation steps are same as that mentioned in chapter 2 except for few changes in the channel model used for the case of time variant fading channels. The general simulation model is shown as in fig:4.1

4.1.1 Time-Variant frequency selective fading

The generation of a time invariant frequency selective fading channel has been explained in the appendix. However, the described channel model does not provide enough support to incorporate doppler spreads and correlation between channel taps at different instants of time required for a time variant fading channel. Hence in this case, the Rayleigh fading envelope is simulated by the Clarke's model allowing us to take into simulation the above aspects. Once a fading envelope using the sum of sinusoids method of Clarke is generated, it can be sampled successively N times for a single, time varying tap that varies every $T_s$secs within an OFDM symbol. Since the succesive samples of a fading envelope have a correlation function,

$$E[h_l[m]^* h_l[m+n]] = J_0(2\pi f_d n T_s) \tag{4.1}$$

where J_0 stands for the Bessel function of zero th order. The time varying tap has the required doppler power sprectrum. To produce L such time varying taps, which are independent of each other, the fading envelope is generated L times keeping the value of ϕ_n in the Clarke's model to be random each time the fading envelope is generated. In

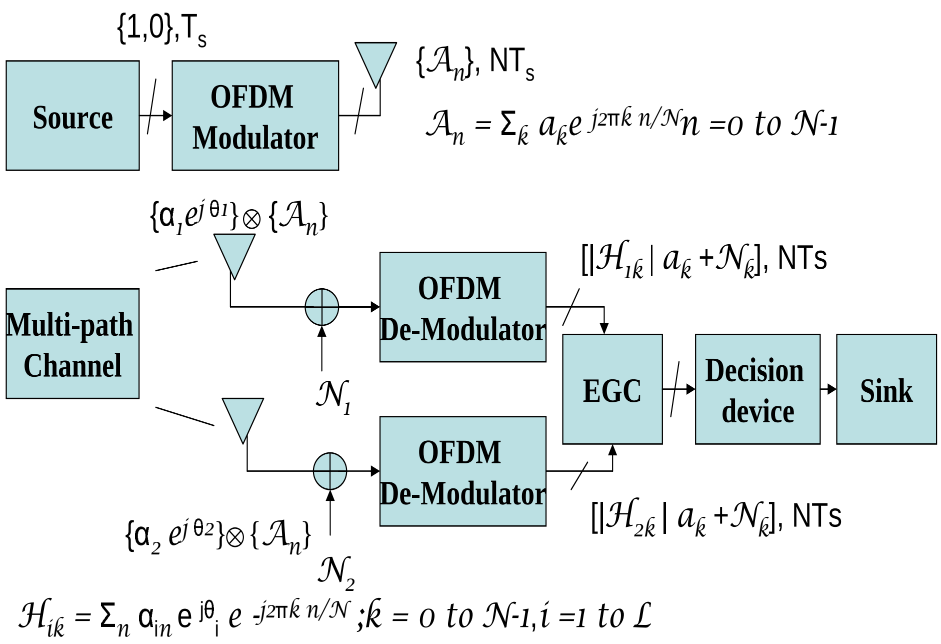

Figure 4.1. Simulation model for OFDM-DivComb system.

order to maintain a correlation between various sub-carriers in the frequency response of the channel an exponential power delay profile is selected,i.e

$$S(\tau) = \alpha e^{-\alpha\tau} \tag{4.2}$$

where $'\tau'$ stands for the delay spread in the channel and the parameter α defines the coherence bandwidth of the channel. Here the correlation between sub-carriers can be calculated from the inverse fourier transform of the power delay profile.

In addition to the changes in simulation of the channel, the following changes are necessary for simulation of diversity techniques; the number of multi-path channels is increased depending on the number of antennas a the receiver and each chanel is assumed to be WSSUS. The number of OFDM demodulators at the receiver are increased based on the number of antennas at the receiver. There is an FFT demodulator corresponding to each antenna leading to an increase in complexity of the receiver. A diversity combining

Table 4.1. System Parameters for simulations

System Parameters	values
System Bandwidth	20MHz
Modulation	2,4-PSK, 16-QAM
Data Payload rate	6-24 Mbp/sec
No of data Sub-carriers	48
No of pilots	4
Sub-carrier frequency spacing	312.5 Khz
IFFT/FFT size	64
OFDM block length	320 μ secs
Preamble duration	16 symbols
Symbols per packet	80
Antennas	2

technique, which uses the estimates of the channel gains to weight the signal on each branch and combines the outputs on each demodulator is used at the receiver.

The parameters for the simulation are given in the table 4.1 below. and the results are provided in the next section. The simulation is organized in the increasing order of complexity and the results are arranged as such. The results were validated in each step.

4.1.2 Simulation Results

The performance of OFDM system with QPSK on an AWGN channel is simulated as the first step. The number of sub-carriers were varied from 64 to 512 and the simulation is carried out with 10^5 bits and error rates for different signal to noise ratios were obtained by averaging over the number of bits. The error rates are plotted in fig:4.2 w.r.t the SNR in dB. Since, OFDM on AWGN channel is like any other single carrier modulation scheme on AWGN channel, the error rates obtained are expected to be similar to a QPSK single carrier system on AWGN channel which is indeed the case. The curve obtained remains the same even when the number of carriers are changed.

Further, the performance of OFDM with QPSK on flat fading channel is simulated. The variance of channel is normalized to one. A slow flat fading channel is implemented as a complex gaussian random variable with a coherence time equal to around 2 OFDM

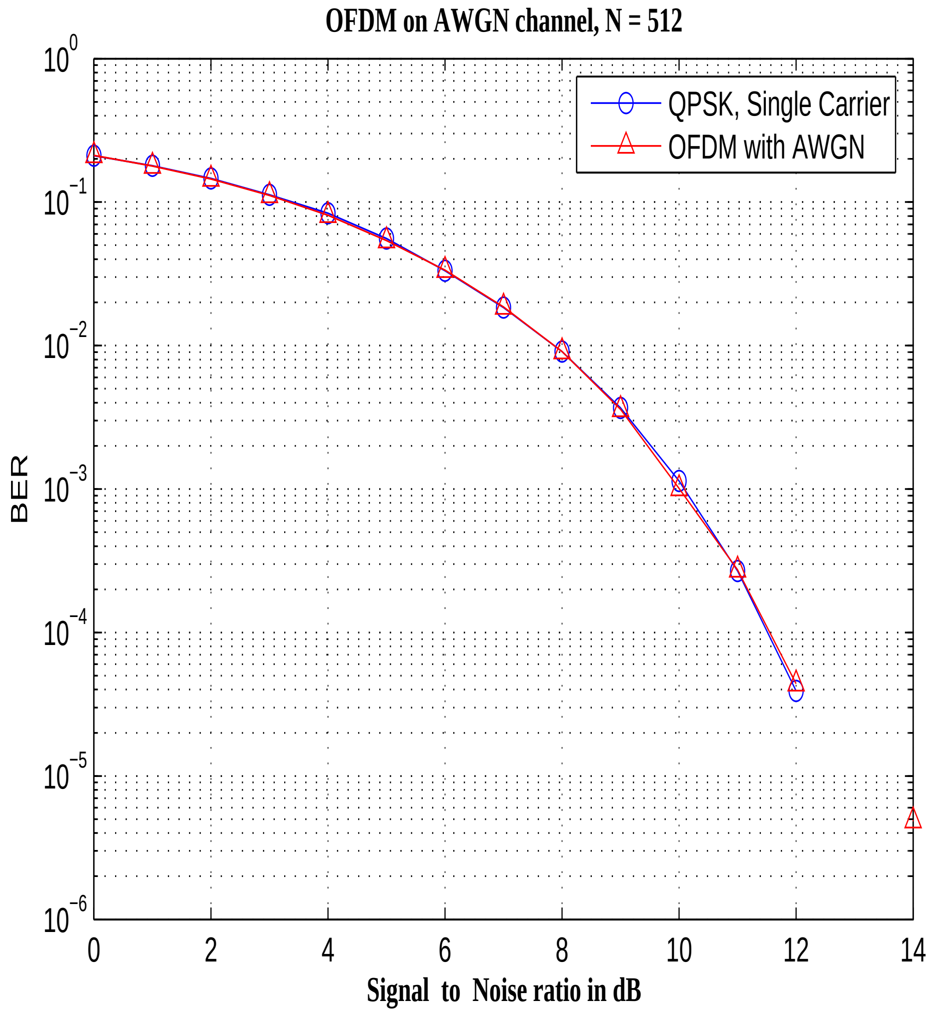

Figure 4.2. BEP of OFDM with QPSK on AWGN channel.

symbols. The sample size is set to be at 80 OFDM blocks, i.e 6400 bits and the simulation is repeated 10^5 times. The error rates are obtained by averaging over the number of bits in each iteration which in turn are averaged over the number of iterations. The zero forcing equalizer taps are updated every two OFDM blocks (i.e every single coherence time interval). The error rates are obtained as in fig:4.3. Vectorizing the steps in simulation decreases the computational complexity.

