

VEHICLE SUSPENSION OPTIMIZATION FOR
STOCHASTIC INPUTS

by

KAILAS VIJAY INAMDAR

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ABSTRACT

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Kailas V. Inamdar, M.S.

The University of Texas at Arlington, 2011

Supervising Professor: David A. Hullender

In the present thesis, a simulation based numerical method has been proposed for optimization of vehicle suspension system for stochastic inputs from random road surface profiles. Road surfaces are classified in based upon the power spectral density functions. The road surface is considered as a stationary stochastic process in time domain assuming constant vehicle speed. Using Fourier transforms, it is possible to generate the road surface elevations as a function of time.

Time domain responses of the output of the suspension system are obtained using transfer function techniques. Optimum values of the damper constant are computed by simulation of the Quarter Car Model for generated stochastic inputs for good road holding and passenger ride comfort. A performance index minimization procedure is developed to find optimum damper constant value considering mutually conflicting requirements of ride comfort and road holding. The handling of the vehicle, cornering force, tractive force depends upon the road holding. The road holding capacity of the vehicle changes with change in vehicle speed as well as road roughness. A quantitative measure for deciding the road holding of the vehicle is defined.

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CHAPTER 1

INTRODUCTION

1. Vehicle Suspension Modeling

Modeling and simulation of the vehicle suspension system is required for predicting the performance and assuring the proper functioning of a system before spending time and money on producing it. A quantitative mathematical model used for the simulation, can accurately predict the performance of the system. If the input to the system is deterministic and can be computed by an explicit mathematical formula then, using the transfer function of the system, the response of the system can be determined.

A vehicle suspension system is a complex vibration system having multiple degrees of freedom [2]. The purpose of the suspension system is to isolate the vehicle body from the road inputs. Various aspects of the dynamics associated with the vehicle put different requirements on the components of the suspension system. Passenger ride comfort requires that the acceleration of the sprung mass be relatively smaller whereas the lateral dynamic performance requires good road holding which needs consistent normal force between the road and the tires. This all has to work within the maximum allowed deflection of the suspension spring and limitations of the dynamic tire deflection [7].

For analyzing the vibration characteristics of the vehicle, equations of motion have to be formulated. Various models from a single degree of freedom model to a complex model having multiple degrees of freedom have been developed for studying the suspension system performance. However, the system is simplified by considering some dominating modes and modal approximations. For instance, a 'Quarter Car Model' has been extensively used to study the dynamic behavior of the vehicle suspension system [4]. This is basically a linear lumped mass parameter model with two degrees of freedom. This model is used to obtain a qualitative

insight into the performance of the suspension, in particular the effects of sprung mass and unsprung mass, stiffness of the suspension spring and tires, damping of the shock absorbers and tires on the vehicle vibration.

The inputs to the road surface are rough road irregularities. These irregularities range from potholes to random variations in the surface elevations along the length of the road. These act as a major source of excitation for the vehicle suspension system. When a vehicle traverses over such a road surface with a certain velocity, the irregularities on the road surface become an input with certain frequencies for the suspension system. Due to the mass imbalances and differences in the stiffness of the suspension and wheel assembly, these excitations from the ground are transmitted to the vehicle body. If the level of vibrations induced in the vehicle exceeds a certain threshold then it makes the ride uncomfortable for the passenger. Secondly, the suspension components are subjected to fluctuating loads due to these vibrations which cause fatigue in the springs and other components of the suspension. Another significant effect that the road excitations have is on the handling of the vehicle. If the amplitude of tire oscillations exceeds a maximum limit then, the vehicle loses its road holding capacity and adversely affects the handling of the vehicle.

All these effects arising from the random road irregularities can be formulated and analyzed using a simulation based approach for design and optimization of the vehicle suspension system [8]. The computer simulation consists of three stages. The first stage is generation of a random input with stochastic properties and characteristics of road profiles that the suspension system would encounter in practice. This input is used to excite the system. The second stage deals with the numerical solution of the differential equations representing the dynamics of the system. The numerical solution obtained by solving the differential equations, by numerical integration method, represents the output of the system. The third stage of the computer simulation consists of processing the output data [3].

1.2 Mathematical Modeling of Road Surface

The road surface is conventionally modeled in the form of step input [7], sine waves or triangular waves with amplitude depending upon the surface elevations and the frequency depending upon the wavelengths of the surface irregularities. However, in practice the road surfaces are of irregular forms and the actual ride behavior of the vehicle cannot be studied considering these forms of input. Therefore, the simplified form of inputs can be used only for the comparative evaluation of different suspension designs [2].

To study the actual ride behavior of the vehicle and performance of the suspension system, the road surface should be modeled as a stochastic process. In practice, the road surface elevations are random in nature. The instantaneous value of the surface elevation at any point above some reference plane cannot be computed by an explicit mathematical relationship or as a function of the distance between the point of elevation and some fixed point of reference. As a consequence of this, when a vehicle is running over the road surface, the excitations imposed to the vehicle forms a set of random data. Therefore the road surface should be modeled as a stochastic process. A random process is the one in which there is no way to predict the exact value at a future instant of time [1]. However, if sufficient knowledge of the basic mechanisms producing the random data is available then, it is possible to describe process with exact mathematical relationships. In the following chapters we will see that by evaluating some fix statistical properties and average values, it is possible to define a stochastic process in deterministic manner.

1.3 Outline of Thesis

Chapter 2 discusses the frequency response characteristics of the quarter car model. These characteristics depend upon the fixed parameters of the system such as vehicle mass, suspension spring stiffness, shock absorber damper constant, mass and stiffness of the tires etc. While the vehicle is moving on a road surface, the input excitations consists of large

number of frequencies. So it is necessary to study the frequency response characteristics of the suspension system.

In chapter 3, procedure for generating random time series with a specified spectral density function is explained. This used for creating road surface profiles.

Chapter 4 gives procedures for finding an optimum value of shock absorber damper constant considering road holding and passenger ride comfort. A quantitative measure for deciding the road holding capacity of the vehicle at different speeds and for different types of the road surfaces is also explained in this chapter.

Chapter 5 contains results and conclusion of the simulations.

CHAPTER 2

FREQUENCY RESPONSE CHARACTERISTICS OF QUARTER CAR MODEL

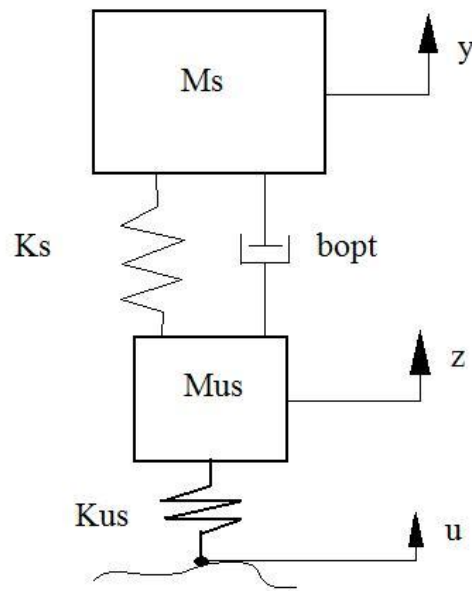


Fig 2.1. Two - degree of freedom Quarter Car Model for passenger car
($M_s = 1814$ kg, $K_{us} = 704$ kN / m)

The 2-Degree-of-Freedom model shown in figure 2.1 includes an unsprung mass representing the wheels and associated components and a sprung mass representing the vehicle body [2]. At a particular point of time, let y and z are the vertical displacements of the sprung and unsprung mass respectively due to the excitation from the rough road surface. The displacements y and z are measured from the static equilibrium positions so that we can neglect the gravity term while writing the equation of motion for the masses. This model is referred as a

'quarter car model'. Usually the vehicle mass and the tire stiffness are the fixed parameters for a given model and other parameters are decided base on the frequency response characteristics. In this chapter these characteristics are studied for different mass ratios, stiffness ratios and damper constants to select values of unsprung mass and suspension spring rate.

By Newton's second law of motion the equations of motion of the system are as follows:-

For the sprung mass,

$$M_s \cdot \ddot{y} + b_{opt} \cdot (\dot{y} - \dot{z}) + K_s \cdot (y - z) = 0 \quad (2.1)$$

For unsprung mass,

$$M_{us} \cdot \ddot{z} + b_{opt} \cdot (\dot{z} - \dot{y}) + K_s \cdot (z - y) + K_{us} \cdot (z - u) = 0 \quad (2.2)$$

To determine natural frequency of the system:-

Considering undamped frequency of the system,

$$M_s \cdot \ddot{y} + K_s \cdot (y - z) = 0 \quad (2.3)$$

$$M_{us} \cdot \ddot{z} + K_s \cdot (z - y) + K_{us} \cdot (z) = 0 \quad (2.4)$$

Solutions can be assumed in the form,

$$y = Y \cdot \cos\{\omega_n \cdot t\}$$

$$z = Z \cdot \cos\{\omega_n \cdot t\}$$

Substituting in above equations,

$$(-M_s \omega_n^2 + K_s) y - K_s \cdot z = 0 \quad (2.5)$$

$$-K_s y + (-M_{us} \omega_n^2 + K_s + K_{us}) z = 0 \quad (2.6)$$

ω_n can be found by solving following determinant:-

$$\begin{vmatrix} (-M_s \omega_n^2 + K_s) & -K_s \\ -K_s & -M_{us} \omega_n^2 + K_s + K_{us} \end{vmatrix} = 0$$

$$\omega_n^4 (M_s M_{us}) + \omega_n^2 (-M_s K_s - M_s K_{us} - M_{us} K_s) + K_s K_{us} = 0 \quad (2.7)$$

$$\therefore \omega_{n1}^2 = \frac{B_1 - \sqrt{B_1^2 - 4 A_1 C_1}}{2 A_1} \quad (2.8)$$

And

$$\omega_{n2}^2 = \frac{B_1 + \sqrt{B_1^2 - 4 A_1 C_1}}{2 A_1} \quad (2.9)$$

Where,

$$A_1 = M_s M_{us}$$

$$B_1 = M_s K_s + M_s K_{us} + M_{us} K_s$$

$$C_1 = K_s K_{us}$$

Substituting the values for the quarter car model shown in figure 3.1, we get,

$$\omega_{n1} = 6.5626 \text{ rad/sec} \quad \text{i.e.} \quad f_{n1} = 1.0445 \text{ Hz}$$

$$\omega_{n1} = 66.19 \text{ rad/sec} \quad \text{i.e.} \quad f_{n1} = 10.53 \text{ Hz}$$

For the passenger car, the mass of the vehicle body is much higher than the mass of the wheel ($M_s/M_{us} = 10.02$) while the stiffness of the suspension spring K_s is much less than that of the wheel ($K_s/K_{us} = 0.125$). Considering this the above two natural frequencies of the sprung and unsprung masses can be determined by an approximate method and are expressed as follows;

Undamped natural frequency of the sprung mass,

$$f_{n1} = \frac{1}{2\pi} \sqrt{\frac{(k_s k_{us}) / (k_s + k_{us})}{m_s}} \quad (2.10)$$

$$f_{n2} = \frac{1}{2\pi} \sqrt{\frac{k_s + k_{us}}{m_{us}}} \quad (2.11)$$

Using these approximate formulae, the same two natural frequencies are computed as,

$$f_{n1} = 1.045 \text{ Hz}$$

$$f_{n2} = 10.527 \text{ Hz}$$

Hence these values are practically identical with the actual undamped natural frequencies of the vehicle and wheels.

The transfer function is computed using Matlab[®] as follows;

$$HY_U = \frac{(5.336 \times 10^9) s + (6.195 \times 10^{10})}{(328333)s^4 + 1995(b)s^3 + (1.4526 \times 10^9)s^2 + 704000(b)s + (6.1925 \times 10^{10})}$$

From the eigenvalues of the characteristic equation, natural frequencies can be found

```
>> damp(HY_U)
```

Eigenvalue	Damping	Freq. (rad/s)
-1.69e+000 + 6.46e+000i	2.54e-001	6.67e+000
-1.69e+000 - 6.46e+000i	2.54e-001	6.67e+000
-2.13e+001 + 6.15e+001i	3.28e-001	6.51e+001
-2.13e+001 - 6.15e+001i	3.28e-001	6.51e+001

The eigenvalues are $(-1.69 \pm j 6.46)$ and $(-21.3 \pm j 61.5)$

Natural frequencies of sprung and unsprung masses are

$$f_{n1} = 1.028 \text{ Hz}$$

$$f_{n2} = 10.36 \text{ Hz}$$

The frequency values calculated by equations (2.10) and (2.11) closely match with the values computed using the transfer function using Matlab[®].

Secondly, the natural frequency of the unsprung mass is higher than that of the sprung mass. For the passenger cars, the damping ratio (ζ) provided by the shock absorbers is usually in the range of (0.2 – 0.4) and the damping ratio of the tires is comparatively less (~ 0.03). As a consequence of this, the difference between the undamped and damped natural frequencies of the masses is negligible and the undamped natural frequencies are commonly used to

characterize the system. The ratio of the natural frequencies of the sprung and unsprung masses plays an important role in deciding the vibration isolation characteristics of the vehicle suspension. For example, consider a situation where the running vehicle hits a bump on the road. The bump on the road can be considered as an impulse input to the suspension system. As the vehicle crosses the bump, the wheel oscillates freely at its natural frequency which in this case is 10.53 Hz. For the sprung mass the natural frequency is 1.045 Hz and the excitation is the vibrations of the unsprung mass. Therefore, the ratio of frequency of excitation to the natural frequency of sprung mass is approximately 10. From the frequency response characteristics, when the ratio of the excitation frequency to the natural frequency is high, the gain of the transfer function is low. So, the amplitude of vibration of the vehicle body would be very low and good vibration isolation can be achieved.

Random road surfaces consist of a wide range of wavelengths. When the vehicle rides over such a road surface at particular speed, the excitation to the vehicle consists of wide range of frequencies. From the transmissibility characteristics of the vehicle, it can be observed that the excitations due to shorter wavelengths of the road (i.e. high frequency inputs) can be isolated effectively since the natural frequency of the sprung mass is low. However, excitations from the larger wavelengths (i.e. the low frequency inputs) can be transmitted to the vehicle body unimpeded or even amplified since the gain of the transfer function is high when the frequency of excitation is close to the natural frequency of the vehicle body.

Evaluation of the overall performance of the suspension system is carried out by considering three main aspects as follows:

1. Road holding
2. Vibration isolation
3. Suspension travel

2.1 Road Holding

The normal force between the road and the tire can be represented by relative displacement between unsprung mass and the road surface elevations, which is also called as the dynamic tire deflection. The dynamic tire deflection represents the normal force acting between the tire and road surface consider the damping of the tires negligible.

