

ANALYSIS OF LAMINATED TUBULAR STRUCTURE
UNDER TORSION

by

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ABSTRACT

ANALYSIS OF LAMINATED TUBULAR STRUCTURE UNDER TORSION

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Laminated composite shells in particular are prominent in bearing various types of loads and are hence used in many engineering structures. The analysis of laminated composites is quite complicated since the material behavior is anisotropic, which gets further intensified in the analysis of complex structures like tubular structures. Finite element method is a useful method for evaluating and conducting the behavior of thin laminated tubular structure subjected to torsion.

In this study, the behavior of shear stress versus angle along the laminated tubular structure cross section was conducted in varies radius ratio which are 1, 0.8, 0.6, 0.4, and 0.2 on different fiber orientation which are symmetry and antisymetry. The results reveal an unsteady behavior pattern on the high curvature section that effected when the radius ratio is reducing to a small number. However, in the high radius ratio the tubular structure shear stress is identical to laminate theory

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CHAPTER 1

INTRODUCTION

1.1 Composite Material Overview

As composite material becomes more and more cost effective, they are beginning to replace many conventional materials in everyday application. For example, the automotive industry is being forced to design lighter car, to increase fleet gas mileage. Composite shafts, for example, golf shafts head to graphite fiber material, commonly called "Broadgood". Aerospace engineering is also changing. Aero planes have traditionally been made out of metal, usually aluminum alloys; now however, engineers are increasingly working with carbon fiber composites. For example, A380, the world's largest passenger aircraft, that these materials have been deployed extensively in primary load carrying structure. The A380 uses composite materials in its wings, which helps enable a 17% lower fuel use per passenger than comparable aircraft.

Laminated composite shells in particular are prominent in bearing various types of loads and are hence used in many engineering structures. The analysis of laminated composites is quite complicated since the material behavior is anisotropic, which gets further intensified in the analysis of complex structures like shells. Laminated composite shells are used in fuselage structures, missiles and spacecraft, jet nozzles etc.

The behavior of material such as a stress concentration or modulus stiffness is important for engineering design because of these environments are guiding an engineer to reach a successive design. Unlike isotropic materials for which the applied normal stress (or shear stress) induces only the normal (or shear) stress, all three displacement components and most of the six stress components are nonzero for general anisotropic materials.

Due to the complexity of the governing equations, the availability of variety of elasticity solutions for laminated composite shells has been scarce in comparison to laminated plates. This paper followed laminated shell theories to calculate stress and stiffness modulus of tubular (ESP. circular) tube made of fiber reinforced composites subjected to torsion.

1.2 Literature Review

In 1968, Lekhniskii [1] used anisotropic elasticity with complex variable method to give close form solution to compute stress distributions around a circular hole in an infinite anisotropic plate. Average laminate elastic properties to compute average stress distribution in the infinite size laminate with hole was used.

In 1971, Whitney [2] developed a shell theory known as Vlasov-Ambartsumyan and applied in order to calculate stresses in anisotropic and laminated cylinders subjected to combination of axial load, torsion, and internal pressure. Comparison to results obtained from exact elasticity theory shows that the shell equations are capable of predicting, with a reasonable degree of accuracy, the large stress gradients found in highly anisotropic tubes.

In 1996, Loughlan and Ata [3] introduced a simple engineering theoretical approach which is able to predict the initial constrained torsional response of a specific class of thin-walled, single-cell, graphite fibrous composite box beams. Comparisons between theory and finite element and between theory and experiment are illustrated in a good agreement.

In 1996, Liu [4] has developed a theory for nonlinear bending of symmetrically laminated, cylindrically orthotropic, shallow conical shells subjected to an axisymmetrically distributed load. Transverse shear effects were also included in his model.

In 1997, Chan and Demirhan [5] derived a closed form analytical solution for evaluation of bending stiffness of composite tubes. Both plate and shell approaches were developed. A conventional approach, which in used the smear property is also used for comparison. They concluded that the laminated shell approach gave a better result than laminated plate approach when the radius of the circular tube became smaller. Their result reveals that for a small radius,

the conventional approach could give an error up to 50% comparing with 11.5% for plate approach and 1.3% for the shell approach.

In 1999, Chouchaoui and Ochoa [6] introduced a general analytical model developed for the stress and displacement of an assembly of several coaxial-laminated hollow circular cylinders that were subjected to internal/extensional pressure, tensile, torsion and bending load.

In 2000, Tarn and Wang [7] presented a developed state space approach to extension, torsion, bending, shearing and pressuring of laminated composite tubes. An analysis of the tube subjected to uniform tractions on the inner and outer surfaces, and axial force, a torque and bending problem at the ends was conducted. The characteristics of the Eigen solution of the system matrix are used to advantage in deriving the fundamental transfer matrices.

In 2001, Lin and Chan [8] developed a simplified closed form expression of stiffness matrices for elliptical composite tube. The results show that the bending stiffness of the tube and the layer stresses are in an excellent agreement with the results obtained from finite element method.

In 2003, Correia and Herskovits [9] have presented a numerical method for the structural analysis of laminated conical shells using a quadrilateral isoperimetric element based on higher order shear deformation theory. The model developed can be used to perform static analysis with arbitrary boundary conditions and loads and for solving Eigen values problems.

In 2007, Rao [10] developed the closed form expressions for determining the displacement and twisting angle of tapered composite tubes. The analytical expressions are developed based on the modified laminated plate theory which includes the tubular wall curvature of the laminate. It is found that the axial deformation and the twisting angle calculated by the current method agree well with the results obtained from the finite element method.

1.3 Objectives of this study

The primary objective of the study is to investigate the behavior of stress along the cross section of laminated tubular structure subjected to torsion. Both simple approximate

analytical method and finite element method were used. Five laminates with different stacking sequences ranging from symmetrical and balanced laminate, antisymmetrical and balance and unsymmetrical and balanced were studied. The tube with a constant of half major axis, 0.5 in and its half minor axis, ranging from 0.5 to 0.1 in.

1.4 Outline of the Thesis

In chapter 2, the convectional thin wall and laminated plate theory under torsion are described, respectively.

Chapter 3 describes the finite element model procedures. The geometry, boundary condition, and loading condition of the tubular structure are included. The developed finite element model is validated at the end of the section.

Chapter 4 presents the comparison results of stress X, Y, Z and shear stress YZ versus angle along the tubular cross section. The behavior of stresses on laminate stacking sequence is described.

Conclusion and future study are presented in chapter 5.

CHAPTER 2

THE THEORY OF THIN-WALLED UNDER TORSION AND LAMINATED SHELL

In this chapter, an analytical equation of thin-walled tubes under torsion and analytical solution for calculating stiffness matrix for laminate plate are reviewed.

2.1 Thin-Walled Tubes Subjected to Torsion

The first assumption is that the wall tube is thin when compared to the tube radius. First rule is the thickness should be less than 10% of the smallest overall external dimension. Second, tube does not have any cut or slice. In other words, it is continuous around the radius. The tube does not need to be circular but it should not have severe reverse curvature. Some acceptable cross sections are shown in the diagram at the left. Also, the thickness does not need to be constant.

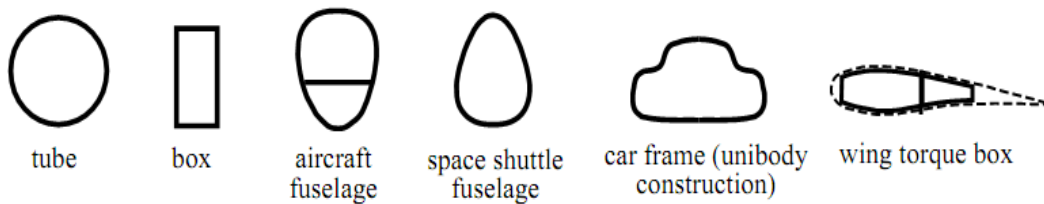


Figure 2.1 Examples of Thin-Walled Tube Cross-Sections

2.1.1. Shear Flow in Thin-Walled Tubes

To analyze thin-walled tubes, the concept of shear flow, q , needs to be understood. When a moment (or torque) load, T , is applied to a thin-walled tube, the load is distributed around the tube as a shear stress, τ . But this is a difficult value to deal with since it is not constant. A better value is the shear flow, which is simply the stress multiplied by the changing thickness, t .

$$q = \tau T \quad (2.1)$$

The shear flow multiplied by an arc length of the tube, ds , will give a force. This force, dF , causes a moment, dT , about the tube center. Summing all the moments caused by all the differential lengths, ds , will give the total applied moment, T .

$$\oint_s r dF = \oint_s r q dS = T \quad (2.2)$$

For a small element, the area of the triangular section, dA_0 , is approximately,

$$dA_0 = 0.5 r dS \quad (2.3)$$

The total area can be calculated by integration over the total tube cross section length, S , giving,

$$\oint_s r dS = 2A_0 \quad (2.4)$$

For a closed section of beam, shear flow is a constant along its contour. Hence, equation 2.2 can be rewritten as;

$$q = \frac{T}{2A_0} \quad (2.5)$$

2.1.2. Angle of Twist in Thin-Walled Tube

The angle of twist for a closed thin tube with a noncircular cross section is developed by using energy methods along with the previous equations. First, at a small element from a typical tube that is undergoing deformation, the stored strain energy can be found from using the basic concept that energy is equal to a force operating through a distance, $U = F \cdot L$. For the small infinitesimal element, ds by dx , the force causing the deformation is dP , $q ds$. At the base, the

deformation is zero and at the free end is (γds) . Thus the average deformation is simply $(\gamma dx)/2$. The total strain energy for the element is

$$dU = [dP] [0.5 (\gamma ds)] = (q ds)(\gamma ds)/2 = q \gamma (ds)^2/2 \quad (2.6)$$

From Hooke's law

$$\gamma = \tau/G \quad (2.7)$$

And,

$$\tau = q/t \quad (2.8)$$

Substitute γ and τ into the energy equation gives,

$$dU = \frac{q^2}{2Gt} ds dx \quad (2.9)$$

Integrated over the length of the tube, L , and around the circumference of the tube, this gives,

$$U = \frac{q^2}{2G} \int_0^L \left(\oint_S \frac{ds}{t} \right) dx = \frac{q^2 L}{2G} \oint_S \frac{ds}{t} \quad (2.10)$$

Apply simply moment $U = T/2$ where θ is the angle of twist, gives,

$$\frac{T\theta}{2} = \left(\frac{T}{2A_0} \right)^2 \frac{L}{2G} \oint_S \frac{ds}{t} \quad (2.11)$$

Rearranging gives,

$$\theta = \frac{TL}{4GA_0^2} \oint_S \frac{ds}{t} \quad (2.12)$$

For constant thickness we got

$$\theta = \frac{TL}{4GA_0^2t} \quad (2.13)$$

2.2 Laminated Plate Theory

The laminated tube considered in a uniform tube, with tubular cross section with an outer radius R_o , inner radius R_i and a length L . The length of the tube is sufficiently larger than its radii. Hence the tube considered is a long tube. In all the derivations the basic assumption of 'Plane remains plane after deformation' is made.

2.2.1. Stress-Strain Relationship of Thin Lamina

In most practical applications of composite material, the laminates are considered as a thin and loaded along the plane of laminates. Since each lamina is a thin layer, one can treat a lamina as a plane stress. This simplification immediately reduces the 6x6 stiffness matrix to 3x3 one.

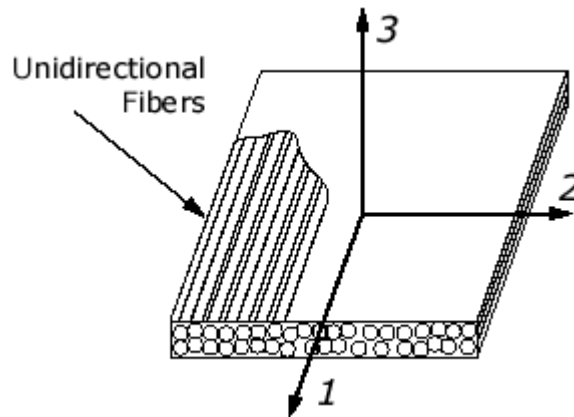


Figure 2.2 Example of Lamina

The thin layer composite is assumed to be under a state of plane stress. This implies that all stress components in the out of plane direction are zero. That is,

$$\sigma_3 = 0$$

$$\tau_{23} = \tau_{43} = 0$$

$$\tau_{13} = \tau_5 = 0 \quad (2.14)$$

Since each lamina is constructed by unidirectional fibers bonded by a matrix, it can be considered as an orthotropic material. Thus, the stress-strain relations on the principal axes of the laminate plane can be expressed by the compliance matrix [S] as

$$[\varepsilon] = [S][\sigma]$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix}$$

Or

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{\nu_{12}}{E_1} & 0 \\ \frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} \quad (2.15)$$

The reduce stiffness matrix [Q], which are function of the elastic constants, can be expressed as such that

$$[\sigma] = [Q][\varepsilon]$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [Q_{1-2}] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Or

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{12}E_1}{1 - \nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} & \frac{1}{1 - \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

(2.16)

Where the subscribes 1, 2 and 3 refer to the properties along the fiber, transverse to the fiber and shear in plane, respectively. E_1 and E_2 are Young's moduli along and transverse to the fiber direction, respectively, ν_{12} is the Poisson's ratio and G_{12} is the shear modulus of the lamina under a loading along the fiber direction.

2.2.2. Transformation of Stress and Strain

Stress and strain at a point in a thin wall of composite tube are defined with respect to an arbitrary coordinate system, e.g. principle coordination. The values of the components of stress or strain change as this coordinate system is rotated. It is necessary to perform this rotation transformation between 2 general coordinate systems or to determine the magnitudes and orientation of the principal stress and strain components.

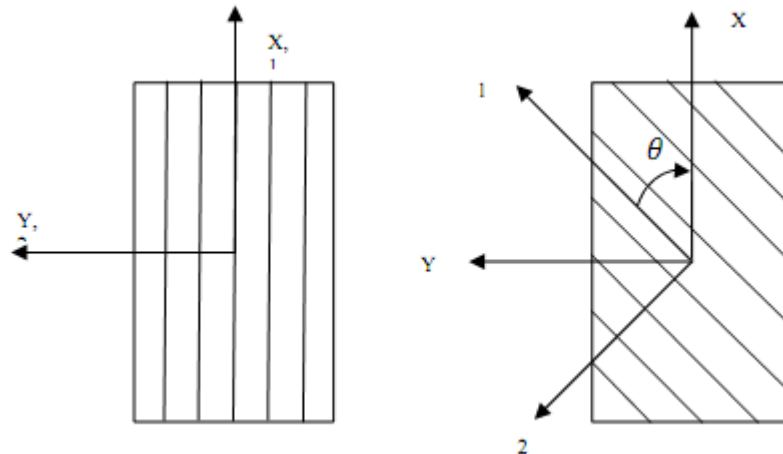


Figure 2.3 Coordinate of Lamina

Consider the fiber oriented with an angle, θ , with respect to lamina axis as shown in

Figure 2.3. Then we can get a stress and strain transformation equation as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T_\sigma(\theta)]_z \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (2.17)$$

and

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [T_\varepsilon(\theta)]_z \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2.18)$$

When we transform the coordinate of plane stress from local to global coordinates, it can be written in the following matrix form

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T_\sigma(-\theta)]_z \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (2.19)$$

Similarly, the strain transform becomes

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = [T_\varepsilon(-\theta)]_z \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (2.20)$$

The stress transformation matrix, $[T_\sigma]_z$, and strain transformation matrix, $[T_\varepsilon]_z$, for a 2D case are given as

$$[T_\sigma(\theta)]_z = \begin{bmatrix} m_z^2 & n_z^2 & 2m_z n_z \\ n_z^2 & m_z^2 & -2m_z n_z \\ -m_z n_z & m_z n_z & m_z^2 - n_z^2 \end{bmatrix} \quad (2.21)$$

and

$$[T_\varepsilon(\theta)]_z = \begin{bmatrix} m_z^2 & n_z^2 & m_z n_z \\ n_z^2 & m_z^2 & -m_z n_z \\ -2m_z n_z & 2m_z n_z & m_z^2 - n_z^2 \end{bmatrix} \quad (2.22)$$

Where $m_z = \cos\theta$ and $n_z = \sin\theta$ the angle θ is positive when the angle from the x-y axis to 1-2 axis is counter clockwise. From equation (2.21) and (2.22), we have:

$$[T_\varepsilon(\theta)]_z = [T_\sigma(-\theta)]_z^T \quad (2.23)$$

Substitute equation 2.21 into 2.19 we have stress – strain in relationship as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T_\sigma(-\theta)]_z \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T_\sigma(-\theta)]_z^T \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Or,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2.24)$$

Where,

$$\bar{Q}_{11} = m_z^4 Q_{11} + 2n_z^2 m_z^2 (Q_{12} + 2Q_{66}) + n_z^4 Q_{22} \quad (2.25)$$

$$\bar{Q}_{12} = 2n_z^2 m_z^2 (Q_{11} + Q_{22} - 4Q_{66}) + (m_z^4 + n_z^4) Q_{12} \quad (2.26)$$

$$\bar{Q}_{22} = n_z^4 Q_{11} + 2n_z^2 m_z^2 (Q_{12} + 2Q_{66}) + m_z^3 n_z (Q_{22} - Q_{22} + 2Q_{66}) \quad (2.27)$$

$$\bar{Q}_{16} = n_z^3 m_z (Q_{11} - Q_{12} - 2Q_{66}) + m_z n_z^3 (Q_{12} - Q_{22} + 2Q_{66}) \quad (2.28)$$

$$\bar{Q}_{26} = n_z^3 m_z (Q_{11} - Q_{12} - 2Q_{66}) + n_z m_z^3 (Q_{12} - Q_{22} + 2Q_{66}) \quad (2.28)$$

$$\bar{Q}_{66} = n_z^2 m_z^2 (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) + (m_z^4 + n_z^4) Q_{66} \quad (2.29)$$

2.3 Laminate Constitutive Equation

2.3.1 Definitions of Strains and Displacements

A displacement of the plate in the x, y, and z direction is designated as u, v, and w. The strains are defined as;

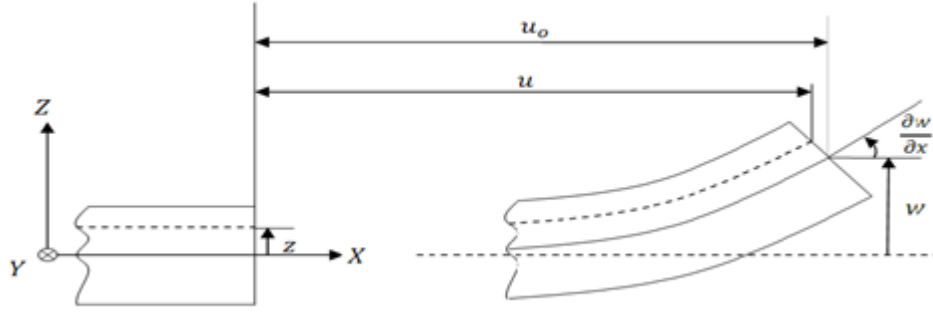


Figure 2.4 Laminate Section Before and After Deformation

$$\varepsilon_x \equiv \frac{\partial u}{\partial x}; \varepsilon_y \equiv \frac{\partial v}{\partial y}; \gamma_{xy} \equiv \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (2.31)$$

The total in-plane displacement at any point in the plate with coordinated Z can be written as

$$u \equiv u_0 - z \frac{\partial w}{\partial x}; v = v_0 - z \frac{\partial w}{\partial y}; w = w_0 \quad (2.32)$$

From equation (2.31) and (2.32):

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \\ \varepsilon_y &= \frac{\partial v}{\partial x} = \frac{\partial v_0}{\partial x} - z \frac{\partial^2 w_0}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} \end{aligned} \quad (2.33)$$

Defining:

$$\frac{\partial u_0}{\partial x} = \varepsilon_x^0; \frac{\partial v_0}{\partial x} = \varepsilon_y^0; \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial x} = \gamma_{xy}^0$$

and

$$- \frac{\partial^2 w_0}{\partial x^2} = K_x; - \frac{\partial^2 w_0}{\partial y^2} = K_y; - 2 \frac{\partial^2 w_0}{\partial x \partial y} = K_{xy} \quad (2.34)$$

To be the plate curvatures will make notation easier. Equation 2.33 can now be written in matrix

from as:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} \quad (2.35)$$

The plate curvature K_x or K_y is rate of change of slope of the bending plate in either the x- or y-direction, respectively. The plate curvature term K_y is the amount of bending in the x direction along the y-axis. Finally, the stress in each K^{th} ply at a distance of z_k from the reference plane in terms of strains and laminate curvatures can be written as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_K = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}_K \left\{ \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z_K \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} \right\} \quad (2.36)$$

2.3.2 Constitutive Equation of Laminated Plate

The stress in each ply varies linearly through the thickness of the laminate this due to the different material properties of the layer resulting from different fiber orientation. Since there exist a discontinuous variation of stresses from a layer to layer in the laminate, it will be convenient to define stress in term of equivalent forces acting at the middle surface. The three stress resultants are therefore:

$$\begin{aligned} N_x &\equiv \sum_{i=1}^n \int_{h_{k-1}}^{h_k} \sigma_x^k \cdot dz \\ N_y &\equiv \sum_{i=1}^n \int_{h_{k-1}}^{h_k} \sigma_y^k \cdot dz \\ N_{xy} &\equiv \sum_{i=1}^n \int_{h_{k-1}}^{h_k} \tau_{xy}^k \cdot dz \end{aligned} \quad (2.37)$$

Following the same procedure, the moment resultants can be defined as:

$$\begin{aligned} M_x &\equiv \sum_{i=1}^n \int_{h_{k-1}}^{h_k} \sigma_x^k \cdot z dz \\ M_y &\equiv \sum_{i=1}^n \int_{h_{k-1}}^{h_k} \sigma_y^k \cdot z dz \\ M_{xy} &\equiv \sum_{i=1}^n \int_{h_{k-1}}^{h_k} \tau_{xy}^k \cdot z dz \end{aligned} \quad (2.38)$$

Where N_x , N_y , N_{xy} are the forces per unit width of the beam M_x , M_y , M_{xy} are the moment per unit width of the beam. Substituting Equation (2.36) in Equation (2.37) and (2.38) the total constitutive equation or load-deformation relations for the laminate can now written as:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \quad (2.39)$$

Where;

$$[A] = \sum_{k=1}^n [\bar{Q}] (h_k - h_{k-1}) \quad (2.40)$$

$$[B] = \frac{1}{2} \sum_{k=1}^n [\bar{Q}] (h_k^2 - h_{k-1}^2) \quad (2.41)$$

$$[D] = \frac{1}{3} \sum_{k=1}^n [\bar{Q}] (h_k^3 - h_{k-1}^3) \quad (2.42)$$

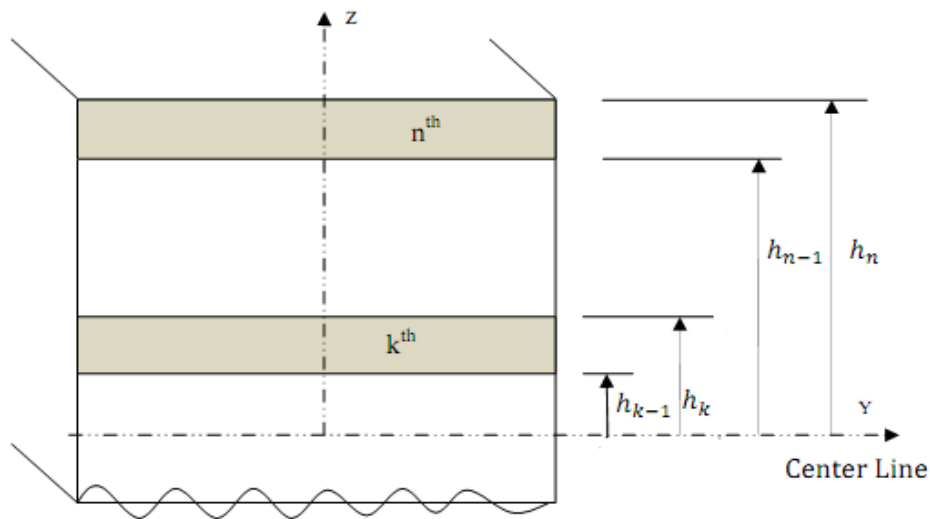


Figure 2.5 Coordinate Notations of individual Plies

h_k and h_{k-1} are the coordinates of the upper and lower surface of the k^{th} lamina as shown in Figure 2.5

A matrix is called extensional stiffness matrix, B matrix is called the coupling stiffness matrix and D matrix is called the bending stiffness. For a symmetrical laminate, it can be proved

that B matrix is a zero matrix however an unsymmetrical laminate B matrix is non-zero because of the coupling stiffness between in-plane and out-of plane are exist.

The inverse of load-deformation relations is used to work easily with strains and curvature of the laminates for any applied load. Laminate compliance matrix can be expressed as:

$$\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} = \begin{bmatrix} a & b \\ b^T & d \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} \quad (2.43)$$

Where

$$\begin{bmatrix} a & b \\ b^T & d \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \quad (2.44)$$

For the given [N] and [M], the mid-plane strains, $[\varepsilon^0]$ and the curvature, [K] can be solved from equation (2.43). By using Equation (2.36), the stresses of each ply can be obtained.

2.3.3 Application of Laminated Plate Theory to Thin-Walled Composite tube

For a thin-walled tube under torsion, a shear stress is induced, Because of stress variation from layer to layer. It is convenient to define equivalent shear force acting on the mid-plane of laminate along cross-section of the tube. Since the shear flow is constant along the contour of the tube, we can consider a laminate of the tube subjected to a shear load.

Since the analysis is computed under torsion load only. All of forces and moments in x and y direction are negligible.

$$N_x = N_y = 0$$

$$M_x = M_y = M_{xy} = 0$$

The total constitutive equation or load-deformation relations for the laminate can now write as:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{b}^T & \mathbf{d} \end{bmatrix}_{6 \times 6} \begin{Bmatrix} 0 \\ 0 \\ N_{xy} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (2.45)$$

Where

$$N_{xy} = q = \frac{T}{2\pi AB} \quad (2.46)$$

From Equation 2.45, the stress of each k_{th} layer in laminate can be calculated from

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \gamma_{xy} \end{bmatrix}_{kth} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}_{kth} \left(\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{kth} + Z_{kth} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} \right) \quad (2.36)$$

CHAPTER 3

FINITE ELEMENT MODEL OF LAMINATED TUBULAR STRUCTURE

The finite element analysis is a computer assisted numerical technique useful in solving for the response of a structure subjected to loading. The model is divided into small blocks called elements. These elements are connected by nodes at which the finite element boundary conditions are applied. A set of algebraic equations are created for each element and combined simultaneous equations are solved. A post processor to the finite element program may be helpful in displaying the stresses and deflections in the form of contours for easy comprehension. A number of finite element packages are available for the analysis of complex structures under various types of loads. A finite element package is chosen depending on the complexity of the model and the results needed.

ANSYS 11.0 is a commercial package that had been used to develop the required 3D finite element model in this study. ANSYS is a versatile finite element package which can be used to analyze structural, thermal, electrical, and contact problems. The preprocessor step consists of selecting the proper elements, providing real constants, material properties, and meshing the model; loads and displacement boundary conditions can also be applied in this module. In this Study the finite element model was developed to conduct the effect of stress and shear stress along the contour of laminated tube with circular cross-section. The detail of preprocessor step will be explained in the later section.

3.1 Geometry and Material properties of Composite Tubular Structure

3.1.1. Geometry of the Tube

The considerable laminated tube in this research has wall thickness, t , is 0.005 in. that consider as a constant along the cross-section. The major radius, a , is 0.5 in and minor radius, b ,

are varies from 0.5 to 0.1 in. The length of the composite tube, L, is 5 in. The geometry of composite tube is depicted in figure 3.1.

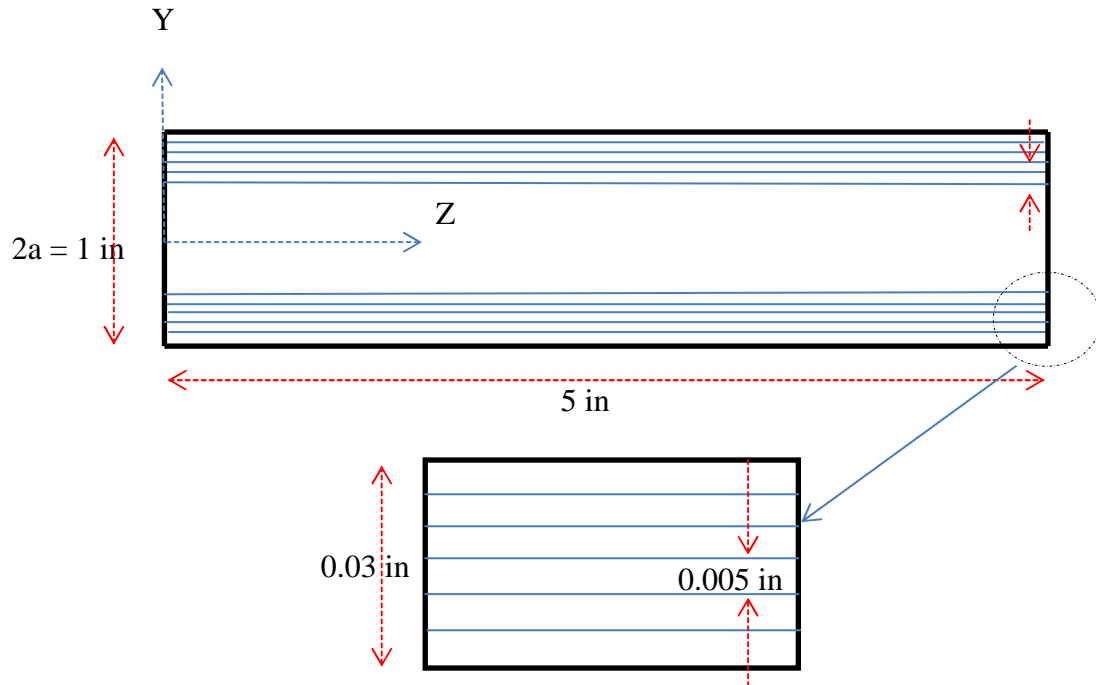


Figure 3.1 2D Dimension of Composite Tubular Structure

The layup of the laminate were conducted in three different categories, symmetrical and balance, anti-symmetrical and balance and symmetrical and unbalance. The staking sequence of symmetrical and balance laminate is $[\pm 45/0]_s$. The staking sequence of anti-symmetrical and balance laminate is $[45/-45/0_2/45/-45]$. The staking sequence of anti-symmetrical and unbalance is $[45/0/90]_s$. All of the laminate consists of 6 layers, with ply thickness 0.005in each.

3.1.2. Material properties

The unidirectional graphite/epoxy AS4/3501-6 is chosen as the composite material used in this research. The unidirectional layer orthotropic material properties are given as follows,

$$E_1 = 21.3 \times 10^6 \text{ psi}, E_2 = 1.5 \times 10^6 \text{ psi}, E_3 = 1.5 \times 10^6 \text{ psi},$$

$$G_{12} = 1.0 \times 10^6 \text{ psi}, G_{23} = 0.54 \times 10^6 \text{ psi}, G_{13} = 1.0 \times 10^6 \text{ psi},$$

$$v_{12} = 0.27 \times 10^6, v_{23} = 0.54 \times 10^6, v_{13} = 0.27 \times 10^6$$

E, G, and v are referring to young's modulus, shear modulus, and Poisson's ratio of the composite material, respectively. The subscripts 1, 2, and 3 are along fiber direction, transverse and perpendicular to the plane, respectively.

3.2 Development of Finite Element Model

3.2.1 Element type used

ANSYS V11 has various shell element like Shell 91Shell 99 is an 8 node, 3D shell element with six degrees of freedom at each node. It is designed for modeling thin to moderately thick plate and shell structure with a side to thickness ratio of roughly 10 or greater. The Shell 99 element allows a total of 100 uniform thickness layers. However, If more than 100 layers are required, we can input our own material properties matrix.

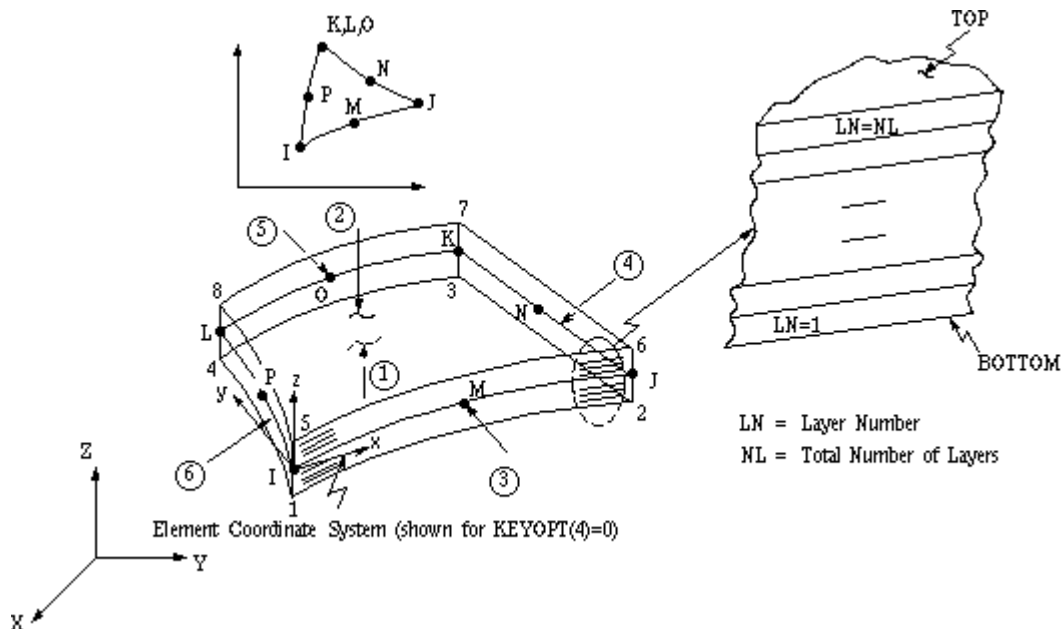


Figure 3.2 SHELL99 Geometry

3.2.2 Modeling

In the present analysis, Shell99 is used to model the composite circular tube. The element has six degrees of freedom at each node which is translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Material properties are set as describe in section 3.2.1. The model start from creating a key points on X-Y plane. The 80 key points were created using K command. The key-points were linked together using BSPLIN command. All the key-points became a set of line that composes of 80 short lines. Those lines need to be glue together as one line using LGLUE command. The line was extruded along the Z direction for 5 inches long. A volume that composes of sub areas were created after extruded processer. The volume was deleted by VDELE command and left over 82 sub areas. The both end areas of the tube were deleted by ADELE command. All of the areas were glue together using AGLUE command. The complete composite circular tube modeling is done as figure below.

3.2.3 Meshing

Meshing can be done in two ways, free and map meshing. In free meshing, the elements that are created are of no definite shape and pattern. In contrast to this, a mapped mesh produces a definite element shape and follows a particular pattern.