The error rates are validated with the theoretical error rates for QPSK on flat fading channel and as expected the the performance of the OFDM system on flat fading channel is similar to that of the theoretical curve. A small degrada on tion of around

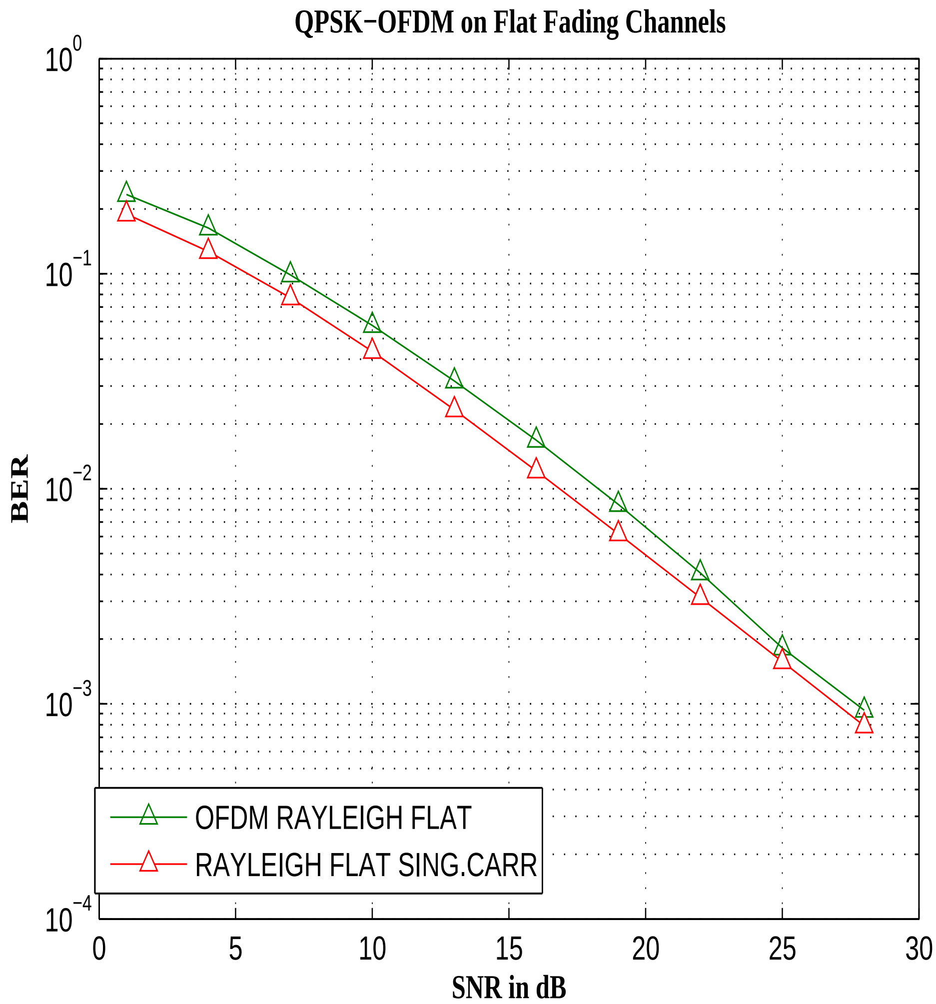

Figure 4.3. BEP of OFDM with QPSK-Rayleigh slow flat fading.

1.2 dB is observed and it is due to a factor of 5/4 accounting for the loss of energy due to the cylcic prefix being transmitted, i.e $80Ec = 64Es$. The demodulated symbols are equalized by using a zero forcing equalizer As long as the channel remains time-invariant post equalization is sufficient else a pre-correction techniques are required [7]. The simulation does not assume the presence of any co-channel interferer in the system and hence there is no enhancement of noise as suggested in[7]. The signal to noise ratio remains the same in case of post-equalization and conventional detection.

A receiver diversity system with $L = 2$ antennas for a BPSK, flat fading, single carrier case is implemented next. This step allows to consider the problem of diver-

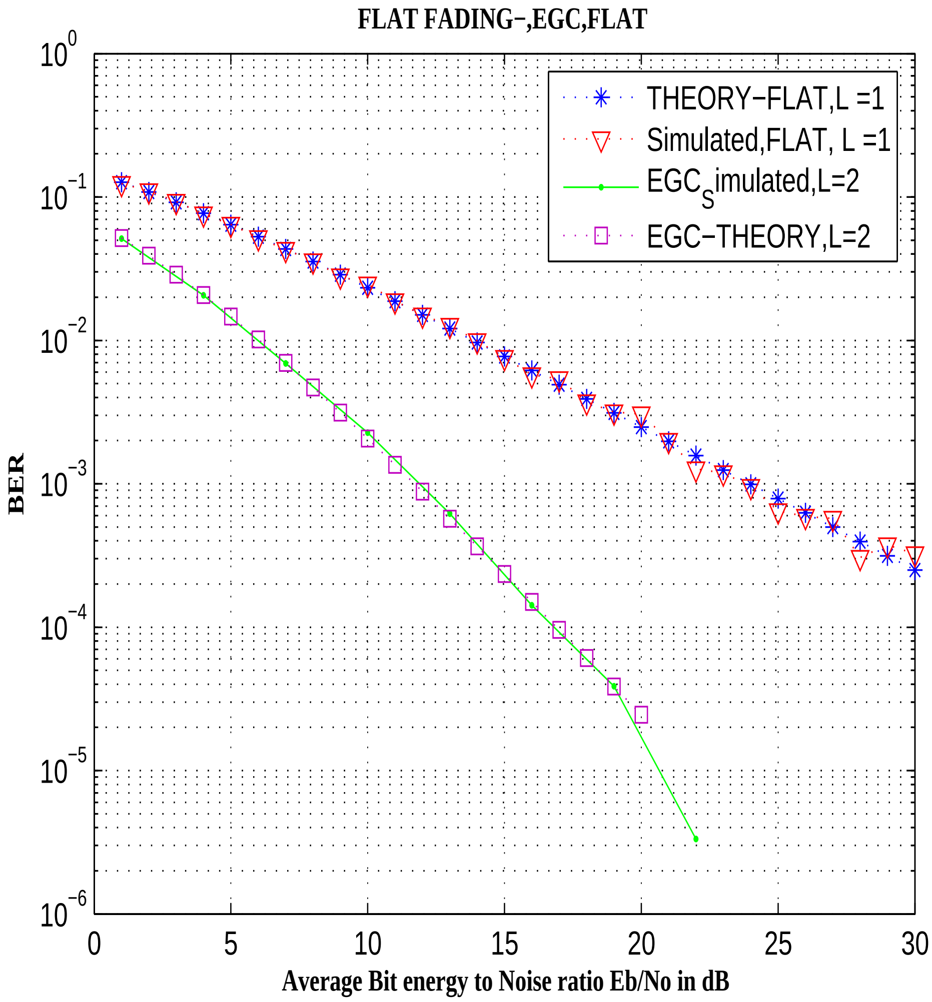

Figure 4.4. BEP of OFDM with BPSK on flat fading-EGC,no diversity.

sity reception separately. If this simulation is validated separately, then the same can be implemented with an OFDM scheme, for frequency selective fading channels. The performance plots for BPSK on the Rayleigh flat fading channel are obtained with no diversity. Performance for an EGC case is compared to the analytical results and the results are well in agreement as shown in the fig:4.4, from which it can be seen that the theoretical and simulated curves fall in place.

An OFDM system on a slow frequency selective fading is then implemented in the next step. The cyclic prefix is ignored as mentioned in case 1. It is assumed to be carrying data and has no cyclic structure when used in simulation. The simulation parameters are set as mentioned in the table and the channel profile is as follows; a Rayleigh frequency selective fading channel is implemented with two taps. The maximum delay spread of the channel is set to 3 symbol times, i.e $[1, 4]$ where 1 stands for the LOS path component. The power in the taps decreases exponentially,in the powers of 10, i.e the power delay profile is set to $[0, -10](dB)$. The frequency selective fading channel is slowly varying, this is achieved by keeping the taps constant over a single OFDM block and the taps selected are uncorrelated with each other. Each of the taps are generated independently from a complex gaussian random source and the channel profile is normalized such that total power in the in channel is unity. More about the channel simulation is mentioned in the appendix.