The ratio of the relative displacement between the unsprung mass and the road surface ($z - u$) to the amplitude of the rough road surface is defined as the dynamic tire deflection ratio. The cornering force, tractive effort and braking effort developed by the tire are related to the normal force acting between the tire and the road surface. When the vehicle system vibrates, this normal force fluctuates and as a consequence, the road holding capacity of the vehicle is affected causing unfavorable effects on handling and performance of the vehicle. Therefore, it becomes necessary to study the effects of different parameters on the dynamic tire deflection ratio.

2.1.1 Effect of ratio of unsprung mass to sprung mass on road holding:

Form figure 2.2, it can be observed that, below the natural frequency of the sprung mass, the ratio of the masses has very little effect on the dynamic tire deflection, in turn on the road holding. As the frequency increases from 1.045 Hz, initially the dynamic tire deflection increases with the lighter unsprung mass but then decreases in the mid-frequency range.

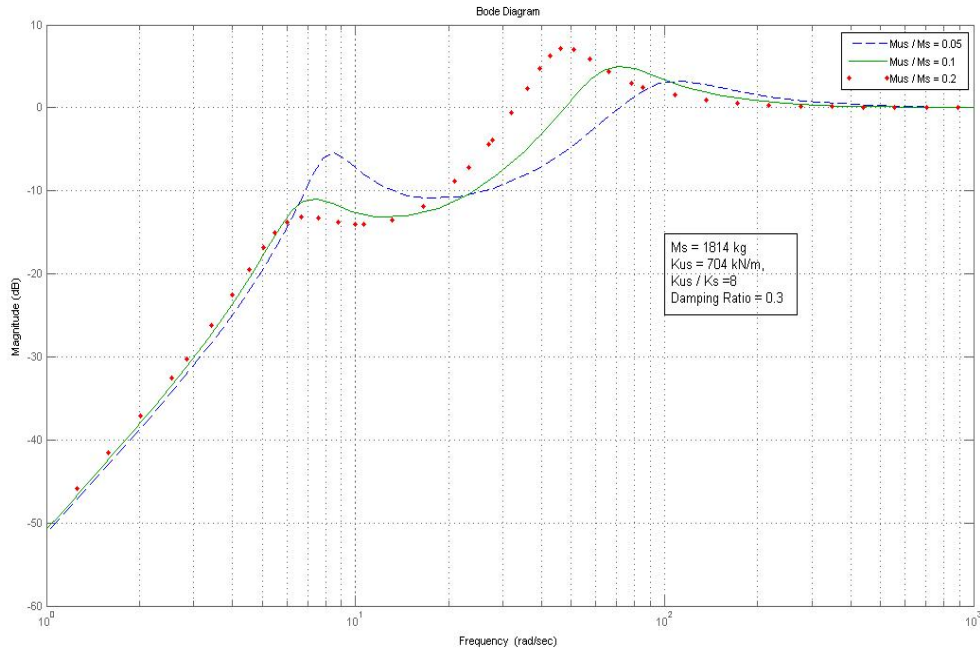


Figure 2.2 Frequency response of dynamic tire deflection for different mass ratios of a quarter car model

Above natural frequency of unsprung mass, the unsprung mass has insignificant effect on the roadholding. Consider that the vehicle is moving with speed ' V ', on a road surface having wavelengths ' l_ω '. So, the excitation frequency for the suspension system would be $f = V/l_\omega$ Hz. If this frequency f , matches the frequency at which the positive value of dynamic tire deflection becomes equal to the static deflection of the tire due to the vehicle weight then, the tire is on the verge of bouncing off the ground. During this part of vibrations, the vehicle will not have any contact with ground. Such situation is very undesirable since it reduces the handling performance.

2.1.2 Effect of ratio of spring stiffness to tire stiffness on road holding:

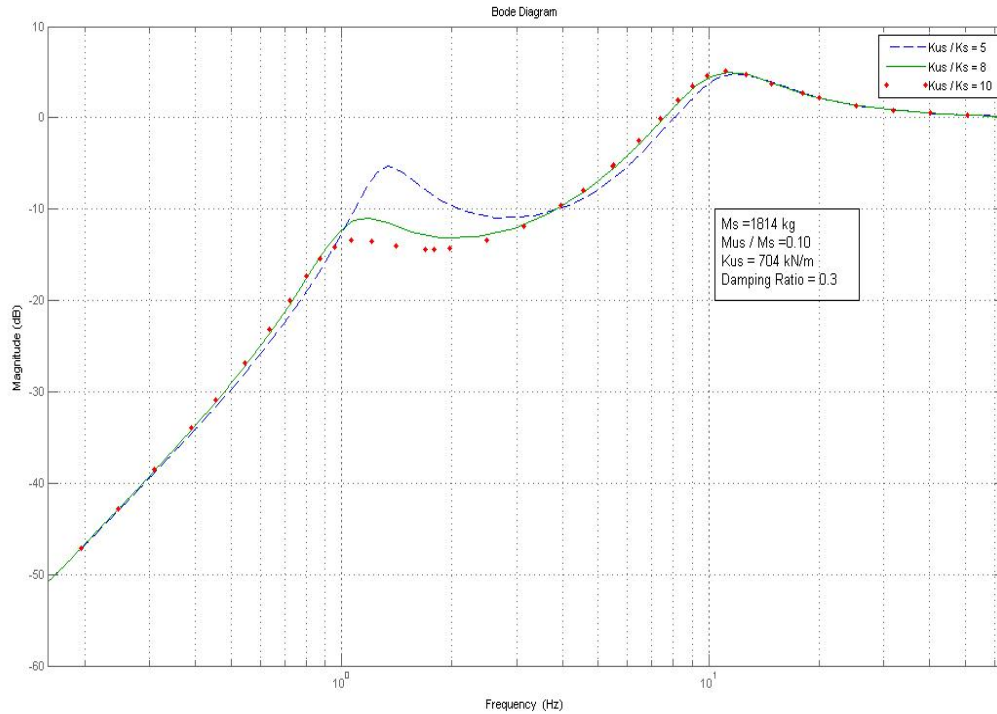


Figure 2.3 Frequency response of dynamic tire deflection ratio for different spring stiffness to tire stiffness ratios of a quarter car model

Figure 2.3 shows that, in the low frequency range, below the natural frequency of the sprung mass and high frequency range, above the natural frequency of the unsprung mass, the effect of the suspension spring stiffness on the dynamic tire deflection ratio is insignificant. Between the natural frequency of sprung mass and the crossover frequency, the dynamic tire deflection ratio is lower with the softer suspension spring. Between the crossover frequency and the natural frequency of the unsprung mass, a stiffer suspension spring provides lower dynamic tire deflection ratio.

2.1.3 Effect of the damping ratio on road holding:

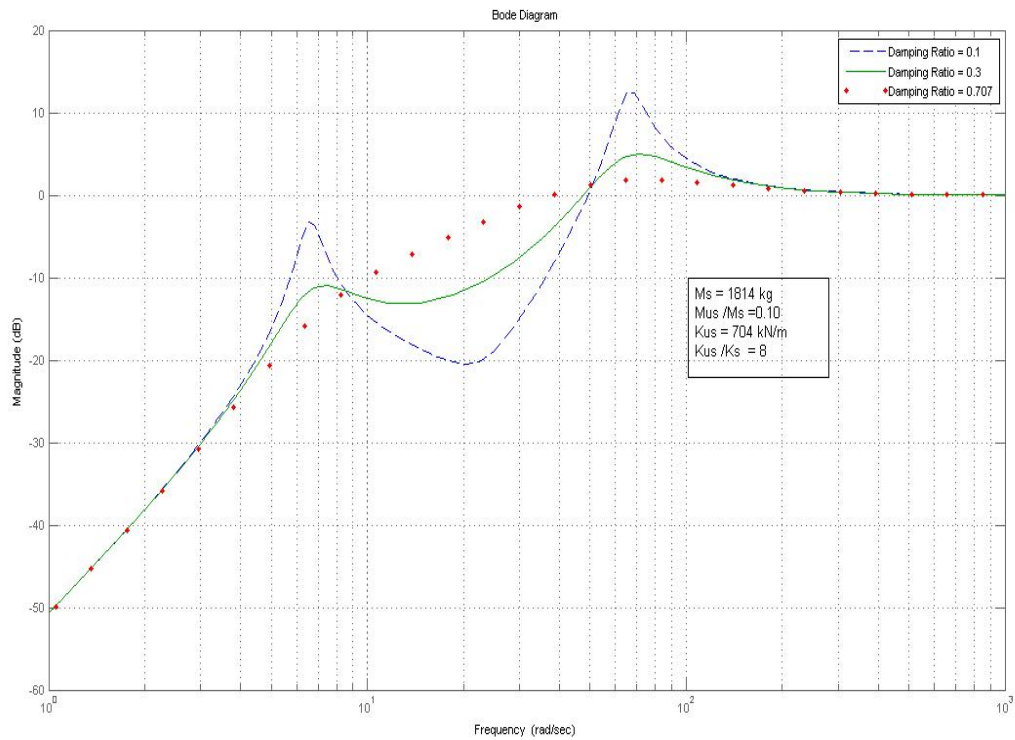


Figure 2.4 Frequency response function of dynamic tire deflection ratio for different spring stiffness to tire stiffness ratios of a quarter car model

From figure 2.4 below the natural frequency of the sprung mass and in the frequency range near the natural frequency of the unsprung mass, the dynamic tire deflection ratio is lower if the damping ratio is higher. However, in the mid-frequency range, lower the damping ratio, lower will be the dynamic tire deflection.

2.2 Vibration Isolation

Vibrations in the vehicle body are usually considered as the vertical displacement of the vehicle body due to elevations on the rough road. The vibration isolation characteristics can be studied using the transfer function between the vehicle displacement and the road input.

2.2.1 Effect of ratio of unsprung mass to sprung mass on vibration isolation:

Fig 2.5 shows the effect of ratio of unsprung mass to sprung mass on the transfer function for the quarter-car model. The mass of the wheel and unsprung parts has negligible effect on the vibration of the vehicle body in the frequency range below 1.045 Hz (natural frequency of the sprung mass). If the frequency of excitation is nearer to 10.53 Hz (natural frequency of the unsprung mass) the gain of the transfer function decreases with the decreasing unsprung mass. It means that, the vibration of the vehicle body is lower with the lighter wheel mass for the same level of excitation in the frequency range between 1.045 Hz to 10.52 Hz. As the frequency becomes more than 10.52 Hz, a lighter mass leads to slightly higher transmissibility.

From the above discussion, it can be concluded that the ratio of the masses has little influence on the vibration of the vehicle body in the low frequency range.

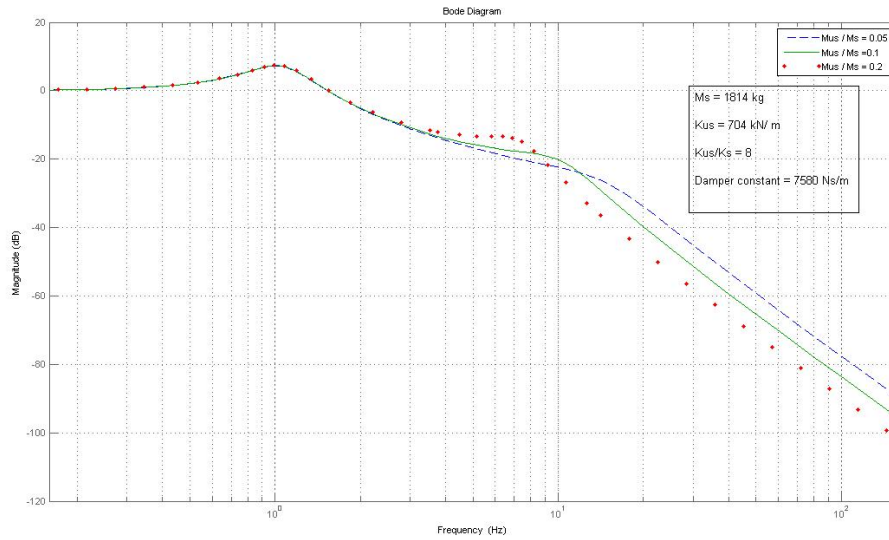


Figure 2.5 Frequency response for different ratios of unsprung mass to sprung mass of a quarter car model

In the mid-frequency range, a lighter wheel assembly will provide better vibration isolation and in the frequency range above the natural frequency of the unsprung mass, there is a slight increase in transmissibility with the lighter wheel assembly.

2.2.2. Effect of ratio of suspension spring stiffness to tire stiffness on vibration isolation:

Figure 2.6 shows the effect of the equivalent tire stiffness K_{us} on the suspension spring stiffness K_s on the transfer function for the vehicle vibrations. For a given tire stiffness, higher value of K_{us}/K_s represents lower suspension spring stiffness. From the frequency response function characteristics, it can be seen that, in the frequency range below 1.045 Hz, the gain of the transfer function decreases with lower ratio of K_{us}/K_s in the frequency range between 1.045 Hz to 10.52 Hz, a higher ratio of K_{us}/K_s provides better vibration isolation. The effect of this ratio on vibration isolation becomes insignificant in the high frequency range.

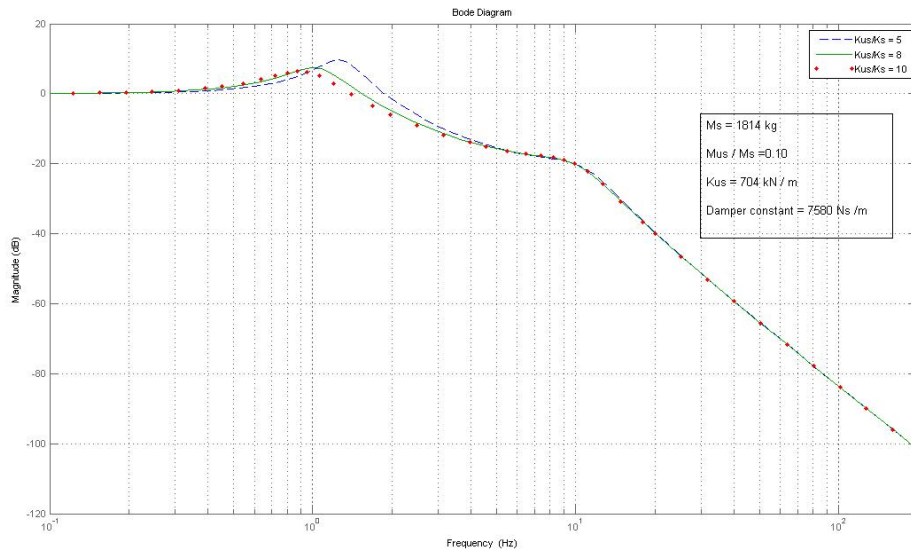


Figure 2.6 Frequency response for different ratios of tire stiffness to suspension spring stiffness of a quarter car model

above 10.52 Hz .