A map meshing is chosen to apply on the composite circular tube model. It is important to divide the areas properly for a good map mashing. Therefore all of the lines along Z direction and along the tubular cross section were divided into 160 and 80 tiny segments respectively before map meshing will be applied. Many tiny square areas are generated around the composite tube which is called an element. The 12,800 elements were generated after applying map meshing into composite circular tube model.

3.2.4 Boundary conditions

Boundary Conditions are the constraints and loads that can simulate the effect of the environment surrounding a body. Loads are applied in the form of forces, moments, pressures, and temperatures. The displacement constraints are applied by restricting the degrees of freedom at the corresponding nodes of a model. There are three boundary conditions are enforced. First, all nodes at one face of the end of the circular tube is constrained along the X Y and Z direction ($UX = UY = UZ = 0$). Second, all of the nodes at the same face is constrained on rotation in X, Y and Z direction ($ROTX = ROTY = ROTZ = 0$). Last, torque was applied in another free end by using constraint equations features that provided in ANSYS 11.0.

3.2.5 Loading condition

Constraint equations provide many useful features in ANSYS, such as tying together dissimilar meshes, representing parts of the system not explicitly modeled, or distributing loads. Two automated methods of generating constraint equations are the CERIG and RBE3 commands. Constraint equations in ANSYS are linear, so they are not valid for large-rotation analyses.

RBE3 is the most common method applied, because it will work for all cases. It uses link elements with appropriately high stiffness. Using RBE3, a rigid region is not formed, but the applied loads are distributed. Because no rigidity is formed, no rotational moments of inertia are transferred to the slave nodes and the key option for Mass21 must be set to ignore rotational inertia effects.

CERIG is the automatic CE generator, the user selects one master node, several slave nodes, and the DOF affected. The master node in this case means that this is the node which

controls the behavior of the rigid region. The Mass21 key option must be set such that it expects both a mass and a rotational inertia even if the rotational inertia is zero.

MPC184 is a rigid link/beam element for use in large rotation problem. These elements can be used to represent rigid connections, even in nonlinear applications. It requires no real constants, no suppression in any DOF, and also has optional material properties of ALPX and DENS for thermal expansion and density, respectively.

3.3 Validation of Finite Element Model and Results

The finite element model developed in this research is validated using isotropic material. Shear stress along the cross section contour of the constant thickness thin wall tube is concerned as the validated variable. The size of the model, meshing, elements, boundary conditions, and loading conditions will remain the same as using in composite case. The isotropic material properties are given as follows,

$$E_1 = E_2 = E_3 = 10 \times 10^6 \text{ psi}$$

$$G_{12} = G_{23} = G_{13} = 3.75 \times 10^6 \text{ psi}$$

$$\nu_{12} = \nu_{23} = \nu_{13} = \frac{E_{11}}{2(1+\nu_{12})}$$

The result is shown in the Graph and table below

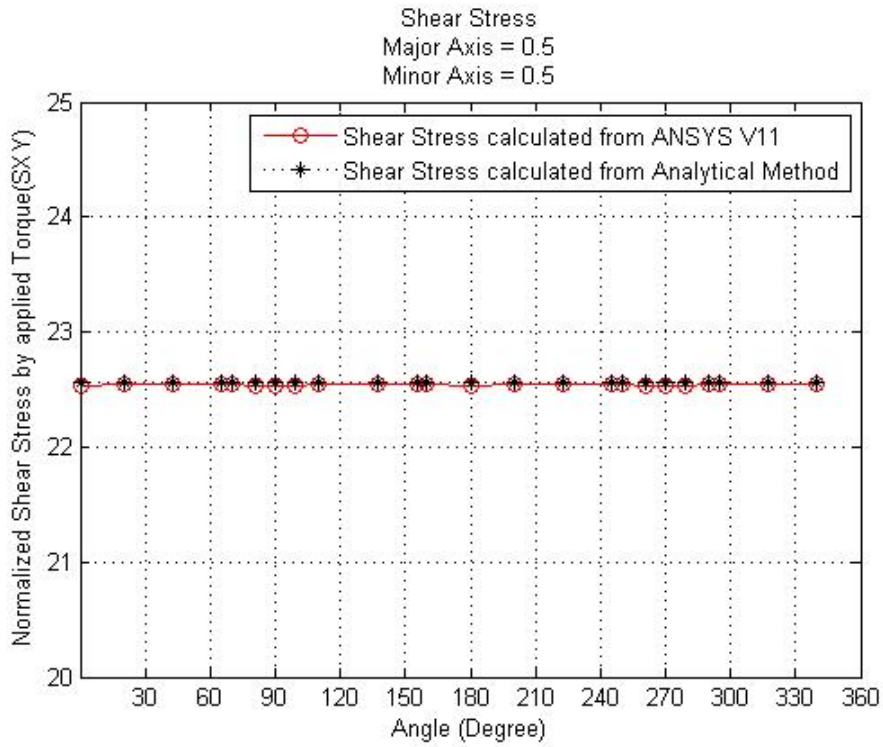


Figure 3.3 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 1$

Table 3.1 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 1$

| ANGLE (deg) | SXY FEM Result (lb-in ²) | SXY Analytical Result (lb-in ²) | % Error |
|-------------|---|--|----------|
| 0 | 22.533 | 22.5581 | 0.111268 |
| 20.25 | 22.549 | 22.5581 | 0.040340 |
| 42.75 | 22.549 | 22.5581 | 0.040340 |
| 65.25 | 22.549 | 22.5581 | 0.040340 |
| 69.75 | 22.549 | 22.5581 | 0.040340 |
| 81 | 22.533 | 22.5581 | 0.111268 |
| 90 | 22.533 | 22.5581 | 0.111268 |
| 99 | 22.533 | 22.5581 | 0.111268 |
| 110.25 | 22.549 | 22.5581 | 0.040340 |
| 137.25 | 22.549 | 22.5581 | 0.040340 |
| 155.75 | 22.549 | 22.5581 | 0.040340 |
| 159.75 | 22.549 | 22.5581 | 0.040340 |
| 180 | 22.533 | 22.5581 | 0.111268 |
| 200.25 | 22.549 | 22.5581 | 0.040340 |

Table 3.1 – Continued

| | | | |
|--------|--------|---------|----------|
| 222.75 | 22.549 | 22.5581 | 0.040340 |
| 245.25 | 22.549 | 22.5581 | 0.040340 |
| 249.75 | 22.549 | 22.5581 | 0.040340 |
| 261 | 22.533 | 22.5581 | 0.111268 |
| 270 | 22.533 | 22.5581 | 0.111268 |
| 279 | 22.533 | 22.5581 | 0.111268 |
| 290.25 | 22.549 | 22.5581 | 0.040340 |
| 294.75 | 22.549 | 22.5581 | 0.040340 |
| 317.25 | 22.549 | 22.5581 | 0.040340 |
| 339.75 | 22.549 | 22.5581 | 0.040340 |

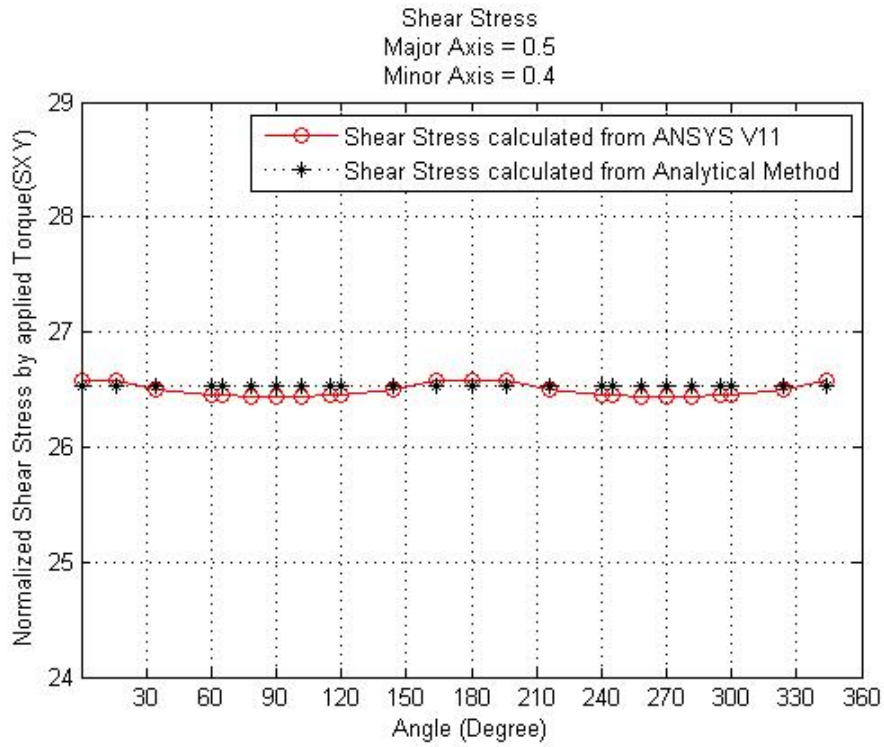


Figure 3.4 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.8$

Table 3.2 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.8$

| ANGLE (deg) | SXY FEM Result (lb-in ²) | SXY Analytical Result (lb-in ²) | % Error |
|-------------|---|--|----------|
| 0 | 26.583 | 26.5266 | 0.212617 |
| 16.432 | 26.574 | 26.5266 | 0.178689 |

Table 3.2 – *Continued*

| | | | |
|---------|--------|---------|----------|
| 34.46 | 26.496 | 26.5266 | 0.115356 |
| 60.03 | 26.46 | 26.5266 | 0.251069 |
| 65.23 | 26.457 | 26.5266 | 0.262378 |
| 78.8 | 26.442 | 26.5266 | 0.318925 |
| 90 | 26.441 | 26.5266 | 0.322695 |
| 101.2 | 26.442 | 26.5266 | 0.318925 |
| 114.77 | 26.457 | 26.5266 | 0.262378 |
| 119.97 | 26.46 | 26.5266 | 0.251069 |
| 143.54 | 26.574 | 26.5266 | 0.178689 |
| 163.57 | 26.496 | 26.5266 | 0.115356 |
| 180 | 26.583 | 26.5266 | 0.212617 |
| 196.432 | 26.574 | 26.5266 | 0.178689 |
| 216.46 | 26.496 | 26.5266 | 0.115356 |
| 240.03 | 26.46 | 26.5266 | 0.251069 |
| 245.23 | 26.457 | 26.5266 | 0.262378 |
| 258.8 | 26.442 | 26.5266 | 0.318925 |
| 270 | 26.441 | 26.5266 | 0.322695 |
| 281.2 | 26.442 | 26.5266 | 0.318925 |
| 294.77 | 26.457 | 26.5266 | 0.262378 |
| 299.97 | 26.46 | 26.5266 | 0.251069 |
| 323.54 | 26.496 | 26.5266 | 0.115356 |
| 343.57 | 26.574 | 26.5266 | 0.178689 |

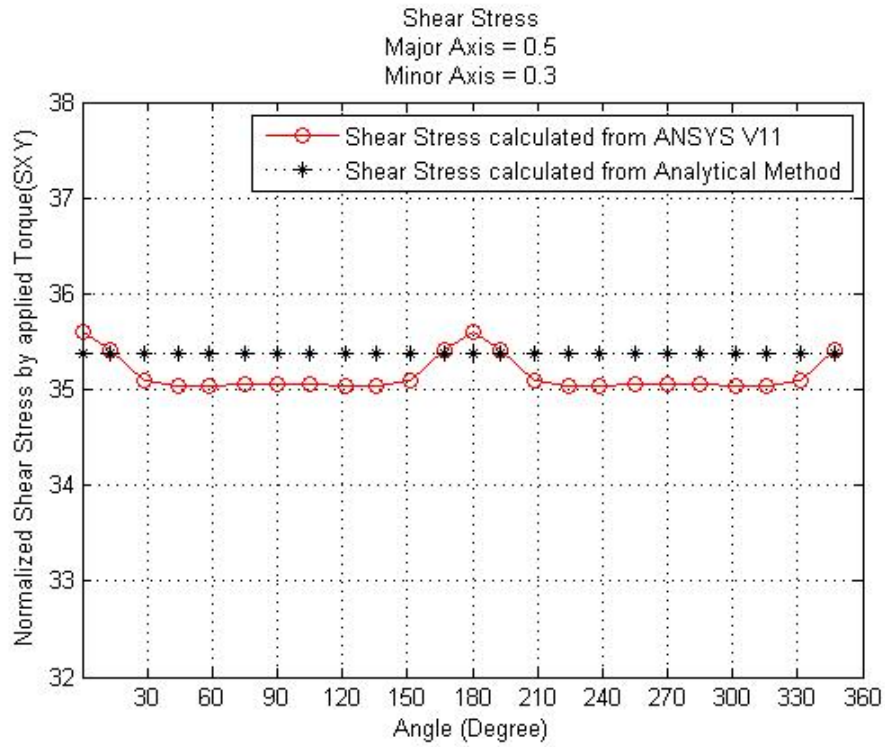


Figure 3.5 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.6$

Table 3.3 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXY FEM Result (lb-in ²) | SXY Analytical Result (lb-in ²) | % Error |
|-------------|---|--|----------|
| 0 | 35.603 | 35.368 | 0.664442 |
| 12.46 | 35.423 | 35.368 | 0.155508 |
| 28.98 | 35.098 | 35.368 | 0.763402 |
| 44.16 | 35.038 | 35.368 | 0.933047 |
| 58.38 | 35.043 | 35.368 | 0.918910 |
| 75.21 | 35.048 | 35.368 | 0.904773 |
| 90 | 35.053 | 35.368 | 0.890636 |
| 104.79 | 35.048 | 35.368 | 0.904773 |
| 121.62 | 35.043 | 35.368 | 0.918910 |
| 135.84 | 35.038 | 35.368 | 0.933047 |
| 151.02 | 35.423 | 35.368 | 0.155508 |
| 167.54 | 35.098 | 35.368 | 0.763402 |
| 180 | 35.603 | 35.368 | 0.664442 |
| 192.46 | 35.423 | 35.368 | 0.155508 |

Table 3.3 – Continued

| | | | |
|--------|--------|--------|----------|
| 208.98 | 35.098 | 35.368 | 0.763402 |
| 224.16 | 35.038 | 35.368 | 0.933047 |
| 238.38 | 35.043 | 35.368 | 0.918910 |
| 255.21 | 35.048 | 35.368 | 0.904773 |
| 270 | 35.053 | 35.368 | 0.890636 |
| 284.79 | 35.048 | 35.368 | 0.904773 |
| 301.62 | 35.043 | 35.368 | 0.918910 |
| 315.84 | 35.038 | 35.368 | 0.933047 |
| 331.02 | 35.098 | 35.368 | 0.763402 |
| 347.54 | 35.423 | 35.368 | 0.155508 |

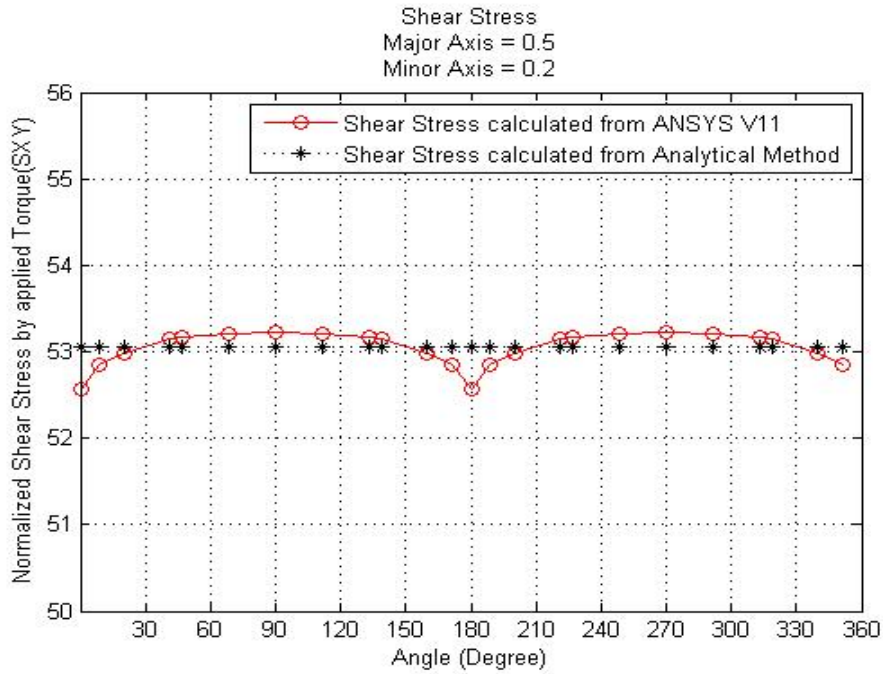


Figure 3.6 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.4$

Table 3.4 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXY FEM Result (lb-in ²) | SXY Analytical Result (lb-in ²) | % Error |
|-------------|--------------------------------------|---|----------|
| 0 | 52.559 | 53.052 | 0.929277 |
| 8.35 | 52.847 | 53.052 | 0.386413 |
| 20.25 | 52.977 | 53.052 | 0.141371 |
| 40.9 | 53.147 | 53.052 | 0.179070 |

Table 3.4 – *Continued*

| | | | |
|--------|--------|--------|----------|
| 47.27 | 53.174 | 53.052 | 0.229963 |
| 68.4 | 53.216 | 53.052 | 0.309131 |
| 90 | 53.228 | 53.052 | 0.331750 |
| 111.6 | 53.216 | 53.052 | 0.309131 |
| 132.73 | 53.174 | 53.052 | 0.229963 |
| 139.1 | 53.147 | 53.052 | 0.179070 |
| 159.75 | 52.847 | 53.052 | 0.386413 |
| 171.65 | 52.977 | 53.052 | 0.141371 |
| 180 | 52.559 | 53.052 | 0.929277 |
| 188.35 | 52.847 | 53.052 | 0.386413 |
| 200.25 | 52.977 | 53.052 | 0.141371 |
| 220.9 | 53.147 | 53.052 | 0.179070 |
| 227.27 | 53.174 | 53.052 | 0.229963 |
| 248.4 | 53.216 | 53.052 | 0.309131 |
| 270 | 53.228 | 53.052 | 0.331750 |
| 291.6 | 53.216 | 53.052 | 0.309131 |
| 312.73 | 53.174 | 53.052 | 0.229963 |
| 319.1 | 53.147 | 53.052 | 0.179070 |
| 339.75 | 52.977 | 53.052 | 0.141371 |
| 351.65 | 52.847 | 53.052 | 0.386413 |

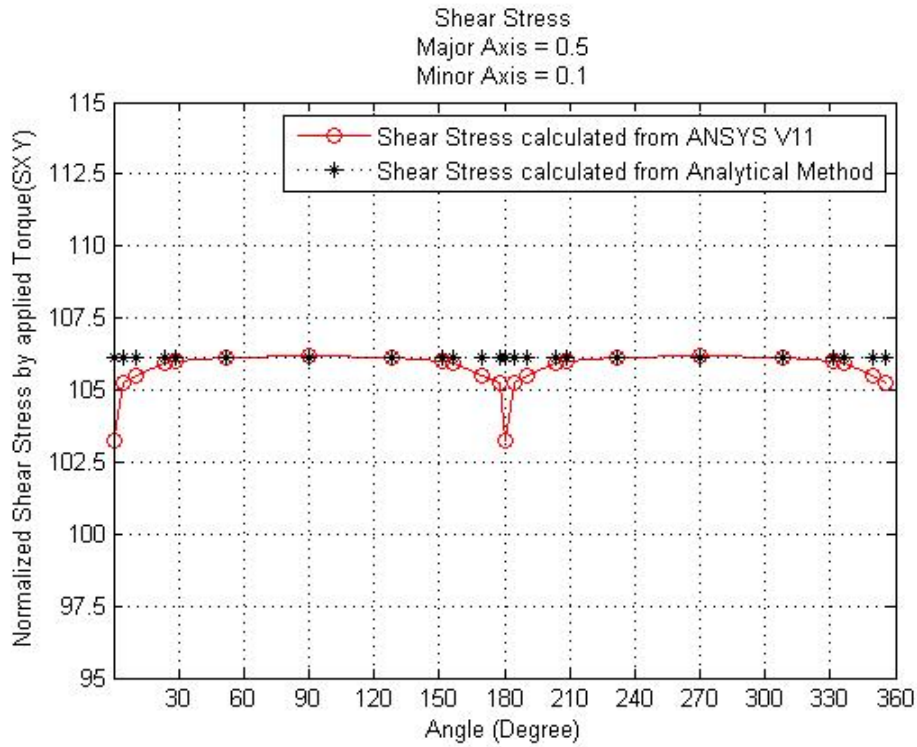


Figure 3.7 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.2$

Table 3.5 Comparison of τ_{xy} in isotropic tube under torsion with $\frac{b}{a} = 0.2$

| ANGLE (deg) | SXY FEM Result (lb-in ²) | SXY Analytical Result (lb-in ²) | % Error |
|-------------|---|--|----------|
| 0 | 103.24 | 106.103 | 2.698321 |
| 4.18 | 105.26 | 106.103 | 0.794511 |
| 10.44 | 105.52 | 106.103 | 0.549466 |
| 23.41 | 105.94 | 106.103 | 0.153624 |
| 28.42 | 106.01 | 106.103 | 0.087651 |
| 51.62 | 106.14 | 106.103 | 0.034872 |
| 90 | 106.17 | 106.103 | 0.063146 |
| 128.38 | 106.14 | 106.103 | 0.034872 |
| 151.58 | 106.01 | 106.103 | 0.087651 |
| 156.59 | 105.94 | 106.103 | 0.153624 |
| 169.56 | 105.26 | 106.103 | 0.794511 |
| 177.59 | 105.52 | 106.103 | 0.549466 |
| 180 | 103.24 | 106.103 | 2.698321 |
| 184.18 | 105.26 | 106.103 | 0.794511 |

Table 3.5 – *Continued*

| | | | |
|--------|--------|---------|----------|
| 190.44 | 105.52 | 106.103 | 0.549466 |
| 203.41 | 105.94 | 106.103 | 0.153624 |
| 208.42 | 106.01 | 106.103 | 0.087651 |
| 231.62 | 106.14 | 106.103 | 0.034872 |
| 270 | 106.17 | 106.103 | 0.063146 |
| 308.38 | 106.14 | 106.103 | 0.034872 |
| 331.58 | 106.01 | 106.103 | 0.087651 |
| 336.59 | 105.94 | 106.103 | 0.153624 |
| 349.56 | 105.52 | 106.103 | 0.549466 |
| 355.82 | 105.26 | 106.103 | 0.794511 |

From the all graphs and tables above, we found that the accuracy of shear stress is affected from radius ratio, as large as radius ratio is as much as errors appear. The highest shear stress error is 0.04 % in the case of minor radius is 0.5in but the biggest error became 2.7% when the minor radius is 0.1in which is about 80% smaller than the major radius, which is 0.5in. All of the error is an impact from surface distortion and curvature that usually has an effect on tubular structure with large aspect ratio.

CHAPTER 4

RESULT

There are two parts which are going to be presented. First, the different results of tubular structure with symmetric layups between finite element method,(FEM) and classical laminated plate theory,(CLPT) are compared. The stacking sequence of $[0]_{6T}$, $[45]_{6T}$, and $[+45/-45/0]_S$ are investigated. The other part exhibits the different results of tubular structure with unsymmetrical layups between finite element method and laminated plate theory which the stacking sequence are $[+45/-45/0/0/45/-45]$ and $[+45/-45/0/+45/-45/0]$.

4.1 The comparison results of Symmetric layups between FEM and CLPT

4.1.1 Symmetric Single fiber orientation $[0]_{6T}$ and $[45]_{6T}$

The $[0]_{6T}$ and $[45]_{6T}$ are no z-displacement in the model. This is because the model is now symmetrically layered, and the material orientation does not cause a curvature effect due to varying moduli on either side of the composite centerline. This can also be seen in the composite constitutive matrices by the null extensional-bending stiffness or B matrix. From the graphs above, we saw the shear stress results that got from FEM are generally agree to the CLPT. The miss match are exist In the high specific curvature region when the radius ratio is high, $\frac{b}{a} = 0.4$ and, $\frac{b}{a} = 0.2$ and the result are present as a graphs below

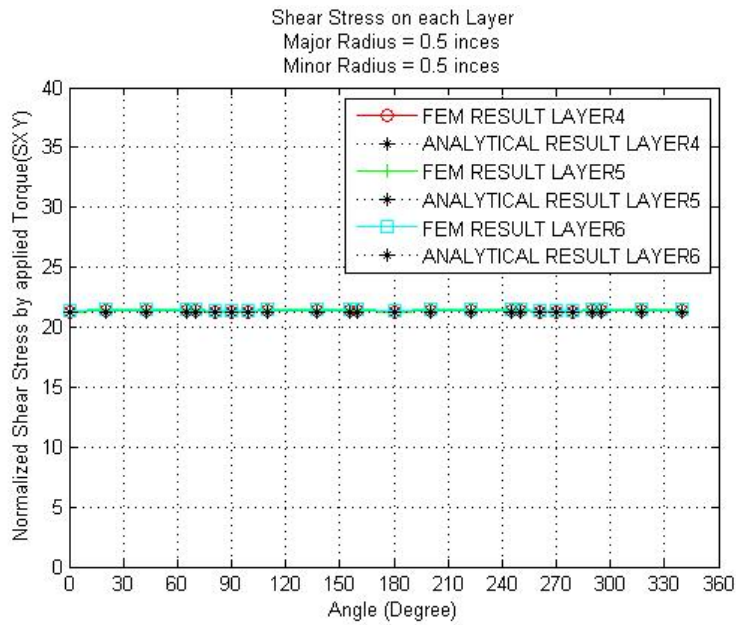
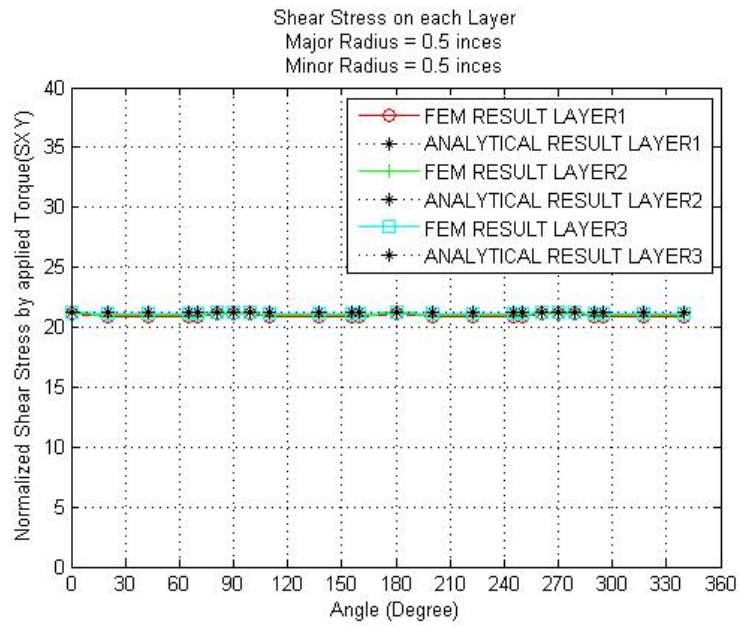


Figure 4.1 The comparison between FEM and CLPT for $[0]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

with $\frac{b}{a} = 1$

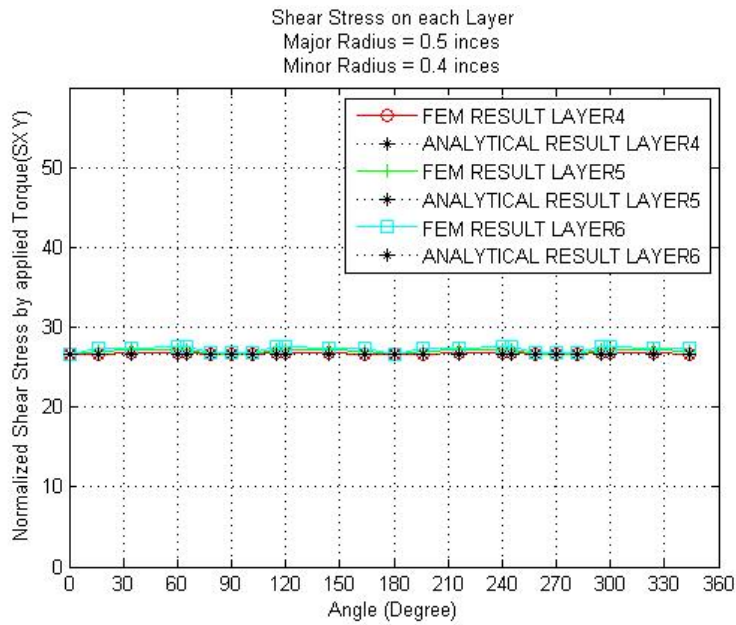
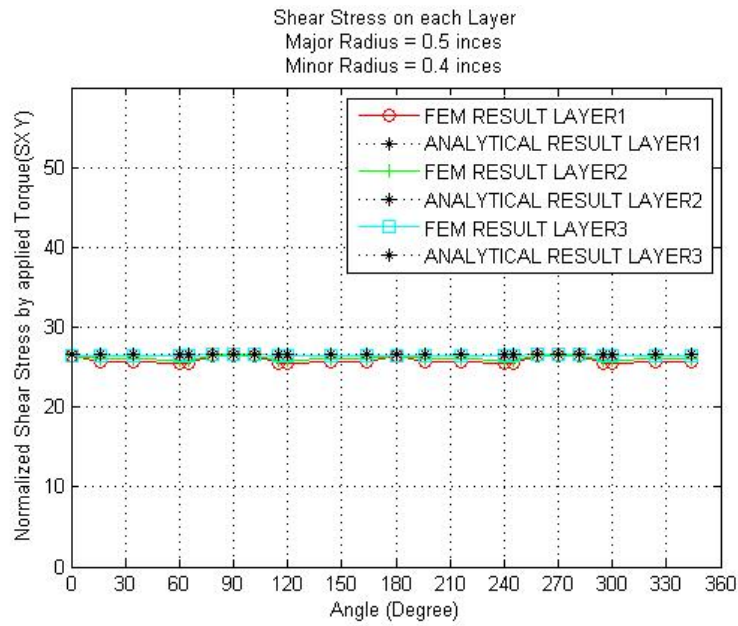


Figure 4.2 The comparison between FEM and CLPT for $[0]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

with $\frac{b}{a} = 0.8$

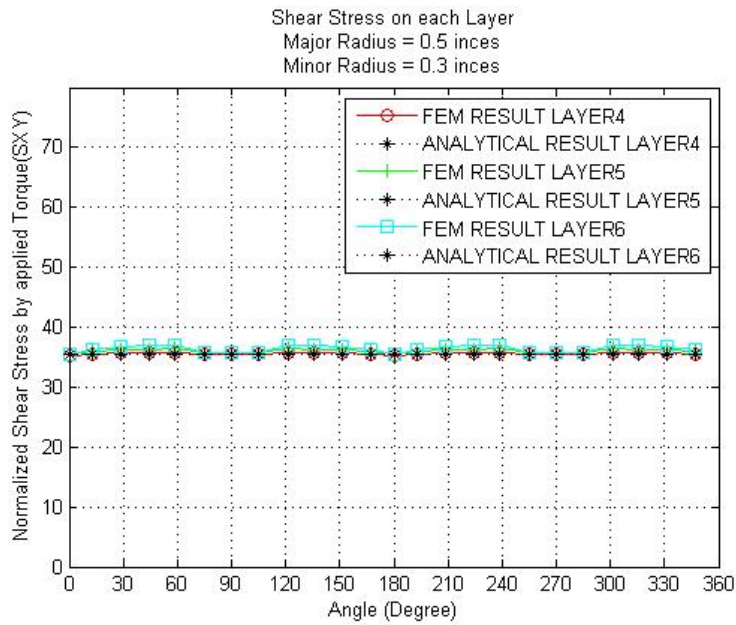
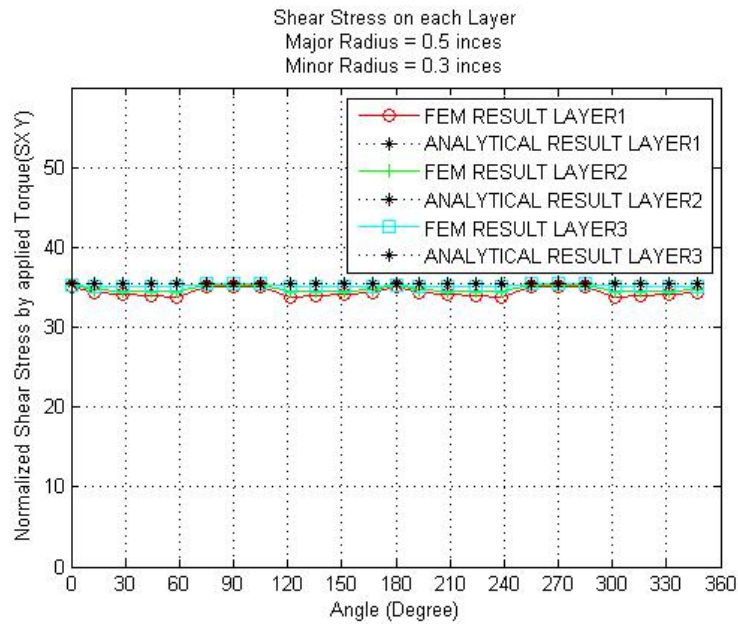


Figure 4.3 The comparison between FEM and CLPT for $[0]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

$$\text{with } \frac{b}{a} = 0.6$$

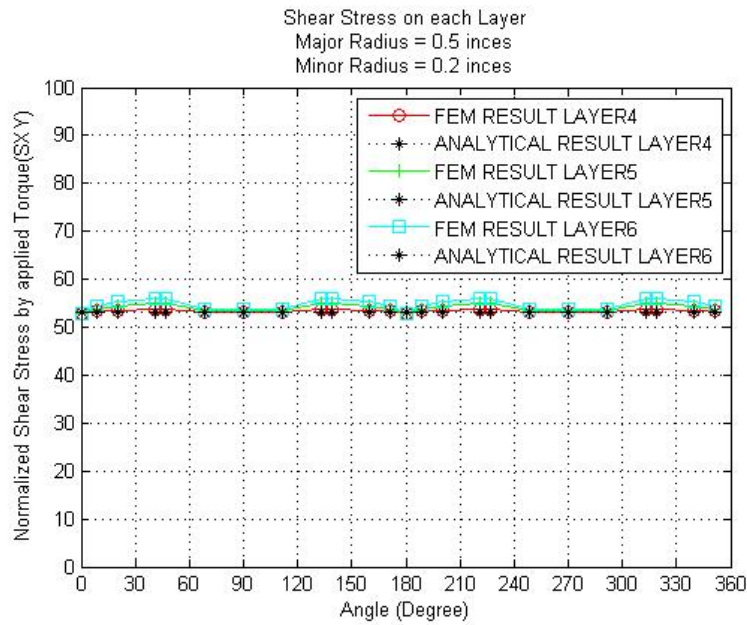
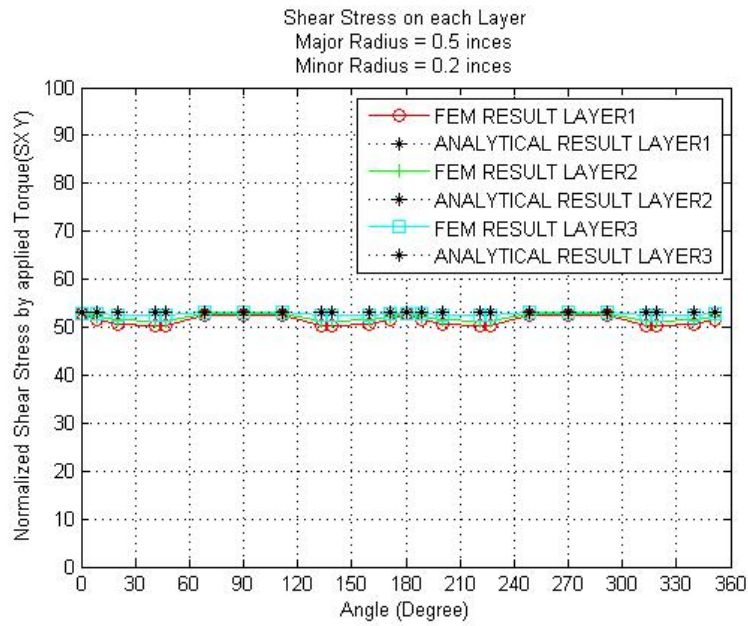