Though BPSK modulation is only used, the simulation can be extended to other modulation schemes. After the parameters of the channel are set as mentioned above, 80 OFDM symbols are transmitted over the channel and the taps are varied after each set of 80 OFDM blocks keeping the above parameters in tact(Quasi-static simulation), leading to a coherence time, T_c of $80 \times 80NT_s$. 10^5 iterations are performed. The simulation is performed with coding and no equalizer,a zero forcing equalizer alone, the performance plots are as obtained in figs:4.5,4.6.

Since a cyclic prefix is not used in the simulation, higher degradation in the performance of the system is expected due to interblock interference. An OFDM system with no equalizers or no diversity reception schemes suffers from severe degradtion in the performance. Employing forward error correction coding in the form of convolution codes with rate $\frac{1}{2}$ system betters the performance but does not improve the performance greatly. An OFDM system with equalization performs better compared to a coded, interleaved OFDM system as can be seen from the plots obtained. Hence equalization can be considered a better alternative for channels with less number of taps for reducing the error floors caused due to IBI. The results for OFDM with equalization are validated with those in[34].

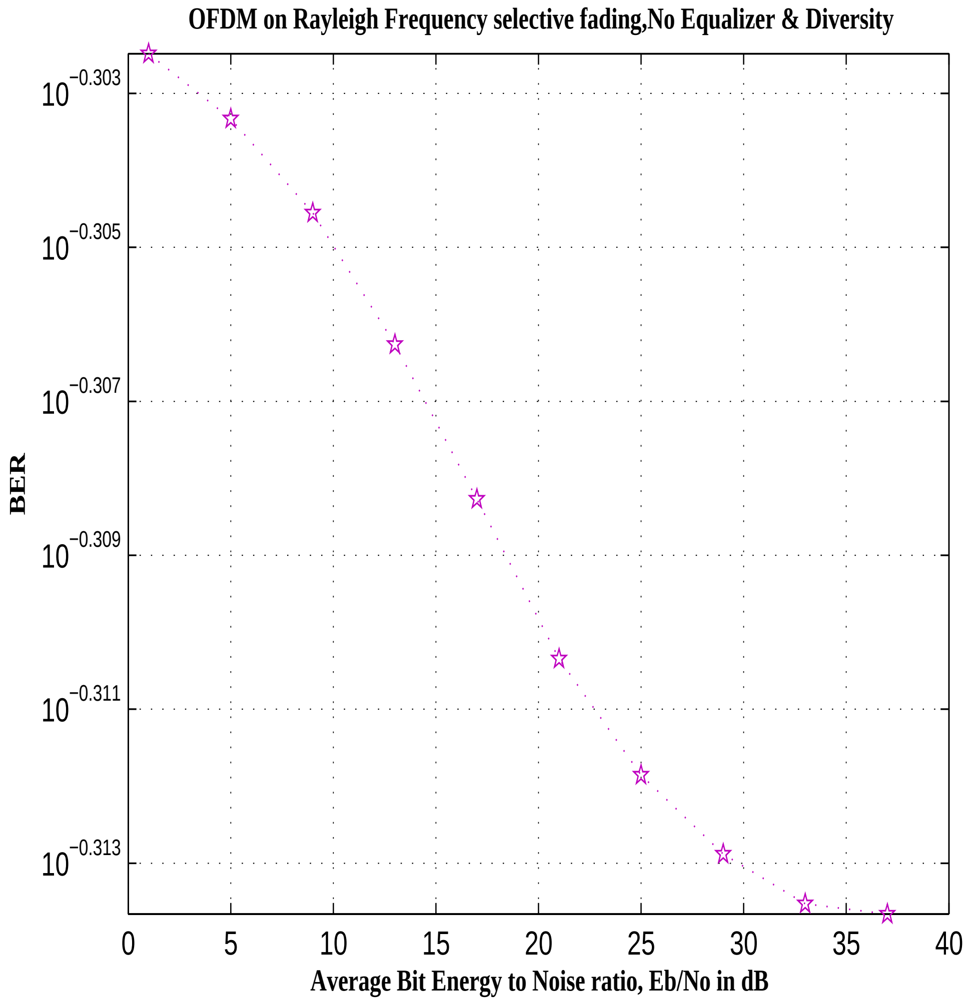

Figure 4.5. BEP of OFDM-BPSK on 2-tap Rayleigh-single antenna no equalizer.

Diversity reception schemes, MRC and EGC with OFDM are later implemented on frequency selective fading channels. The performance plots obtained are shown as in the fig:4.7. It can be seen that the diversity schemes out perform equalizers and maximize the performance gain. MRC performs marginally better than the EGC scheme for an $L = 2$ case at the expense of complexity. The error floors are reduced by twice in this case, re-affirming the fact that diversity is a better alternative to improve the performance of the system.

The next set of results are obtained for an OFDM system with cyclic prefix included. As it can be see that the length of the cyclic prefix is greater than the maximum delay

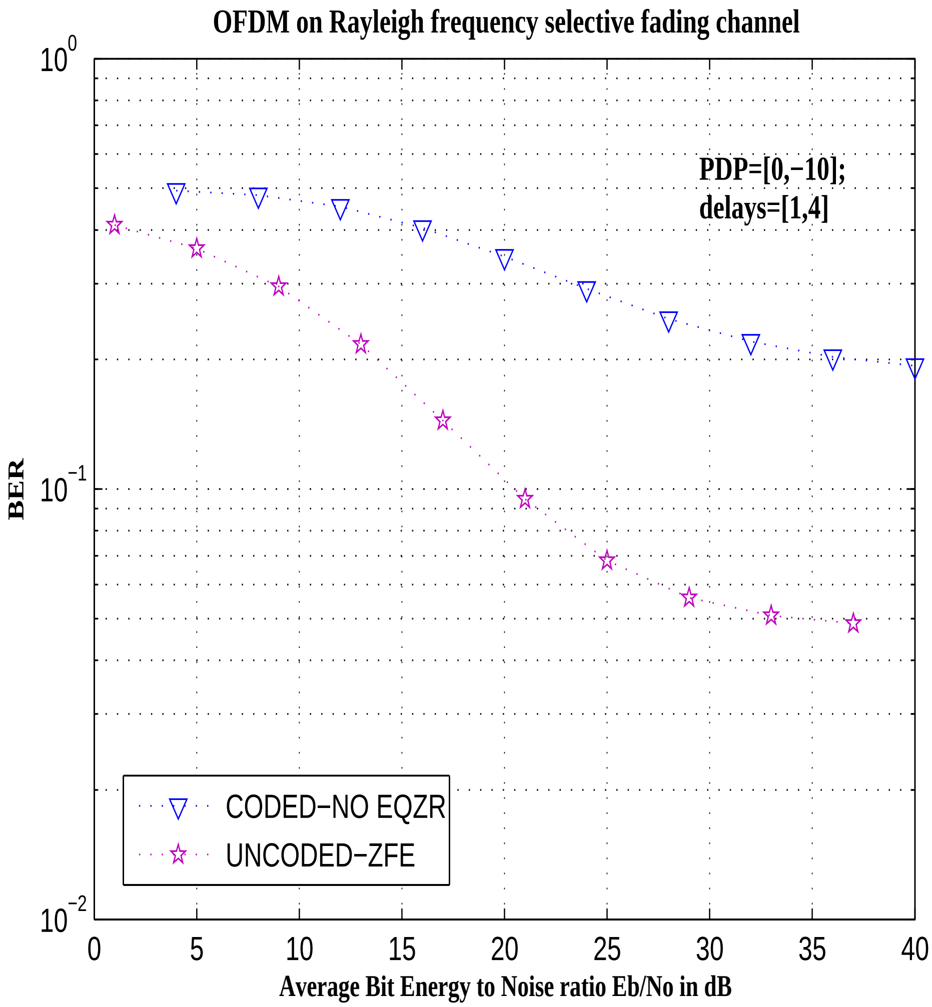

Figure 4.6. BEP:OFDM-BPSK, 2-tap Rayleigh-uncoded ZFE, COFDM.

spread of the channel, the ISI can be eliminated completely and the performance of the system is obtained similar to the flat fading channels as obtained in [16]. Thus it can be seen that an OFDM system reduces a frequency selective fading channel into a set of flat fading sub-channels. The results in fig:4.8 were compared to the theoretical error rates for flat fading channel for a BPSK system and are in coherence with a 1 to 1.25 dB degradation which results due to the loss of energy in transmission due to the presence of cyclic prefix.