From the discussion above, it can be concluded that a softer suspension spring provides better vibration isolation in mid- to high frequency range but with this there is some penalty in terms of transmissibility in lower frequency range, below the natural frequency of the sprung mass.

2.2.3 Effect of damping ratio of suspension shock absorber on vibration isolation:

Figure 2.7 shows the effect of damping ratio (ζ) on the transfer function of the vehicle vibration. In the frequency range close to 1.045 Hz , the gain of the transfer function decreases with increase in the damping ratio. Whereas, in the frequency range between 1.045 Hz to 10.52 Hz the TF gain will be lower if the damping ratio is lower. Again near around 10.52 Hz the damping ration has not much effect on the vibration isolation. Above 10.52 Hz, lower the damping ratio, lower is the transmissibility.

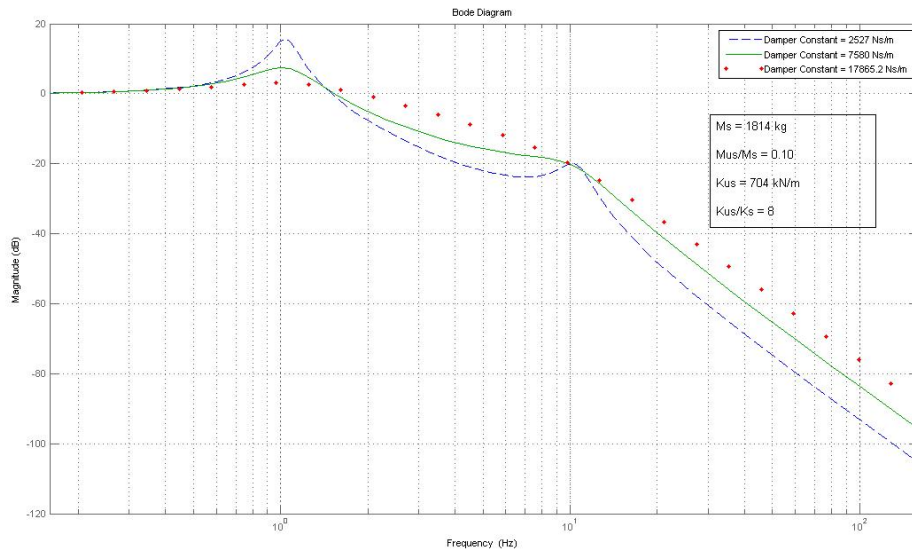


Figure 2.7 Frequency response for different damping ratios of shock absorber of a quarter car model

From this it can be concluded that in the frequency range close to the natural frequency of the vehicle body, a high damping ratio is required. However, lower damping ratio is required to provide better vibration isolation in the mid- to high frequency range.

2.3 Suspension Travel

The relative displacement between sprung and unsprung mass is defined as the 'suspension travel'. The space required to accommodate the spring between road bumps and rebound stops is termed as 'rattle space'.

2.3.1 Effect of ratio of unsprung mass to sprung mass on suspension travel:

'Suspension travel ratio' is defined as the ratio of maximum relative displacement between sprung and unsprung mass ($y-z$) to the amplitude of the road profile. In the frequency range below 1.045 Hz the ratio M_{us}/M_s has little effect on the transfer function. Between 1.045 Hz to 10.52 Hz the gain of the transfer function increases with the increase in the mass

ratio. Whereas, above 10.52 Hz, the suspension travel decreases with the increasing mass ratio of unsprung and sprung mass.

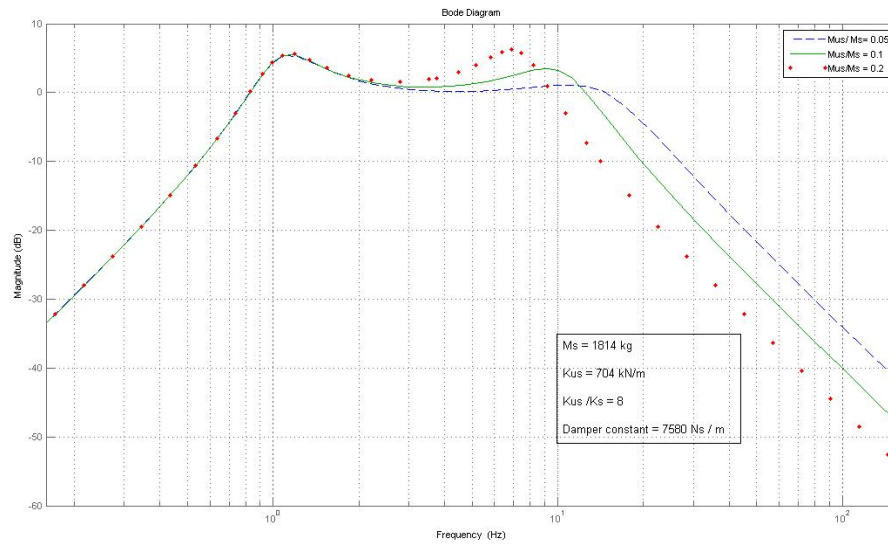


Figure 2.8. Frequency response of suspension travel of a quarter car model for different ratios of unsprung mass to sprung mass

It can be concluded from the figure 2.8 that, the lower weight of the wheel assembly decreases suspension travel in the mid-frequency range but increases the suspension travel in the frequency range above the natural frequency of unsprung mass. However it does not have significant effect on the suspension travel in the low frequency range.

2.3.2 Effect of ratio of suspension spring stiffness to tire stiffness on suspension travel:

From the figure 2.9, in the frequency range below 1.045 Hz, higher ratio of the tire stiffness to the suspension spring stiffness leads to a high value of suspension travel.

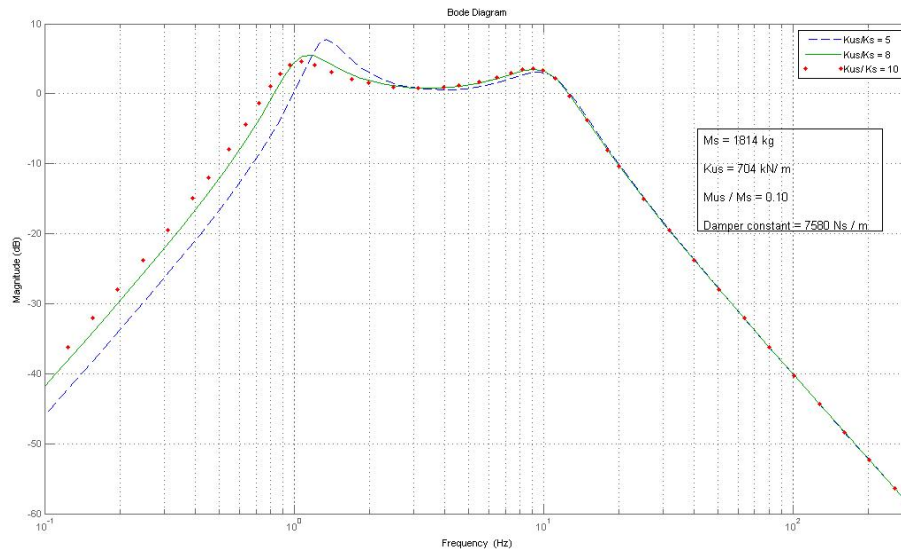


Figure 2.9 Frequency response of suspension travel for different ratios of suspension spring stiffness to tire stiffness of a quarter car model

In the frequency range between 1.045 Hz to 10.52 Hz , initially the suspension travel decreases with increasing ratio of K_{us} / K_s but then decreases with frequency approaching 10.52 Hz. The frequency at which this change takes place is called as the “crossover frequency” [2] and for this suspension system it is 3 Hz.

Form this, it can be concluded that, in the low frequency range, the suspension travel is larger if the suspension spring is softer. In the mid-frequency range, from the natural frequency of the sprung mass to the crossover frequency, the softer spring leads to lower value of suspension travel and between crossover frequency and natural frequency of unsprung mass the suspension travel goes higher if the spring has lower stiffness. However the ratio K_{us} / K_s has negligible effect on the suspension travel in the high frequency range.

2.3.3 Effect of damping ratio of the shock absorber on suspension travel:

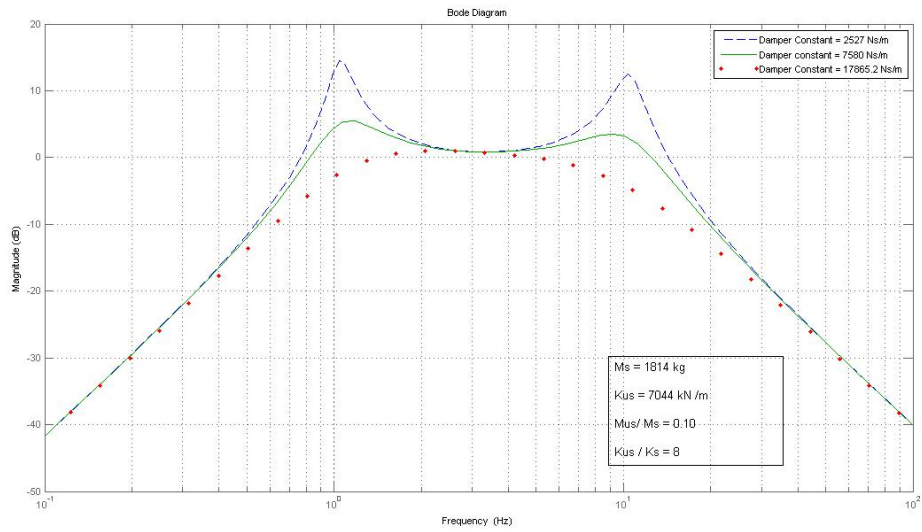


Figure 2.9 Frequency response for different damping ratios of suspension travel of a quarter car model

From the Figure 2.9, it can be observed that over the entire frequency range, higher damping ratio (ζ) provides lower suspension travel. So, to reduce the suspension travel higher damping ratio is required.

CHAPTER 3

GENERATING RANDOM ROAD INPUTS

As seen previously, road surfaces are practically defined by random functions. Further, if the statistical properties of the road surface derived from one portion of the road can be used to define the properties of the entire section of the road then the road surface can be assumed to be a stationary stochastic process provided that the velocity of the vehicle is constant [2]. If the velocity is changing with time then the road surface becomes a non-stationary stochastic process [9].

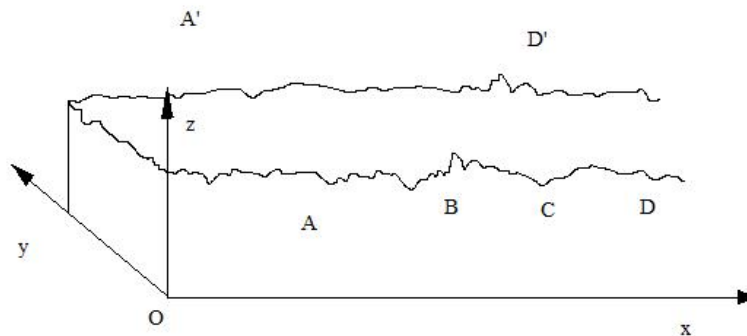


Figure 3.1 Road surface elevation as a random function

Consider figure 3.1. If the statistical properties of the road profile on one plane such as AD are same as those on any other plane, such as A'D' then the road surface is considered as an ergodic process.

A stationary stochastic process is said to be ergodic if its mean value μ_x , and autocorrelation function $R_x(\tau)$ can be obtained by time averaging a single time record of the process instead of averaging an ensemble of time functions. Consider the k^{th} sample function

of the stochastic process $x(t)$. The mean value $\mu_x(k)$ and the autocorrelation function $R_x(\tau, k)$ of the k^{th} sample function are given by,

$$\mu_x(k) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_k(t) \cdot dt \quad (3.1a)$$

$$R_x(\tau, k) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_k(t) \cdot x_k(t + \tau) \cdot dt \quad (3.1b)$$

If for a stationary stochastic process, $\mu_x(k)$ and $R_x(\tau, k)$ defined in equation (3.1) do not differ when computed over different sample functions, the stochastic process is said to be 'ergodic'.

This assumption of the road surface being stationary ergodic process helps to simplify mathematical modeling of the road surfaces. When a surface profile is considered as a random function then, it can be characterized by power spectral density functions. The power spectral density of random road surfaces is determined using digital spectral density analyzers. These analyzers work on the principle of filtering-squaring-averaging technique. The input signal from the road surface is passed through a highly selective narrow band pass filter with a specific center frequency. The instantaneous value of the filtered signal is squared and an average of this squared instantaneous value is obtained as the mean square value. The mean square value is divided by the bandwidth to get the average power spectral density at the specific center frequency as per equation (3.4). By varying the center frequency of the narrow band pass filter, power spectral densities at a series of center frequencies can be obtained and a graph of power spectral density versus frequency can be plotted.

Consider a harmonic component $u_n(x)$ with amplitude U_n and wavelength $l_{\omega n}$.

$$u_n(x) = U_n \cdot \sin\left(\frac{2\pi x}{l_{\omega n}}\right)$$

$$u_n(x) = U_n \cdot \sin(\Omega_n \cdot x) \quad (3.2)$$

Where, Ω_n - circular spatial frequency of the harmonic component

$$\Omega_n = \frac{2\pi}{l_{\omega n}} \quad (3.3)$$

Circular spatial frequency is expressed in rad/m

Let, $G(n \Omega_0)$ be the power spectral density at frequency $n\Omega_0$ in the frequency interval $\Delta\Omega$.

$$\therefore G(n \Omega_0) \cdot \Delta\Omega = \frac{U_n^2}{2} = u_n^2 \quad (3.4)$$

Therefore, the discrete power spectral density becomes,

$$G(n \Omega_0) = \frac{U_n^2}{2 \cdot \Delta\Omega} = \frac{\overline{u_n^2}}{\Delta \Omega} \quad (3.5)$$

3.1 Power spectral density (PSD) in terms of autocorrelation function

PSD functions are defined for frequencies $(-\infty, \infty)$ and denoted by $G(f)$.