Figure 4.4 The comparison between FEM and CLPT for $[0]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

$$\text{with } \frac{b}{a} = 0.4$$

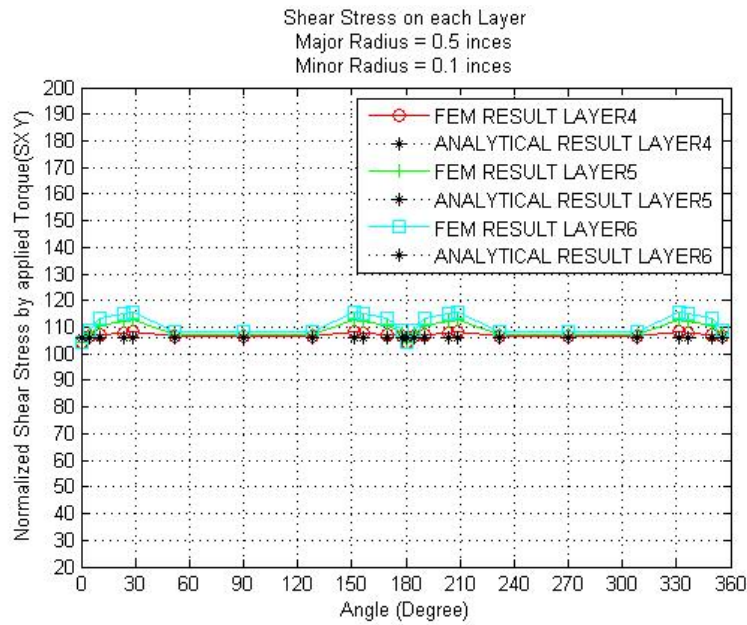
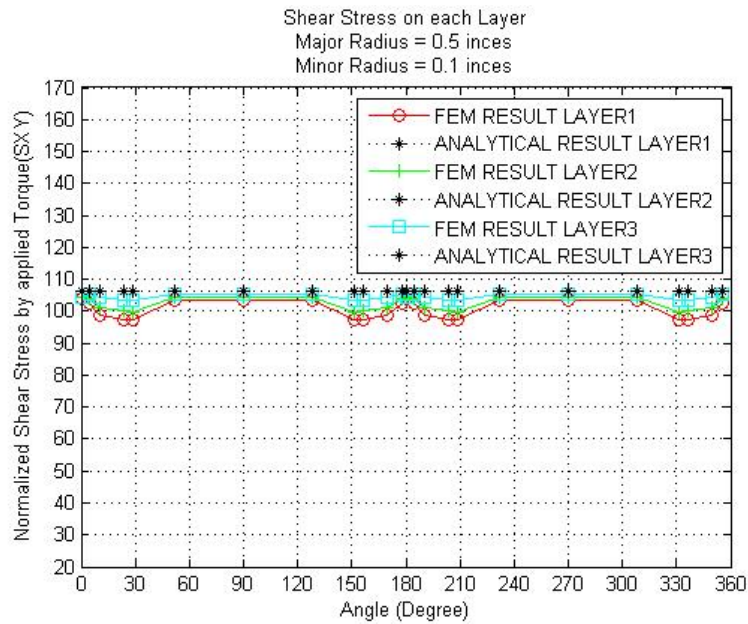


Figure 4.5 The comparison between FEM and CLPT for $[0]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

$$\text{with } \frac{b}{a} = 0.2$$

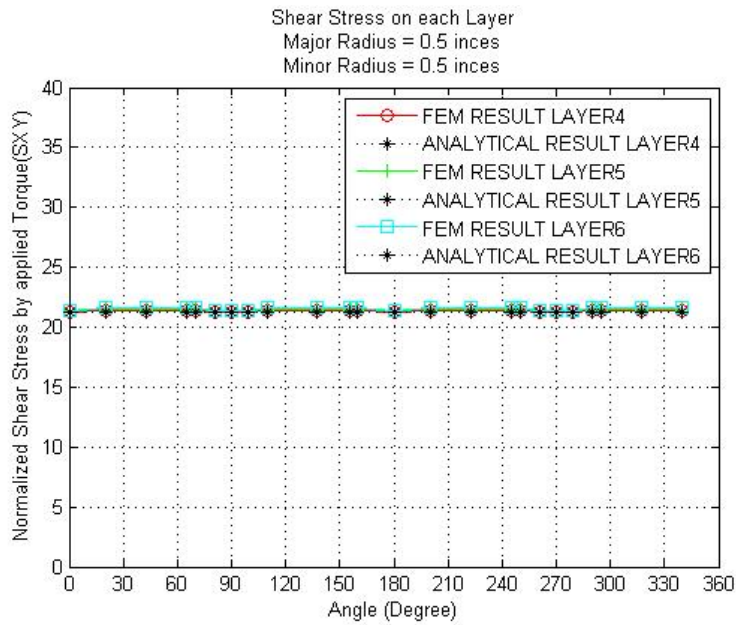
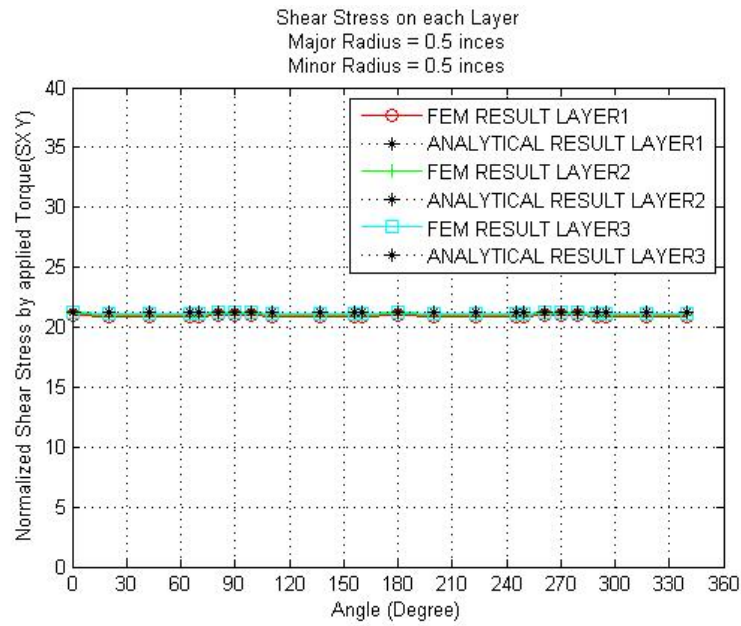
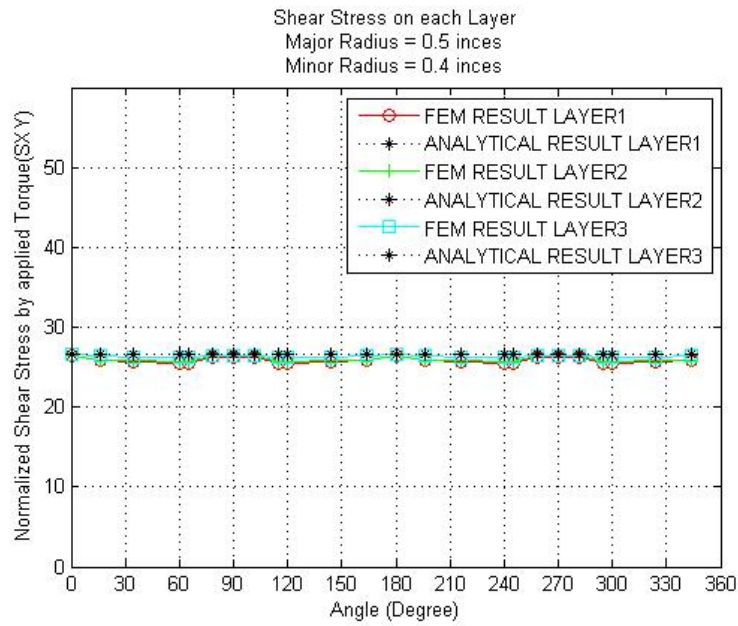
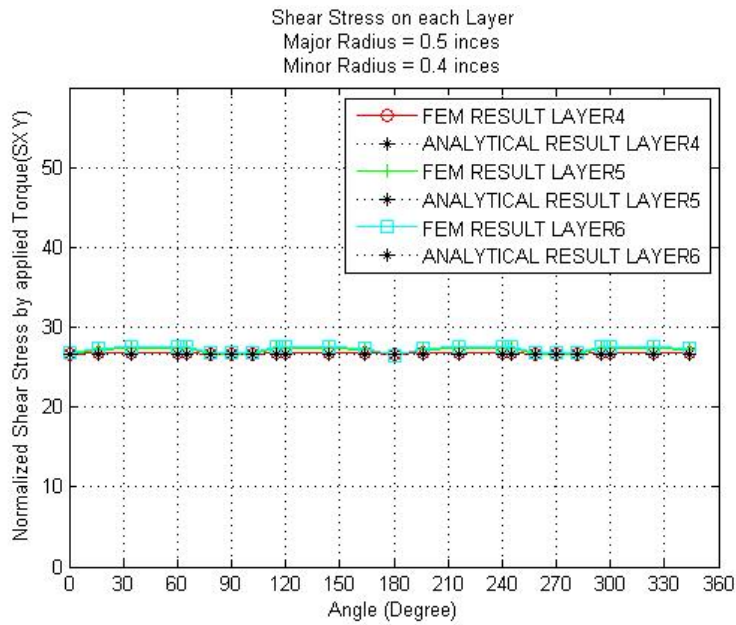


Figure 4.6 The comparison between FEM and CLPT for $[45]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

with $\frac{b}{a} = 1$



(a)



(b)

Figure 4.7 The comparison between FEM and CLPT for $[45]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

with $\frac{b}{a} = 0.8$

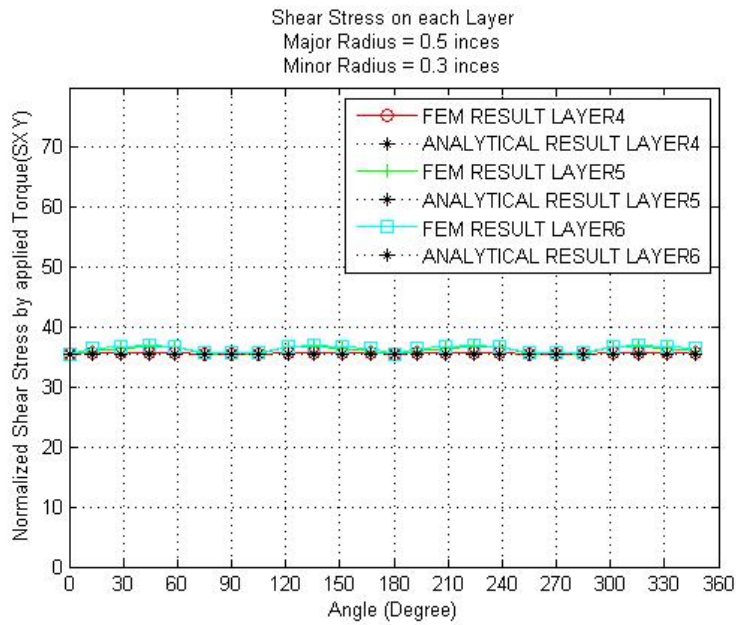
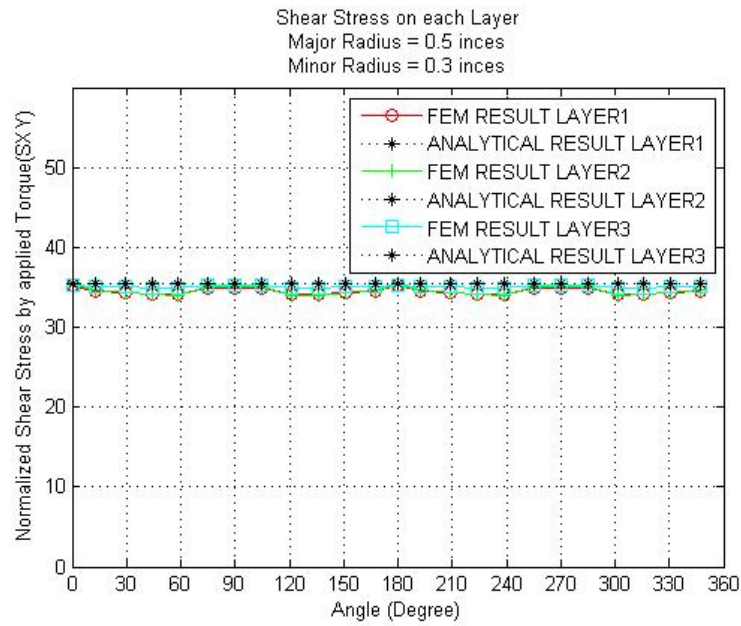
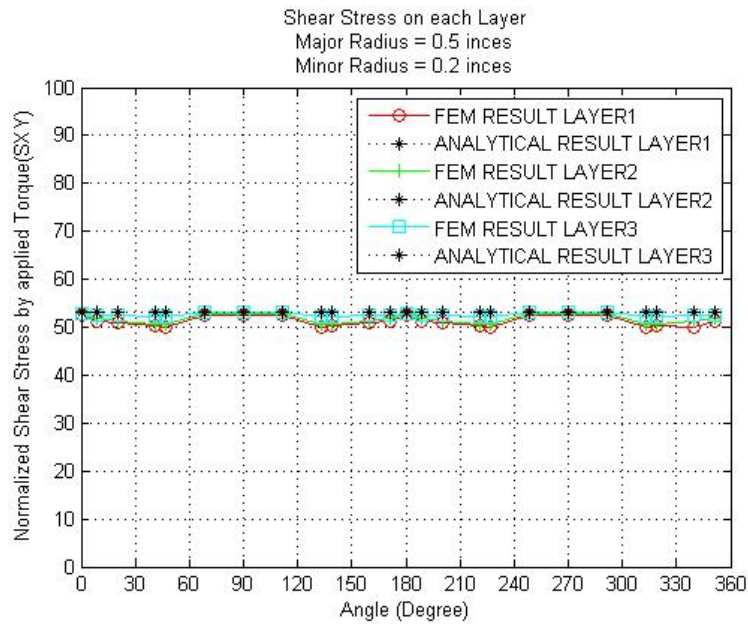
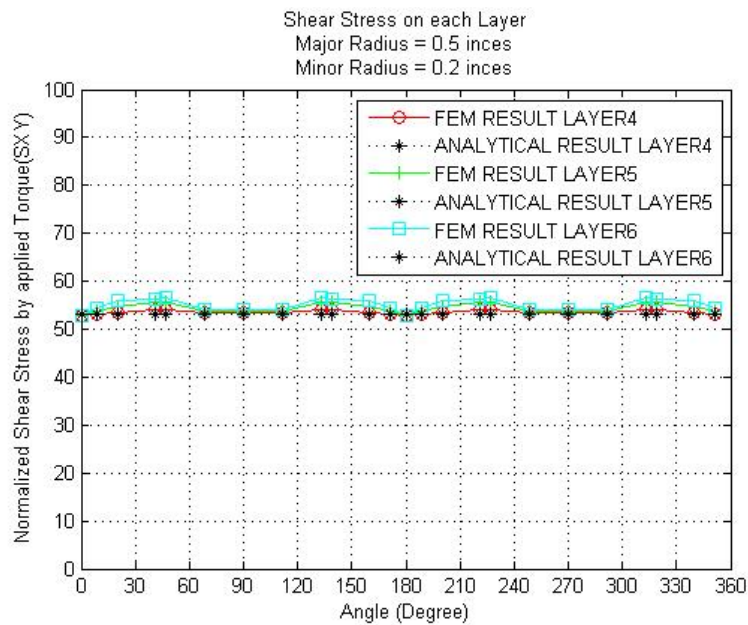


Figure 4.8 The comparison between FEM and CLPT for $[45]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

$$\text{with } \frac{b}{a} = 0.6$$



(a)



(b)

Figure 4.9 The comparison between FEM and CLPT for $[45]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

$$\text{with } \frac{b}{a} = 0.4$$

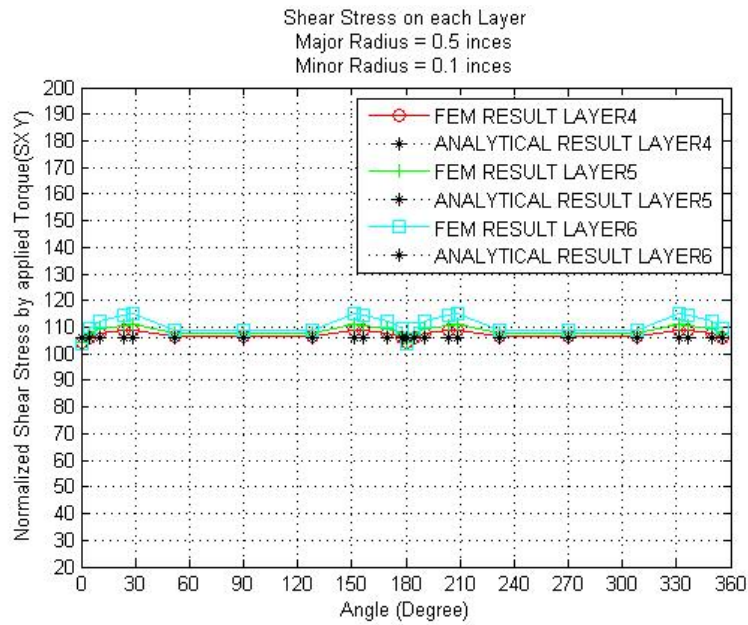
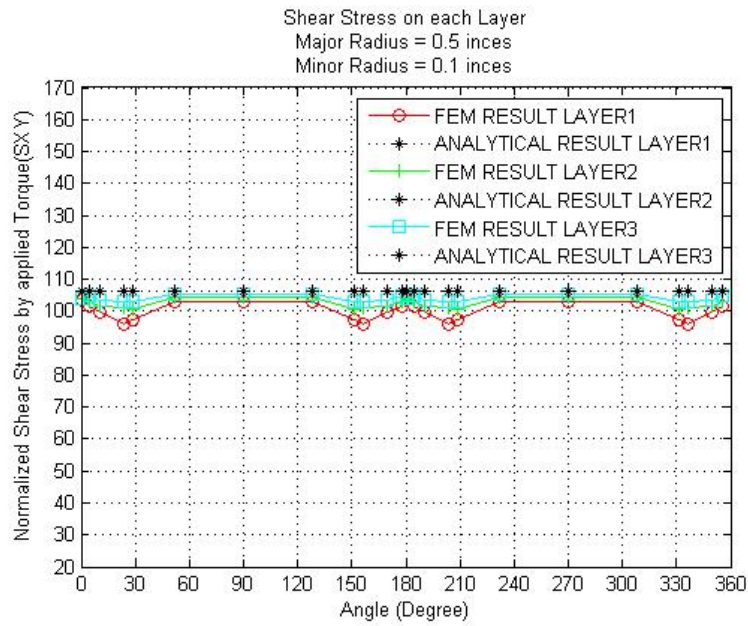
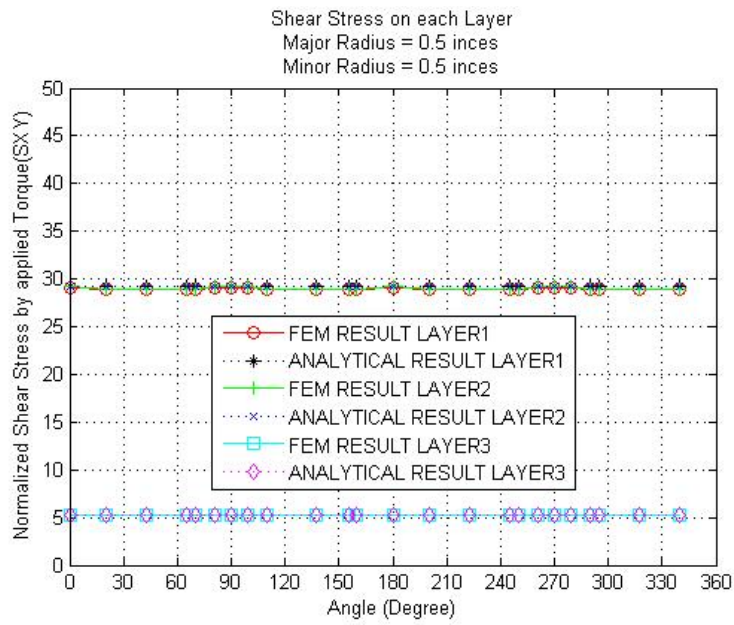


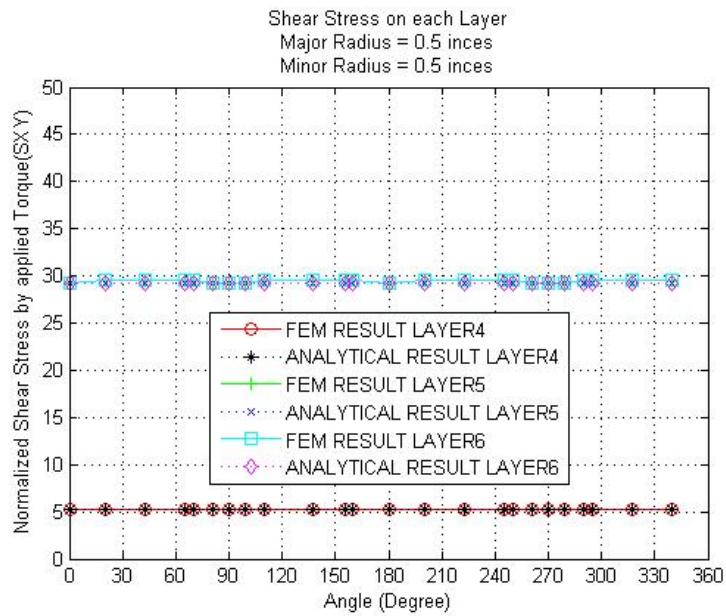
Figure 4.10 The comparison between FEM and CLPT for $[45]_{6T}$ from layer (a) 1 - 3 and (b) 3 - 6

$$\text{with } \frac{b}{a} = 0.2$$

4.1.2 Symmetric & Balance Multi fiber orientation $[\pm 45/0]_s$



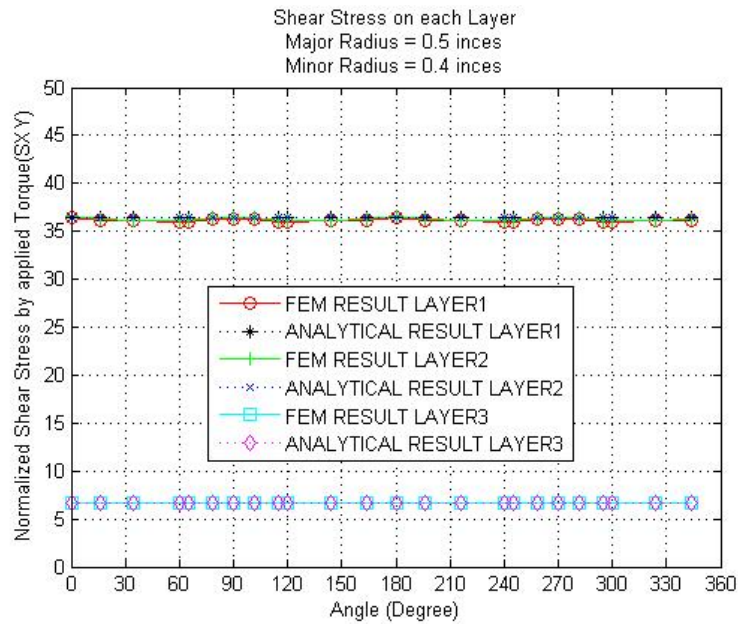
(a)



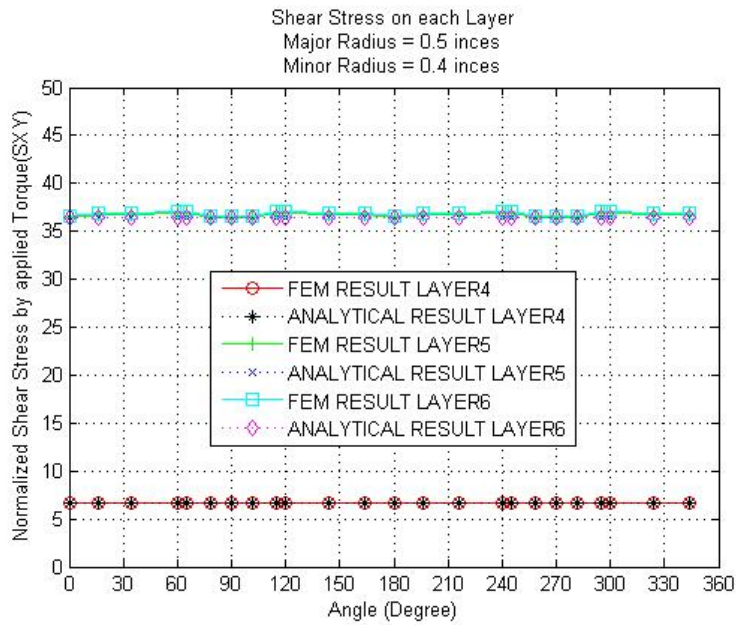
(b)

Figure 4.11 The comparison between FEM and CLPT for $[\pm 45/0]_s$ from layer (a) 1-3 and (b) 3-6

with $\frac{b}{a} = 1$



(a)



(b)

Figure 4.12 The comparison between FEM and CLPT for $[\pm 45/0]_S$ from layer (a) 1-3 and (b) 3-6

with $\frac{b}{a} = 0.8$

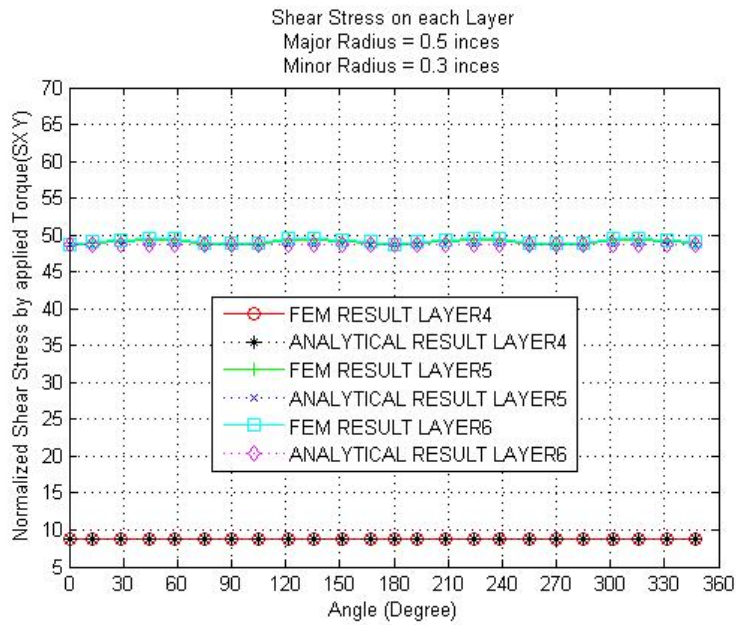
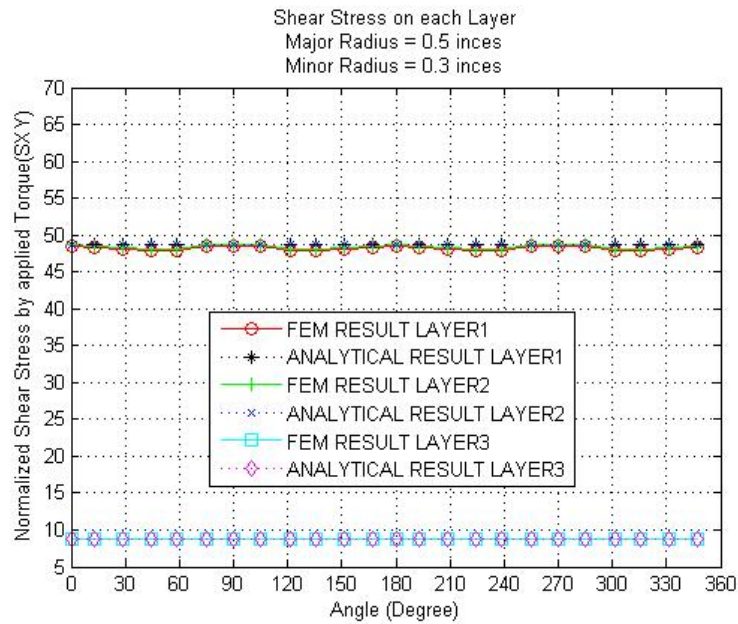
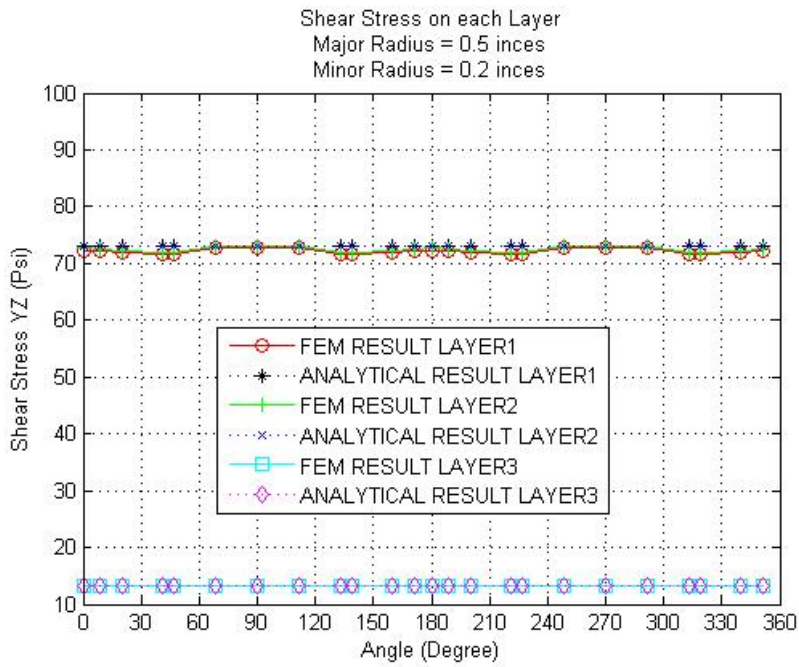
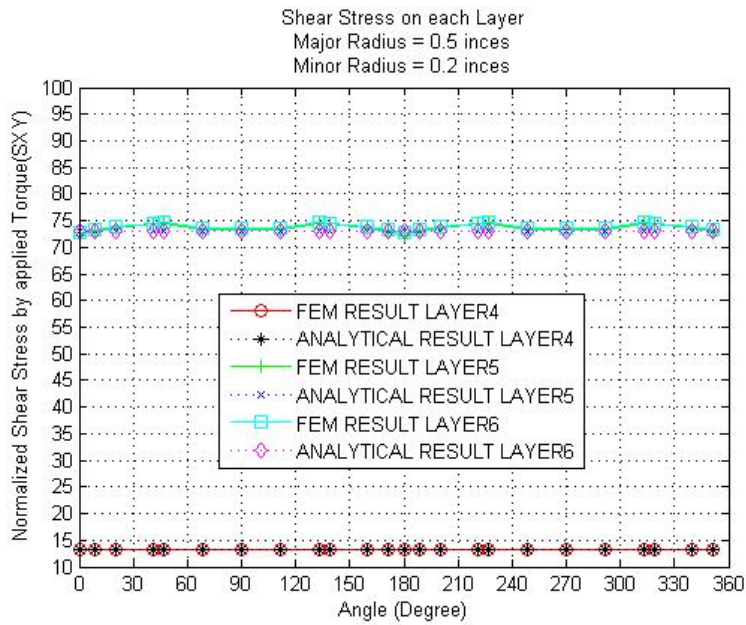


Figure 4.13 The comparison between FEM and CLPT for $[\pm 45/0]_S$ from layer (a) 1-3 and (b) 3-6

$$\text{with } \frac{b}{a} = 0.6$$



(a)



(b)

Figure 4.14 The comparison between FEM and CLPT for $[\pm 45/0]_S$ from layer (a) 1-3 and (b) 3-6

with $\frac{b}{a} = 0.4$

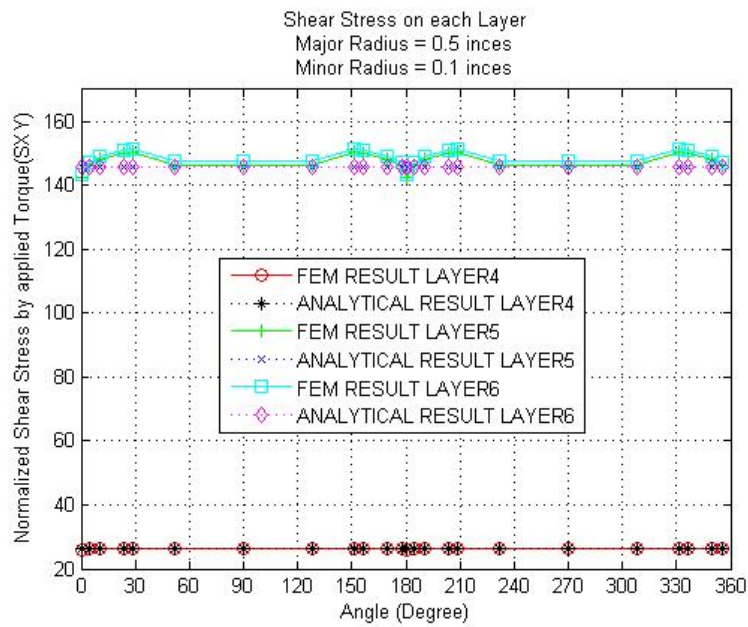
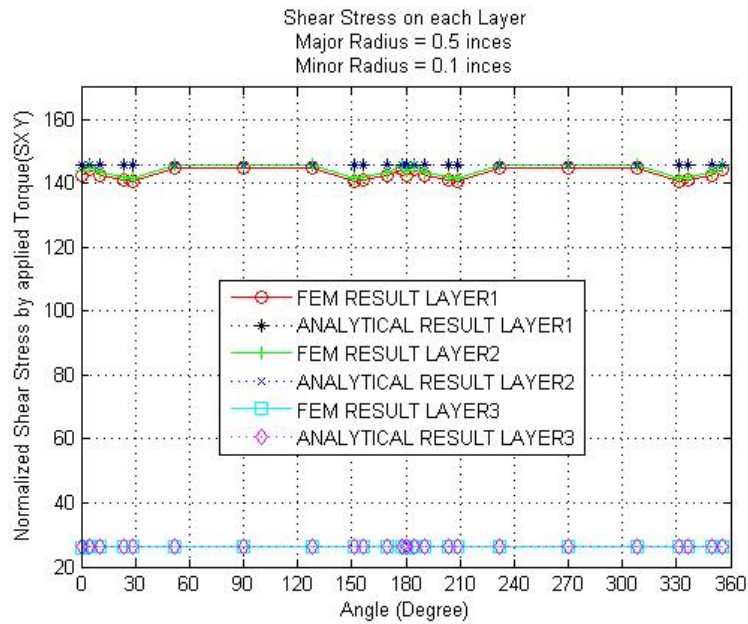


Figure 4.15 The comparison between FEM and CLPT for $[\pm 45/0]_S$ from layer (a) 1-3 and (b) 3-6

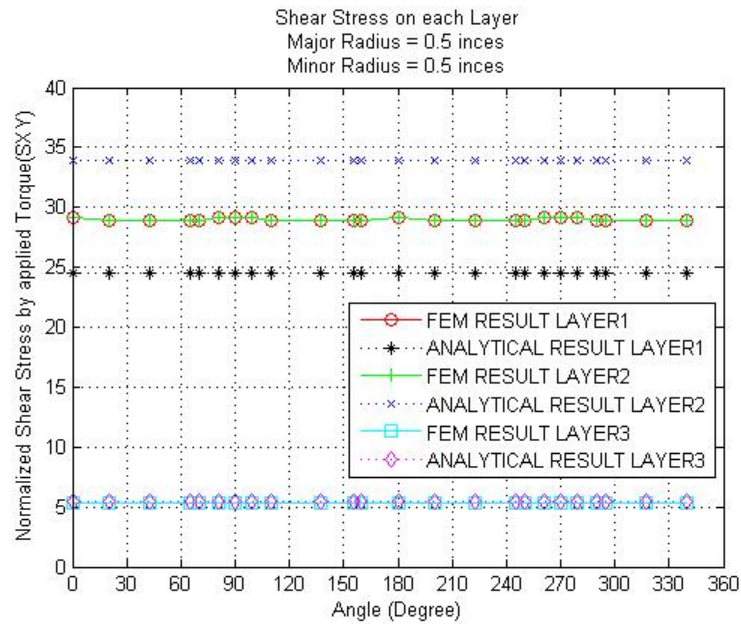
$$\text{with } \frac{b}{a} = 0.2$$

The $[\pm 45/0]_s$ is symmetric and balance layup. There is no z-displacement in the model and the material orientation does not cause a curvature effect due to varying moduli on either side of the composite centerline. This can also be seen in the composite constitutive matrices by the null extensional-bending stiffness or B matrix. The shear stress results that got from FEM are generally agrees to the CLPT as the previous cases. The miss matches are emerged on 45 and -45 degree ply when the radius ratio is high, $\frac{b}{a} = 0.4$ and, $\frac{b}{a} = 0.2$ on the high curvature region due to the severe configuration.