Also it can be seen that OFDM with cyclic prefix greater than the length of the maximum delay spread of the channel peforms better than that without the cyclic prefix.

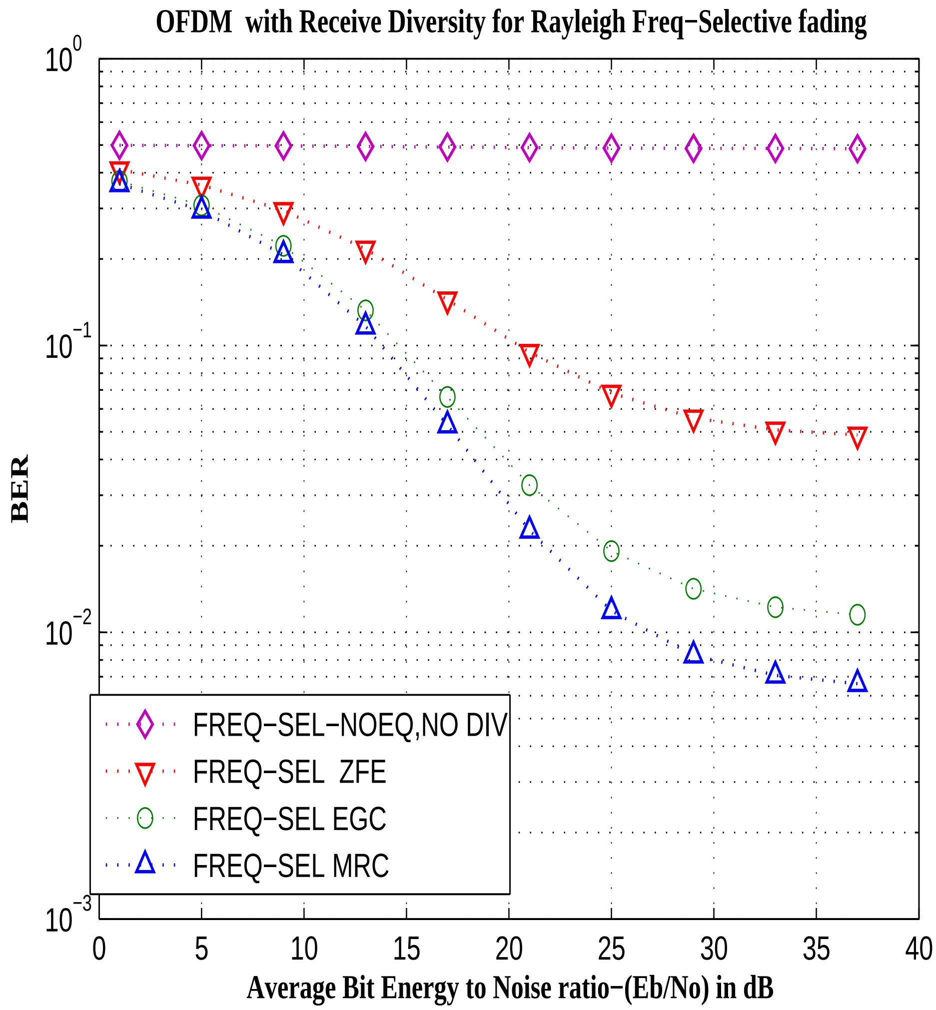

Figure 4.7. BEP OFDM,COFDM-BPSK on 2-tap Rayleigh-EGC,MRC.

An equal gain combining scheme with L =2 antennas proposed in chapter 3 is later implemented with cylic prefix and the performance plots are compared to that with a single antenna. The performance of this system,in fig:4.9 is close to that of EGC on flat fading channels as expected. The model of the channel used for all the schemes is a 2-tapped channel as mentioned previously. This also supports the statement that diversity combining schemes are a viable option for improving the performance of a communication system. The performance of equal gain combining scheme with OFDM is compared to that without any diversity reception and considerable diversity gains are obtained.

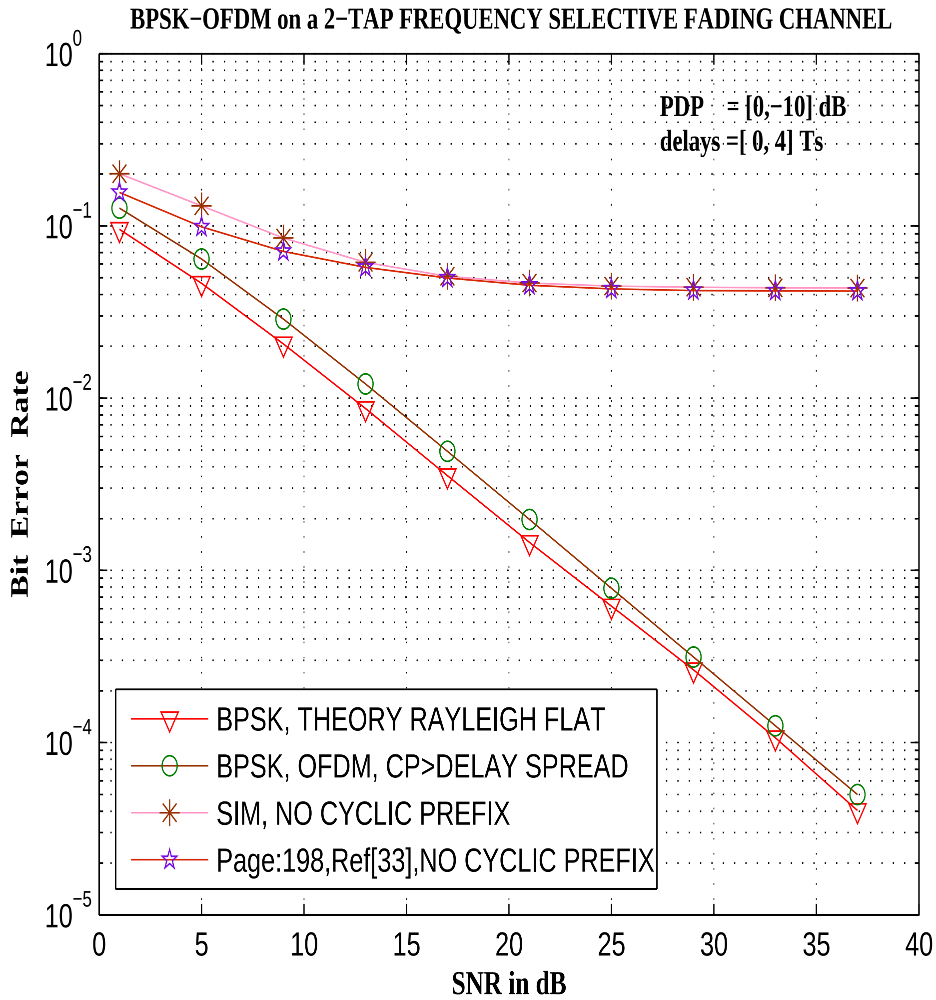

Figure 4.8. BEP-OFDM-BPSK on Rayleigh 2-tap-multi-path channel.

Finally, the effect of inter carrier interference in OFDM system on fast fading time variant channels has been simulated and the performance with and without diversity-equalization schemes proposed in chapter 3 was investigated. A time variant multipath fading channel is iplemented as matrix with random entries with each entry being obtained from a fast fading Rayliegh random process simulated by the Clarke's model. Since the taps are varying for each sub carrier in the OFDM symbol, the net effect can be simulated by arranging the taps of the channel in each row of a matrix and then cyclically shifting the postions of the vaying taps in each row.