Mathematically the PSD is Fourier transform of the autocorrelation function.

$$\therefore G(f) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-j2\pi f\tau} \cdot d\tau \quad (3.6)$$

Which will exist if $R(\tau)$ exists and if,

$$\int_{-\infty}^{\infty} |R(\tau)| \cdot d\tau < \infty$$

The inverse Fourier transform of $S(f)$ gives,

$$R(\tau) = \int_{-\infty}^{\infty} G(f) \cdot e^{j2\pi f\tau} \cdot df \quad (3.7)$$

This relationship between the PSD and autocorrelation function is very important and used for generating random time series with specific PSD.

3.2 Road surface classification by ISO

Various organizations have characterized the road surface roughness over the years.

The International Organization for Standardization (ISO) has proposed road roughness classification based on the power spectral density, as shown in Figure 3.2 [2]

The relationship between the road surface PSD and spatial frequency can be approximated as,

$$G_u(\Omega) = C_{sp} \cdot \Omega^{-N} \quad (3.8)$$

Where, $G_u(\Omega)$ is the power spectral density of the elevation of road surface profile and C_{sp} and N are constants. Fitting the expression to the curves obtained by measured data produces the values of C_{sp} and N as given in table 3.1 [2].

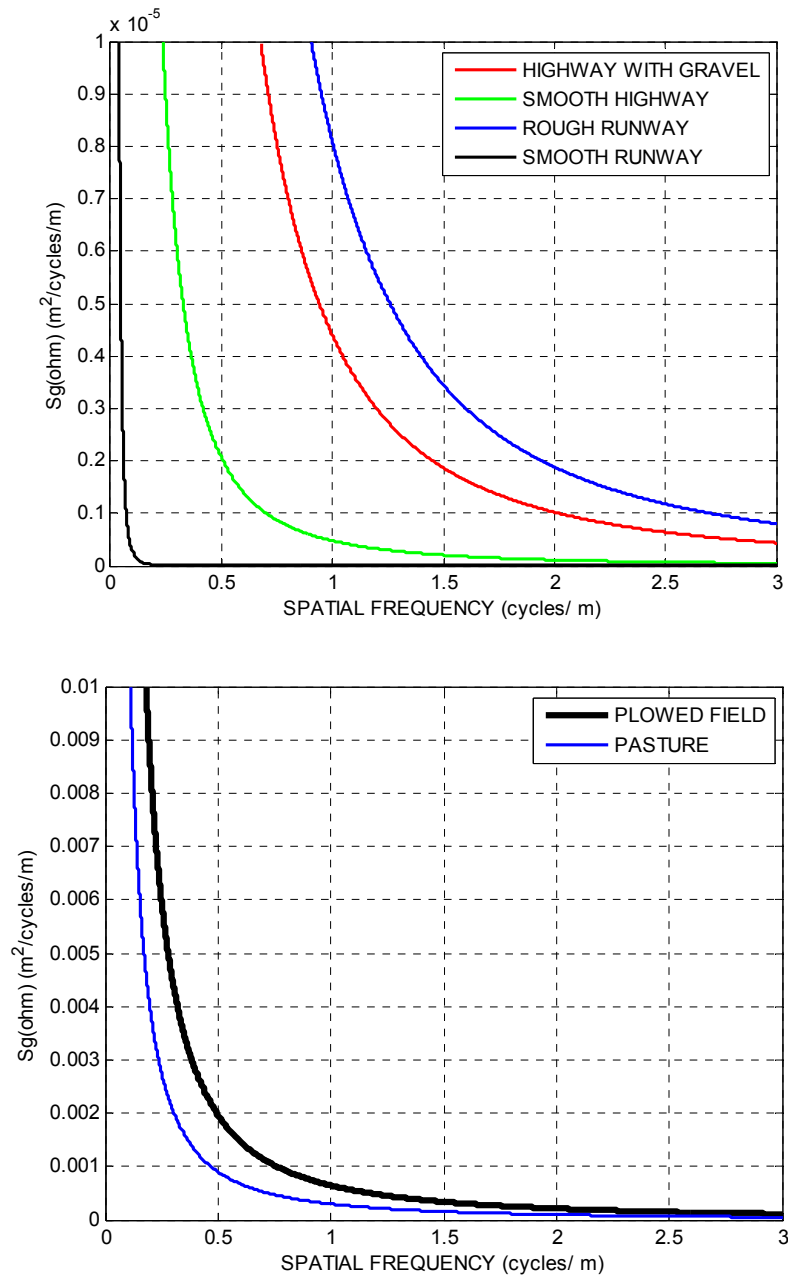


Figure 3.2 Road roughness classification by ISO

Table 3.1 Values for constants C_{sp} and N for different types of road surfaces

Sr. No.	Description	N	C_{sp}	C'_{sp}
1	Smooth Runway	3.8	4.3×10^{-11}	1.6×10^{-11}
2	Rough Runway	2.1	8.1×10^{-6}	2.3×10^{-5}
3	Smooth Highway	2.1	4.8×10^{-7}	1.2×10^{-6}
4	Highway with gravel	2.1	4.4×10^{-6}	1.1×10^{-5}
5	Pasture	1.6	3.0×10^{-4}	1.6×10^{-3}
6	Plowed Field	1.6	6.5×10^{-4}	3.4×10^{-3}

(C_{sp} is used for calculating road surface PSD in $m^2/$ cycles/ m whereas, C'_{sp} is used for calculating PSD in $ft^2/$ cycles/ ft.) [2]

Since we are considering that the vehicle is moving at a constant speed and the road surface being an ergodic process, it is more convenient to express the PSD of the road surface in terms of temporal frequency rather than in terms of spatial frequency since the vehicle vibration is a function of time.

The relationship between spatial frequency and temporal frequency is given as follows;

$$f = \Omega \times V \quad (3.9)$$

The transformation of the PSD of the road surface from space domain to the frequency domain considering the road surface as an ergodic process is given as,

$$G_u(f) = \frac{G_u(\Omega)}{V} \quad (3.10)$$

3.3 Generating random road profile using Fourier transforms

The procedure explained in the paper 'Generation of random time series with a specified spectral density function' by Dr. D. A. Hullender [3] is followed for creating rough road surfaces. By using the approach described in this paper, it is possible to generate a random sequence of numbers which already has the desired frequency characteristics. For using this method, the function need not be analytical and may be defined by simply a series of frequencies at each of the values. Random road surface should be generated such that it will have desired statistical properties. Otherwise, in evaluating the output characteristics of the vehicle suspension, it will be impossible to completely isolate and understand the performance characteristics of the system. In this section the procedure for this is explained. A Matlab[®] code is written for generating random road profile with specific PSD.

The first step of generating a random time series of road surface elevations, $u(t)$ with a specific power spectral density, is to generate the discrete Fourier transform $U(f_k)$ of $u(t)$ based on the desired spectral density function. By taking inverse discrete Fourier transform of $U(f_k)$, the random sequence $u(t)$ is obtained. This can be done by generating random phase angles for each of the Fourier terms of $U(f_k)$.

Equations for Fourier and inverse Fourier transform are given as below, respectively,

$$U(f) = \int_{-\infty}^{+\infty} u(t) \cdot e^{-j2\pi ft} \cdot dt \quad (3.11)$$

$$u(t) = \int_{-\infty}^{+\infty} U(f) \cdot e^{j2\pi ft} \cdot df \quad (3.12)$$

The respective discrete forms of these integrals are given by,

$$U(f_k) = h \cdot \sum_{n=0}^{N-1} u(nh) \cdot e^{-j2\pi f_k nh} \quad (3.13)$$

$$u(kh) = \Delta f \cdot \sum_{n=0}^{N-1} U(n \cdot \Delta f) \cdot e^{j2\pi n k/N} \quad (3.14)$$

Where, $f_k = \frac{k}{Nh}$, $\Delta f = \frac{1}{Nh}$ and $k = 0, 1, 2, \dots, (N - 1)$

The Fourier transform $U(f_k)$, is a complex number.

$$\therefore U(f_k) = R(f_k) + j \cdot I(f_k) \quad (3.15)$$

The relationship for the estimator for the PSD [2],

$$\hat{G}_x(f_k) = \frac{2}{Nh} |U(f_k)|^2 \quad (3.16)$$

Substituting equation (3.14) in equation (3.15) we get,

$$\hat{G}_x(f_k) = \frac{2}{Nh} \{R^2(f_k) + I^2(f_k)\} \quad (3.17)$$

Thus, the objective is to generate the real and imaginary parts for a Fourier transform which then substituted into the right side of the equation (3.16) gives the required spectral density function at each frequency f_k . Let,

$$R(f_k) = \left[\sqrt{\hat{G}_x(f_k) \frac{Nh}{2}} \cdot \cos \theta \right]$$

$$I(f_k) = \left[\pm j \cdot \sqrt{\hat{G}_x(f_k) \frac{Nh}{2}} \cdot \sin \theta \right]$$

It can be easily derived that substituting these values in equation(3.17), desired result is obtained independent of θ . By using the random values with uniform probability density between 0 to 2π , for angle θ , a random road surface elevations $u(kh)$ is obtained from inverse Fourier transform of $U(f_k)$.

3.3.1 Generating road surfaces using Matlab[®]:

The Matlab[®] m-file 'rough_road_input.m' is used for generating road surface using Matlab. The Matlab command to generate a random road surface having specific PSD from the Table 3.1, is '[U,t] = rough_road_input(N,H)'. 'N' is the number of points to be generated to make up the random series. Since, discrete Fourier transform is used, N should be a power of 2. Time interval between the points is H. So, the total duration time for the simulation

is (NH) sec and the time increment is (H) sec. Thus, the highest frequency generated will be $\frac{1}{2H}$ Hz. The frequency resolution will be $\frac{1}{NH}$ Hz.

3.3.2 Computing PSD using Matlab[®]:

For example, if a road profile for a smooth highway is of interest for studying behavior of the suspension system then first the PSD for a smooth highway is required to be calculated using values in Table 4.1. Let, G_u be the PSD of the smooth highway. From (3.7)

$$G_u = \frac{C_{sp}}{\Omega^N}$$

$$\therefore G_u = \frac{4.8 \times 10^{-7}}{\Omega^{2.1}}$$

The frequency response characteristics of the suspension systems are obtained in terms of temporal frequency. Therefore, it is convenient to compute the PSD of the road in terms of temporal frequency. By using relationships given by equations (3.8) and (3.9) the PSD becomes,

$$G_u = \frac{4.8 \times 10^{-7}}{V \cdot (f/V)^{2.1}}$$

This PSD is used for generating the road profile for smooth highway in 'rough_road_input.m' file.

3.3.3 Computing N and H for input to 'rough_road_input.m' file:

For computing the numbers N and H, the minimum and maximum frequencies of interest are required to be decided. This is done using the frequency response characteristics of the suspension as explained in chapter 2. The general procedure for deciding the time step increment (H) and the total number of time steps required (N) for the simulation of the suspension system for stochastic inputs is explained below with an example.

Consider that effect of random vibrations on the vibration isolation is of interest. The frequency response characteristics are as shown in figure 2.5 for all the parameters of the suspension system fixed except the damper constant. Let the damper constant be equal to 7580 Ns/ m ($\zeta = 0.3$ approx). The transfer function for this system is computed as,

$$HY_U = \frac{(5.336 \times 10^9) s + (6.195 \times 10^{10})}{(328333)s^4 + (5.041 \times 10^6)s^3 + (1.453 \times 10^9)s^2 + (1.779 \times 10^9)s + (6.195 \times 10^{10})}$$

Eigenvalues for this transfer function are obtained by using `damp(HY_U)` command in Matlab[®].

```
>> damp(HY_U)
```

Eigenvalue	Damping	Freq. (rad/s)
-1.69e+000 + 6.46e+000i	2.54e-001	6.67e+000
-1.69e+000 - 6.46e+000i	2.54e-001	6.67e+000
-2.13e+001 + 6.15e+001i	3.28e-001	6.51e+001
-2.13e+001 - 6.15e+001i	3.28e-001	6.51e+001

The eigenvalues are, $(-1.69 \pm j 6.46)$ and $(-21.3 \pm j 61.5)$.

As a rule of thumb, the time increment for each step of the simulation should be less than one tenth of the inverse of magnitude of the maximum eigenvalue. Thus, it is made sure that the higher frequency modes are taken care of during the simulation. This is particularly important in case of stiff systems where there is a large difference in the eigenvalues of the modes.

$$\therefore (H) < \frac{1}{10 \times |(-21.3 \pm j 61.5)|}$$

$$\therefore (H) < 0.0015364 \text{ sec} \quad (3.18)$$

Secondly, the simulation should run at least for time duration equal to 5 times the maximum value of the time constant to get the steady state response. From the eigenvalues, the time constants are calculated as,

$$\tau_1 = \frac{1}{r_1} = 0.5917 \text{ sec}$$

$$\tau_2 = \frac{1}{r_2} = 0.0469 \text{ sec}$$

$$\therefore (N \times H) > 5 \tau_1 \quad \text{i.e.}$$

$$\therefore (N \times H) > 2.9585 \text{ sec} \quad (3.19)$$

Discrete Fourier transform is used in the algorithm for generating random time series with specific spectral density, it is necessary that N is in terms of 2^n where, n is a positive integer.

Using the relationships in (3.17) and (3.18) and the necessary condition for N, N and H are calculated as follows;

$$N = 2048 \quad \text{and} \quad H = 0.0015 \text{ sec}$$

$$\text{Total time of simulation} = N \times H = 3.072 \text{ sec}$$

$$\text{And, Frequency resolution} = \frac{1}{N \times H} = 0.3255 \text{ Hz}$$

First natural frequency of the system is 1.69 Hz. With frequency resolution of 0.3255 Hz, we would get maximum 5 points in between the minimum generated frequency and first natural frequency of the system. Therefore the frequency resolution needs to be less. Let the frequency resolution be less than 0.05 Hz so that there are more than 25 points between the minimum generated frequency and the first natural frequency.

$$\therefore \frac{1}{N \times H} < 0.05 \text{ Hz}$$

$$\therefore N > 1.3333 \times 10^4$$

$$\therefore N = 2^{14} = 16384$$

Thus with $N = 16384$ and $H = 0.0015$ the frequency resolution becomes 0.0407 Hz. The total time of the simulation becomes 24.5760 sec.