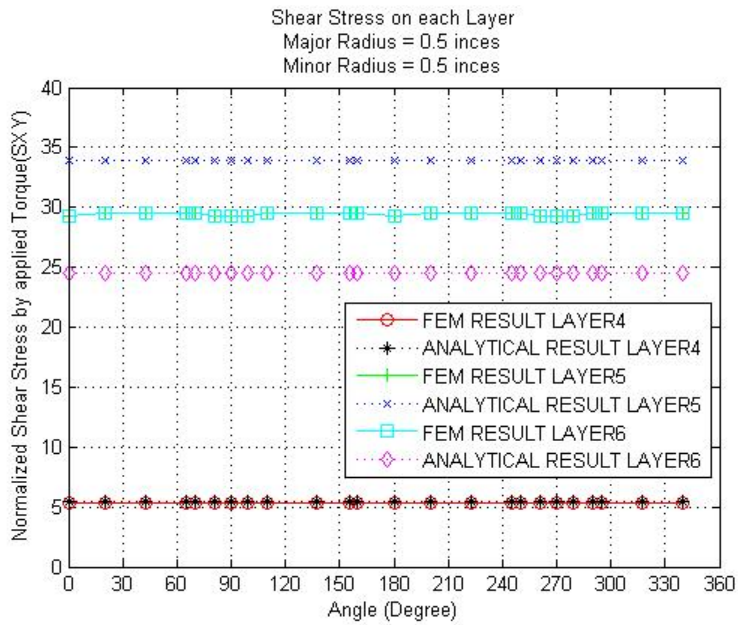
4.2 The comparison results of Unsymmetric layups between FEM and CLPT

4.2.1 Anti-Symmetric & Balanced Multi fiber orientation [+45/-45/0/0/45/-45]

The $[+45/-45/0/0/45/-45]$ is anti-symmetric and balanced layup. The model is now unsymmetrically layered, and the material orientation cause a curvature effect due to varying moduli on either side of the composite centerline. The composite constitutive matrices are no longer zero which means extensional-bending stiffness or B matrix. These caused the miss match of shear stress between FEM and CLPT on the 45 and -45 degree plies which impact from the effect of the extensional-bending stiffness and curvature. The spike pattern appear on the high curvature region when the radius ratio is high for example $\frac{b}{a} = 0.4$ and, $\frac{b}{a} = 0.2$ because of the effect of curvatures



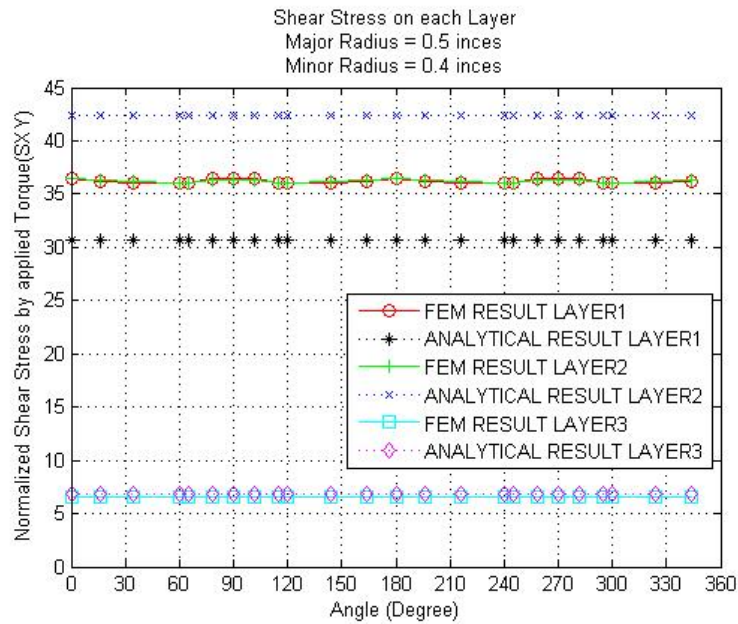
(a)



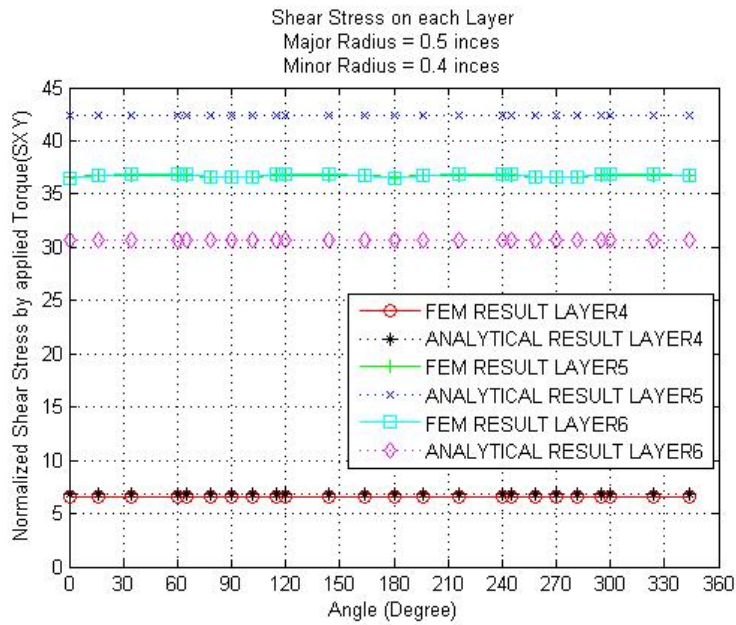
(b)

Figure 4.16 The comparison between FEM and CLPT for [+45/-45/0/0/45/-45] from layer (a) 1-3

and (b) 3-6 with $\frac{b}{a} = 1$



(a)



(b)

Figure 4.17 The comparison between FEM and CLPT for [+45/-45/0/0/45/-45] from layer (a) 1-3

and (b) 3-6 with $\frac{b}{a} = 0.8$

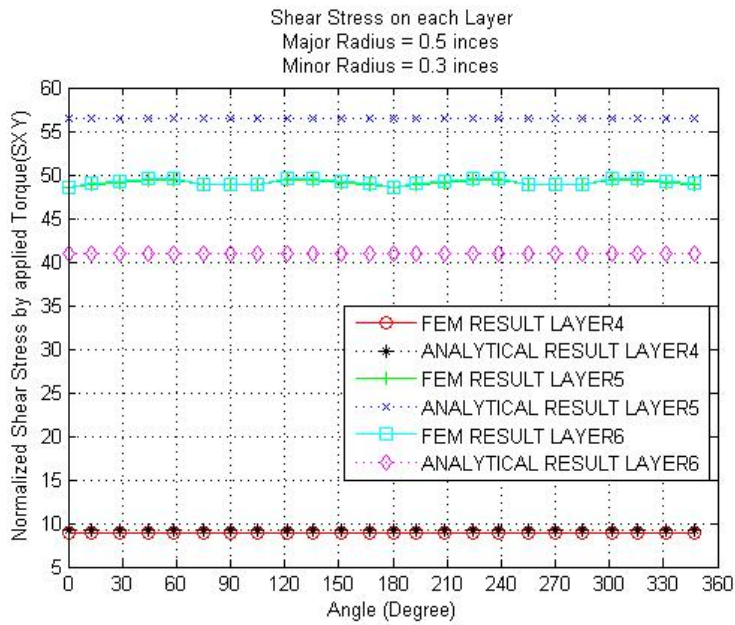
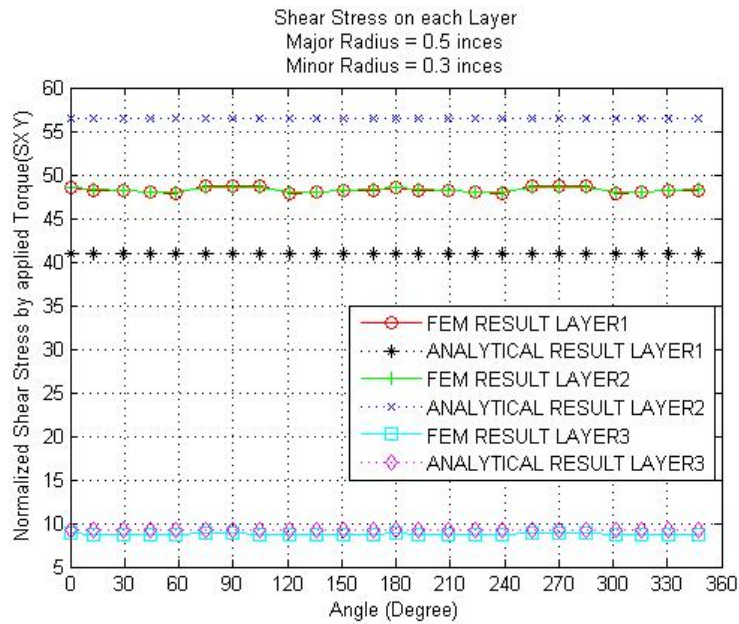


Figure 4.18 The comparison between FEM and CLPT for [+45/-45/0/0/45/-45] from layer (a) 1-3

and (b) 3-6 with $\frac{b}{a} = 0.6$

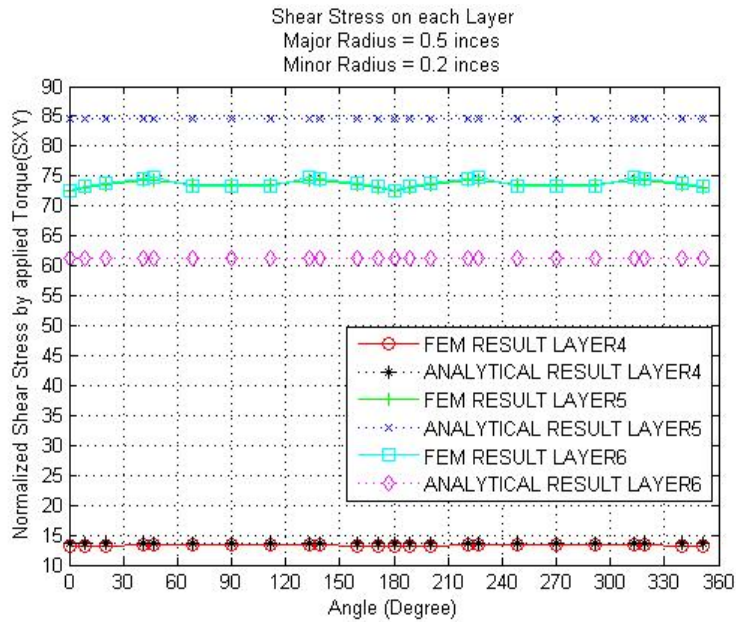
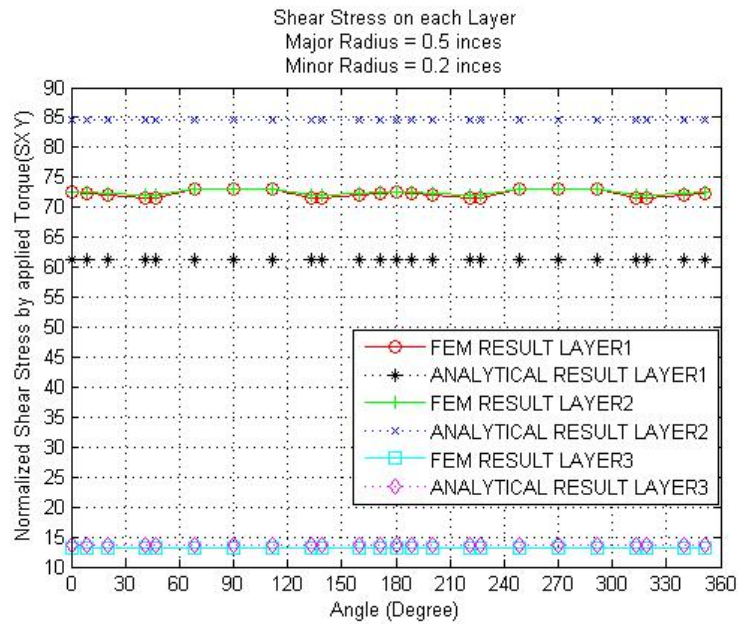


Figure 4.19 The comparison between FEM and CLPT for [+45/-45/0/0/45/-45] from layer (a) 1-3

and (b) 3-6 with $\frac{b}{a} = 0.4$

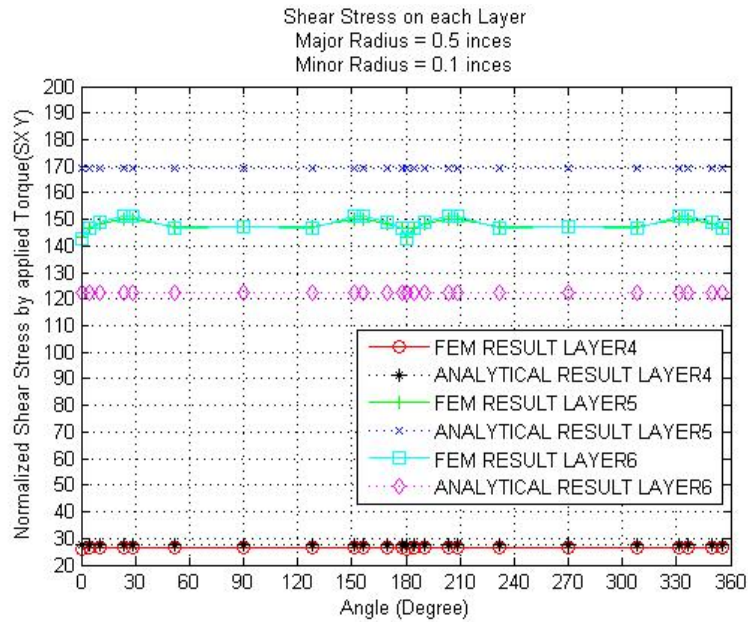
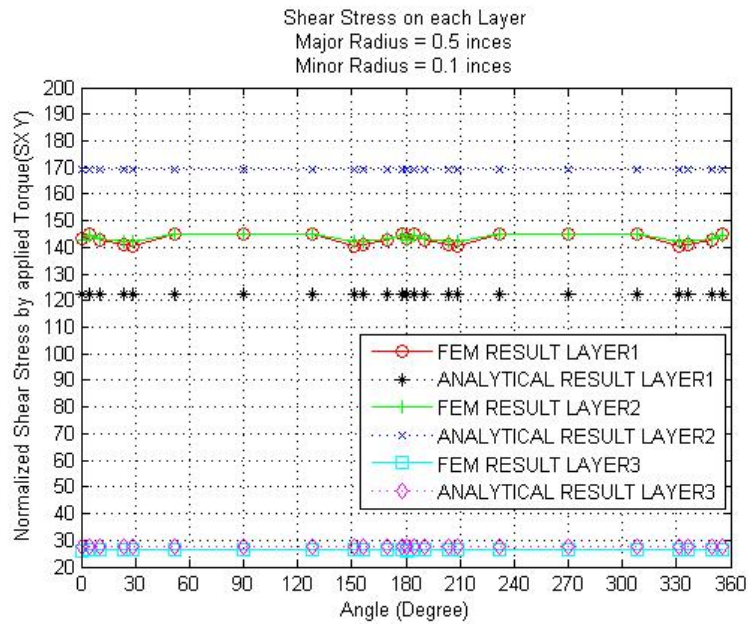


Figure 4.20 The comparison between FEM and CLPT for [+45/-45/0/0/45/-45] from layer (a) 1-3

and (b) 3-6 with $\frac{b}{a} = 0.2$

4.2.2 Unsymmetric & Balanced Multi fiber orientation $[\pm 45/0]_{2T}$

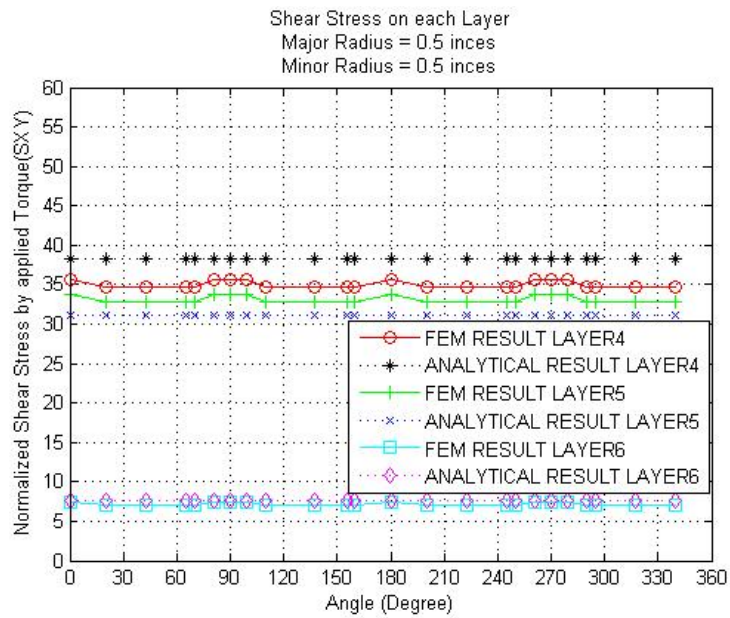
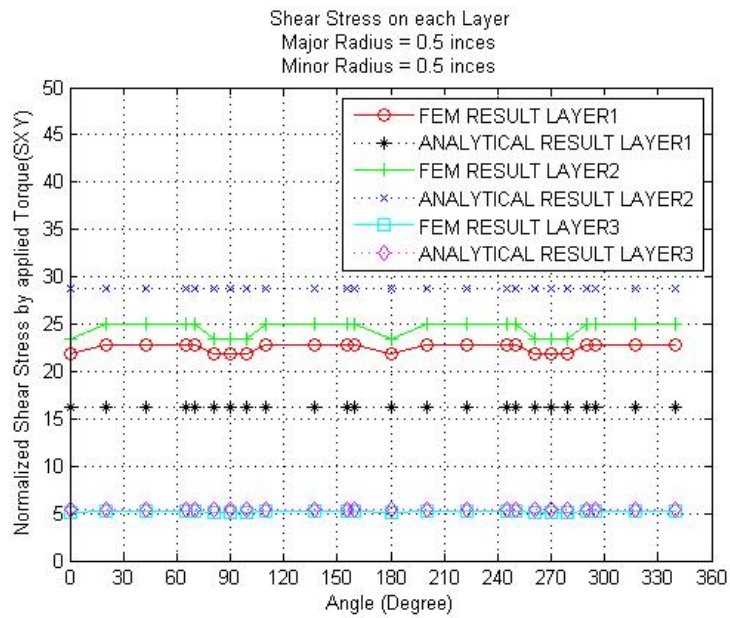
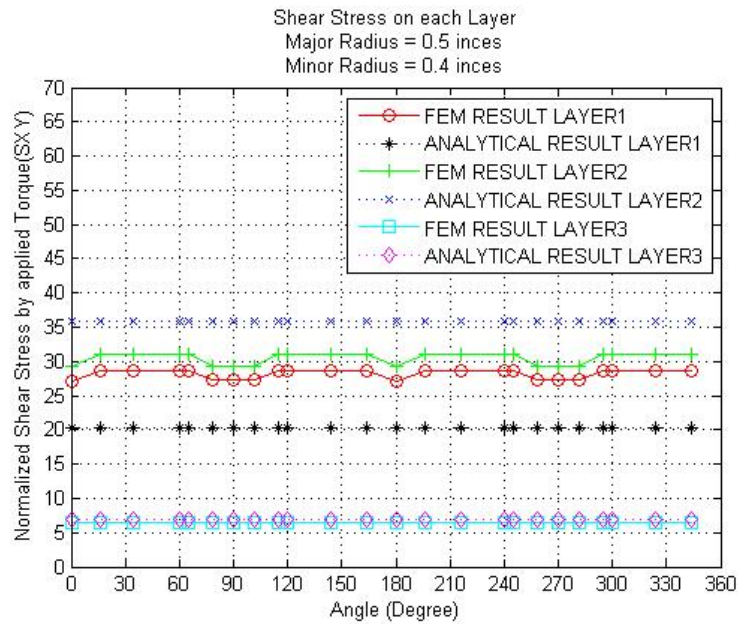
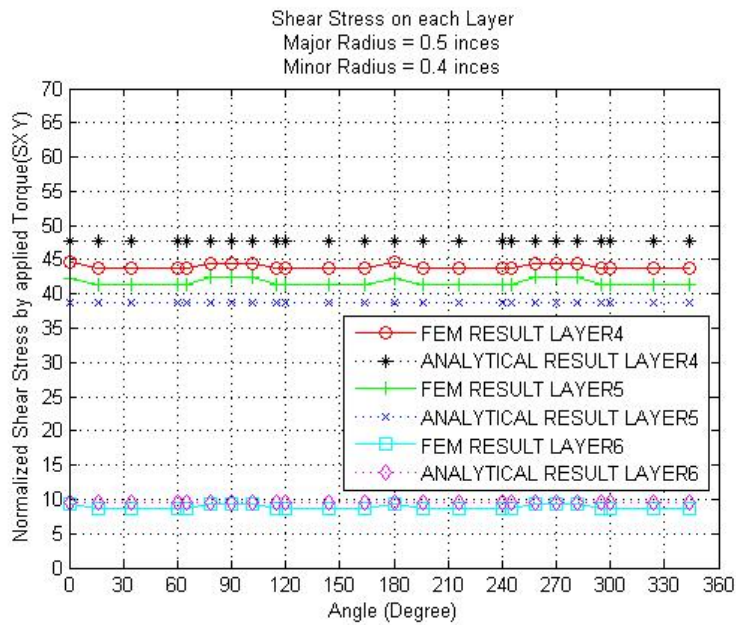


Figure 4.21 The comparison between FEM and CLPT for $[\pm 45/0]_{2T}$ from layer (a) 1-3 and (b) 3-6

with $\frac{b}{a} = 1$



(a)



(b)

Figure 4.22 The comparison between FEM and CLPT for $[\pm 45/0]_{2T}$ from layer (a) 1-3 and (b) 3-6

with $\frac{b}{a} = 0.8$

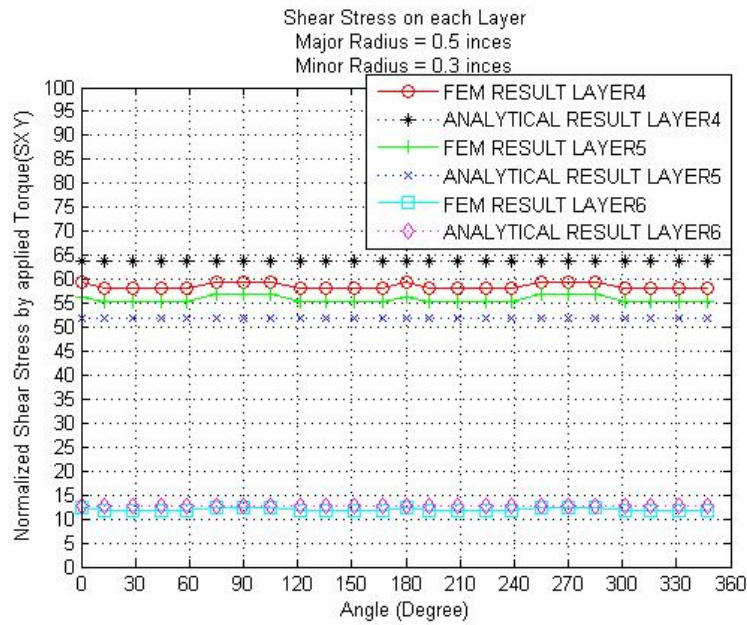
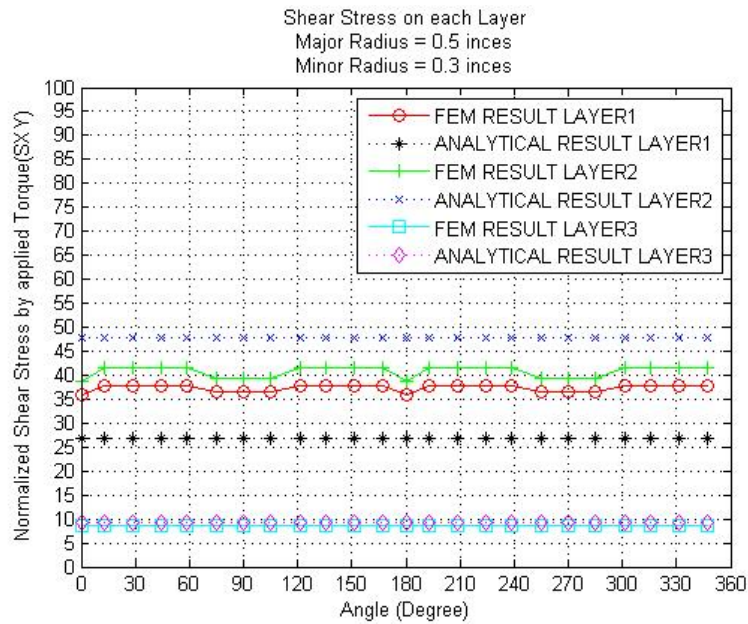
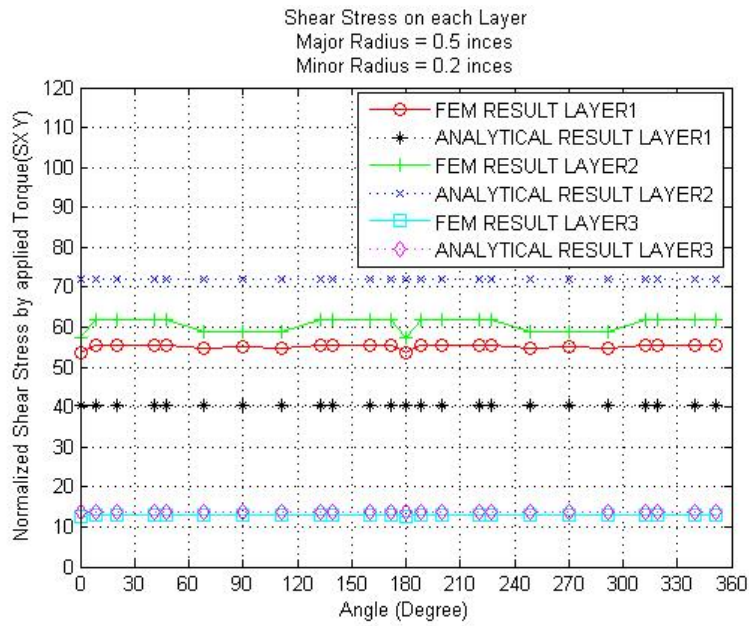
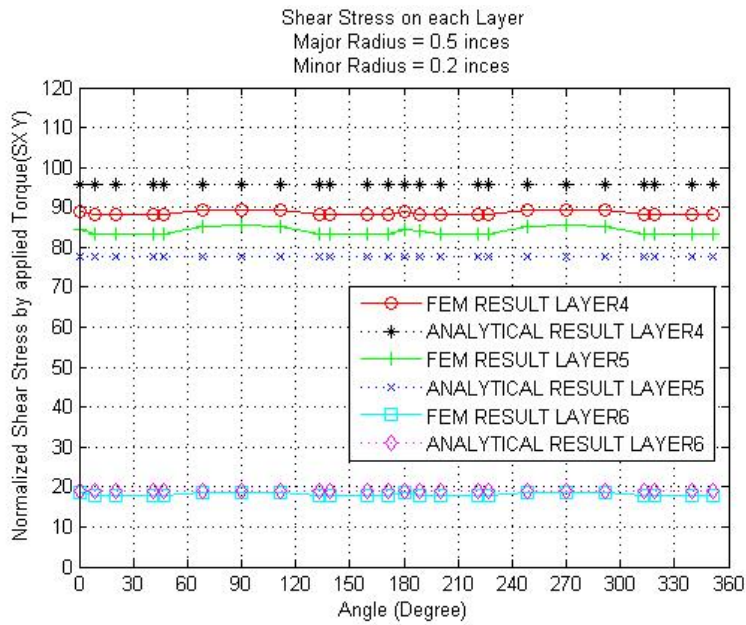


Figure 4.23 The comparison between FEM and CLPT for $[\pm 45/0]_{2T}$ from layer (a) 1-3 and (b) 3-6

$$\text{with } \frac{b}{a} = 0.6$$



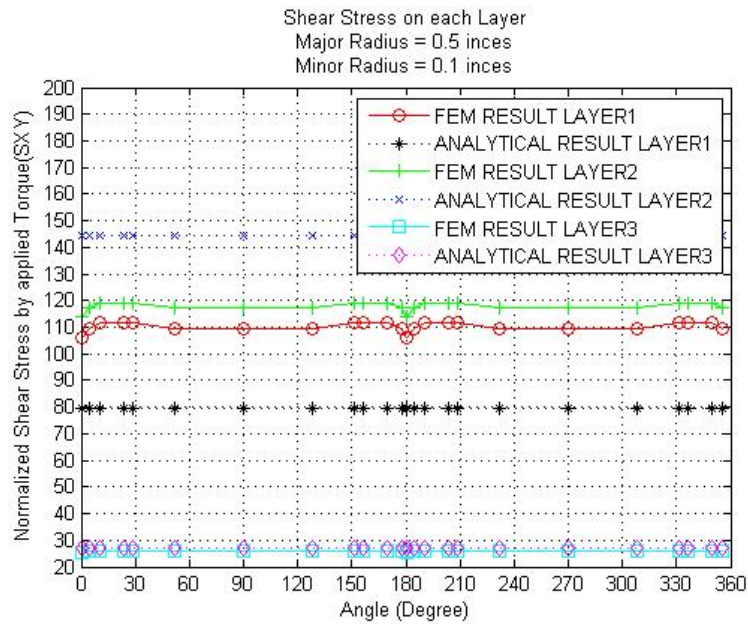
(a)



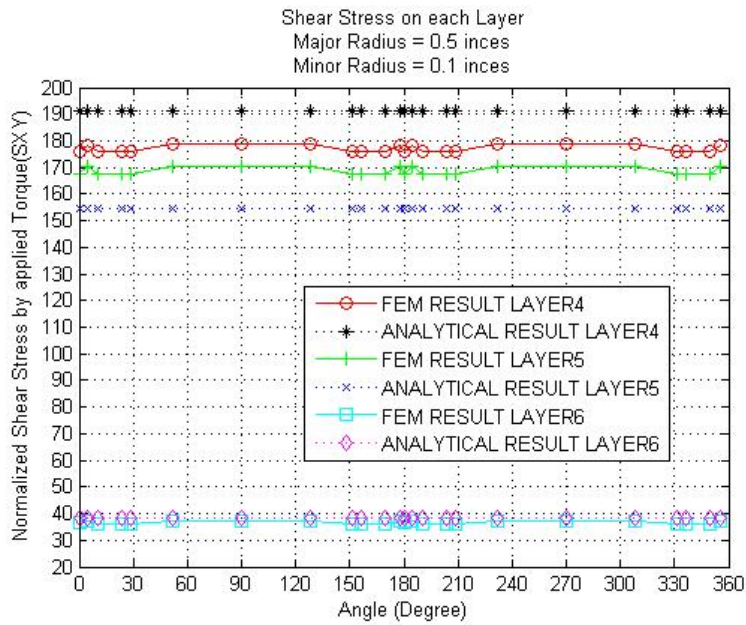
(b)

Figure 4.24 The comparison between FEM and CLPT for $[\pm 45/0]_{2T}$ from layer (a) 1-3 and (b) 3-6

with $\frac{b}{a} = 0.4$



(a)



(b)

Figure 4.25 The comparison between FEM and CLPT for $[\pm 45/0]_{2T}$ from layer (a) 1-3 and (b) 3-6

$$\text{with } \frac{b}{a} = 0.2$$

The $[\pm 45/0]_{2T}$ is unsymmetric and balanced layup. The model is now unsymmetrically layered, and the material orientation cause a curvature effect due to varying moduli on either side of the composite centerline. The composite constitutive matrices are no longer zero which means extensional-bending stiffness or B matrix. These caused the miss match of shear stress between FEM and CLPT on the 45 and -45 degree plies which impact from the effect of the extensional-bending stiffness and curvature. The spike pattern appear on the high curvature region when the radius ratio is high for example $\frac{b}{a} = 0.4$ and, $\frac{b}{a} = 0.2$ because of the effect of curvatures

CHAPTER 5

CONCLUSION AND RECOMMENDATION

The behavior of shear stress was investigated for circular and elliptical laminated tubular cross sections with symmetric and unsymmetrical layups includes change of ply orientation along the circumference of the tube. The results of shear stress are exhibited corresponding to the angle along the circumference of the tube. Several elliptical tubes with different radius ratio are investigated based on a major radius is equal to 0.5in. The results reveal an unusual behavior pattern in the high curvature section of large aspect ratio elliptical cross sections. However, in the circular cross section shape the shear stress are identical to laminated plate theory

The numerical model of laminated tubular structure was developed under the commercial finite element program called ANSYS CLASSIC V.11. The tubular models are validated using isotropic material, aluminum. Shear stress along the tube circumferences with constant thickness are compared by the result of FEM with analytic thin wall under torsion. Shear stresses are identical in circular and elliptical tube with large aspect ratio, 1, 0.8, and 0.6. However, the shear stress result in the elliptical tube with small aspect ratio is about 2.7% off from the theory of thin wall under torsion.