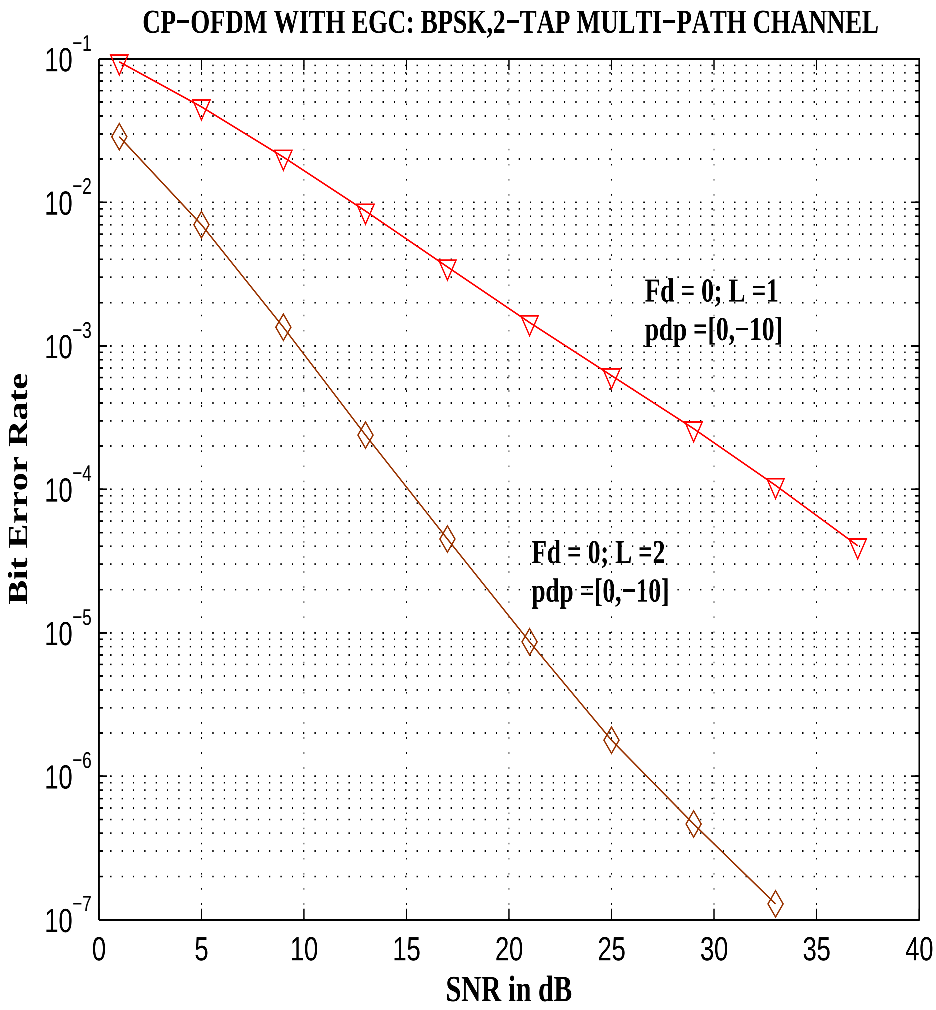

Figure 4.9. BEP-EGC-OFDM on Rayleigh 2-tap-multi-path channel.

Each OFDM symbol is multiplied by such a channel matrix. The rest of the parameters are set as mentioned earlier and the simulation is performed iteratively with 80 such OFDM blocks The equalizers used for ICI mitigation were the Zero forcing equalizers. Combining is done post-FFT. Only 10^5 iterations are possible due to paucity of resources and time. The average error rate plots are obtained as in fig:4.10

For validation, the results are compared with the performance plots obtained by Wang in [35],[1].It can be seen that for a single antenna with the same system parameters taken in [35], the results obtained are as in fig4.11. As mentioned in previous chapters, employing diversity equalzation techniques improve the performance of such a scheme and

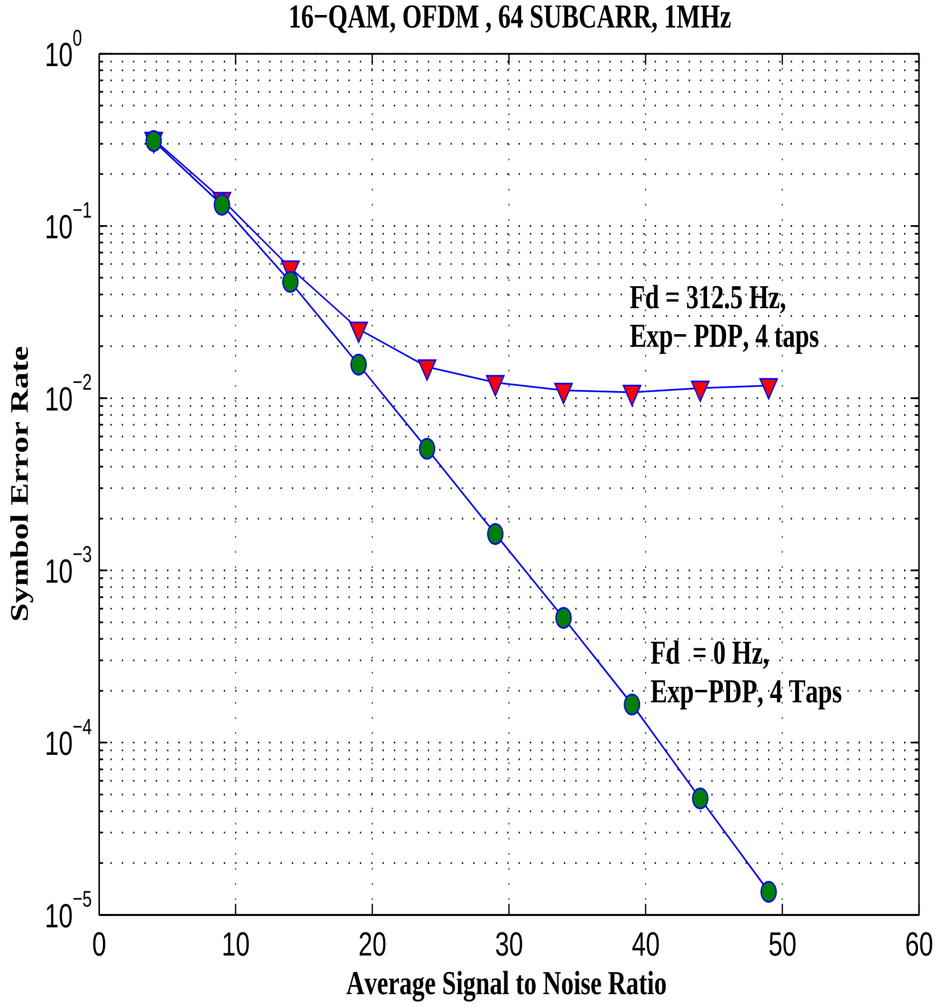

Figure 4.10. SEP OFDM-16QAM on 4-tap Rayleigh-channel.

there is a signifiacnt reduction in error floors produced due to ICI as shown in fig:4.12. However the error floors produced cannot entirely be elimnated in all the cases due to the presence of ICI. Hence diversity-equalization schemes are favourable to use only for channels with smaller Doppler spreads.

The performance can be compared to the SVD technique which is the optimum single antenna scheme that eliminates ICI in the system by projecting the received signal onto an eigen basis of a given channel matrix. The magnitude of the error floors obtained are specified by the normalized Doppler spread,i.e F_dNT_s for all the time vari-

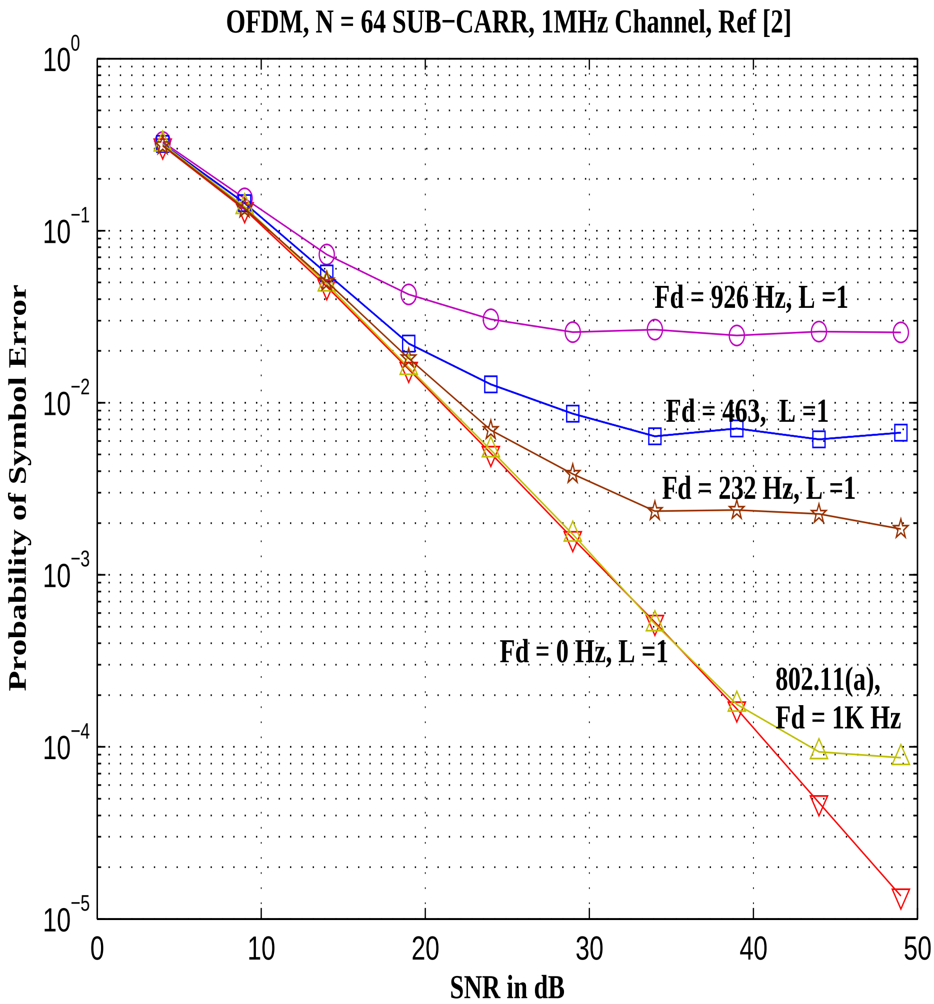

Figure 4.11. SEP OFDM-16QAM System parameters from Ref[1].

ant frequency selective fading channels with an exponential channel profile as mentioned [22],[16].