These values are used to generate the PSD and the random road surface profile for a smooth highway. The Matlab command for this is,

```
>> [U,t]=rough_road_input(16384,0.0015);
```

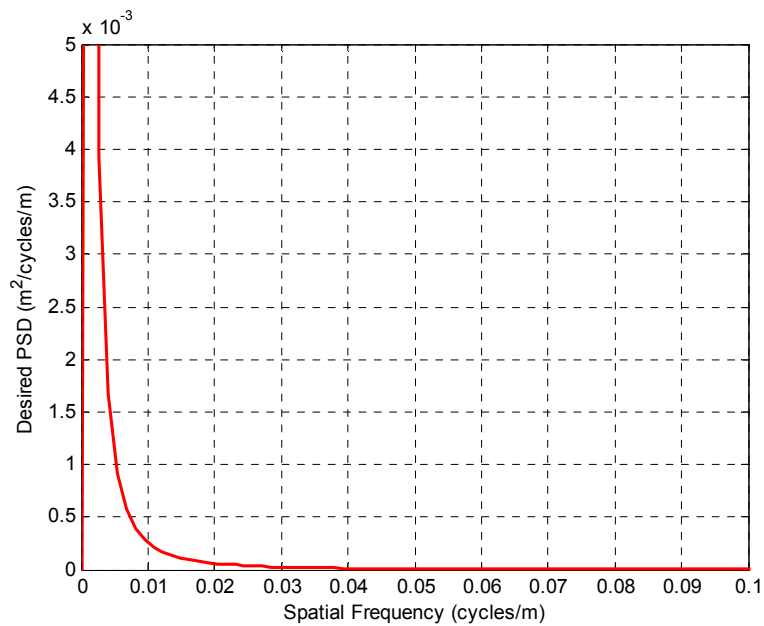


Figure 3.3 PSD for a smooth highway

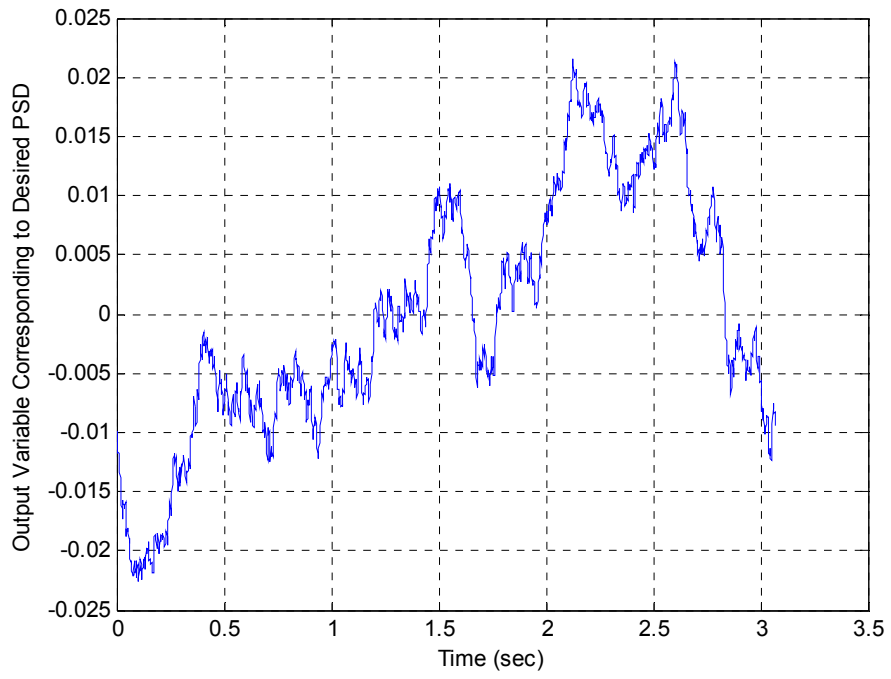


Figure 3.4 Road profile for a smooth highway

CHAPTER 4

OPTIMIZATION OF DAMPER CONSTANT

The performance characteristics which are of most interest while designing a vehicle suspension system are ride comfort, road holding and suspension travel [6]. Among the three characteristics, road holding and ride comfort are chosen for this study. All the suspension system parameters such as sprung mass, unsprung mass, suspension spring stiffness and tire stiffness are fixed depending upon the frequency response characteristics of the quarter car model as explained in chapter 2. So, the damper constant of the shock absorber is selected for optimization.

4.1 Optimization for road holding

Because of the vibrations in the vehicle suspension system while moving on a road surface, normal force between road and the tire fluctuates. This normal force is responsible for the road holding capacity of the vehicle. The tractive effort, braking effort and cornering force also depends upon the normal force between the road and tires. Therefore it necessary to maintain a minimum value of tire force for better handling of the vehicle.

Since the damping ratio of the tire is very small compared to the damping ratio of the suspension system, it can be assumed that the normal force between the road and tire is directly proportional to the relative displacement between the two. Mathematically, for the quarter car model,

$$F_{\text{tire}} = K_s \cdot (z - y) \quad (4.1)$$

Therefore, it is possible to evaluate the road holding of the vehicle using the dynamic tire deflection, $(z - y)$. The values of the fixed parameters for the quarter car model are considered as shown in figure 2.1. Ratio of unsprung mass to sprung mass is considered as 0.1. Ratio of

tire stiffness to suspension spring stiffness is considered as 8. The values of these ratios are taken from the frequency response characteristics explained in chapter 2.

4.1.1 Computation of transfer function for the Quarter Car Model:

Matlab[®] is used for computing the dynamic tire deflection of the quarter-car model for different types of road surfaces given by table 3.1. First the equations of motions are derived for the model using D'Alembert's principle. The free body diagram of the model shown in figure 2.1 is as follows;

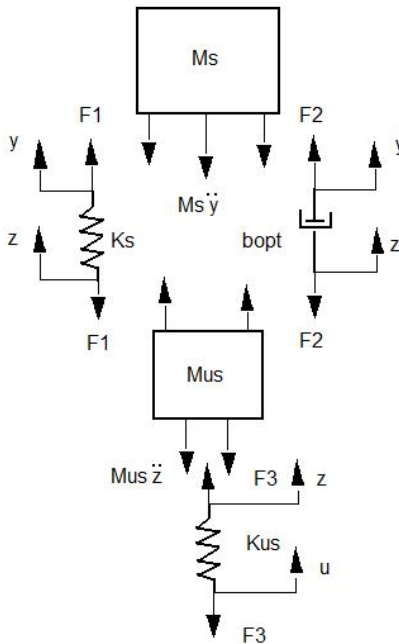


Figure 4.1 Free body diagram of the quarter-car model

Considering the displacements of the masses are measured from the static equilibrium position so that the weights of the masses can be excluded from the equations for simplicity. Equations of motion are

$$M_s \ddot{y} + F_1 + F_2 = 0 \quad (4.2)$$

$$F_1 = K_1 \cdot (y - z) \quad (4.3)$$

$$F_2 = b_{opt} \cdot (\dot{y} - \dot{z}) \quad (4.4)$$

$$M_{us} \ddot{z} + F_3 - F_1 - F_2 = 0 \quad (4.5)$$

$$F_3 = K_2 \cdot (z - u) \quad (4.6)$$

$$dtr = (z - u) \quad (4.7)$$

Thus 6 equations are available for finding 6 unknowns namely y, F_1, F_2, F_3, z and dtr

By taking Laplace transform of above equations and using Matlab[®], the transfer function between road elevations u and the dynamic tire deflection dtr is obtained as follows;

$$H_{dtr_U} = \frac{-\{328333 s^4 + 1995 (b)s^3 + (1.7556 \times 10^8)s^2\}}{(328333)s^4 + 1995(b)s^3 + (1.4526 \times 10^9)s^2 + 704000 (b)s + (6.1925 \times 10^{10})}$$

4.1.2 Time domain response of the quarter car model using Matlab[®]:

The time response of linear time invariant models for arbitrary inputs can be obtained using the Matlab command '`lsim(SYS_TF,U,t)`'. Where, SYS_TF is the transfer function of the system, U is the generated input and t is the time vector having same dimensions as those of the generated input matrix U .

4.2 Optimum damper constant for minimum dynamic tire deflection

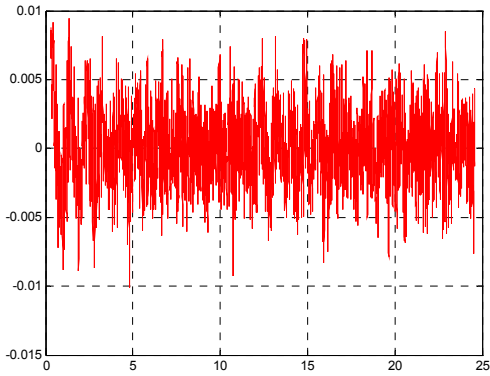
4.2.1 Optimization by simulation method:

For simulating the dynamic tire deflection for the input from the rough road surface, the road surface is generated as per the procedure described in sections 3.4.1 and 3.4.2. A smooth highway is considered for this case. The optimum damper constant for the vehicle is to be computed such that the RMS value of the dynamic tire deflection is kept to the minimum. This way it can be ensured that the normal force between the road and the tires is kept to the minimum. So, the objective function is [6],

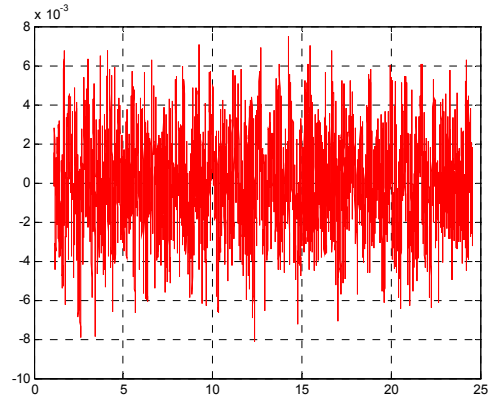
$$\min f(dtr) = \text{RMS}(dtr)$$

For optimizing the damper constant, the RMS value dynamic tire deflection is computed by running the simulation using above transfer function. Matlab[®] file '`DYN_TIRE_DEFLECTION_NUMERICAL.m`' is used for the simulation and calculation of the RMS value. A Matlab[®] file '`rmsval.m`' is written for the calculation of RMS value using

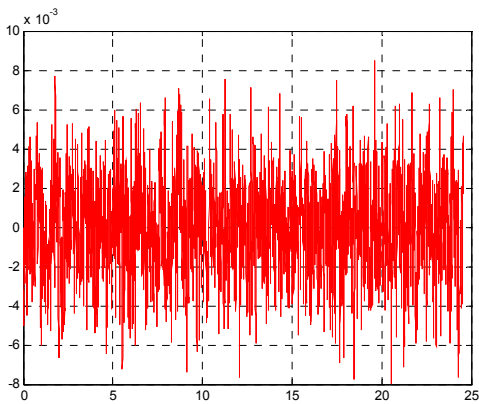
equation 2.2. The damper constant is varied with a step of 450 Ns/ m from 5050 Ns/ m to 12700 Ns/ m and for each value the RMS dynamic tire deflection is computed. Following figure illustrates some of the time history records of the output for the tire deflection for the smooth highway.



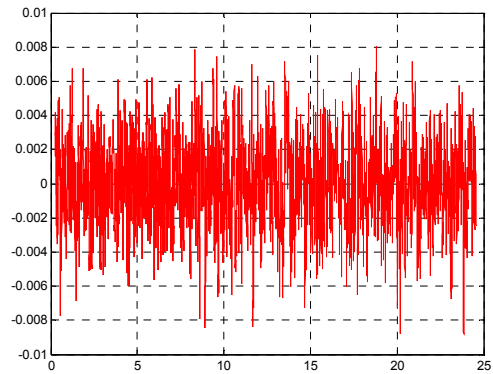
Dynamic tire deflection for $b = 5050$ Ns/ m
RMS (dtr) = 5.2 mm



Dynamic tire deflection for $b = 8200$ Ns/ m
RMS (dtr) = 2.9 mm



Dynamic tire deflection for $b = 10000$ Ns/ m
RMS (dtr) = 2.4 mm



Dynamic tire deflection for $b = 12700$ Ns/ m
RMS (dtr) = 3.4 mm

Figure 4.2 Time responses for dynamic tire deflection for different damper constants

To find the optimum value of the damper constant such that the minimum dynamic tire deflection, a graph of the tire deflection versus the damper constant is plotted.

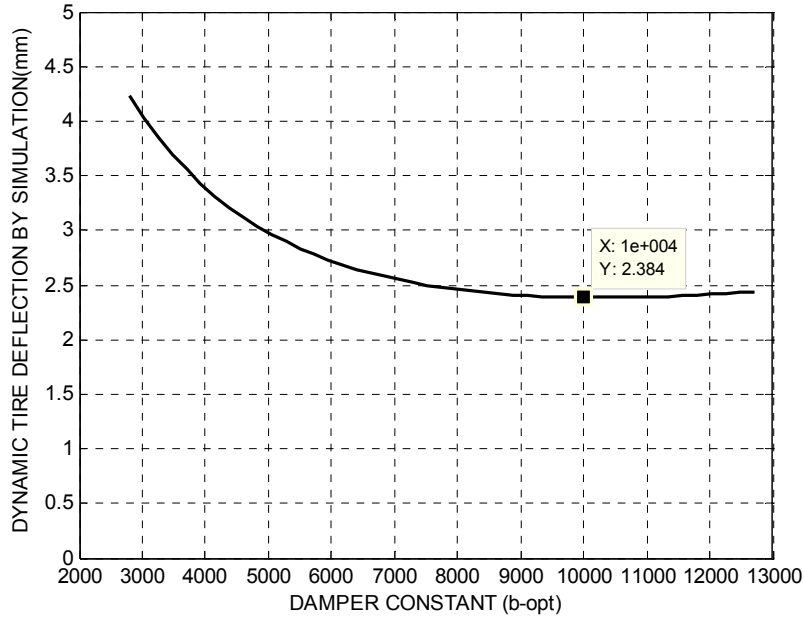


Figure 4.3 Optimization of the damper constant by simulation

From the graph in the figure 4.3, the minimum RMS value of the dynamic tire deflection occurs at the damper constant of the shock absorber equal to 10000 Ns/ m.

Simulations are run number of times to observe the change in minimum value of RMS dynamic tire deflection and optimum damper constant for a smooth highway. Table 4.1 contains the results obtained from the simulation.

From the table, it can be concluded that the value of the optimum damper constant changes depending upon the RMS dynamic tire deflection when the vehicle is running over a smooth highway with a constant speed. Further, simulation results are obtained for different types of the road surfaces to find out the damper constant required for keeping the dynamic tire

deflection to the minimum. It is observed that the damper constant values for good road holding are in the same range as computed in the Table 5.1

Table 4.1 Simulation results for optimum damper constant values for minimum dynamic tire deflection

Simulation No.	Minimum RMS Dynamic Tire Deflection (mm)	Optimum Damper Constant (Ns/ m)
1	2.311	10000
2	2.133	9775
3	2.234	9775
4	2.18	9775
5	2.414	10000
6	2.273	10000
7	2.375	10000
8	2.378	10000
9	2.168	10000
10	2.178	10000

4.2.2. Optimization by analytical method:

We are interested in computing the RMS value of the dynamic tire deflection. In terms of the power spectral density function, the mean square value of $y(t)$ is given by,

$$\Psi_y^2 = \int_0^\infty G_y(f) . df$$

Thus, the mean square value is the total area under the plot of the power spectral density versus frequency. An important application of the power spectral density function is to determine the frequency composition of the physical data.