The symmetric layups which includes $[0]_{6T}$, $[45]_{6T}$, and $[\pm 45/0]_S$ were investigated. The results of shear stress in laminated tube with symmetric layups obtained by the finite element method generally agree with those obtained from laminated plate theory because the effect of curvature does not play a significant role in the laminate tube due to the behavior of symmetrical layups. The extensional-bending coupling stiffness matrix is equal to zero. However, when the minor over major radius ratio is getting small the unusual pattern also exist on the high curvature region.

The shear stress in the unsymmetrical case as $[45/-45/0_2/45/-45]$ and $[45/-45/0/45/-45/0]$ which obtained from FEM exhibit the miss match result with the shear stress obtained from CSLP. The miss match exists due to the behavior of unsymmetrical layups that calculating based on laminated plated theory, inducing the effect of curvature due to the extensional-bending coupling stiffness matrix is not equal to zero anymore. However, this effect is not affecting much on the 0 degree ply. Developing the lamination theory under torsion should be developed in the future study based on shell laminated by using this research as a reference. Furthermore, this command code will be used to study tubular structure under thermal effect by applying the thermal condition into the MP command which attached in the APPENDIX C.

APPENDIX A

MATLAB PROGRAMS FOR LAMINATED PLAT THEORY

```

clc, clear;

%Given Constants%
E1=21.3e6;
E2=1.5e6;
E3=1.5e6;
G12=1.0e6;
G23=0.54e6;
G13=1.0e6;
v12=0.27;
v21=v12*E2/E1;
v23=0.54;
v13=0.27;
t=0.005;

%Compliance Matrix [S]1-2 %
S11=1/E1;
S12=-v21/E2;
S21=-v12/E1;
S22=1/E2;
S66=1/G12;
S14=0; S15=0; S16=0;
S24=0; S25=0; S26=0;
S34=0; S35=0; S36=0;
S41=0; S42=0; S43=0; S45=0; S46=0;
S51=0; S52=0; S53=0; S54=0; S56=0;
S61=0; S62=0; S63=0; S64=0; S65=0;
S=[S11, S12, S16;S21, S22, S26;S61, S62, S66];

%Whole Stiffness Matrix [Q]1-2 %
Q=inv(S)

%Convert [Q]1-2 to [Q_0]x-y%
m1=cos(0); n1=sin(0);
Tstress_0=[m1^2, n1^2, 2*m1*n1; n1^2, m1^2, -2*m1*n1; -m1*n1, m1*n1,
m1^2-n1^2];
Tinvstress_0 = inv(Tstress_0);
Tstrain_0=[m1^2, n1^2, m1*n1; n1^2, m1^2, -m1*n1; -2*m1*n1, 2*m1*n1,
m1^2-n1^2];
Q_0=(Tinvstress_0*Q)*Tstrain_0

%Convert [Q]1-2 to [Q_45]x-y%
m2=cos(pi/4); n2=sin(pi/4);
Tstress_45=[m2^2, n2^2, 2*m2*n2; n2^2, m2^2, -2*m2*n2; -m2*n2, m2*n2,
m2^2-n2^2];
Tinvstress_45= inv(Tstress_45);
Tstrain_45=[m2^2, n2^2, m2*n2; n2^2, m2^2, -m2*n2; -2*m2*n2, 2*m2*n2,
m2^2-n2^2];
Q_45=(Tinvstress_45*Q)*Tstrain_45

%Convert [Q]1-2 to [Q_-45]x-y%
m3=cos(-pi/4); n3=sin(-pi/4);

```

```

Tstress_neg45=[m3^2, n3^2, 2*m3*n3; n3^2, m3^2, -2*m3*n3; -m3*n3,
m3*n3, m3^2-n3^2];
Tinvstress_neg45= inv(Tstress_neg45);
Tstrain_neg45=[m3^2, n3^2, m3*n3; n3^2, m3^2, -m3*n3; -2*m3*n3,
2*m3*n3, m3^2-n3^2];
Q_neg45=(Tinvstress_neg45*Q)*Tstrain_neg45

%Convert [Q]1-2 to [Q_90]x-y%
m4=cos(pi/2); n4=sin(pi/2);
Tstress_90=[m3^2, n3^2, 2*m3*n3; n3^2, m3^2, -2*m3*n3; -m3*n3, m3*n3,
m3^2-n3^2];
Tinvstress_90= inv(Tstress_90);
Tstrain_90=[m3^2, n3^2, m3*n3; n3^2, m3^2, -m3*n3; -2*m3*n3, 2*m3*n3,
m3^2-n3^2];
Q_90=(Tinvstress_90*Q)*Tstrain_90

%Matrix [A];

%For [A] [45/-45/0/0/-45/45]%
A = (Q_45+Q_neg45+Q_0+Q_0+Q_neg45+Q_45)*t

%Matrix [B];
%k14b = 13*t^2;
%k13b = 11*t^2;
%k12b = 9*t^2;
%k11b = 7*t^2;
k6b = 5*t^2;
k5b = 3*t^2;
k4b = t^2;
k3b = -t^2;
k2b = -3*t^2;
k1b = -5*t^2;
%k4b = -7*t^2;
%k3b = -9*t^2;
%k2b = -11*t^2;
%k1b = -13*t^2;

%For [B] [45/-45/0/0/-45/45]%
B = (Q_45*k1b+Q_neg45*k2b+Q_0*k3b+Q_0*k4b+Q_neg45*k5b+Q_45*k6b)/2

%Matrix [D];
%k14d = 127*t^3;
%k13d = 91*t^3;
%k12d = 61*t^3;
%k11d = 37*t^3;
k6d = 19*t^3;
k5d = 7*t^3;
k4d = t^3;
k3d = t^3;
k2d = 7*t^3;

```

```

k1d = 19*t^3;
%k4d = 37*t^3;
%k3d = 61*t^3;
%k2d = 91*t^3;
%k1d = 127*t^3;

%For [D] [45/-45/0/90/90/0/-45/45]%
D = (Q_45*k1d+Q_neg45*k2d+Q_0*k3d+Q_0*k4d+Q_neg45*k5d+Q_45*k6d)/3

%Matrix [a], [b], and [d]%
Z = D-(B/A)*B;
a = inv(A)-(-inv(A)*B)*inv(Z)*(B*inv(A));
b = (-inv(A)*B)*inv(Z);
c = b';
d = inv(Z);

ABD = [A B; B D];
abd = [a b; b d];

%Calculate N_th and M_th
syms Nx Ny Nxy Kx Ky Kxy Mx My Mxy z;
Nx=0; Ny=0; Nxy=0.637; Kx=0; Ky=0; Kxy=0; Mx=0; My=0; Mxy=0;

%Calculated Epsilon(0) and Curvature%
N = [Nx;Ny;Nxy]
M = [Mx;My;Mxy]
e_0=[a b]*[N; M]
k = [b d]*[N; M]

%Calculated Principle Strain%
Strain_45_TOP6 = e_0+(0.005*3)*k
Strain_45_BOT6 = e_0+(0.005*2)*k

Strain_neg45_TOP5 = e_0+(0.005*2)*k
Strain_neg45_BOT5 = e_0+(0.005)*k

Strain_0_TOP4 = e_0+(0.005)*k
Strain_0_BOT4 = e_0

Strain_0_TOP3 = e_0
Strain_0_BOT3 = e_0+(-0.005)*k

Strain_neg45_TOP2 = e_0+(-0.005)*k
Strain_neg45_BOT2 = e_0+(-0.005*2)*k

Strain_45_TOP1 = e_0+(-0.005*2)*k
Strain_45_BOT1 = e_0+(-0.005*3)*k
%-----%

%Calculated Principle Stress%
Stress_45_TOP6 =Q_45*Strain_45_TOP6

```

Stress_45_BOT6 =Q_45*Strain_45_BOT6
Stress_neg45_TOP5 = Q_neg45*Strain_neg45_TOP5
Stress_neg45_BOT5 = Q_neg45*Strain_neg45_BOT5

Stress_0_TOP4 = Q_0*Strain_0_TOP4
Stress_0_BOT4 = Q_0*Strain_0_BOT4

Stress_0_TOP3 = Q_0*Strain_0_TOP3
Stress_0_BOT3 = Q_0*Strain_0_BOT3

Stress_neg45_TOP2 = Q_neg45*Strain_neg45_TOP2
Stress_neg45_BOT2 = Q_neg45*Strain_neg45_BOT2

Stress_45_TOP1 = Q_45*Strain_45_TOP1
Stress_45_BOT1 = Q_45*Strain_45_BOT1

%-----%

APPENDIX B

FINITE ELEMENT CODE : ANSYS V.11 PROGRAMS

| | |
|--|----------------------------------|
| FINISH | keyopt,1,2,0 |
| /CLEAR | keyopt,1,3,0 |
| /FILENAME,SYMETRY | keyopt,1,4,0 |
| /TITLE,Tubular Structure Under Torsion | keyopt,1,5,2 |
| /prep7 | keyopt,1,6,5 |
| /OUTPUT, CCT, OUT, | keyopt,1,8,1 |
| !! Basic Parameter Setup !! | keyopt,1,9,0 |
| *SET,radi,0.5 | keyopt,1,10,0 |
| *SET,semiradi,0.1 | keyopt,1,11,0 |
| *SET,length,5 | |
| *SET,t,0.005 !-----* | r,1 |
| *SET,torque,1 | RMODIF,1,1,6, |
| *SET,force,2 | RMODIF,1,13, 1, 45, t, |
| | RMODIF,1,16, 1, -45, t, |
| !! Material Properties Setup!! | RMODIF,1,19, 1, 0, t, |
| et,1,shell99 | RMODIF,1,22, 1, 0, t, |
| mp,ex,1,21.3e6 | RMODIF,1,25, 1, -45, t, |
| mp,ey,1,1.5e6 | RMODIF,1,28, 1, 45, t, |
| mp,ez,1,1.5e6 | |
| mp,prxy,1,0.27 | !! Coordinate Preparation !! |
| mp,prxz,1,0.27 | local,11,1,,,,,,,,,semiradi/radi |
| mp,pryz,1,0.54 | CSYS,11 |
| mp,gxy,1,1e6 | K,1:80,radi,0:360:4.5 |
| mp,gyz,1,0.54e6 | |
| mp,gxz,1,1e6 | !! Laminated Tube Modeling !! |
| | k,100001,0,0,0 |

| | |
|---------------------------|-------------------------------|
| k,100002,0,0,length | d,all,uy,0 |
| | d,all,uz,0 |
| *do,i,1,79,1 | d,all,rotx,0 |
| bsplin,i,i+1 | d,all,roty,0 |
| *enddo | d,all,rotz,0 |
| bsplin,80,1 | allsel |
| lglue,all | !! Apply Loading Condition !! |
| al,all | et,2,MPC184 |
| VOFFST,1,length, , | keyopt,2,1,1 |
| VDELE,1 | keyopt,2,2,1 |
| ADELE,1,2 | type,2 |
| AGLUE,ALL | real,2 |
| | E,2, 8 |
| lsel,s,length,,length,,,0 | E,2, 168 |
| lesize,all,,,80 | E,2, 169 |
| cm,sideline,line | E,2, 170 |
| allsel,all,line | E,2, 171 |
| nkpt,,100001 | E,2, 652 |
| nkpt,,100002 | E,2, 812 |
| amesh,all | E,2, 813 |
| nrotat,all | E,2, 814 |
| | E,2, 1136 |
| nset,s,loc,z,0 | E,2, 1296 |
| nset,u,node,1 | E,2, 1297 |
| d,all,ux,0 | E,2, 1298 |

| | |
|-----------|-----------|
| E,2, 1620 | E,2, 4685 |
| E,2, 1780 | E,2, 4686 |
| E,2, 1781 | E,2, 5008 |
| E,2, 1782 | E,2, 5168 |
| E,2, 2104 | E,2, 5169 |
| E,2, 2264 | E,2, 5170 |
| E,2, 2265 | E,2, 5492 |
| E,2, 2266 | E,2, 5652 |
| E,2, 2588 | E,2, 5653 |
| E,2, 2748 | E,2, 5654 |
| E,2, 2749 | E,2, 5976 |
| E,2, 2750 | E,2, 6136 |
| E,2, 3072 | E,2, 6137 |
| E,2, 3232 | E,2, 6138 |
| E,2, 3233 | E,2, 6460 |
| E,2, 3234 | E,2, 6620 |
| E,2, 3556 | E,2, 6621 |
| E,2, 3716 | E,2, 6622 |
| E,2, 3717 | E,2, 6944 |
| E,2, 3718 | E,2, 7104 |
| E,2, 4040 | E,2, 7105 |
| E,2, 4200 | E,2, 7106 |
| E,2, 4201 | E,2, 7428 |
| E,2, 4202 | E,2, 7588 |
| E,2, 4524 | E,2, 7589 |
| E,2, 4684 | E,2, 7590 |

| | |
|------------|------------|
| E,2, 7912 | E,2, 10977 |
| E,2, 8072 | E,2, 10978 |
| E,2, 8073 | E,2, 11300 |
| E,2, 8074 | E,2, 11460 |
| E,2, 8396 | E,2, 11461 |
| E,2, 8556 | E,2, 11462 |
| E,2, 8557 | E,2, 11784 |
| E,2, 8558 | E,2, 11944 |
| E,2, 8880 | E,2, 11945 |
| E,2, 9040 | E,2, 11946 |
| E,2, 9041 | E,2, 12268 |
| E,2, 9042 | E,2, 12428 |
| E,2, 9364 | E,2, 12429 |
| E,2, 9524 | E,2, 12430 |
| E,2, 9525 | E,2, 12752 |
| E,2, 9526 | E,2, 12912 |
| E,2, 9848 | E,2, 12913 |
| E,2, 10008 | E,2, 12914 |
| E,2, 10009 | E,2, 13236 |
| E,2, 10010 | E,2, 13396 |
| E,2, 10332 | E,2, 13397 |
| E,2, 10492 | E,2, 13398 |
| E,2, 10493 | E,2, 13720 |
| E,2, 10494 | E,2, 13880 |
| E,2, 10816 | E,2, 13881 |
| E,2, 10976 | E,2, 13882 |

| | |
|------------|------------|
| E,2, 14204 | E,2, 17269 |
| E,2, 14364 | E,2, 17270 |
| E,2, 14365 | E,2, 17592 |
| E,2, 14366 | E,2, 17752 |
| E,2, 14688 | E,2, 17753 |
| E,2, 14848 | E,2, 17754 |
| E,2, 14849 | E,2, 18076 |
| E,2, 14850 | E,2, 18236 |
| E,2, 15172 | E,2, 18237 |
| E,2, 15332 | E,2, 18238 |
| E,2, 15333 | E,2, 18560 |
| E,2, 15334 | E,2, 18720 |
| E,2, 15656 | E,2, 18721 |
| E,2, 15816 | E,2, 18722 |
| E,2, 15817 | E,2, 19044 |
| E,2, 15818 | E,2, 19204 |
| E,2, 16140 | E,2, 19205 |
| E,2, 16300 | E,2, 19206 |
| E,2, 16301 | E,2, 19528 |
| E,2, 16302 | E,2, 19688 |
| E,2, 16624 | E,2, 19689 |
| E,2, 16784 | E,2, 19690 |
| E,2, 16785 | E,2, 20012 |
| E,2, 16786 | E,2, 20172 |
| E,2, 17108 | E,2, 20173 |
| E,2, 17268 | E,2, 20174 |

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|------------|------------|
| E,2, 20496 | E,2, 23561 |
| E,2, 20656 | E,2, 23562 |
| E,2, 20657 | E,2, 23884 |
| E,2, 20658 | E,2, 24044 |
| E,2, 20980 | E,2, 24045 |
| E,2, 21140 | E,2, 24046 |
| E,2, 21141 | E,2, 24368 |
| E,2, 21142 | E,2, 24528 |
| E,2, 21464 | E,2, 24529 |
| E,2, 21624 | E,2, 24530 |
| E,2, 21625 | E,2, 24852 |
| E,2, 21626 | E,2, 25012 |
| E,2, 21948 | E,2, 25013 |
| E,2, 22108 | E,2, 25014 |
| E,2, 22109 | E,2, 25336 |
| E,2, 22110 | E,2, 25496 |
| E,2, 22432 | E,2, 25497 |
| E,2, 22592 | E,2, 25498 |
| E,2, 22593 | E,2, 25820 |
| E,2, 22594 | E,2, 25980 |
| E,2, 22916 | E,2, 25981 |
| E,2, 23076 | E,2, 25982 |
| E,2, 23077 | E,2, 26304 |
| E,2, 23078 | E,2, 26464 |
| E,2, 23400 | E,2, 26465 |
| E,2, 23560 | E,2, 26466 |

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|------------|------------|
| E,2, 26788 | E,2, 29853 |
| E,2, 26948 | E,2, 29854 |
| E,2, 26949 | E,2, 30176 |
| E,2, 26950 | E,2, 30336 |
| E,2, 27272 | E,2, 30337 |
| E,2, 27432 | E,2, 30338 |
| E,2, 27433 | E,2, 30660 |
| E,2, 27434 | E,2, 30820 |
| E,2, 27756 | E,2, 30821 |
| E,2, 27916 | E,2, 30822 |
| E,2, 27917 | E,2, 31144 |
| E,2, 27918 | E,2, 31304 |
| E,2, 28240 | E,2, 31305 |
| E,2, 28400 | E,2, 31306 |
| E,2, 28401 | E,2, 31628 |
| E,2, 28402 | E,2, 31788 |
| E,2, 28724 | E,2, 31789 |
| E,2, 28884 | E,2, 31790 |
| E,2, 28885 | E,2, 32112 |
| E,2, 28886 | E,2, 32272 |
| E,2, 29208 | E,2, 32273 |
| E,2, 29368 | E,2, 32274 |
| E,2, 29369 | E,2, 32596 |
| E,2, 29370 | E,2, 32756 |
| E,2, 29692 | E,2, 32757 |
| E,2, 29852 | E,2, 32758 |

| | |
|------------|---------------|
| E,2, 33080 | E,2, 36144 |
| E,2, 33240 | E,2, 36145 |
| E,2, 33241 | E,2, 36146 |
| E,2, 33242 | E,2, 36468 |
| E,2, 33564 | E,2, 36628 |
| E,2, 33724 | E,2, 36629 |
| E,2, 33725 | E,2, 36630 |
| E,2, 33726 | E,2, 36952 |
| E,2, 34048 | E,2, 37112 |
| E,2, 34208 | E,2, 37113 |
| E,2, 34209 | E,2, 37114 |
| E,2, 34210 | E,2, 37436 |
| E,2, 34532 | E,2, 37596 |
| E,2, 34692 | E,2, 37597 |
| E,2, 34693 | E,2, 37598 |
| E,2, 34694 | E,2, 37920 |
| E,2, 35016 | E,2, 38080 |
| E,2, 35176 | E,2, 38081 |
| E,2, 35177 | E,2, 38082 |
| E,2, 35178 | E,2, 38403 |
| E,2, 35500 | E,2, 38404 |
| E,2, 35660 | E,2, 38405 |
| E,2, 35661 | f,2,mz,torque |
| E,2, 35662 | allsel,all |
| E,2, 35984 | |

APPENDIX C

SHEAR STRESS RESULTS ALONG CIRCUMFERENCES

Table C.1 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 20.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 42.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 65.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 69.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 81 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 90 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 99 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 110.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 137.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 155.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 159.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 180 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 200.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 222.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 245.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 249.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 261 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 270 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 279 | 21.101 | 21.2333 | 0.6230779 | 21.16 | 21.2333 | 0.34521247 | 21.214 | 21.2333 | 0.09089496 |
| 290.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |

Table C.1 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 294.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 317.25 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |
| 339.75 | 20.906 | 21.2333 | 1.54144669 | 20.991 | 21.2333 | 1.14113209 | 21.085 | 21.2333 | 0.69843124 |

Table C.2 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 20.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 42.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 65.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 69.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 81 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 90 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 99 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 110.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 137.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 155.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 159.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 180 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 200.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 222.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 245.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 249.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |

Table C.2 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 261 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 270 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 279 | 21.263 | 21.2333 | -0.1398746 | 21.306 | 21.2333 | -0.3423867 | 21.344 | 21.2333 | -0.5213509 |
| 290.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 294.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 317.25 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |
| 339.75 | 21.379 | 21.2333 | -0.6861863 | 21.404 | 21.2333 | -0.8039259 | 21.468 | 21.2333 | -1.1053393 |

Table C.3 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| ANGL E (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-----------------|--------------------------|----------------------------------|-------------------|--------------------------|----------------------------------|-------------------|--------------------------|----------------------------------|-------------------|
| | FEM Result (lb-in) | Analytica l Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytica l Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytica l Result (lb-in) | Miss Match (%) |
| 0 | 26.358 | 26.5333 | 0.66067922 | 26.427 | 26.5333 | 0.40062864 | 26.486 | 26.5333 | 0.17826656 |
| 16.432 | 25.678 | 26.5333 | 3.22349651 | 26.012 | 26.5333 | 1.96470096 | 26.345 | 26.5333 | 0.70967426 |
| 34.46 | 25.611 | 26.5333 | 3.47600939 | 25.98 | 26.5333 | 2.08530413 | 26.35 | 26.5333 | 0.69083001 |
| 60.03 | 25.552 | 26.5333 | 3.69837148 | 25.955 | 26.5333 | 2.17952535 | 26.359 | 26.5333 | 0.65691037 |
| 65.23 | 25.544 | 26.5333 | 3.72852227 | 25.952 | 26.5333 | 2.1908319 | 26.36 | 26.5333 | 0.65314152 |
| 78.8 | 26.393 | 26.5333 | 0.52876951 | 26.475 | 26.5333 | 0.21972389 | 26.55 | 26.5333 | -0.0629398 |
| 90 | 26.394 | 26.5333 | 0.52500066 | 26.476 | 26.5333 | 0.21595505 | 26.552 | 26.5333 | -0.0704775 |
| 101.2 | 26.393 | 26.5333 | 0.52876951 | 26.475 | 26.5333 | 0.21972389 | 26.55 | 26.5333 | -0.0629398 |
| 114.77 | 25.544 | 26.5333 | 3.72852227 | 25.952 | 26.5333 | 2.1908319 | 26.36 | 26.5333 | 0.65314152 |
| 119.97 | 25.552 | 26.5333 | 3.69837148 | 25.955 | 26.5333 | 2.17952535 | 26.359 | 26.5333 | 0.65691037 |
| 143.54 | 25.611 | 26.5333 | 3.47600939 | 25.98 | 26.5333 | 2.08530413 | 26.35 | 26.5333 | 0.69083001 |
| 163.57 | 25.678 | 26.5333 | 3.22349651 | 26.012 | 26.5333 | 1.96470096 | 26.345 | 26.5333 | 0.70967426 |
| 180 | 26.358 | 26.5333 | 0.66067922 | 26.427 | 26.5333 | 0.40062864 | 26.486 | 26.5333 | 0.17826656 |

Table C.3 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 196.43 | 25.678 | 26.5333 | 3.22349651 | 26.012 | 26.5333 | 1.96470096 | 26.345 | 26.5333 | 0.70967426 |
| 216.46 | 25.611 | 26.5333 | 3.47600939 | 25.98 | 26.5333 | 2.08530413 | 26.35 | 26.5333 | 0.69083001 |
| 240.03 | 25.552 | 26.5333 | 3.69837148 | 25.955 | 26.5333 | 2.17952535 | 26.359 | 26.5333 | 0.65691037 |
| 245.23 | 25.544 | 26.5333 | 3.72852227 | 25.952 | 26.5333 | 2.1908319 | 26.36 | 26.5333 | 0.65314152 |
| 258.8 | 26.393 | 26.5333 | 0.52876951 | 26.475 | 26.5333 | 0.21972389 | 26.55 | 26.5333 | -0.0629398 |
| 270 | 26.394 | 26.5333 | 0.52500066 | 26.476 | 26.5333 | 0.21595505 | 26.552 | 26.5333 | -0.0704775 |
| 281.2 | 26.393 | 26.5333 | 0.52876951 | 26.475 | 26.5333 | 0.21972389 | 26.55 | 26.5333 | -0.0629398 |
| 294.77 | 25.544 | 26.5333 | 3.72852227 | 25.952 | 26.5333 | 2.1908319 | 26.36 | 26.5333 | 0.65314152 |
| 299.97 | 25.552 | 26.5333 | 3.69837148 | 25.955 | 26.5333 | 2.17952535 | 26.359 | 26.5333 | 0.65691037 |
| 323.54 | 25.611 | 26.5333 | 3.47600939 | 25.98 | 26.5333 | 2.08530413 | 26.35 | 26.5333 | 0.69083001 |
| 343.57 | 25.678 | 26.5333 | 3.22349651 | 26.012 | 26.5333 | 1.96470096 | 26.345 | 26.5333 | 0.70967426 |

Table C.4 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[0]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 0.8$

| ANGL E (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-----------------|--------------------------|----------------------------------|-------------------|--------------------------|----------------------------------|-------------------|--------------------------|----------------------------------|-------------------|
| | FEM Result (lb-in) | Analytica l Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytica l Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytica l Result (lb-in) | Miss Match (%) |
| 0 | 26.536 | 26.5333 | -0.0101759 | 26.578 | 26.5333 | -0.1684675 | 26.61 | 26.5333 | -0.2890707 |
| 16.432 | 26.679 | 26.5333 | -0.5491213 | 27.013 | 26.5333 | -1.8079168 | 27.347 | 26.5333 | -3.0667124 |
| 34.46 | 26.72 | 26.5333 | -0.7036441 | 27.089 | 26.5333 | -2.0943494 | 27.459 | 26.5333 | -3.4888235 |
| 60.03 | 26.762 | 26.5333 | -0.8619358 | 27.166 | 26.5333 | -2.3845507 | 27.569 | 26.5333 | -3.9033969 |
| 65.23 | 26.769 | 26.5333 | -0.8883177 | 27.177 | 26.5333 | -2.4260081 | 27.586 | 26.5333 | -3.9674673 |
| 78.8 | 26.619 | 26.5333 | -0.3229904 | 26.682 | 26.5333 | -0.5604278 | 26.739 | 26.5333 | -0.7752522 |
| 90 | 26.621 | 26.5333 | -0.3305281 | 26.685 | 26.5333 | -0.5717344 | 26.743 | 26.5333 | -0.7903276 |
| 101.2 | 26.619 | 26.5333 | -0.3229904 | 26.682 | 26.5333 | -0.5604278 | 26.739 | 26.5333 | -0.7752522 |
| 114.77 | 26.769 | 26.5333 | -0.8883177 | 27.177 | 26.5333 | -2.4260081 | 27.586 | 26.5333 | -3.9674673 |

Table C.4 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 119.97 | 26.762 | 26.5333 | -0.8619358 | 27.166 | 26.5333 | -2.3845507 | 27.569 | 26.5333 | -3.9033969 |
| 143.54 | 26.72 | 26.5333 | -0.7036441 | 27.089 | 26.5333 | -2.0943494 | 27.459 | 26.5333 | -3.4888235 |
| 163.57 | 26.679 | 26.5333 | -0.5491213 | 27.013 | 26.5333 | -1.8079168 | 27.347 | 26.5333 | -3.0667124 |
| 180 | 26.536 | 26.5333 | -0.0101759 | 26.578 | 26.5333 | -0.1684675 | 26.61 | 26.5333 | -0.2890707 |
| 196.43 | 26.679 | 26.5333 | -0.5491213 | 27.013 | 26.5333 | -1.8079168 | 27.347 | 26.5333 | -3.0667124 |
| 216.46 | 26.72 | 26.5333 | -0.7036441 | 27.089 | 26.5333 | -2.0943494 | 27.459 | 26.5333 | -3.4888235 |
| 240.03 | 26.762 | 26.5333 | -0.8619358 | 27.166 | 26.5333 | -2.3845507 | 27.569 | 26.5333 | -3.9033969 |
| 245.23 | 26.769 | 26.5333 | -0.8883177 | 27.177 | 26.5333 | -2.4260081 | 27.586 | 26.5333 | -3.9674673 |
| 258.8 | 26.619 | 26.5333 | -0.3229904 | 26.682 | 26.5333 | -0.5604278 | 26.739 | 26.5333 | -0.7752522 |
| 270 | 26.621 | 26.5333 | -0.3305281 | 26.685 | 26.5333 | -0.5717344 | 26.743 | 26.5333 | -0.7903276 |
| 281.2 | 26.619 | 26.5333 | -0.3229904 | 26.682 | 26.5333 | -0.5604278 | 26.739 | 26.5333 | -0.7752522 |
| 294.77 | 26.769 | 26.5333 | -0.8883177 | 27.177 | 26.5333 | -2.4260081 | 27.586 | 26.5333 | -3.9674673 |
| 299.97 | 26.762 | 26.5333 | -0.8619358 | 27.166 | 26.5333 | -2.3845507 | 27.569 | 26.5333 | -3.9033969 |
| 323.54 | 26.72 | 26.5333 | -0.7036441 | 27.089 | 26.5333 | -2.0943494 | 27.459 | 26.5333 | -3.4888235 |
| 343.57 | 26.679 | 26.5333 | -0.5491213 | 27.013 | 26.5333 | -1.8079168 | 27.347 | 26.5333 | -3.0667124 |

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Table C.5 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[0]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.136 | 35.3667 | 0.65230853 | 35.222 | 35.3667 | 0.40914193 | 35.292 | 35.3667 | 0.21121564 |
| 12.46 | 34.331 | 35.3667 | 2.92846095 | 34.732 | 35.3667 | 1.79462602 | 35.132 | 35.3667 | 0.6636186 |
| 28.98 | 34.061 | 35.3667 | 3.69189096 | 34.585 | 35.3667 | 2.21027124 | 35.108 | 35.3667 | 0.73147905 |
| 44.16 | 33.858 | 35.3667 | 4.26587722 | 34.482 | 35.3667 | 2.50150565 | 35.106 | 35.3667 | 0.73713408 |
| 58.38 | 33.833 | 35.3667 | 4.33656519 | 34.471 | 35.3667 | 2.53260836 | 35.108 | 35.3667 | 0.73147905 |

Table C.5 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 75.21 | 35.163 | 35.3667 | 0.57596553 | 35.288 | 35.3667 | 0.22252571 | 35.406 | 35.3667 | -0.1111215 |
| 90 | 35.163 | 35.3667 | 0.57596553 | 35.29 | 35.3667 | 0.21687067 | 35.409 | 35.3667 | -0.119604 |
| 104.79 | 35.163 | 35.3667 | 0.57596553 | 35.288 | 35.3667 | 0.22252571 | 35.406 | 35.3667 | -0.1111215 |
| 121.62 | 33.833 | 35.3667 | 4.33656519 | 34.471 | 35.3667 | 2.53260836 | 35.108 | 35.3667 | 0.73147905 |
| 135.84 | 33.858 | 35.3667 | 4.26587722 | 34.482 | 35.3667 | 2.50150565 | 35.106 | 35.3667 | 0.73713408 |
| 151.02 | 34.061 | 35.3667 | 3.69189096 | 34.585 | 35.3667 | 2.21027124 | 35.108 | 35.3667 | 0.73147905 |
| 167.54 | 34.331 | 35.3667 | 2.92846095 | 34.732 | 35.3667 | 1.79462602 | 35.132 | 35.3667 | 0.6636186 |
| 180 | 35.136 | 35.3667 | 0.65230853 | 35.222 | 35.3667 | 0.40914193 | 35.292 | 35.3667 | 0.21121564 |
| 192.46 | 34.331 | 35.3667 | 2.92846095 | 34.732 | 35.3667 | 1.79462602 | 35.132 | 35.3667 | 0.6636186 |
| 208.98 | 34.061 | 35.3667 | 3.69189096 | 34.585 | 35.3667 | 2.21027124 | 35.108 | 35.3667 | 0.73147905 |
| 224.16 | 33.858 | 35.3667 | 4.26587722 | 34.482 | 35.3667 | 2.50150565 | 35.106 | 35.3667 | 0.73713408 |
| 238.38 | 33.833 | 35.3667 | 4.33656519 | 34.471 | 35.3667 | 2.53260836 | 35.108 | 35.3667 | 0.73147905 |
| 255.21 | 35.163 | 35.3667 | 0.57596553 | 35.288 | 35.3667 | 0.22252571 | 35.406 | 35.3667 | -0.1111215 |
| 270 | 35.163 | 35.3667 | 0.57596553 | 35.29 | 35.3667 | 0.21687067 | 35.409 | 35.3667 | -0.119604 |
| 284.79 | 35.163 | 35.3667 | 0.57596553 | 35.288 | 35.3667 | 0.22252571 | 35.406 | 35.3667 | -0.1111215 |
| 301.62 | 33.833 | 35.3667 | 4.33656519 | 34.471 | 35.3667 | 2.53260836 | 35.108 | 35.3667 | 0.73147905 |
| 315.84 | 33.858 | 35.3667 | 4.26587722 | 34.482 | 35.3667 | 2.50150565 | 35.106 | 35.3667 | 0.73713408 |
| 331.02 | 34.061 | 35.3667 | 3.69189096 | 34.585 | 35.3667 | 2.21027124 | 35.108 | 35.3667 | 0.73147905 |
| 347.54 | 34.331 | 35.3667 | 2.92846095 | 34.732 | 35.3667 | 1.79462602 | 35.132 | 35.3667 | 0.6636186 |

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Table C.6 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[0]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.344 | 35.3667 | 0.06418467 | 35.381 | 35.3667 | -0.0404335 | 35.398 | 35.3667 | -0.0885013 |