However, the SVD based equalization scheme is difficult to be realized practically. It invloves transmit precoding which is performed based on the estimated channel coeffecients provided by the receiver. The accuracy of these channel gains at high speeds and the computational complexity make it a highly impractical option. Turning on the diversity antenna when the signal to noise ratio is low or at high speeds is the best possible alternative in such cases. The SVD technique cannot be implemented as a post FFT diversity combining scheme as there will be two transmit pre-coding vectors corresponding

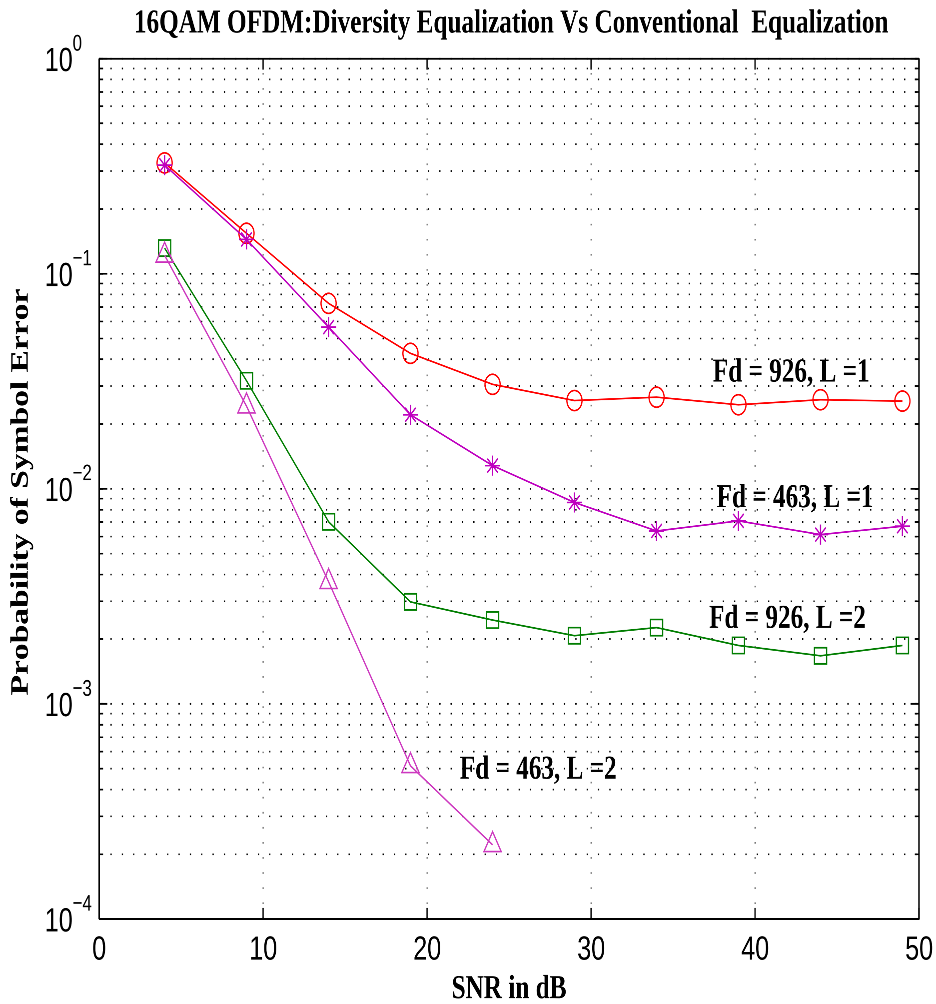

Figure 4.12. SEP OFDM-16QAM System parameters from Ref[1].

to each multi-path channel to be applied in order to diagonalize the channel. Schemes based on Karhunen-Loeve expansion theorem, which involve the spectral decompostion of the covariance matrix of the channel can also be considered here for comparison. In all the cases the estimation of channel gains on a fast varying channel is a difficult task. In such cases predicting the channel coefficients at the transmitter apriori based on past channel conditions can help. Hence eliminating Inter carrier interference from the system like IBI is almost an impossible task.

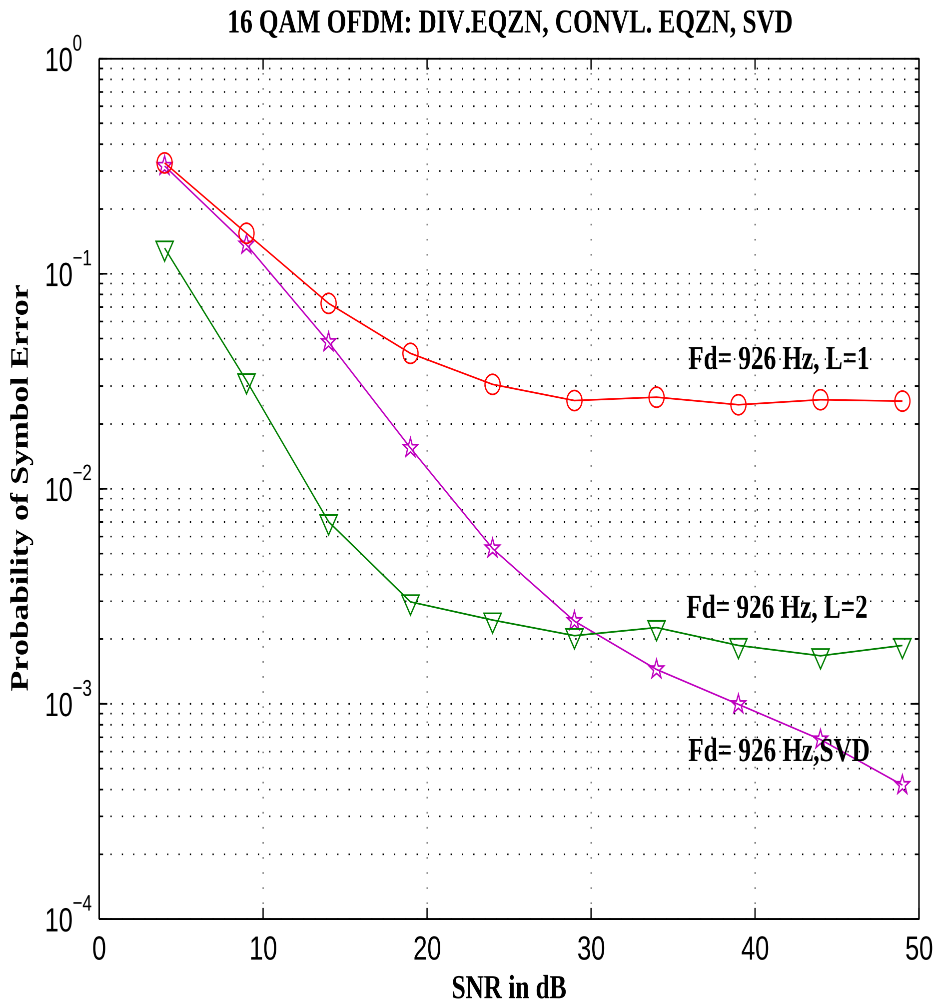

Figure 4.13. SEP OFDM-16QAM System parameters from Ref[1].