The frequency composition of the data bears important relationship with the basic characteristics of the physical system involved. For example, consider a system having transfer

function $H(f)$. Let $G_u(f)$ be the power spectral density of the input stationary random signal to this system and $G_y(f)$ be the power spectral density of the output of interest. The relationship between these is given as [9],

$$G_y(f) = |H(f)|^2 \times G_u(f)$$

Let,

G_U - Input PSD of the road surface

H_{dtr_U} - Transfer function for the dynamic tire deflection

$G_{d_{tire}}$ - Output PSD of dynamic tire deflection

Using above results , the output PSD of the dynamic tire deflection can be expressed as,

$$\therefore G_{d_{tire}}(s) = |H_{dtr_U}(s)| \times |H_{dtr_U}(-s)| \times G_U(s) \quad (4.8)$$

$E[dtr^2(t)]$ - mean square value of the dynamic tire deflection,

$$\therefore E[dtr^2(t)] = \frac{1}{2\pi j} \int_{j\omega_{min}}^{j\omega_{max}} G_{d_{tire}}(s) \cdot ds \quad (4.9)$$

Substituting equation (4.8) into equation (4.9),

$$\therefore E[dtr^2(t)] = \frac{1}{2\pi j} \int_{j\omega_{min}}^{j\omega_{max}} \{|H_{dtr_U}(s)| \times |H_{dtr_U}(-s)| \times G_U(s)\} \cdot ds$$

Substituting $s = 2\pi j f$ and using the frequency limits as, $f_{min} = 0.5 \text{ Hz}$ and $f_{max} = 300 \text{ Hz}$

The PSD of the road surface is given by,

$$G_U = \frac{C_{sp}}{\Omega^N}$$

Where,

$$C_{sp} = 4.8 \times 10^{-7}$$

$$N = 2.1$$

} For smooth highway

$$\Omega = \frac{f}{V}$$

$$V = 30 \text{ m/sec}$$

With these values, the road surface PSD is comes out to be,

$$GU(f) = \frac{2.0234 \times 10^{-5}}{f^{2.1}}$$

For solving the integration analytically, the road PSD function is approximated as,

$$GU(f) \approx \frac{2.0234 \times 10^{-5}}{f^2} \quad (4.10)$$

The transfer function for the dynamic tire deflection is,

$$H_{dtr_U} = \frac{-\{328333 s^4 + 1995 (b)s^3 + (1.7556 \times 10^8)s^2\}}{(328333)s^4 + 1995(b)s^3 + (1.4526 \times 10^9)s^2 + 704000 (b)s + (6.1925 \times 10^{10})} \quad (4.11)$$

Where, $b = b_{opt}$ (damper constant of the shock absorber to be optimized)

Substituting equation (4.10) and (4.11) in equation (4.8), we get the output PSD for the dynamic tire deflection. Then by substituting this value in equation (4.8), the mean square value can be obtained.

The integration is solved using Matlab® symbolic math. 'DYNAMIC_TIRE_DEFLECTION_ANALYTICAL.m' file is used to plot the dynamic tire deflection against the damping ratio. From the graph the optimum value of the damper constant can be found as 10000 Ns/ m.

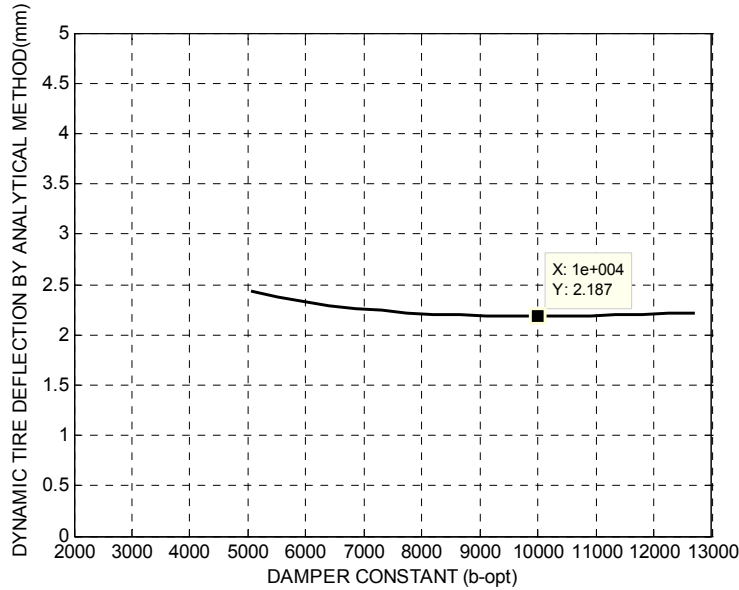


Figure 4.4 Optimization of damper constant by analytical method

Comparing the results obtained by simulation of the quarter car model and by the integration of the output PSD over the frequency range of interest i.e. by analytical method, it can be concluded that the simulation method gives fairly accurate results and can be effectively used for design and optimization of the damper constant of the shock absorber.

4.3 Optimization for Passenger Ride Comfort

The objective is to formulate optimum value of damper constant such that the vertical acceleration of the vehicle body is kept to the minimum. This is important in deciding the passenger ride comfort. The objective function is [6],

$$\min f(\ddot{y}) = \text{RMS}(\ddot{y})$$

Using the equations of motions for the quarter car model, transfer function between the road inputs and the vertical acceleration of the vehicle body is computed as,

$$HY_{dd_U} = \frac{704000 (b)s^3 + (6.1925 \times 10^{10}) s^2}{(328333)s^4 + 1995(b)s^3 + (1.4526 \times 10^9)s^2 + 704000 (b)s + (6.1925 \times 10^{10})}$$

Eigenvalues of the transfer function are calculated using 'damp(HYddm_U)' command in Matlab®. Eigenvalues are;

```
>> damp(HYddm_U)
```

Eigenvalue	Damping	Freq. (rad/s)
-3.02e+000 + 6.22e+000i	4.37e-001	6.91e+000
-3.02e+000 - 6.22e+000i	4.37e-001	6.91e+000
-3.56e+001 + 5.18e+001i	5.66e-001	6.28e+001
-3.56e+001 - 5.18e+001i	5.66e-001	6.28e+001

The eigenvalues are, $(-3.02 \pm j6.22)$ and $(-35.6 \pm j51.8)$

From the eigenvalues, the same values of N and H can be used for the simulation. A smooth highway is considered for the simulation. Vehicle speed is assumed to be 30 m/ sec. To find the optimum value of the damper constant, the simulation is run by varying the damper constant between 2350 Ns/ m to 12700 Ns/m with an increment of 225 Ns/m. The Matlab file 'VEH_ACCN_NUMERICAL.m' is used for the simulation. Figure 4.6 illustrates the output acceleration.

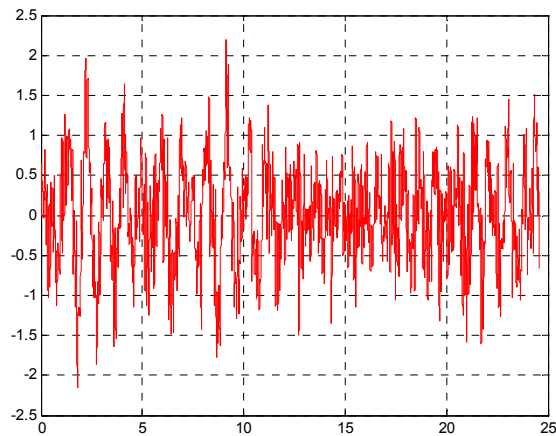


Figure 4.5 Vertical acceleration of the vehicle mass ($\text{RMS}(\ddot{y}) = 0.6115 \text{ m/s}^2$)

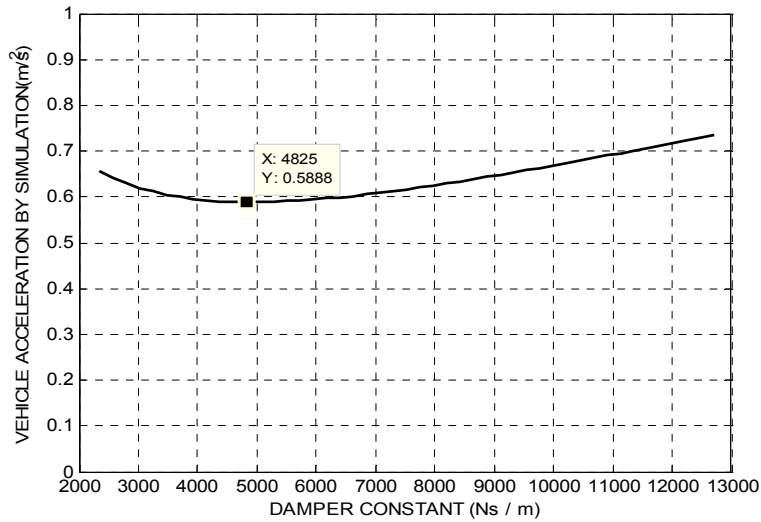


Figure 4.6 Optimum damper constant for vehicle body acceleration by numerical method

A graph of the damper constant versus RMS acceleration is plotted as shown in figure 4.6 to find the minimum value of the RMS vertical acceleration and the corresponding damper constant value. In this case the damping coefficient is 4825 Ns/ m.

Similar results are obtained for different types of road surfaces. It is observed that with increase in the roughness of the road the RMS vertical acceleration of the vehicle mass increases but the optimum value of the damper constant is in the same range as in the table 4.2.

By following the same procedure as explained in section 4.2.2, the optimum value of damper constant for minimum RMS vehicle acceleration is computed by analytical method (by solving the integration for output PSD of vehicle acceleration). Following figure shows result of the analytical method

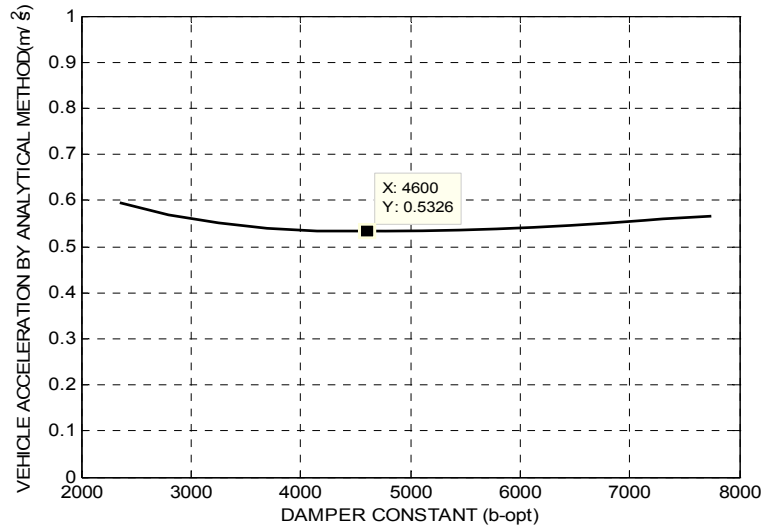


Figure 4.7 Optimum damper constant for vehicle body acceleration by analytical method

4.4 Performance Index Minimization Procedure

From the results in section 4.2 and 4.3, it can be observed that the value of damper constant required to keep the tire deflections and vehicle mass acceleration to the minimum is not a constant. Ranges of this value required for road holding and ride comfort are different. For good road holding the damper constant needs to be in a higher range equal to 10000 Ns/ m whereas to achieve good ride comfort the value should be in the range of 4825 Ns/ m..

These mutually conflicting conditions generate the need for evaluation of the performance index which would take into consideration the road holding of the vehicle as well as the passenger ride comfort at a time in optimization procedure.

For specified values for the maximum allowed acceleration \ddot{y}_{\max} maximum allowed dynamic tire deflection dtr_{\max} and by calculating the mean square values for acceleration (\ddot{y}_{ms}) and dynamic tire deflection (dtr_{ms}), the performance index is defined as,

$$PI = \left[\frac{\ddot{y}_{ms}}{\ddot{y}_{\max}} \right] + \left[\frac{dtr_{ms}}{dtr_{\max}} \right] \quad (4.12)$$

Since the mean square values are divided by the respective maximum values, the performance index is a dimensionless quantity. However, the values in the denominator need not be the maximum values. These values can be used to normalize the dynamic tire deflection and sprung mass acceleration and hence can be selected conveniently.

A Matlab® code 'PERFORMANCE_INDEX_MIN.m' is written for computing the damper constant corresponding to the minimum value of performance index. In this m-file, basically the mean square values of the dynamic tire deflection and vehicle acceleration are computed by simulation. Values of maximum sprung mass acceleration and maximum dynamic tire deflection are selected as $\ddot{y}_{\max} = 0.8 \text{ m/s}^2$ and $dtr_{\max} = 0.0278 \text{ m}$ respectively. Then by calculating the performance index as defined by equation (4.12), a graph of performance index versus damper constant is plotted. Figure 4.8 illustrates the simulation result for performance index minimization procedure.

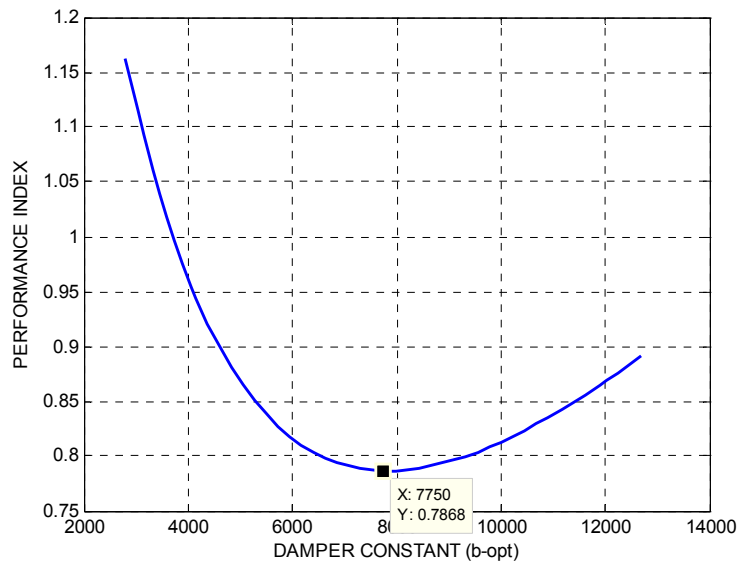


Figure 4.8 Performance Index Minimization Procedure

Table 4.2 Comparison of results for different optimization criteria

Optimization Criterion	Damper Constant by Simulation (Ns / m)
Dynamic Tire Deflection	10000
Vertical Acceleration of Sprung Mass	4825
Performance Index	7750

Table 4.2 shows comparison between the optimum damper constant values obtained using different optimization criteria. The values obtained using performance index are shifted in the range of the values obtained using the sprung mass acceleration. This is because the ratio $[\ddot{y}_{ms}/\ddot{y}_{max}]$ has a higher value than $[dtr_{ms}/dtr_{max}]$. Therefore, more weight is given to sprung mass acceleration than tire deflection in the performance index.