Table C.6 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 12.46 | 35.532 | 35.3667 | -0.4673888 | 35.933 | 35.3667 | -1.6012238 | 36.333 | 35.3667 | -2.7322312 |
| 28.98 | 35.632 | 35.3667 | -0.7501407 | 36.155 | 35.3667 | -2.2289329 | 36.679 | 35.3667 | -3.7105526 |
| 44.16 | 35.73 | 35.3667 | -1.0272375 | 36.354 | 35.3667 | -2.7916091 | 36.978 | 35.3667 | -4.5559806 |
| 58.38 | 35.745 | 35.3667 | -1.0696503 | 36.382 | 35.3667 | -2.8707796 | 37.019 | 35.3667 | -4.6719089 |
| 75.21 | 35.517 | 35.3667 | -0.424976 | 35.62 | 35.3667 | -0.7162104 | 35.716 | 35.3667 | -0.9876522 |
| 90 | 35.521 | 35.3667 | -0.4362861 | 35.626 | 35.3667 | -0.7331756 | 35.724 | 35.3667 | -1.0102724 |
| 104.79 | 35.517 | 35.3667 | -0.424976 | 35.62 | 35.3667 | -0.7162104 | 35.716 | 35.3667 | -0.9876522 |
| 121.62 | 35.745 | 35.3667 | -1.0696503 | 36.382 | 35.3667 | -2.8707796 | 37.019 | 35.3667 | -4.6719089 |
| 135.84 | 35.73 | 35.3667 | -1.0272375 | 36.354 | 35.3667 | -2.7916091 | 36.978 | 35.3667 | -4.5559806 |
| 151.02 | 35.632 | 35.3667 | -0.7501407 | 36.155 | 35.3667 | -2.2289329 | 36.679 | 35.3667 | -3.7105526 |
| 167.54 | 35.532 | 35.3667 | -0.4673888 | 35.933 | 35.3667 | -1.6012238 | 36.333 | 35.3667 | -2.7322312 |
| 180 | 35.344 | 35.3667 | 0.06418467 | 35.379 | 35.3667 | -0.0347785 | 35.398 | 35.3667 | -0.0885013 |
| 192.46 | 35.532 | 35.3667 | -0.4673888 | 35.933 | 35.3667 | -1.6012238 | 36.333 | 35.3667 | -2.7322312 |
| 208.98 | 35.632 | 35.3667 | -0.7501407 | 36.155 | 35.3667 | -2.2289329 | 36.679 | 35.3667 | -3.7105526 |
| 224.16 | 35.73 | 35.3667 | -1.0272375 | 36.354 | 35.3667 | -2.7916091 | 36.978 | 35.3667 | -4.5559806 |
| 238.38 | 35.745 | 35.3667 | -1.0696503 | 36.382 | 35.3667 | -2.8707796 | 37.019 | 35.3667 | -4.6719089 |
| 255.21 | 35.517 | 35.3667 | -0.424976 | 35.62 | 35.3667 | -0.7162104 | 35.716 | 35.3667 | -0.9876522 |
| 270 | 35.521 | 35.3667 | -0.4362861 | 35.626 | 35.3667 | -0.7331756 | 35.724 | 35.3667 | -1.0102724 |
| 284.79 | 35.517 | 35.3667 | -0.424976 | 35.62 | 35.3667 | -0.7162104 | 35.716 | 35.3667 | -0.9876522 |
| 301.62 | 35.745 | 35.3667 | -1.0696503 | 36.382 | 35.3667 | -2.8707796 | 37.019 | 35.3667 | -4.6719089 |
| 315.84 | 35.73 | 35.3667 | -1.0272375 | 36.354 | 35.3667 | -2.7916091 | 36.978 | 35.3667 | -4.5559806 |
| 331.02 | 35.632 | 35.3667 | -0.7501407 | 36.155 | 35.3667 | -2.2289329 | 36.679 | 35.3667 | -3.7105526 |
| 347.54 | 35.532 | 35.3667 | -0.4673888 | 35.933 | 35.3667 | -1.6012238 | 36.333 | 35.3667 | -2.7322312 |

Table C.7 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.4$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|---------------------|----------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in)3 | Analytical Result (lb-in)4 | Miss Match (%) |
| 0 | 52.676 | 53.0667 | 0.73624326 | 52.809 | 53.0667 | 0.48561527 | 52.902 | 53.0667 | 0.31036413 |
| 8.35 | 51.646 | 53.0667 | 2.67719681 | 52.194 | 53.0667 | 1.64453414 | 52.743 | 53.0667 | 0.60998705 |
| 20.25 | 50.671 | 53.0667 | 4.51450721 | 51.616 | 53.0667 | 2.73372944 | 52.562 | 53.0667 | 0.95106724 |
| 40.9 | 50.369 | 53.0667 | 5.08360233 | 51.206 | 53.0667 | 3.50634202 | 52.443 | 53.0667 | 1.17531333 |
| 47.27 | 50.284 | 53.0667 | 5.24377811 | 51.157 | 53.0667 | 3.59867864 | 52.43 | 53.0667 | 1.1998108 |
| 68.4 | 52.505 | 53.0667 | 1.05847923 | 52.755 | 53.0667 | 0.587374 | 52.996 | 53.0667 | 0.13322856 |
| 90 | 52.497 | 53.0667 | 1.0735546 | 52.751 | 53.0667 | 0.59491169 | 52.995 | 53.0667 | 0.13511298 |
| 111.6 | 52.505 | 53.0667 | 1.05847923 | 52.755 | 53.0667 | 0.587374 | 52.996 | 53.0667 | 0.13322856 |
| 132.73 | 50.284 | 53.0667 | 5.24377811 | 51.157 | 53.0667 | 3.59867864 | 52.43 | 53.0667 | 1.1998108 |
| 139.1 | 50.369 | 53.0667 | 5.08360233 | 51.206 | 53.0667 | 3.50634202 | 52.443 | 53.0667 | 1.17531333 |
| 159.75 | 50.671 | 53.0667 | 4.51450721 | 51.616 | 53.0667 | 2.73372944 | 52.562 | 53.0667 | 0.95106724 |
| 171.65 | 51.646 | 53.0667 | 2.67719681 | 52.194 | 53.0667 | 1.64453414 | 52.743 | 53.0667 | 0.60998705 |
| 180 | 52.676 | 53.0667 | 0.73624326 | 52.809 | 53.0667 | 0.48561527 | 52.902 | 53.0667 | 0.31036413 |
| 188.35 | 51.646 | 53.0667 | 2.67719681 | 52.194 | 53.0667 | 1.64453414 | 52.743 | 53.0667 | 0.60998705 |
| 200.25 | 50.671 | 53.0667 | 4.51450721 | 51.616 | 53.0667 | 2.73372944 | 52.562 | 53.0667 | 0.95106724 |
| 220.9 | 50.369 | 53.0667 | 5.08360233 | 51.206 | 53.0667 | 3.50634202 | 52.443 | 53.0667 | 1.17531333 |
| 227.27 | 50.284 | 53.0667 | 5.24377811 | 51.157 | 53.0667 | 3.59867864 | 52.43 | 53.0667 | 1.1998108 |
| 248.4 | 52.505 | 53.0667 | 1.05847923 | 52.755 | 53.0667 | 0.587374 | 52.996 | 53.0667 | 0.13322856 |
| 270 | 52.497 | 53.0667 | 1.0735546 | 52.751 | 53.0667 | 0.59491169 | 52.995 | 53.0667 | 0.13511298 |
| 291.6 | 52.505 | 53.0667 | 1.05847923 | 52.755 | 53.0667 | 0.587374 | 52.996 | 53.0667 | 0.13322856 |
| 312.73 | 50.284 | 53.0667 | 5.24377811 | 51.157 | 53.0667 | 3.59867864 | 52.43 | 53.0667 | 1.1998108 |

Table C.7 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 319.1 | 50.369 | 53.0667 | 5.08360233 | 51.206 | 53.0667 | 3.50634202 | 52.443 | 53.0667 | 1.17531333 |
| 339.75 | 50.671 | 53.0667 | 4.51450721 | 51.616 | 53.0667 | 2.73372944 | 52.562 | 53.0667 | 0.95106724 |
| 351.65 | 51.646 | 53.0667 | 2.67719681 | 52.194 | 53.0667 | 1.64453414 | 52.743 | 53.0667 | 0.60998705 |

Table C.8 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 52.955 | 53.0667 | 0.21048982 | 52.968 | 53.0667 | 0.18599235 | 52.942 | 53.0667 | 0.23498729 |
| 8.35 | 53.292 | 53.0667 | -0.42456 | 53.84 | 53.0667 | -1.4572227 | 54.389 | 53.0667 | -2.4917698 |
| 20.25 | 53.507 | 53.0667 | -0.8297105 | 54.452 | 53.0667 | -2.6104883 | 55.398 | 53.0667 | -4.3931505 |
| 40.9 | 53.68 | 53.0667 | -1.1557154 | 54.917 | 53.0667 | -3.486744 | 56.014 | 53.0667 | -5.5539538 |
| 47.27 | 53.703 | 53.0667 | -1.199057 | 54.976 | 53.0667 | -3.5979249 | 56.049 | 53.0667 | -5.6199085 |
| 68.4 | 53.226 | 53.0667 | -0.3001883 | 53.446 | 53.0667 | -0.7147609 | 53.657 | 53.0667 | -1.1123737 |
| 90 | 53.23 | 53.0667 | -0.3077259 | 53.455 | 53.0667 | -0.7317206 | 53.67 | 53.0667 | -1.1368711 |
| 111.6 | 53.226 | 53.0667 | -0.3001883 | 53.446 | 53.0667 | -0.7147609 | 53.657 | 53.0667 | -1.1123737 |
| 132.73 | 53.703 | 53.0667 | -1.199057 | 54.976 | 53.0667 | -3.5979249 | 56.049 | 53.0667 | -5.6199085 |
| 139.1 | 53.68 | 53.0667 | -1.1557154 | 54.917 | 53.0667 | -3.486744 | 56.014 | 53.0667 | -5.5539538 |
| 159.75 | 53.507 | 53.0667 | -0.8297105 | 54.452 | 53.0667 | -2.6104883 | 55.398 | 53.0667 | -4.3931505 |
| 171.65 | 53.292 | 53.0667 | -0.42456 | 53.84 | 53.0667 | -1.4572227 | 54.389 | 53.0667 | -2.4917698 |
| 180 | 52.955 | 53.0667 | 0.21048982 | 52.968 | 53.0667 | 0.18599235 | 52.942 | 53.0667 | 0.23498729 |
| 188.35 | 53.292 | 53.0667 | -0.42456 | 53.84 | 53.0667 | -1.4572227 | 54.389 | 53.0667 | -2.4917698 |
| 200.25 | 53.507 | 53.0667 | -0.8297105 | 54.452 | 53.0667 | -2.6104883 | 55.398 | 53.0667 | -4.3931505 |
| 220.9 | 53.68 | 53.0667 | -1.1557154 | 54.917 | 53.0667 | -3.486744 | 56.014 | 53.0667 | -5.5539538 |
| 227.27 | 53.703 | 53.0667 | -1.199057 | 54.976 | 53.0667 | -3.5979249 | 56.049 | 53.0667 | -5.6199085 |

Table C.8 – Continued

| | | | | | | | | | |
|--------|--------|---------|-----------|--------|---------|-----------|--------|---------|------------|
| 248.4 | 53.226 | 53.0667 | -0.300188 | 53.446 | 53.0667 | -0.714760 | 53.657 | 53.0667 | -1.1123737 |
| 270 | 53.23 | 53.0667 | -0.307725 | 53.455 | 53.0667 | -0.731720 | 53.67 | 53.0667 | -1.1368711 |
| 291.6 | 53.226 | 53.0667 | -0.300188 | 53.446 | 53.0667 | -0.714760 | 53.657 | 53.0667 | -1.1123737 |
| 312.73 | 53.703 | 53.0667 | -1.199057 | 54.976 | 53.0667 | -3.597924 | 56.049 | 53.0667 | -5.6199085 |
| 319.1 | 53.68 | 53.0667 | -1.155715 | 54.917 | 53.0667 | -3.486744 | 56.014 | 53.0667 | -5.5539538 |
| 339.75 | 53.507 | 53.0667 | -0.829710 | 54.452 | 53.0667 | -2.610488 | 55.398 | 53.0667 | -4.3931505 |
| 351.65 | 53.292 | 53.0667 | -0.42456 | 53.84 | 53.0667 | -1.457222 | 54.389 | 53.0667 | -2.4917698 |

Table C.9 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 103.86 | 106.103 | 2.11426035 | 104.23 | 106.103 | 1.76554358 | 104.44 | 106.103 | 1.56762325 |
| 4.18 | 102.61 | 106.103 | 3.29235754 | 103.88 | 106.103 | 2.09541079 | 105.15 | 106.103 | 0.89846404 |
| 10.44 | 98.937 | 106.103 | 6.75407834 | 101.07 | 106.103 | 4.74377329 | 104.21 | 106.103 | 1.78439313 |
| 23.41 | 97.584 | 106.103 | 8.02925074 | 100.063 | 106.103 | 5.69284838 | 103.54 | 106.103 | 2.41585323 |
| 28.42 | 97.175 | 106.103 | 8.41472414 | 99.817 | 106.103 | 5.92469791 | 103.46 | 106.103 | 2.49125145 |
| 51.62 | 103.37 | 106.103 | 2.57607445 | 104.38 | 106.103 | 1.62417192 | 105.37 | 106.103 | 0.69111894 |
| 90 | 103.3 | 106.103 | 2.64204789 | 104.33 | 106.103 | 1.6712958 | 105.35 | 106.103 | 0.70996849 |
| 128.38 | 103.37 | 106.103 | 2.57607445 | 104.38 | 106.103 | 1.62417192 | 105.37 | 106.103 | 0.69111894 |
| 151.58 | 97.175 | 106.103 | 8.41472414 | 99.817 | 106.103 | 5.92469791 | 103.46 | 106.103 | 2.49125145 |
| 156.59 | 97.584 | 106.103 | 8.02925074 | 100.063 | 106.103 | 5.69284838 | 103.54 | 106.103 | 2.41585323 |
| 169.56 | 98.937 | 106.103 | 6.75407834 | 101.07 | 106.103 | 4.74377329 | 104.21 | 106.103 | 1.78439313 |
| 177.59 | 102.61 | 106.103 | 3.29235754 | 103.88 | 106.103 | 2.09541079 | 105.15 | 106.103 | 0.89846404 |
| 180 | 103.86 | 106.103 | 2.11426035 | 104.23 | 106.103 | 1.76554358 | 104.44 | 106.103 | 1.56762325 |

Table C.9 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|---------|---------|------------|--------|---------|------------|
| 184.18 | 102.61 | 106.103 | 3.29235754 | 103.88 | 106.103 | 2.09541079 | 105.15 | 106.103 | 0.89846404 |
| 190.44 | 98.937 | 106.103 | 6.75407834 | 101.07 | 106.103 | 4.74377329 | 104.21 | 106.103 | 1.78439313 |
| 203.41 | 97.584 | 106.103 | 8.02925074 | 100.063 | 106.103 | 5.69284838 | 103.54 | 106.103 | 2.41585323 |
| 208.42 | 97.175 | 106.103 | 8.41472414 | 99.817 | 106.103 | 5.92469791 | 103.46 | 106.103 | 2.49125145 |
| 231.62 | 103.37 | 106.103 | 2.57607445 | 104.38 | 106.103 | 1.62417192 | 105.37 | 106.103 | 0.69111894 |
| 270 | 103.3 | 106.103 | 2.64204789 | 104.33 | 106.103 | 1.6712958 | 105.35 | 106.103 | 0.70996849 |
| 308.38 | 103.37 | 106.103 | 2.57607445 | 104.38 | 106.103 | 1.62417192 | 105.37 | 106.103 | 0.69111894 |
| 331.58 | 97.175 | 106.103 | 8.41472414 | 99.817 | 106.103 | 5.92469791 | 103.46 | 106.103 | 2.49125145 |
| 336.59 | 97.584 | 106.103 | 8.02925074 | 100.063 | 106.103 | 5.69284838 | 103.54 | 106.103 | 2.41585323 |
| 349.56 | 98.937 | 106.103 | 6.75407834 | 101.07 | 106.103 | 4.74377329 | 104.21 | 106.103 | 1.78439313 |
| 355.82 | 102.61 | 106.103 | 3.29235754 | 103.88 | 106.103 | 2.09541079 | 105.15 | 106.103 | 0.89846404 |

Table C.10 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[0]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 104.5 | 106.103 | 1.51107458 | 104.39 | 106.103 | 1.61474714 | 104.13 | 106.103 | 1.85979135 |
| 4.18 | 106.42 | 106.103 | -0.2984827 | 107.69 | 106.103 | -1.4954295 | 108.96 | 106.103 | -2.6923762 |
| 10.44 | 107.35 | 106.103 | -1.174987 | 110.49 | 106.103 | -4.1343672 | 113.63 | 106.103 | -7.0937473 |
| 23.41 | 108.02 | 106.103 | -1.8064471 | 112.5 | 106.103 | -6.0287475 | 114.98 | 106.103 | -8.3660923 |
| 28.42 | 108.1 | 106.103 | -1.8818453 | 112.75 | 106.103 | -6.2643669 | 115.39 | 106.103 | -8.7525082 |
| 51.62 | 106.34 | 106.103 | -0.2230845 | 107.29 | 106.103 | -1.1184384 | 108.23 | 106.103 | -2.0043674 |
| 90 | 106.35 | 106.103 | -0.2325093 | 107.33 | 106.103 | -1.1561375 | 108.3 | 106.103 | -2.0703409 |
| 128.38 | 106.34 | 106.103 | -0.2230845 | 107.29 | 106.103 | -1.1184384 | 108.23 | 106.103 | -2.0043674 |
| 151.58 | 108.1 | 106.103 | -1.8818453 | 112.75 | 106.103 | -6.2643669 | 115.39 | 106.103 | -8.7525082 |

Table C.10 - Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 156.59 | 108.02 | 106.103 | -1.8064471 | 112.5 | 106.103 | -6.0287475 | 114.98 | 106.103 | -8.3660923 |
| 169.56 | 107.35 | 106.103 | -1.174987 | 110.49 | 106.103 | -4.1343672 | 113.63 | 106.103 | -7.0937473 |
| 177.59 | 106.42 | 106.103 | -0.2984827 | 107.69 | 106.103 | -1.4954295 | 108.96 | 106.103 | -2.6923762 |
| 180 | 104.5 | 106.103 | 1.51107458 | 104.39 | 106.103 | 1.61474714 | 104.13 | 106.103 | 1.85979135 |
| 184.18 | 106.42 | 106.103 | -0.2984827 | 107.69 | 106.103 | -1.4954295 | 108.96 | 106.103 | -2.6923762 |
| 190.44 | 107.35 | 106.103 | -1.174987 | 110.49 | 106.103 | -4.1343672 | 113.63 | 106.103 | -7.0937473 |
| 203.41 | 108.02 | 106.103 | -1.8064471 | 112.5 | 106.103 | -6.0287475 | 114.98 | 106.103 | -8.3660923 |
| 208.42 | 108.1 | 106.103 | -1.8818453 | 112.75 | 106.103 | -6.2643669 | 115.39 | 106.103 | -8.7525082 |
| 231.62 | 106.34 | 106.103 | -0.2230845 | 107.29 | 106.103 | -1.1184384 | 108.23 | 106.103 | -2.0043674 |
| 270 | 106.35 | 106.103 | -0.2325093 | 107.33 | 106.103 | -1.1561375 | 108.3 | 106.103 | -2.0703409 |
| 308.38 | 106.34 | 106.103 | -0.2230845 | 107.29 | 106.103 | -1.1184384 | 108.23 | 106.103 | -2.0043674 |
| 331.58 | 108.1 | 106.103 | -1.8818453 | 112.75 | 106.103 | -6.2643669 | 115.39 | 106.103 | -8.7525082 |
| 336.59 | 108.02 | 106.103 | -1.8064471 | 112.5 | 106.103 | -6.0287475 | 114.98 | 106.103 | -8.3660923 |
| 349.56 | 107.35 | 106.103 | -1.174987 | 110.49 | 106.103 | -4.1343672 | 113.63 | 106.103 | -7.0937473 |
| 355.82 | 106.42 | 106.103 | -0.2984827 | 107.69 | 106.103 | -1.4954295 | 108.96 | 106.103 | -2.6923762 |

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Table C.11 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[45]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 20.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 42.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 65.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 69.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |

Table C.11 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 81 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 90 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 99 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 110.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 137.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 155.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 159.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 180 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 200.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 222.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 245.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 249.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 261 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 270 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 279 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 290.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 294.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 317.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 339.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |

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Table C.12 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[45]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 1$

| | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |

Table C.12 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 20.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 42.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 65.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 69.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 81 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 90 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 99 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 110.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 137.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 155.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 159.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 180 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 200.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 222.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 245.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 249.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 261 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 270 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 279 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 290.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 294.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 317.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 339.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |

Table C.13 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[45]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 26.443 | 26.5333 | 0.34032706 | 26.483 | 26.5333 | 0.1895731 | 26.557 | 26.5333 | -0.0893217 |
| 16.432 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |
| 34.46 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 60.03 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 65.23 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |
| 78.8 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 90 | 26.304 | 26.5333 | 0.86419707 | 26.36 | 26.5333 | 0.65314152 | 26.466 | 26.5333 | 0.25364353 |
| 101.2 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 114.77 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |
| 119.97 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 143.54 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 163.57 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |
| 180 | 26.443 | 26.5333 | 0.34032706 | 26.483 | 26.5333 | 0.1895731 | 26.557 | 26.5333 | -0.0893217 |
| 196.432 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |
| 216.46 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 240.03 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 245.23 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |
| 258.8 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 270 | 26.304 | 26.5333 | 0.86419707 | 26.36 | 26.5333 | 0.65314152 | 26.466 | 26.5333 | 0.25364353 |
| 281.2 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 294.77 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |

Table C.13 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 299.97 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 323.54 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 343.57 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |

Table C.14 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[45]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 26.62 | 26.5333 | -0.3267592 | 26.672 | 26.5333 | -0.5227394 | 26.712 | 26.5333 | -0.6734933 |
| 16.432 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |
| 34.46 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 60.03 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 65.23 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 78.8 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 90 | 26.563 | 26.5333 | -0.1119348 | 26.652 | 26.5333 | -0.4473624 | 26.732 | 26.5333 | -0.7488703 |
| 101.2 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 114.77 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 119.97 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 143.54 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 163.57 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |
| 180 | 26.62 | 26.5333 | -0.3267592 | 26.672 | 26.5333 | -0.5227394 | 26.512 | 26.5333 | 0.08027648 |
| 196.432 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |
| 216.46 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 240.03 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 245.23 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |

Table C.14 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 258.8 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 270 | 26.563 | 26.5333 | -0.1119348 | 26.652 | 26.5333 | -0.4473624 | 26.732 | 26.5333 | -0.7488703 |
| 281.2 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 294.77 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 299.97 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 323.54 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 343.57 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |

Table C.15 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[45]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.6$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.179 | 35.3667 | 0.53072523 | 35.279 | 35.3667 | 0.24797338 | 35.36 | 35.3667 | 0.0189443 |
| 12.46 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.6523085 |
| 28.98 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.9605080 |
| 44.16 | 34.135 | 35.3667 | 3.48265459 | 34.125 | 35.3667 | 3.51092977 | 34.915 | 35.3667 | 1.2771901 |
| 58.38 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.3252579 |
| 75.21 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.1546652 |
| 90 | 34.963 | 35.3667 | 1.14146924 | 35.142 | 35.3667 | 0.63534342 | 35.309 | 35.3667 | 0.1631478 |
| 104.79 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.1546652 |
| 121.62 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.3252579 |
| 135.84 | 34.135 | 35.3667 | 3.48265459 | 34.025 | 35.3667 | 3.79368163 | 34.915 | 35.3667 | 1.2771901 |
| 151.02 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.9605080 |
| 167.54 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.6523085 |
| 180 | 35.179 | 35.3667 | 0.53072523 | 35.279 | 35.3667 | 0.24797338 | 35.36 | 35.3667 | 0.0189443 |

Table C.15 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 192.46 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.65230853 |
| 208.98 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.96050805 |
| 224.16 | 34.135 | 35.3667 | 3.48265459 | 34.125 | 35.3667 | 3.51092977 | 34.915 | 35.3667 | 1.27719013 |
| 238.38 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.32525794 |
| 255.21 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.15466526 |
| 270 | 34.963 | 35.3667 | 1.14146924 | 35.142 | 35.3667 | 0.63534342 | 35.309 | 35.3667 | 0.16314782 |
| 284.79 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.15466526 |
| 301.62 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.32525794 |
| 315.84 | 34.135 | 35.3667 | 3.48265459 | 34.125 | 35.3667 | 3.51092977 | 34.915 | 35.3667 | 1.27719013 |
| 331.02 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.96050805 |
| 347.54 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.65230853 |

Table C.16 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[45]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 0.6$

| ANGL E (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.421 | 35.3667 | -0.1535343 | 35.462 | 35.3667 | -0.2694625 | 35.483 | 35.3667 | -0.3288404 |
| 12.46 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |
| 28.98 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 44.16 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 58.38 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |
| 75.21 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 90 | 35.467 | 35.3667 | -0.2836001 | 35.614 | 35.3667 | -0.6992453 | 35.75 | 35.3667 | -1.0837879 |
| 104.79 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 121.62 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |

Table C.16 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 135.84 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 151.02 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 167.54 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |
| 180 | 35.421 | 35.3667 | -0.1535343 | 35.462 | 35.3667 | -0.2694625 | 35.483 | 35.3667 | -0.3288404 |
| 192.46 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |
| 208.98 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 224.16 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 238.38 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |
| 255.21 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 270 | 35.467 | 35.3667 | -0.2836001 | 35.614 | 35.3667 | -0.6992453 | 35.75 | 35.3667 | -1.0837879 |
| 284.79 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 301.62 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |
| 315.84 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 331.02 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 347.54 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |

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Table C.17 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[45]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 52.529 | 53.0667 | 1.01325313 | 52.682 | 53.0667 | 0.72493673 | 52.789 | 53.0667 | 0.52330369 |
| 8.35 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |
| 20.25 | 50.957 | 53.0667 | 3.97556283 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 40.9 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 47.27 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |

Table C.17 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 68.4 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 90 | 52.423 | 53.0667 | 1.21300175 | 52.76 | 53.0667 | 0.5779519 | 53.084 | 53.0667 | -0.0326005 |
| 111.6 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 132.73 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 139.1 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 159.75 | 50.957 | 53.0667 | 3.97556283 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 171.65 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |
| 180 | 52.529 | 53.0667 | 1.01325313 | 52.682 | 53.0667 | 0.72493673 | 52.789 | 53.0667 | 0.52330369 |
| 188.35 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |
| 200.25 | 50.957 | 53.0667 | 3.97556283 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 220.9 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 227.27 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 248.4 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 270 | 52.423 | 53.0667 | 1.21300175 | 52.76 | 53.0667 | 0.5779519 | 53.084 | 53.0667 | -0.0326005 |
| 291.6 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 312.73 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 319.1 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 339.75 | 49.957 | 53.0667 | 5.85998376 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 351.65 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |

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Table C.18 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[45]_{6T}$ between FEM and

CLPT with $\frac{b}{a} = 0.4$

| | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 52.85 | 53.0667 | 0.40835401 | 52.866 | 53.0667 | 0.37820328 | 52.836 | 53.0667 | 0.43473591 |

Table C.18 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 8.35 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |
| 20.25 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 40.9 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 47.27 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 68.4 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 90 | 53.395 | 53.0667 | -0.6186554 | 53.692 | 53.0667 | -1.1783284 | 53.975 | 53.0667 | -1.7116195 |
| 111.6 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 132.73 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 139.1 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 159.75 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 171.65 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |
| 180 | 52.85 | 53.0667 | 0.40835401 | 52.866 | 53.0667 | 0.37820328 | 52.836 | 53.0667 | 0.43473591 |
| 188.35 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |
| 200.25 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 220.9 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 227.27 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 248.4 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 270 | 53.395 | 53.0667 | -0.6186554 | 53.692 | 53.0667 | -1.1783284 | 53.975 | 53.0667 | -1.7116195 |
| 291.6 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 312.73 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 319.1 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 339.75 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 351.65 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |

Table C.19 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[45]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 103.47 | 106.103 | 2.48182667 | 103.9 | 106.103 | 2.07656124 | 104.14 | 106.103 | 1.85036658 |
| 4.18 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |
| 10.44 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 23.41 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 28.42 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |
| 51.62 | 103.2 | 106.103 | 2.73629567 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 90 | 103.14 | 106.103 | 2.79284433 | 104.36 | 106.103 | 1.64302147 | 105.55 | 106.103 | 0.52147294 |
| 128.38 | 103.2 | 106.103 | 2.73629567 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 151.58 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |
| 156.59 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 169.56 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 177.59 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |
| 180 | 103.47 | 106.103 | 2.48182667 | 103.9 | 106.103 | 2.07656124 | 104.14 | 106.103 | 1.85036658 |
| 184.18 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |
| 190.44 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 203.41 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 208.42 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |
| 231.62 | 103.2 | 106.103 | 2.73629567 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 270 | 103.14 | 106.103 | 2.79284433 | 104.36 | 106.103 | 1.64302147 | 105.55 | 106.103 | 0.52147294 |
| 308.38 | 103.14 | 106.103 | 2.79284433 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 331.58 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |

Table C.19 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|---------|---------|------------|--------|---------|------------|
| 336.59 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 349.56 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 355.82 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |

Table C.20 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[45]_{6T}$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 104.2 | 106.103 | 1.79381791 | 104.08 | 106.103 | 1.90691524 | 103.78 | 106.103 | 2.18965857 |
| 4.18 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |
| 10.44 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 23.41 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 28.42 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 51.62 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 90 | 106.72 | 106.103 | -0.581226 | 107.87 | 106.103 | -1.6650755 | 108.99 | 106.103 | -2.7206505 |
| 128.38 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 151.58 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 156.59 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 169.56 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 177.59 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |
| 180 | 104.2 | 106.103 | 1.79381791 | 104.08 | 106.103 | 1.90691524 | 103.78 | 106.103 | 2.18965857 |
| 184.18 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |
| 190.44 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 203.41 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 208.42 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |

Table C.20 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 231.62 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 270 | 106.72 | 106.103 | -0.581226 | 107.87 | 106.103 | -1.6650755 | 108.99 | 106.103 | -2.7206505 |
| 308.38 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 331.58 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 336.59 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 349.56 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 355.82 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |

Table C.21 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_S$ between FEM and CLPT with $\frac{b}{a} = 1$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 20.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 42.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 65.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 69.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 81 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 90 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 99 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 110.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 137.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 155.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 159.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 180 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |

Table C.21 - *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 200.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 222.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 245.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 249.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 261 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 270 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 279 | 21.052 | 21.2333 | 0.85384749 | 21.135 | 21.2333 | 0.46295206 | 21.21 | 21.2333 | 0.1097333 |
| 290.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 294.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 317.25 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |
| 339.75 | 20.863 | 21.2333 | 1.74395878 | 20.989 | 21.2333 | 1.15055126 | 21.115 | 21.2333 | 0.55714373 |

Table C.22 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_s$ between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 20.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 42.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 65.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 69.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 81 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 90 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 99 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 110.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |

Table C.22 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 137.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 155.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 159.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 180 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 200.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 222.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 245.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 249.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 261 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 270 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 279 | 21.277 | 21.2333 | -0.2058088 | 21.337 | 21.2333 | -0.4883838 | 21.389 | 21.2333 | -0.7332822 |
| 290.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 294.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 317.25 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |
| 339.75 | 21.341 | 21.2333 | -0.5072221 | 21.566 | 21.2333 | -1.5668784 | 21.612 | 21.2333 | -1.7835193 |

Table C.23 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_s$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 26.443 | 26.5333 | 0.34032706 | 26.483 | 26.5333 | 0.1895731 | 26.557 | 26.5333 | -0.0893217 |
| 16.432 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |
| 34.46 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 60.03 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 65.23 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |

Table C.23 – Continued

| | | | | | | | | | |
|---------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 78.8 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 90 | 26.304 | 26.5333 | 0.86419707 | 26.36 | 26.5333 | 0.65314152 | 26.466 | 26.5333 | 0.25364353 |
| 101.2 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 114.77 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |
| 119.97 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 143.54 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 163.57 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |
| 180 | 26.443 | 26.5333 | 0.34032706 | 26.483 | 26.5333 | 0.1895731 | 26.557 | 26.5333 | -0.0893217 |
| 196.432 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |
| 216.46 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 240.03 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 245.23 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |
| 258.8 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 270 | 26.304 | 26.5333 | 0.86419707 | 26.36 | 26.5333 | 0.65314152 | 26.466 | 26.5333 | 0.25364353 |
| 281.2 | 26.308 | 26.5333 | 0.84912167 | 26.363 | 26.5333 | 0.64183498 | 26.469 | 26.5333 | 0.24233699 |
| 294.77 | 25.522 | 26.5333 | 3.81143695 | 25.616 | 26.5333 | 3.45716515 | 26.206 | 26.5333 | 1.23354426 |
| 299.97 | 25.548 | 26.5333 | 3.71344688 | 25.638 | 26.5333 | 3.37425047 | 26.217 | 26.5333 | 1.19208692 |
| 323.54 | 25.723 | 26.5333 | 3.05389831 | 25.776 | 26.5333 | 2.85414931 | 26.284 | 26.5333 | 0.93957404 |
| 343.57 | 25.887 | 26.5333 | 2.43580708 | 25.906 | 26.5333 | 2.36419895 | 26.344 | 26.5333 | 0.71344311 |

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Table C.24 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_s$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 26.62 | 26.5333 | -0.3267592 | 26.672 | 26.5333 | -0.5227394 | 26.712 | 26.5333 | -0.6734933 |

Table C.24 – *Continued*

| | | | | | | | | | |
|---------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 16.432 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |
| 34.46 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 60.03 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 65.23 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 78.8 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 90 | 26.563 | 26.5333 | -0.1119348 | 26.652 | 26.5333 | -0.4473624 | 26.732 | 26.5333 | -0.7488703 |
| 101.2 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 114.77 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 119.97 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 143.54 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 163.57 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |
| 180 | 26.62 | 26.5333 | -0.3267592 | 26.672 | 26.5333 | -0.5227394 | 26.512 | 26.5333 | 0.08027648 |
| 196.432 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |
| 216.46 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 240.03 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 245.23 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 258.8 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 270 | 26.563 | 26.5333 | -0.1119348 | 26.652 | 26.5333 | -0.4473624 | 26.732 | 26.5333 | -0.7488703 |
| 281.2 | 26.565 | 26.5333 | -0.1194725 | 26.653 | 26.5333 | -0.4511312 | 26.732 | 26.5333 | -0.7488703 |
| 294.77 | 26.796 | 26.5333 | -0.9900766 | 27.386 | 26.5333 | -3.2136975 | 27.576 | 26.5333 | -3.9297788 |
| 299.97 | 26.796 | 26.5333 | -0.9900766 | 27.375 | 26.5333 | -3.1722402 | 27.554 | 26.5333 | -3.8468641 |
| 323.54 | 26.791 | 26.5333 | -0.9712324 | 27.298 | 26.5333 | -2.8820388 | 27.505 | 26.5333 | -3.6621905 |
| 343.57 | 26.783 | 26.5333 | -0.9410816 | 27.221 | 26.5333 | -2.5918374 | 27.459 | 26.5333 | -3.4888235 |