4.2 Accuracy and Confidence Intervals

Though Monte Carlo simulations empirically determine the probability of bit error, they are computationally costlier than any other simulation methods. The cost of the simulation is related to the number of times an experiment is performed to obtain a given reliability which in turn, depends on the spread of an estimated value for a given number of trials. Hence variance of the estimate is a measure of reliability. A more accurate measure of reliability would be confidence intervals as one can associate probability to an interval. The MC method of estimating BER uses a relative frequency approach and the BER is obtained as the ratio of the number of errors from N bits processed. Let r be the number of errors after N bits are demoodulated then P_e is obtained as,

$$P_e = \frac{r}{N} \tag{4.3}$$

In order to determine the confidence intervals, one needs to find the upper and lower bounds P_U and P_L for the sample mean P_e such that $P_L \leq P_e \leq P_U$ and the difference between between the upper and lower bounds, $P_U - P_L$ is as small as possible. A smaller confidence interval means an accurate estimate of the BER. Since the sample mean P_e is a random variable, each interval can be associated with a probability which is often termed as the confidence level for the interval considered and is defined as,

$$Prob\{P_L \leq P_e \leq P_U\} = 1 - \alpha \tag{4.4}$$

Since P_e is binomial, the above interval can be split into two one-sided intervals and P_U, P_L can further be shown[36] to be the solutions of the equations,

$$\sum_{k=0}^{r} \frac{N!}{k!(N-k)!} {P_U}^k (1 - P_U)^{N-k} = \alpha/2 \tag{4.5}$$

$$\sum_{k=r}^{N} \frac{N!}{k!(N-k)!} {P_L}^k (1 - P_L)^{N-k} = \alpha/2 \tag{4.6}$$

A binomial distribution converges to a normal distribution as $N \to \infty$. Therefore the estimate P_e can be treated as a normal random variable. However this assumption is valid only for higher error rates. For lower rates a Poisson approximation can be used. The normal approximation to P_e for large N yields the confidence interval for P_e as follows,

$$Prob\{P_L \leq P_e \leq P_U\} = 1 - \alpha \tag{4.7}$$

$$P_{L/U} = 10^{-k}\{1 + \frac{d^2}{2\eta}[1 \pm \sqrt{\frac{4\eta}{d^2} + 1}]\} \tag{4.8}$$

where $\eta = \frac{N}{10^k}$ and d is given by,

$$\int_{-d}^{d} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy = 1 - \alpha \tag{4.9}$$

The confidence intervals are plotted w.r.t N as in [36],the number of bits processed, for different confidence levels from 90 to 99 percent, It can be seen that the value of N required to obtain P_e for these confidence levels is $10/P_e$ with the confidence interval $(2P_e, 0.5P_e)$. However these bounds are valid only if the ratio r/N is considerably large, i.e the number of errors counted are high. For smaller ratios of r/N, the binomial distribution of P_e can be approximated by using a Poisson distribution. As mentioned in[37], P_U and P_L can be calculated for small values of r/N from equations which are to be evaluated numerically,

$$\sum_{k=0}^{r} \frac{(NP_U)^k e^{-NP_U}}{k!} \leq \frac{1-\alpha}{2}. \tag{4.10}$$

$$\sum_{k=r}^{N} \frac{(NP_L)^k e^{-NP_L}}{k!} \leq \frac{1-\alpha}{2}. \tag{4.11}$$

According to the above approximations, since the number of trials considered for all the simulations in the estimation of BER is 10^5, the estimated BER is within the interval $(2P_e, 0.5P_e)$ with a 90 to 95 percent confidence for the cases whose estimated BER is greater than 10^{-4}. Since the ratio r/N is large enough the approximation based on Central limit theorem(CLT)is used. For BERs lower than 10^{-4} for the same confidence levels i.e 99 percent, the error bars can be determined from the equations based on Poisson approximation or can be evaluated by CLT yielding a wider confidence interval. The CLT yields a confidence interval of $(10^{-(k-1)}, 10^{-(k+1)})$ for a BER of 10^{-k}. Since the inerval is too large for any conclusions, lowering the confidence level, will decrease the length of the confidence interval. Considering a 60 percent confidence level would yield a confidence interval of $(2.2 \times 10^{-(k-1)}, 4.4 \times 10^{-(k+1)})$ and 80 percent level an interval of $(3.343 \times 10^{-(k-1)}, 2.99 \times 10^{-(k+1)})$ for a BER of 10^{-k}. Polynomial approximations proposed in[37] provide confidence intervals for all values of r/N accurately until $N = 10^6$.

4.3 Conclusion

In this section the results obtained by M.C simulation of the proposed system have been presented and validated based on the previously published results.

CHAPTER 5

CONCLUSIONS

In this chapter the conclusions on the work done in the thesis are provided followed by the contributions. Finally some areas for future work have been mentioned.

- The performance of OFDM systems on an AWGN channel is similar to the performance of any other single carrier modulation scheme on AWGN channel. Increase in the number of sub-carriers does not effect the performance of such cases.
- An OFDM system performs similar to that of a single-carrier flat fading channel and on a slow frequency selective fading channel experiences higher degradation in the absence of a cyclic prefix causing an error floor at a BER of 5×10^{-2} at an SNR of 24 dB compared to an SNR of 8 dB for the same BER, with out IBI.
- Diversity combining systems are better alternatives compared to equalization schemes in reducing the error floors due to IBI and the error floor is reduced by a factor of 2, i.e to 10^{-2} at 30dB when compared to a zero forcing equalizer (5×10^{-2})at 30dB.
- OFDM system with cyclic pre-fix on a time invariant frequency selective fading channel eliminates IBI and hence the error floors, whereas on a time variant multi-path channel irreducible error floors still exist and are between 10^{-2} to 10^{-3} between 20 to 30 dB.
- Diversity with equalization provides significant gains, i.e 8×10^{-2} at 11 dB corresponding to 21 dB for the same BER, compared to a single antenna system with equalizers on a time variant fading channel. Also these schemes reduce the error floors by $7 - 10$ times at SNR between 10 to 30db for 5 to 10 percent doppler spread w.r.t sub-carrier spacing. Hence diversity equalizers are better alternatives for reducing error floors at higher data rates.

5.1 Contributions

- The performance of an OFDM system on AWGN, slow flat, slow and fast frequency selective Rayleigh fading channels with considerable Doppler spreads has been analyzed through M.C simulations.
- System models for various diversity combining schemes such as MRC, EGC on slow frequency selective fading channels proposed with and without cyclic prefix have been analyzed and simulated for a dual antenna case.
- Performance of a dual antenna OFDM system with diversity-equalizers is compared to that of a single antenna system with equalizer on a time varying multi-path Rayleigh frequency selective fading channel and it is found that diversity-equalizers reduce the error floor by around 10 times but do not eliminate it.

5.2 Future work

An interesting area and a more natural development to the above work is to have generalized systematic methods to calculate the error probability for OFDM on time variant frequency selective fading channels in the presence of diversity reception techniques. Also another area to explore is to evaluate the performance of MIMO-OFDM systems using simulations. Performance of transmit diversity and space time codes and space frequency codes with OFDM are also some direct extensions of the work.

APPENDIX A

SIMULATION OF A MULTI-TAP CHANNEL

In this chapter the procedure to simulate a Rayleigh slow frequency selective fading channel has been summarized.

A multi-tap channel with L-independent taps can be generated from a complex Gaussian distributed random process. The channel model used is derived in chapter1.

Let $x(t) + jy(t)$ be the response of the channel at the receiver to a transmitted transmitted signal along the non LOS path and A(t) be the the direct path component of the transmitted wave. The total signal at the receiver is

$$r(t) = A(t) + x(t) + jy(t). \tag{A.1}$$

Where $x(t), y(t)$ are gaussian random variables with zero mean and variance 1/2. The envelope of the signal is obtained by taking the magnitude of $r(t)$;

Let $r(t) = R(t)\cos\theta(t) + j\sin\theta(t)$ then,

$$A(t) + x(t) = R(t)\cos\theta(t) \tag{A.2}$$

$$y(t) = R(t)\sin\theta(t) \tag{A.3}$$

The envelope of $r(t)$, i.e $R(t)$ can be calculated as $R(t) = \sqrt{[A(t) + x(t)]^2 + [y(t)]^2}$. This is called Rician fading envelope of the signal. In order for a signal to undergo Rayleigh fading, the signal has to be multiplied by the fading envelope. Let d(t) be the transmitted signal,At the receiver the faded signal is obtained as

$$r(t) = R(t)d(t). \tag{A.4}$$

The signal power at the receiver has to be normalized to one so that the calibration of noise remains unaffected. The signal at the output of the channel has to have the same power with which it is transmitted. The power of the combined signal can be calculated as

$$P_r = E[A(t) + x(t) + y(t)]^2 \tag{A.5}$$

P_r can be expanded and since the mean of $x(t)$ and $y(t)$ are both zero, the power in the envelope is obtained to be

$$P_r = E[A(t)]^2 + 2\sigma^2 \tag{A.6}$$

The power in the the line of sight component P_d be $\|A\|^2$,

K-parameter is defined as

$$K = \frac{\|A\|^2}{2\sigma^2} \tag{A.7}$$

Let the total input power in the signal be P_i,then $P_i = P_d + 2\sigma^2$, Using the above equations for K and P_i,

$$\|A\| = \sqrt{\frac{P_i K}{K+1}} \tag{A.8}$$

$$\sigma = \sqrt{\frac{P_i}{K+1}} \tag{A.9}$$

Using the above constants an L -tap channel can be obtained with the following steps