4.5 Computation of road holding of the vehicle

If the relative displacement between the tire and the road surface elevations during the random vibrations of the wheel is such that the positive value of dynamic tire deflection becomes equal to static tire deflection then, the normal force between the tire and the road becomes zero and the tire tends to jump of the ground. In such a situation the vehicle loses its road holding capacity to a great extent and in turn handling of the vehicle is affected badly. Therefore this is an unfavorable condition for the vehicle.

To obtain a quantitative insight in deciding the road holding of the vehicle the number of times the positive value of the dynamic tire deflection crosses the static tire deflection is calculated. This number gives the information about how many times the tire could lose contact

with the ground per second. This can be calculated using following equation [10]. Let, p_{dtire}^+ be the number of positive crossings of the static tire deflection by the dynamic tire deflection.

$$\therefore p_{dtire}^+ = \frac{1}{2\pi} \cdot \frac{\dot{\sigma}_{dtire}}{\sigma_{dtire}} e^{\left(-\frac{d_{st}}{2\sigma_{dtire}^2}\right)} \quad (4.12)$$

Where,

σ_{dtire} - Standard deviation of the dynamic tire deflection

$\dot{\sigma}_{dtire}$ - First derivative of standard deviation of the dynamic tire deflection

d_{st} - Static tire deflection

Since all the equations of motion for the quarter car model are developed from the static equilibrium position, the tires lose contact with the ground when the positive value of the dynamic tire deflection becomes equal to the static tire deflection. The static deflection of the tire is calculated as,

$$K_{us} \cdot d_{st} = (M_s + M_{us}) \cdot g$$

$$\therefore d_{st} = 0.0278 \text{ m}$$

The standard deviation of the dynamic tire deflection is computed by running the simulation for of the quarter car model for a smooth highway for different values of the damper constant. Substituting these values in equation (4.12) the number of positive crossings of the tire deflection per second is calculated for each damper constant.

For example, consider the damper constant, $b = 9550 \text{ Ns/ m}$

The output dynamic tire deflection by simulation is as shown in figure 4.9

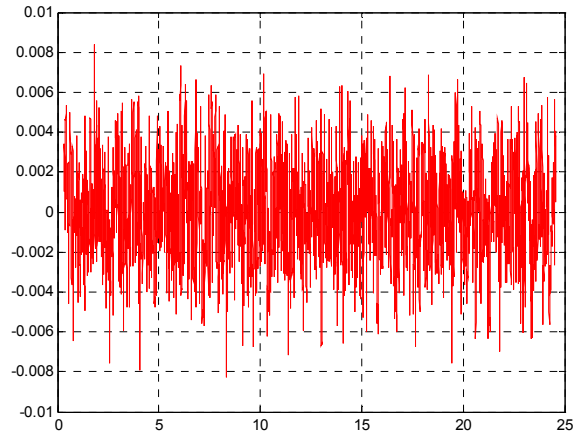


Figure 4.9 Dynamic tire deflection for $b = 9550 \text{ Ns/m}$

Standard deviation, $\sigma_{\text{dtire}} = 0.002334 \text{ m}$

First derivative of standard deviation, $\dot{\sigma}_{\text{dtire}} = 0.0104$

By substituting these values in equation (5.12) ,

$$p_{\text{dtire}}^+ = \frac{1}{2\pi} \cdot \frac{0.0104}{0.002334} e\left(-\frac{0.0278}{2(0.002334)^2}\right)$$

$$p_{\text{dtire}}^+ = 8.1736 \times 10^{-37}$$

This means that when the vehicle is running on a smooth highway at a speed of 30 m/sec, the positive value dynamic tire deflection $(z-u)$ crosses the static tire deflection d_{st} 1.1884×10^{-29} times per second. This number is too small which implies that the vehicle has good road holding.

4.5.1 Effect of vehicle speed on road holding:

The speed of the vehicle has significant effect on the road holding. To study the effect of the vehicle speed in road holding, the number p_{dtire}^+ is obtained for different speeds of the vehicle. The PSD of the smooth highway is computed at particular speeds and output is

generated by simulation. The standard deviations for each output are then used for computing the value of p_{dtire}^+ by equation (4.12) .

Table 4.3 Road Holding at Different Vehicle Speeds

Sr. No.	Vehicle speed (m/sec)	P_{dtire}^+
1	10	9.5453×10^{-91}
2	20	3.9972×10^{-44}
3	30	3.0218×10^{-30}
4	40	7.5786×10^{-24}
5	50	2.2385×10^{-18}
6	60	1.4390×10^{-15}

From table 4.3, it can be concluded that as speed of the vehicle increases, number of positive crossings of the static tire deflections by the dynamic tire deflections increases and in turn the road holding of the vehicle decreases.

4.5.2 Effect of road surface on road holding:

The effect of different types of road surfaces on the road holding of the vehicle is presented in this section. PSD for different types of road surfaces is computed as per the values given by table 4.1. Using this PSD the corresponding road surfaces are generated and the by simulation in MATLAB[®], the number of positive crossings of the dynamic tire deflection is computed for each of the road surfaces.

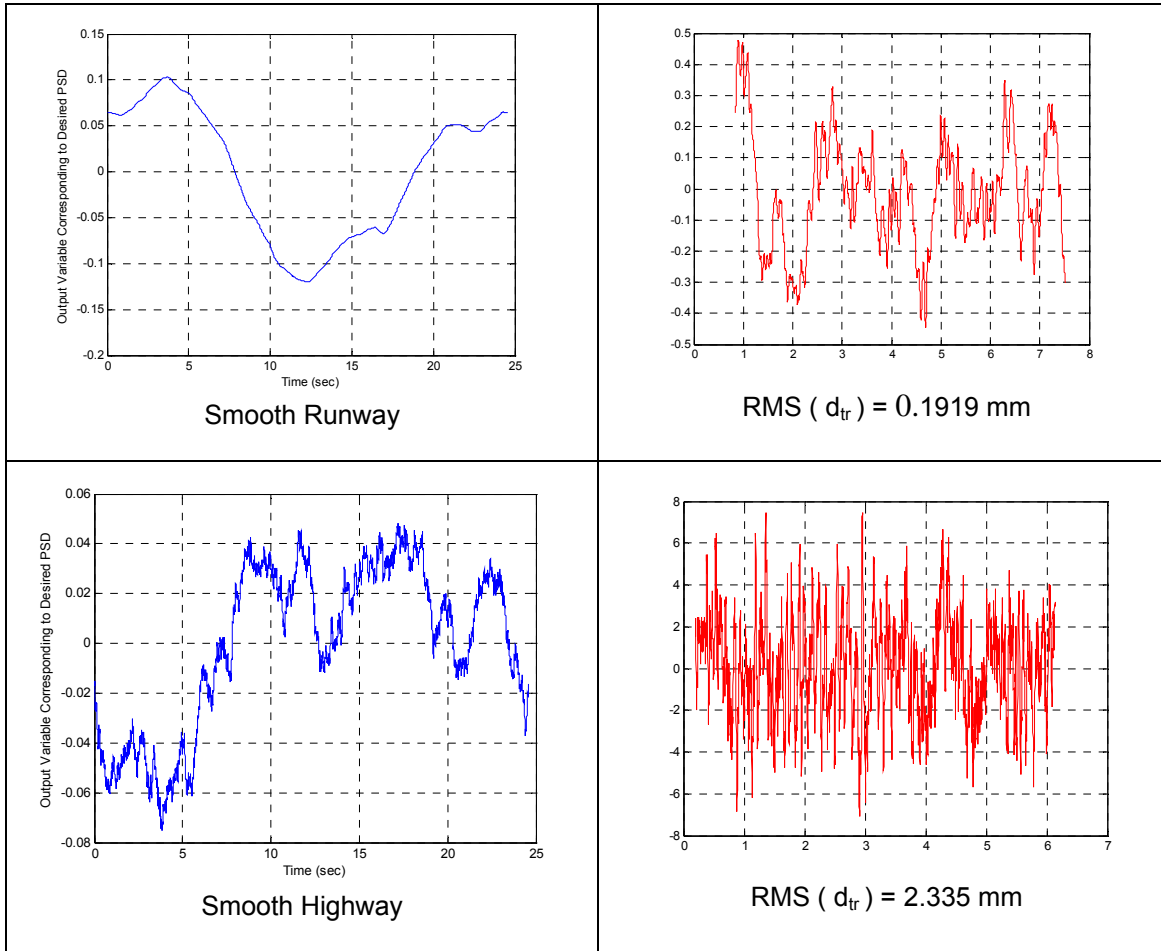


Figure 4.10 Dynamic tire deflection for smooth runway and smooth highway

Figure 4.10 illustrates the results of the simulations for different types of road surfaces classified by ISO. The speed of the vehicle is assumed to be 30 m/ sec for all the simulations and the optimum damper constant value is selected as obtained in section 4.2.1

The value of p_{dtire}^+ is computed for each type of road surface, using above values of vehicle speed and damper constant. Table 4.4 illustrates the same.

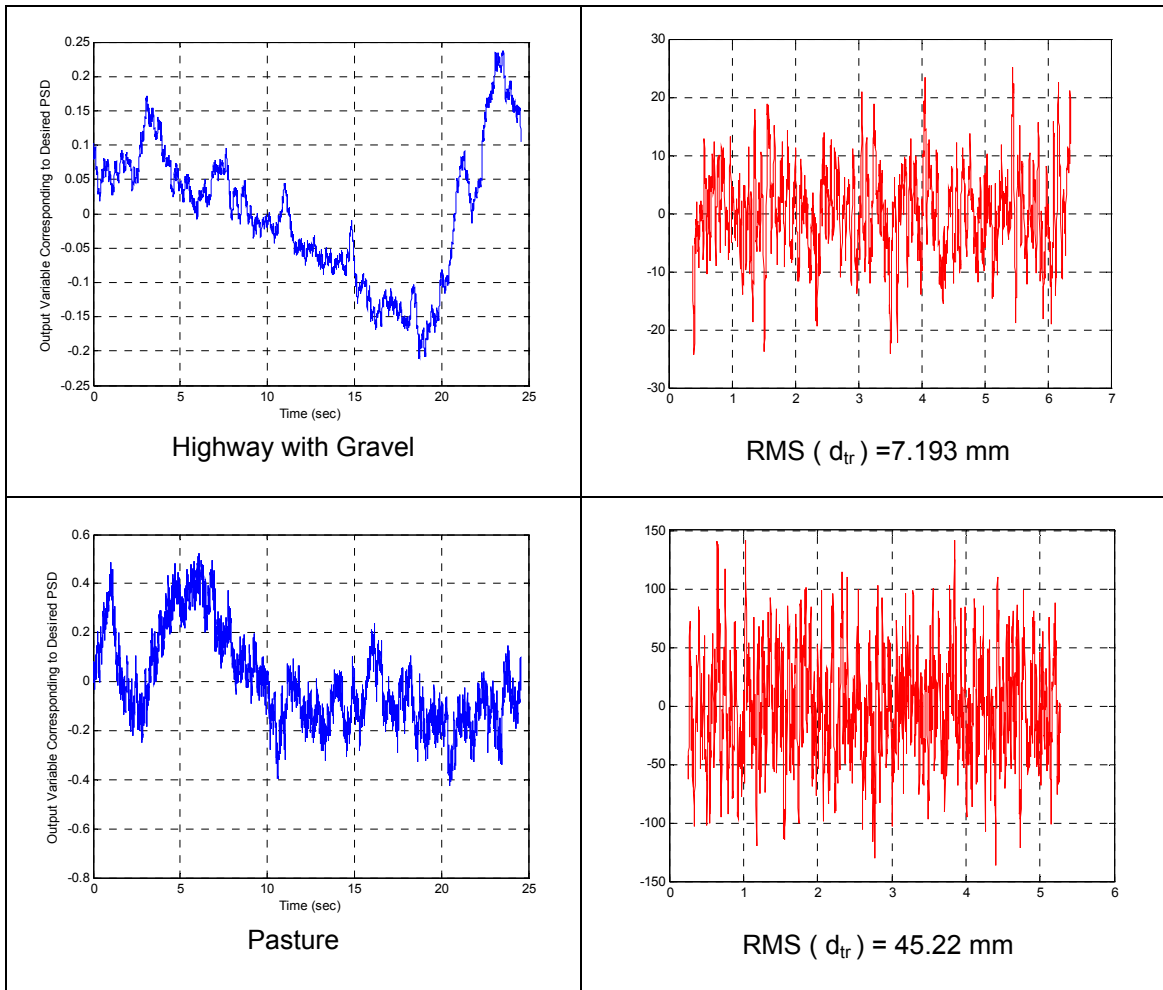


Figure 4.11 Dynamic tire deflection for highway with gravel and pasture

Table 4.4 Road Holding for different road surfaces

Sr. No.	Type of Road	P_{dtire}^+
1	Smooth Runway	0
3	Smooth Highway	1.1884×10^{-29}
4	Highway with Gravel	4.7795×10^{-4}
5	Pasture	0.9131

From the table 4.4, it can be observed that as the roughness and irregularities of the road surface increases, the number of positive crossings of the static tire deflection by the positive value of the dynamic tire deflection increases. That is the tire tends to leave contact with the ground more times per second. As a result, the road holding of the vehicle decreases.