Table C.25 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_S$ between FEM and CLPT with $\frac{b}{a} = 0.6$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.179 | 35.3667 | 0.53072523 | 35.279 | 35.3667 | 0.24797338 | 35.36 | 35.3667 | 0.01894437 |
| 12.46 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.65230853 |
| 28.98 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.96050805 |
| 44.16 | 34.135 | 35.3667 | 3.48265459 | 34.125 | 35.3667 | 3.51092977 | 34.915 | 35.3667 | 1.27719013 |
| 58.38 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.32525794 |
| 75.21 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.15466526 |
| 90 | 34.963 | 35.3667 | 1.14146924 | 35.142 | 35.3667 | 0.63534342 | 35.309 | 35.3667 | 0.16314782 |
| 104.79 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.15466526 |
| 121.62 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.32525794 |
| 135.84 | 34.135 | 35.3667 | 3.48265459 | 34.025 | 35.3667 | 3.79368163 | 34.915 | 35.3667 | 1.27719013 |
| 151.02 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.96050805 |
| 167.54 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.65230853 |
| 180 | 35.179 | 35.3667 | 0.53072523 | 35.279 | 35.3667 | 0.24797338 | 35.36 | 35.3667 | 0.01894437 |
| 192.46 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.65230853 |
| 208.98 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.96050805 |
| 224.16 | 34.135 | 35.3667 | 3.48265459 | 34.125 | 35.3667 | 3.51092977 | 34.915 | 35.3667 | 1.27719013 |
| 238.38 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.32525794 |
| 255.21 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.15466526 |
| 270 | 34.963 | 35.3667 | 1.14146924 | 35.142 | 35.3667 | 0.63534342 | 35.309 | 35.3667 | 0.16314782 |
| 284.79 | 34.971 | 35.3667 | 1.11884909 | 35.147 | 35.3667 | 0.62120582 | 35.312 | 35.3667 | 0.15466526 |
| 301.62 | 34.06 | 35.3667 | 3.69471848 | 34.025 | 35.3667 | 3.79368163 | 34.898 | 35.3667 | 1.32525794 |

Table C.25 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 315.84 | 34.135 | 35.3667 | 3.48265459 | 34.125 | 35.3667 | 3.51092977 | 34.915 | 35.3667 | 1.27719013 |
| 331.02 | 34.348 | 35.3667 | 2.88039314 | 34.337 | 35.3667 | 2.91149584 | 35.027 | 35.3667 | 0.96050805 |
| 347.54 | 34.562 | 35.3667 | 2.27530417 | 34.649 | 35.3667 | 2.02931006 | 35.136 | 35.3667 | 0.65230853 |

Table C.26 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_s$ between FEM and CLPT with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.421 | 35.3667 | -0.1535343 | 35.462 | 35.3667 | -0.2694625 | 35.483 | 35.3667 | -0.3288404 |
| 12.46 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |
| 28.98 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 44.16 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 58.38 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |
| 75.21 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 90 | 35.467 | 35.3667 | -0.2836001 | 35.614 | 35.3667 | -0.6992453 | 35.75 | 35.3667 | -1.0837879 |
| 104.79 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 121.62 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |
| 135.84 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 151.02 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 167.54 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |
| 180 | 35.421 | 35.3667 | -0.1535343 | 35.462 | 35.3667 | -0.2694625 | 35.483 | 35.3667 | -0.3288404 |
| 192.46 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |
| 208.98 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 224.16 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 238.38 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |

Table C.26 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 255.21 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 270 | 35.467 | 35.3667 | -0.2836001 | 35.614 | 35.3667 | -0.6992453 | 35.75 | 35.3667 | -1.0837879 |
| 284.79 | 35.467 | 35.3667 | -0.2836001 | 35.611 | 35.3667 | -0.6907628 | 35.744 | 35.3667 | -1.0668227 |
| 301.62 | 35.817 | 35.3667 | -1.2732316 | 36.737 | 35.3667 | -3.8745487 | 36.656 | 35.3667 | -3.6455197 |
| 315.84 | 35.805 | 35.3667 | -1.2393014 | 36.694 | 35.3667 | -3.7529654 | 36.884 | 35.3667 | -4.2901939 |
| 331.02 | 35.716 | 35.3667 | -0.9876522 | 36.405 | 35.3667 | -2.9358125 | 36.694 | 35.3667 | -3.7529654 |
| 347.54 | 35.623 | 35.3667 | -0.724693 | 36.109 | 35.3667 | -2.098867 | 36.596 | 35.3667 | -3.4758685 |

Table C.27 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_S$ between FEM and CLPT with $\frac{b}{a} = 0.4$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 52.529 | 53.0667 | 1.01325313 | 52.682 | 53.0667 | 0.72493673 | 52.789 | 53.0667 | 0.52330369 |
| 8.35 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |
| 20.25 | 50.957 | 53.0667 | 3.97556283 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 40.9 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 47.27 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 68.4 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 90 | 52.423 | 53.0667 | 1.21300175 | 52.76 | 53.0667 | 0.5779519 | 53.084 | 53.0667 | -0.0326005 |
| 111.6 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 132.73 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 139.1 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 159.75 | 50.957 | 53.0667 | 3.97556283 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 171.65 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |
| 180 | 52.529 | 53.0667 | 1.01325313 | 52.682 | 53.0667 | 0.72493673 | 52.789 | 53.0667 | 0.52330369 |

Table C.27 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 188.35 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |
| 200.25 | 50.957 | 53.0667 | 3.97556283 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 220.9 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 227.27 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 248.4 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 270 | 52.423 | 53.0667 | 1.21300175 | 52.76 | 53.0667 | 0.5779519 | 53.084 | 53.0667 | -0.0326005 |
| 291.6 | 52.429 | 53.0667 | 1.20169523 | 52.76 | 53.0667 | 0.5779519 | 53.078 | 53.0667 | -0.021294 |
| 312.73 | 50.034 | 53.0667 | 5.71488334 | 50.483 | 53.0667 | 4.86877835 | 52.253 | 53.0667 | 1.53335331 |
| 319.1 | 50.234 | 53.0667 | 5.33799916 | 50.562 | 53.0667 | 4.7199091 | 52.265 | 53.0667 | 1.51074026 |
| 339.75 | 49.957 | 53.0667 | 5.85998376 | 51.17 | 53.0667 | 3.57418117 | 52.383 | 53.0667 | 1.28837859 |
| 351.65 | 51.267 | 53.0667 | 3.39139234 | 51.921 | 53.0667 | 2.15898106 | 52.574 | 53.0667 | 0.92845419 |

Table C.28 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_s$ between FEM and CLPT with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 52.85 | 53.0667 | 0.40835401 | 52.866 | 53.0667 | 0.37820328 | 52.836 | 53.0667 | 0.43473591 |
| 8.35 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |
| 20.25 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 40.9 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 47.27 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 68.4 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 90 | 53.395 | 53.0667 | -0.6186554 | 53.692 | 53.0667 | -1.1783284 | 53.975 | 53.0667 | -1.7116195 |
| 111.6 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 132.73 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |

Table C.28 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 139.1 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 159.75 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 171.65 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |
| 180 | 52.85 | 53.0667 | 0.40835401 | 52.866 | 53.0667 | 0.37820328 | 52.836 | 53.0667 | 0.43473591 |
| 188.35 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |
| 200.25 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 220.9 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 227.27 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 248.4 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 270 | 53.395 | 53.0667 | -0.6186554 | 53.692 | 53.0667 | -1.1783284 | 53.975 | 53.0667 | -1.7116195 |
| 291.6 | 53.382 | 53.0667 | -0.5941579 | 53.672 | 53.0667 | -1.14064 | 53.948 | 53.0667 | -1.6607402 |
| 312.73 | 54.024 | 53.0667 | -1.8039562 | 55.794 | 53.0667 | -5.1393812 | 56.565 | 53.0667 | -6.5922697 |
| 319.1 | 53.968 | 53.0667 | -1.6984286 | 55.671 | 53.0667 | -4.9075974 | 56.374 | 53.0667 | -6.2323453 |
| 339.75 | 53.596 | 53.0667 | -0.997424 | 54.809 | 53.0667 | -3.2832266 | 56.022 | 53.0667 | -5.5690292 |
| 351.65 | 53.228 | 53.0667 | -0.3039571 | 53.882 | 53.0667 | -1.5363684 | 54.535 | 53.0667 | -2.7668952 |

Table C.29 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_s$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 103.47 | 106.103 | 2.48182667 | 103.9 | 106.103 | 2.07656124 | 104.14 | 106.103 | 1.85036658 |
| 4.18 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |
| 10.44 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 23.41 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 28.42 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |

Table C.29 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|---------|---------|------------|--------|---------|------------|
| 51.62 | 103.2 | 106.103 | 2.73629567 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 90 | 103.14 | 106.103 | 2.79284433 | 104.36 | 106.103 | 1.64302147 | 105.55 | 106.103 | 0.52147294 |
| 128.38 | 103.2 | 106.103 | 2.73629567 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 151.58 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |
| 156.59 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 169.56 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 177.59 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |
| 180 | 103.47 | 106.103 | 2.48182667 | 103.9 | 106.103 | 2.07656124 | 104.14 | 106.103 | 1.85036658 |
| 184.18 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |
| 190.44 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 203.41 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 208.42 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |
| 231.62 | 103.2 | 106.103 | 2.73629567 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 270 | 103.14 | 106.103 | 2.79284433 | 104.36 | 106.103 | 1.64302147 | 105.55 | 106.103 | 0.52147294 |
| 308.38 | 103.14 | 106.103 | 2.79284433 | 104.38 | 106.103 | 1.62417192 | 105.54 | 106.103 | 0.53089772 |
| 331.58 | 97.483 | 106.103 | 8.12444099 | 100.659 | 106.103 | 5.13113164 | 102.84 | 106.103 | 3.07558766 |
| 336.59 | 96.066 | 106.103 | 9.45993197 | 100.989 | 106.103 | 4.82011398 | 102.91 | 106.103 | 3.00961422 |
| 349.56 | 99.661 | 106.103 | 6.07172444 | 101.631 | 106.103 | 4.21504326 | 103.6 | 106.103 | 2.35930456 |
| 355.82 | 101.64 | 106.103 | 4.20656096 | 103.15 | 106.103 | 2.78341955 | 104.66 | 106.103 | 1.36027814 |

Table C.30 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_S$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 104.2 | 106.103 | 1.79381791 | 104.08 | 106.103 | 1.90691524 | 103.78 | 106.103 | 2.18965857 |

Table C.30 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 4.18 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |
| 10.44 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 23.41 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 28.42 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 51.62 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 90 | 106.72 | 106.103 | -0.581226 | 107.87 | 106.103 | -1.6650755 | 108.99 | 106.103 | -2.7206505 |
| 128.38 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 151.58 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 156.59 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 169.56 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 177.59 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |
| 180 | 104.2 | 106.103 | 1.79381791 | 104.08 | 106.103 | 1.90691524 | 103.78 | 106.103 | 2.18965857 |
| 184.18 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |
| 190.44 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 203.41 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 208.42 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 231.62 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 270 | 106.72 | 106.103 | -0.581226 | 107.87 | 106.103 | -1.6650755 | 108.99 | 106.103 | -2.7206505 |
| 308.38 | 106.68 | 106.103 | -0.5435269 | 107.8 | 106.103 | -1.599102 | 108.89 | 106.103 | -2.6264028 |
| 331.58 | 109.01 | 106.103 | -2.7395001 | 111.19 | 106.103 | -4.7941016 | 115.36 | 106.103 | -8.7242338 |
| 336.59 | 108.83 | 106.103 | -2.5698541 | 110.76 | 106.103 | -4.3888362 | 114.68 | 106.103 | -8.083349 |
| 349.56 | 107.57 | 106.103 | -1.3823321 | 109.54 | 106.103 | -3.2390133 | 112.51 | 106.103 | -6.0381722 |
| 355.82 | 106.16 | 106.103 | -0.0534385 | 107.67 | 106.103 | -1.4765799 | 109.18 | 106.103 | -2.8997213 |

Table C.31 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 20.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 42.75 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 65.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 69.75 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 81 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 90 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 99 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 110.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 137.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 155.75 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 159.75 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 180 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 200.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 222.75 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 245.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 249.75 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 261 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 270 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 279 | 21.878 | 16.1715 | -35.287388 | 23.463 | 28.7945 | 18.51568876 | 5.1479 | 5.4681 | 5.855781716 |
| 290.25 | 22.819 | 16.1715 | -41.106267 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |

Table C.31 – Continued

| | | | | | | | | | |
|--------|--------|---------|-------------|-------|---------|-------------|--------|--------|-------------|
| 294.75 | 22.819 | 16.1715 | -41.1062672 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 317.25 | 22.819 | 16.1715 | -41.1062672 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |
| 339.75 | 22.819 | 16.1715 | -41.1062672 | 25.05 | 28.7945 | 13.00421956 | 5.2317 | 5.4681 | 4.323256707 |

Table C.32 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 20.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 42.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 65.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 69.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 81 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 90 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 99 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 110.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 137.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 155.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 159.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 180 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 200.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 222.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 245.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 249.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |

Table C.32 – Continued

| | | | | | | | | | |
|--------|--------|---------|-------------|--------|---------|-------------|--------|--------|-------------|
| 261 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 270 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 279 | 35.596 | 38.2377 | 6.908626826 | 33.81 | 31.0586 | -8.85873799 | 7.3645 | 7.6695 | 3.976791186 |
| 290.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 294.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 317.25 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |
| 339.75 | 34.699 | 38.2377 | 9.254479218 | 32.819 | 31.0586 | -5.66799533 | 7.1105 | 7.6695 | 7.288610731 |

Table C.33 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.8$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 27.167 | 20.2081 | -34.4361914 | 29.197 | 35.9818 | 18.85619952 | 6.4239 | 6.8329 | 5.985745438 |
| 16.432 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.526 | 6.8329 | 4.491504339 |
| 34.46 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.532 | 6.8329 | 4.403693893 |
| 60.03 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.5381 | 6.8329 | 4.314419939 |
| 65.23 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.539 | 6.8329 | 4.301248372 |
| 78.8 | 27.368 | 20.2081 | -35.4308420 | 29.317 | 35.9818 | 18.52269759 | 6.4546 | 6.8329 | 5.536448653 |
| 90 | 27.373 | 20.2081 | -35.4555846 | 29.32 | 35.9818 | 18.51436004 | 6.4552 | 6.8329 | 5.527667608 |
| 101.2 | 27.368 | 20.2081 | -35.4308420 | 29.317 | 35.9818 | 18.52269759 | 6.4546 | 6.8329 | 5.536448653 |
| 114.77 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.539 | 6.8329 | 4.301248372 |
| 119.97 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.5381 | 6.8329 | 4.314419939 |
| 143.54 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.532 | 6.8329 | 4.403693893 |
| 163.57 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.526 | 6.8329 | 4.491504339 |
| 180 | 27.167 | 20.2081 | -34.4361914 | 29.197 | 35.9818 | 18.85619952 | 6.4239 | 6.8329 | 5.985745438 |

Table C.33 – Continued

| | | | | | | | | | |
|---------|--------|---------|-------------|--------|---------|-------------|--------|--------|-------------|
| 196.432 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.526 | 6.8329 | 4.491504339 |
| 216.46 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.532 | 6.8329 | 4.403693893 |
| 240.03 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.5381 | 6.8329 | 4.314419939 |
| 245.23 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.539 | 6.8329 | 4.301248372 |
| 258.8 | 27.368 | 20.2081 | -35.4308420 | 29.317 | 35.9818 | 18.52269759 | 6.5398 | 6.8329 | 4.289540312 |
| 270 | 27.373 | 20.2081 | -35.4555846 | 29.32 | 35.9818 | 18.51436004 | 6.4552 | 6.8329 | 5.527667608 |
| 281.2 | 27.368 | 20.2081 | -35.4308420 | 29.317 | 35.9818 | 18.52269759 | 6.4546 | 6.8329 | 5.536448653 |
| 294.77 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.539 | 6.8329 | 4.301248372 |
| 299.97 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.5381 | 6.8329 | 4.314419939 |
| 323.54 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.532 | 6.8329 | 4.403693893 |
| 343.57 | 28.588 | 20.2081 | -41.4680252 | 31.148 | 35.9818 | 13.43401386 | 6.526 | 6.8329 | 4.491504339 |

Table C.34 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.8$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 44.577 | 47.7821 | 6.707742021 | 42.308 | 38.6184 | -9.55399498 | 9.2385 | 9.5839 | 3.603960809 |
| 16.432 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 34.46 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 60.03 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 65.23 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 78.8 | 44.52 | 47.7821 | 6.827033554 | 42.375 | 38.6184 | -9.72748741 | 9.226 | 9.5839 | 3.73438788 |
| 90 | 44.517 | 47.7821 | 6.833312056 | 42.376 | 38.6184 | -9.73007685 | 9.2256 | 9.5839 | 3.738561546 |
| 101.2 | 44.52 | 47.7821 | 6.827033554 | 42.375 | 38.6184 | -9.72748741 | 9.226 | 9.5839 | 3.73438788 |
| 114.77 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |

Table C.34 – Continued

| | | | | | | | | | |
|---------|--------|---------|-------------|--------|---------|-------------|--------|--------|-------------|
| 119.97 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 143.54 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 163.57 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 180 | 44.577 | 47.7821 | 6.707742021 | 42.308 | 38.6184 | -9.55399498 | 9.2385 | 9.5839 | 3.603960809 |
| 196.432 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 216.46 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 240.03 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 245.23 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 258.8 | 44.52 | 47.7821 | 6.827033554 | 42.375 | 38.6184 | -9.72748741 | 9.226 | 9.5839 | 3.73438788 |
| 270 | 44.517 | 47.7821 | 6.833312056 | 42.376 | 38.6184 | -9.73007685 | 9.2256 | 9.5839 | 3.738561546 |
| 281.2 | 44.52 | 47.7821 | 6.827033554 | 42.375 | 38.6184 | -9.72748741 | 9.226 | 9.5839 | 3.73438788 |
| 294.77 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 299.97 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 323.54 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |
| 343.57 | 43.863 | 47.7821 | 8.202025445 | 41.45 | 38.6184 | -7.33225612 | 8.735 | 9.5839 | 8.857563205 |

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Table C.35 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|---------------------------------|--|-----------------------------|---------------------------------|--|-----------------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) ² | Analytical Result (lb-in) ² | Miss Match (%) ² | FEM Result (lb-in) ³ | Analytical Result (lb-in) ³ | Miss Match (%) ³ |
| 0 | 35.941 | 26.9356 | -33.4330774 | 38.642 | 47.9607 | 19.42986654 | 8.5305 | 9.1077 | 6.337494647 |
| 12.46 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6702 | 9.1077 | 4.8036277 |
| 28.98 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6977 | 9.1077 | 4.501685387 |
| 44.16 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7285 | 9.1077 | 4.163509997 |
| 58.38 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7333 | 9.1077 | 4.110807339 |

Table C.35 – Continued

| | | | | | | | | | |
|--------|--------|---------|-------------|--------|---------|-------------|--------|--------|-------------|
| 75.21 | 36.566 | 26.9356 | -35.7534266 | 39.194 | 47.9607 | 18.2789242 | 8.6488 | 9.1077 | 5.038593717 |
| 90 | 36.581 | 26.9356 | -35.8091150 | 39.207 | 47.9607 | 18.25181868 | 8.6512 | 9.1077 | 5.012242388 |
| 104.79 | 36.566 | 26.9356 | -35.7534266 | 39.194 | 47.9607 | 18.2789242 | 8.6488 | 9.1077 | 5.038593717 |
| 121.62 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7333 | 9.1077 | 4.110807339 |
| 135.84 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7285 | 9.1077 | 4.163509997 |
| 151.02 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6977 | 9.1077 | 4.501685387 |
| 167.54 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6702 | 9.1077 | 4.8036277 |
| 180 | 35.941 | 26.9356 | -33.4330774 | 38.642 | 47.9607 | 19.42986654 | 8.5305 | 9.1077 | 6.337494647 |
| 192.46 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6702 | 9.1077 | 4.8036277 |
| 208.98 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6977 | 9.1077 | 4.501685387 |
| 224.16 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7285 | 9.1077 | 4.163509997 |
| 238.38 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7333 | 9.1077 | 4.110807339 |
| 255.21 | 36.566 | 26.9356 | -35.7534266 | 39.194 | 47.9607 | 18.2789242 | 8.6488 | 9.1077 | 5.038593717 |
| 270 | 36.581 | 26.9356 | -35.8091150 | 39.207 | 47.9607 | 18.25181868 | 8.6512 | 9.1077 | 5.012242388 |
| 284.79 | 36.566 | 26.9356 | -35.7534266 | 39.194 | 47.9607 | 18.2789242 | 8.6488 | 9.1077 | 5.038593717 |
| 301.62 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7333 | 9.1077 | 4.110807339 |
| 315.84 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.7285 | 9.1077 | 4.163509997 |
| 331.02 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6977 | 9.1077 | 4.501685387 |
| 347.54 | 37.831 | 26.9356 | -40.4498136 | 41.415 | 47.9607 | 13.64804934 | 8.6702 | 9.1077 | 4.8036277 |

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Table C.36 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is [+45/-45/0/0/45/-45]

between FEM and CLPT with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 59.4 | 63.6895 | 6.735019116 | 56.327 | 51.7319 | -8.88252702 | 12.327 | 12.7746 | 3.503827909 |

Table C.36 – *Continued*

| | | | | | | | | | |
|--------|--------|---------|-------------|--------|---------|-------------|--------|---------|-------------|
| 12.46 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 28.98 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 44.16 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 58.38 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 75.21 | 59.521 | 63.6895 | 6.545034896 | 56.75 | 51.7319 | -9.70020432 | 12.351 | 12.7746 | 3.315955098 |
| 90 | 59.523 | 63.6895 | 6.541894661 | 56.76 | 51.7319 | -9.71953475 | 12.351 | 12.7746 | 3.315955098 |
| 104.79 | 59.521 | 63.6895 | 6.545034896 | 56.75 | 51.7319 | -9.70020432 | 12.351 | 12.7746 | 3.315955098 |
| 121.62 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 135.84 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 151.02 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 167.54 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 180 | 59.4 | 63.6895 | 6.735019116 | 56.327 | 51.7319 | -8.88252702 | 12.327 | 12.7746 | 3.503827909 |
| 192.46 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 208.98 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 224.16 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 238.38 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 255.21 | 59.521 | 63.6895 | 6.545034896 | 56.75 | 51.7319 | -9.70020432 | 12.351 | 12.7746 | 3.315955098 |
| 270 | 59.523 | 63.6895 | 6.541894661 | 56.76 | 51.7319 | -9.71953475 | 12.351 | 12.7746 | 3.315955098 |
| 284.79 | 59.521 | 63.6895 | 6.545034896 | 56.75 | 51.7319 | -9.70020432 | 12.351 | 12.7746 | 3.315955098 |
| 301.62 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 315.84 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 331.02 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |
| 347.54 | 57.998 | 63.6895 | 8.936323884 | 55.212 | 51.7319 | -6.72718380 | 11.907 | 12.7746 | 6.791602085 |

Table C.37 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.4$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|---------------------------------|--|-----------------------------|---------------------------------|--|-----------------------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) ² | Analytical Result (lb-in) ² | Miss Match (%) ² | FEM Result (lb-in) ³ | Analytical Result (lb-in) ³ | Miss Match (%) ³ |
| 0 | 53.578 | 40.4161 | -32.5659823 | 57.469 | 71.9637 | 20.14168254 | 12.75 | 13.6659 | 6.702083288 |
| 8.35 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 12.973 | 13.6659 | 5.070284431 |
| 20.25 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.031 | 13.6659 | 4.645870378 |
| 40.9 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.103 | 13.6659 | 4.119011554 |
| 47.27 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.114 | 13.6659 | 4.038519234 |
| 68.4 | 54.89 | 40.4161 | -35.8122134 | 58.975 | 71.9637 | 18.0489608 | 13.029 | 13.6659 | 4.660505345 |
| 90 | 54.923 | 40.4161 | -35.8938640 | 59.011 | 71.9637 | 17.99893557 | 13.034 | 13.6659 | 4.623917927 |
| 111.6 | 54.89 | 40.4161 | -35.8122134 | 58.975 | 71.9637 | 18.0489608 | 13.029 | 13.6659 | 4.660505345 |
| 132.73 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.114 | 13.6659 | 4.038519234 |
| 139.1 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.103 | 13.6659 | 4.119011554 |
| 159.75 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.031 | 13.6659 | 4.645870378 |
| 171.65 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 12.973 | 13.6659 | 5.070284431 |
| 180 | 53.578 | 40.4161 | -32.5659823 | 57.469 | 71.9637 | 20.14168254 | 12.75 | 13.6659 | 6.702083288 |
| 188.35 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 12.973 | 13.6659 | 5.070284431 |
| 200.25 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.031 | 13.6659 | 4.645870378 |
| 220.9 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.103 | 13.6659 | 4.119011554 |
| 227.27 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.114 | 13.6659 | 4.038519234 |
| 248.4 | 54.89 | 40.4161 | -35.8122134 | 58.975 | 71.9637 | 18.0489608 | 13.029 | 13.6659 | 4.660505345 |
| 270 | 54.923 | 40.4161 | -35.8938640 | 59.011 | 71.9637 | 17.99893557 | 13.034 | 13.6659 | 4.623917927 |
| 291.6 | 54.89 | 40.4161 | -35.8122134 | 58.975 | 71.9637 | 18.0489608 | 13.029 | 13.6659 | 4.660505345 |
| 312.73 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.114 | 13.6659 | 4.038519234 |

Table C.37 – Continued

| | | | | | | | | | |
|--------|--------|---------|-------------|--------|---------|-------------|--------|---------|-------------|
| 319.1 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.103 | 13.6659 | 4.119011554 |
| 339.75 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 13.031 | 13.6659 | 4.645870378 |
| 351.65 | 55.456 | 40.4161 | -37.2126454 | 61.904 | 71.9637 | 13.97885323 | 12.973 | 13.6659 | 5.070284431 |

Table C.38 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 88.929 | 95.5643 | 6.943283214 | 84.381 | 77.6222 | -8.70730280 | 18.488 | 19.1679 | 3.547076101 |
| 8.35 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 20.25 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 40.9 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 47.27 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 68.4 | 89.518 | 95.5643 | 6.326944267 | 85.373 | 77.6222 | -9.98528771 | 18.585 | 19.1679 | 3.041021708 |
| 90 | 89.527 | 95.5643 | 6.317526524 | 85.393 | 77.6222 | -10.0110535 | 18.585 | 19.1679 | 3.041021708 |
| 111.6 | 89.518 | 95.5643 | 6.326944267 | 85.373 | 77.6222 | -9.98528771 | 18.585 | 19.1679 | 3.041021708 |
| 132.73 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 139.1 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 159.75 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 171.65 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 180 | 88.929 | 95.5643 | 6.943283214 | 84.381 | 77.6222 | -8.70730280 | 18.488 | 19.1679 | 3.547076101 |
| 188.35 | 88.095 | 95.5643 | 7.815994048 | 84.238 | 77.6222 | -8.52307716 | 17.895 | 19.1679 | 6.640790071 |
| 200.25 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 220.9 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 227.27 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |

Table C.38 – Continued

| | | | | | | | | | |
|--------|--------|---------|-------------|--------|---------|-------------|--------|---------|-------------|
| 248.4 | 89.518 | 95.5643 | 6.326944267 | 85.373 | 77.6222 | -9.98528771 | 18.585 | 19.1679 | 3.041021708 |
| 270 | 89.527 | 95.5643 | 6.317526524 | 85.393 | 77.6222 | -10.0110535 | 18.585 | 19.1679 | 3.041021708 |
| 291.6 | 89.518 | 95.5643 | 6.326944267 | 85.373 | 77.6222 | -9.98528771 | 18.585 | 19.1679 | 3.041021708 |
| 312.73 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 319.1 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 339.75 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |
| 351.65 | 88.095 | 95.5643 | 7.815994048 | 83.21 | 77.6222 | -7.19871377 | 17.895 | 19.1679 | 6.640790071 |

Table C.39 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.2$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 105.82 | 79.5253 | -33.0645719 | 113.68 | 144.6572 | 21.41421236 | 25.326 | 27.324 | 7.312252964 |
| 4.18 | 109.29 | 79.5253 | -37.4279631 | 117.07 | 144.6572 | 19.07074103 | 26.02 | 27.324 | 4.772361294 |
| 10.44 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.063 | 27.324 | 4.614990485 |
| 23.41 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.108 | 27.324 | 4.450300102 |
| 28.42 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.116 | 27.324 | 4.421021812 |
| 51.62 | 109.19 | 79.5253 | -37.3022170 | 117.2 | 144.6572 | 18.9808734 | 26.009 | 27.324 | 4.812618943 |
| 90 | 109.24 | 79.5253 | -37.3650901 | 117.25 | 144.6572 | 18.94630893 | 26.012 | 27.324 | 4.801639584 |
| 128.38 | 109.19 | 79.5253 | -37.3022170 | 117.2 | 144.6572 | 18.9808734 | 26.009 | 27.324 | 4.812618943 |
| 151.58 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.116 | 27.324 | 4.421021812 |
| 156.59 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.108 | 27.324 | 4.450300102 |
| 169.56 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.063 | 27.324 | 4.614990485 |
| 177.59 | 109.29 | 79.5253 | -37.4279631 | 117.07 | 144.6572 | 19.07074103 | 26.02 | 27.324 | 4.772361294 |
| 180 | 105.82 | 79.5253 | -33.0645719 | 113.68 | 144.6572 | 21.41421236 | 25.326 | 27.324 | 7.312252964 |

Table C.39 – Continued

| | | | | | | | | | |
|--------|--------|---------|-------------|--------|----------|-------------|--------|--------|-------------|
| 184.18 | 109.29 | 79.5253 | -37.4279631 | 117.07 | 144.6572 | 19.07074103 | 26.02 | 27.324 | 4.772361294 |
| 190.44 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.063 | 27.324 | 4.614990485 |
| 203.41 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.108 | 27.324 | 4.450300102 |
| 208.42 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.116 | 27.324 | 4.421021812 |
| 231.62 | 109.19 | 79.5253 | -37.3022170 | 117.2 | 144.6572 | 18.9808734 | 26.009 | 27.324 | 4.812618943 |
| 270 | 109.24 | 79.5253 | -37.3650901 | 117.25 | 144.6572 | 18.94630893 | 26.012 | 27.324 | 4.801639584 |
| 308.38 | 109.19 | 79.5253 | -37.3022170 | 117.2 | 144.6572 | 18.9808734 | 26.009 | 27.324 | 4.812618943 |
| 331.58 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.116 | 27.324 | 4.421021812 |
| 336.59 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.108 | 27.324 | 4.450300102 |
| 349.56 | 111.95 | 79.5253 | -40.7728106 | 119.05 | 144.6572 | 17.70198787 | 26.063 | 27.324 | 4.614990485 |
| 355.82 | 109.29 | 79.5253 | -37.4279631 | 117.07 | 144.6572 | 19.07074103 | 26.02 | 27.324 | 4.772361294 |

Table C.40 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is [+45/-45/0/0/45/-45] between FEM and CLPT with $\frac{b}{a} = 0.2$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 176.14 | 191.3314 | 7.939836326 | 167.69 | 154.4299 | -8.58648487 | 36.725 | 38.3249 | 4.174570579 |
| 4.18 | 178.43 | 191.3314 | 6.742960121 | 170.43 | 154.4299 | -10.3607526 | 37.105 | 38.3249 | 3.183048097 |
| 10.44 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 23.41 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 28.42 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 51.62 | 178.77 | 191.3314 | 6.565257976 | 170.4 | 154.4299 | -10.3413263 | 37.143 | 38.3249 | 3.083895848 |
| 90 | 178.74 | 191.3314 | 6.580937577 | 170.37 | 154.4299 | -10.3219001 | 37.13 | 38.3249 | 3.117816354 |
| 128.38 | 178.77 | 191.3314 | 6.565257976 | 170.4 | 154.4299 | -10.3413263 | 37.143 | 38.3249 | 3.083895848 |
| 151.58 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |

Table C.40 – Continued

| | | | | | | | | | |
|--------|--------|----------|-------------|--------|----------|-------------|--------|---------|-------------|
| 156.59 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 169.56 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 177.59 | 178.43 | 191.3314 | 6.742960121 | 170.43 | 154.4299 | -10.3607526 | 37.105 | 38.3249 | 3.183048097 |
| 180 | 176.14 | 191.3314 | 7.939836326 | 167.69 | 154.4299 | -8.58648487 | 36.725 | 38.3249 | 4.174570579 |
| 184.18 | 178.43 | 191.3314 | 6.742960121 | 170.43 | 154.4299 | -10.3607526 | 37.105 | 38.3249 | 3.183048097 |
| 190.44 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 203.41 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 208.42 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 231.62 | 178.77 | 191.3314 | 6.565257976 | 170.4 | 154.4299 | -10.3413263 | 37.143 | 38.3249 | 3.083895848 |
| 270 | 178.74 | 191.3314 | 6.580937577 | 170.37 | 154.4299 | -10.3219001 | 37.13 | 38.3249 | 3.117816354 |
| 308.38 | 178.77 | 191.3314 | 6.565257976 | 170.4 | 154.4299 | -10.3413263 | 37.143 | 38.3249 | 3.083895848 |
| 331.58 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 336.59 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 349.56 | 176.01 | 191.3314 | 8.007781263 | 167.51 | 154.4299 | -8.46992713 | 36.181 | 38.3249 | 5.594013292 |
| 355.82 | 178.43 | 191.3314 | 6.742960121 | 170.43 | 154.4299 | -10.3607526 | 37.105 | 38.3249 | 3.183048097 |

Table C.41 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 20.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 42.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 65.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 69.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |

Table C.41 – Continued

| | | | | | | | | | |
|--------|--------|-------|------------|--------|-------|------------|--------|--------|------------|
| 81 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 90 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 99 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 110.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 137.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 155.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 159.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 180 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 200.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 222.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 245.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 249.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 261 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 270 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 279 | 29.11 | 29.21 | 0.34234851 | 29.209 | 29.21 | 0.00342349 | 5.2898 | 5.2855 | -0.0813546 |
| 290.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 294.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 317.25 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |
| 339.75 | 28.856 | 29.21 | 1.21191373 | 28.939 | 29.21 | 0.92776446 | 5.2718 | 5.2855 | 0.2591997 |

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Table C.42 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 1$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |

Table C.42 – Continued

| | | | | | | | | | |
|--------|--------|--------|------------|--------|-------|------------|--------|-------|------------|
| 20.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 42.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 65.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 69.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 81 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 90 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 99 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 110.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 137.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 155.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 159.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 180 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 200.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 222.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 245.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 249.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 261 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 270 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 279 | 5.2898 | 5.2855 | -0.0813546 | 29.191 | 29.21 | 0.06504622 | 29.278 | 29.21 | -0.232797 |
| 290.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 294.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 317.25 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |
| 339.75 | 5.2718 | 5.2855 | 0.2591997 | 29.487 | 29.21 | -0.9483054 | 29.57 | 29.21 | -1.2324546 |

Table C.43 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 36.417 | 36.4976 | 0.22083644 | 36.53 | 36.4976 | -0.088773 | 6.6152 | 6.6049 | -0.1559448 |
| 16.432 | 36.183 | 36.4976 | 0.86197449 | 36.285 | 36.4976 | 0.58250406 | 6.5974 | 6.6049 | 0.11355206 |
| 34.46 | 36.096 | 36.4976 | 1.10034632 | 36.203 | 36.4976 | 0.80717636 | 6.5913 | 6.6049 | 0.20590774 |
| 60.03 | 35.997 | 36.4976 | 1.37159704 | 36.106 | 36.4976 | 1.07294726 | 6.5842 | 6.6049 | 0.31340369 |
| 65.23 | 35.982 | 36.4976 | 1.41269563 | 36.09 | 36.4976 | 1.11678576 | 6.5831 | 6.6049 | 0.33005799 |
| 78.8 | 36.356 | 36.4976 | 0.38797072 | 36.485 | 36.4976 | 0.03452282 | 6.6089 | 6.6049 | -0.0605611 |
| 90 | 36.355 | 36.4976 | 0.39071062 | 36.482 | 36.4976 | 0.04274254 | 6.6086 | 6.6049 | -0.056019 |
| 101.2 | 36.358 | 36.4976 | 0.3824909 | 36.483 | 36.4976 | 0.04000263 | 6.6088 | 6.6049 | -0.0590471 |
| 114.77 | 35.985 | 36.4976 | 1.40447591 | 36.086 | 36.4976 | 1.12774539 | 6.5831 | 6.6049 | 0.33005799 |
| 119.97 | 36.001 | 36.4976 | 1.36063741 | 36.101 | 36.4976 | 1.08664679 | 6.5842 | 6.6049 | 0.31340369 |
| 143.54 | 36.101 | 36.4976 | 1.08664679 | 36.197 | 36.4976 | 0.8236158 | 6.5913 | 6.6049 | 0.20590774 |
| 163.57 | 36.187 | 36.4976 | 0.85101486 | 36.281 | 36.4976 | 0.59346368 | 6.5974 | 6.6049 | 0.11355206 |
| 180 | 36.417 | 36.4976 | 0.22083644 | 36.53 | 36.4976 | -0.088773 | 6.6152 | 6.6049 | -0.1559448 |
| 196.432 | 36.183 | 36.4976 | 0.86197449 | 36.285 | 36.4976 | 0.58250406 | 6.5974 | 6.6049 | 0.11355206 |
| 216.46 | 36.096 | 36.4976 | 1.10034632 | 36.203 | 36.4976 | 0.80717636 | 6.5913 | 6.6049 | 0.20590774 |
| 240.03 | 35.997 | 36.4976 | 1.37159704 | 36.106 | 36.4976 | 1.07294726 | 6.5842 | 6.6049 | 0.31340369 |
| 245.23 | 35.982 | 36.4976 | 1.41269563 | 36.09 | 36.4976 | 1.11678576 | 6.5831 | 6.6049 | 0.33005799 |
| 258.8 | 36.356 | 36.4976 | 0.38797072 | 36.485 | 36.4976 | 0.03452282 | 6.6089 | 6.6049 | -0.0605611 |
| 270 | 36.355 | 36.4976 | 0.39071062 | 36.482 | 36.4976 | 0.04274254 | 6.6086 | 6.6049 | -0.056019 |
| 281.2 | 36.358 | 36.4976 | 0.3824909 | 36.483 | 36.4976 | 0.04000263 | 6.6088 | 6.6049 | -0.0590471 |
| 294.77 | 35.985 | 36.4976 | 1.40447591 | 36.086 | 36.4976 | 1.12774539 | 6.5831 | 6.6049 | 0.33005799 |

Table C.43 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|--------|------------|
| 299.97 | 36.001 | 36.4976 | 1.36063741 | 36.101 | 36.4976 | 1.08664679 | 6.5842 | 6.6049 | 0.31340369 |
| 323.54 | 36.101 | 36.4976 | 1.08664679 | 36.197 | 36.4976 | 0.8236158 | 6.5913 | 6.6049 | 0.20590774 |
| 343.57 | 36.187 | 36.4976 | 0.85101486 | 36.281 | 36.4976 | 0.59346368 | 6.5974 | 6.6049 | 0.11355206 |

Table C.44 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.8$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 6.6152 | 6.6049 | -0.1559448 | 36.469 | 36.4976 | 0.07836132 | 36.592 | 36.4976 | -0.2586471 |
| 16.432 | 6.5974 | 6.6049 | 0.11355206 | 36.772 | 36.4976 | -0.7518303 | 36.889 | 36.4976 | -1.0723993 |
| 34.46 | 6.5913 | 6.6049 | 0.20590774 | 36.833 | 36.4976 | -0.9189645 | 36.938 | 36.4976 | -1.2066547 |
| 60.03 | 6.5842 | 6.6049 | 0.31340369 | 36.904 | 36.4976 | -1.1134979 | 36.993 | 36.4976 | -1.3573495 |
| 65.23 | 6.5831 | 6.6049 | 0.33005799 | 36.915 | 36.4976 | -1.1436368 | 37.002 | 36.4976 | -1.3820087 |
| 78.8 | 6.6089 | 6.6049 | -0.0605611 | 36.498 | 36.4976 | -0.001096 | 36.588 | 36.4976 | -0.2476875 |
| 90 | 6.6086 | 6.6049 | -0.056019 | 36.498 | 36.4976 | -0.001096 | 36.588 | 36.4976 | -0.2476875 |
| 101.2 | 6.6088 | 6.6049 | -0.0590471 | 36.498 | 36.4976 | -0.001096 | 36.588 | 36.4976 | -0.2476875 |
| 114.77 | 6.5831 | 6.6049 | 0.33005799 | 36.915 | 36.4976 | -1.1436368 | 37.002 | 36.4976 | -1.3820087 |
| 119.97 | 6.5842 | 6.6049 | 0.31340369 | 36.904 | 36.4976 | -1.1134979 | 36.993 | 36.4976 | -1.3573495 |
| 143.54 | 6.5913 | 6.6049 | 0.20590774 | 36.833 | 36.4976 | -0.9189645 | 36.938 | 36.4976 | -1.2066547 |
| 163.57 | 6.5974 | 6.6049 | 0.11355206 | 36.772 | 36.4976 | -0.7518303 | 36.889 | 36.4976 | -1.0723993 |
| 180 | 6.6152 | 6.6049 | -0.1559448 | 36.469 | 36.4976 | 0.07836132 | 36.592 | 36.4976 | -0.2586471 |
| 196.432 | 6.5974 | 6.6049 | 0.11355206 | 36.772 | 36.4976 | -0.7518303 | 36.889 | 36.4976 | -1.0723993 |
| 216.46 | 6.5913 | 6.6049 | 0.20590774 | 36.833 | 36.4976 | -0.9189645 | 36.938 | 36.4976 | -1.2066547 |
| 240.03 | 6.5842 | 6.6049 | 0.31340369 | 36.904 | 36.4976 | -1.1134979 | 36.993 | 36.4976 | -1.3573495 |
| 245.23 | 6.5831 | 6.6049 | 0.33005799 | 36.915 | 36.4976 | -1.1436368 | 37.002 | 36.4976 | -1.3820087 |

Table C.44 – Continued

| | | | | | | | | | |
|--------|--------|--------|------------|--------|---------|------------|--------|---------|------------|
| 258.8 | 6.6089 | 6.6049 | -0.0605611 | 36.498 | 36.4976 | -0.001096 | 36.588 | 36.4976 | -0.2476875 |
| 270 | 6.6086 | 6.6049 | -0.056019 | 36.498 | 36.4976 | -0.001096 | 36.588 | 36.4976 | -0.2476875 |
| 281.2 | 6.6088 | 6.6049 | -0.0590471 | 36.498 | 36.4976 | -0.001096 | 36.588 | 36.4976 | -0.2476875 |
| 294.77 | 6.5831 | 6.6049 | 0.33005799 | 36.915 | 36.4976 | -1.1436368 | 37.002 | 36.4976 | -1.3820087 |
| 299.97 | 6.5842 | 6.6049 | 0.31340369 | 36.904 | 36.4976 | -1.1134979 | 36.993 | 36.4976 | -1.3573495 |
| 323.54 | 6.5913 | 6.6049 | 0.20590774 | 36.833 | 36.4976 | -0.9189645 | 36.938 | 36.4976 | -1.2066547 |
| 343.57 | 6.5974 | 6.6049 | 0.11355206 | 36.772 | 36.4976 | -0.7518303 | 36.889 | 36.4976 | -1.0723993 |

Table C.45 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.6$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 48.453 | 48.6481 | 0.40104341 | 48.634 | 48.6481 | 0.02898366 | 8.8048 | 8.8037 | -0.0124947 |
| 12.46 | 48.267 | 48.6481 | 0.78338106 | 48.435 | 48.6481 | 0.43804383 | 8.7915 | 8.8037 | 0.1385781 |
| 28.98 | 48.108 | 48.6481 | 1.11021808 | 48.273 | 48.6481 | 0.77104758 | 8.7864 | 8.8037 | 0.19650829 |
| 44.16 | 47.925 | 48.6481 | 1.48638899 | 48.073 | 48.6481 | 1.18216333 | 8.7804 | 8.8037 | 0.26466145 |
| 58.38 | 47.896 | 48.6481 | 1.54600077 | 48.039 | 48.6481 | 1.25205301 | 8.7795 | 8.8037 | 0.27488442 |
| 75.21 | 48.52 | 48.6481 | 0.26331964 | 48.687 | 48.6481 | -0.079962 | 8.8221 | 8.8037 | -0.209003 |
| 90 | 48.522 | 48.6481 | 0.25920848 | 48.683 | 48.6481 | -0.0717397 | 8.8223 | 8.8037 | -0.2112748 |
| 104.79 | 48.524 | 48.6481 | 0.25509732 | 48.682 | 48.6481 | -0.0696841 | 8.822 | 8.8037 | -0.2078671 |
| 121.62 | 47.905 | 48.6481 | 1.52750056 | 48.031 | 48.6481 | 1.26849764 | 8.7794 | 8.8037 | 0.27602031 |
| 135.84 | 47.935 | 48.6481 | 1.4658332 | 48.063 | 48.6481 | 1.20271912 | 8.7804 | 8.8037 | 0.26466145 |
| 151.02 | 48.12 | 48.6481 | 1.08555113 | 48.26 | 48.6481 | 0.79777011 | 8.7864 | 8.8037 | 0.19650829 |
| 167.54 | 48.275 | 48.6481 | 0.76693643 | 48.426 | 48.6481 | 0.45654404 | 8.7914 | 8.8037 | 0.13971398 |
| 180 | 48.453 | 48.6481 | 0.40104341 | 48.634 | 48.6481 | 0.02898366 | 8.8048 | 8.8037 | -0.0124947 |

Table C.45 – Continued

| | | | | | | | | | |
|--------|--------|---------|------------|--------|---------|------------|--------|--------|------------|
| 192.46 | 48.267 | 48.6481 | 0.78338106 | 48.435 | 48.6481 | 0.43804383 | 8.7915 | 8.8037 | 0.1385781 |
| 208.98 | 48.108 | 48.6481 | 1.11021808 | 48.273 | 48.6481 | 0.77104758 | 8.7864 | 8.8037 | 0.19650829 |
| 224.16 | 47.925 | 48.6481 | 1.48638899 | 48.073 | 48.6481 | 1.18216333 | 8.7804 | 8.8037 | 0.26466145 |
| 238.38 | 47.896 | 48.6481 | 1.54600077 | 48.039 | 48.6481 | 1.25205301 | 8.7795 | 8.8037 | 0.27488442 |
| 255.21 | 48.52 | 48.6481 | 0.26331964 | 48.687 | 48.6481 | -0.079962 | 8.8221 | 8.8037 | -0.209003 |
| 270 | 48.522 | 48.6481 | 0.25920848 | 48.683 | 48.6481 | -0.0717397 | 8.8223 | 8.8037 | -0.2112748 |
| 284.79 | 48.524 | 48.6481 | 0.25509732 | 48.682 | 48.6481 | -0.0696841 | 8.822 | 8.8037 | -0.2078671 |
| 301.62 | 47.905 | 48.6481 | 1.52750056 | 48.031 | 48.6481 | 1.26849764 | 8.7794 | 8.8037 | 0.27602031 |
| 315.84 | 47.935 | 48.6481 | 1.4658332 | 48.063 | 48.6481 | 1.20271912 | 8.7804 | 8.8037 | 0.26466145 |
| 331.02 | 48.12 | 48.6481 | 1.08555113 | 48.26 | 48.6481 | 0.79777011 | 8.7864 | 8.8037 | 0.19650829 |
| 347.54 | 48.275 | 48.6481 | 0.76693643 | 48.426 | 48.6481 | 0.45654404 | 8.7914 | 8.8037 | 0.13971398 |

Table C.46 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.6$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 8.8048 | 8.8037 | -0.0124947 | 48.477 | 48.6481 | 0.35170952 | 48.674 | 48.6481 | -0.0532395 |
| 12.46 | 8.7915 | 8.8037 | 0.1385781 | 48.864 | 48.6481 | -0.4437994 | 49.056 | 48.6481 | -0.8384706 |
| 28.98 | 8.7864 | 8.8037 | 0.19650829 | 49.084 | 48.6481 | -0.8960268 | 49.244 | 48.6481 | -1.2249194 |
| 44.16 | 8.7804 | 8.8037 | 0.26466145 | 49.348 | 48.6481 | -1.4386996 | 49.467 | 48.6481 | -1.6833134 |
| 58.38 | 8.7795 | 8.8037 | 0.27488442 | 49.392 | 48.6481 | -1.529145 | 49.503 | 48.6481 | -1.7573143 |
| 75.21 | 8.8221 | 8.8037 | -0.209003 | 48.761 | 48.6481 | -0.2320748 | 48.875 | 48.6481 | -0.4664108 |
| 90 | 8.8223 | 8.8037 | -0.2112748 | 48.768 | 48.6481 | -0.2464639 | 48.879 | 48.6481 | -0.4746331 |
| 104.79 | 8.822 | 8.8037 | -0.2078671 | 48.761 | 48.6481 | -0.2320748 | 48.875 | 48.6481 | -0.4664108 |
| 121.62 | 8.7794 | 8.8037 | 0.27602031 | 49.392 | 48.6481 | -1.529145 | 49.503 | 48.6481 | -1.7573143 |

Table C.46 – Continued

| | | | | | | | | | |
|--------|--------|--------|------------|--------|---------|------------|--------|---------|------------|
| 135.84 | 8.7804 | 8.8037 | 0.26466145 | 49.348 | 48.6481 | -1.4386996 | 49.467 | 48.6481 | -1.6833134 |
| 151.02 | 8.7864 | 8.8037 | 0.19650829 | 49.084 | 48.6481 | -0.8960268 | 49.244 | 48.6481 | -1.2249194 |
| 167.54 | 8.7914 | 8.8037 | 0.13971398 | 48.864 | 48.6481 | -0.4437994 | 49.056 | 48.6481 | -0.8384706 |
| 180 | 8.8048 | 8.8037 | -0.0124947 | 48.477 | 48.6481 | 0.35170952 | 48.674 | 48.6481 | -0.0532395 |
| 192.46 | 8.7915 | 8.8037 | 0.1385781 | 48.864 | 48.6481 | -0.4437994 | 49.056 | 48.6481 | -0.8384706 |
| 208.98 | 8.7864 | 8.8037 | 0.19650829 | 49.084 | 48.6481 | -0.8960268 | 49.244 | 48.6481 | -1.2249194 |
| 224.16 | 8.7804 | 8.8037 | 0.26466145 | 49.348 | 48.6481 | -1.4386996 | 49.467 | 48.6481 | -1.6833134 |
| 238.38 | 8.7795 | 8.8037 | 0.27488442 | 49.392 | 48.6481 | -1.529145 | 49.503 | 48.6481 | -1.7573143 |
| 255.21 | 8.8221 | 8.8037 | -0.209003 | 48.761 | 48.6481 | -0.2320748 | 48.875 | 48.6481 | -0.4664108 |
| 270 | 8.8223 | 8.8037 | -0.2112748 | 48.768 | 48.6481 | -0.2464639 | 48.879 | 48.6481 | -0.4746331 |
| 284.79 | 8.822 | 8.8037 | -0.2078671 | 48.761 | 48.6481 | -0.2320748 | 48.875 | 48.6481 | -0.4664108 |
| 301.62 | 8.7794 | 8.8037 | 0.27602031 | 49.392 | 48.6481 | -1.529145 | 49.503 | 48.6481 | -1.7573143 |
| 315.84 | 8.7804 | 8.8037 | 0.26466145 | 49.348 | 48.6481 | -1.4386996 | 49.467 | 48.6481 | -1.6833134 |
| 331.02 | 8.7864 | 8.8037 | 0.19650829 | 49.084 | 48.6481 | -0.8960268 | 49.244 | 48.6481 | -1.2249194 |
| 347.54 | 8.7914 | 8.8037 | 0.13971398 | 48.864 | 48.6481 | -0.4437994 | 49.056 | 48.6481 | -0.8384706 |

Table C.47 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|----------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|--------------------------|---------------------------------|-------------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 72.3 | 72.995 | 0.95212001 | 72.702 | 72.995 | 0.40139736 | 13.16 | 13.21 | 0.37850114 |
| 8.35 | 72.275 | 72.995 | 0.98636893 | 72.633 | 72.995 | 0.49592438 | 13.166 | 13.21 | 0.333081 |
| 20.25 | 71.966 | 72.995 | 1.40968559 | 72.28 | 72.995 | 0.97951915 | 13.166 | 13.21 | 0.333081 |
| 40.9 | 71.683 | 72.995 | 1.79738338 | 71.931 | 72.995 | 1.45763408 | 13.173 | 13.21 | 0.28009084 |
| 47.27 | 71.643 | 72.995 | 1.85218166 | 71.878 | 72.995 | 1.5302418 | 13.175 | 13.21 | 0.26495079 |

Table C.47 – Continued

| | | | | | | | | | |
|--------|--------|--------|------------|--------|--------|------------|--------|-------|------------|
| 68.4 | 72.882 | 72.995 | 0.15480512 | 73.152 | 72.995 | -0.2150832 | 13.258 | 13.21 | -0.3633611 |
| 90 | 72.893 | 72.995 | 0.1397356 | 73.155 | 72.995 | -0.2191931 | 13.26 | 13.21 | -0.3785011 |
| 111.6 | 72.886 | 72.995 | 0.1493253 | 73.148 | 72.995 | -0.2096034 | 13.258 | 13.21 | -0.3633611 |
| 132.73 | 71.652 | 72.995 | 1.83985204 | 71.87 | 72.995 | 1.54120145 | 13.175 | 13.21 | 0.26495079 |
| 139.1 | 71.694 | 72.995 | 1.78231386 | 71.922 | 72.995 | 1.4699637 | 13.173 | 13.21 | 0.28009084 |
| 159.75 | 71.98 | 72.995 | 1.3905062 | 72.268 | 72.995 | 0.99595863 | 13.166 | 13.21 | 0.333081 |
| 171.65 | 72.283 | 72.995 | 0.97540927 | 72.626 | 72.995 | 0.50551408 | 13.166 | 13.21 | 0.333081 |
| 180 | 72.3 | 72.995 | 0.95212001 | 72.702 | 72.995 | 0.40139736 | 13.16 | 13.21 | 0.37850114 |
| 188.35 | 72.275 | 72.995 | 0.98636893 | 72.633 | 72.995 | 0.49592438 | 13.166 | 13.21 | 0.333081 |
| 200.25 | 71.966 | 72.995 | 1.40968559 | 72.28 | 72.995 | 0.97951915 | 13.166 | 13.21 | 0.333081 |
| 220.9 | 71.683 | 72.995 | 1.79738338 | 71.931 | 72.995 | 1.45763408 | 13.173 | 13.21 | 0.28009084 |
| 227.27 | 71.643 | 72.995 | 1.85218166 | 71.878 | 72.995 | 1.5302418 | 13.175 | 13.21 | 0.26495079 |
| 248.4 | 72.882 | 72.995 | 0.15480512 | 73.152 | 72.995 | -0.2150832 | 13.258 | 13.21 | -0.3633611 |
| 270 | 72.893 | 72.995 | 0.1397356 | 73.155 | 72.995 | -0.2191931 | 13.26 | 13.21 | -0.3785011 |
| 291.6 | 72.886 | 72.995 | 0.1493253 | 73.148 | 72.995 | -0.2096034 | 13.258 | 13.21 | -0.3633611 |
| 312.73 | 71.652 | 72.995 | 1.83985204 | 71.87 | 72.995 | 1.54120145 | 13.175 | 13.21 | 0.26495079 |
| 319.1 | 71.694 | 72.995 | 1.78231386 | 71.922 | 72.995 | 1.4699637 | 13.173 | 13.21 | 0.28009084 |
| 339.75 | 71.98 | 72.995 | 1.3905062 | 72.268 | 72.995 | 0.99595863 | 13.166 | 13.21 | 0.333081 |
| 351.65 | 72.283 | 72.995 | 0.97540927 | 72.626 | 72.995 | 0.50551408 | 13.166 | 13.21 | 0.333081 |

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Table C.48 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.4$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 13.16 | 13.21 | 0.37850114 | 72.322 | 72.995 | 0.92198096 | 72.653 | 72.995 | 0.46852524 |

Table C.48 – *Continued*

| | | | | | | | | | |
|--------|--------|-------|------------|--------|--------|------------|--------|--------|------------|
| 8.35 | 13.166 | 13.21 | 0.333081 | 73.058 | 72.995 | -0.0863073 | 73.399 | 72.995 | -0.5534626 |
| 20.25 | 13.166 | 13.21 | 0.333081 | 73.659 | 72.995 | -0.9096513 | 73.958 | 72.995 | -1.3192684 |
| 40.9 | 13.173 | 13.21 | 0.28009084 | 74.355 | 72.995 | -1.8631413 | 74.599 | 72.995 | -2.1974108 |
| 47.27 | 13.175 | 13.21 | 0.26495079 | 74.467 | 72.995 | -2.0165765 | 74.702 | 72.995 | -2.3385163 |
| 68.4 | 13.258 | 13.21 | -0.3633611 | 73.314 | 72.995 | -0.4370162 | 73.568 | 72.995 | -0.7849853 |
| 90 | 13.26 | 13.21 | -0.3785011 | 73.338 | 72.995 | -0.4698952 | 73.588 | 72.995 | -0.8123844 |
| 111.6 | 13.258 | 13.21 | -0.3633611 | 73.314 | 72.995 | -0.4370162 | 73.568 | 72.995 | -0.7849853 |
| 132.73 | 13.175 | 13.21 | 0.26495079 | 74.467 | 72.995 | -2.0165765 | 74.702 | 72.995 | -2.3385163 |
| 139.1 | 13.173 | 13.21 | 0.28009084 | 74.355 | 72.995 | -1.8631413 | 74.599 | 72.995 | -2.1974108 |
| 159.75 | 13.166 | 13.21 | 0.333081 | 73.659 | 72.995 | -0.9096513 | 73.958 | 72.995 | -1.3192684 |
| 171.65 | 13.166 | 13.21 | 0.333081 | 73.058 | 72.995 | -0.0863073 | 73.399 | 72.995 | -0.5534626 |
| 180 | 13.16 | 13.21 | 0.37850114 | 72.322 | 72.995 | 0.92198096 | 72.653 | 72.995 | 0.46852524 |
| 188.35 | 13.166 | 13.21 | 0.333081 | 73.058 | 72.995 | -0.0863073 | 73.399 | 72.995 | -0.5534626 |
| 200.25 | 13.166 | 13.21 | 0.333081 | 73.659 | 72.995 | -0.9096513 | 73.958 | 72.995 | -1.3192684 |
| 220.9 | 13.173 | 13.21 | 0.28009084 | 74.355 | 72.995 | -1.8631413 | 74.599 | 72.995 | -2.1974108 |
| 227.27 | 13.175 | 13.21 | 0.26495079 | 74.467 | 72.995 | -2.0165765 | 74.702 | 72.995 | -2.3385163 |
| 248.4 | 13.258 | 13.21 | -0.3633611 | 73.314 | 72.995 | -0.4370162 | 73.568 | 72.995 | -0.7849853 |
| 270 | 13.26 | 13.21 | -0.3785011 | 73.338 | 72.995 | -0.4698952 | 73.588 | 72.995 | -0.8123844 |
| 291.6 | 13.258 | 13.21 | -0.3633611 | 73.314 | 72.995 | -0.4370162 | 73.568 | 72.995 | -0.7849853 |
| 312.73 | 13.175 | 13.21 | 0.26495079 | 74.467 | 72.995 | -2.0165765 | 74.702 | 72.995 | -2.3385163 |
| 319.1 | 13.173 | 13.21 | 0.28009084 | 74.355 | 72.995 | -1.8631413 | 74.599 | 72.995 | -2.1974108 |
| 339.75 | 13.166 | 13.21 | 0.333081 | 73.659 | 72.995 | -0.9096513 | 73.958 | 72.995 | -1.3192684 |
| 351.65 | 13.166 | 13.21 | 0.333081 | 73.058 | 72.995 | -0.0863073 | 73.399 | 72.995 | -0.5534626 |

Table C.49 Comparison of τ_{xy} along the mid cross-section of the tube for layer 1, 2, 3 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| | SXYLAYER 1 | | | SXYLAYER 2 | | | SXYLAYER 3 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| ANGLE (deg) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 142.5 | 145.95 | 2.36382323 | 143.74 | 145.95 | 1.5142172 | 26.046 | 26.412 | 1.38573376 |
| 4.18 | 144.23 | 145.95 | 1.17848578 | 145.32 | 145.95 | 0.43165468 | 26.323 | 26.412 | 0.33696804 |
| 10.44 | 142.53 | 145.95 | 2.34326824 | 143.51 | 145.95 | 1.67180541 | 26.254 | 26.412 | 0.59821293 |
| 23.41 | 140.98 | 145.95 | 3.40527578 | 141.89 | 145.95 | 2.78177458 | 26.205 | 26.412 | 0.78373467 |
| 28.42 | 140.77 | 145.95 | 3.54916067 | 141.66 | 145.95 | 2.9393628 | 26.199 | 26.412 | 0.80645161 |
| 51.62 | 144.81 | 145.95 | 0.78108941 | 145.87 | 145.95 | 0.05481329 | 26.435 | 26.412 | -0.0870816 |
| 90 | 144.8 | 145.95 | 0.78794108 | 145.86 | 145.95 | 0.06166495 | 26.436 | 26.412 | -0.0908678 |
| 128.38 | 144.81 | 145.95 | 0.78108941 | 145.88 | 145.95 | 0.04796163 | 26.435 | 26.412 | -0.0870816 |
| 151.58 | 140.75 | 145.95 | 3.56286399 | 141.68 | 145.95 | 2.92565947 | 26.199 | 26.412 | 0.80645161 |
| 156.59 | 140.97 | 145.95 | 3.41212744 | 141.91 | 145.95 | 2.76807126 | 26.205 | 26.412 | 0.78373467 |
| 169.56 | 142.51 | 145.95 | 2.35697157 | 143.54 | 145.95 | 1.65125043 | 26.255 | 26.412 | 0.59442678 |
| 177.59 | 144.22 | 145.95 | 1.18533744 | 145.34 | 145.95 | 0.41795135 | 26.324 | 26.412 | 0.33318189 |
| 180 | 142.5 | 145.95 | 2.36382323 | 143.74 | 145.95 | 1.5142172 | 26.046 | 26.412 | 1.38573376 |
| 184.18 | 144.23 | 145.95 | 1.17848578 | 145.32 | 145.95 | 0.43165468 | 26.323 | 26.412 | 0.33696804 |
| 190.44 | 142.53 | 145.95 | 2.34326824 | 143.51 | 145.95 | 1.67180541 | 26.254 | 26.412 | 0.59821293 |
| 203.41 | 140.98 | 145.95 | 3.40527578 | 141.89 | 145.95 | 2.78177458 | 26.205 | 26.412 | 0.78373467 |
| 208.42 | 140.77 | 145.95 | 3.54916067 | 141.66 | 145.95 | 2.9393628 | 26.199 | 26.412 | 0.80645161 |
| 231.62 | 144.81 | 145.95 | 0.78108941 | 145.87 | 145.95 | 0.05481329 | 26.435 | 26.412 | -0.0870816 |
| 270 | 144.8 | 145.95 | 0.78794108 | 145.86 | 145.95 | 0.06166495 | 26.436 | 26.412 | -0.0908678 |
| 308.38 | 144.81 | 145.95 | 0.78108941 | 145.88 | 145.95 | 0.04796163 | 26.435 | 26.412 | -0.0870816 |
| 331.58 | 140.75 | 145.95 | 3.56286399 | 141.68 | 145.95 | 2.92565947 | 26.199 | 26.412 | 0.80645161 |

Table C.49 – Continued

| | | | | | | | | | |
|--------|--------|--------|------------|--------|--------|------------|--------|--------|------------|
| 336.59 | 140.97 | 145.95 | 3.41212744 | 141.91 | 145.95 | 2.76807126 | 26.205 | 26.412 | 0.78373467 |
| 349.56 | 142.51 | 145.95 | 2.35697157 | 143.54 | 145.95 | 1.65125043 | 26.255 | 26.412 | 0.59442678 |
| 355.82 | 144.22 | 145.95 | 1.18533744 | 145.34 | 145.95 | 0.41795135 | 26.324 | 26.412 | 0.33318189 |

Table C.50 Comparison of τ_{xy} along the mid cross-section of the tube for layer 4, 5, 6 and stacking sequence is $[\pm 45/0]_{2T}$ between FEM and CLPT with $\frac{b}{a} = 0.2$

| ANGLE (deg) | SXYLAYER 4 | | | SXYLAYER 5 | | | SXYLAYER 6 | | |
|-------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|--------------------|---------------------------|----------------|
| | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) | FEM Result (lb-in) | Analytical Result (lb-in) | Miss Match (%) |
| 0 | 26.046 | 26.412 | 1.38573376 | 142.5 | 145.95 | 2.36382323 | 143.38 | 145.95 | 1.76087701 |
| 4.18 | 26.323 | 26.412 | 0.33696804 | 146.03 | 145.95 | -0.0548133 | 147.02 | 145.95 | -0.7331278 |
| 10.44 | 26.254 | 26.412 | 0.59821293 | 148.06 | 145.95 | -1.4457006 | 149.02 | 145.95 | -2.1034601 |
| 23.41 | 26.205 | 26.412 | 0.78373467 | 150.06 | 145.95 | -2.8160329 | 151.02 | 145.95 | -3.4737924 |
| 28.42 | 26.199 | 26.412 | 0.80645161 | 150.36 | 145.95 | -3.0215827 | 151.32 | 145.95 | -3.6793422 |
| 51.62 | 26.435 | 26.412 | -0.0870816 | 146.33 | 145.95 | -0.2603631 | 147.43 | 145.95 | -1.0140459 |
| 90 | 26.436 | 26.412 | -0.0908678 | 146.38 | 145.95 | -0.2946214 | 147.48 | 145.95 | -1.0483042 |
| 128.38 | 26.435 | 26.412 | -0.0870816 | 146.33 | 145.95 | -0.2603631 | 147.43 | 145.95 | -1.0140459 |
| 151.58 | 26.199 | 26.412 | 0.80645161 | 150.36 | 145.95 | -3.0215827 | 151.32 | 145.95 | -3.6793422 |
| 156.59 | 26.205 | 26.412 | 0.78373467 | 150.06 | 145.95 | -2.8160329 | 151.02 | 145.95 | -3.4737924 |
| 169.56 | 26.255 | 26.412 | 0.59442678 | 148.06 | 145.95 | -1.4457006 | 149.02 | 145.95 | -2.1034601 |
| 177.59 | 26.324 | 26.412 | 0.33318189 | 146.03 | 145.95 | -0.0548133 | 147.02 | 145.95 | -0.7331278 |
| 180 | 26.046 | 26.412 | 1.38573376 | 142.5 | 145.95 | 2.36382323 | 143.38 | 145.95 | 1.76087701 |
| 184.18 | 26.323 | 26.412 | 0.33696804 | 146.03 | 145.95 | -0.0548133 | 147.02 | 145.95 | -0.7331278 |
| 190.44 | 26.254 | 26.412 | 0.59821293 | 148.06 | 145.95 | -1.4457006 | 149.02 | 145.95 | -2.1034601 |
| 203.41 | 26.205 | 26.412 | 0.78373467 | 150.06 | 145.95 | -2.8160329 | 151.02 | 145.95 | -3.4737924 |
| 208.42 | 26.199 | 26.412 | 0.80645161 | 150.36 | 145.95 | -3.0215827 | 151.32 | 145.95 | -3.6793422 |

Table C.50 – *Continued*

| | | | | | | | | | |
|--------|--------|--------|------------|--------|--------|------------|--------|--------|------------|
| 231.62 | 26.435 | 26.412 | -0.0870816 | 146.33 | 145.95 | -0.2603631 | 147.43 | 145.95 | -1.0140459 |
| 270 | 26.436 | 26.412 | -0.0908678 | 146.38 | 145.95 | -0.2946214 | 147.48 | 145.95 | -1.0483042 |
| 308.38 | 26.435 | 26.412 | -0.0870816 | 146.33 | 145.95 | -0.2603631 | 147.43 | 145.95 | -1.0140459 |
| 331.58 | 26.199 | 26.412 | 0.80645161 | 150.36 | 145.95 | -3.0215827 | 151.32 | 145.95 | -3.6793422 |
| 336.59 | 26.205 | 26.412 | 0.78373467 | 150.06 | 145.95 | -2.8160329 | 151.02 | 145.95 | -3.4737924 |
| 349.56 | 26.255 | 26.412 | 0.59442678 | 148.06 | 145.95 | -1.4457006 | 149.02 | 145.95 | -2.1034601 |
| 355.82 | 26.324 | 26.412 | 0.33318189 | 146.03 | 145.95 | -0.0548133 | 147.02 | 145.95 | -0.7331278 |

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BIOGRAPHICAL INFORMATION

Thana Chomtid was born in Bangkok, Thailand, in March 1984. He was study at Phadungsit Phittaya School for kindergarten. He moved to Saint John when he was grade 1 and smoved again to Samsem Vitayalai Scholl at 7grade. He finished grade 12 from Samsem Witayalai Scholl in Bangkok, Thailand. He pursued and received his Bachelor Degree in Aerospace Engineering from King Mongkut's University of Technology North Bangkok, Bangkok, Thailand, in March 2007. In 2008, he continued his Master Degree in Aerospace Engineering at The University of Texas at Arlington, Texas, USA.