- 1.The channel taps can be simulated by scaling the complex random variables generated by the randn function with the above constants. To generate a channel with L taps; Two L by 1 vectors can be used. One for the LOS component with its elements set to ones. Another vector with L Complex gaussian distributed random variables with mean zero and variance 1.
- 2.The vector with ones is scaled by the constants$\|A\|_i$ in each tap and the other by σ_i depending on the input power in each tap. These two vectors are added element by element to form a set of L Rician random variables. One can verify if the power profile of the taps varies according to the required conditions.
- 3.Another vector is made such that its elements are zero at the places where the taps are present for the channel and the ones at the places where the taps are non-zero. Replace all the ones with the L taps generated previously to obtain the multi tap channel with L independent taps.

- 4.For a Rician envelope of a signal, i.e the output of a Rician flat fading channel; scale the noise added signal by$\|A\|$ and add to the same noise added signal scaled byσ. The resultant signal is the output of the channel.

REFERENCES

[1] T.Wang, J. Proakis, E.Masry, and J.Zeidler, "Performance degradation of ofdm systems due to doppler spreading," *IEEE Trans. Commun.*, vol. 5, pp. 1422–1432, June 2006.

[2] A. Goldsmith, *Wireless Communications.* Cambridge: Cambridge University Press, 2005.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications.* Cambridge, UK: Cambridge University Press, 2005.

[4] G. Stuber, *Principles of Mobile Communications.* Norwell, MA: Kluwer, 1996.

[5] J. Proakis, *Digital Communications.* New York, NY: McGraw-Hill, 1995.

[6] L. Litwin and M. Pugel, "The principles of ofdm," January 2001, article id:145279.

[7] L. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 33, pp. 665–675, Apr. 1985.

[8] S. Weinstein and P. Ebert, "Data transmission by frequency-division multiplexing using discrete fourier transform," *IEEE Trans. Commun.*, vol. 19, pp. 628–634, Oct. 1971.

[9] Z. Jhun, "Analysis of coded ofdm system over frequency selective fading channels," M.S thesis, Texas A and M University, College Station, Texas, Nov. 2003.

[10] G. B. Zhiqiang Liu, Yan Xin, "Linear constellation precoding for ofdm with maximum multipath diversity and coding gains," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 416–427, Mar. 2003.

[11] Y. Mostofi and D. C. Cox, "Average error rate analysis for pilot-aided ofdm receivers with frequency domain interpolation."

[12] M.-X. Chang and Y. Su, "Performance analysis of equalized ofdm systems in rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 1-4, pp. 721–732, Oct. 2002.

[13] W. S. Burchill, "Performance and receiver structures for ofdm on rayliegh fading channels," Ph.d Thesis, The University of British Columbia, Vancouver,Canada, Mar. 1997.

[14] T. Tjhung and F.Adachi, "Error proabability analysis for 16 star-qam in frequency-selective ricean fading with diversity reception," *IEEE Trans. Veh. Technol.*, vol. 47, no. 3, pp. 924–935, Aug. 1998.

[15] T. Staley, R. North, and J. Zeidler, "Performance evaluation for multi-channel reception of coherent mpsk over frequency selective fading channels," *IEEE Trans. Veh. Technol.*, vol. 50-4, pp. 877–893, July 2001.

[16] R. Prasad, *OFDM for Wireless Communications Systems.* Boston, MA: Artech House, 2004.

[17] M.-X. Chang and Y. Su, "Performance analysis of equalized ofdm systems in rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 1-4, pp. 721–732, Oct. 2002.

[18] N. Beaulieu and A. Abu-Dayya, "Analysis of equal gain diversity on nakagami fading channels," *IEEE Trans. Broadcast.*, vol. 48-3, pp. 223–229, Sept. 2002.

[19] X. Ma, "Channel estimation of ofdm in wireless applications," Ph.d Thesis, Princeton University, NewJersey,Princeton, Nov. 2004.

[20] S. Crohn and E. Bonek, "Error floor mechanisms in indoor digital portable communications," *IEEE*, 1992.

[21] E.Leung and P.Ho, "A successive interference cancellation scheme for an ofdm system," *In Proc IEEE Commn*, pp. 7–11, June 1998.

[22] T. Tjhung and F.Adachi, "Ber performance of ofdm-dmpsk system in frequency-selective rician fading with diversity reception," *IEEE Trans. Veh. Technol.*, vol. 49, pp. 1216–1226, July 2000.

[23] D. Brennan, "Linear diversity combining techniques," in *In Proc. IEEE*, Feb 2003, pp. 332–356.

[24] M. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. Hoboken, NJ: Wiley, 2005.

[25] V. Prabhu, "Psk performance with imperfect carrier phase recovery," *IEEE Trans. Aerosp. Electron. Syst.*, vol. ASE-12, pp. 275–286, Mar. 1976.

[26] S. Chennakeshu and J. B. Anderson, "Error rates for rayleigh fading multichanel reception of mpsk signals," *IEEE Trans. Commun.*, vol. 43, pp. 338–346, Apr. 1995.

[27] N. Beaulieu, "An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of rayleigh random variables," *IEEE Trans. Commun.*, vol. 38, pp. 1463–1474, Sept. 1990.

[28] Q. Zhang, "Probability of error for equal-gain combiners over rayleigh channels: Some closed-form solutions," *IEEE Trans. Commun.*, vol. 45, pp. 270–273, Mar. 1997.

[29] G. Karagiannidis, "Moments based approach to the peformance analysis of equal gain diversity in nakagami-m fading channels," *IEEE Trans. Commun.*, vol. 52-5, pp. 685–691, May 2004.

[30] C. Tellambura, A.Annamalai, and V. Bhargava, "Equal gain diversity receiver performance in wireless channels," *IEEE Trans. Commun.*, vol. 48, no. 10, pp. 1732–1745, Oct. 2000.

[31] I. Alfalujah, "Performance analysis of psk systems in the presence of slow fading, imperfect carrier phase recovery and awgn," *In Proc IEEE Commn*, vol. 152, no. 152, pp. 903–911, Dec. 2005.

[32] M. Najib and V. Prabhu, "Lower bounds on error performance for bpsk and qpsk systems with imperfect carrier phase recovery," in *Proc. IEEE Int. Conf. Commun. Technology*, Atlanta,GA, June 1998, pp. 1253–1258.

[33] R. Gray. Toeplitz and circulant matrices: A review. Internet draft. [Online]. Available: http://ee.stanford.edu/ gray/toeplitz.pdf

[34] R.Prasad and H. Harada, *Simulation and Software Radio for Mobile Communications.* Norwood MA: Artech House.Inc, 2002.

[35] T. Wang and J. Proakis, "Techniques for suppression of inter carrier interference in ofdm systems," in *Proc. IEEE Wireless Communications and Networking Conference,2005*, Mar. 2005, pp. 39–44.

[36] M. C. Jeruchim, "Techniques for estimating the bit error rate in the simulation of digital communication systems," *IEEE J. Select. Areas Commun.*, vol. 2, no. 1, pp. 153–170, Jan. 1984.

[37] K.Kosbar and T.Chang, "Interval estimation and monte-carlo simulation of digital communication systems," in *Proc. IEEE MILCOM,1992*, Nov. 1992, pp. 3.3.1–3.3.5.

BIOGRAPHICAL STATEMENT

Sai Sravan Bharadwaj Karri was born in Srikakulam, India, in 1983. He received his B.Tech. degree from Jawaharlal Nehru Technological University-Hyderabad,India in June 2004. He joined the University of Texas at Arlington in Fall 2005 and received his M.S degree in Electrical Engineering in Spring 2008. He was the recipient of Dean's Graduate scholarship and worked as Graduate Teaching Assistant for Linear Systems and Fundamentals of Telecommunication Systems at UT, Arlington. He also interned at Qualcomm,San Diego and joined as a Engineer in 2008. His current research interest include the analysis and performance improvement of OFDM systems for mobile cellular systems,Coding Theory and Linear Algebra. He is an associate member of IEEE.