CHAPTER 5

RESULTS AND CONCLUSION

- 1) This thesis illustrates a computer simulation based method for optimizing the suspension system parameters for stationary stochastic inputs from uneven road surfaces. A computer program using Matlab[®] has been created for generating random road surfaces with a given power spectral density. Different performance characteristics of the suspension system such as vertical acceleration of vehicle body, dynamic tire deflection can be evaluated by numerical methods by exciting the system for generated random inputs.
- 2) The output results for the dynamic tire deflection are computed by analytical method by integrating the PSD of the output between the limits of frequency range. The results obtained by analytical method closely match with the results of numerical method. So, it can be concluded that the numerical method of generating random time series can be effectively used for simulation of the suspension system considering a quarter car model.
- 3) For good road holding of the vehicle, an optimum value of the damper constant is computed. Results are obtained for number of simulations and it is observed that the damper constant for good road holding is 10000 Ns/ m for the quarter car model under consideration running on a smooth highway. Therefore good road holding and handling requires a higher value of damping.
- 4) For passenger ride comfort, the optimum value of the damper constant obtained by simulation results is 4825 Ns/ m over the entire frequency range. Therefore to achieve better passenger ride comfort, lower value of shock absorber damping is required.
- 5) Optimization procedure by taking into consideration effect of vehicle mass acceleration and dynamic tire deflection at the same time is possible by defining the performance index (PI)

as per equation (5.12). Performance Index is a dimensionless quantity and the optimum value of the damper constant can be found by minimizing the performance index.

- 6) A quantitative measure for deciding the roadholding of the vehicle has been proposed. A number (p_{dtire}^+) defined by equation (4.12) gives the frequency of crossing the static tire deflection by the dynamic tire deflections per second and is used to evaluate the road holding of the vehicle
- 7) Value of p_{dtire}^+ is computed for different speeds of the vehicle for a smooth highway. It is observed that the road holding of the tires decrease with increase in the speed.
- 8) Value of p_{dtire}^+ is computed for different types of road surfaces such as smooth runway, smooth highway, highway with gravel and pasture. It is shown numerically that as the road roughness increases, the road holding of the tires decrease.

APPENDIX A

MATLAB[®] FILES FOR USED FOR GENERATING STOCHASTIC
INPUTS AND OUTPUT RESPONSES
IN TIME DOMAIN

1. 'rough_road_input.m' File for Generating Random Road Surfaces

```
% function[U,t]=rough_road_input(N,H)
% Generates N points in time U(t) at a time interval of H with zero
mean.
% N must be a power of 2! The N points will be generated
% so as to have a desired one-sided PSD defined in the function
% road_psd_smooth_highway(f). f is the frequency in Hertz. The
frequency resolution will be 1/NH. The smallest frequency will be 0
Hz and the largest frequency will be 1/2H Hz. Note, the frequency
1/2H is referred to as the folding frequency.

function[U,t]=rough_road_input(N,H)
no2=N/2;
% Step 1: Generate frequencies and PSD values at each frequency
f=0:1/(N*H):1/(2*H);
%Note, the first frequency is zero but we don't
%compute GY at f=0 to avoid potential computational problems. GY at
f=0
%should be zero for a zero mean process.
GU(1)=0;
% If the desired PSD is defined in road_psd_smooth_highway, then use
the next 3 lines.
for m1=2:no2+1
% USE NEXT LINE IF SIMULTAION IS FOR SMOOTH RUNWAY
%GU(m1)=road_psd_smooth_runway(f(m1));
% USE NEXT LINE IF SIMULTAION IS FOR ROUGH RUNWAY
%GU(m1)=road_psd_rough_runway(f(m1));
% USE NEXT LINE IF SIMULTAION IS FOR SMOOTH HIGHWAY
GU(m1)=road_psd_smooth_highway(f(m1));
% USE NEXT LINE IF SIMULTAION IS FOR HIGHWAY WITH GRAVEL
%GU(m1)=road_psd_gravel_highway(f(m1));
% USE NEXT LINE IF SIMULTAION IS FOR PASTURE IN FIELD
%GU(m1)=road_psd_pasture(f(m1));
end
% Step 1: Generate N/2 random phase angles with uniform denisty
% between 0 and 2pi
TH=random('unif',0,2*pi,no2,1);
% Step 2: Generate the appropriate amplitude at each frequency using
% the random phase angles.
for m=2:no2
C=sqrt(GU(m)*N*H/2);
CTH=cos(TH(m-1))*C;
STH=sin(TH(m-1))*C;
U(m)=CTH+j*STH;
k=N+2-m;
U(k)=CTH-j*STH;
end
% Step 3: Make sure the N/2+1 value is real that the negative
frequency values
% will be the mirror image of the positive frequency values.
no2p1=no2+1;
```

```

C=sqrt(GU(no2p1)*N*H/2);
CTH=cos(TH(no2))*C;
U(no2p1)=CTH+j*0.e0;
% Make sure the zero frequency value is zero to achieve a zero mean
time series.
U(1)=0.e0+j*0.e0;
% Generate Y(t), the inverse transform of the N Fourier Transform
values Y(f).
[U]=ifft(U);
% Rescale to get the correct time series corresponding to this desired
PSD.
for m=1:N
    U(m)=real(U(m)/H);
end
% Plot the PSD and time series
%PLOT OF ROAD ELEVATIONS VS TIME
t=0:H:(N-1)*H;
t=t';
U=U';
figure
plot(t,U);
xlabel('Time (sec)')
ylabel('Output Variable Corresponding to Desired PSD')
grid on
%PLOT OF ROAD ELEVATIONS VS DISTANCE
%figure
%dist=30.*t;
%plot(dist,U);
%xlabel('Distance (m)')
%ylabel('Output Variable Corresponding to Desired PSD')
grid on
figure
plot((f/30),GU,'r','LineWidth',2)
xlabel('Spatial Frequency (cycles/m)')
ylabel('Desired PSD (m^2/cycles/m)')
grid on
% AXIS COMMAND FOR SMOOTH RUNWAY
%axis([0 0.02 0 4])
% AXIS COMMAND FOR ROUGH RUNWAY
axis([0 0.02 0 2])
% AXIS COMMAND FOR SMOOTH HIGHWAY
%axis([0 0.2 0 3e-4])
% AXIS COMMAND FOR HIGHWAY WITH GRAVEL
%axis([0 0.02 0 1.2])
% AXIS COMMAND FOR PASTURE
%axis([0 0.05 0 1.5])

```

2. Matlab® file for Computing PSD at Each Sampled Frequency for a Smooth Runway

```

%PSD for SMOOTH RUNWAY AT VELOCITY = 30 m /sec
function[GU]=road_psd_smooth_runway(f)
N=3.8;
Csp=4.3e-11;
GU=(Csp)./(30.*(f/30).^N);

```

3. Matlab® file for Computing PSD at Each Sampled Frequency for a Smooth Highway

```

%PSD for SMOOTH HIGHWAY AT VELOCITY = 34 m /sec
function[GU]=road_psd_smooth_highway(f)
N=2.1;
Csp=4.8e-07;
GU=(Csp)./(30.*(f/30).^N);

```

4. Matlab® file for Computing PSD at Each Sampled Frequency for a Highway with Gravel

```

%PSD for HIGHWAY WITH GRAVEL AT VELOCITY = 30 m /sec
function[GU]=road_psd_gravel_highway(f)
N=2.1;
Csp=4.4e-6;
GU=(Csp)./(30.*(f/30).^N);

```

5. Matlab® file for Computing PSD at Each Sampled Frequency for a Pasture

```

%PSD for SMOOTH HIGHWAY AT VELOCITY = 34 m /sec
function[GU]=road_psd_pasture(f)
N=1.6;
Csp=3.0e-04;
GU=(Csp)./(30.*(f/30).^N);

```

6. Matlab® file for Computing Transfer Function for Dynamic Tire Deflection, Generating Output in Time Domain and Computing p_{dtire}^+

```

% m-file for suspension optimization problem
% Quarter car model with two degrees of freedom is considered.
% sprung mass (vehicle mass)= 1814 kg
% ratio of unsprung mass to sprung mass = 0.1 (unsprung mass=181 kg)
% shock absorber damper constant= b (to be optimized)
% equivalent tire stiffness (K2) = 704 kN/m
% ratio of Spring stiffness to tire stiffness = 8
%(suspension spring stiffness (K1)= 88 kN/m)
% input road PSD (Upsd)= Gu=Csp/(f/V).^N;

clear
clc
syms Ms b K1 K2 Mus U s
digits(5)

```

```

H=solve('Ms*s^2*Y+F1+F2=0','F1=K1*(Y-Z)','F2=s*b*(Y-Z)','Mus*s^2*Z+F3-F1-F2=0','F3=K2*(Z-U)','dtr=(Z-U)','Y,Z,F1,F2,F3,dtr');

%T.F BETWEEN DYNAMIC TIRE DEFLECTION AND ROAD INPUT
Hdtr=collect(H.dtr,s);
Hdtr_U=collect(Hdtr,U);
Hdtr_U=vpa(subs(Hdtr_U,[Ms K1 Mus K2],[1814 88000 181 704000]));
pretty(Hdtr_U);

% GENERATING RANDOM INPUT Y FOR SPECIFIC ROAD ROUGHNESS PSD
[U,t]=rough_road_input(2048,0.0015);

% T.F. FOR OPTIMUM DAMPER CONSTANT
b=9550;
NUM=[-328333,-1995*b,-1.7556e8,0,0];
DEN=[328333,1995*b,1.4526e9,704000*b,6.1952e10];
Hdtr_U=tf(NUM,DEN)
figure
bode(Hdtr_U)
grid on

% SIMULATION OF DYNAMIC TIRE DEFLECTION (Z-U)
figure
t=0:0.0015:3.0705;
D_tire_dyn=lsim(Hdtr_U,U,t);
plot(t,D_tire_dyn,'r','LineWidth',1.1);
%axis([0.15 3.5 -0.01 0.01])
grid on
D_tire_dyn_rms=rmsval(D_tire_dyn)

% PROBABILITY OF THAT THE RANDOM TIRE DEFLECTION CROSSES A CERTAIN
MAXIMUM
% VALUE
d=0.0278;
mv=mean(D_tire_dyn(5:2048)) % SELECT THE M.V. FROM THE DATA STATISTICS
OF OUTPUT
sdv=std(D_tire_dyn(5:2048)) % SELECT THE S.DV. FROM THE DATA STATISTICS
OF OUTPUT
npd=((sdv*(2*pi)/0.0104)^-1)*(exp(-(d^2)/(2*sdv^2)))

```

7. Matlab® file for Performance Index Minimization Procedure:

```

%% OPTIMIZING THE DAMPER CONSTNT CONSIDERING BOTH OF THE VEHICLE
%% ACCELERATION AS WELL AS DYNAMIC TIRE DEFLECTION BY DEFINING A COST
%% FUNCTION

```

```

clear
clc

```

```

% GENERATING RANDOM INPUT Y FOR SPECIFIC ROAD ROUGHNESS PSD
[U,t]=rough_road_input(16384,0.0015);
n=1;

```

```

% MAXIMUM ALLOWED TIRE DEFLECTION AND VEH ACCN

```

```

dtr_max= 0.002;
ydd_max=0.80;

```

```

%RMS VEHICLE ACCELERATION

```

```

for b1=2800:225:12700
    NUM1=[704000*b1,6.1952e10,0,0];
    DEN1=[328333,1995*b1,1.4526e9,704000*b1,6.1952e10];
    HYddm_U=tf(NUM1,DEN1);

```

```

    % SIMULATION OF VEHICLE ACCELERATION

```

```

    %figure
    t=0:0.0015:24.5745;
    VEH_ACCN=lsim(HYddm_U,U,t');
    VEH_ACCN_rms_numerical(:,n)=rmsval(VEH_ACCN);
    if n<45
        n=n+1;
    end
end

```

```

% PLOT OF RMS VALUE OF THE VEHICLE ACCELERATION VS DAMPER CONSTANT

```

```

%b=2800:225:12700;
%figure
%plot(b,VEH_ACCN_rms_numerical,'k','LineWidth',2);
%xlabel('DAMPER CONSTANT (b-opt)')
%ylabel('VEHICLE FORCE BY SIMULATION(m)')
%grid on

```

```

% RMS DYNAMIC TIRE DEFLECTION

```

```

m=1;

```

```

for b=2800:225:12700
    NUM=[-328333,-1995*b,-1.7556e8,0,0];
    DEN=[328333,1995*b,1.4526e9,704000*b,6.1952e10];
    HRH_U=tf(NUM,DEN);

```

```

    % SIMULATION OF DYNAMIC TIRE DEFLECTION

```

```

    %figure

```

```

t=0:0.0015:24.5745;
TIRE_DFLN=lsim(HRH_U,U,t');
TIRE_DFLN_rms_numerical(:,m)=rmsval(TIRE_DFLN);
if m<45
m=m+1;
end
end

% PLOT OF RMS VALUE OF THE DYNAMIC TIRE DEFLECTION VS DAMPER CONSTANT
%b=2800:225:12700;
%figure
%plot(b,TIRE_DFLN_rms_numerical,'k','LineWidth',2);
%xlabel('DAMPER CONSTANT (b-opt)')
%ylabel('DYNAMIC TIRE DEFLECTION BY SIMULATION(m)')
%grid on

% COST FUNCTION
mstd=TIRE_DFLN_rms_numerical.^2;
msydd=VEH_ACCN_rms_numerical.^2;

CF=(mstd./(dtr_max^2))+(msydd./(ydd_max)^2);

%PI=(ydd_max^2/dtr_max^2).*(mstd)+msydd

b=2800:225:12700;
figure
plot(b,PI,'b','LineWidth',1.5);
xlabel('DAMPER CONSTANT (b-opt)')
ylabel('COST FUNCTION')
grid on

```

8. Matlab® file for Calculating First Derivative of Standard Deviation of Dynamic tire Deflection ($\dot{\sigma}_{dtire}$):

```

syms s
TF=(-328333*s^5-1.905e7*s^4-
1.756e8*s^3)/(328333*s^4+1.905e7*s^4+1.453e9*s^2+6.723e9*s+6.195e10);
syms f
%SMOOTH RUNWAY
GUU=5.8804e-7/f^4;
msval=msv(TF,GUU,0.5,300);
sigma_dot=sqrt(real(msval))
%SMOOTH HIGHWAY
GUU=2.0234e-5/f^2;
msval=msv(TF,GUU,0.5,300);
sigma_dot=sqrt(real(msval))
%GRAVEL HIGHWAY
GUU=1.8547e-4/f^2;
msval=msv(TF,GUU,0.5,300);

```

```
sigma_dot=sqrt(real(msval))
%SMOOTH HIGHWAY
GUU=2.3088e-3/f^1.5;
msval=msv(TF,GUU,0.5,300);
sigma_dot=sqrt(real(msval))
```


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BIOGRAPHICAL INFORMATION

Kailas has completed his bachelor in Mechanical Engineering from VIT, University of Pune, India. He has worked as an engineer in Larsen & Toubro Limited, India in design and engineering of process gas boiler systems. His research interests include applied mechanics, multi-body dynamics and noise and vibration analysis.